Chapter 5

APPLYING NEWTON'S LAWS

楊 本 立 副教授

Newton's Laws – Problem Solving Hints

1. Identify the system. (Who is the key role of the problem?)
2. Draw **free-body diagram(s)** for the object(s) in an inertial frame.
   - Include only forces acting on the object, not including internal forces and "fictitious" forces.
3. Establish coordinate system.
4. Categorize the problem
   - Newton's 1st law, Equilibrium ($\Sigma \boldsymbol{F} = 0$)
   - Newton's Second Law ($\Sigma \boldsymbol{F} = m\,\boldsymbol{a}$)
5. Solve for the unknown(s).

Outline

1. Using Newton's 1st law: particles in equilibrium
2. Using Newton's 2nd law: dynamics of particles
3. Frictional forces
4. Dynamics of circular motion
5. Motion in non-inertial Frames
6. The fundamental forces of Nature

1. Using Newton's 1st law: particles in equilibrium
2. Using Newton's 2nd law: dynamics of particles
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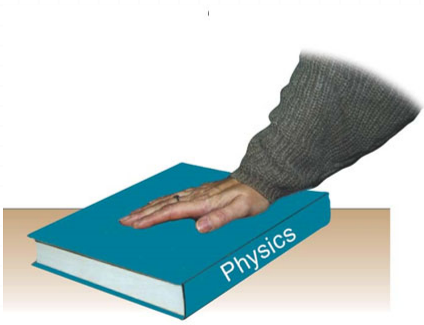
1. Using Newton's 1st law: particles in equilibrium

Ex.



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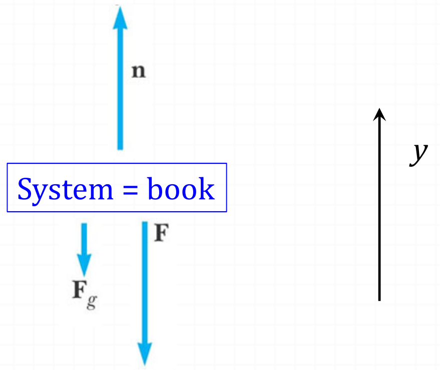
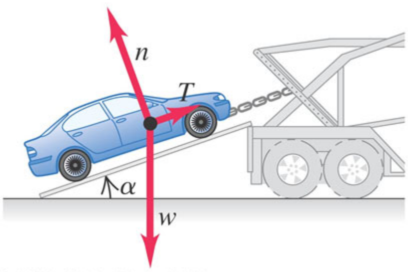


$$\sum F_y = n\hat{j} + (-)F_g\hat{j} + (-)F\hat{j} = 0$$

$$\Rightarrow n = F_g + F$$

Ex.

System = car



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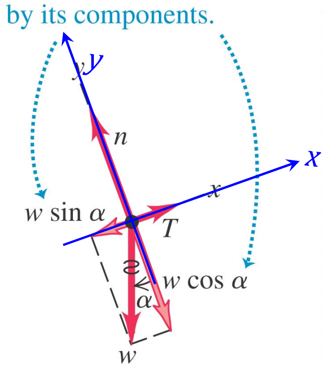

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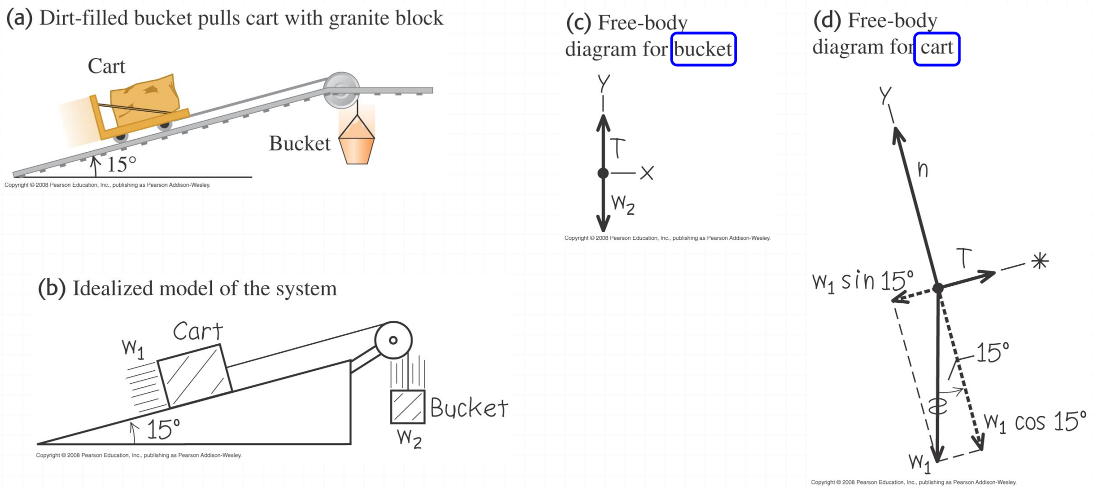
Ex.



1. Using Newton's 1st law: particles in equilibrium
2. Using Newton's 2nd law: dynamics of particles
3. Frictional forces
4. Dynamics of circular motion
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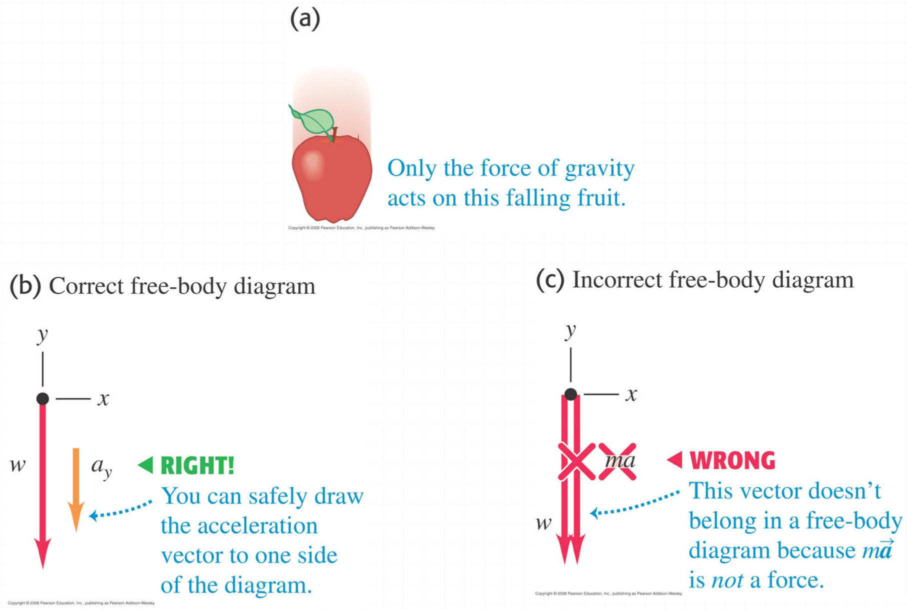
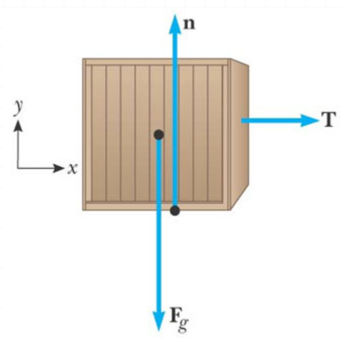
6. The fundamental forces of Nature

2. Using Newton's 2nd law: dynamics of particles



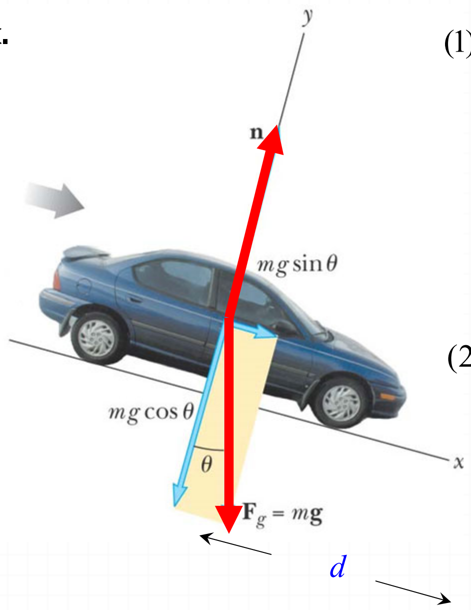
Ex.



$$\sum F_x = T\hat{i} = ma_x\hat{i}$$

$$\sum F_y = n\hat{j} + (-)F_g\hat{j} = 0 \rightarrow n = F_g$$

Ex.



