國立交通大學 電子物理系 NCTU Electrophysics

# CHAPTER 6

## WORK AND KINETIC ENERGY

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## Outline

1. Systems and Environments
2. Work
3. Kinetic Energy
4. Work-Kinetic Energy Theorem
5. Power

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## 1. Systems and environments



## System and Environment

- A *system* is a small portion of the Universe. A valid system may
  - be a single object or particle;
  - be a collection of objects or particles;
  - be a region of space;
  - vary in size and shape.
- The *environment* is the rest of the Universe.

$$\text{The Universe} = \underbrace{\text{system} + \text{environment}}_{\substack{\text{may interact via} \\ \text{force \& energy transfer etc.}}}$$

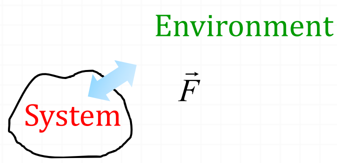


## Introduction to Energy

- Why can a car move by gasoline without any external driving forces?
- Change of the state variable $v$,

$$\vec{v}_f - \vec{v}_i = \vec{a}\Delta t = \frac{\vec{F}}{m}\Delta t$$

$$v_f^2 - v_i^2 = 2\vec{a}\cdot\Delta\vec{r} = 2\frac{\vec{F}}{m}\cdot\Delta\vec{r}$$




- Define new **state variables**:

$$\text{Momentum } \Delta\vec{P} \equiv m\Delta\vec{v} = \vec{F}\Delta t = \text{ Impulse}$$

$$\text{Kinetc Energy } \Delta K \equiv \frac{1}{2}m\Delta v^2 = \vec{F}\cdot\Delta\vec{r} \equiv \text{ Work}$$

- **Energy** is a **state variable** of a system, an abstract concept compared to Newton's Laws.
  - the ability to do "work".
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- Useful to analyze the problems where the force is not constant or well-defined.



## Why Energy?

- No need to know the details of the forces.
- Energy is a scalar; force is a vector.
- A more profound physics can be understood by the energy approach.
  - Lagrangian Mechanics: $L \equiv K - U$. $K$: kinetic energy
  - Hamiltonian Mechanics: $H \equiv K + U$. $U$: potential energy

(http://en.wikipedia.org/wiki/Lagrangian)

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# 2. Work

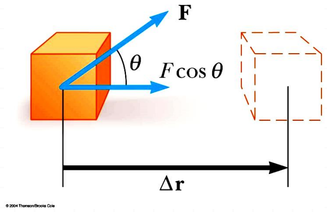


# Definition of Work

- "Work" is an energy transfer.
  - $W$ is "+" : energy is transferred to system
  - $W$ is "–" : energy is transferred from system
- Usually expressed as "Work is done by … (environment) on … (system)"

Work done by a "constant" force:

$$W \equiv \vec{F} \cdot \Delta\vec{r}$$




$\Delta\vec{r}$ : displacement for point where the force applied

SI Unit: newton-meter (N-m) = joule (J)



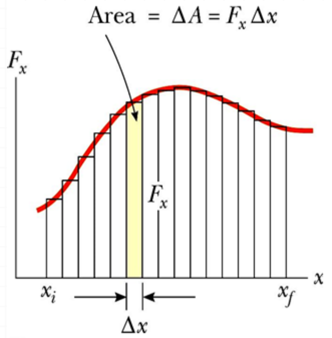
Work done by a varying force

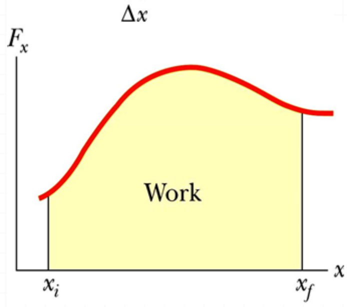
$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

$$\Rightarrow \quad W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

$$W = \int_{x_i}^{x_f} F_x dx \quad \text{(1-D)}$$

$$= \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} \quad \text{(3-D)}$$



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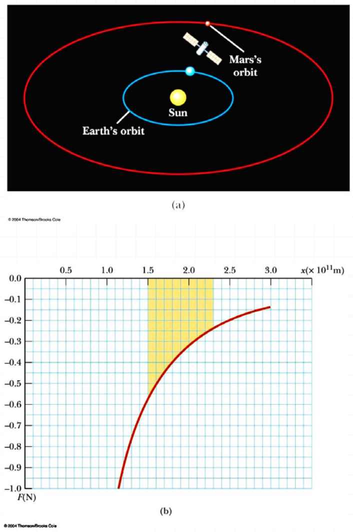



