Robot Motion Analysis

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Content

- Introduction
- Matrix Representation
- Transformations
- Standard Robot coordinate System
- Numericals

Robot Kinematics: Position Analysis

INTRODUCTION

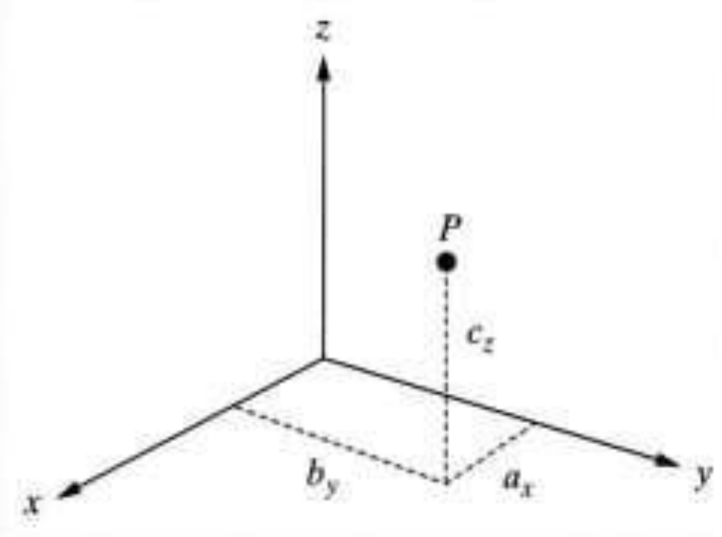
- Forward Kinematics:
  to determine where the robot's hand is?
  (If all joint variables are known)

- Inverse Kinematics:
  to calculate what each joint variable is?
  (If we desire that the hand be located at a particular point)

Matrix Representation

- Representation Of A Point In Space

- A point *P* in space :
  3 coordinates relative to a reference frame



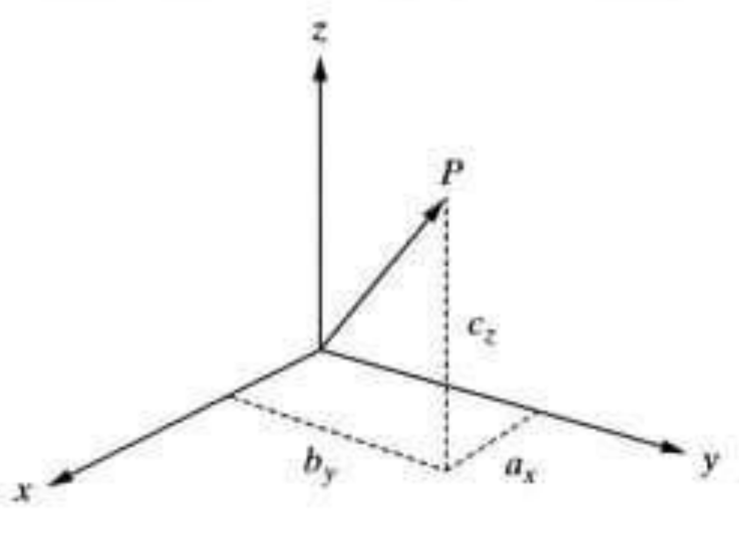
Representation of a point in space

$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Matrix Representation

-Representation of a Vector in Space

- A Vector *P* in space :
  3 coordinates of its tail and of its head



Representation of a vector in space

$$\overline{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

$$\overline{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Where w is Scale factor

