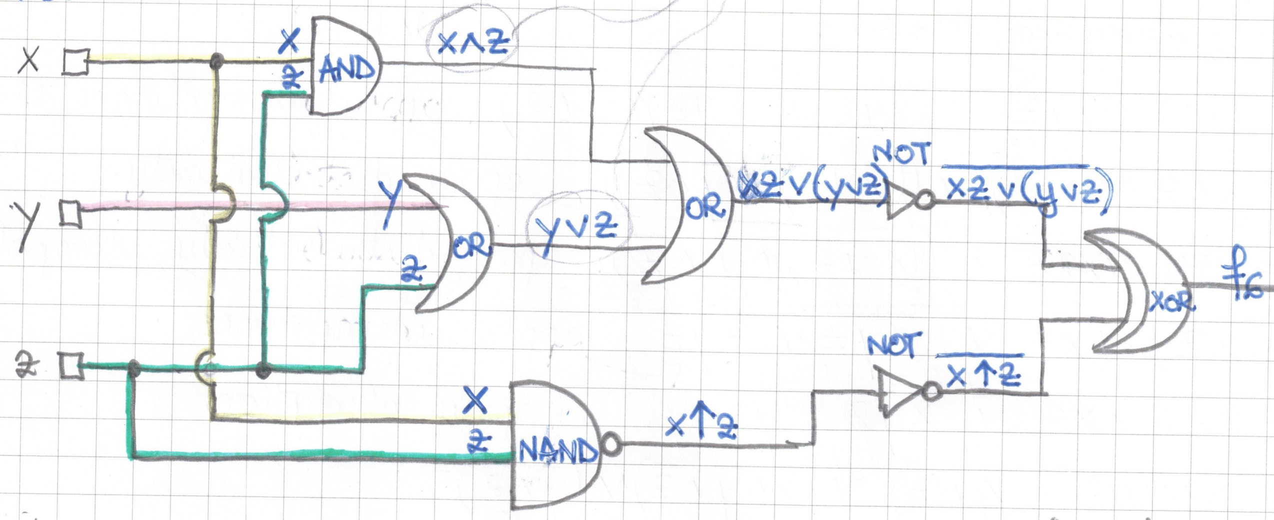
Ex 1 Write the corresponding Boolean fn associated to the following logic circuit, then simplify it and draw simpl equivalent circuit using only basic gates.

1.6.

write output at each gate



$$f_6(x,y,z) = \overline{(xz\vee(y\vee z))} \oplus \overline{x\uparrow z}$$

replace ⊕

$$=\left(\overline{\overline{xz\vee(y\vee z)}}\right)\left(\overline{x\uparrow z}\right)$$
$$\vee\left(\overline{xz\vee(y\vee z)}\right)(x\uparrow z)$$

$$=(xz\vee y\vee z)\left(\overline{x\uparrow z}\right)\vee$$
$$\vee\ \overline{xz\vee y\vee z}\ (x\uparrow z)$$

replace ↑

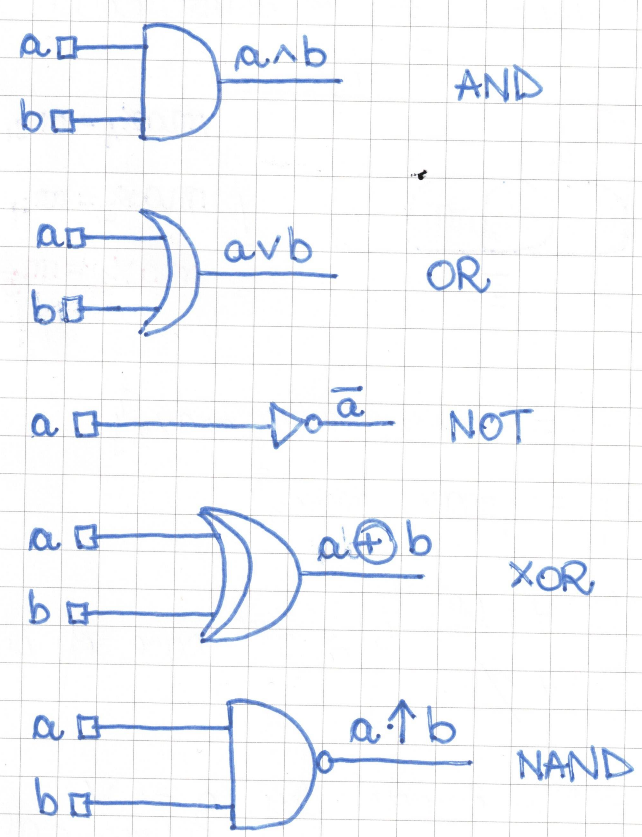
$$=(xz\vee y\vee z)\left(\overline{\overline{x\wedge z}}\right)\vee$$
$$\vee\left(\overline{xz\vee y\vee z}\right)(\bar{x}\vee\bar{z})$$

$$=(xz\vee y\vee z)(x\wedge z)\vee$$
$$\vee((\bar{x}\vee\bar{z})\wedge\bar{y}\wedge\bar{z})(\bar{x}\vee\bar{z})$$

$$=(xz\vee y\vee z)\,xz\,\vee$$
$$\vee((\bar{x}\vee\bar{z})\wedge\bar{y}\wedge\bar{z})(\bar{x}\vee\bar{z})$$