Gravitational force:

$$\vec{F}(x) = -\frac{c}{x^2}\hat{x}, \quad c: \text{ constant}$$

Work done by Sun for moving an object from $x = a$ to $x = b$

$$W = \int_a^b F(x)dx$$

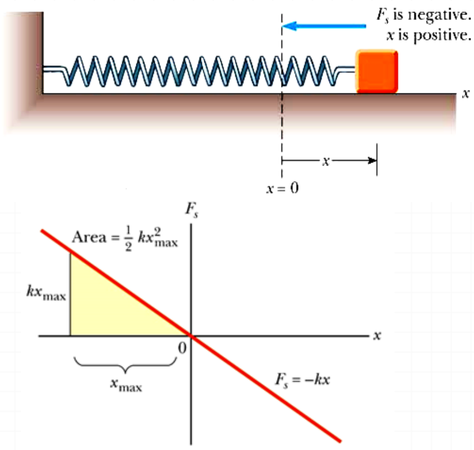
$$= -\int_a^b \frac{c}{x^2}dx$$

$$= \frac{c}{x}\bigg|_a^b$$

$$= \left(\frac{1}{b} - \frac{1}{a}\right)c$$



**Ex:**



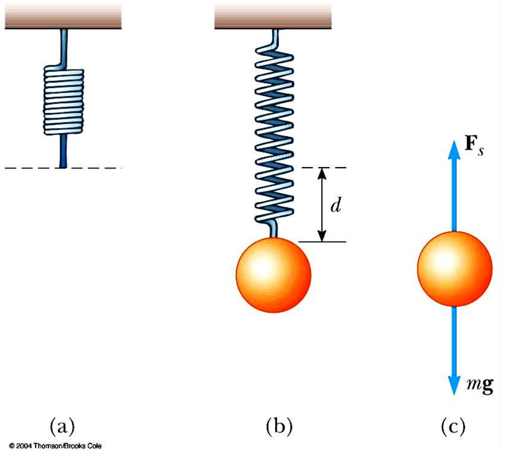

**Hook's law:**

$$\vec{F}_s = -kx\hat{i}$$

- Force done by the spring **on** the block.
- $k$ is spring constant
- "$x$" is relative to equilibrium $x = 0$ position

Work done **by the spring**: $W_s = \int_{x_i}^{x_f}(-kx)dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

**Ex:**




$$k = \frac{mg}{d}$$

$$W_s = 0 - \frac{1}{2}kd^2 = -\frac{1}{2}kd^2$$



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## 3. Kinetic energy

# Kinetic Energy

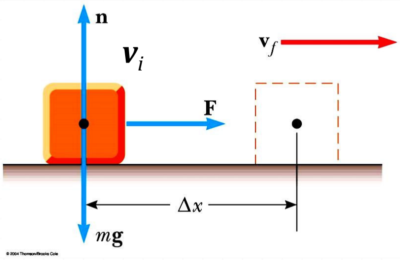
Work done on a system:

$$W = \int_{x_i}^{x_f} \left(\sum F_x\right) dx = \int_{x_i}^{x_f} (ma_x)\, dx$$

$$= \int_{x_i}^{x_f} \left(m\frac{dv}{dt}\right) dx = \int_{x_i}^{x_f} \left(m\frac{dv}{dx}\frac{dx}{dt}\right) dx$$

$$= \int_{v_i}^{v_f} (mv)\, dv$$

$$\Rightarrow \quad W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$




Kinetic energy: $K \equiv \frac{1}{2}mv^2$ depends on inertial reference frame!



For one-object system:

$$v_f^2 = v_i^2 + 2\vec{a}\cdot\Delta\vec{r}$$

$$\overset{\times m}{\Rightarrow} \quad mv_f^2 = mv_i^2 + 2\underbrace{m\vec{a}}_{\vec{F}}\cdot\Delta\vec{r}$$

$$\Rightarrow \quad \vec{F}\cdot\Delta\vec{r} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

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# 4. Work-kinetic energy theorem



## Work-Kinetic Energy Theorem

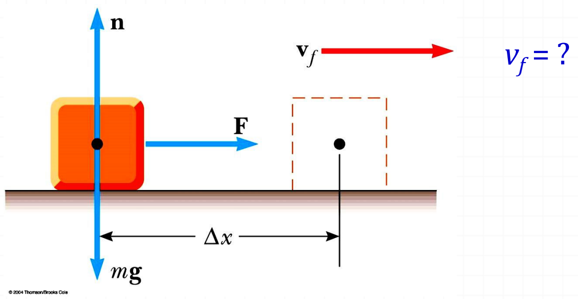
- When work is done on a system and if the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system. (Also valid for rotational motion)

$$W_{\text{external}} = K_f - K_i = \Delta K$$

😎

1. Always valid for a "one-particle" system, but not guaranteed for a system containing more than one particle.
2. Valid only in any inertial reference frame.