(1) $a_x = ?$

$$\begin{cases} \sum F_x = mg\sin\theta = ma \\ \sum F_y = 0 = n - mg\cos\theta \end{cases}$$

$$\Rightarrow a_x = g\sin\theta$$ Indep. of mass.

(2) $t = ?$

$$d = \frac{1}{2}a_x t^2$$

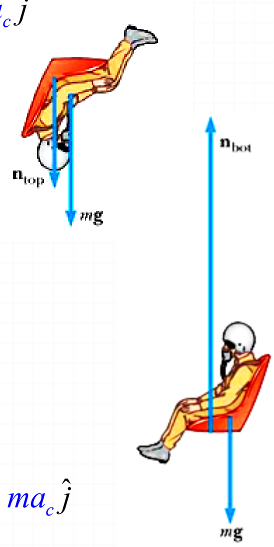
$$\Rightarrow t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

Indep. of mass.

Ex.

$$(-)n_{\text{top}}\hat{j} + (-)mg\hat{j} = (-)ma_c\hat{j}$$

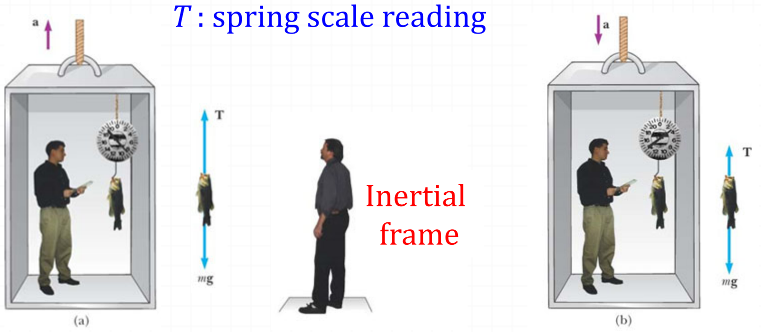
DO NOT draw the centripetal force in the free-body diagram!



$$n_{\text{bot}}\hat{j} + (-)mg\hat{j} = ma_c\hat{j}$$

Ex. Apparent weight in an accelerating elevator

T : spring scale reading



$T\hat{j} - mg\hat{j} = ma\hat{j}$
$\Rightarrow T = m(g+a)$

$T\hat{j} - mg\hat{j} = (-)ma\hat{j}$
$\Rightarrow T = m(g-a)$

p.s.,
If a = g, $\Rightarrow T$ =0
weightless.

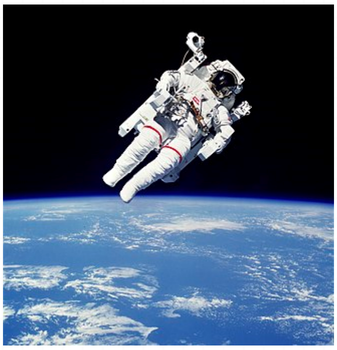
😎 "Zero g" or weightless can be checked by using the acceleration sensor in your smart phone doing free fall motion.

Apparent weightless

For an astronaut in orbit,

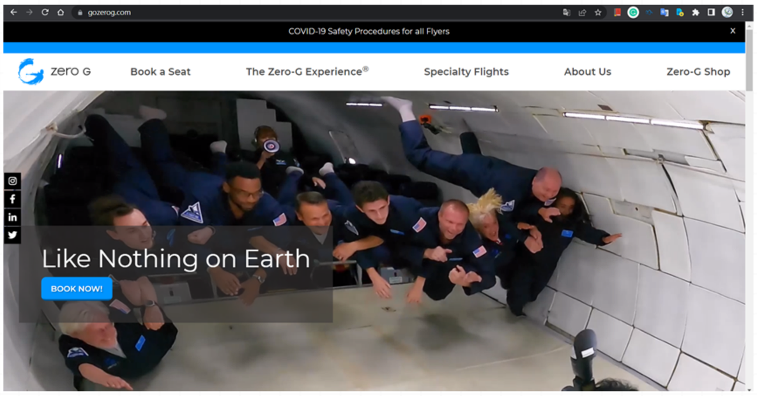
$$mg + n = ma_c$$

$$\Rightarrow \quad n = 0$$



Weightless!

😎 Not due to outside the pull of the earth's gravity!

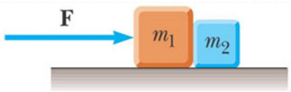


https://www.gozerog.com/

https://en.wikipedia.org/wiki/Reduced-gravity_aircraft

😎 Can be checked by using the acceleration sensor in your smart phone!

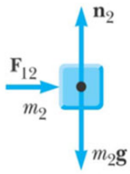
Ex.



$a = ?$

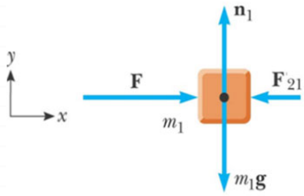
$F_{12} = ?$

System = m_2



$$F_{12} = m_2 a_{2x}$$

System = m_1



$$F - F_{21} = m_1 a_{1x}$$

System = $m_1 + m_2$

$$F = ma = (m_1 + m_2)a$$

$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

$$\Rightarrow a_1 = a_2 = a = \frac{F}{m_1 + m_2}$$

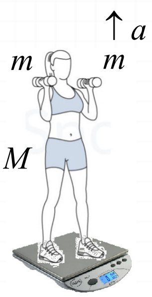
$$\Rightarrow F_{12} = F_{21} = \left(\frac{m_2}{m_1 + m_2}\right) F < F$$

😎 Similarly, the tension of a rope is not uniform if the mass of the rope can't be ignored.

Q:

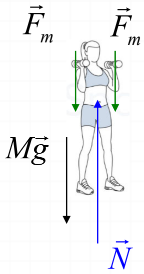
$$T_N \underset{<}{\overset{>}{=}} T_1\ ?$$

Ex.



Scale reading?

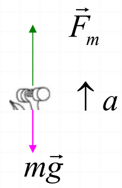
$Mg + 2m(g+a)$



$$\vec{N} + M\vec{g} + 2\vec{F}_m = 0$$

$$\Rightarrow N\hat{j} - Mg\hat{j} - 2F_m\hat{j} = 0$$

$$\Rightarrow N = Mg + 2m(g+a)$$



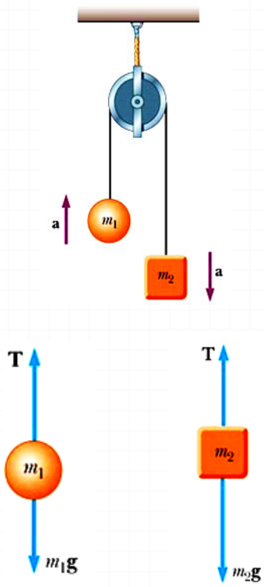
$$\vec{F}_m + m\vec{g} = m\vec{a}$$

$$\Rightarrow F_m\hat{j} - mg\hat{j} = ma\hat{j}$$

$$\Rightarrow F_m = m(g+a)$$

$$Mg + 2m(g - a)$$

Ex.