Scale Factor w

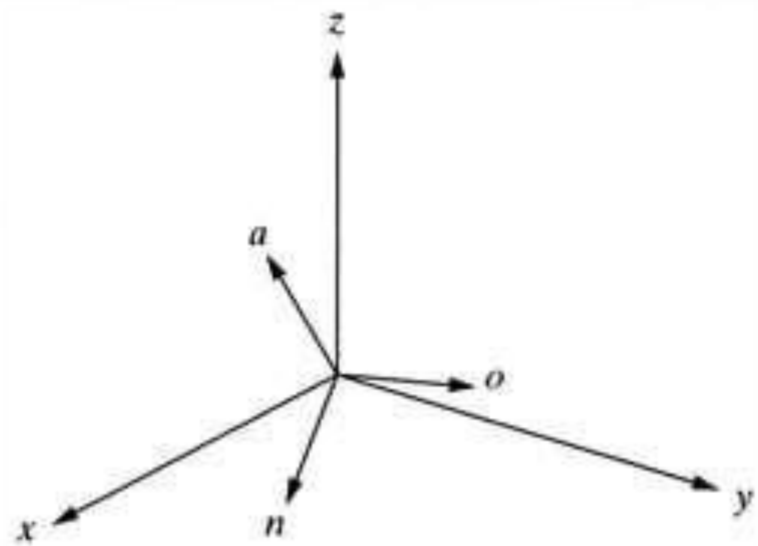
- It can Change overall size of vector similar to zooming function in computer graphics.

- When $w=1$,
  - Size of components remain unchanged
- When $w=0$,
  - It represent a vector whose length is infinite but it represents the direction so called as directional vector

Matrix Representation

-Representation of a Frame at the Origin of a Fixed-Reference Frame

- Each Unit Vector is mutually perpendicular. : normal, orientation, approach vector

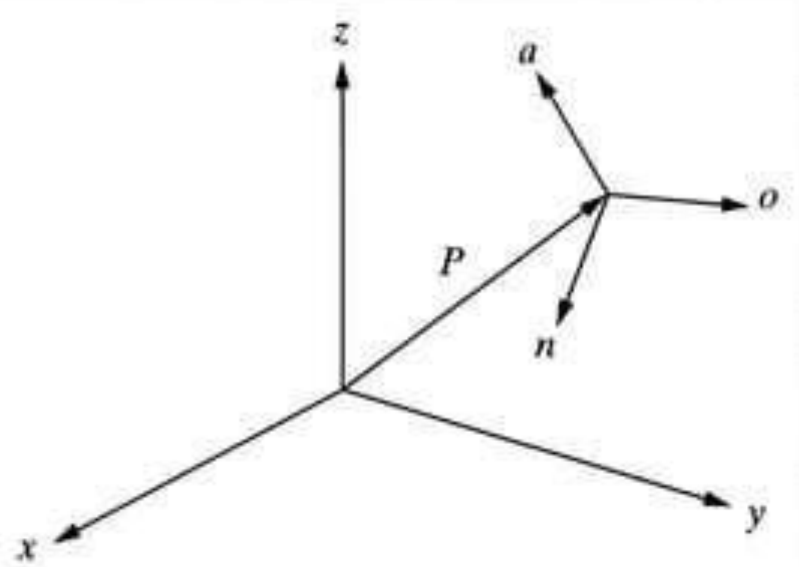


Representation of a frame at the origin of the reference frame

$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

Representation of a Frame in a Fixed Reference Frame

- Each Unit Vector is mutually perpendicular. :
  normal, orientation, approach vector

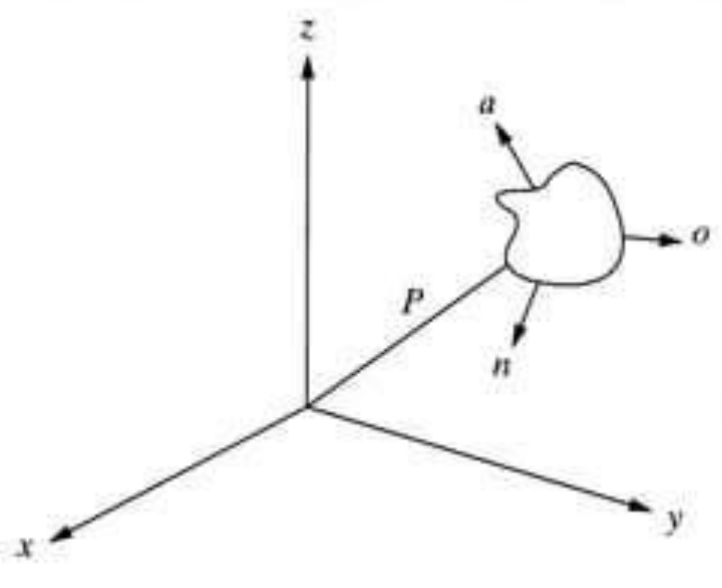


Representation of a frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Rigid Body

- An object can be represented in space by attaching a frame to it and representing the frame in space.



