information to navigate robustly in an array of environments, as we shall see in the case studies below.

5.5 Map Representation

The problem of representing the environment in which the robot moves is a dual of the problem of representing the robot's possible position or positions. Decisions made regarding the environmental representation can have impact on the choices available for robot position representation. Often the fidelity of the position representation is bounded by the fidelity of the map.

Three fundamental relationships must be understood when choosing a particular map representation:

1. The precision of the map must appropriately match the precision with which the robot needs to achieve its goals.
2. The precision of the map and the type of features represented must match the precision and data types returned by the robot's sensors.
3. The complexity of the map representation has direct impact on the computational complexity of reasoning about mapping, localization, and navigation.

In the following sections, we identify and discuss critical design choices in creating a map representation. Each such choice has great impact on the relationships listed above and on the resulting robot localization architecture. As we shall see, the choice of possible map representations is broad. Selecting an appropriate representation requires understanding all of the trade-offs inherent in that choice as well as understanding the specific context in which a particular mobile robot implementation must perform localization. In general, the environmental representation and model can be roughly classified as presented in chapter 4, section 4.3.

5.5.1 Continuous representations

A continuous-valued map is one method for *exact* decomposition of the environment. The position of environmental features can be annotated precisely in continuous space. Mobile robot implementations to date use continuous maps only in 2D representations, as further dimensionality can result in computational explosion.

A common approach is to combine the exactness of a continuous representation with the compactness of the *closed-world assumption*. This means that one assumes that the representation will specify all environmental objects in the map, and that any area in the map that is devoid of objects has no objects in the corresponding portion of the environment. Thus, the total storage needed in the map is proportional to the density of objects in the environment, and a sparse environment can be represented by a low-memory map.

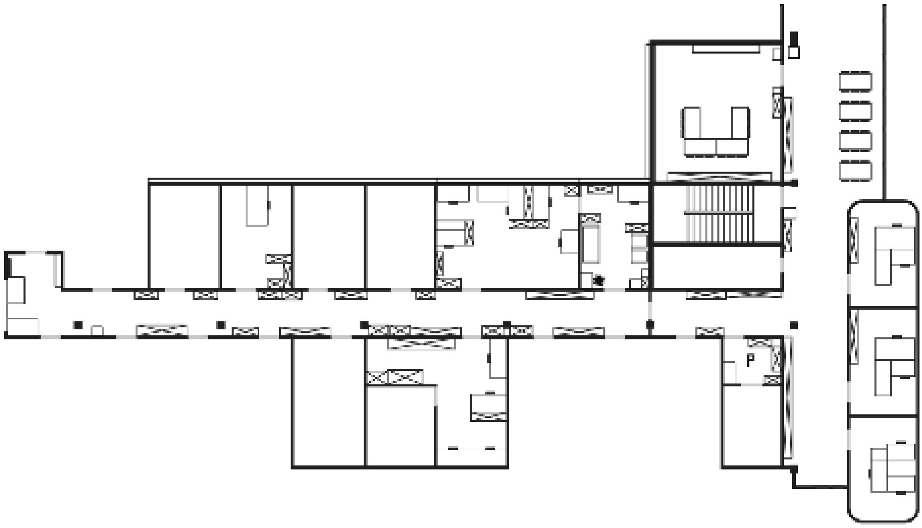

Figure 5.12
A continuous representation using polygons as environmental obstacles.

One example of such a representation, shown in figure 5.12, is a 2D representation in which polygons represent all obstacles in a continuous-valued coordinate space. This is similar to the method used by Latombe [21, 98] and others to represent environments for mobile robot path-planning techniques.

In the case of [21, 98], most of the experiments are in fact simulations run exclusively within the computer's memory. Therefore, no real effort would have been expended to attempt to use sets of polygons to describe a real-world environment, such as a park or office building.

In other work in which real environments must be captured by the maps, one sees a trend toward selectivity and abstraction. The human map maker tends to capture on the map, for localization purposes, only objects that can be detected by the robot's sensors and, furthermore, only a subset of the features of real-world objects.

It should be immediately apparent that geometric maps can capably represent the physical locations of objects without referring to their texture, color, elasticity, or any other such secondary features that do not relate directly to position and space. In addition to this level of simplification, a mobile robot map can further reduce memory usage by capturing only aspects of object geometry that are immediately relevant to localization. For example, all objects may be approximated using very simple convex polygons, sacrificing map felicity for the sake of computational speed.

