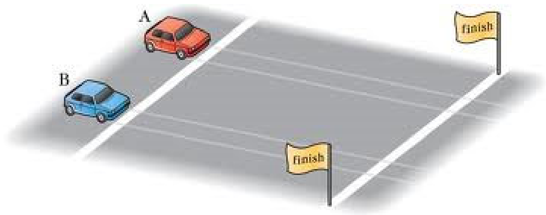
Chapter 2

Motion along a Straight Line



楊 本 立 副教授

Outline

1. Definition of Quantities in Kinematics
   - Position, Velocity, and Speed
   - Instantaneous Velocity and Speed
   - Acceleration
2. One-Dimensional Motion with Constant Acceleration
   - Kinematic equations
3. Freely Falling Objects
4. Kinematic Equations Derived from Calculus

Kinematics (運動學)

- Describe particle motion in terms of space and time.
- Motion
  - Translation
  - Rotation
  - Vibration
- Quantities
  - Position
  - Displacement
  - Velocity
  - Acceleration

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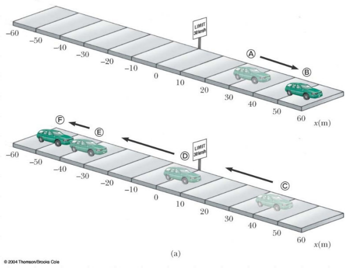
1. Definition of Quantities in Kinematics

Definition of Quantities in Kinematics

- Position, x
- Displacement, $\Delta x \equiv x_f - x_i$
- Distance s
- Average velocity, $\bar{v}_x \equiv \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$
- Average speed = $s / \Delta t$
- Instantaneous velocity, $v_x \equiv \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
- Instantaneous speed $\equiv |v_x|$
- Average acceleration, $\bar{a}_x \equiv \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{xf} - v_{xi}}{t_f - t_i}$
- Instantaneous acceleration, $a_x \equiv \lim_{\Delta t \to 0} \dfrac{\Delta v_x}{\Delta t} = \dfrac{dv_x}{dt} = \dfrac{d^2 x}{dt^2}$

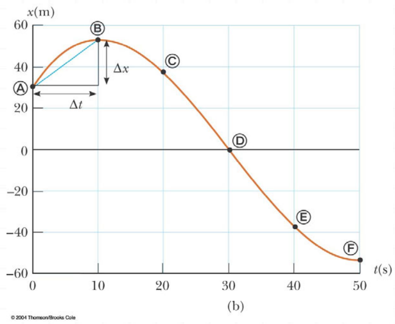
Position: the location with respect to the frame of reference.



Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Position of the Car at Various Times

Position-Time Graph



The goal is to find $x = x(t) = ?$

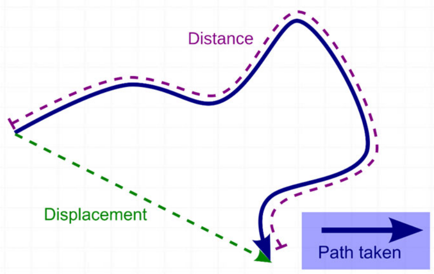
😎 Modern physics:

- More precise definitions of space and time will require "relativity". Space and time are not independent each other.

- Motion of small particles like atoms is subject to "uncertainty principle" and is described by "Quantum Mechanics".

Displacement & distance

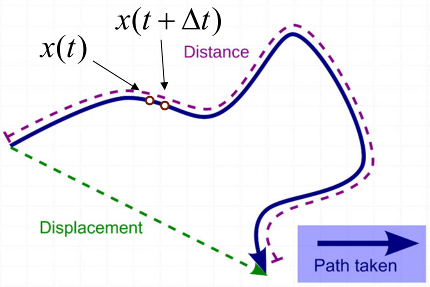
- Displacement: the change in position $\Delta x \equiv x_f - x_i$.
- Distance: the length of a path followed by a particle, s.