Representation of an object in space

$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous Transformation Matrices

- A transformation matrices must be in square form.
  - It is much easier to calculate the inverse of square matrices.
  - To multiply two matrices, their dimensions must match.

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformations

- A transformation is defined as making a movement in space.

- Types of Transformation are:
  - A pure translation
  - A pure rotation
  - A combination of translation and rotation

Homogeneous Transformations

- **Homogeneous transformations** combine rotation and displacement into a single transformation matrix:

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

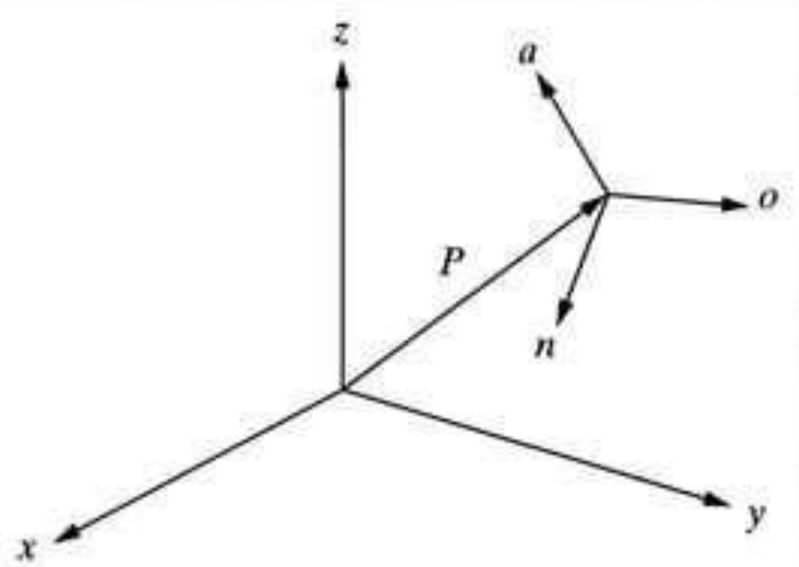
This does the displacemen

This does the rotation

$$= \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Frame in a Fixed Reference Frame

- Each Unit Vector is mutually perpendicular. : normal, orientation, approach vector