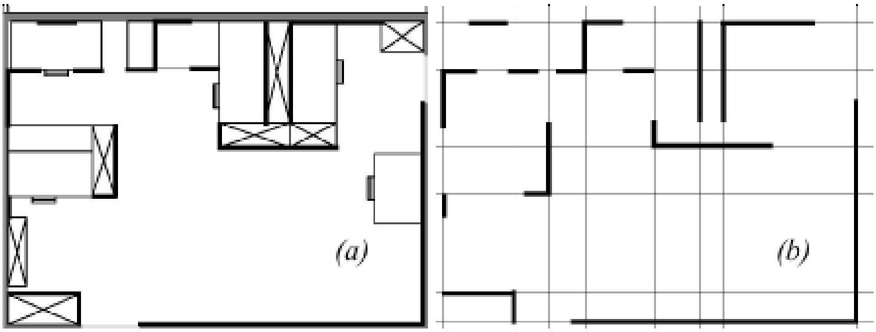

Figure 5.13
Example of a continuous-valued line representation of EPFL. (a) Real map. (b) Representation with a set of infinite lines.

One excellent example involves line extraction. Many indoor mobile robots rely upon laser rangefinding devices to recover distance readings to nearby objects. Such robots can automatically extract best-fit lines from the dense range data provided by thousands of points of laser strikes. Given such a line extraction sensor, an appropriate continuous mapping approach is to populate the map with a set of infinite lines. The continuous nature of the map guarantees that lines can be positioned at arbitrary positions in the plane and at arbitrary angles. The abstraction of real environmental objects such as walls and intersections captures only the information in the map representation that matches the type of information recovered by the mobile robot's rangefinding sensor.

Figure 5.13 shows a map of an indoor environment at EPFL using a continuous line representation. Note that the only environmental features captured by the map are straight lines, such as those found at corners and along walls. This represents not only a sampling of the real world of richer features but also a simplification, for an actual wall may have texture and relief that is not captured by the mapped line.

The impact of continuous map representations on position representation is primarily positive. In the case of single-hypothesis position representation, that position may be specified as any continuous-valued point in the coordinate space, and therefore extremely high accuracy is possible. In the case of multiple-hypothesis position representation, the continuous map enables two types of multiple position representation.

In one case, the possible robot position may be depicted as a geometric shape in the hyperplane, such that the robot is known to be within the bounds of that shape. This is shown in figure 5.29, in which the position of the robot is depicted by an oval bounding area.

Yet, the continuous representation does not disallow representation of position in the form of a discrete set of possible positions. For instance, in [62] the robot position belief state is captured by sampling nine continuous-valued positions from within a region near the robot's best-known position. This algorithm captures, within a continuous space, a discrete sampling of possible robot positions.

In summary, the key advantage of a continuous map representation is the potential for high accuracy and expressiveness with respect to the environmental configuration as well as the robot position within that environment. The danger of a continuous representation is that the map may be computationally costly. But this danger can be tempered by employing abstraction and capturing only the most relevant environmental features. Together with the use of the *closed-world assumption*, these techniques can enable a continuous-valued map to be no more costly, and sometimes even less costly, than a standard discrete representation.

5.5.2 Decomposition strategies

In the section above, we discussed one method of simplification, in which the continuous map representation contains a set of infinite lines that approximate real-world environmental lines based on a 2D slice of the world. Basically this transformation from the real world to the map representation is a filter that removes all nonstraight data and furthermore extends line segment data into infinite lines that require fewer parameters.

A more dramatic form of simplification is *abstraction*: a general decomposition and selection of environmental features. In this section, we explore decomposition as applied in its more extreme forms to the question of map representation.

Why would one radically decompose the real environment during the design of a map representation? The immediate disadvantage of decomposition and abstraction is the loss of fidelity between the map and the real world. Both qualitatively, in terms of overall structure, and quantitatively, in terms of geometric precision, a highly abstract map does not compare favorably to a high-fidelity map.

Despite this disadvantage, decomposition and abstraction may be useful if the abstraction can be planned carefully so as to capture the relevant, *useful* features of the world while discarding all other features. The advantage of this approach is that the map representation can potentially be minimized. Furthermore, if the decomposition is hierarchical, such as in a pyramid of recursive abstraction, then reasoning and planning with respect to the map representation may be computationally far superior to planning in a fully detailed world model.

