$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

1

Eigen Values = $\det(A - \lambda I) = 0$

$$(A - \lambda I) = \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

1) Eigen Values

1st term: $(4-\lambda)\begin{vmatrix} -4-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$

2nd term: $+8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$

3rd term: $-1(-1)\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$

4th term: $-2\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ 1 & -13 & -14 \end{vmatrix}$

$= (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) -$

$8(-2\lambda^2 - 22\lambda + 370) - 1(\lambda^3 + 390) +$

$2(\cdot\lambda^2 + 22\lambda + 275)$

$= \cancel{\lambda}^4 + 13\lambda^3 - 214\lambda^2 - 835\lambda + 35000 = 0$

Eigen values:

$\lambda_1 \simeq -21.125$

$\lambda_2 \simeq -5.604$

$\lambda_3 \simeq 2.675$

$\lambda_4 \simeq 11.054$

Eigen vector for $\lambda_3 = 2.675$

$(A - \lambda I)\vec{V} = 0$

$$A - 2.675 I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

let $\vec{V} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

Augmented Matrix:

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

Dividing first Row by 1.325

$$= \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

Elimination:

- $R_2 + 2 \cdot R_1$
- $R_4 + R_1$

$$\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 0.4004 & -3.5094 & -7.0189 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.9623 & -14.7547 & -17.1844 & 0 \end{array}$$

* Divide R_2 by 0.4004:

$$\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 0.4004 & -3.5094 & -7.0189 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.9623 & -14.7547 & -17.1844 & 0 \end{array}$$

Eliminate using new R_2:

- $R_3 - 10 \cdot R_2$
- $R_4 + 6.9623 \cdot R_2$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5044 & 0 \\ 0 & 1 & -8.768 & -17.533 & 0 \\ 0 & 0 & 90.005 & 165.33 & 0 \\ 0 & 0 & 46.257 & 105.3 & 0 \end{array}\right]$$

Substitution:

Row$_3$: $90.005z + 165.33w = 0$

$$-z = \frac{-165.33}{90.005} w$$

$$\underline{z \simeq -1.837w}$$

Row 2: $y - 8.768z - 17.533w = 0$

$$y = 8.768(-1.837w) + 17.533w$$

$$= \underline{16.107w}$$

Row 1: $x + 6.0377y - 0.7547z - 1.5094w = 0$

$$x = -6.0377y + 0.7547z + 1.5094w$$

$$x \simeq -97.352w$$

let $\omega = 1$,

$$\vec{V_3} = \begin{pmatrix} x \\ y \\ z \\ \omega \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

$$\|\vec{V_3}\| = \sqrt{9741.1}$$

$$\simeq 98.69$$

$$\vec{V_3}^{norm} \simeq \frac{1}{98.69} \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.987 \\ -0.1683 \\ -0.019 \\ 0.010 \end{pmatrix}$$

$$\vec{V_3}^{norm} = \begin{pmatrix} 0.987 \\ -0.163 \\ -0.019 \\ 0.010 \end{pmatrix}$$

$$\lambda_3 = 2.675$$

$$\vec{V_3} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix} \quad \vec{V_3}^{norm} = \begin{pmatrix} 0.987 \\ -0.163 \\ -0.014 \\ 0.010 \end{pmatrix}$$