# Logic Exercises 1

We employ the conventions about binding priorities of the logical connectives from propositional logic put forward in Convention 1.3.

1. Use $\neg$, $\rightarrow$, $\wedge$ and $\vee$ to express the following declarative sentences in propositional logic; in each case, state what your propositional atoms p, q, r etc. mean:

    (a) If the sun shines today, then it won't shine tomorrow.

    (b) Robert was jealous of Yvonne, or he was not in a good mood.

    (c) If the barometer falls, then either it will rain or it will snow.

    (d) Alzheimer's disease will not be cured unless its cause is determined and a new drug for it is found.

    (e) No shoes, no shirt, no service.

2. The formulas below use the binding priorities from Convention 1.3. Reinsert as many brackets as possible *(except the outermost ones)*. For example, $p \wedge q \rightarrow r$, is changed into $(p \wedge q) \rightarrow r$, since $\wedge$ binds more tightly than $\rightarrow$.

    (a) $\neg p \wedge q \rightarrow r$

    (b) $(p \rightarrow q) \wedge \neg(r \vee p \rightarrow q)$

    (c) $p \vee (\neg q \rightarrow p \wedge r)$

    (d) Why is the expression $p \vee q \wedge r$ problematic?

3. Remove as many brackets as possible from the formulas below, using the binding priorities from Convention 1.3 as well as associativity of $\wedge$ and $\vee$.

    (a) $(\neg(p \wedge (q \wedge r)) \rightarrow (\neg(p) \vee \neg(r)))$

    (b) $((\neg(p \wedge (q \vee r)) \rightarrow \neg(p)) \vee \neg(r))$

4. Draw the parse trees of the following formulas:

    (a) $\neg((\neg q \wedge (p \rightarrow r)) \wedge (r \rightarrow q))$

    (b) $(p \wedge q) \rightarrow (\neg r \vee (q \rightarrow r))$

5. Draw the parse tree of a propositional logic formula ϕ which is a disjunction whose disjuncts are both conjunctions.

6. Draw the parse tree of $\neg(s \to (\neg(p \to (q \vee \neg s))))$, and list all its subformulas.

7. For the following parse tree, find the propositional logic formula that it represents.
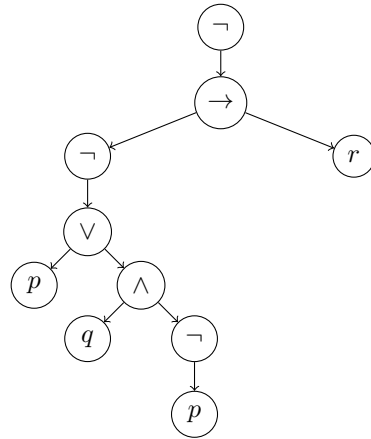
8. Determine the truth value of the formula represented by the parse tree in exercise 7 by means of the parse tree in a bottom-up fashion, for the following values of p, q and r.

   (a) p is F, q is T, and r is F.
   (b) p is F, q is F, and r is F.

9. For the following formulas, compute the complete truth table, and indicate whether it is a tautology, a contingent formula, or a contradiction:

   (a) $((p \to \neg q) \to \neg p) \to q$
   (b) $(p \to q) \vee (p \to \neg q)$

10. Which of the following formulas are semantically equivalent to $p \to (q \vee r)$?

    (a) $q \vee (\neg p \vee r)$
    (b) $q \wedge \neg r \to p$
    (c) $p \wedge \neg r \to q$

11. Is bi-implication $\leftrightarrow$ associative? If so, prove this via a truth table. If not, give a counterexample.

12. Go to `https://infinity.few.vu.nl/logic/` and try your hand at some of the interactive exercises for week 1.