CHAPTER 14

Fluid Mechanics

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Outline

1. Fluid statics
   - Pressure
   - Buoyant Forces and Archimedes's Principle
2. Fluid Dynamics
   - Bernoulli's Equation
   - Flow of viscous fluids in pipes
   - Applications of Fluid Dynamics
3. Surface tension

1. Fluid statics – Pressure

Fluid

- A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container.
- Both liquids and gases are fluids.
- Fluids do not sustain shearing stresses or tensile stresses.

Pressure

3 The force exerted by a static fluid on an object is always perpendicular to the surfaces of the object.

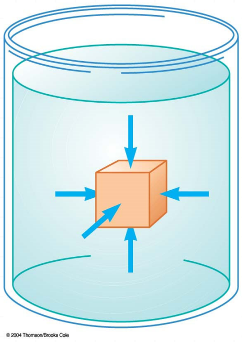
Pressure:

$$P = \frac{F}{A_\perp}, \text{ a scalar}$$

SI unit: 1 Pascal (Pa) ≡ 1 N/m^2

The atmospheric pressure

P_0 = 1.00 atm = 1.013 × 10^5 Pa = 1.013 bar

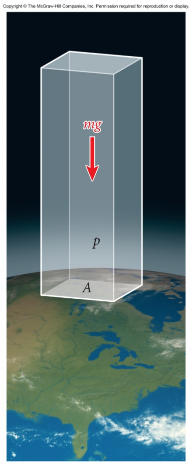


Q: How do you estimate the total mass of the air on the Earth? if the Earth radius is known.

$$F = PA$$

$$\Rightarrow \int \rho g dV = 1 \text{ atm} \times 4\pi R_E^2$$

$$\Rightarrow m = \int \rho dV \approx 1 \text{ atm} \times 4\pi R_E^2 / g$$



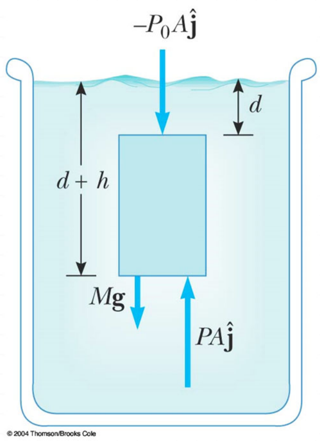
😎 The gas pressure in a closed container is dependent on temperature. However, the ambient pressure is mainly caused by gravity, in addition to temperature. If without gravity, the air will escape from the Earth.

Variation of Pressure with Depth

$$\sum \vec{F} = 0$$

$$PA - P_0 A - \rho A h g = 0$$

$$\Rightarrow P = P_0 + \rho g h$$

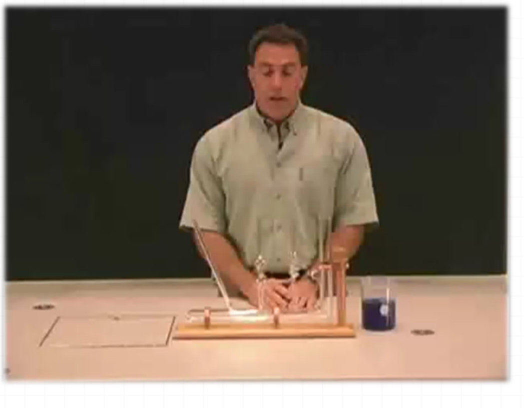


p.s., The pressure variation comes from gravitation.

Pascal's Law

3 The pressure in a fluid depends on depth and on the value of P_0.

3 An increase in pressure at the surface must be transmitted to every other point in the fluid.

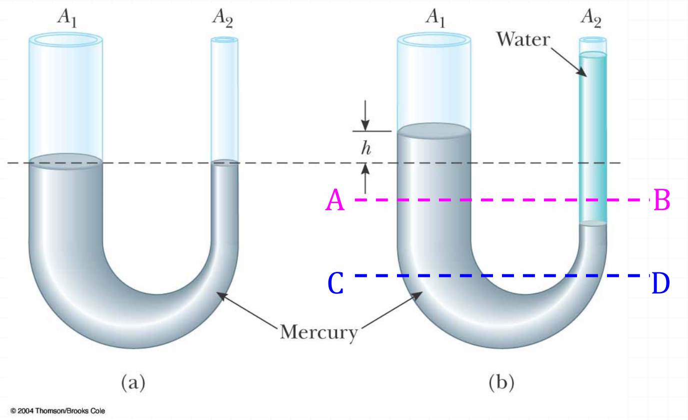


Pascal vase

Q: You have invented a spacesuit with a straw passing through the faceplate so that you can drink from a glass while on the surface of a planet. Out on the surface of the Moon, you attempt to drink through the straw from an open glass of water. The value of g on the Moon is about one sixth of that on Earth.

Compared to the difficulty in drinking through a straw on Earth, you find drinking through a straw on the Moon to be (a) easier (b) equally difficult (c) harder (d) impossible.

Q:

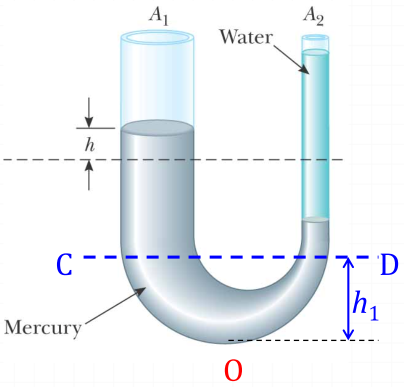


$>$ $\qquad$ $>$

$P_A = P_B$? $\qquad$ $P_C = P_D$?