A standard, lossless form of *opportunistic decomposition* is termed *exact cell decomposition*. This method, introduced by Latombe [21], achieves decomposition by selecting boundaries between discrete cells based on geometric criticality.

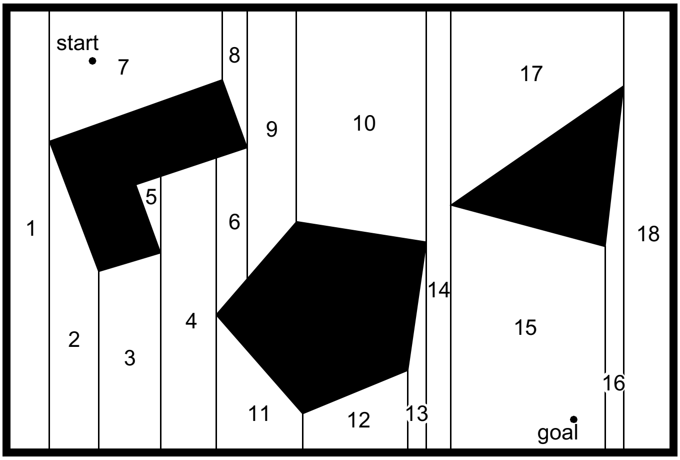

Figure 5.14
Example of exact cell decomposition.

Figure 5.14 depicts an exact decomposition of a planar workspace populated by polygonal obstacles. The map representation tessellates the space into areas of free space. The representation can be extremely compact because each such area is actually stored as a single node, resulting in a total of only eighteen nodes in this example.

The underlying assumption behind this decomposition is that the particular position of a robot within each area of free space does not matter. What matters is the robot's ability to traverse from each area of free space to the adjacent areas. Therefore, as with other representations we will see, the resulting graph captures the adjacency of map locales. If indeed the assumptions are valid and the robot does not care about its precise position within a single area, then this can be an effective representation that nonetheless captures the connectivity of the environment.

Such an exact decomposition is not always appropriate. Exact decomposition is a function of the particular environment obstacles and free space. If this information is expensive to collect or even unknown, then such an approach is not feasible.

An alternative is *fixed decomposition*, in which the world is tessellated, transforming the continuous real environment into a discrete approximation for the map. Such a transformation is demonstrated in figure 5.15, which depicts what happens to obstacle-filled and free areas during this transformation. The key disadvantage of this approach stems from its *inexact* nature. It is possible for narrow passageways to be lost during such a transformation, as shown in figure 5.15. Formally, this means that fixed decomposition is sound but not complete. Yet another approach is adaptive cell decomposition, as presented in figure 5.16.