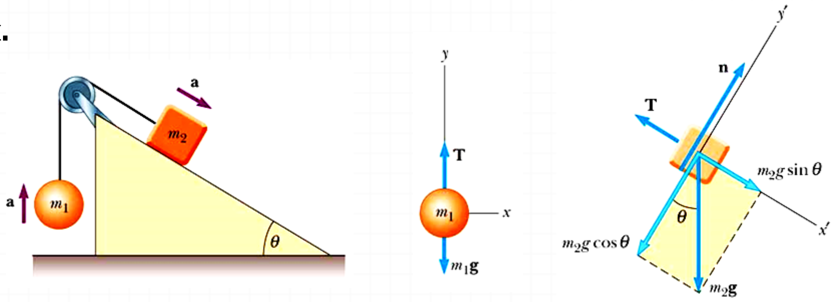


The Atwood machine a = ?
(tension and pulley mass are ignored)

$$\begin{cases} T - m_1 g = m_1 a \\ -m_2 g + T = -m_2 a \end{cases}$$

$$\Rightarrow \begin{cases} a = \dfrac{m_2 - m_1}{m_1 + m_2} g \\ T = \dfrac{2m_1 m_2}{m_1 + m_2} g \end{cases}$$

Ex.



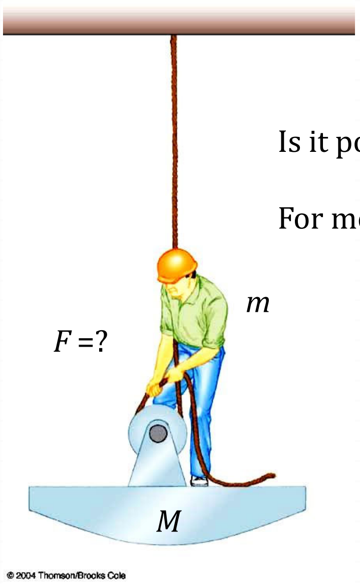
(1) $T - m_1 g = m_1 a$

(2) $\begin{cases} m_2 g \sin\theta - T = m_2 a \\ n - m_2 g \cos\theta = 0 \end{cases}$

$$\Rightarrow \begin{cases} a = \dfrac{m_2 g \sin\theta - m_1 g}{m_1 + m_2} \\ T = \dfrac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2} \end{cases}$$

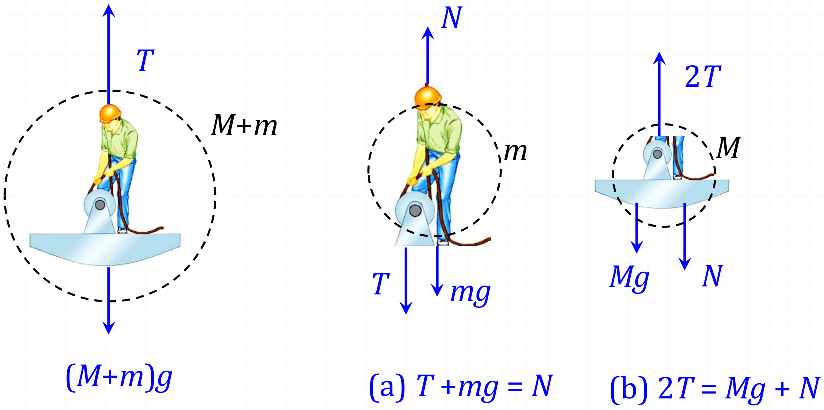
p.s., θ=90º → Atwood machine.

Ex.



Is it possible to lift upward?

For moving upward steadily, ***F*** = ?

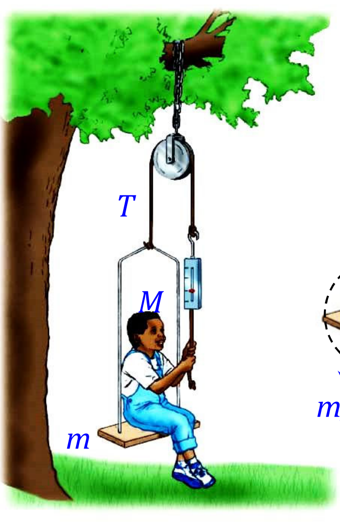


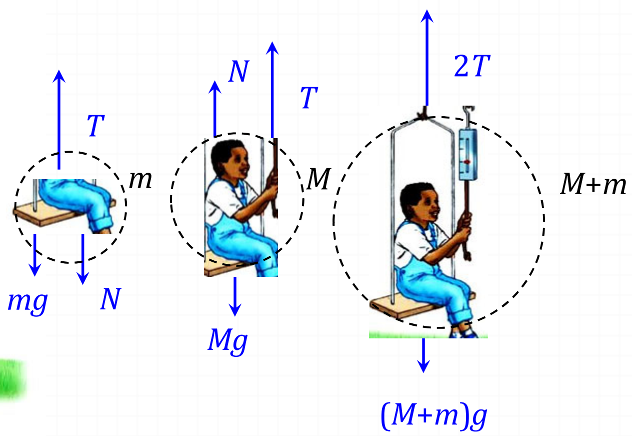
(a) $T + mg = N$ (b) $2T = Mg + N$

$\Rightarrow T = (M+m)g$ $\Rightarrow T = (M+m)g$

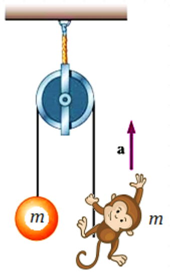
Ex.

Draw free-body diagrams for the chair, child, and chair+child.





Q: What will happen to the mass m, if the monkey climbs up at an acceleration of a?



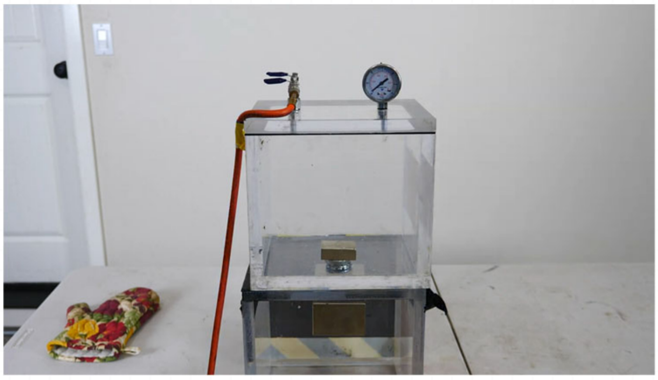
Ans:
The mass will move up at the same acceleration.

3. Frictional forces

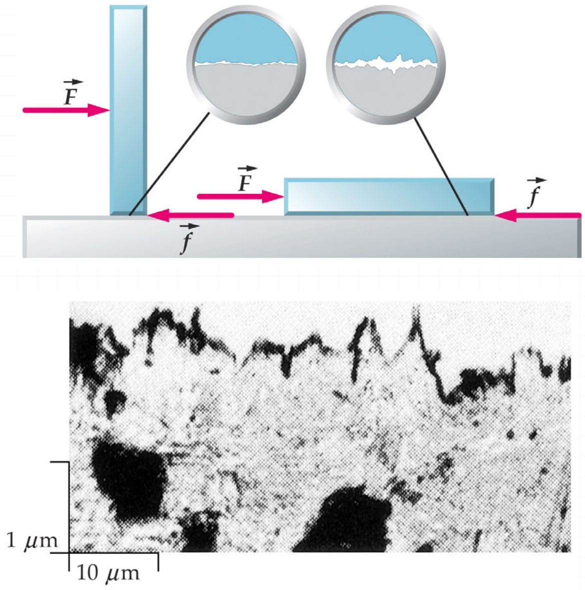
Friction force

- The frictional force is an *electromagnetic force* acting between the surface atoms or molecules.
  - A complex question at the atomic level.
  - Intermolecular bonding (e.g., cold weld).
  - Physical collisions impedes the motion.

