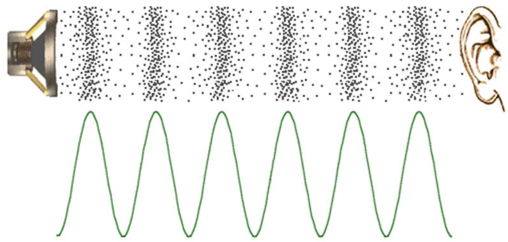
國立交通大學 電子物理系 NCTU Electrophysics



Chapter 16

SOUND AND HEARING

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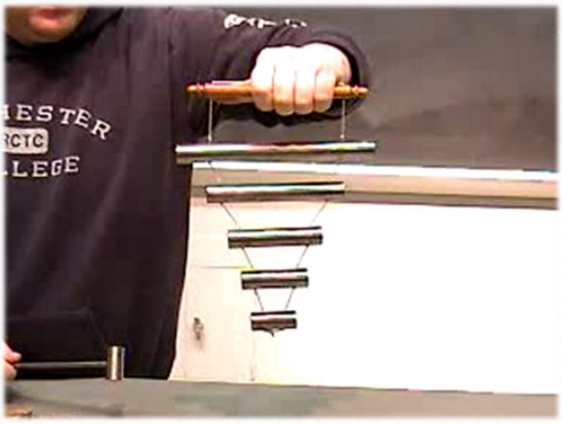
Outline

1. Sound waves
2. Sound wave function
3. Sound speed
4. Sound wave equation
5. Sound intensity
6. Standing sound wave
7. Resonance and sound
8. The Doppler effect
9. Shock waves
10. Beats

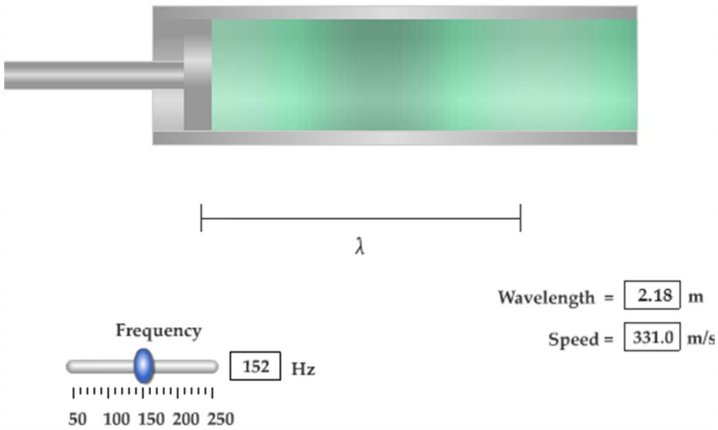
1. Sound waves

Sound Waves

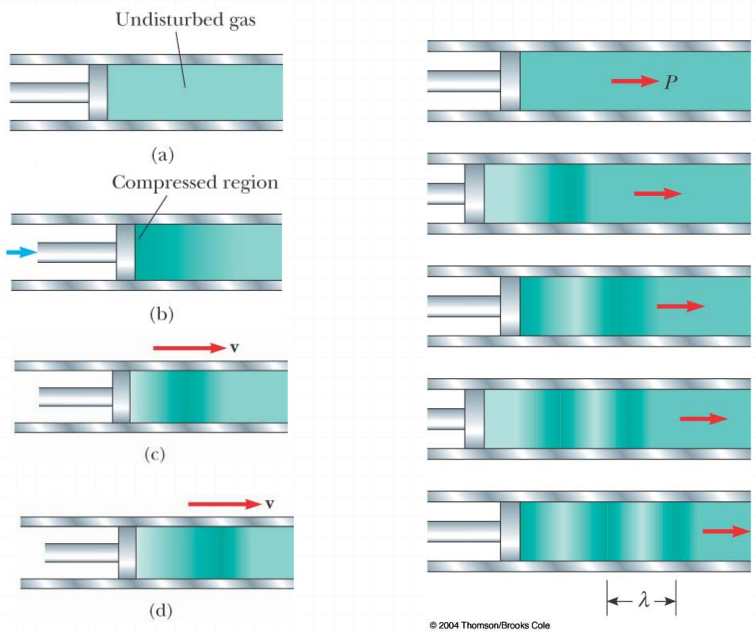
- Sound waves are longitudinal waves.
- Audible waves within the sensitivity of the human ear are approximately **20 Hz to 20 kHz**. **Why?**
- Infrasonic waves have frequencies below the audible range.
- Ultrasonic waves have frequencies above the audible range.



HESTER
RCTC
LLEGE



λ
Wavelength = 2.18 m
Speed = 331.0 m/s
Frequency
152 Hz
50 100 150 200 250



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😎 The piston's frequency determines the frequency of the sound wave. The displacement of the piston controls the amplitude, not the wavelength.