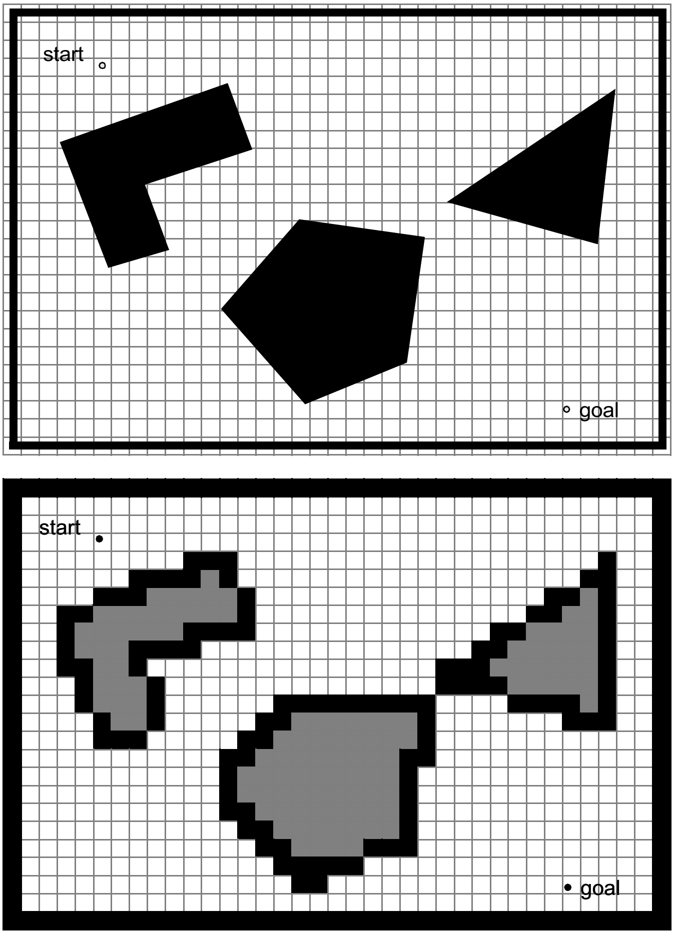

Figure 5.15
Fixed decomposition of the same space (narrow passage disappears).

The concept of fixed decomposition is extremely popular in mobile robotics; it is perhaps the single most common map representation technique currently utilized. One very popular version of fixed decomposition is known as the *occupancy grid* representation [112]. In an occupancy grid, the environment is represented by a discrete grid, where each cell is either filled (part of an obstacle) or empty (part of free space). This method is of particular value when a robot is equipped with range-based sensors because the range values

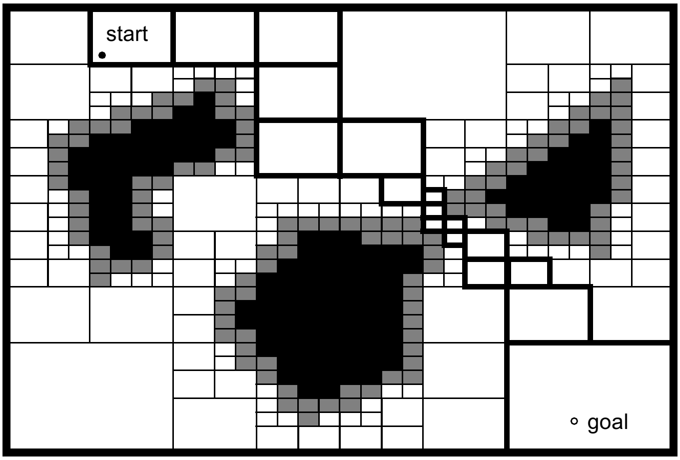

Figure 5.16
Example of adaptive (approximate variable-cell) decomposition of an environment [21]. The rectangle, bounding the free space, is decomposed into four identical rectangles. If the interior of a rectangle lies completely in free space or in the configuration space obstacle, it is not decomposed further. Otherwise, it is recursively decomposed into four rectangles until some predefined resolution is attained. The white cells lie outside the obstacles, the black inside, and the gray are part of both regions.

of each sensor, combined with the absolute position of the robot, can be used directly to update the filled or empty value of each cell.

In the occupancy grid, each cell may have a counter, whereby the value 0 indicates that the cell has not been "hit" by any ranging measurements and, therefore, it is likely free space. As the number of ranging strikes increases, the cell's value is incremented and, above a certain threshold, the cell is deemed to be an obstacle. The values of cells are commonly discounted when a ranging strike travels *through* the cell, striking a further cell. By also discounting the values of cells over time, both hysteresis and the possibility of transient obstacles can be represented using this occupancy grid approach. Figure 5.17 depicts an occupancy grid representation in which the darkness of each cell is proportional to the value of its counter. One commercial robot that uses a standard occupancy grid for mapping and navigation is the Cye robot [163].

There remain two main disadvantages of the occupancy grid approach. First, the size of the map in robot memory grows with the size of the environment and if a small cell size is used, this size can quickly become untenable. This occupancy grid approach is not compatible with the *closed-world assumption*, which enabled continuous representations to have potentially very small memory requirements in large, sparse environments. In contrast, the

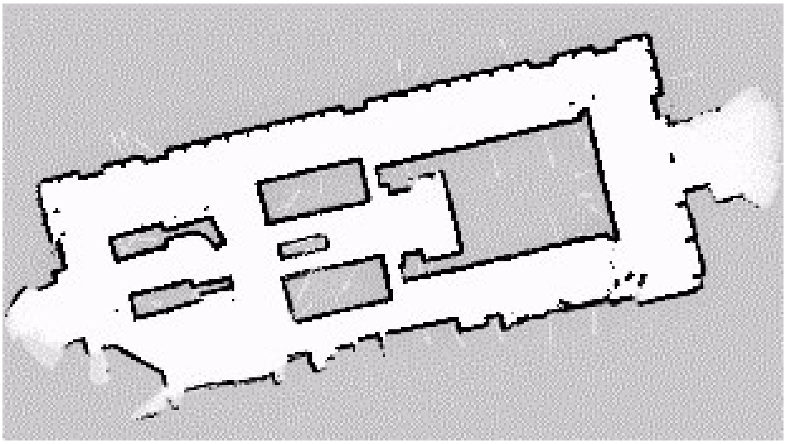

Figure 5.17
Example of an occupancy grid map representation (courtesy of S. Thrun [145]).