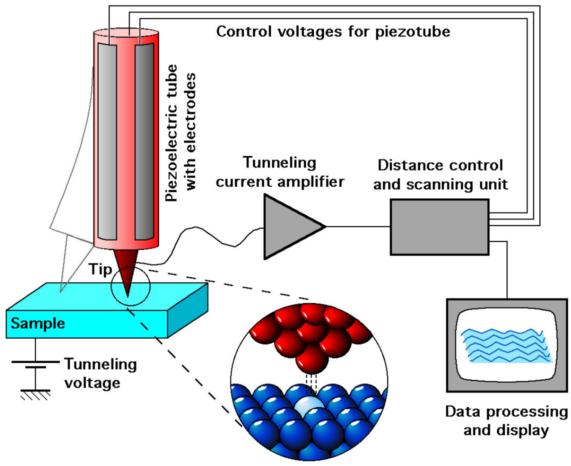
Cold weld



https://youtu.be/pavp6dps6q0

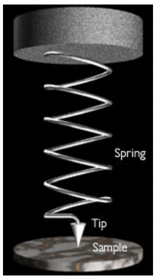


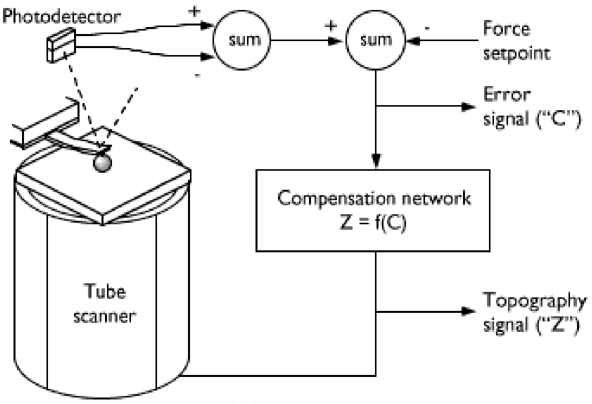
Scanning Tunneling Microscopy



http://www.iap.tuwien.ac.at/www/surface/STM_Gallery/stm_schematic.html

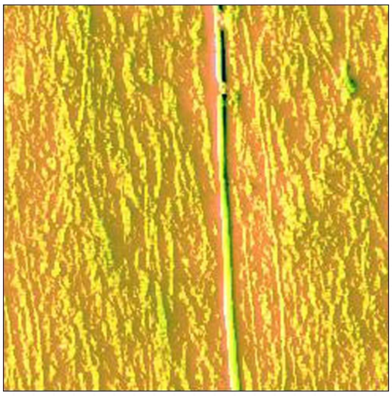
Atomic Force Microscopy





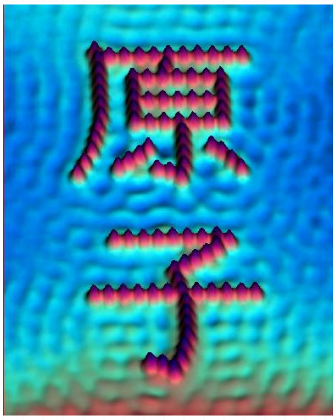
http://stm2.nrl.navy.mil/how-afm/how-afm.html

AFM image



10x10um Steel, polished

STM image



Iron on Copper (111)

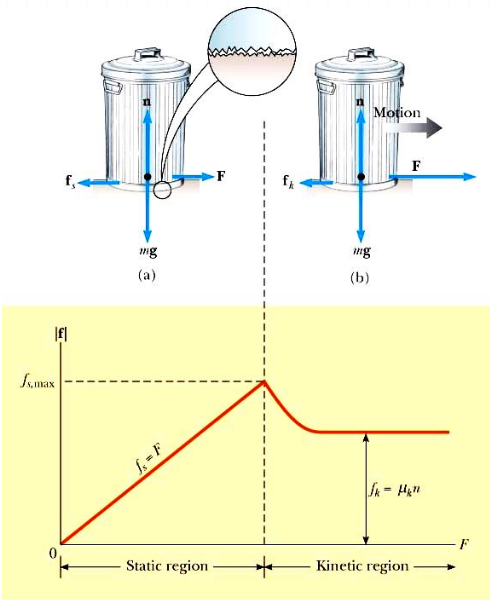
http://www.pacificnanotech.com/85B_Gallery_G_1.html http://www.almaden.ibm.com/vis/stm/atomo.html

Types of friction

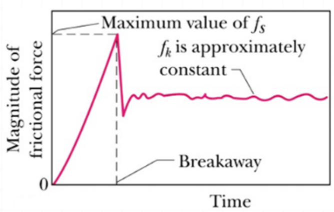
- Static friction: $f_s \le \mu_s\, n$; n: normal force
- Kinetic friction: $f_k = \mu_k\, n$; $\mu_{s,k}$: coefficient
- $f_k \le f_{s,max}$
- μ_s and μ_k :
  - Nearly constant (usually 0< μ_s, μ_k <1, but can be > 1)

Ideally,

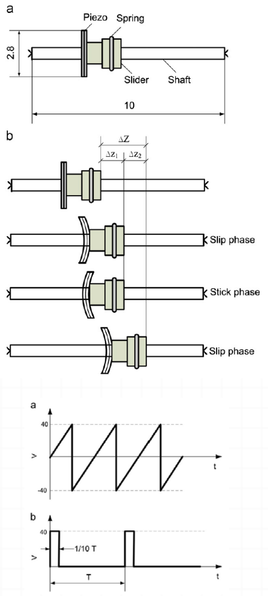
- Independent of the contact area!
- Independent of speed (for μ_k case)



(c)



Inertial piezoelectric motor



[https://www.youtube.com/watch?v=i-Dm-5EfSAk]

[Mechanical Systems and Signal Processing 36 (2013) 110–117]

Coefficients of Friction[a]

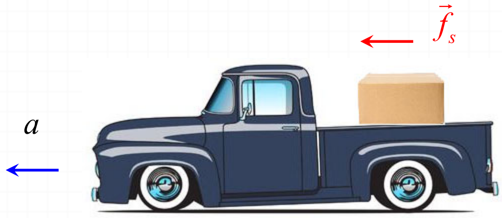
	μ_s $\geq$	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

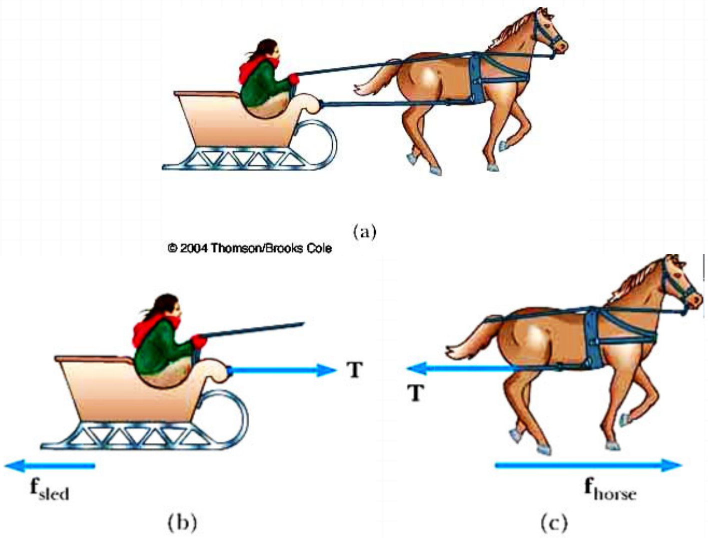
Direction of Friction Force

- The frictional force is opposite to the object's intended inertial motion or the impeding motion "*relative to the contact surface*".

Q:



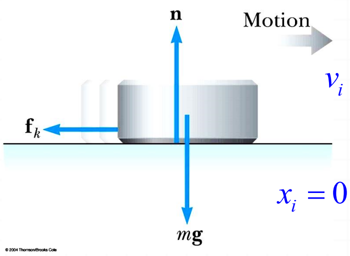
Which direction is for the friction force on the box?



