

Logic Exercises 6

1. Use the predicates

$$\begin{aligned}A(x, y) &: x \text{ admires } y \\P(x) &: x \text{ is a professor}\end{aligned}$$

and the constant

$$m : \text{ Mary}$$

to translate the following three phrases into predicate logic.

- (a) Mary admires every professor.
 - (b) Some professor admires Mary.
 - (c) Mary admires herself.
2. Find appropriate predicates and their specifications to translate the following phrases into predicate logic.
- (a) All red things are in the box.
 - (b) Only red things are in the box.
 - (c) Every enemy of Fred is also an enemy of John.
3. Use the predicates

$$\begin{aligned}A(x, y) &: x \text{ attended } y \\S(x) &: x \text{ is a student} \\L(x) &: x \text{ is a lecture}\end{aligned}$$

to translate the following phrases into predicate logic.

- (a) No student attended every lecture.
 - (b) No lecture was attended by every student.
 - (c) Each lecture was attended by some student.
 - (d) For each pair of lectures, some student attended both lectures.
 - (e) If two students attended some lecture together, then they attended exactly the same lectures.
4. Consider the model consisting of the set of elements $\{a, b, c\}$ together with a unary predicate D and a binary predicate E , defined by

$$\begin{aligned}D(b) \quad D(c) \\E(a, a) \quad E(a, b) \quad E(a, c) \quad E(b, c) \quad E(c, a)\end{aligned}$$

Say for each of the following formulas whether it holds for this model.

- (a) $\forall x \exists y E(x, y)$
 - (b) $\exists x \forall y E(x, y)$
 - (c) $\forall x \forall y (E(x, y) \vee E(y, x))$
 - (d) $\forall x (D(x) \vee E(x, x))$
 - (e) $\forall x (\forall y E(x, y) \rightarrow D(x))$
 - (f) $\forall x (\forall y E(x, y) \rightarrow \neg D(x))$
5. For each of the following pairs of formulas, either argue that they are semantically equivalent, or give a model on which their truth values are different. In cases where semantic equivalence does not hold, explain moreover whether one of the formulas semantically entails the other.
- (a) $\forall x (C(x) \vee D(x))$ and $\forall x C(x) \vee \forall x D(x)$
 - (b) $\forall x (C(x) \wedge D(x))$ and $\forall x C(x) \wedge \forall x D(x)$
 - (c) $\exists x \exists y R(x, y)$ and $\exists y \exists x R(y, x)$
 - (d) $\exists x \forall y R(x, y)$ and $\exists x \forall y R(y, x)$
 - (e) $\forall x \exists y C(x, y)$ and $\forall x C(x, y)$
 - (f) $\exists x (P(x) \rightarrow Q(x))$ and $\exists x (P(x) \rightarrow \exists x Q(x))$
 - (g) $\exists x (P(x) \rightarrow Q(x))$ and $\exists x (\exists x P(x) \rightarrow Q(x))$