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Blow air with sound wave

https://fb.watch/fO1kz9VVzH/

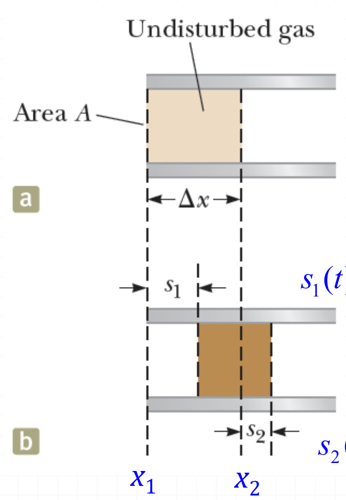
2. Sound wave function

Sound wave function

Displacement: $s(x,t) = s_{max}\cos(kx - \omega t)$

Pressure: $P(x,t) = P_{max}\sin(kx - \omega t)$

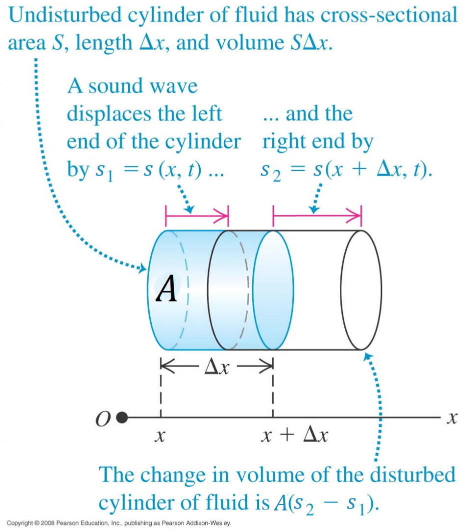
Displacement



$$s_1(t) = s_{\max}\cos(kx_1 - \omega t)$$

$$s_2(t) = s_{\max}\cos(kx_2 - \omega t)$$

Pressure

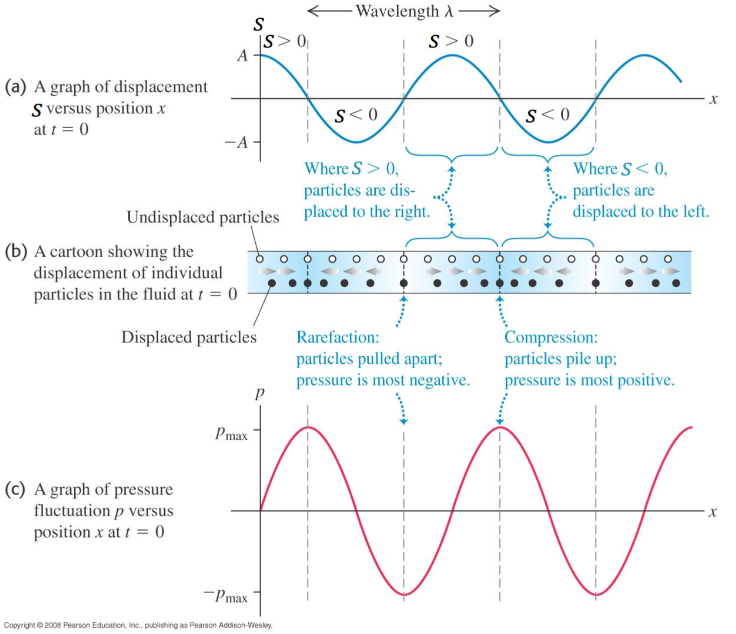


$$\begin{cases} \Delta V = A\Delta s \\ V = A\Delta x \\ B = -\dfrac{P}{\Delta V / V} \end{cases} \quad \Rightarrow \quad P = -B\frac{\Delta V}{V} = -B\frac{ds}{dx}$$

Displacement: $s(x,t) = s_{\max}\cos(kx - \omega t)$

$\Rightarrow$ Pressure: $P(x,t) = P_{\max}\sin(kx - \omega t)$

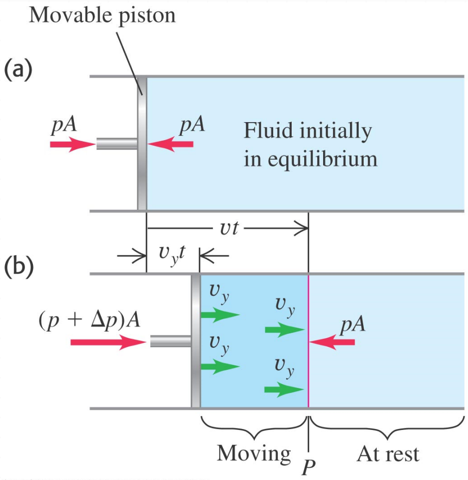
$$P_{\max} = Bks_{\max}$$



Q: If you blow across the top of an empty soft-drink bottle, a pulse of sound travels down through the air in the bottle. At the moment the pulse reaches the bottom of the bottle, the correct descriptions of the displacement of elements of air from their equilibrium positions and the pressure of the air at this point are (a) the displacement and pressure are both at a maximum (b) the displacement and pressure are both at a minimum (c) the displacement is zero and the pressure is a maximum (d) the displacement is zero and the pressure is a minimum

3. Sound speed

Sound speed



Applying impulse-momentum theorem

$$\text{Momentum} = (\rho A v t) v_x$$

$$\text{Impulse} = \Delta P A t$$

$$\Delta P = -B\frac{\Delta V}{V} = B\frac{A v_x t}{A v t}$$

$$\Rightarrow \quad v = \sqrt{\frac{B}{\rho}}$$

Sound speed in a fluid

3 The bulk modulus of the fluid is B.

3 The density of the fluid is ρ.

3 The speed of sound in that medium is

$$v = \sqrt{\frac{B}{\rho}}.$$

Sound speed in a gas

3 Sound speed in an ideal gas:

$$v = \sqrt{\frac{\gamma k_B T}{m}}, \quad \gamma \equiv \frac{C_P}{C_V} \quad \text{ratio of specific heats}$$

- It is closely related to molecular speed.

3 For air, the relationship between the speed and temperature is

$$v = (331\ \mathrm{m/s})\sqrt{1 + \frac{T_C}{273^\circ\ \mathrm{C}}}.$$

- The 331 m/s is the speed at 0° C.
- T_C is the air temperature in Celsius.

Speed of Sound in Various Media

Medium	v (m/s)	Medium	v (m/s)	Medium	v (m/s)
Gases		**Liquids at 25°C**		**Solids**[a]	
Hydrogen (0°C)	1 286	Glycerol	1 904	Pyrex glass	5 640
Helium (0°C)	972	Seawater	1 533	Iron	5 950
Air (20°C)	343	Water	1 493	Aluminum	6 420
Air (0°C)	331	Mercury	1 450	Brass	4 700
Oxygen (0°C)	317	Kerosene	1 324	Copper	5 010
		Methyl alcohol	1 143	Gold	3 240
		Carbon tetrachloride	926	Lucite	2 680
				Lead	1 960
				Rubber	1 600

[a]Values given are for propagation of longitudinal waves in bulk media. Speeds for longitudinal waves in thin rods are smaller, and speeds of transverse waves in bulk are smaller yet.