Kinetic Static

Q: Regarding the friction force between the tires and the ground, what is the difference between the front-wheel drive and rear-wheel drive vehicles? Which one is easier to do back end whip or drifting?

Ex.



$v_i = v$ $\Longrightarrow$ $\mu_k = ?$ $v_f = 0$

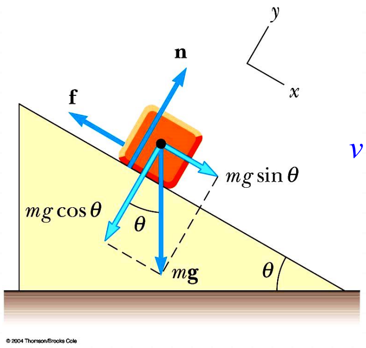
$x_i = 0$ x_f

$$\begin{cases} -F_k = ma_x \\ n - mg = 0 \end{cases} \qquad \begin{aligned} &-\mu_k n = -\mu_k mg = ma_x \\ &\Rightarrow a_x = -\mu_k g \end{aligned}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

$$0 = v^2 - 2\mu_k g x_f \quad \Rightarrow \quad \mu_k = \frac{v^2}{2gx_f}$$

Ex.



$v = 0$

$\mu_s = ?$

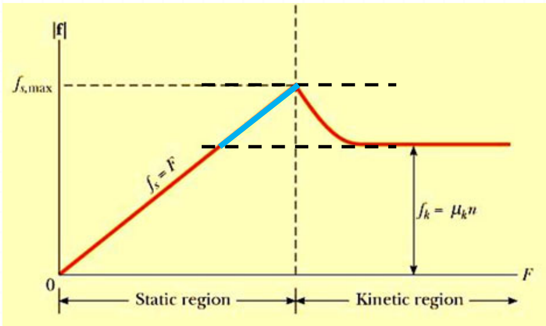
$$\begin{cases} mg\sin\theta - F_s = ma_x = 0 \\ n - mg\cos\theta = ma_y = 0 \end{cases}$$

$$\Rightarrow F_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = (\tan\theta)n$$

$$F_{s,\max} = (\tan\theta_c)n = \mu_s n \quad \Rightarrow \mu_s = \tan\theta_c$$

ABS-Automobile Antilock Braking Systems

- ABS is to maintain rolling contact between tires and ground, i.e., no slipping, so that the car can stop in a minimized distance.
- The friction force is larger if it is static friction.
  - Slipping, $f = f_k$
  - No slipping, $f = f_s$



ABS $\rightarrow$ To make $f_k \not< f < f_{s,\max}$, not $f = f_k$!

v_i m $v_f = 0$


$f \leftarrow$ $\longleftrightarrow$ d

$$-f = ma \Rightarrow a = -\frac{f}{m}$$

$$f = \mu n = \mu mg$$

$$f = f_{s,\max} \text{ or } f_k$$

$$\mu = \mu_s \text{ or } \mu_k$$

$$\Rightarrow a = -\mu g$$

$$v_f^2 = v_i^2 + 2ad$$

$$0 = v_i^2 + 2ad$$

$$\Rightarrow d = \frac{-v_i^2}{2a}$$

$$\Rightarrow d = \frac{v_i^2}{2g\mu}$$

1. d is indep. of m
(In reality, this may not be true,
because μ_s may change if the tires get hotter
for the heavier car than for the lighter car.)
2. $d \propto v_i^2$
3. $d_k = \dfrac{v_i^2}{2g\mu_k} > d_s = \dfrac{v_i^2}{2g\mu_s}$

Fluid Resistance and Terminal Speed

- Liquid and gas are fluid, which can exert a **fluid resistance**, *R*, on a moving object.
  - The magnitude of *R* depends on the medium and the object's shape and size.
  - *R* nearly always increases with increasing speed.
  - The direction of *R* is opposite to the direction of motion of the object relative to the medium.

- Two special cases:
  - ***R*** is proportional to ***v***.
    - Good approximation for *slow* motions or *small* objects.
  - ***R*** is proportional to $\boldsymbol{v}^2$.
    - Good approximation for *high* speed and *large* objects.

Non-Newtonian Fluid

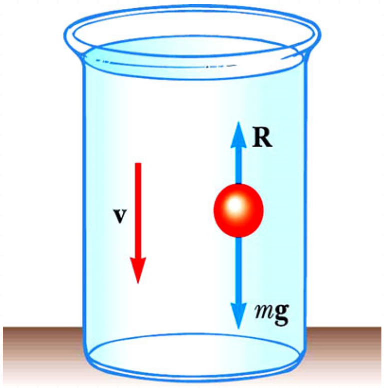
https://www.youtube.com/watch?v=RIUEZ3AhrVE

- The resistive force can be expressed as $R = -\, b\, v$
  - b depends on the property of the medium, and on the shape and dimensions of the object
- Analysis:

$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow -mg\hat{j} + (bv\hat{j}) = -m\frac{dv}{dt}\hat{j}$$

$$\Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \vec{g} - \frac{b}{m}\vec{v}$$



(Buoyant force is ignored here)

Terminal Speed

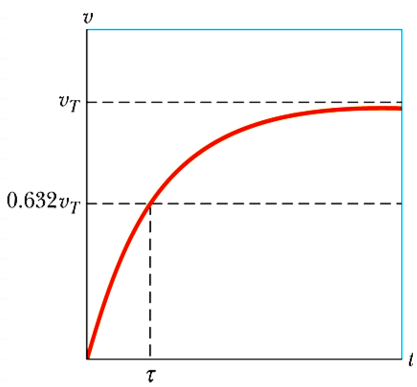
- The **terminal speed**:

$$a = 0 \quad \Rightarrow \quad v_T = \frac{mg}{b}$$

- Solving the differential equation gives

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

  - τ is the ***time constant*** and $\tau = m/b$.



$$\frac{dv}{dt} = g - \frac{b}{m}v \qquad \text{1st-order differential equation}$$

$$\Rightarrow \frac{dv}{g - \frac{b}{m}v} = dt \quad \Rightarrow \int_{v_i}^{v_f} \frac{dv}{g - \frac{b}{m}v} = \int_{t_i}^{t_f} dt$$

$$\Rightarrow -\frac{m}{b}\ln(g - \frac{bv}{m})\Big|_{v_i}^{v_f} = t\Big|_{t_i}^{t_f}$$

$$\Rightarrow \left[-\frac{m}{b}\ln(g - \frac{bv_f}{m})\right] - \left[-\frac{m}{b}\ln(g - \frac{bv_i}{m})\right] = t_f - t_i$$

$$t_i = 0,\ v_i = 0;\ t_f = t,\ v_f = v$$

$$\Rightarrow v(t) = \frac{mg}{b}\left(1 - e^{-\frac{t}{m/b}}\right)$$

Air drag force: $R = \frac{1}{2}D\rho A v^2$

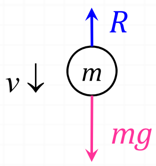
(See Serway Problem 8.32)

D: drag coefficient

ρ: air density

A: cross-section area

v: velocity



$$\sum F = mg - \frac{1}{2}D\rho A v^2 = ma$$

$$\Rightarrow a = g - \left(\frac{D\rho A}{2m}\right)v^2$$

$$a = 0 \quad \Rightarrow \quad v_T = \sqrt{\frac{2mg}{D\rho A}}$$

Note: mg never $< \frac{1}{2}D\rho A v^2$, assuming still air.