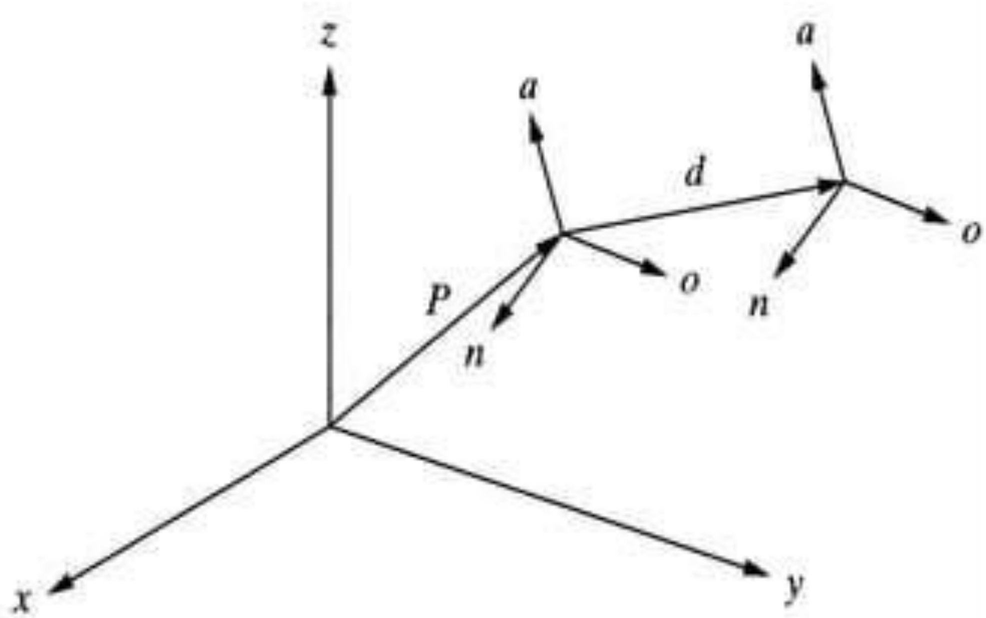


Representation of a frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Translation

- If a frame moves in space without any change in its orientation



$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of an pure translation in space

Numerical Problem-1

A frame F has been moved 10 units along y-axis and 5 units along z-axis of reference frame. Find new location of frame.

$$F = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.439 & 3 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer:

$$F_{new} = Trans(d_x, d_y, d_z) \times F_{old} = Trans(0, 10, 5) \times F_{old}$$

Numerical Problem-1

$$F_{new} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.439 & 3 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

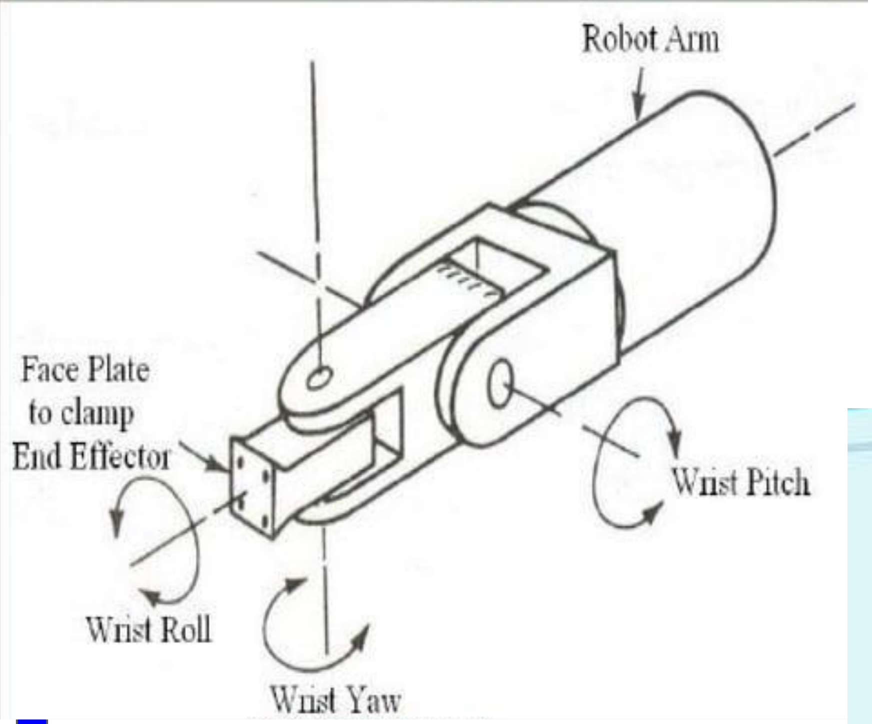
$$= \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.439 & 13 \\ -0.766 & 0 & 0.643 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Axis of Rotation	Euler Angle Name
x	Roll
y	Pitch
z	Yaw

