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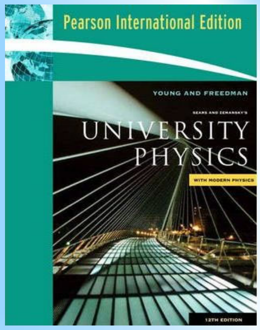
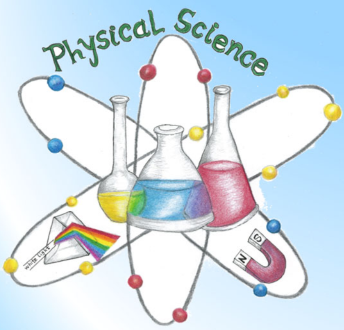


Chapter 1



Units, Physical Quantities, and Vectors

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Outline

1. Introduction to Physics
2. Quantities and Measurement in Physics
   - Quantities, Units, Dimensional analysis
   - Number notation
   - Estimates and Order-of-magnitude calculations
   - Uncertainty, significant figures
3. Model Building
4. Vectors

1. Introduction to Physics

What is Physics for?

- To find the limited number of **fundamental laws** that govern natural phenomena.
  - "Everything should be made as simple as possible, but not simpler." --- Albert Einstein

  Unified field theory?

- To use these laws to develop **theories** that can predict the results of future experiments.

- Express the laws in the language of **mathematics.**

☞ The goal of Physics is to find the mechanisms and equations that can describe the Nature.

For example,

Classical mechanics: $\vec{F} = m\vec{a}$

Thermodynamics: e.g., $PV = nRT$ (state of equation)

Electrodynamics: Maxwell's equations

Quantum mechanics: $H\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ (Schrödinger eq.)

What does Physics have?

- Classical physics (before 1900)
- Modern physics (after 1900)

Classical Physics

- **Classical Mechanics**
  - Major developments by Newton, and continuing through the latter part of the 19^{th} century.
- **Thermodynamics**
- **Optics**
- **Electromagnetism**

All of these were not developed well until the latter part of the 19^{th} century.

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Modern Physics

- Began near the end of the 19^{th} century
- Phenomena that could not be explained by classical physics
- Includes theories of relativity and quantum mechanics

Range of validity!

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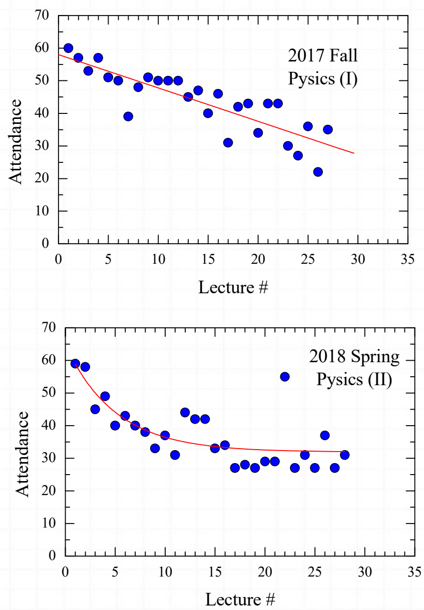
Evolution of Science

- Experimental observation → explained by theory
- Theoretical predication → verified by experiment
- Some statistical phenomena (經驗法則), no theory to predict or explain. (e.g., 80/20 rule)

😎 **Physics is an experimental science.**



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2. Quantities and Measurement in Physics

Physical Quantities

- In **mechanics**, three *fundamental quantities* are used
  - Length
  - Mass
  - Time
- Will also use *derived quantities*
  - These are other quantities can be expressed in terms of these.

    e.g., velocity=length/time

Standards of Quantities

- "Standard" → Universal, reproducible, not changing with time.
- SI – Systéme International (metric system)

Base Units

Table 1.1 Unit Names and Abbreviations for the Base Units of the SI System of Units

Unit	Abbreviation	Base Unit for
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	current
kelvin	K	temperature
mole	mol	amount of a substance
candela	cd	luminous intensity

😎 Charge is not a base unit!

(Bauer/Westfall, University Physics with Modern Physics)