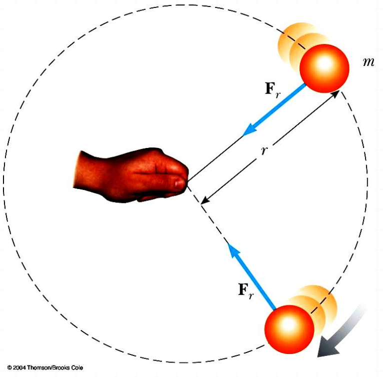
For a spherical object, $m \propto r^3$, $A \propto r^2$

$$\Rightarrow v_T \propto \sqrt{r}$$

1. Using Newton's 1st law: particles in equilibrium
2. Using Newton's 2nd law: dynamics of particles
3. Frictional forces
4. Dynamics of circular motion
5. Motion in non-inertial frames
6. The fundamental forces of Nature

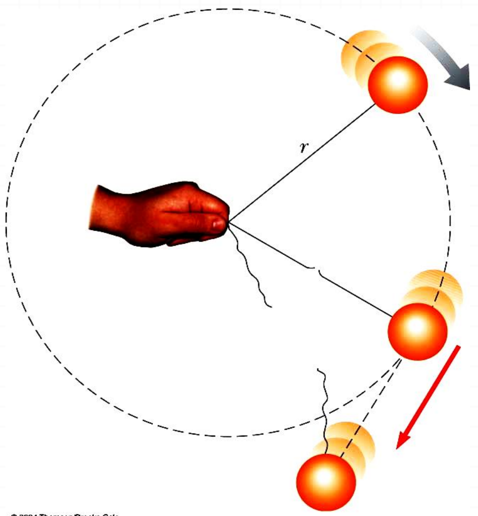
4. Dynamics of circular motion

Uniform Circular Motion

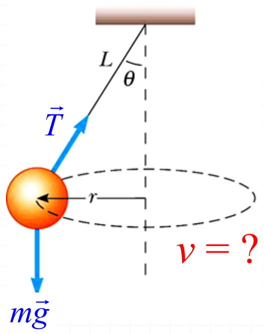


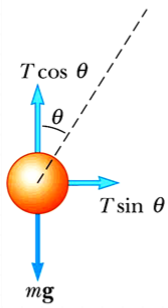
$$\vec{a}_c \to \vec{F}_c$$

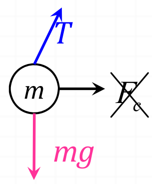
$$\sum \vec{F} = \vec{F}_c = m\vec{a}_c = (-)m\frac{v^2}{r}\hat{r}$$



Ex.







$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = ma_c = \dfrac{mv^2}{r} \end{cases} \Rightarrow \tan\theta = \frac{v^2}{rg}, \quad r = L\sin\theta$$

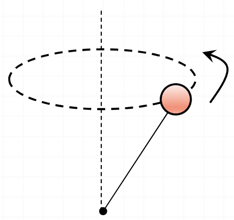
$$\Rightarrow v = \sqrt{Lg\sin\theta\tan\theta}, \quad \text{indep. of mass.}$$

p.s.,

- "Centripetal Force" is not a new kind of force. It acts just in the role of a force that causes a circular motion.
  - Don't draw another vector ma_c for the centripetal force.

Q:

A mass is attached to a rope. Is this circular motion possible?

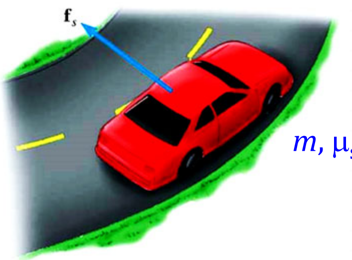


No.

(X)

(O)

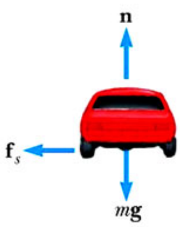
Ex.



m, μ_s

$$v_{\max} = ?$$

$$f_{s,\max} = m\frac{v_{\max}^2}{r}$$

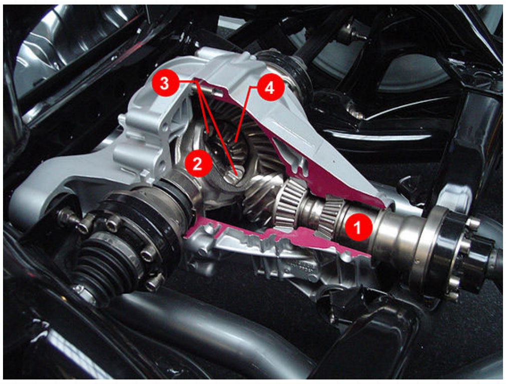


$$\mu_s n = \mu_s mg = m\frac{v_{\max}^2}{r}$$

$$v_{\max} = \sqrt{\mu_s gr}$$

indep. of mass.

差速器

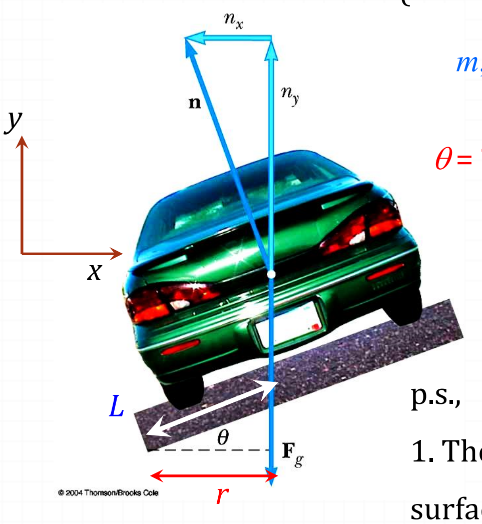


https://zh.wikipedia.org/wiki/%E5%B7%AE%E9%80%9F%E5%99%A8

https://www.youtube.com/watch?v=DXSgeMlcSU4

Ex.

(No friction, e.g., 磁浮列車)



m, v

$\theta = ?$

$$\begin{cases} F_x = n\sin\theta = \dfrac{mv^2}{r} \\ n\cos\theta = mg \end{cases}$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

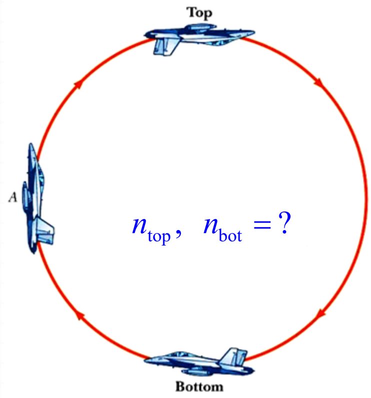
p.s.,

1. The x-axis is NOT chosen along the surface here.

2. a_c is $\dfrac{mv^2}{r}$, NOT $\dfrac{mv^2}{L}$

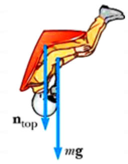
Ex.

Bottom



n_{top}, $n_{\text{bot}} = ?$

Top



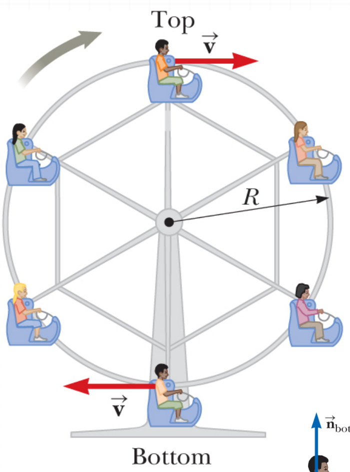
$$n_{\text{bot}}\hat{j} - mg\hat{j} = m\frac{v^2}{r}\hat{j}$$

$$\Rightarrow n_{\text{bot}} = m\left(g + \frac{v^2}{r}\right)$$

$$-n_{\text{top}}\hat{j} - mg\hat{j} = -m\frac{v^2}{r}\hat{j}$$

$$\Rightarrow n_{\text{top}} = m\left(\frac{v^2}{r} - g\right)$$

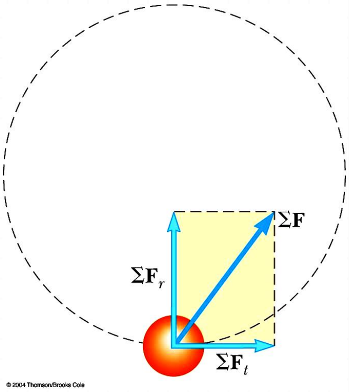
Ex.