$>$ $\qquad$ $>$

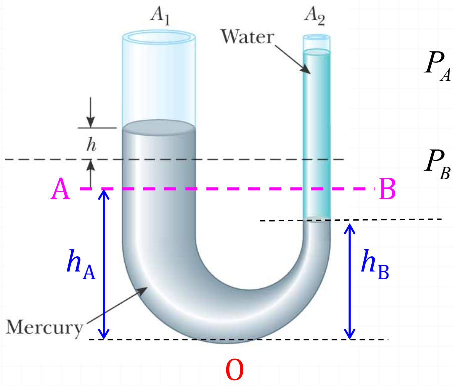
Ans:



$$P_C = P_0 - \rho_{Hg} g h_1$$

$$P_D = P_0 - \rho_{Hg} g h_1$$

$$\Rightarrow \quad P_C = P_D$$



$$P_A = P_0 - \rho_{Hg} g h_A$$

$$P_B = P_0 - \rho_{Hg} g h_B - \rho_{H_2O} g (h_A - h_B)$$

$$P_A - P_B = \rho_{Hg} g (h_B - h_A) + \rho_{H_2O} g (h_A - h_B)$$

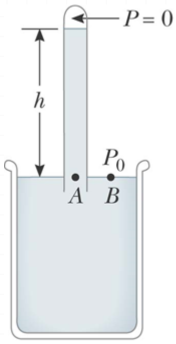
$$= (h_A - h_B) g (\rho_{H_2O} - \rho_{Hg}) < 0$$

$$\Rightarrow \quad P_A - P_B < 0$$

Pressure Measurements

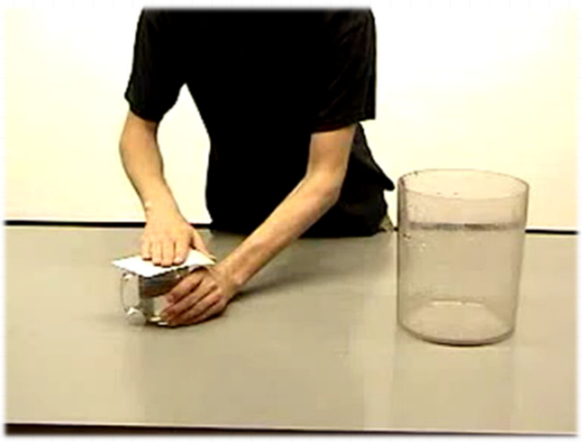
Barometer

- Invented by Torricelli
- 1 atm = 760 mm (of Hg)
- 1 mm Hg = 1 torr



$$P_0 = \rho_{Hg} g h$$

$$h = \frac{P_0}{\rho_{Hg} g} = \frac{1.013\times10^5\ \text{Pa}}{\left(13.6\times10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 0.76\ \text{m}$$



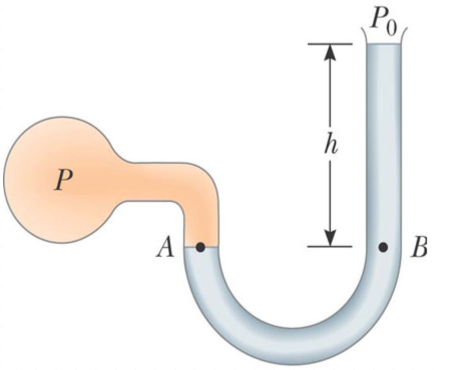
Manometer

3 Absolute pressure:

$$P = P_0 + \rho g h$$

3 Gauge pressure:

$$P - P_0 = \rho g h$$



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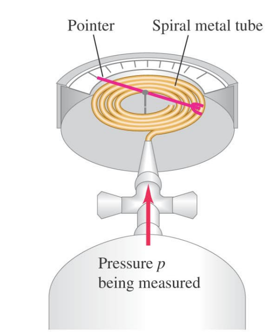
Bourdon gauge



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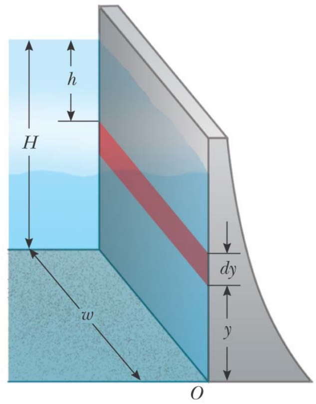
Ex. The force on a Dam

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.



Ans:

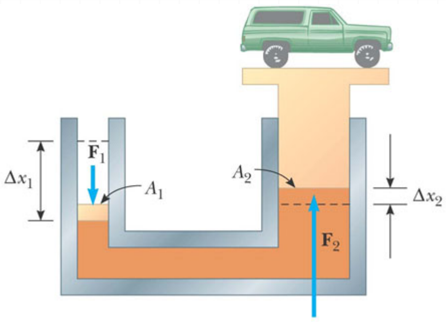
$$P = (P_0 + \rho g h) - P_0 = \rho g (H - y)$$

$$dF = P\,dA = \rho g (H - y) w\,dy$$

$$F = \int dF = \int_0^H \rho g (H - y) w\,dy$$

$$= \frac{1}{2} \rho g w H^2$$

A hydraulic press



(1) $P_1 = P_2$

(assuming A_1 and A_2 at same height)

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = \left(\frac{A_1}{A_2}\right) F_2$$

(2) $A_1 \Delta x_1 = A_2 \Delta x_2$

Work done by force:

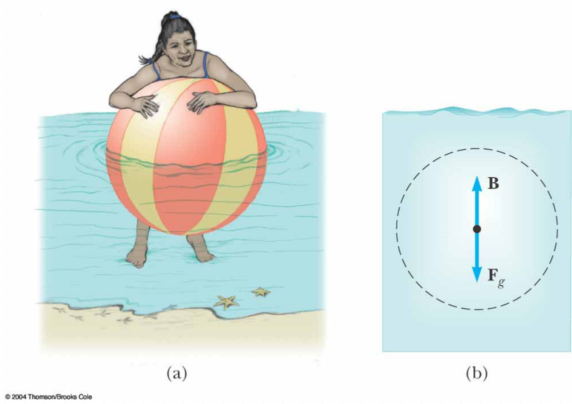
$$\Rightarrow F_1 \Delta x_1 = F_2 \Delta x_2$$

Energy is conserved.

