Logic Exercises 4

1. Group the following eight formulas into three semantic equivalence classes:

$$p \to q \qquad p \to (p \lor q) \qquad p \to (p \to q) \qquad (p \to q) \to q$$
$$\neg q \to p \qquad (p \land q) \to p \qquad \neg(\neg q \land p) \qquad p \lor q$$

2. Show using the axioms for semantic equivalence that:

   (a) $\neg\bot \;\equiv\; \top$

   (b) $(\phi \lor \psi) \lor (\phi \land \psi) \;\equiv\; \phi \lor \psi$

   (c) $\phi \lor (\phi \land \psi) \;\equiv\; \phi$

   (d) $\phi \land (\phi \lor \psi) \;\equiv\; \phi$

   (e) $\phi \lor (\psi \land (\neg\phi \lor \chi)) \;\equiv\; \phi \lor \psi$

3. Transform the semantic equivalence in exercise 2(e) into an equation for sets, and argue that this equation indeed holds for sets.

4. Consider the formula $\neg(p \land \neg q) \;\land\; (p \lor r)$.

   (a) Provide the truth table for this formula.

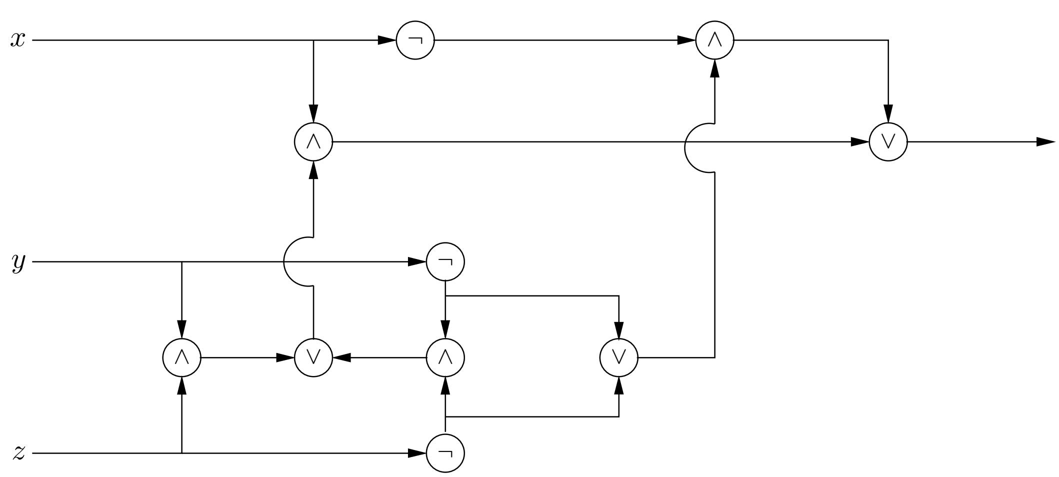
   (b) Use the truth table you constructed in (a) to turn this formula into a DNF.

   (c) Derive with the axioms for semantic equivalence that the formula above and the DNF you constructed in (b) are semantically equivalent.

5. Construct a logic circuit corresponding to the formula $(\phi \land \neg\psi) \lor (\psi \land (\chi \lor \neg\phi))$.

6. Construct a logic circuit corresponding to the formula $\phi \leftrightarrow \psi$, using only AND, OR and NOT gates.

7. Give the formula that corresponds to the following logic circuit:



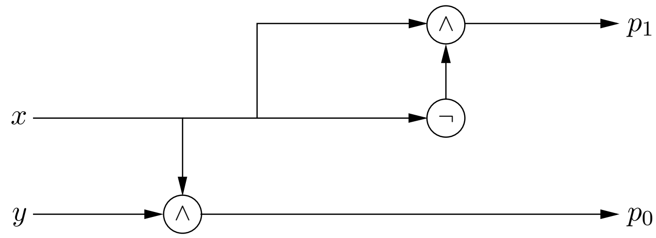
8. (a) Write the decimal number 13 as a binary number.

   (b) What is the result of the following binary addition?

$$\begin{array}{cccccc l} 1 & 0 & 0 & 0 & 1 & 1 & \\ 1 & 0 & 1 & 1 & 1 & 0 & + \\ \hline \end{array}$$

   Also state how this addition is represented in a decimal representation.

9. Explain why the logic circuit below expresses the result $p_1\, p_0$ of multiplying two bits x and y in binary arithmetic.



10. Consider the (binary) *product* of two 2-bits numbers $x_1\, x_0$ and $y_1\, y_0$, yielding a 4-bits number $c_3\, c_2\, c_1\, c_0$. Express the bits c_3, c_2, c_1 and c_0 in terms of x_1, x_0, y_1, y_0 and $\wedge, \vee, \oplus, \neg$.