GATES

identify gates given



$a\oplus b = \bar{a}b \vee a\bar{b}$

$a\uparrow b = \overline{a\wedge b} = \bar{a}\vee\bar{b}$

write formulas you use

$= xz\,(xz \vee y \vee z) \vee ((\overline{x}\vee\overline{z}) \wedge \overline{y} \wedge \overline{z})(\overline{x}\vee\overline{z})$ **ABSORPTION**

$=(x\wedge z)((x\wedge z)\vee y\vee z)\vee(\overline{x}\vee\overline{z})((\overline{x}\vee\overline{z})\wedge\overline{y}\wedge\overline{z})$ $\quad U\wedge(U\vee V)\equiv U$

$=(x\wedge z)\wedge((x\wedge z)\vee y\vee z)\vee$ $\quad U\vee(U\wedge V)\equiv U$

$\vee((\overline{x}\vee\overline{z})\wedge(\overline{x}\vee\overline{z})\wedge\overline{y}\wedge\overline{z})$ **IDEMPOTENCE**

apply absorption $U=z$ $\quad U\wedge U\equiv U$

$=(x\wedge z)(y\vee z)\vee((\overline{x}\vee\overline{z})\wedge(\overline{x}\vee\overline{z})\wedge\overline{y}\wedge\overline{z})$ apply idempotence $U=\overline{x}\vee\overline{z}$

$=(x\wedge z\wedge(y\vee z))\vee((\overline{x}\vee\overline{z})\wedge\overline{y}\wedge\overline{z})$ apply distributivity

$= xzy \vee xzz \vee ((\overline{x}\vee\overline{z})\wedge\overline{y}\wedge\overline{z})$ apply distributivity, idempotence

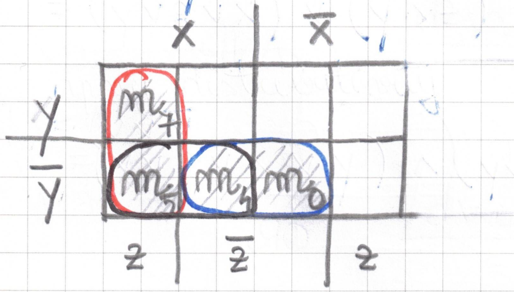
$= xzy \vee xz \vee \overline{x}\overline{y}\overline{z}\vee\overline{z}\overline{y}\overline{z}$ apply idempotence

* $= xzy\vee xz\vee\overline{x}\overline{y}\overline{z}\vee\overline{y}\overline{z}$ apply absorption

$=(x\wedge y\wedge z)\vee(x\wedge z)\vee(\overline{x}\wedge\overline{y}\wedge\overline{z})\vee(\overline{y}\wedge\overline{z})$

$=(x\wedge z)\vee(\overline{y}\wedge\overline{z})$

$=xz\vee\overline{y}\overline{z}$ ← we must check that it is the simplest form



$max_1 = m_4\vee m_0=\overline{y}\overline{z}$

$max_2=m_4\vee m_5=x\overline{y}$

$max_3=m_5\vee m_7=xz$

* $\overset{111}{xzy}$: m_7 $\qquad$ $\overset{000}{\overline{x}\overline{y}\overline{z}}$: m_0

xz : $\overset{111}{xyz}$ (m_7), $\overset{101}{x\overline{y}z}$ (m_5) $\qquad$ $\overline{y}\overline{z}$: $\overline{x}\overline{y}\overline{z}$ m_0, $x\overline{y}\overline{z}$ m_4

$M(f)=\{max_1, max_2, max_3\}$

$C(f)=\{max_1, max_3\}$

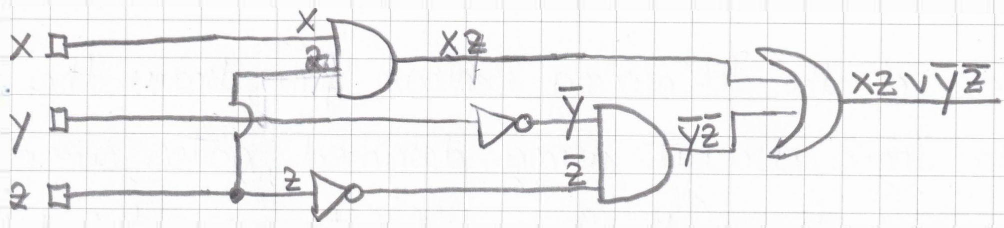
$\left.\begin{array}{l} M(f)\neq C(f) \\ C(f)\neq\emptyset \end{array}\right\} \Rightarrow 2^{nd}$ case of simplification

