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ANALYTICAL TRANSIT LIGHT CURVES FOR LIMB-DARKENED STARS

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ABSTRACT

We derive analytical, closed form solutions for the light curve of a planet transiting a star with a limb darkening profile of arbitrary order. We provide updated expressions for the linear and quadratically limb darkened cases that are numerically stable over the entire domain.

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1. INTRODUCTION

Currently I just copy-pasted some relevant stuff from the starry paper. Let's expand and add to this.

2. REPARAMETRIZATION

2.1. *Linear limb darkening ($n = 2$)*

From Mandel & Agol (2002), the total flux visible during the occultation of a body whose surface map is given by $I(x, y) = \sqrt{1 - x^2 - y^2}$ may be computed as

$$s_2 = \frac{2\pi}{3}\left(1 - \frac{3\Lambda}{2} - \Theta(r - b)\right) \tag{1}$$

where $\Theta(\cdot)$ is the Heaviside step function and

$$\Lambda = \begin{cases} \frac{1}{9\pi\sqrt{br}}\left[\frac{(r+b)^2 - 1}{r+b}\Big(-2r\left(2(r+b)^2 + (r+b)(r-b) - 3\right)K(k^2) \right. \\ \left. \qquad + 3(b-r)\,\Pi\big(k^2(b+r)^2,\, k^2\big)\Big) - 4br(4 - 7r^2 - b^2)E(k^2)\right] & k^2 < 1 \\ \\ \frac{2}{9\pi}\left[(1-(r+b)^2)\left(\sqrt{1-(b-r)^2}\,K\left(\frac{1}{k^2}\right) + 3\left(\frac{b-r}{(b+r)\sqrt{1-(b-r)^2}}\right)\right.\right. \\ \left.\left. \qquad \times\,\Pi\left(\frac{1}{k^2(b+r)^2}, \frac{1}{k^2}\right)\right) - \sqrt{1-(b-r)^2}(4 - 7r^2 - b^2)E\left(\frac{1}{k^2}\right)\right] & k^2 \geq 1 \end{cases} \tag{2}$$

with

$$k^2 = \frac{1 - r^2 - b^2 + 2br}{4br}\,. \tag{3}$$

In the expressions above, $K(\cdot)$, $E(\cdot)$, and $\Pi(\cdot, \cdot)$ are the complete elliptic integrals of the first, second kind, and third kind, respectively, defined as

$$\begin{aligned} K(k^2) &\equiv \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\varphi}{\sqrt{1 - k^2\sin^2\varphi}} \\ E(k^2) &\equiv \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2\sin^2\varphi}\,\mathrm{d}\varphi \\ \Pi(n, k^2) &\equiv \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\varphi}{(1 - n\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}}\,. \end{aligned} \tag{4}$$

The solution for s_2 (Equation 1) becomes unstable as $b \to r$ because the elliptic integral Π diverges. In the vicinity of $b = r$ we use Equation (17.7.14) in **?** to express $\Pi(n, k^2)$ in terms of Heuman's Lambda function.

The s_2 term is also unstable when r (and b) become much greater than unity, since in this limit $k^2 \to 0$ and $E(k^2) \to K(k^2) \to \frac{\pi}{2}$. Since s_2 depends on (among other things) a function of the difference between these two elliptic integrals, roundoff error in their computation leads to catastrophic cancellation in the result. In order to circumvent this, we re-write Equation (2) in terms of $E(k^2) - K(k^2)$ and Taylor expand the expression when $r > 1$ to high order in k^2.

3. QUADRATIC LIMB-DARKENING

Any radially symmetric specific intensity profile can be expressed as a sum over the $m = 0$ spherical harmonics. In particular, the radial intensity profile of a quadratically limb-darkened star,

$$I(\mu) = 1 - u_1(1 - \mu) - u_2(1 - \mu)^2 \,, \tag{5}$$

where $\mu = z = \sqrt{1 - x^2 - y^2}$ and u_1 and u_2 are the limb darkening coefficients, can be written in terms of spherical harmonics by re-writing Equation (5) as

$$I(x, y) = (1 - u_1 - 2u_2) + (u_1 + 2u_2)z + u_2 x^2 + u_2 y^2 \,. \tag{6}$$

Performing the change of basis to spherical harmonics (Luger et al. 2018), we have

$$I(x, y) = \frac{2\sqrt{\pi}}{3}(3 - 3u_1 + 4u_2)\, Y_{0,0} + \frac{2\sqrt{\pi}}{\sqrt{3}}(u_1 + 2u_2)\, Y_{1,0} - \frac{4\sqrt{\pi}}{3\sqrt{5}} u_2\, Y_{2,0} \,. \tag{7}$$

Thus, quadratic limb darkening can be expressed exactly as the sum of the first three $m = 0$ spherical harmonics.

REFERENCES

Luger, R., et al. 2018, ApJ, XX, YY

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