Instantaneous Velocity

- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx}{dt}$$



p.s.,

$\frac{dx}{dt}$: derivative of x with respect to t

e.g.,

$$x(t) = At^2$$

$$x(t+\Delta t) = A(t+\Delta t)^2 = At^2 + 2At\Delta t + A(\Delta t)^2$$

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{2At\Delta t + A(\Delta t)^2}{\Delta t} = 2At$$

Instant speed

Instant speed = the magnitude of instant velocity, $|v_x|$

Average Velocity

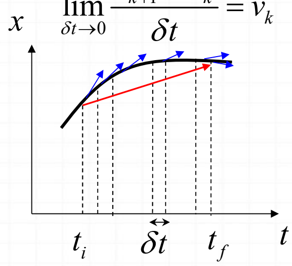
- The average velocity is rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

Why is it called "average" ?

Proof:

$$\lim_{\delta t \to 0} \frac{x_{k+1} - x_k}{\delta t} = v_k$$



$$\overline{v} = \frac{\sum_{k=1}^{n} v_k}{n} = \frac{\sum_{k=1}^{n} \left(\lim_{\delta t \to 0} \frac{x_{k+1} - x_k}{\delta t} \right)}{(t_f - t_i) / \delta t} = \frac{\sum_{k=1}^{n} (x_{k+1} - x_k)}{t_f - t_i}$$

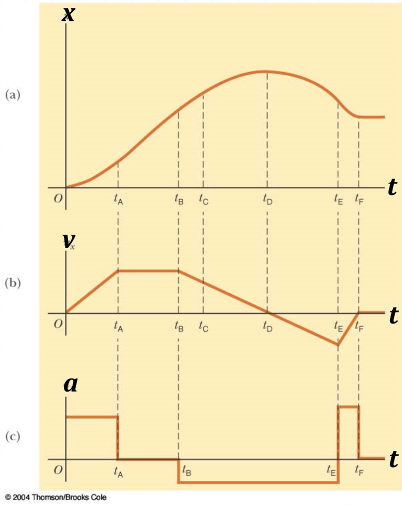
$$= \frac{x_f - x_i}{t_f - t_i}$$

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

Once $x = x(t)$ is known, the $v(t)$ and $a(t)$ can be obtained.



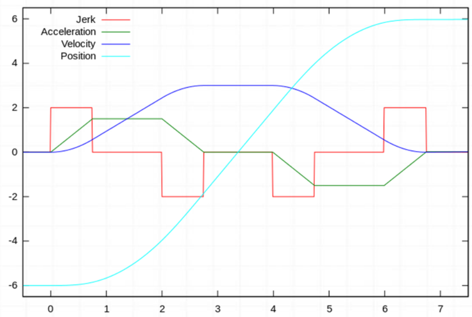
P.s.,

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

Can we define another quantity as $\frac{d^3 x}{dt^3}$? (Jerk) (Serway, Problem 2.33)

- Air bags are triggered by Jerk?!



https://en.wikipedia.org/wiki/Jerk_(physics)

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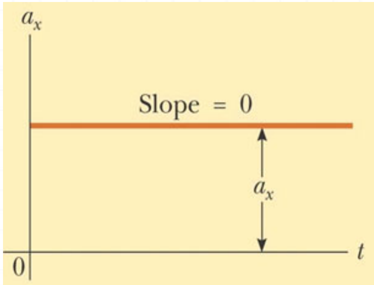
2. One-dimensional motion with constant acceleration

One-dimensional motion with constant acceleration



(1) $t_i = 0,\ t_f = t$ $\qquad a_x = \dfrac{v_{xf} - v_{xi}}{t - 0}$

$$\boxed{\Rightarrow v_{xf} = v_{xi} + a_x t}$$

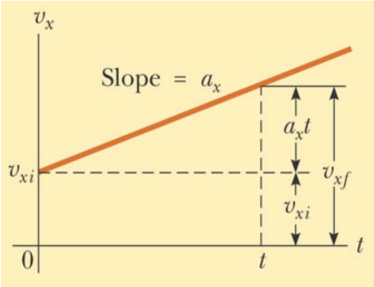
(2) Average velocity

$$\boxed{\bar{v}_x = \frac{v_{xi}+v_{xf}}{2}} = \frac{x_f - x_i}{t-0} \qquad \boxed{\Rightarrow x_f = x_i + \bar{v}_x t} \qquad \Leftarrow \text{Only for constant } a$$

$$\text{p.s., } \bar{v}_x = \frac{\sum_{k=1}^{n} v_{xk}}{n} = \frac{\sum_{k=1}^{n} v_{xk}}{t/\delta t}$$

$$= \frac{1}{t}\left(\underbrace{\sum_{k=1}^{n} v_{xk}\delta t}_{\substack{\text{area under } v\text{-}t \text{ curve} \\ \Delta x}} \right)$$

