$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

Where x is

$$\det (A - \lambda I) = 0$$

$$\begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -4-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -18 & -14 & -13-\lambda \end{vmatrix} = 0$$

1st term: $\quad 4-\lambda \begin{vmatrix} -4-\lambda & -2 & -4 \\ 10 & 5+\lambda & -10 \\ -13 & -14 & 13-\lambda \end{vmatrix}$

2nd term: $\quad 8 \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$

3rd term:

$$-1(-1)\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$$

4th term:

$$\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

Substituting and Simplifying the Minors

$$\Rightarrow (4-\lambda)(\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8$$
$$(2\lambda^2 - 22\lambda + 370) - 1(\lambda^3 + 390 + 2(\lambda^2 + 22\lambda + 275$$
$$= \lambda^4 + 73\lambda^3 - 214\lambda^2 - 835\lambda + 3500 = 0$$

Find the Eigen Value yields

$\lambda_1 = 21.125, \quad \lambda_2 = -5.604, \quad \lambda_3 = 2.675$

$\lambda_4 = 11.054$

Finding eigen Vectors

Substituting $\lambda = -5.604$

$$A + 5.604 I =$$

$$\begin{bmatrix} 9.504 & 8 & -1 & -2 \\ -2 & 1.604 & -2 & -4 \\ 0 & 10 & 10.604 & 10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix}$$

$$\text{let } \vec{V} = \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix}$$

$$\begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & 1.604 & -2 & -4 \\ 0 & 10 & 10.604 & 10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix} , \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gaussian Elimination

$Raw_1 ;\ 9.604 V_1 + 8 V_2 - V_3 - 2V_4 = 0$

$$V_1 = \frac{-8V_2 + V_3 + 2V_4}{9.604}$$

$Raw_2 = -2V_1 + 1.604 V_2 - 2 V_3 - 4V_4 = 0$

$$-2\left(\frac{-8V_2 + V_3 + 2V_4}{9.804}\right) + 1.604V_2 - 2V_3 - 4V_3 = 0$$

Row$_3$: $10V_2 + 10.604V_3 + 10V_4 = 0$

$\cancel{N_2 + 1.\cancel{680}}$

$V_2 + 1.604V_3 \overline{\overline{=}} V_4 = 0$

$N_2 = -1.604V_3 + V_4$

Row$_4$: $-V_1 - 13V_2 - 14V_3 - 7.396V_4 = 0$

$V_4 = t$ (a free parameter)

Row$_3$: $N_2 = -1.604V_3 + t$

Row$_1$ to find V_1 in terms of V_3 and t

Row$_4$: to solve for N_3 in terms of t

$$N = \begin{vmatrix} -0.48 \\ 1 \\ -0.52 \\ 1 \end{vmatrix}$$

Final Eigen Vectors for $\lambda = -1.604$

$$\underline{=} \begin{vmatrix} -0.48 \\ 1 \\ -0.52 \\ 1 \end{vmatrix}$$