1. Fluid statics

– Buoyant force & Archimedes's principle

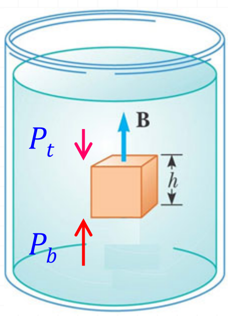
Buoyant Force



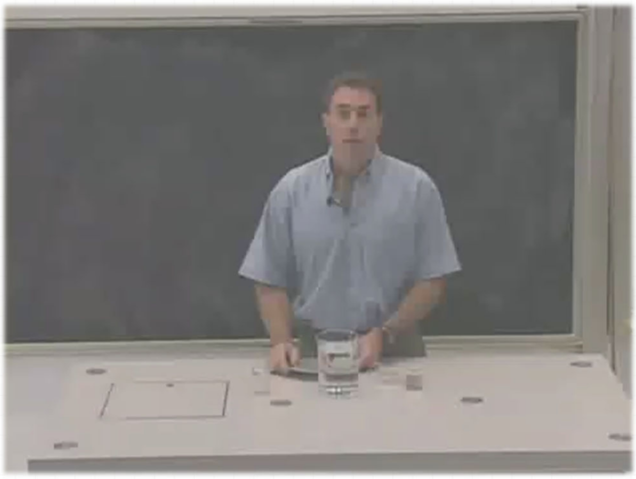
$$\vec{B} = \vec{F}_g$$

Archimedes's Principle

3 The magnitude of the buoyant force always equals the weight of the fluid displaced by the object, $B = M_{fluid}\, g$.

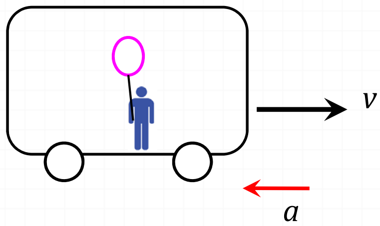


$$\begin{aligned} B &= (P_b - P_t) A \\ &= (\rho_{fluid}\, g h) A \\ &= \rho_{fluid}\, g V \\ &= M_{fluid}\, g \end{aligned}$$



Q: An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared to the force needed to hold the apple just below the surface, the force needed to hold it at a deeper point is (a) larger (b) the same (c) smaller (d) impossible to determine.

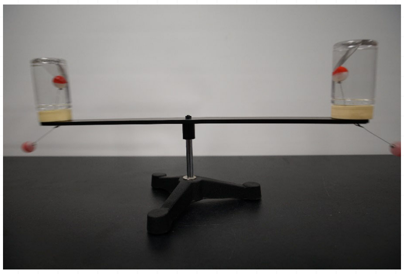
Q:



What will be the movement of the person and the balloon when the bus stops suddenly?

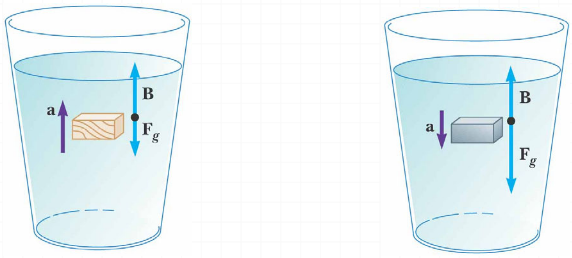
Person -> forward

Balloon -> backward



https://physics.nyu.edu/~physlab/Demos/updatedEquipment/kinematicsForcesEnergy/forceParadox.html

Totally submerged object



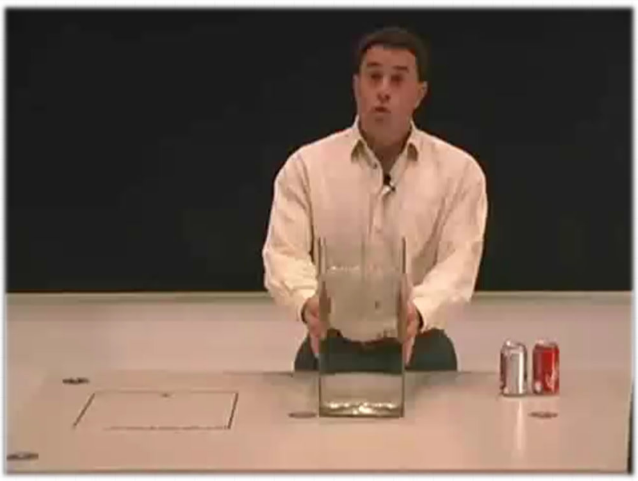
$$B = \rho_{\text{fluid}} g V = \rho_{\text{fluid}} g V_{\text{obj}}$$

$$F_g = mg = \rho_{\text{obj}} g V_{\text{obj}}$$

The net force:

