Exercises on Chapter 2: Relations

1. In the universe $U := \{a, b, c, d, e\}$ we are given the sets

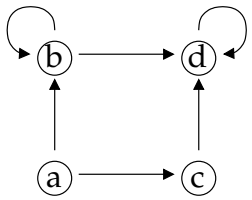
$$A := \{b, c\} \qquad B := \{a, b, c, e\} \qquad C := \{a, c, d\} \qquad D := \{a, c, e\}$$

   (a) Draw a picture of the sets $B \times C$ and $A \times D$: draw a horizontal and a vertical axis, put the elements of U along these axes, and indicate with the symbol $\times$ which points in the picture are in $B \times C$, and with the symbol + which ones are in $A \times D$ (see slide 4 of the lecture).
   (b) How many elements are there in the set $(B \times C) \cap (A \times D)$, and how many elements are there in the sets $(B \times C) \setminus (A \times D)$ and $B \times C$?
   (c) Use your answers in part (b) to verify that

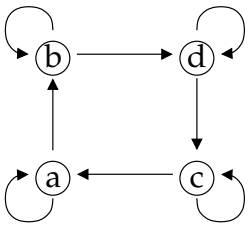
$$\#((B \times C) \cap (A \times D)) + \#((B \times C) \setminus (A \times D)) = \#(B \times C)$$

2. Consider the relation $R := \{\langle 0, 0\rangle, \langle 0, 1\rangle, \langle 1, 1\rangle, \langle 1, 2\rangle, \langle 2, 1\rangle\}$ in the set $\{0, 1, 2\}$.
   (a) Draw a directed graph representation of R.
   (b) Is the relation R reflexive? transitive? symmetric? anti-symmetric?
   (c) Explicitly list the elements of R^{-1} in the curly-bracket notation.

3. Using the directed graph representation, we define two relations R and S in the set $\{a, b, c, d\}$ as follows:



relation R



relation S

   (a) Explicitly list the elements of R and S in the curly-bracket notation.
   (b) Give the matrix representations of the relations R and S.
   (c) Determine the relations $R \circ S$ and $S \circ R$ and write down an explicit listing of their elements (in the curly-bracket notation). *Hint:* it can help to draw a Venn diagram of the composition first.
   (d) Draw directed graph representations of the relations $R \circ S$ and $S \circ R$.
   (e) Is the relation $R \circ S$ reflexive? transitive? symmetric? anti-symmetric?
   (f) Is the relation $S \circ R$ reflexive? transitive? symmetric? anti-symmetric?

4. Let R and S be two relations in a set of 4 elements, described by the following matrices (with rows and columns ordered in the same way):

$$R: \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad S: \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Determine the matrix representation of the composite relation $R \circ S$.
Hint: you can use a Venn diagram to picture the composition $R \circ S$ first.

5. In this exercise, we work in a domain of *People*.
   (a) Give an appropriate simplified name to each of the following relations:
      i. the relation $IsMarriedTo^{-1}$
      ii. the relation $(IsSiblingOf \circ IsChildOf)^{-1}$
      iii. the relation $IsParentOf \circ IsSiblingOf \circ IsChildOf$
   (b) Compare the two relations *IsBrotherOf* and $IsBrotherOf \circ IsBrotherOf$.
      i. Is it true that $IsBrotherOf \subseteq (IsBrotherOf \circ IsBrotherOf)$?
      ii. Is it true that $(IsBrotherOf \circ IsBrotherOf) \subseteq IsBrotherOf$?
   (c) Express each of the relations below in terms of the relations *IsParentOf* and *IsTheSamePersonAs* using the operations of inverse, composition, union, intersection, complement and / or difference:
      i. the relation *IsGreatGrandChildOf*
      ii. the relation *IsSiblingOf*

6. In this exercise we describe a date as a triple of the form $\langle d, m, y \rangle$, where $d \in \{01, 02, \ldots, 31\}$, $m \in \{01, 02, \ldots, 12\}$ and $y \in \{1990, 1991, 1992, \ldots\}$. For example, $\langle 13, 04, 1995 \rangle$ denotes the 13^{th} of April, 1995. Let the set *Dates* consist of all triples $\langle d, m, y \rangle$ that correspond to actual dates. We define the relation *Period* in the set *Dates* as the set of all pairs of dates $\langle d_1, d_2 \rangle$ such that d_1 does not lie (strictly) ahead of d_2, and d_2 lies at most 21 days ahead of d_1 in time.
   (a) Which of the following pairs of dates are elements of *Period* ?

$$\langle\, \langle 08, 10, 1998 \rangle, \langle 20, 09, 1998 \rangle \,\rangle$$
$$\langle\, \langle 10, 10, 1998 \rangle, \langle 01, 11, 1998 \rangle \,\rangle$$
$$\langle\, \langle 10, 11, 1998 \rangle, \langle 01, 12, 1998 \rangle \,\rangle$$

   (b) Is the relation *Period* reflexive? transitive? symmetric? anti-symmetric?
   (c) What is the interpretation of the relation $Period \circ Period$?