occupancy grid must have memory set aside for every cell in the matrix. Furthermore, any fixed decomposition method such as this imposes a geometric grid on the world *a priori*, regardless of the environmental details. This can be inappropriate in cases where geometry is not the most salient feature of the environment.

For these reasons, an alternative, called *topological* decomposition, has been the subject of some exploration in mobile robotics. Topological approaches avoid direct measurement of geometric environmental qualities, instead concentrating on characteristics of the environment that are most relevant to the robot for localization.

Formally, a topological representation is a graph that specifies two things: *nodes* and the *connectivity* between those nodes. Insofar as a topological representation is intended for the use of a mobile robot, nodes are used to denote areas in the world and arcs are used to denote adjacency of pairs of nodes. When an arc connects two nodes, then the robot can traverse from one node to the other without requiring traversal of any other intermediary node.

Adjacency is clearly at the heart of the topological approach, just as adjacency in a cell decomposition representation maps to geometric adjacency in the real world. However, the topological approach diverges in that the nodes are not of fixed size or even specifications of free space. Instead, nodes document an area based on any sensor discriminant such that the robot can recognize entry and exit of the node.

Figure 5.18 depicts a topological representation of a set of hallways and offices in an indoor environment. In this case, the robot is assumed to have an intersection detector, perhaps using sonar and vision to find intersections between halls and between halls and

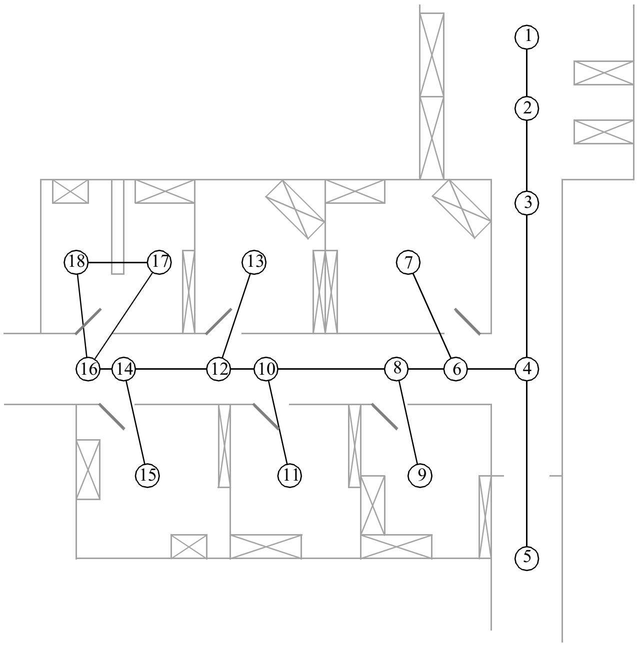

Figure 5.18
A topological representation of an indoor office area.

rooms. Note that nodes capture geometric space, and arcs in this representation simply represent connectivity.

Another example of topological representation is the work of Simhon and Dudek [134], in which the goal is to create a mobile robot that can capture the most interesting aspects of an area for human consumption. The nodes in their representation are visually striking locales rather than route intersections.