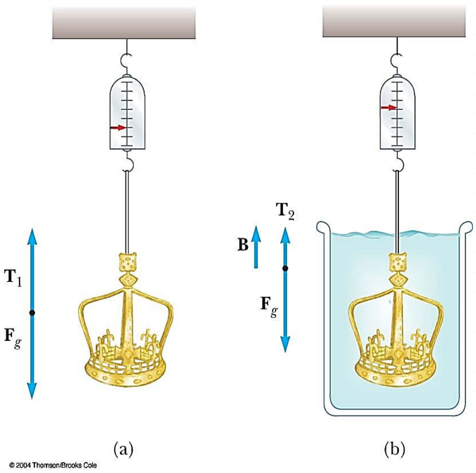
$$F_g - B = (\rho_{\text{obj}} - \rho_{\text{fluid}}) g V_{\text{obj}}$$

$\Rightarrow$ The motion of an object in a fluid is determined by the densities of the fluid and the object



Galileo thermometer

Ex.



$$B + T_2 - F_g = 0$$

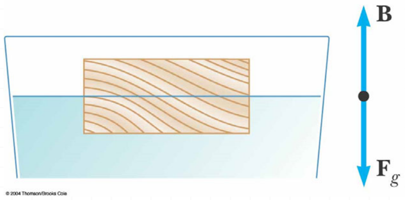
$$\rho_w g V_w + T_2 - \rho_c g V_c = 0$$

$$V_w = V_c$$

$$\Rightarrow \rho_c = \rho_w + \frac{T_2}{g V_w}$$

Use ρ_c to determine whether the crown is real or not.

Floating Object



V_{fluid} : object's volume in fluid

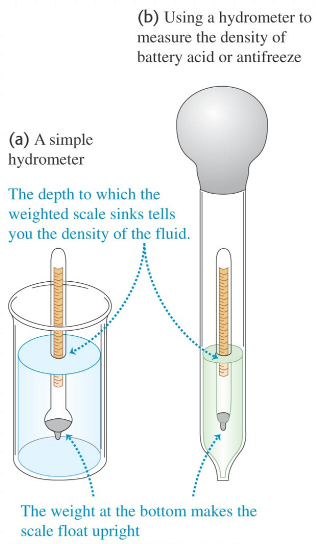
V_{obj} : object's volume

$$B = \rho_{\text{fluid}} g V_{\text{fluid}}$$

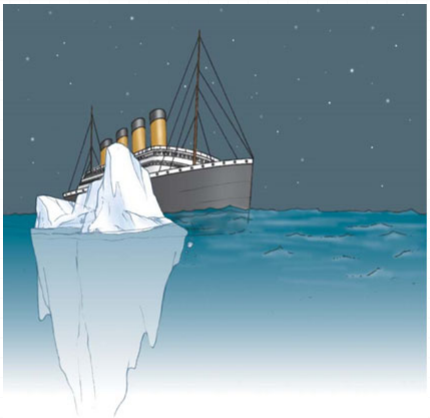
$$F_g = mg = \rho_{\text{obj}} g V_{\text{obj}}$$

$$B = F_g \quad \Rightarrow \quad \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{obj}}}$$

Hydrometer



Q:



$$\frac{V_{water}}{V_{ice}} = ?$$

$$\frac{V_{water}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{917\ \text{kg/m}^3}{1030\ \text{kg/m}^3} = 89\%$$

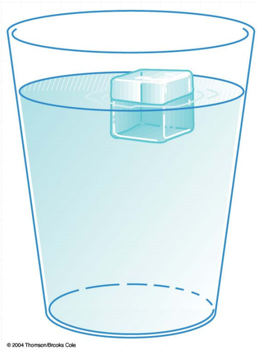
Q: A glass of water contains a single floating ice cube. When the ice melts, does the water level (a) go up (b) go down (c) remain the same?

$$m_{ice} = \rho_{water} V_{\text{melted water}}$$

$$F_g = B$$

$$\Rightarrow \quad V'_{\text{ice in water}} = \frac{m_{ice} g}{\rho_{water} g} = \frac{\rho_{water} V_{\text{melted water}}}{\rho_{water}}$$

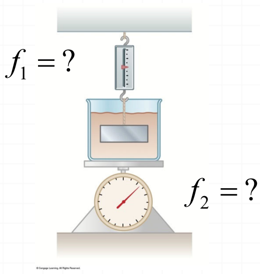
$$= V_{\text{melted water}}$$



😎 Why would the sea level rise if iceberg melts?

https://www.facebook.com/theactionlabofficial/

Ex.



ρ_0 : liquid density

ρ_b : block density

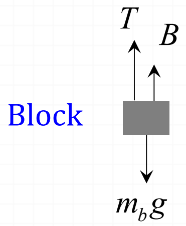
m_b : block mass

m_0 : liquid mass

Ans:

(1) $f_1 = ?$

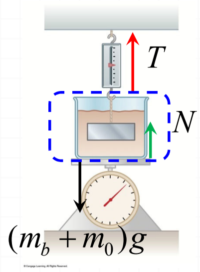
System = m_b



$$f_1 = T$$
$$= m_b g - B$$
$$= m_b g - \rho_0 V_b g$$
$$= m_b g - \rho_0 \frac{m_b}{\rho_b} g$$
$$\Rightarrow \quad f_1 = m_b g - \frac{\rho_0}{\rho_b} m_b g$$

Note that, $\rho_b = \rho_0 \quad \Rightarrow \quad f_1 = 0$

(2) $f_2 = ?$



System = $m_b + m_0$

Buoyant force is an internal force here.