**Ex:**



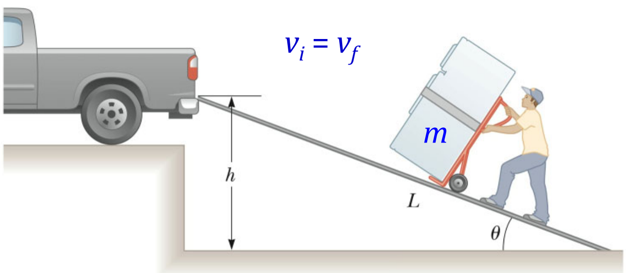

System = $m$

$$W = F\Delta x$$

$$W = \Delta K = \frac{1}{2}mv_f^2 - 0 \qquad \Rightarrow v_f = \sqrt{\frac{2F\Delta x}{m}}$$



**Ex:**




System = $m$

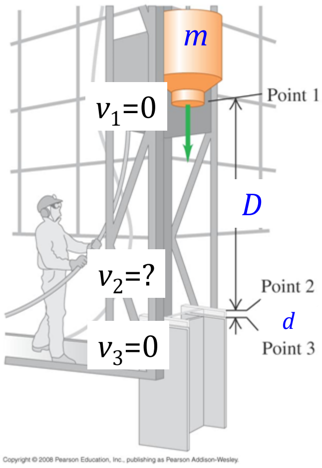
$$v_i = v_f \qquad \Rightarrow \quad \Delta K = 0 = W = W_{\text{man}} + W_{\text{gravity}}$$

$$W_{\text{gravity}} = \int_0^h \left(-mg\hat{j}\right)\cdot(dx\hat{i} + dy\hat{j}) = -mgh$$

$$W_{\text{man}} = mg\sin\theta L = mgh, \text{ indep. of } L$$

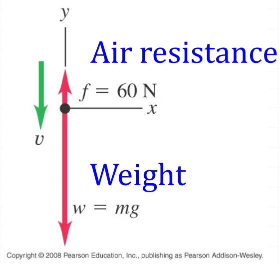
$\Rightarrow$ Less force, but same work.

(a)




$v_2 = ?$ & $n = ?$ at point 2.

(b) Free-body diagram for falling hammerhead

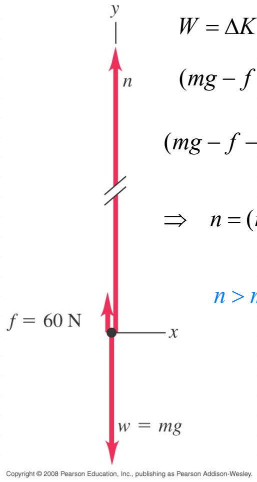



$W = \Delta K$

$(mg - f)D = \frac{1}{2}mv_2{}^2 - 0$

$\Rightarrow v_2 = \sqrt{\frac{2(mg-f)D}{m}}$

(c) Free-body diagram for hammerhead pushing I-beam




$W = \Delta K$

$(mg - f - n)d = 0 - \frac{1}{2}mv_2^2$

$(mg - f - n)d = -(mg - f)D$

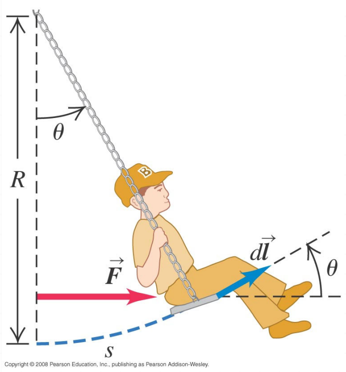
$\Rightarrow \quad n = (mg - f)(1 + \frac{D}{d})$

$n > mg$, if $f = 0$.