Speed of Sound in a Solid Rod

- The Young's modulus of the material is Y.
- The density of the material is ρ.
- The speed of sound in the rod is

$$v = \sqrt{\frac{Y}{\rho}}.$$

Speed of Mechanical Waves

3 The speed of sound waves in a medium depends on the compressibility and the density of the medium.

3 The compressibility can sometimes be expressed in terms of the elastic modulus of the material.

3 The speed of all mechanical waves follows a general form:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

E.g, for sea water wave,

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\Delta P / \left(\Delta V/V\right)}{\rho}} = \sqrt{\frac{\rho g h / \left(\Delta V/V\right)}{\rho}}$$

$$\Rightarrow v \propto \sqrt{gh}$$

4. Sound wave equation

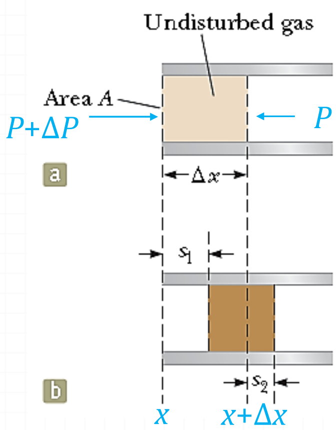
Sound wave equation

$$\begin{cases} F = (P+\Delta P)A - PA = \left[P(x) - P(x+\Delta x)\right]A = -\left(\dfrac{\partial P}{\partial x}\Delta x\right)A \\ ma = \left(\rho V\right)\dfrac{\partial^2 s}{\partial t^2} = \rho A \Delta x \dfrac{\partial^2 s}{\partial t^2} \end{cases}$$

$$\Rightarrow -\left(\frac{\partial P}{\partial x}\Delta x\right)A = \rho A \Delta x \frac{\partial^2 s}{\partial t^2} \Rightarrow -\frac{\partial P}{\partial x} = \rho \frac{\partial^2 s}{\partial t^2}$$

$$B = -\frac{P}{\Delta V / V} = -\frac{P}{A\Delta s / A\Delta x} \Rightarrow P = -B\frac{\partial s}{\partial x}$$

$$\Rightarrow \frac{B}{\rho}\left(\frac{\partial^2 s}{\partial x^2}\right) = \left(\frac{\partial^2 s}{\partial t^2}\right)$$



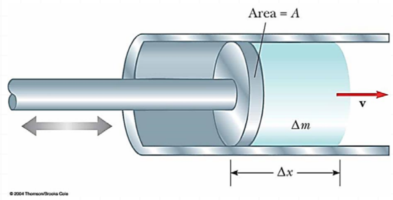
Compare with $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} \Rightarrow v = \sqrt{\dfrac{B}{\rho}}$

5. Sound Intensity

Intensity of Periodic Sound Waves

$$\text{Power} = \vec{F}\cdot\vec{v}_x = P(x,t)A\frac{\partial s(x,t)}{\partial t}$$

$$\begin{cases} s(x,t) = s_{\max}\cos(kx-\omega t) \\ P(x,t) = \underbrace{Bks_{\max}}_{P_{\max}}\sin(kx-\omega t) \end{cases}$$



$$\text{Intensity } I = \frac{(\text{Power})_{\text{average}}}{\text{area}} = \underbrace{B}_{=\rho v^2}\ \underbrace{k}_{=\frac{\omega}{v}}\ s_{\max}^2\,\omega\underbrace{\langle\sin^2(kx-\omega t)\rangle}_{=1/2}$$

$$\Rightarrow\quad I = \underbrace{\frac{1}{2}\rho(s_{\max}\omega)^2}_{\text{Energy density}}v \quad \text{or} \quad I = \frac{P_{\max}^2}{2B}v$$

Ex. Hearing Limits

The faintest sounds the human ear can detect at a frequency of 1 kHz correspond to an intensity of about 1.00×10^{-12} W/m^2—the so-called threshold of hearing. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about 1.00 W/m^2—the threshold of pain. Determine the pressure amplitude and displacement amplitude associated with these two limits.

Ans:

$$P_{\max} = \sqrt{2\rho v I} = \sqrt{2(1.20\ \text{kg/m}^3)(343\ \text{m/s})(1.00\times10^{-12}\ \text{W/m}^2)}$$
$$= 2.87\times10^{-5}\ \text{N/m}^2$$

$$1\ \text{atm} \sim 10^5\ \text{N/m}^2$$

$$s_{\max} = \frac{P_{\max}}{\rho v \omega} = \frac{2.87\times10^{-5}\ \text{N/m}^2}{(1.20\ \text{kg/m}^3)(343\ \text{m/s})(2\pi\times1000\ \text{Hz})}$$
$$= 1.11\times10^{-11}\ \text{m}$$

The decibel scale

3 The range of intensities detectible by the human ear is very large.

3 It is convenient to use a logarithmic scale to determine the intensity level, β,

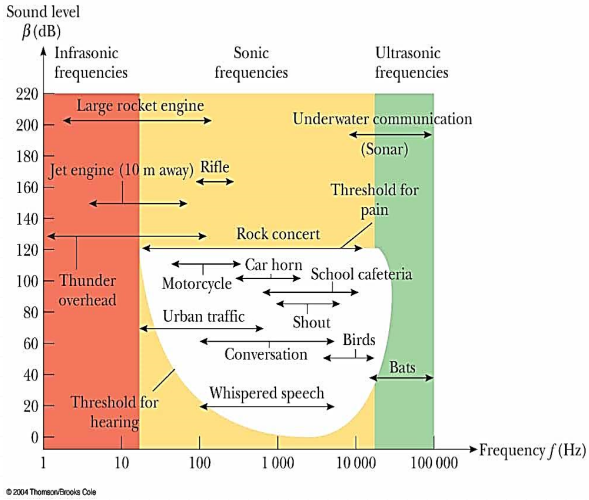
$$\beta = 10 \log\left(\frac{I}{I_o}\right)$$

- I_0 is called the reference intensity
- The threshold of hearing, $I_0 = 1.00 \times 10^{-12}$ W/ m^2.
- β is in decibels (dB).