Table 1.2 Common SI Derived Units

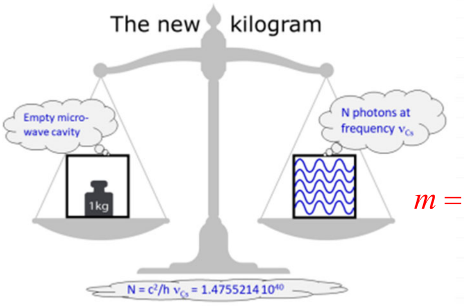
Derived or Dimensionless Unit	Name	Symbol	Equivalent	Expressions
Absorbed dose	gray	Gy	J/kg	$m^2\ s^{-2}$
Angle	radian	rad	—	—
Capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
Catalytic activity	katal	kat	—	s^{-1} mol
Dose equivalent	sievert	Sv	J/kg	$m^2\ s^{-2}$
Electric charge	coulomb	C	—	s A
Electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
Electric potential	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
Electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
Energy	joule	J	N m	$m^2\ kg\ s^{-2}$
Force	newton	N	—	$m\ kg\ s^{-2}$
Frequency	hertz	Hz	—	s^{-1}
Illuminance	lux	lx	lm/m^2	m^{-2} cd
Inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Luminous flux	lumen	lm	cd sr	cd
Magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}\ A^{-1}$
Magnetic field	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
Power	watt	W	J/s	$m^2\ kg\ s^{-3}$
Pressure	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
Radioactivity	becquerel	Bq	—	s^{-1}
Solid angle	steradian	sr	—	—
Temperature	degree Celsius	°C	—	K

New Definition of kg

"Guide for teachers and students to explain the new kg"
Wolfgang Ketterle, Alan Jamison, MIT

https://www.rle.mit.edu/cua_pub/ketterle_group/The%20new%20kg/Kilogram%20for%20students%20and%20teachers.pdf

Old kg
(Pt-Ir alloy)



$$m = \frac{h\nu_{Cs}}{c^2}$$

Conversion of Units

- Units can be treated like algebraic quantities that can cancel each other out.

  e.g.,

  1 inch = 2.54 cm

  15.0 in = (15.0 in)(2.54 cm/in) = 38.1 cm

Dimensional Analysis

- Dimension specifies the physical nature of a quantity.
  - Length [L], Mass [M], Time [T]
  - Can be treated as algebraic quantities.
- Dimensional analysis is to check the correctness of an equation or to assist in deriving an equation.
  - Both sides of equation must have the same dimensions.
  - E.g., $S = \frac{1}{2} a t^n, \quad n = ?$

$$[L] = [L/T^2][T^n] \qquad \Rightarrow n = 2$$

Number Notation

- When writing out numbers with many digits, spacing in groups of three will be used.
  - No commas,

    e.g.,

    25 100

    5.123 456 789 12

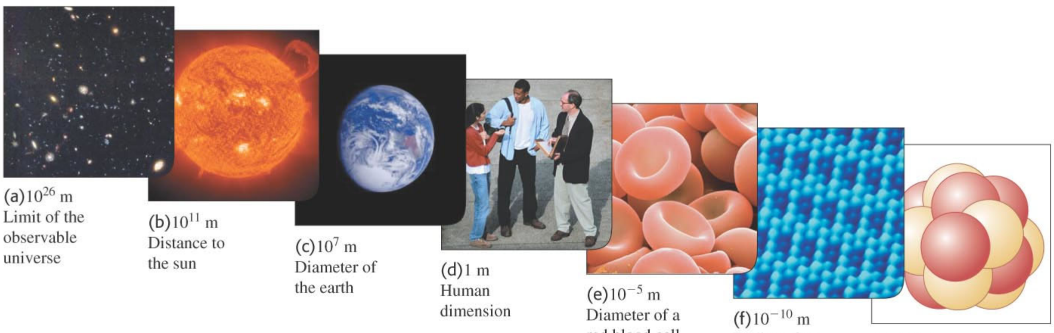
Powers of ten

- Examples:
  - 1 mm = 10^{-3} m
  - 1 mg = 10^{-3} g

Scientific notation : $M \times 10^n$

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y



(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

(d) 1 m Human dimension

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

Estimates and Order-of-Magnitude Calculations

- Approximation based on a number of assumptions.
- Expressed in scientific notation : M x 10^n
- Order of magnitude is the power of 10 that applies

$$M < \sqrt{10} \approx 3.162 \quad \Rightarrow \quad \text{set } M \text{ to } 1$$
$$M > 3.162 \quad \Rightarrow \quad \text{set } M \text{ to } 10$$

e.g.,

$$0.0086 = 8.6\times10^{-3} \sim 10^{-2}$$
$$0.0021 = 2.1\times10^{-3} \sim 10^{-3}$$
$$720 = 7.2\times10^{2} \sim 10^{3}$$

Back-of-the-envelope calculation!

Ex. Estimate the number of breaths in a human lifetime

Number of breaths per minute: ~ 10 breaths/min

Lifetime: ~ 70 yr

$$= 70 \text{ yr} \, \frac{\overset{400}{\cancel{365}} \text{ days}}{\text{yr}} \frac{\overset{25}{\cancel{24}} \text{ hr}}{\text{day}} \frac{60 \text{ min}}{\text{hr}}$$

$$\approx (70 \text{ yr})(6\times10^{5} \text{ min/yr})$$

$$\approx 4\times10^{7} \text{ min}$$

Total breaths: ~ (10 breaths/min)($4\text{x}10^7$ min)=$4\text{x}10^8$ breaths

Ex. Estimate the number of Stars in the Milky Way ?

