Project 4 (0.2 points)

- *Input*: non-zero natural numbers k and n with $k \leq n$

- *Output*:

  1. the number of k-dimensional subspaces of the vector space $\mathbb{Z}_2^n$ over $\mathbb{Z}_2$
  2. a basis of each such subspace (for $1 \leq k \leq n \leq 6$)

Example: The vector space $\mathbb{Z}_2^3$ over $\mathbb{Z}_2$ has 8 vectors, namely $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$. Any 2-dimensional subspace has a basis with two vectors. There are $C_7^2 = 21$ possibilities to choose 2 vectors out of the 8 vectors of $\mathbb{Z}_2^3$, but some of them will generate the same subspace. Only 7 choices will generate different subspaces.

- *Input*: $k = 2$, $n = 3$

- *Output*:

  1. the number of 2-dimensional subspaces of the vector space $\mathbb{Z}_2^3$ over $\mathbb{Z}_2$ is 7
  2. a basis of each such subspace is:

  ((0,0,1),(0,1,0))   ((0,1,0),(1,0,1))
  ((0,0,1),(1,0,0))
  ((0,0,1),(1,1,0))   ((0,1,1),(1,0,0))
  ((0,1,0),(1,0,0))
                                       ((0,1,1),(1,0,1))

Note:

- Any (reasonable) programming language may be used.

- The solutions will consist of the source code with comments (do not send executable files!) and at least 5 relevant input and output files, and will be sent to the e-mail address: septimiu.crivei@ubbcluj.ro.

- If necessary, you will be asked to explain your solution.

- The first 25 solutions will be rewarded.

- The final deadline is January 12, 2025.