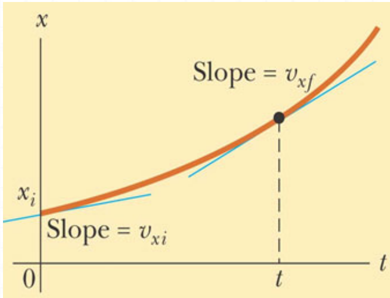
$$= \frac{1}{t}\left[\frac{1}{2}\left(v_{xi}+v_{xf}\right)\cdot t\right] = \frac{1}{2}\left(v_{xi}+v_{xf}\right)$$



(3) $$x_f = x_i + \bar{v}_x t$$

$$\Rightarrow x_f = x_i + \frac{1}{2}\left[v_{xi} + \left(v_{xi} + a_x t\right)\right]t$$

$$\boxed{\Rightarrow x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$



(4) $x_f = x_i + \bar{v}_x t$

$$\Rightarrow x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$$= \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

$$\boxed{\Rightarrow v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)}$$

Kinematic Equations - summary

(x, v, a, t)

Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Non-uniform acceleration case

- Even the smoothest sports car does not move with constant acceleration.
- A typical car takes twice as long to accelerate from 50 km/h to 100 km/h as it does to accelerate from 0 to 50 km/h. Why?

Assume the engine output power is fixed,

$$P = F(t)v(t) = ma(t)v(t) = const.$$

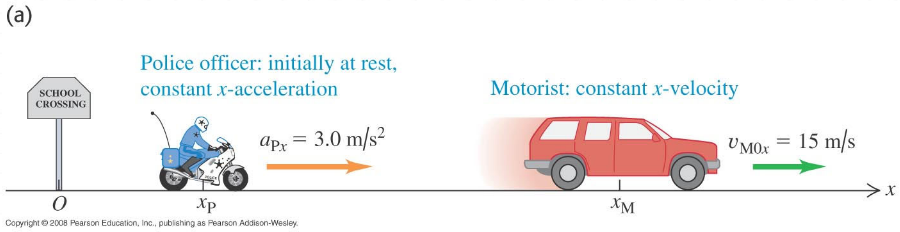
$$\frac{P}{m} = \frac{dv}{dt}v \qquad \int_0^t \frac{P}{m}dt = \int_0^v vdv \qquad \frac{P}{m}t = \frac{1}{2}v^2$$

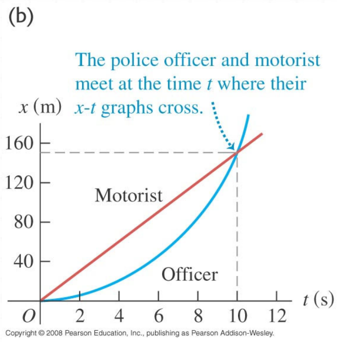
$$\Rightarrow \quad t = \frac{m}{2P}v^2$$

$$v = 0 \to v_0 \quad \Rightarrow \quad \Delta t_1 = \frac{m}{2P}v_0^2$$

$$v = v_0 \to 2v_0 \quad \Rightarrow \quad \Delta t_2 = \frac{m}{2P}(2v_0)^2 - \frac{m}{2P}v_0^2 = 3\Delta t_1 > \Delta t_1$$

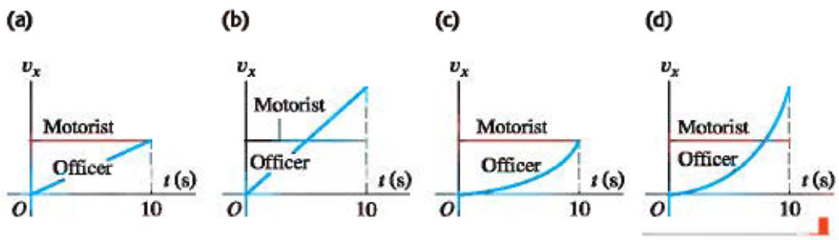
Ex.