Facts:

1. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m .
2. The distance between Sun and nearest Star is 4 x 10^{16} m.

Assume 4 x 10^{16} m is a typical distance between stars.

Galaxy volume:

$$\pi \times \left(10^{21}\,\mathrm{m}\right)^2 \times 10^{19}\,\mathrm{m} \sim 10^{61}\,\mathrm{m}^3$$

Volume for finding a star:

$$\frac{4}{3}\pi\left(\frac{4}{2}\times 10^{16}\,\mathrm{m}\right)^3 \sim 10^{49}\,\mathrm{m}^3$$

$$\Rightarrow \text{Number of stars: } 10^{61}/10^{49} = 10^{12}$$

Uncertainty (error) in Measurements

- There is uncertainty in every measurement - this uncertainty carries over through the calculations.
- Absolute uncertainty

$$\text{e.g., } \left(5.5 \pm 0.1\right)\text{cm or } 5.5(1)\text{ cm}$$

- Percent uncertainty

$$\text{e.g., } \left(5.5 \pm \frac{0.1}{5.5}\right)\text{cm} = (5.5 \pm 0.18\%)\text{cm}$$

- **Propagation of Uncertainty** (See Serway, Appendices-20)

Significant Figures

- A significant figure is one that is reliably known in a measurement.
- The significant figures include the first estimated digit.
- Zeros may or may not be significant.
  - Those used to position the decimal point are not significant
  - To remove ambiguity, use scientific notation.

Significant Figures, examples

- **"0.0075 m"** has 2 significant figures.
  - The leading zeros are placeholders only.
  - Can write in scientific notation 7.5×10^{-3} m for 2 significant figures.
- **"10.0 m"** has 3 significant figures.
  - The decimal point gives information about the reliability of the measurement.
- **"1500 m"** is ambiguous.
  - Use 1.5×10^3 m for 2 significant figures.
  - Use 1.50×10^3 m for 3 significant figures.
  - Use 1.500×10^3 m for 4 significant figures.

Operations with Significant Figures – Adding or Subtracting

- When adding or subtracting, the number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.

  - Example: $13\underline{5}$ cm + $3.2\underline{5}$ cm = $13\underline{8}$ cm

Operations with Significant Figures – Multiplying or Dividing

- When multiplying or dividing, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures.

  - Example:

$$\underset{(4)}{25.5\underline{7}}\text{ m} \times \underset{(3)}{2.4\underline{5}}\text{ m} = (62.\underline{6465})\text{m}^2 = \underset{(3)}{62.\underline{6}}\text{ m}^2$$

$$\underset{(2)}{5.\underline{1}} \times \underset{(1)}{\underline{2}} \times \underset{(2)}{9.\underline{3}} = (\underline{94.86}) = \underset{(1)}{\underline{9}} \times 10^1$$

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3. Model building

Model Building

- If physicists cannot interact with some phenomenon directly, they often imagine a model for a physical system that is related to the phenomenon, and make predictions about the behavior of the system.
  - Geometric model (e.g., geometric construction of star's size and distance)
  - Simplification model (e.g., uniform, isotropic, symmetry, resistance-free, ...)
  - Analysis model (e.g., <u>particle</u>, system, rigid object, <u>wave</u>)
  - Structural model (e.g., atomic model, standard model, Drude model)

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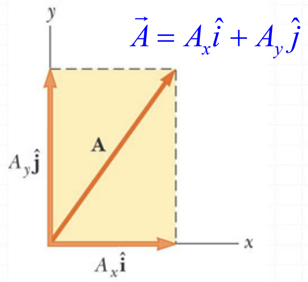
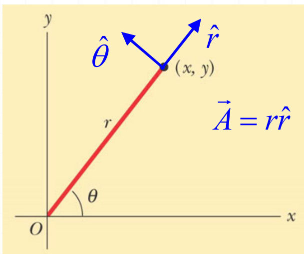
Coordinate Systems

Coordinates	Expressions	Unit vectors
Cartesian (rectangular)	(x, y, z)	$\hat{i}, \hat{j}, \hat{k}$
Polar	(r, θ)	$\hat{r}, \hat{\theta}$
Cylindrical	(r, θ, z)	$\hat{r}, \hat{\theta}, \hat{z}$
Spherical	(r, ϕ, θ)	$\hat{r}, \hat{\phi}, \hat{\theta}$



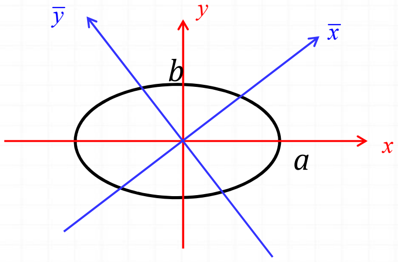
Rectangular coordinate



Polar coordinate

Proper choice of the coordinates can simplify the calculation.