$\Rightarrow$ We denote by $g=\bigvee_{max\in C(f)} max$ the disjunction of central monoms belonging to every simplified form

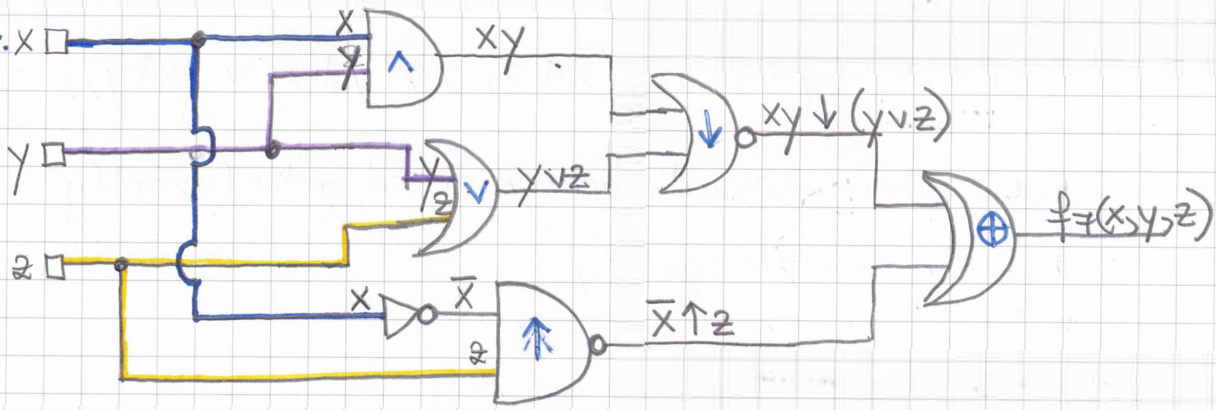
$g = max_1 \vee max_3 \Rightarrow max_1, max_3$ cover all minterms $\Rightarrow$

$$\Rightarrow f_6^S(x,y,z) = \cancel{m}max_1 \vee max_3$$

$$= xz \vee \overline{y}\,\overline{z}$$



1.7.



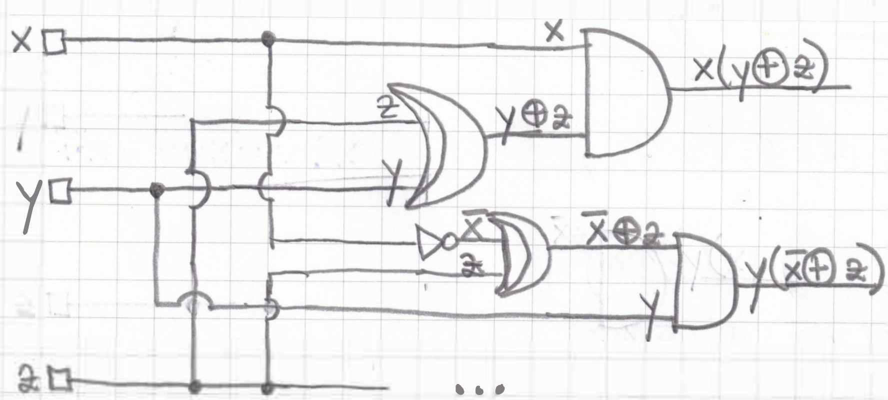
$$f_7(x,y,z) = (xy \downarrow (y \vee z)) \oplus (\overline{x} \uparrow z)$$

$x \downarrow y = \overline{x \vee y} = \overline{x}\,\overline{y}$

Ex 2 For each of the following Boolean fns, draw the corresponding logic circuit using derived gates, simplify the function and draw the logic circuits associated to all simplified forms of the initial fn using basic gates.