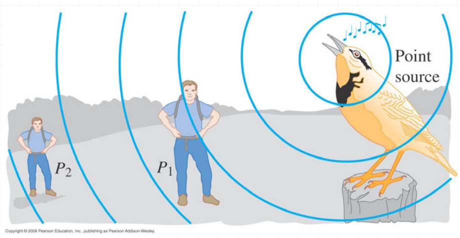
Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Loudness and Frequency

- There is a complex relationship between loudness and frequency.
- The lower curve of the white area shows the threshold of hearing.
- The upper curve shows the threshold of pain.



Ex. Consider an idealized model with a bird (treated as a point source) emitting constant sound power, with intensity inversely proportional to the square of the distance from the bird. By how many decibels does the sound intensity level drop when you move twice as far away from the bird?



Ans:

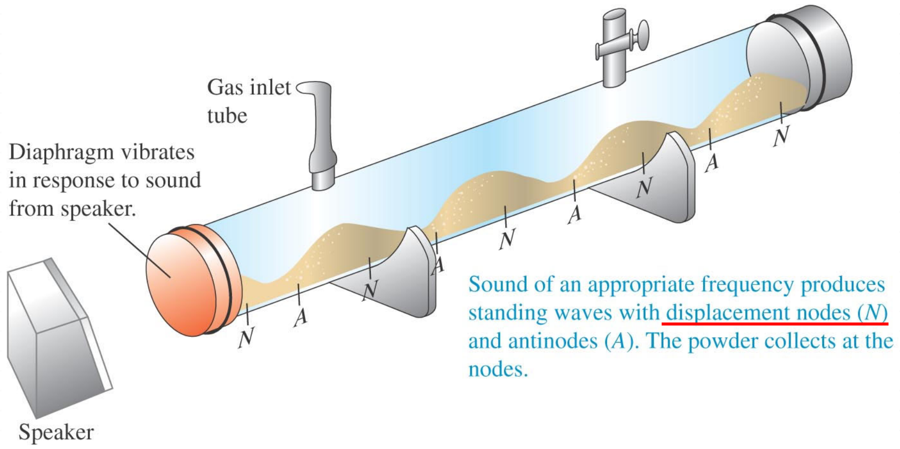
$$\beta_2 - \beta_1 = 10(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}) = 10\log\frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

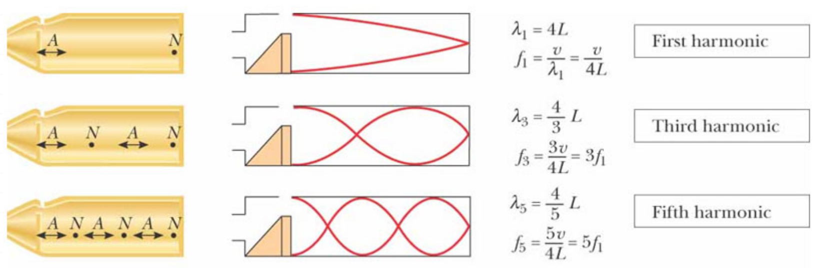
$$\beta_2 - \beta_1 = 10\log\frac{r_1^2}{r_2^2} = 10\log\frac{r_1^2}{(2r_1)^2} = -6 \text{ dB}$$

6. Standing sound wave

Standing Waves in Air Columns



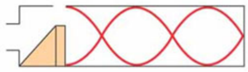
Standing Waves in a Tube Closed at One End



Closed at one end, open at the other

- The closed end is a displacement node.
- The open end is a displacement antinode.
- The fundamental corresponds to ¼ λ.
- The frequencies are $f_n = nf_1 = n\,(v/4L)$, where $n = 1, 3, 5, \ldots$

$$y_1(x,t) = A\sin(kx - \omega t) \quad \rightarrow$$



$$\leftarrow \quad y_2(x,t) = A\sin(kx + \omega t + \pi)$$

$$y(x,t) = y_1(x,t) + y_2(x,t) = 2A\sin(kx + \frac{\pi}{2})\cos(\omega t + \frac{\pi}{2})$$

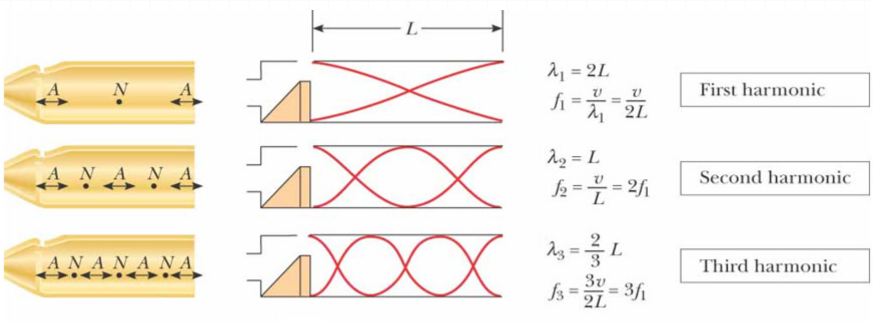
Boundary conditions: $y(L) = 0$

$$\sin(kL + \frac{\pi}{2}) = 0$$

$$kL + \frac{\pi}{2} = m\pi, \quad m = \text{integer}$$

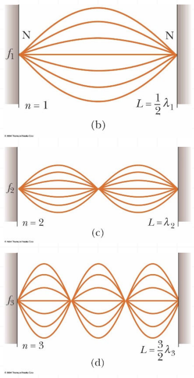
$$\frac{2\pi}{\lambda_m} L = (m - \frac{1}{2})\pi \quad \Rightarrow \quad L = \left(m - \frac{1}{2}\right)\left(\frac{\lambda_m}{2}\right) = n\frac{\lambda_n}{4}, \quad n = 1,3,5,\ldots$$