😇 追到的時候兩車速度會一樣?

Q: Which one is the correct *v*-*t* diagram?

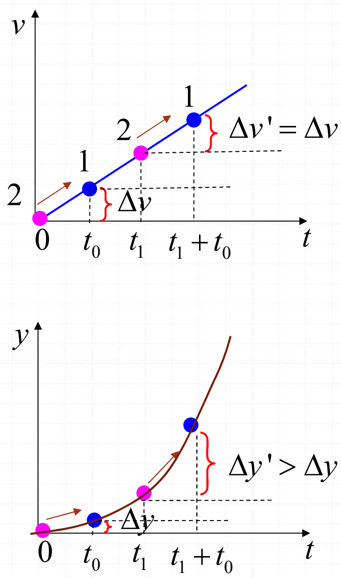


3. Freely Falling Objects

Freely Falling Objects

- A *freely falling object* is any object moving freely under the influence of gravity alone.
- The magnitude of free fall acceleration is g = 9.80 m/s^2.
  - g decreases with increasing altitude.
  - g varies with latitude.
  - 9.80 m/s^2 is the average at the Earth's surface.

Q: A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?



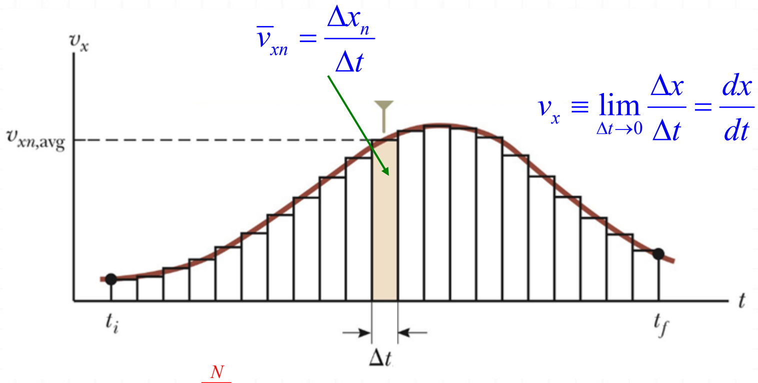
Why is the water dripping not continuous in the lower portion?

4. Kinematic Equations – Derived from Calculus

Kinematic Equations -Derived from Calculus

$$\bar{v}_{xn} = \frac{\Delta x_n}{\Delta t}$$

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

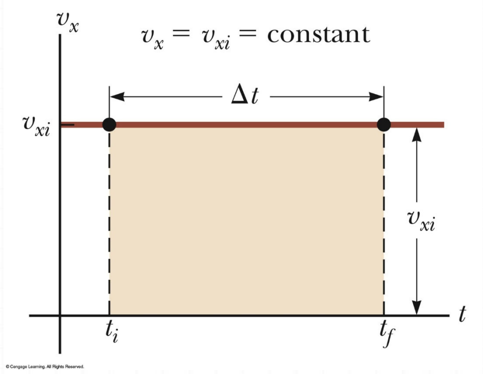


$$\Delta x = \bar{v}(t_f - t_i) = \lim_{\substack{\Delta t \to 0 \\ N \to \infty}} \sum_{n=1}^{N} \bar{v}_{xn} \Delta t = \text{area}$$

$$= \int_{t_i}^{t_f} v_x(t)dt = \int_{t_i}^{t_f} \frac{dx}{dt} dt = \int_{x_i}^{x_f} dx = x\Big|_{x_i}^{x_f} = x_f - x_i$$

$$v_x(t) = \frac{dx(t)}{dt} \quad \leftrightarrow \quad \Delta x = \int_{t_i}^{t_f} v_x(t)dt$$