$$T + N - m_b g - m_0 g = 0$$
$$\Rightarrow f_2 = N = m_b g + m_0 g - (1 - \frac{\rho_0}{\rho_b}) m_b g$$
$$= m_0 g + \underbrace{\frac{\rho_0}{\rho_b} m_b g}_{\text{Buoyant}}$$

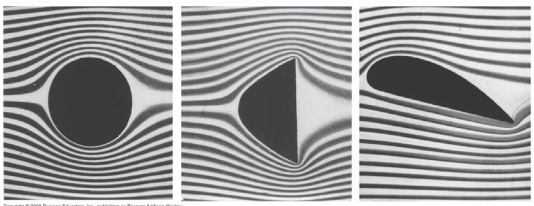
2. Fluid dynamics

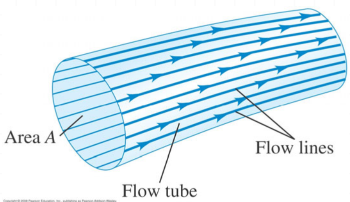
– Bernoullie's equation

Types of Fluid Flow

3 **Laminar flow**

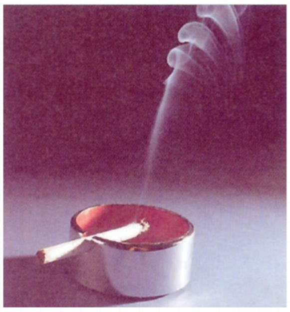
- Steady flow.
- In a smooth path called a streamline.
- The paths of the different particles never cross each other.





3 **Turbulent flow**

- An irregular flow characterized by small whirlpool like regions.
- Turbulent flow occurs when the particles go above some critical speed.



https://fb.watch/8RNkssEGun/

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Viscosity

- The degree of internal friction in the fluid.
- This internal friction, *viscous force*, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other.
- It causes part of the kinetic energy of a fluid to be converted to internal energy.

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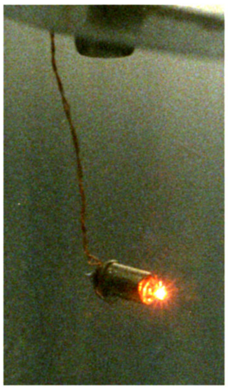
Superfluid - ^4He

Liquid helium enters superfluid phase transition at 2.17K.

Zero viscosity

https://www.youtube.com/watch?v=-7PNacL4n8g



Glowing filaments in liquid He. Left: normal liquid. Right: superfluid.

No bubbling in superfluid liquid helium!

(Prof. Kimitoshi Kono's lab)

Ideal Fluid Flow

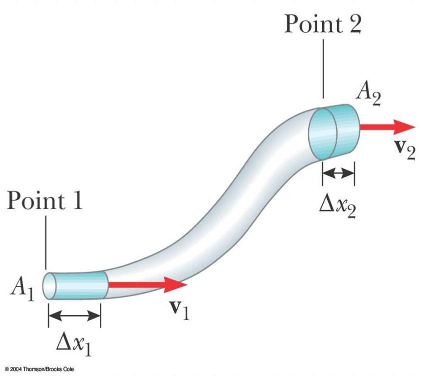
(1) *The fluid is nonviscous* – internal friction is neglected.

(2) *The flow is steady* – the velocity of each point remains constant.

(3) *The fluid is incompressible* – the density remains constant.

(4) *The flow is irrotational* – the fluid has no angular momentum about any point.

Equation of Continuity



Mass flow rate: $\dfrac{dm}{dt} = \dfrac{\rho A dx}{dt} = \rho A v$

Volume flow rate: $\dfrac{dV}{dt} = \dfrac{A dx}{dt} = A v$

(1) Mass conservation ⇒ $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

(2) Incompressibility ⇒ $\rho_1 = \rho_2 = \rho$

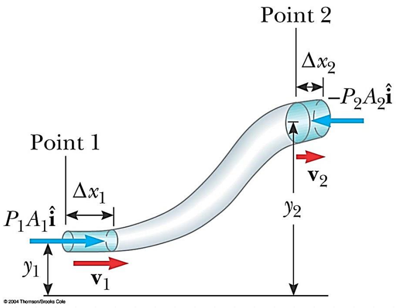
$$\Rightarrow \quad A_1 v_1 = A_2 v_2 = const.$$

Equation of continuity (incompressible fluid)

For compressible fluid, $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Why is the diameter of the water stream different?

Bernoulli's Equation



$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$$

$$W_2 = F_2\Delta x_2 = -P_2A_2\Delta x_2 = -P_2V$$

$$\Rightarrow \quad W = W_1 + W_2 = (P_1 - P_2)V$$

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\Delta U = mgy_2 - mgy_1$$

Mechanical energy conservation:

$$W = \Delta K + \Delta U$$

$$(P_1 - P_2)V = (\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2) + (mgy_2 - mgy_1)$$

$$\rho = m / V$$

$$\Rightarrow \quad P_1 - P_2 = (\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2) + (\rho g y_2 - \rho g y_1)$$

$$\Rightarrow \quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho g y = \text{const.}$

Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho g y = \text{const.}$

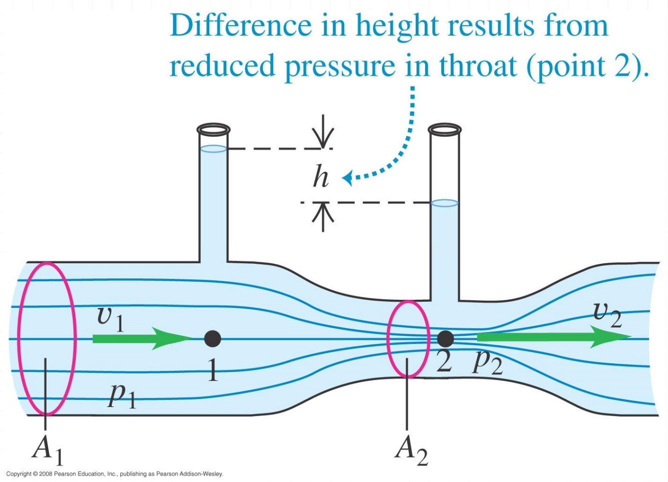
😎 It is a result of energy conservation.