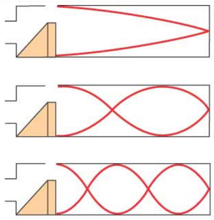
Standing Waves in an Open Tube

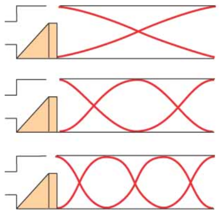


Open at both ends

- Both ends are displacement antinodes.
- The fundamental frequency is $v/2L$.
- The higher harmonics are $f_n = nf_1 = n\,(v/2L)$, where $n = 1, 2, 3, \ldots$







(1) $L = n\left(\frac{\lambda_n}{2}\right)$ $\quad n = 1,\ 2,\ 3,\ \ldots$

$$f_n = \frac{v}{\lambda_n}$$

$$f_n = n\left(\frac{v}{2L}\right) = nf_1$$

$$f_1 = \frac{v}{2L}$$

(2) $L = (2m-1)\left(\frac{\lambda_m}{4}\right)$ $\quad m = 1,\ 2,\ 3,\ \ldots$

$$f_n = (2m-1)\left(\frac{v}{4L}\right) = nf_1, \quad n = 1,\ 3,\ 5,\ \ldots$$

$$f_1 = \frac{v}{4L}$$

(3) $L = n\left(\frac{\lambda_n}{2}\right)$ $\quad n = 1,\ 2,\ 3,\ \ldots$

$$f_n = n\left(\frac{v}{2L}\right) = nf_1$$

$$f_1 = \frac{v}{2L}$$

Q: A pipe open at both ends resonates at a fundamental frequency f_{open}. When one end is covered and the pipe is again made to resonate, the fundamental frequency is f_{closed}. Which of the following expressions describes how these two resonant frequencies compare?

(a) $f_{closed} = f_{open}$

(b) $f_{closed} = ½\, f_{open}$

(c) $f_{closed} = 2\, f_{open}$

(d) $f_{closed} = {}^{2}/_{3} f_{open}$

Temperature effect on Instruments

As the temperature rises:

3 Sounds produced by air columns become sharp.

- Higher frequency.
- Higher speed due to the higher temperature.

3 Sounds produced by strings become flat.

- Lower frequency.
- The strings expand due to the higher temperature, and their tension decreases.

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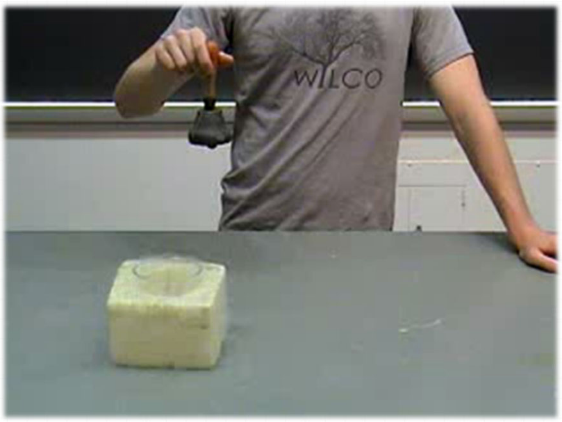

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Q: Balboa Park in San Diego has an outdoor organ. When the air temperature increases, the fundamental frequency of one of the organ pipes

(a) stays the same

(b) goes down

(c) goes up

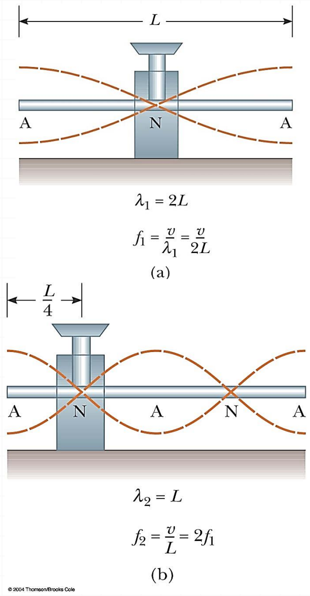
(d) is impossible to determine.

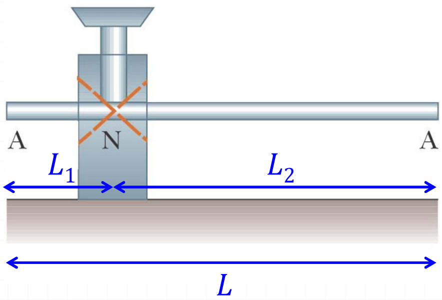
A: (c)

Standing Waves in Rods

3 A rod is clamped in the middle.

- The clamp forces a displacement node.
- The ends of the rod are free to vibrate and so will correspond to displacement antinodes.





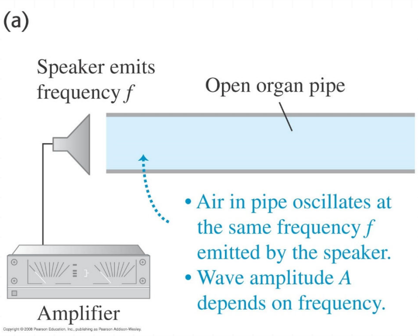
$$L_1 = (n - \frac{1}{2})\left(\frac{\lambda}{2}\right)$$

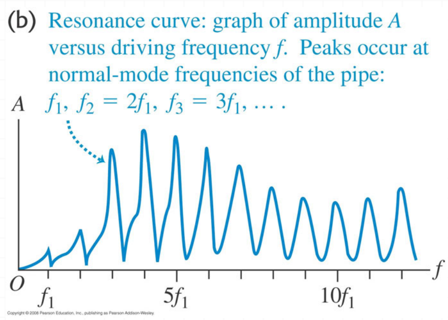
$$L_2 = (m - \frac{1}{2})\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow \quad L = L_1 + L_2 = (m + n - 1)\left(\frac{\lambda}{2}\right)$$