Ex.



$$p\bar{x}^2 + q\bar{y}^2 + r\overline{xy} = 1 \quad \Leftrightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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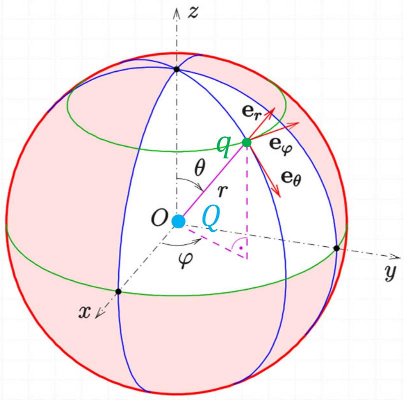
Ex.



$$\vec{F}_{\text{on } q}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(x^2+y^2+z^2)}\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}} \quad \text{in Cartesian coordinate}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r} \quad \text{in spherical coordinate}$$

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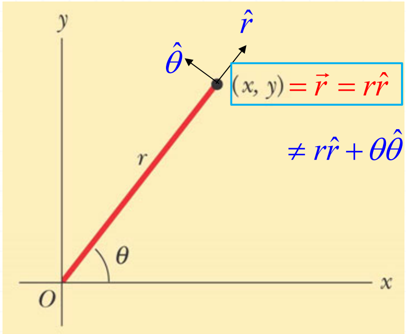
Transformation between polar and Cartesian coordinates

$$\begin{cases} x \\ y \end{cases} \Leftrightarrow \begin{cases} r \\ \theta \end{cases}$$

$x = r\cos\theta$
$y = r\sin\theta$

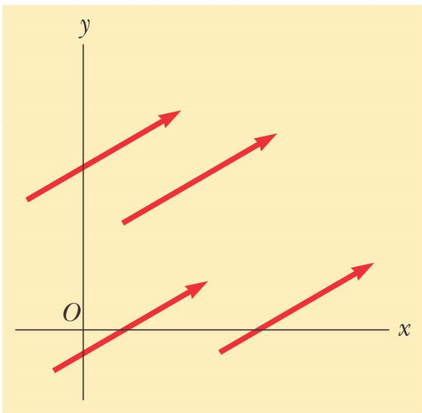
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



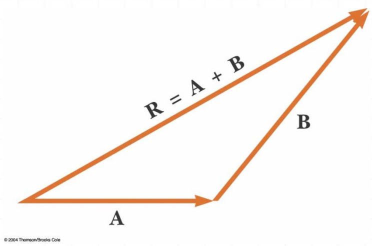
Equality of Two Vectors

- Two vectors are *equal* if they have the same magnitude and the same direction.
- A = B if *A* = *B* and they point along parallel lines.
- All of the vectors shown are equal.

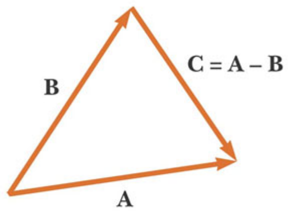


Operations on Vectors

Vector addition

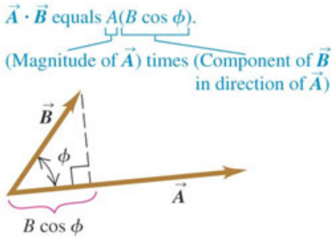


Vector subtraction



Scalar product

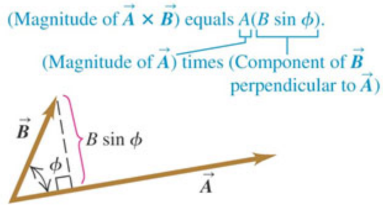
$$\vec{A}\cdot\vec{B} = AB\cos\phi$$
$$= A_xB_x + A_yB_y + A_zB_z$$



Vector product

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$$



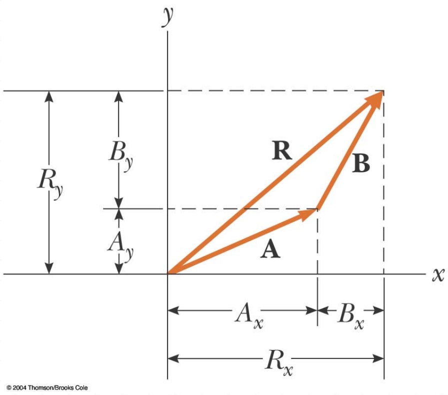
Solving vector problems

- **Graphical method**

  e.g., scale drawings

- **Algebraic Method**

  e.g., Trigonometry



$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$= R\cos\theta\hat{i} + R\sin\theta\hat{j}$$

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$\tan\theta = \frac{A_y + B_y}{A_x + B_x}$$