😎 The constant depends on the initial conditions of P, ρ, and y.

Bernoulli's equation reduces to Pascal's law when $v = 0$.

The **Venturi tube** can be used to measure the flow speed of an incompressible fluid.

Q: Determine the flow speed at point 2 if the height h is known.



Ans:

Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho g y = \text{const.}$

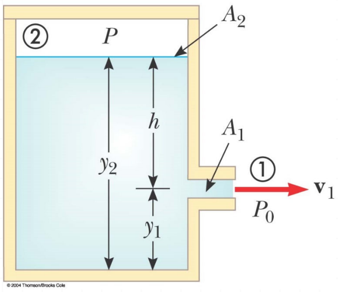
$$y_1 = y_2$$

Bernoulli's eq. $\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Continuity eq. $\Rightarrow A_1 v_1 = A_2 v_2$

$$\Rightarrow v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}} = A_1\sqrt{\frac{2\rho g h}{\rho(A_1^2 - A_2^2)}}$$

Ex. An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom. The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P. Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.



Ans:

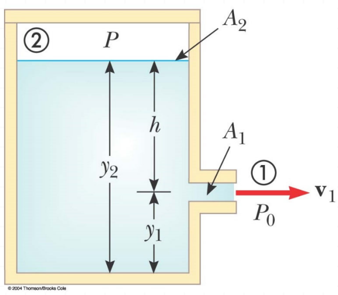
(1) $A_1 \ll A_2$ & $A_1 v_1 = A_2 v_2$

$\Rightarrow v_2 \ll v_1$

(2) $P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \cancel{\frac{1}{2}\rho v_2^2} + \rho g y_2$

$$y_2 - y_1 = h \quad \Rightarrow \quad v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

Torricelli's law: $P = P_0 \Rightarrow v_1 = \sqrt{2gh}$



😎 If the tank is open to the atmosphere and sitting on a table, what position of the hole would cause the water to land on the table at the farthest distance from the tank?

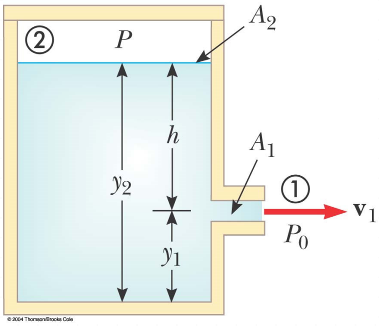
$$0 = y_1 + 0 - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2y_1}{g}}$$

$$x_f = x_i + v_{xi}t = 0 + \sqrt{2g(y_2 - y_1)}\sqrt{\frac{2y_1}{g}}$$

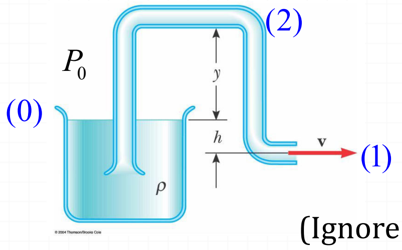
$$= 2\sqrt{(y_2 y_1 - y_1^2)}$$

$$\Rightarrow \quad \frac{dx_f}{dy_1} = \frac{1}{2}(2)(y_2 y_1 - y_1^2)^{-1/2}(y_2 - 2y_1) = 0$$

$$\Rightarrow \quad y_1 = \frac{1}{2}y_2$$



Ex.



$v = ?$

$y_{\max} = ?$

(Ignore the water vapor pressure)

$$(1)\ \ P_0 + 0 + 0 = P_0 - \rho g h + \frac{1}{2}\rho v^2 \quad \Rightarrow \quad v = \sqrt{2gh} \qquad \text{indep. of } y.$$

$$(2)\ \ P_0 + 0 + 0 = {\color{red}P} + \rho g y + \frac{1}{2}\rho v^2 \quad \Rightarrow \quad y = \frac{P_0 - P - \frac{1}{2}\rho v^2}{\rho g}$$

$$P \ge 0,\ v \ge 0 \quad \Rightarrow \quad y_{\max} \le \frac{P_0}{\rho g}$$

2. Fluid dynamics

– Flow of viscous fluids in pipes

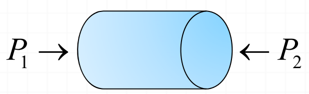
Viscosity

In reality, the viscosity of fluids is not negligible.

The pressure differential to keep fluid moving at a fixed speed in a horizontal pipe is

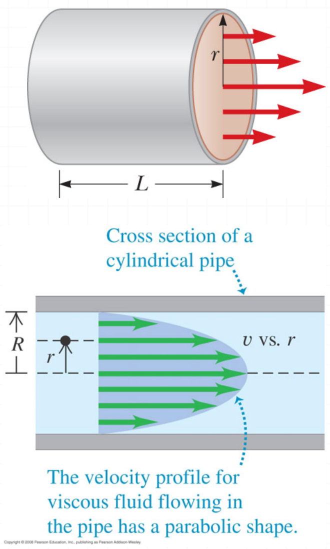
$$\Delta P = I_V R,$$

I_V : the volume flow rate ($= Av$)
R : the measure of resistance



$$\Delta P = P_1 - P_2 \neq 0$$

- It is a typical transport equation, similar to the electrical current flow $\Delta V = IR$, and the diffusion due to concentration difference.