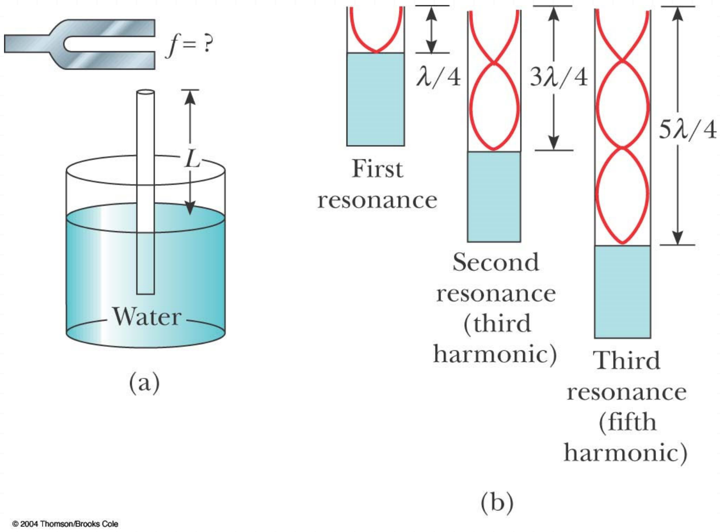
$$n,\ m = 1,\ 2,\ 3,\ \ldots \text{ and } m > n\ (\because L_2 > L_1)$$

7. Resonance and sound





Determining frequency by the tuning fork



Helmholtz resonance

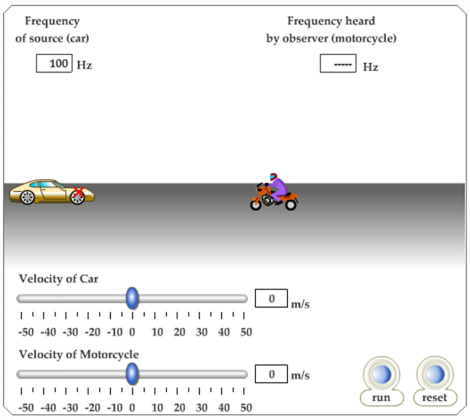
https://www.youtube.com/watch?v=PZVeJ2rh6ts

https://en.wikipedia.org/wiki/Helmholtz_resonance

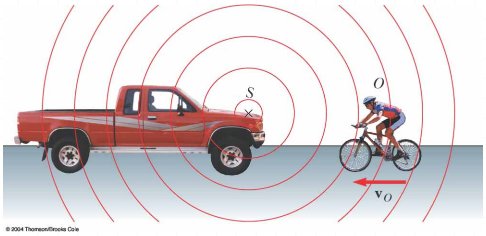
8. The Doppler effect

The Doppler Effect

3 The Doppler effect is the apparent change in frequency (or wavelength) that occurs because of motion of the source or observer of a wave.



Case 1: Source at rest & observer moving



v : wave speed

λ : wavelength

v_0 : observer speed

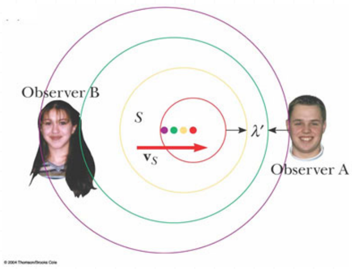
$\vec{v}\,' = \vec{v} - \vec{v}_0,$ wave speed relative to observer

$\lambda' = \lambda,$ wavelength relative to observer

$f\,' = \dfrac{v'}{\lambda'},$ frequency relative to observer

$$\Rightarrow f\,' = \frac{|\vec{v} - \vec{v}_0|}{\lambda} = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v}|/f} = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v}|} f$$

Case 2: Source moving & observer at rest



v : wave speed

λ : wavelength

v_s : source speed

$\vec{v}\,' = \vec{v},$ wave speed relative to observer

$\lambda',$ wavelength relative to observer

$$\Rightarrow \lambda' = \underbrace{\lambda \pm \Delta\lambda}_{\substack{\text{"}-\text{": observer A} \\ \text{"}+\text{": observer B}}} = \frac{v}{f} \pm v_s T = \left|\frac{\vec{v}}{f} - \frac{\vec{v}_s}{f}\right| = \frac{|\vec{v} - \vec{v}_s|}{f}$$

$f\,' = \dfrac{v'}{\lambda'},$ frequency relative to observer $\Rightarrow f\,' = \dfrac{|\vec{v}|}{|\vec{v} - \vec{v}_s|} f$

Doppler Effect for mechanical waves

$$f' = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v} - \vec{v}_s|} f$$

v : wave speed

v_0 : observer speed

v_s : source speed

The speeds are all relative to the media that carries the wave.

Q. You are at an outdoor concert with a wind blowing at 10 m/s from the performers toward you. Is the sound you hear Doppler shifted? If so, is it shifted to lower or higher frequencies?

Ans: No.

∵ The speeds in the Doppler shift formula are all relative to the media, so that it doesn't matter whether the wind is blowing or not. Only the relative motion of the source and observer matter.

Doppler effect for electromagnetic waves

$$f' = \sqrt{\left|\frac{\vec{c}+\vec{v}}{\vec{c}-\vec{v}}\right|} f$$

In the observer's reference frame,

c : light speed

v : source speed relative to the observer

Q: Consider detectors of water waves at three locations A, B, and C in Figure. Which of the following statements is true?

(a) The wave speed is highest at location A.

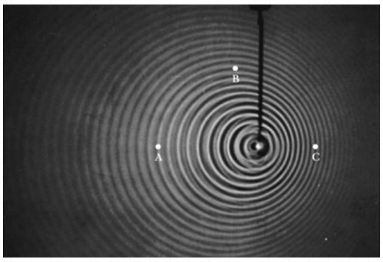
(b) The wave speed is highest at location C.

