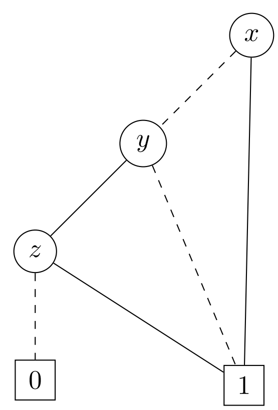
Logic Exercises 5

1. Represent the formula $(x \wedge y) \oplus z$ by a binary decision tree, against the variable order x, y, z, and transform it into a reduced OBDD.

2. Consider the ternary majority function: $m(x, y, z) = 1$ if and only if at least two of the arguments x, y, z are equal to 1. Represent this function by a binary decision tree, against some variable order, and transform it into a reduced OBDD.

3. Give a formula that corresponds to the following reduced OBDD:



4. Let O_1 and O_2 be OBDDs, against the same variable order.

   (a) Explain how an OBDD for $O_1 \oplus O_2$ can be constructed from O_1 and O_2.

   (b) Give an example to show that given reduced OBDDs O_1 and O_2, the constructed OBDD for $O_1 \oplus O_2$ is not always reduced.

5. Consider the variable order x, y, z.

   (a) Give the reduced OBDDs for $\neg(x \vee y)$ and for z.

   (b) Build an OBDD for $\neg(x \vee y) \vee z$ from the reduced OBDDs for $\neg(x \vee y)$ and for z; if needed, transform it into a reduced OBDD.

   (c) Build an OBDD for $\exists y\, (\neg(x \vee y) \vee z)$ from the reduced OBDD for $\neg(x \vee y) \vee z$; if needed, transform it into a reduced OBDD.

   (d) Build an OBDD for $\forall y\, (\neg(x \vee y) \vee z)$ from the reduced OBDD for $\neg(x \vee y) \vee z$; if needed, transform it into a reduced OBDD.

6. Say for each of the following statements whether it holds or not. Motivate your answers. In (d-g), $\phi(x)$ denotes a propositional formula in which x is the only propositional variable to occur (freely).

   (a) $\exists x\,(x \to y)$ is a tautology.

   (b) $\forall x\,(x \to y)$ is a tautology.

   (c) $\forall x\,\exists y\,(x \to y) \;\equiv\; \forall x\,(x \to y)$.

   (d) $\exists x\,\phi(x)$ is either a tautology or a contradiction.

   (e) $x \wedge \exists x\,\phi(x)$ is either a tautology or a contradiction.

   (f) $\forall x\,\phi(x) \;\equiv\; \forall y\,\phi(y)$.
   ($\phi(y)$ is obtained by renaming each (free) occurrence of x in $\phi(x)$ into y.)

   (g) $\phi(x) \equiv \phi(y)$.

7. Consider the variable order x, y, z.

   (a) Construct the reduced OBDD for the formula $(x \wedge y) \vee (\neg x \wedge \neg z)$ from its binary decision tree.

   (b) Construct OBBDs for $\forall x\,((x \wedge y) \;\vee\; (\neg x \wedge \neg z))$ and $\exists x\,((x \wedge y) \;\vee\; (\neg x \wedge \neg z))$, using the OBDD you constructed in (a).