In order to navigate using a topological map robustly, a robot must satisfy two constraints. First, it must have a means for detecting its current position in terms of the nodes of the topological graph. Second, it must have a means for traveling between nodes using robot motion. The node sizes and particular dimensions must be optimized to match the sensory discrimination of the mobile robot hardware. This ability to "tune" the representation to the robot's particular sensors can be an important advantage of the topological approach. However, as the map representation drifts further away from true geometry, the expressiveness of the representation for accurately and precisely describing a robot position is lost. Therein lies the compromise between the discrete cell-based map representations and the topological representations. Interestingly, the continuous map representation has

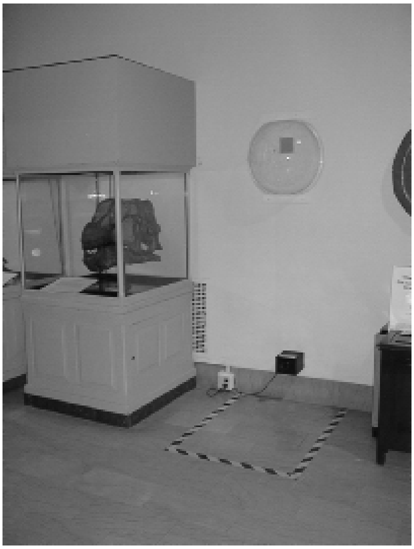

Figure 5.19
An artificial landmark used by Chips during autonomous docking.

the potential to be both compact like a topological representation and precise as with all direct geometric representations.

Yet, a chief motivation of the topological approach is that the environment may contain important nongeometric features – features that have no ranging relevance but are useful for localization. In chapter 4 we described such whole-image vision-based features.

In contrast to these whole-image feature extractors, often spatially localized landmarks are artificially placed in an environment to impose a particular visual-topological connectivity upon the environment. In effect, the artificial landmark can impose artificial structure. Examples of working systems operating with this landmark-based strategy have also demonstrated success. Latombe's landmark-based navigation research [99] has been implemented on real-world indoor mobile robots that employ paper landmarks attached to the ceiling as the locally observable features. Chips, the museum robot, is another robot that uses man-made landmarks to obviate the localization problem. In this case, a bright pink square serves as a landmark with dimensions and color signature that would be hard to accidentally reproduce in a museum environment [118]. One such museum landmark is shown in figure 5.19.

In summary, range is clearly not the only measurable and useful environmental value for a mobile robot. This is particularly true with the advent of color vision, as well as laser rangefinding, which provides reflectance information in addition to range information. Choosing a map representation for a particular mobile robot requires, first, understanding the sensors available on the mobile robot, and, second, understanding the mobile robot's functional requirements (e.g., required goal precision and accuracy).

5.5.3 State of the art: current challenges in map representation

The sections above describe major design decisions in regard to map representation choices. There are, however, fundamental real-world features that mobile robot map representations do not yet represent well. These continue to be the subject of open research, and several such challenges are described below.

The real world is dynamic. As mobile robots come to inhabit the same spaces as humans, they will encounter moving people, cars, strollers, and the transient obstacles placed and moved by humans as they go about their activities. This is particularly true when one considers the home environment with which domestic robots will someday need to contend.

The map representations described above do not, in general, have explicit facilities for identifying and distinguishing between permanent obstacles (e.g., walls, doorways, etc.) and transient obstacles (e.g., humans, shipping packages, etc.). The current state of the art in terms of mobile robot sensors is partly to blame for this shortcoming. Although vision research is rapidly advancing, robust sensors that discriminate between moving animals and static structures *from a moving reference frame* are not yet available. Furthermore, estimating the motion vector of transient objects remains a research problem.

Usually, the assumption behind the above map representations is that all objects on the map are effectively static. Partial success can be achieved by discounting mapped objects over time. For example, occupancy grid techniques can be more robust to dynamic settings by introducing temporal discounting, effectively treating transient obstacles as noise. The more challenging process of map creation is particularly fragile to environmental dynamics; most mapping techniques generally require that the environment be free of moving objects during the mapping process. One exception to this limitation involves topological representations. Because precise geometry is not important, transient objects have little effect on the mapping or localization process, subject to the critical constraint that the transient objects must not change the topological connectivity of the environment. Still, neither the occupancy grid representation nor a topological approach is actively recognizing and representing transient objects as distinct from both sensor error and permanent map features.

As vision sensing provides more robust and more informative content regarding the transience and motion details of objects in the world, mobile roboticists will in time propose representations that make use of that information. A classic example involves occlusion by human crowds. Museum tour guide robots generally suffer from an extreme amount of occlusion. If the robot's sensing suite is located along the robot's body, then the robot is effectively blind when a group of human visitors completely surround the robot. This is because its map contains only environmental features that are, at that point, fully hidden from the robot's sensors by the wall of people. In the best case, the robot should recognize its occlusion and make no effort to localize using these invalid sensor readings. In the worst case, the robot will localize with the fully occluded data, and will update its location incorrectly. A vision sensor that can discriminate the local conditions of the robot (e.g,. we are surrounded by people) can help eliminate this error mode.

