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$A\vec{V} = \lambda\vec{V}$

$\det(A - \lambda I) = 0$

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ 1 & -13 & -14 & -13 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2+\lambda & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix} = 0$$

$x(-x)$

$$\det\begin{pmatrix} 4-\lambda & -1 & -2 \\ -2 & -2 & -4 \\ -1 & -14 & -13-\lambda \end{pmatrix}$$

$$(4-\lambda)\det\begin{pmatrix} -2 & -4 \\ -14 & -13-\lambda \end{pmatrix} + 1\det\begin{pmatrix} -2 & -4 \\ -1 & -13-\lambda \end{pmatrix} - 2\det\begin{pmatrix} -2 & -2 \\ -1 & -14 \end{pmatrix}$$

$$(4-\lambda)((+26+2\lambda)+50) + ((26+2\lambda)+4) - 2(28+2)$$

$$(4-\lambda)(2\lambda+30) + (2\lambda+30) - 56$$

$$(4-\lambda)(2\lambda-30) + (2\lambda+22) - 52$$

$$8\lambda - 120 - 2\lambda^2 + 30\lambda + 2\lambda + 22 - 52$$

$$-2\lambda^2 + 40\lambda - 150$$

$$\det\begin{pmatrix} 4-\lambda & 8 & -2 \\ -2 & -9-\lambda & -4 \\ -1 & -13 & -13-\lambda \end{pmatrix}$$

$$= (4-\lambda)\det\begin{pmatrix} -9-\lambda & -4 \\ -13 & -13-\lambda \end{pmatrix} - 8\det\begin{pmatrix} -2 & -4 \\ -1 & -13-\lambda \end{pmatrix} + (-2)\det\begin{pmatrix} -2 & -9-\lambda \\ -1 & -13 \end{pmatrix}$$

$$(4-\lambda)((-9-\lambda)(-13-\lambda)-(52)) - 8((+26+2\lambda)-(4)) - 2(26-(9+\lambda))$$

$$(4-\lambda)(117+9\lambda+13\lambda+\lambda^2-52) - 8(26+2\lambda-4) - 2(26-9-\lambda)$$

$$(4-\lambda)(\lambda^2+22\lambda+65) - 8(2\lambda+22) - 2(-\lambda+17)$$

$(4\lambda^2 + 88\lambda + 260 - \lambda^3 - 22\lambda^2 - 65\lambda) - 16\lambda - 176 + 2\lambda - 34$

$-\lambda^3 - 18\lambda^2 + 23\lambda + 260 - 16\lambda - 176 + 2\lambda - 34$

$-\lambda^3 - 18\lambda^2 - 9\lambda + 50$

$$\det\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ -1 & -13 & -14 \end{pmatrix} =$$

$$(4-\lambda)\det\begin{pmatrix} -9-\lambda & -2 \\ -13 & -14 \end{pmatrix} - 8\det\begin{pmatrix} -2 & -2 \\ -1 & -14 \end{pmatrix} + (-1)\det\begin{pmatrix} -2 & -9-\lambda \\ -1 & -13 \end{pmatrix}$$

$(4-\lambda)((126+14\lambda)-(26)) - 8(28-2) - (26-(9+\lambda))$

$(4-\lambda)(14\lambda + 100) - 224 + 16 - (26-9-\lambda)$

$56\lambda + 400 - 14\lambda^2 - 100\lambda - 224 + 16 - 17 + \lambda$

$-14\lambda^2 - 43\lambda + 175$

for $\lambda = -21.125$

$$\begin{bmatrix} 25.125 & 8 & -1 & -2 & 0 \\ -2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ -2 & 12.125 & -2 & -4 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ -1 & -13 & -14 & 8.125 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 12.761 & -2.080 & -4.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 1 & -12.682 & -14.040 & 8.045 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 27.754 & -6.741 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 27.754 & -6.741 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & -16.106 & 3.912 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & -0.163 & -0.326 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.318 & 0 & -0.089 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0.027 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 + 0.027 x_4 = 0 \\ x_2 - 0.365 x_4 = 0 \\ x_3 - 0.243 x_4 = 0 \end{cases}$$

$$V_1 = \begin{pmatrix} -0.27 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix}$$

This equation $\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$ has no real possible roots but we tried estimating to nearest values.