Project 5 (0.2 points)

- *Input*: natural numbers $m, n \geq 2$

- *Output*:

  1. the number of matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in *reduced* (that is, each column containing a leading 1 has zeros everywhere else) echelon form
  2. the matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in reduced echelon form (for $2 \leq m, n \leq 5$)

Example:

- *Input*: $m = 2$, $n = 3$

- *Output*:

  1. the number of matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form is 15
  2. the matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form are (the leading 1's are framed):

$$
\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad
\begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
\begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 0 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \end{pmatrix} \quad
\begin{pmatrix} 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad
\begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
\begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad
\begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad
\begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \quad
\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
$$

Note:

- Any (reasonable) programming language may be used.

- The solutions will consist of the source code with comments (do not send executable files!) and at least 5 relevant input and output files, and will be sent to the e-mail address: septimiu.crivei@ubbcluj.ro.

- If necessary, you will be asked to explain your solution.

- The first 25 solutions will be rewarded.

- The final deadline is January 12, 2025.