A second open challenge in mobile robot localization involves the traversal of open spaces. Existing localization techniques generally depend on local measures such as range, thereby demanding environments that are somewhat densely filled with objects that the sensors can detect and measure. Wide-open spaces such as parking lots, fields of grass, and indoor atriums such as those found in convention centers pose a difficulty for such systems because of their relative sparseness. Indeed, when populated with humans, the challenge is exacerbated because any mapped objects are almost certain to be occluded from view by the people.

Once again, more recent technologies provide some hope of overcoming these limitations. Both vision and state-of-the-art laser rangefinding devices offer outdoor performance with ranges of up to a hundred meters and more. Of course, GPS performs even better. Such long-range sensing may be required for robots to localize using distant features.

This trend teases out a hidden assumption underlying most topological map representations. Usually, topological representations make assumptions regarding spatial locality: a node contains objects and features that are themselves within that node. The process of map creation thus involves making nodes that are, in their own self-contained way, recognizable by virtue of the objects contained within the node. Therefore, in an indoor environment, each room can be a separate node, and this is reasonable because each room will have a layout and a set of belongings that are unique to that room.

However, consider the outdoor world of a wide-open park. Where should a single node end and the next node begin? The answer is unclear because objects that are far away from the current node, or position, can yield information for the localization process. For example, the hump of a hill at the horizon, the position of a river in the valley, and the trajectory of the sun all are nonlocal features that have great bearing on one's ability to infer current position. The spatial locality assumption is violated and, instead, replaced by a visibility criterion: the node or cell may need a mechanism for representing objects that are measurable and visible from that cell. Once again, as sensors improve and, in this case, as outdoor locomotion mechanisms improve, there will be greater urgency to solve problems associated with localization in wide-open settings, with and without GPS-type global localization sensors.

We end this section with one final open challenge that represents one of the fundamental academic research questions of robotics: sensor fusion. A variety of measurement types are possible using off-the-shelf robot sensors, including heat, range, acoustic and light-based reflectivity, color, texture, friction, and so on. Sensor fusion is a research topic closely related to map representation. Just as a map must embody an environment in sufficient detail for a robot to perform localization and reasoning, sensor fusion demands a representation of the world that is sufficiently general and expressive that a variety of sensor types can have their data correlated appropriately, strengthening the resulting percepts well beyond that of any individual sensor's readings.

Perhaps the only general implementation of sensor fusion to date is that of neural network classifier. Using this technique, any number and any type of sensor values may be jointly combined in a network that will use whatever means necessary to optimize its classification accuracy. For the mobile robot that must use a human-readable internal map representation, no equally general sensor fusion scheme has yet been born. It is reasonable to expect that, when the sensor fusion problem is solved, integration of a large number of disparate sensor types may easily result in sufficient discriminatory power for robots to achieve real-world navigation, even in wide-open and dynamic circumstances such as a public square filled with people.

5.6 Probabilistic Map-Based Localization

5.6.1 Introduction

As stated earlier, multiple-hypothesis position representation is advantageous because the robot can explicitly track its own beliefs regarding its possible positions in the environment. Ideally, the robot's *belief state* will change, over time, as is consistent with its motor outputs and perceptual inputs. One geometric approach to multiple-hypothesis representation, mentioned earlier, involves identifying the possible positions of the robot by specifying a polygon in the environmental representation [98]. This method does not provide any indication of the relative chances between various possible robot positions.

Probabilistic techniques differ from this because they explicitly identify probabilities with the possible robot positions, and for this reason these methods have been the focus of recent research. In the following sections we present two classes of probabilistic localization. The first class, *Markov localization*, uses an explicitly specified probability distribution across all possible robot positions. The second method, *Kalman filter localization*, uses a Gaussian probability density representation of robot position and scan matching for localization. Unlike Markov localization, Kalman filter localization does not independently consider each possible pose in the robot's configuration space. Interestingly, the Kalman filter localization process results from the Markov localization axioms if the robot's position uncertainty is assumed to have a Gaussian form [3, pp. 43-44].

Before discussing each method in detail, we present the general robot localization problem and solution strategy. Consider a mobile robot moving in a known environment. As it