$$\sum \vec{F} = -mg\hat{j} + n_{\text{top}}\hat{j} = -m\frac{v^2}{r}\hat{j}$$

$$\sum \vec{F} = -mg\hat{j} + n_{\text{bot}}\hat{j} = m\frac{v^2}{r}\hat{j}$$

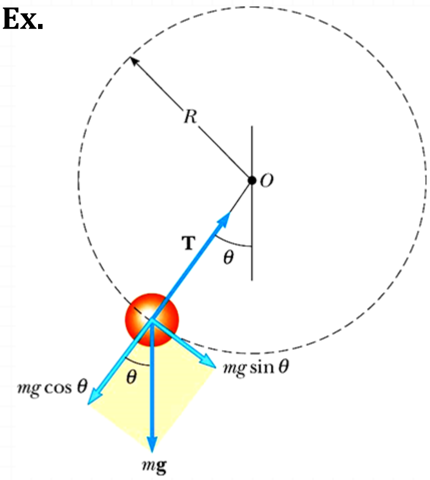
Nonuniform circular motion



$$\sum \vec{F} = \sum \vec{F}_t + \sum \vec{F}_r$$

$$\vec{a} = \vec{a}_t + \vec{a}_r$$

Ex.



1. a_t = ?
2. T = ?
3. At top if T=0, then v_{top}=?
4. What happen if $v < v_{top}$?

$$(1)\quad mg\sin\theta = ma_t$$

$$\Rightarrow\ a_t = g\sin\theta$$

$$(2)\quad -T\hat{r} + mg\cos\theta\hat{r} = -m\frac{v^2}{R}\hat{r}$$

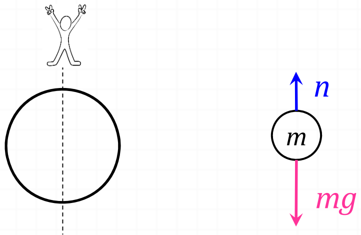
$$\Rightarrow\ T = m(\frac{v^2}{R} + g\cos\theta)$$

$$(3)\quad 0 = m(\frac{v_{\text{top}}^2}{R} + g\cos 180^0)$$

$$\Rightarrow\ v_{\text{top}} = \sqrt{gR}$$

Q: Assume the Earth is a uniform sphere, will the weight of an object be the same for at the equator and the poles?

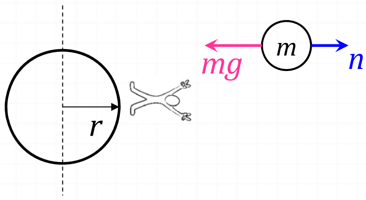
(1) At poles:



$$mg - n = 0$$

$$\Rightarrow \quad n = mg$$

(2) At equator:



$$mg - n = ma_c$$

$$\Rightarrow \quad n = m(g - a_c)$$

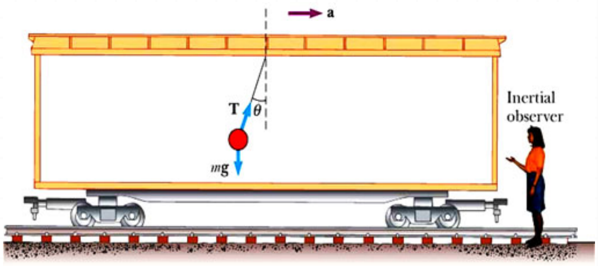
$$\text{p.s., } a_c = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2$$

5. Motion in non-inertial frames

Motion in Accelerated Frames

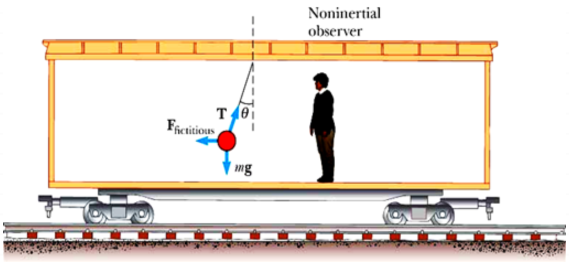
- A *fictitious force* can result from an accelerated frame of reference.
  - A fictitious force appears to act on an object in the same way as a real force, but you cannot identify a second object for the fictitious force, i.e., NOT in a pair of the act-reaction forces.

A fictitious force due to the change of the train speed



$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = ma \end{cases}$$

$$\Rightarrow a = g\tan\theta$$



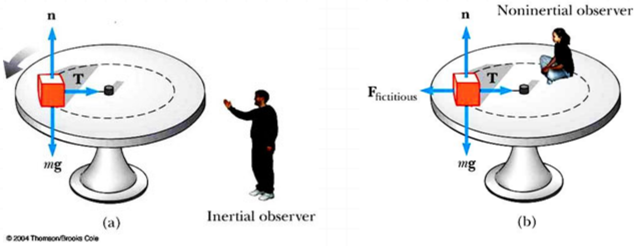
$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = m\underbrace{a}_{=0} = 0 \quad (\text{x}) \end{cases}$$

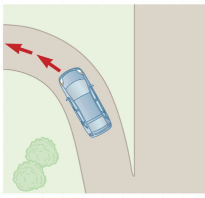
$$T\sin\theta + (F_{\text{fictitious}}) = m\underbrace{a}_{=0} = 0$$

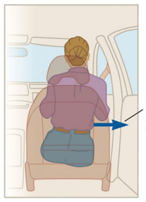
$$\Rightarrow F_{\text{fictitious}} = -T\sin\theta$$

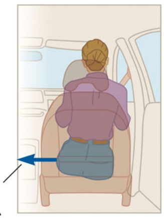
A fictitious force due to the changing direction of the car's velocity:

"Centrifugal force"





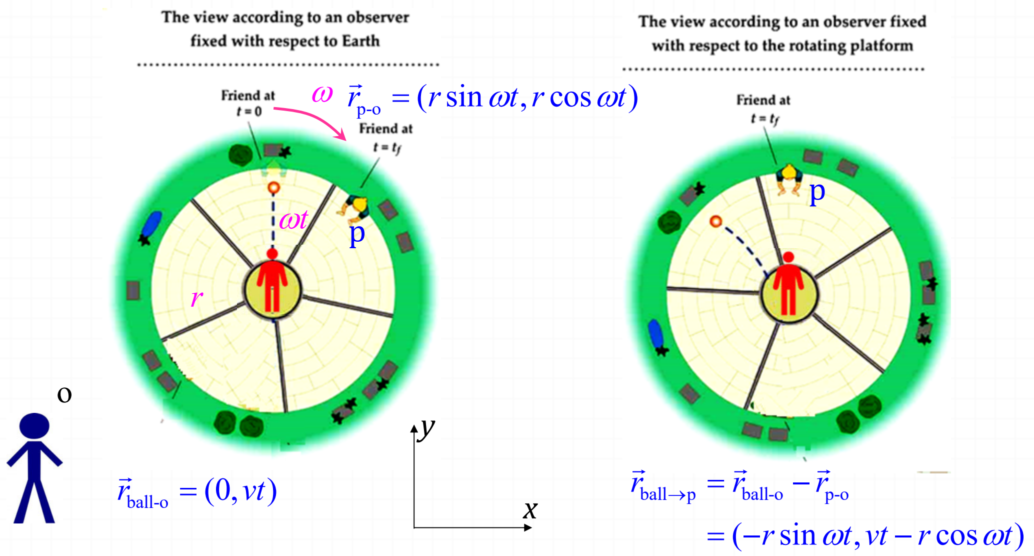




$$\begin{cases} T = ma_c = m\dfrac{v^2}{r} \\ n = mg \end{cases}$$

$$\underbrace{T}_{\neq 0} = m\underbrace{a_c}_{=0} \quad (\mathrm{x})$$
$$\Rightarrow T + F_{\text{fictitious}} = 0$$
$$\Rightarrow F_{\text{fictitious}} = -T$$

😎 Fictitious force is in the direction opposite to the acceleration of the non-inertial frame.