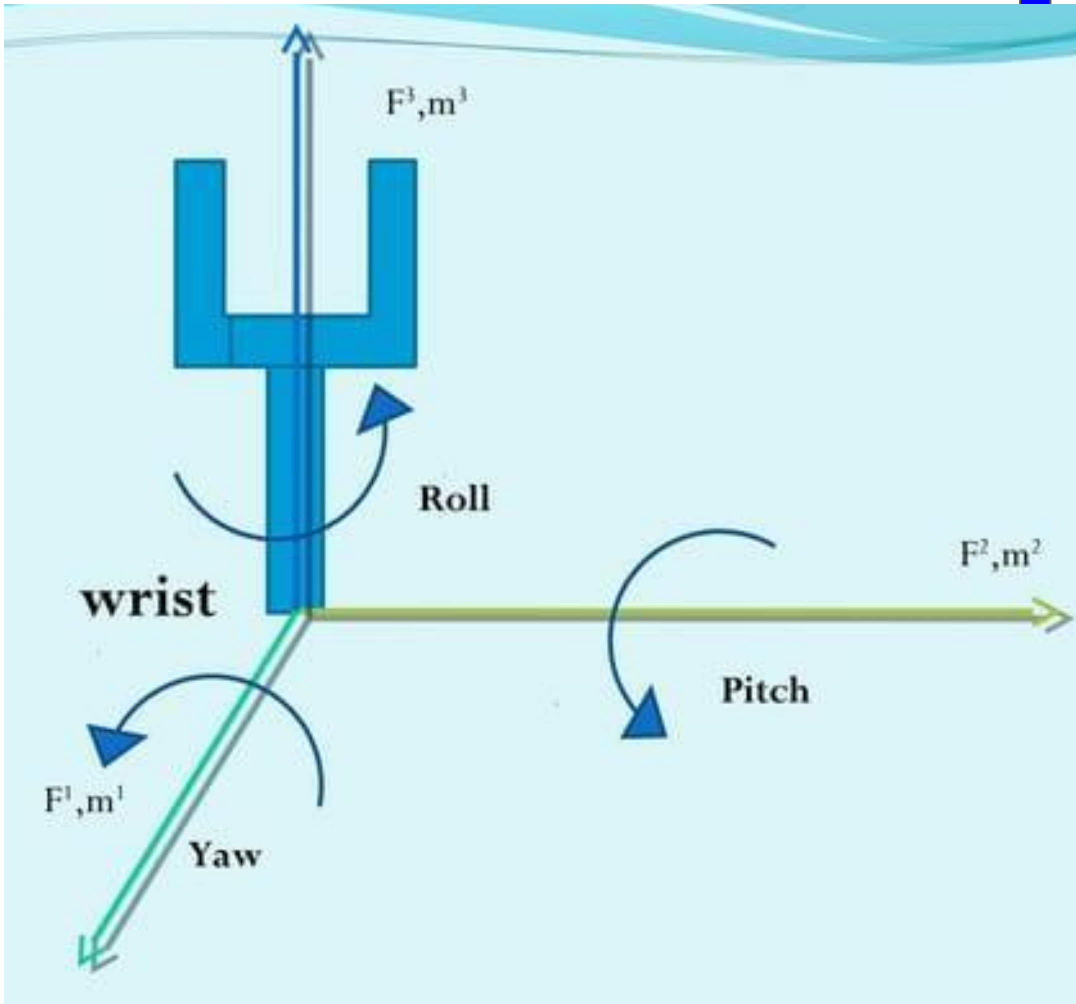
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

