Logic Exercises 3

1. Which of the following five formulas are in disjunctive normal form (DNF), and which ones are in conjunctive normal form (CNF)?

$$p \wedge \neg(q \vee r)\,, \quad p \vee q \vee \neg r\,, \quad p \wedge q \wedge \neg r\,, \quad p \vee (q \wedge \neg r)\,, \quad p \wedge (q \vee \neg r)$$

2. Construct formulas in DNF and in CNF that are semantically equivalent to the formula ϕ, based on its truth table:

p	q	r	ϕ
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

3. (a) Express the formula $p \to q$ using only the Sheffer stroke $|$ (and p and q).
   (b) Express the formula $p \leftrightarrow q$ using only the Sheffer stroke $|$ (and p and q).
   (c) Build a tautology and a contradiction using only the Sheffer stroke (and a propositional variable).

4. (a) Show that $\{\neg, \to\}$ is an adequate system of connectives (meaning that every truth table can be expressed using a formula built from only these connectives and some propositional variables).
   (b) Argue that $\{\wedge, \vee, \to\}$ is not an adequate system of connectives.
   (c) Is $\{\neg, \leftrightarrow\}$ an adequate system of connectives?

5. (a) Determine a CNF of the formula $(\neg q \wedge r) \to (p \wedge \neg q)$ by applying the algorithm `CNF`.
   (b) Do the same for the formula $\neg\neg p \to \neg(p \to \neg p)$.
   (c) Apply the criterion for determining whether a CNF is a tautology (which was treated in the lecture) to the CNFs you constructed in (a) and (b).
   (d) Formulate an analogous criterion to determine whether a DNF is a contradiction.

6. Is a DNF a contradiction if and only if each disjunct contains literals p and $\neg p$ for some p? If so, argue that this is the case. If not, give a counterexample.

7. Apply the DPLL procedure to the following CNFs, to check if they are satisfiable.

   (a) $p \wedge \neg p$

   (b) $p \vee \neg p$

   (c) $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q)$

   (d) $(p \vee q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge r$

   (e) $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

   (f) $(p \vee q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee r)$