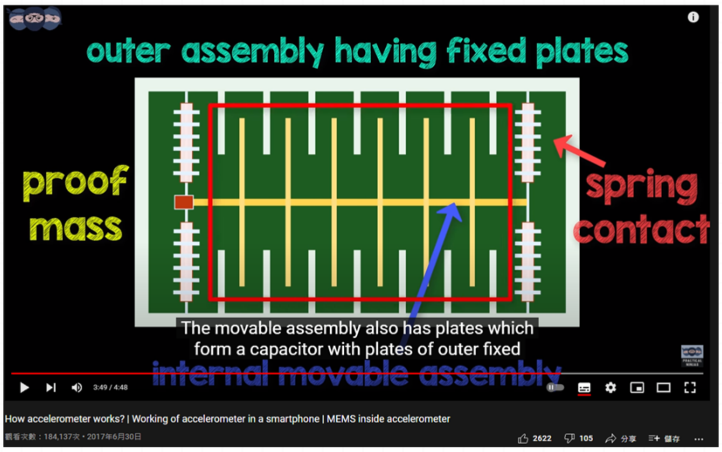


$\vec{r}_{\text{p-o}} = (r\sin\omega t, r\cos\omega t)$

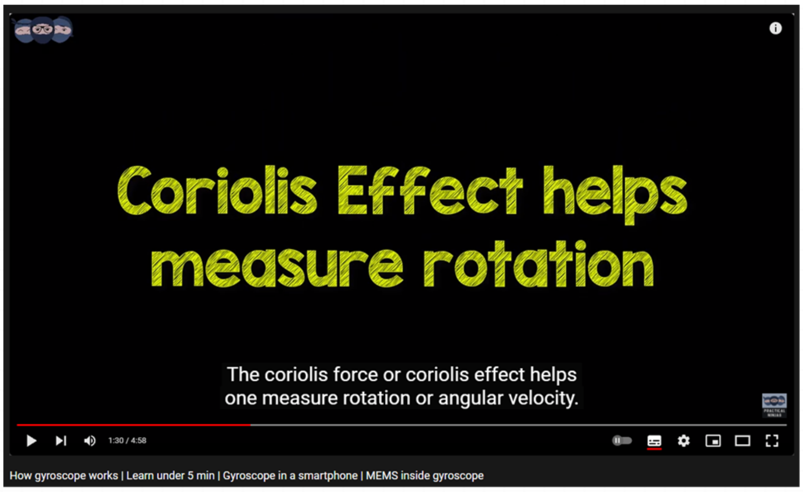
$\vec{r}_{\text{ball-o}} = (0, vt)$

$$\vec{r}_{\text{ball}\to\text{p}} = \vec{r}_{\text{ball-o}} - \vec{r}_{\text{p-o}} = (-r\sin\omega t, vt - r\cos\omega t)$$

A fictitious force due to the change of the radial position in a rotating coordinate: **"Coriolis force"**



https://www.youtube.com/watch?v=T_iXLNkkjFo&t=0s



https://www.youtube.com/watch?v=ti4HEgd4Fgo

1. Using Newton's 1st law: particles in equilibrium
2. Using Newton's 2nd law: dynamics of particles
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4. Dynamics of circular motion
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6. The fundamental forces of Nature

6. The fundamental forces of nature

Fundamental Forces in Nature

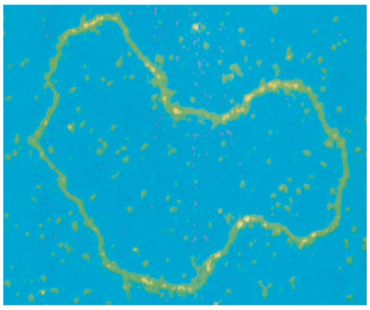
- Strong force (nuclear force): between subatomic particles inside nuclei.
- Electromagnetic force: between two charges
- Weak force: in certain radioactive decay processes
- Gravitational force: between two masses

p.s.

- All are field forces.
- Strong force is ~100 times larger than electroweak force.
- Open questions: a force unification theory? or a 5th fundamental force for “dark matter”?

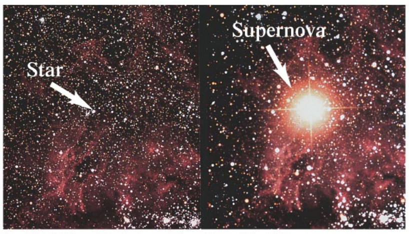
(a) Gravitational forces hold planets together.

(b) Electromagnetic forces hold molecules together.

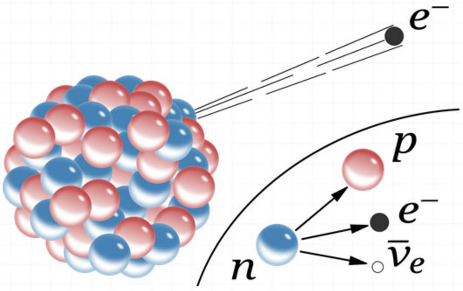


(c) Strong forces release energy to power the sun.

(d) Weak forces play a role in exploding stars.

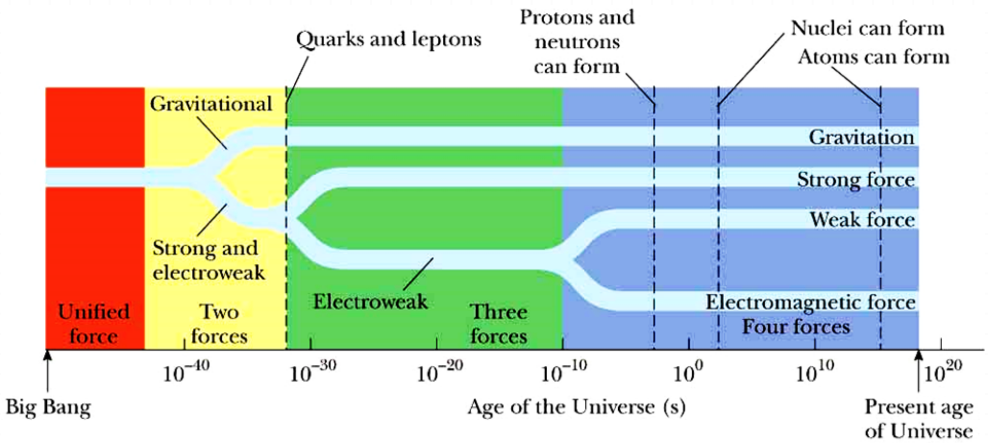


β decay



https://en.wikipedia.org/wiki/Beta_decay

Interaction	Relative Strength	Range of Force	Mediating Field Particle	Mass of Field Particle (GeV/c^2)
Nuclear	1	Short (≈ 1 fm)	Gluon	0
Electromagnetic	10^{-2}	∞	Photon	0
Weak	10^{-5}	Short ($\approx 10^{-3}$ fm)	$W^{\pm}$, Z^0 bosons	80.4, 80.4, 91.2
Gravitational	10^{-39}	∞	Graviton	0



Quarks and leptons
Protons and neutrons can form
Nuclei can form
Atoms can form
Gravitational
Gravitation
Strong force
Weak force
Strong and electroweak
Electroweak
Electromagnetic force
Unified force
Two forces
Three forces
Four forces
10^{-40}
10^{-30}
10^{-20}
10^{-10}
10^{0}
10^{10}
10^{20}
Big Bang
Age of the Universe (s)
Present age of Universe

2022/9/23

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