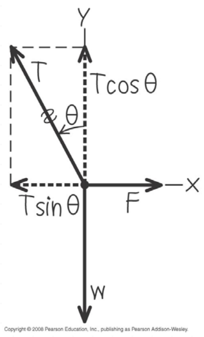
(a)

$W_F = ?\ W_T = ?\ W_g = ?$




Varying $F$ to keep $v$ fixed

(b) Free-body diagram for Throckmorton (neglecting the weight of the chains and seat)




$\begin{cases} T\cos\theta = mg \\ T\sin\theta = F \end{cases} \quad \Rightarrow \quad F = mg\tan\theta$

$W_F = \int \vec{F}\cdot d\vec{\ell}$

$= \int_0^\theta F\cos\theta d\ell = \int_0^\theta mg\tan\theta\cos\theta R d\theta$

$= mgR\int_0^\theta \sin\theta d\theta = mgR(1-\cos\theta)$

$W_T = \int \vec{T}\cdot d\vec{\ell} = 0$

$W_g = \int \vec{w}\cdot d\vec{\ell} = \int -mg\sin\theta d\ell$

$= -mg\int_0^\theta \sin\theta R d\theta$

$= -mgR(1-\cos\theta)$

$\Rightarrow \quad W = W_F + W_T + W_g = 0 = \Delta K$

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# 5. Power



# Power

- The time rate at which work is done or energy is transferred is called *power.*
  - Instantaneous power: $P \equiv \frac{dE}{dt}$
  - Average power: $P \equiv \frac{\Delta E}{\Delta t}$
  - SI unit: watt; 1 watt = 1 joule / second = 1 kg · m$^2$ / s$^3$
  - A unit of power in the US Customary system is horsepower; 1 hp = 746 W
- Units of power can also be used to express units of work or energy.
  - 1 kWh = (1000 W)(3600 s) = 3.6 MJ

If the energy transfer is done by work, then

Average power: $\bar{P} = \dfrac{W}{\Delta t}$

Instantaneous power: $P = \lim\limits_{\Delta t \to 0} \dfrac{W}{\Delta t} = \dfrac{dW}{dt} = \vec{F} \cdot \vec{v}$

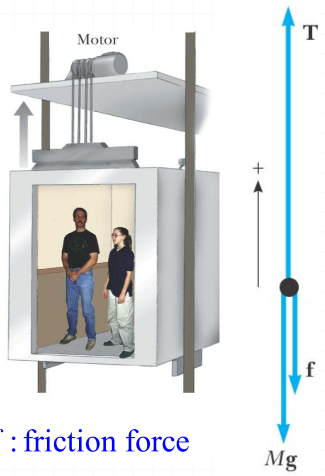
$$W = \int \vec{F} \cdot d\vec{r}$$

$$\Rightarrow \quad dW = \vec{F} \cdot d\vec{r}$$

$$\Rightarrow \quad P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$



**Ex:**




$f$ : friction force

(b)

1. $a = 0$, $v$ is const.
2. $a \neq 0$

Power = ?

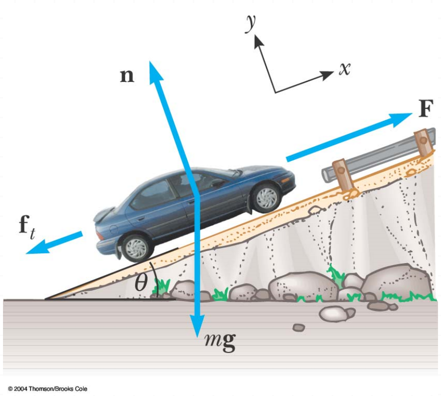
1. $T - f - Mg = Ma = 0 \quad \Rightarrow \quad T = f + Mg$

$$P = \vec{F} \cdot \vec{v} = Tv = \left(f + Mg\right)v$$

2. $T - f - Mg = Ma \quad \Rightarrow \quad T = M\left(g + a\right) + f$

$$P = \vec{F} \cdot \vec{v} = Tv = \left[f + M(g + a)\right]v$$

**Ex:**



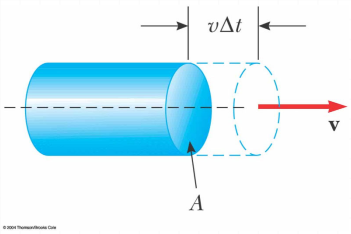

Engine power = ?

$$F - f_t - mg\sin\theta = ma$$
$$\Rightarrow \quad F = ma + mg\sin\theta + f_t$$

$$P = \vec{F}\cdot\vec{v} = \left(ma + mg\sin\theta + f_t\right)v$$



**Ex:**




Calculate air drag force $f$ = ?

System = air

$$W_{\text{ext}} = \Delta K$$

$$\Rightarrow \quad f\left(v\Delta t\right) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad f = \frac{1}{2}m\frac{v}{\Delta t}$$

By $m = \rho\Delta V = \rho\left(Av\Delta t\right)$

$$\Rightarrow \quad f = \frac{1}{2}\rho A v^2$$