The resistance R can be shown as

$$R = \frac{8\eta L}{\pi r^4} \qquad \eta : \text{viscosity (unit: poise)}$$

$$\Delta P = I_V R, \quad \Rightarrow \quad \Delta P = \frac{8\eta L}{\pi r^4} I_V$$

$$\Rightarrow \quad \Delta P \propto \frac{1}{r^4} \quad !$$

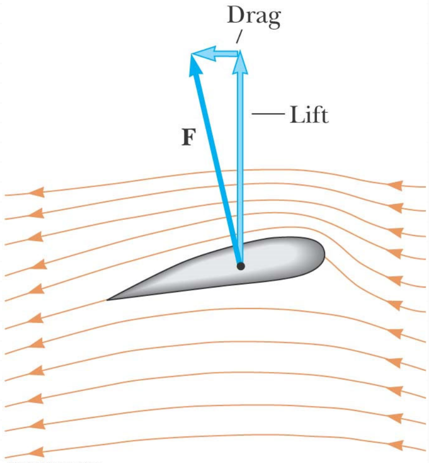
1. Fluid statics
   - Pressure
   - Buoyant Forces and Archimedes's Principle
2. Fluid Dynamics
   - Bernoulli's Equation
   - Flow of viscous fluids in pipes
   - Applications of Fluid Dynamics
3. Surface tension

2. Fluid dynamics

– Applications of fluid dynamics

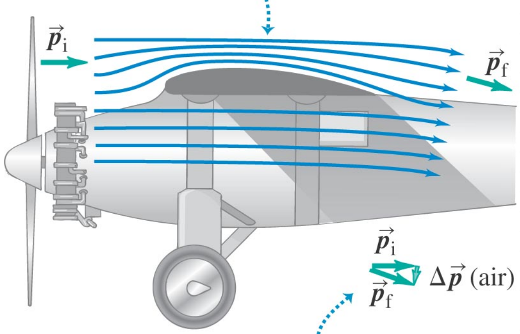
Applications of Fluid Dynamics

- Streamline flow around a moving airplane wing.
- *Lift* is the upward force on the wing from the air.
- *Drag* is the resistance.
- The lift depends on the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal.



(a) Flow lines around an airplane wing

The flowlines of air moving over the top of the wing are crowded together, so the flow speed is higher and the pressure consequently lower.



An equivalent explanation: The wing's shape imparts a net downward momentum to the air, so the reaction force on the airplane is upward.

(b) Computer simulation of airflow around an airplane wing

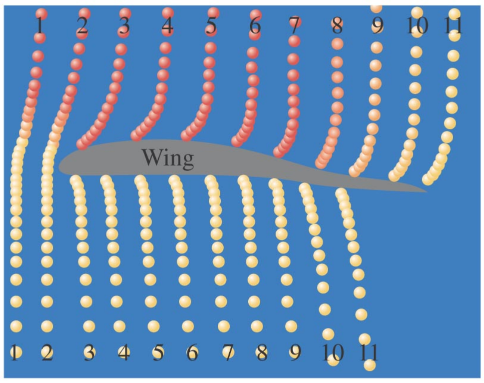
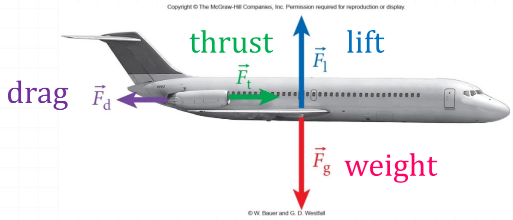
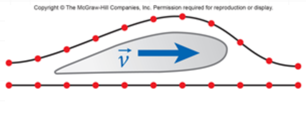


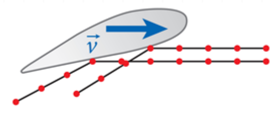
Image of air parcels flowing around a wing, showing that the air goes much faster over the top than over the bottom (and that air parcels which are together at the leading edge of the wing do *not* meet up at the trailing edge!)





(a)

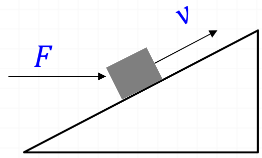
Bernoulli’s effect

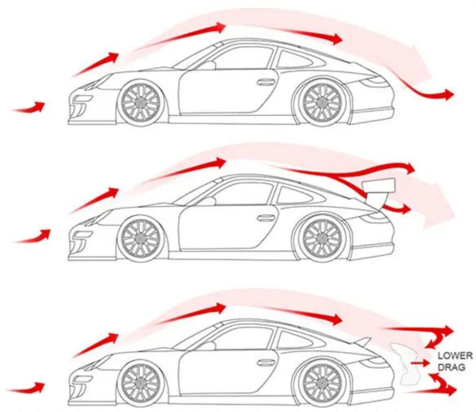


(b)