(c) The detected wavelength is largest at location B.

(d) The detected wavelength is largest at location C.

(e) The detected frequency is highest at location C.

(f) The detected frequency is highest at location A.



Q: You stand on a platform at a train station and listen to a train approaching the station at a constant velocity. While the train approaches, but before it arrives, you hear

(a) the intensity and the frequency of the sound both increasing

(b) the intensity and the frequency of the sound both decreasing

(c) the intensity increasing and the frequency decreasing

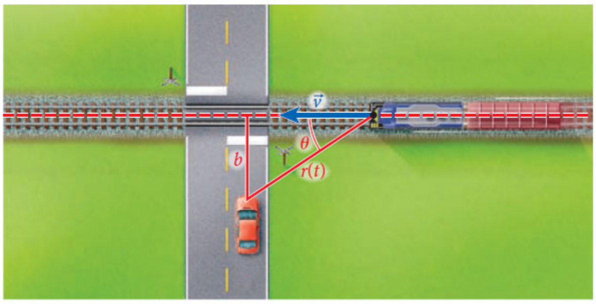
(d) the intensity decreasing and the frequency increasing

(e) the intensity increasing and the frequency remaining the same

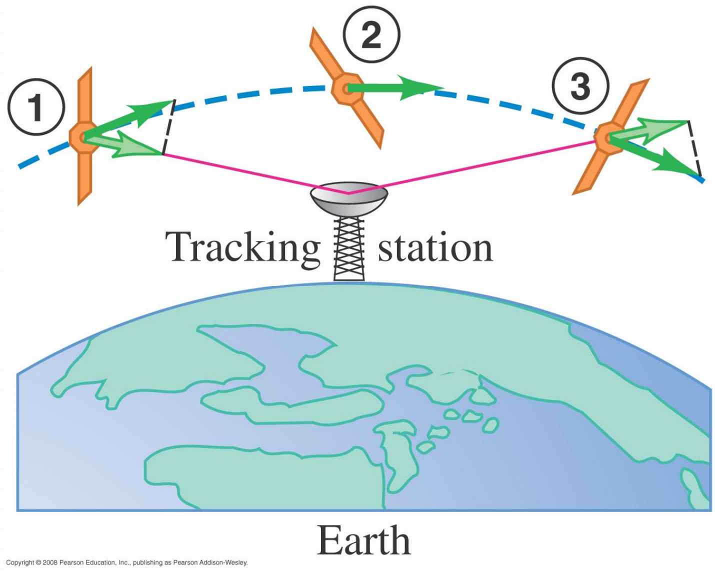
(f) the intensity decreasing and the frequency remaining the same.

Doppler effect in 2-dimension

$t > 0$ $t = 0$ $t < 0$



$$f_{\text{observer}} = \left| \frac{\vec{v}}{\vec{v} - \vec{v}_s} \right| f = \left| \frac{v}{v - v_s \cos\theta} \right| f = \frac{v}{v + \dfrac{v^2 t}{\sqrt{b^2 + v^2 t^2}}}$$



Ex. Your clock radio awakens you with a steady and irritating sound of frequency f Hz. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, h m from the ground. Assume the speed of sound is v_s m/s. As you listen to the falling clock radio, what frequency do you hear just before you hear the radio striking the ground?

Ans:

$$v_s = v_{yi} + a_y t = 0 - gt = -gt$$

$$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2 = h + 0 - \frac{1}{2} g t^2 = 0 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

$$f' = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v} - \vec{v}_s|} f \quad \Rightarrow \quad f' = \left[\frac{v + 0}{v - (-\sqrt{2gh})}\right] f = \left(\frac{v}{v + \sqrt{2gh}}\right) f$$

Ex. A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1 400 Hz. The speed of sound in the water is 1 533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.

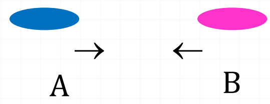
(A) What frequency is detected by an observer riding sub B as the subs approach each other?

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

Ans:

(A)

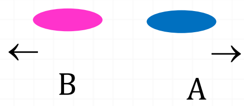
$$f_B = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v} - \vec{v}_s|} f_A$$



$$= \frac{1533 - (-9)}{1533 - (+8)} 1400 = 1416 \text{ Hz}$$

(B)

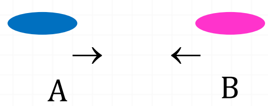
$$f_B = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v} - \vec{v}_s|} f_A$$



$$= \frac{-1533 - (-9)}{-1533 - (+8)} 1400 = 1385 \text{ Hz}$$

What If? While the subs are approaching each other, some of the sound from sub A will reflect from sub B and return to sub A. If this sound were to be detected by an observer on sub A, what is its frequency?

$$f_A' = \frac{|\vec{v} - \vec{v}_0|}{|\vec{v} - \vec{v}_s|} f_B$$



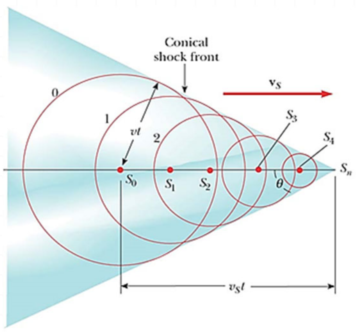
$$= \frac{-1533 - (+8)}{-1533 - (-9)} 1416 = 1432 \text{ Hz}$$

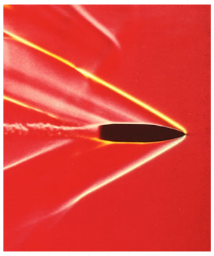
1. Sound waves
2. Sound wave function
3. Sound speed
4. Sound wave equation
5. Sound intensity
6. Standing sound wave
7. Resonance and sound
8. The Doppler effect
9. Shock waves
10. Beats

9. Shock waves

Shock Wave

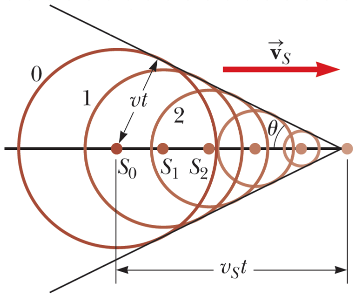
3 The speed of the source v_s *exceeds* the speed of the wave v.