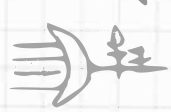
2.7. $f_7(x,y,z) = x(y \oplus z) \lor y(\bar{x} \oplus z) \lor x(y \downarrow z) \lor (x \downarrow y)\bar{z}$

Step 1: draw logic circuit usin derived gates (exactly like the function given)



draw circuit for $x(y \downarrow z)$

$(x \downarrow y)z$

and then 

Step 2: simplify fn

$f(x,y,z) = x(y\bar{z} \lor \bar{y}z) \lor y(xz \lor \bar{x}\bar{z}) \lor x(\bar{y} \land \bar{z}) \lor (\bar{x} \land \bar{y})\bar{z}$

$= \underbrace{\overset{110}{xy\bar{z}}}_{m_6} \lor \underbrace{\overset{010}{\bar{x}y\bar{z}}}_{m_2} \lor \underbrace{\overset{111}{xyz}}_{m_7} \lor \underbrace{\overset{101}{x\bar{y}z}}_{m_5} \lor \underbrace{\overset{100}{x\bar{y}\bar{z}}}_{m_4} \lor \underbrace{\overset{000}{\bar{x}\bar{y}\bar{z}}}_{m_0}$

x \ yz	00	01	11	10
0	m_0			m_2
1	m_4	m_5	m_7	m_6

max_2 max_3 max_1

x \ yz	00	01	11	10
0	m_0			m_2
1	m_4	m_5	m_7	m_6

not okay, can double factorise

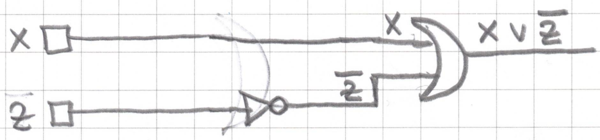
$max_1 = m_0 \vee m_4 \vee m_2 \vee m_6 = \overline{z}$

$max_2 = m_4 \vee m_5 \vee m_7 \vee m_6 = x$

$M(f) = C(f) = \{max_1, max_2\} \Rightarrow 1^{st}$ case of simplification

$\Rightarrow f_7^S(x,y,z) = max_1 \vee max_2$

$= x \vee \overline{z}$



$$f_7(x,y,z) = (xy \downarrow (y \vee z)) \oplus (\bar{x} \uparrow z)$$

$$= (\bar{y} \wedge \bar{z}) \oplus (x \vee \bar{z})$$

$$= xy \vee y\bar{z} \vee xz$$

	x	x	$\bar{x}$	$\bar{x}$
y	m_7	m_6	m_2	
$\bar{y}$	m_5			
	z	$\bar{z}$	$\bar{z}$	z

xy: $\overset{111}{xyz}$, $\overset{110}{xy\bar{z}}$

$y\bar{z}$: $xy\bar{z}$, $\overset{010}{\bar{x}y\bar{z}}$

xz: xyz, $\overset{101}{x\bar{y}z}$

$$a \downarrow b = \overline{a \vee b} = \bar{a}\bar{b}$$

$$xy \downarrow (y \vee z) =$$

$$= \overline{xy} \wedge \overline{y \vee z}$$

$$= (\bar{x} \vee \bar{y}) \wedge \bar{y} \wedge \bar{z}$$

$$= \bar{y} \wedge \bar{z}$$

$$\bar{x} \uparrow z = \overline{\bar{x} \wedge z} = x \vee \bar{z}$$

$$a \uparrow b = \overline{a \wedge b} = \bar{a} \vee \bar{b}$$

$max_1 = m_6 \vee m_2 = y\bar{z}$

$max_2 = m_6 \vee m_7 = xy$

$max_3 = m_7 \vee m_5 = xz$

$$a \oplus b = \bar{a}b \vee a\bar{b}$$

$$\bar{y}\bar{z} \oplus (x \vee \bar{z}) =$$

$$= (\overline{\bar{y}\bar{z}} \wedge (x \vee \bar{z})) \vee$$

$$\vee (\bar{y}\bar{z} \wedge \overline{x \vee \bar{z}})$$

$$= ((y \vee z) \wedge (x \vee \bar{z})) \vee \underbrace{\bar{y}\bar{z}\bar{x}z}_{0} \; {}^{*}$$

$$= (y \vee z) \wedge (x \vee \bar{z})$$

$$= xy \vee y\bar{z} \vee xz \vee \underbrace{z\bar{z}}_{0}$$

$$= xy \vee y\bar{z} \vee xz$$

$M(f) = \{max_1, max_2, max_3\}$

$C(f) = \{max_1, max_3\}$

$\left.\begin{array}{l} M(f) \neq C(f) \\ C(f) \neq \emptyset \end{array}\right\} \Rightarrow 2^{nd}$ case of simplification

$$g = \bigvee_{m \in C(f)} m = max_1 \vee max_3$$

$$= y\bar{z} \vee xz$$

→ all minterms covered ⇒ 1 unique simplified form

$$f_7^S(x,y,z) = y\bar{z} \vee xz$$

$^{*}\; x \wedge \bar{x} = 0, \; x \vee \bar{x} = 1$

$$\left[\begin{array}{l} x \wedge 0 = 0 \wedge x = 0 \\ x \vee 0 = 0 \vee x = x \end{array}\right.$$