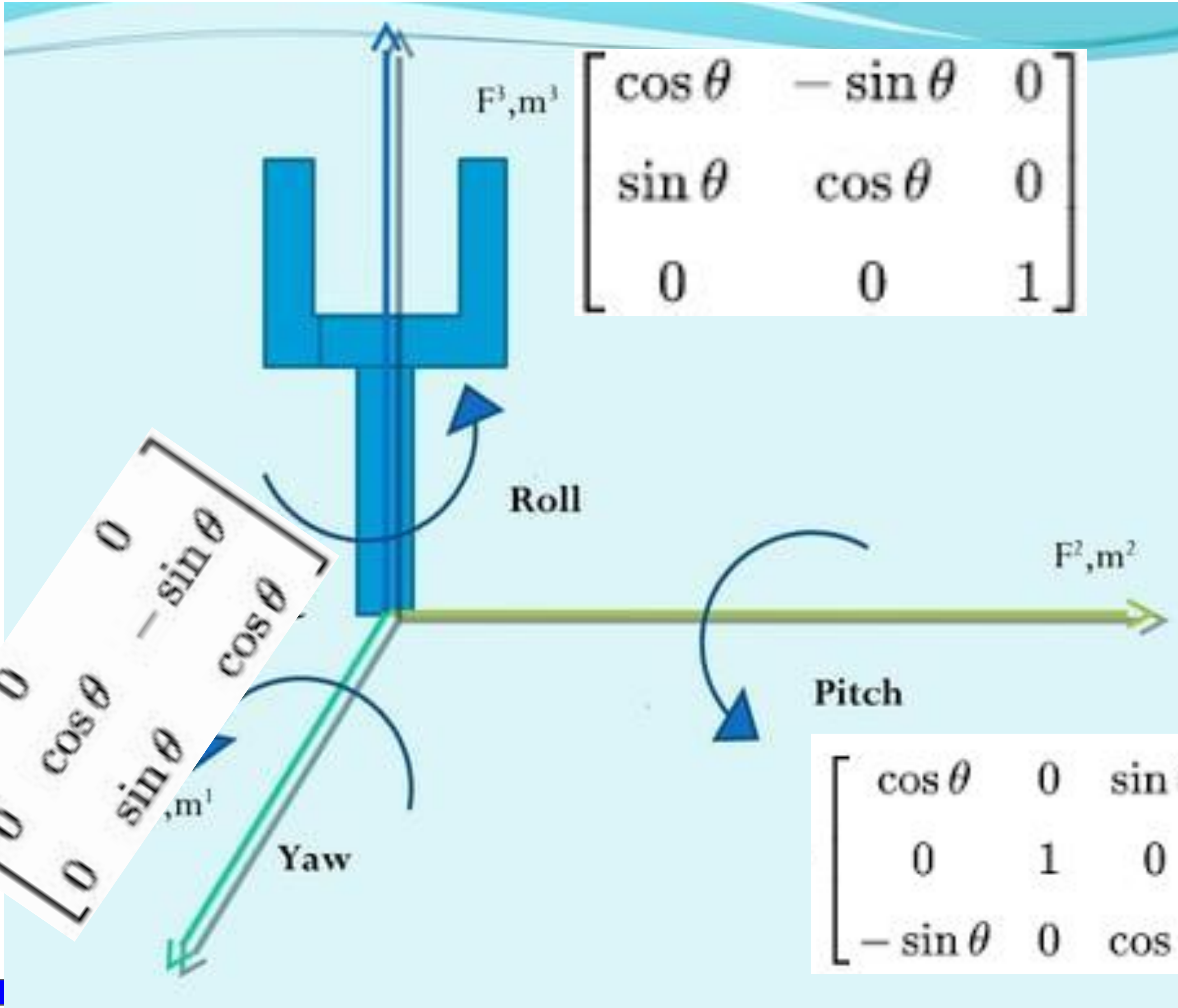
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$



