Exercises on Chapter 1: Sets

1. Express each of the following sets in the curly-bracket notation by means of a description of the elements:
   (a) The set of all integers that are divisible by 7 or 11.
   (b) The set of all real roots of the polynomial $x^3 - 6x^2 + 11x - 6$.
   (c) The set of all solutions $\langle x, y \rangle$ of the system of equations

$$\begin{cases} 3x - y = 0, \\ x + 2y = 7. \end{cases}$$

2. In this exercise we consider the universe $U := \{1, 2, \ldots, 30\}$ with the three subsets *MultiplesOf2*, *MultiplesOf3* and *MultiplesOf5*, defined as suggested by their names. Express each of the sets below as a union, intersection or difference of the three defined sets and/or their complements:
   (a) Multiples of 2 that are not a multiple of 5.
   (b) Multiples of 6.
   (c) Odd numbers that are divisible by 3.
   (d) $\{30\}$.

3. In the universe $\mathbb{R}$ of real number, consider the sets

$$A := \{x \colon 0 < x \leq 2\} \qquad \text{and} \qquad B := \{x \colon 1 \leq x < 3\}.$$

   Give explicit descriptions of the sets $A \cup B$, $A \cap B$, $A \setminus B$ and $B \setminus A$.

4. For each pair of sets below, use Venn diagrams to determine whether or not the two sets are equal. For each set, clearly illustrate how the set is constructed, by drawing a new Venn diagram for every set operation used in the construction of the set.
   (a) $(A \setminus (B \cup C))'$ and $A' \cap (B \cup C)$.
   (b) $(A \setminus (B \cap C))'$ and $A' \cup (B \cap C)$.
   (c) $(A \cup B) \setminus C$ and $(A \setminus C) \cup (B \setminus C)$.
   (d) $A \cup (B \mathbin{\Delta} C)$ and $(A \cup B) \mathbin{\Delta} (A \cup C)$.

5. In a universe U with 20 elements, we have three sets A, B and C about which we have the following information:

$$\#A = 7, \qquad \#B = 8, \qquad \#C = 9$$
$$\#(A \cap B) = \#(A \cap C) = \#(B \cap C) = 3$$
$$\#(A \cap B \cap C) = 1.$$

   Use a Venn diagram to determine the following numbers:

$$\#(A \cup B \cup C), \qquad \#(A' \cap B' \cap C'), \qquad \#(A \cup B)'.$$

6. Suppose that in a certain population of 1 000 people, 100 persons have a disease D. Of the people that have this disease, 45 exhibit the symptom A and 70 the symptom B, while 15 of the patients show neither symptom. Of the people that do not have the disease, 90 exhibit symptom A and 265 symptom B, while 585 do not show either symptom. Draw a Venn diagram for this situation (using sets A, B and D as subsets of the entire population), and use this diagram to determine the following numbers:
   (a) The number of people that have disease D and exhibit both symptoms.
   (b) The number of people that do not have disease D and exhibit only the symptom A (not symptom B).
   (c) The total number of people that do not exhibit either symptom.

7. Prove each of the equalities below using the algebra of sets. Formulas that contain the symmetric difference Δ must first be converted into unions, intersections and complements using the definition $A \Delta B = (A \cup B) \cap (A \cap B)'$.
   (a) $((A \cap B) \cup C')' = (A' \cap C) \cup (B' \cap C)$.
   (b) $(A \cup B)' \cup (A \cup C)' = A' \cap (B \cap C)'$.
   (c) $A \Delta A' = U$.
   (d) $A' \Delta B' = A \Delta B$.

8. From an infinite number of nickels n (5 cents), dimes d (10 cents) and quarters q (25 cents), three coins are sampled in sequence. A sample such as "nickel, dime, nickel" is represented symbolically as "*ndn*".
   (a) Give an explicit description of the universe U of all possible samples (represented in the symbolic notation), and determine $\#U$.
   (b) Let A be the subset of U of all samples that form a sum of at least 50 cents. Explicitly list all the elements of A, and determine $\#A$.
   (c) Let B be the subset of U of all samples that do not contain a quarter. Determine the number of elements of B.
   (d) Let C be the subset of U of all samples that contain exactly one quarter. Explain why $\{A, B, C\}$ is a partition of U, and use this fact together with your answers to (a), (b) and (c) to determine $\#C$.