Collision effect

⇒ Similar to





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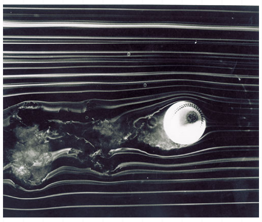
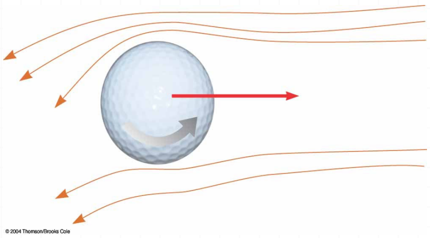
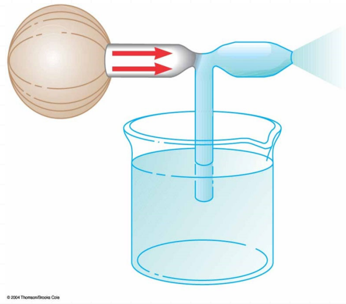




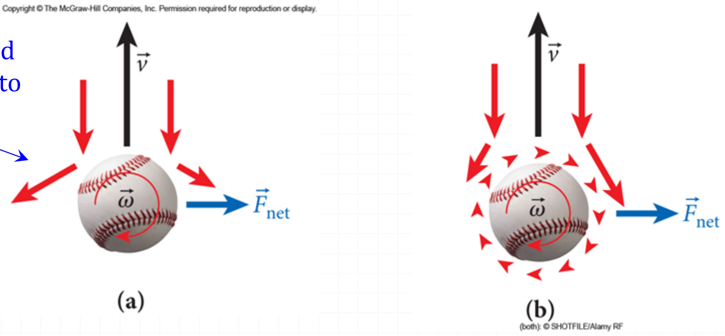


Golf ball

Atomizer

A clockwise spinning ball experiences a clockwise deflection.

Air speed relative to the ball



Collision effect

Magnus effect

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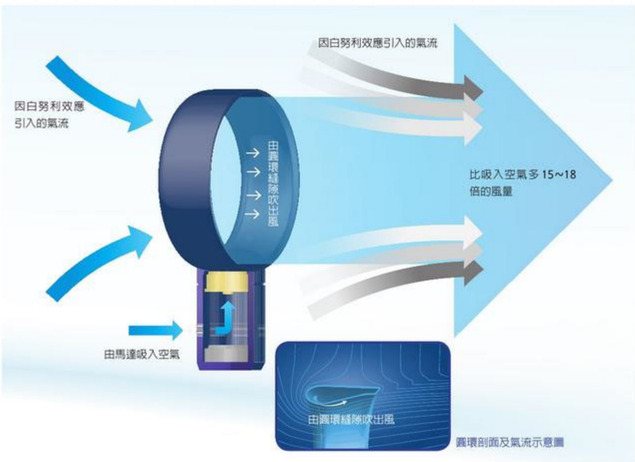

https://kknews.cc/news/qjmnklr.html

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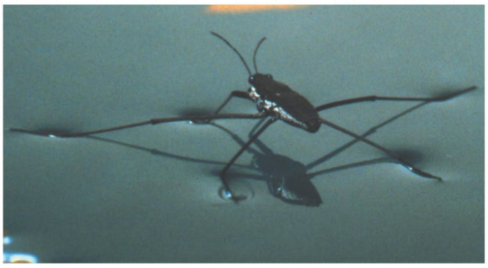
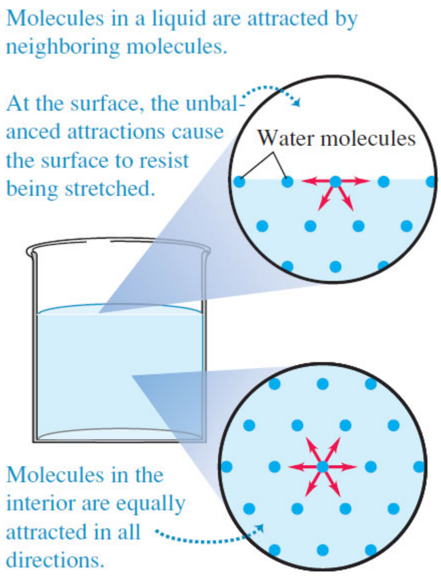
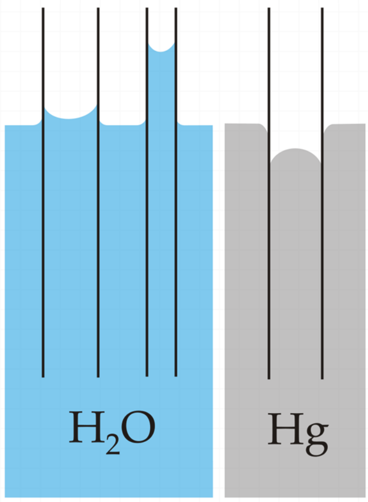
3. Surface tension

Surface tension:

- A molecule at the surface of a liquid is attracted into the bulk liquid, which tends to reduce the liquid's surface area.
- Surface tension can be changed with temperature or by adding certain chemical into the liquid.



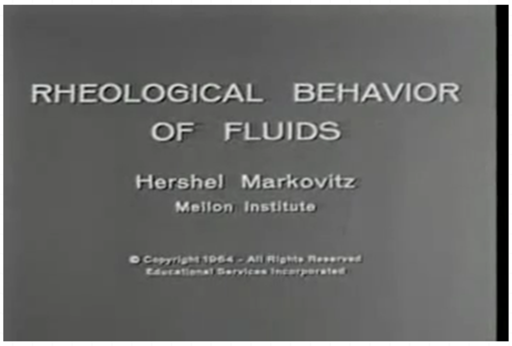




Capillary force vs. surface tension

https://www.youtube.com/watch?v=Ol6bBB3zuGc

Rheological Behavior of Fluids



Fun with Non-Newtonian Fluid

https://www.youtube.com/watch?v=RIUEZ3AhrVE