Robot Arm
Face Plate
to clamp
End Effector
Wrist Pitch
Wrist Roll
Wrist Yaw



F³,m³
Roll
F²,m²
wrist
Pitch
F¹,m¹
Yaw



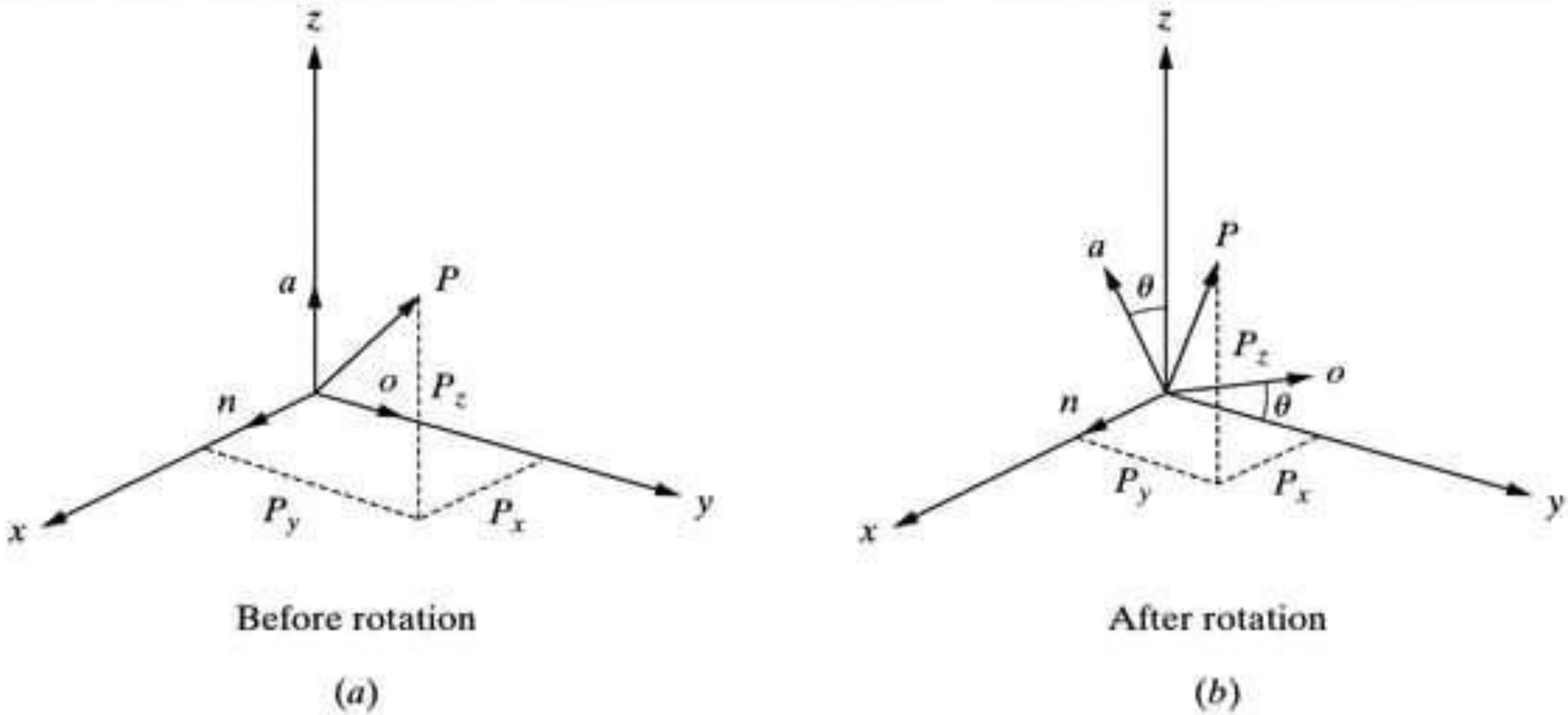
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Pure Rotation about an Axis

Assumption : The frame is at the origin of the reference frame and parallel to it.



Coordinates of a point in a rotating frame before and after rotation.

Pure Rotation about an Axis

$$Rot(z,\theta) = \left[\begin{array}{ccc|c} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combined Transformations

- Combined Transformation consist of a number of successive translations and rotations about fixed reference frame axes.
- The order of matrices written is the opposite of the order of transformations performed.
- If order of matrices changes then final position of robot also changes

Numerical Problem (Forward Kinematics)-2

A point p(7,3,1) is attached to frame and subjected to following transformations. Find coordinate of point relative to reference frame.

1. Rotation of 90° about z-axis
2. Followed by rotation of 90 about y-axis
3. Followed by translation of [4,-3,7].

Answer: The matrix equation is given as

$$p_{xyz} = Trans(4, -3, 7)Rot(y, 90)Rot(z, 90)p_{noa}$$

Numerical Problem-2

$$p_{xyz} = Trans(4,-3,7)Rot(y,90)Rot(z,90)p_{noa}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$