Sonic boom

Mach Number



3 The ratio v_s / v is referred to as the *Mach number*.

3 The relationship between the Mach angle θ and the Mach number is $\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$, which can be used to measure the speeds of the wave and the source.

$v_s < v$

$v_s = v$

$v_s > v$

Cherenkov radiation:
The electromagnetic radiation emitted when a charged particle (such as an electron) passes through a dielectric medium at a speed greater than the *phase velocity* of light in that medium. The characteristic blue glow of an underwater nuclear reactor is due to Cherenkov radiation. It is named for Soviet physicist Pavel Cherenkov, who shared the 1958 Nobel Prize in Physics for its discovery.

(Wikipedia)

10. Beats

Spatial and Temporal Interference

- Spatial interference occurs when the amplitude of the oscillation in a medium varies with the position in space of the element.
- Temporal interference occurs when the amplitude has a temporal alternation between constructive and destructive interference.

Beats

- Temporal interference will occur when the interfering waves have slightly different frequencies.
- Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies.

$$\begin{cases} y_1(x,t) = A_1 \sin(k_1 x - \omega_1 t + \phi_1) \\ y_2(x,t) = A_2 \sin(k_2 x - \omega_2 t + \phi_2) \end{cases}$$

To be simple, set $A_1 = A_2 = A,\ \phi_1 = \phi_2 = 0.$

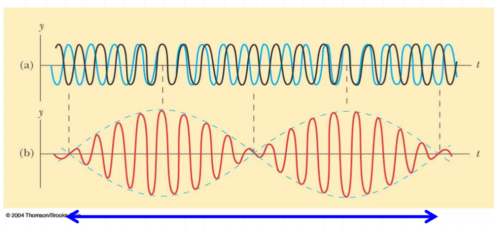
At $x = 0$, $\begin{cases} y_1(0,t) = A\sin(\omega_1 t) \\ y_2(0,t) = A\sin(\omega_2 t) \end{cases}$

$$\Rightarrow\ y(0,t) = y_1(0,t) + y_2(0,t) = 2A\cos\left[\frac{1}{2}(\omega_1 - \omega_2)t\right]\sin\left[\frac{1}{2}(\omega_1 + \omega_2)t\right]$$

$$\Rightarrow\ A_{\text{resultant}} = 2A\cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]$$

- The amplitude of the resultant wave varies in time according to

$$A_{\text{resultant}} = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$$

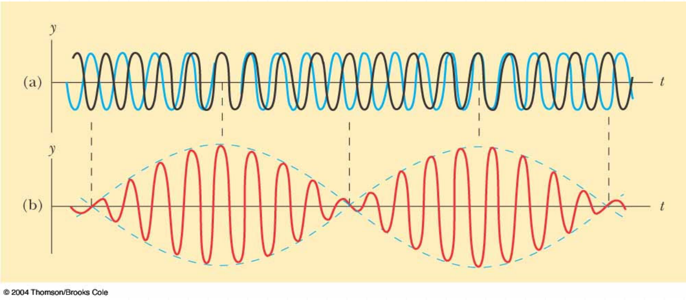


$$f = \frac{f_1 - f_2}{2}$$

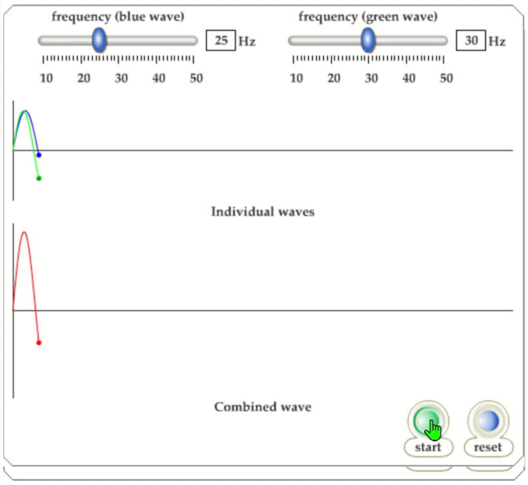
- The number of beats per second, the beat frequency, is

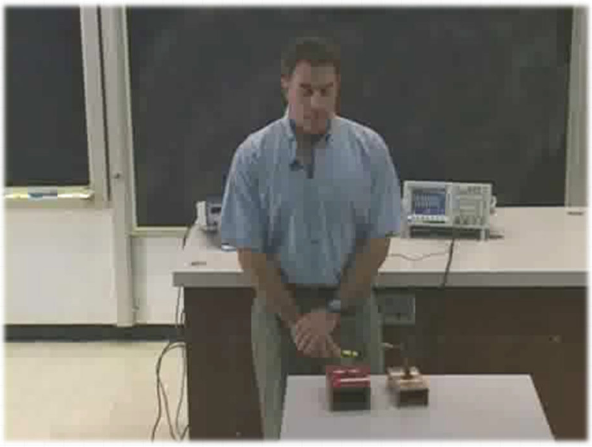
$$f_{\text{beat}} = 2f = |f_1 - f_2|.$$

Beat Frequency



- The number of amplitude maxima one hears per second is the beat frequency.
- It equals the difference between the frequencies of the two sources.
- The human ear can detect a beat frequency up to about 20 beats/sec.





Q: You are tuning a guitar by comparing the sound of the string with that of a standard tuning fork. You notice a beat frequency of 5 Hz when both sounds are present. You tighten the guitar string and the beat frequency rises to 8 Hz. In order to tune the string exactly to the tuning fork, you should

(a) continue to tighten the string

(b) loosen the string

(c) impossible to determine.

A: (b)