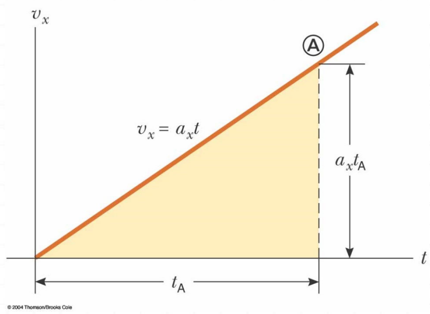
Ex.



$$\Delta x = v_{xi}\Delta t = \text{area}$$

$$= \int_{t_i}^{t_f} v_{xi}dt = v_{xi}t\Big|_{t_i}^{t_f} = v_{xi}t_f - v_{xi}t_i = v_{xi}\Delta t$$

Ex.



$$\Delta x = \text{area } = \frac{1}{2}t_A \cdot a_x t_A = \frac{1}{2}a_x t_A^2$$

$$= \int_{t_i}^{t_f} v_x dt = \int_{t_i}^{t_f} a_x t dt = a_x \int_{t_i}^{t_f} t dt = a_x \frac{1}{2}t^2\Big|_{t_i}^{t_f} = \frac{1}{2}a_x t_A^2$$

Kinematic Equations – General Calculus Form

$$(1)\ a_x = \frac{dv_x}{dt} \qquad \Rightarrow \int_0^t a_x dt = \int_{v_i}^{v_f} \frac{dv_x}{dt} dt$$

$$= a_x \int_0^t dt = \int_{v_{xi}}^{v_{xf}} dv_x$$

$$= a_x (t - 0) = v_{xf} - v_{xi} \qquad \Rightarrow v_{xf} = v_{xi} + a_x t$$

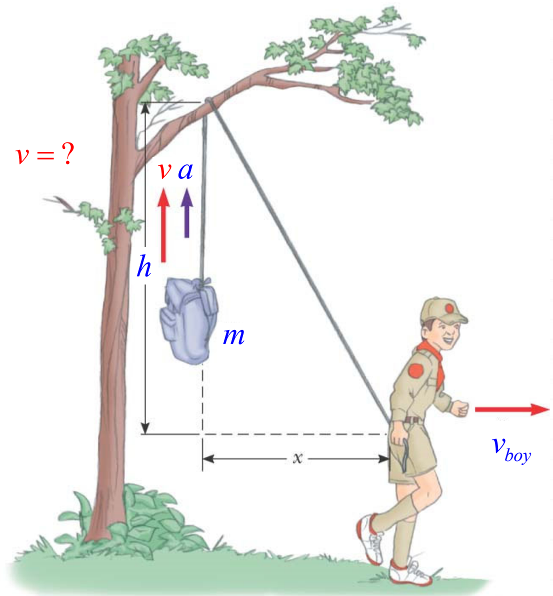
$$(2)\ v_x = \frac{dx}{dt} \qquad \Rightarrow \int_0^t v_x dt = \int_{x_i}^{x_f} \frac{dx}{dt} dt$$

$$= \int_0^t \left(v_{xi} + a_x t\right) dt = \int_{x_i}^{x_f} dx$$

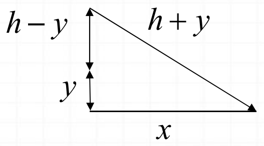
$$= \int_0^t v_{xi} dt + a_x \int_0^t t dt = x_f - x_i$$

$$= v_{xi}(t - 0) + a_x \frac{1}{2} t^2 \Big|_0^t = x_f - x_i \qquad \Rightarrow x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Ex.



Method#1



$$(h+y)^2 = x^2 + h^2$$

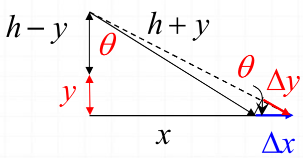
$$h + y = \sqrt{x^2 + h^2}$$

$$\Rightarrow y = \sqrt{x^2 + h^2} - h$$

$$v = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{x}{\sqrt{x^2 + h^2}} v_{\text{boy}}$$

v is dependent on x! ⇨ $a \neq 0$

Method#2



$$v = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \approx \lim_{\Delta t \to 0} \frac{\Delta x \sin\theta}{\Delta t}$$

$$= \sin\theta \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \frac{x}{\sqrt{x^2 + h^2}} v_{\text{boy}}$$

Application: 3D printer