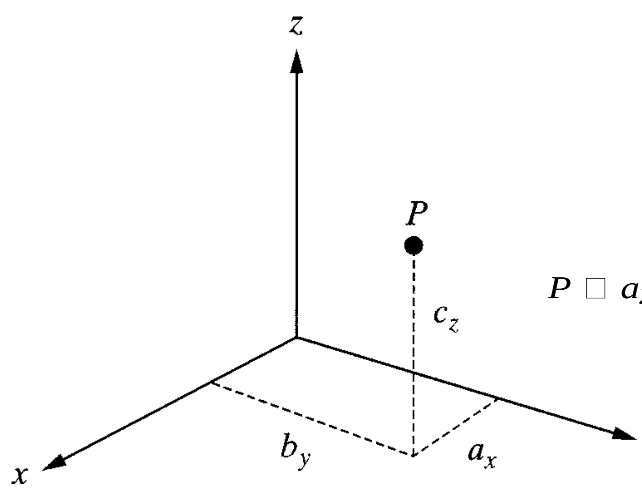
MATRIX REPRESENTATION

Representation of a Point in Space

A point *P* in space: 3 coordinate relative to a reference frame

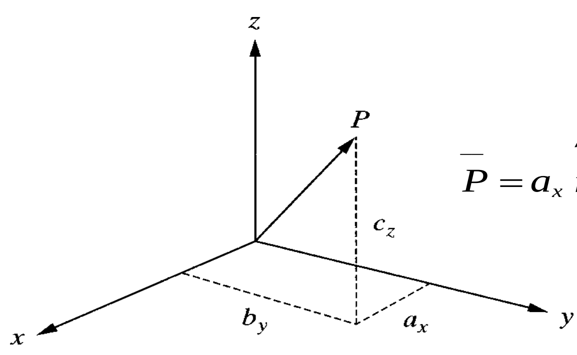


$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Fig 2.5 Representation of a point in space

Representation of a Vector in Space

A Vector *P* in space: 3 coordinates of its tail and of its head



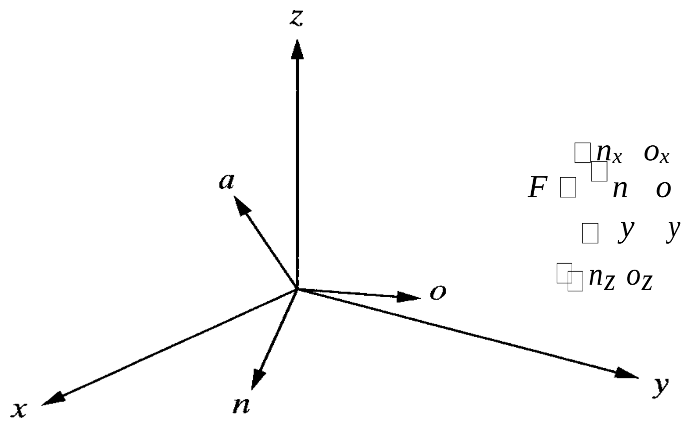
$$\bar{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Fig 2.6 Representation of a vector in space

$$\bar{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Representation of a Frame at the Origin of a Fixed-Reference Frame

Each Unit Vector is mutually perpendicular: normal, orientation, approach vector



$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

Fig. 2.7 Representation of a frame at the origin of the reference frame

Representation of a Frame in a Fixed Reference Frame

Each Unit Vector is mutually perpendicular: normal, orientation, approach vector

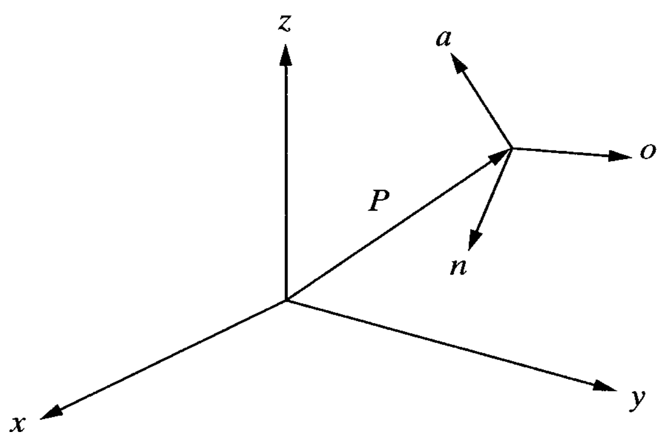


Fig.2.8 Representation of a frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Rigid Body

An object can be represented in space by attaching a frame to it and representing the frame in space.

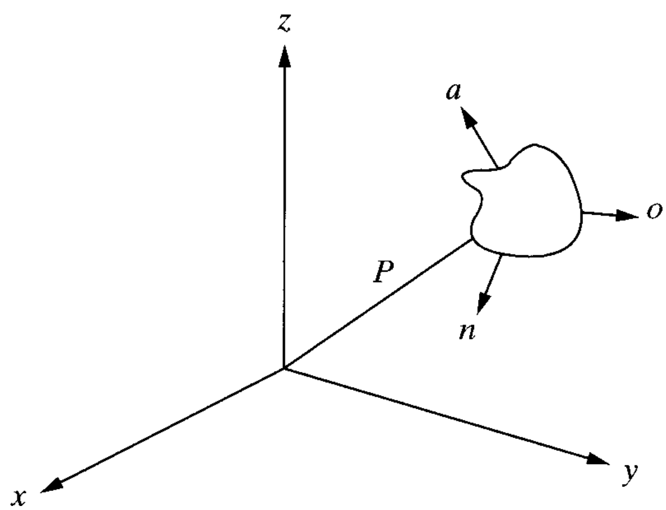


Fig. 2.9 Representation of an object in space

$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HOMOGENEOUS TRANSFORMATION MATRICES

Transformation matrices must be in square form. It is much easier to calculate the inverse of square matrices. To multiply two matrices, their dimensions must match.

Representation of a Pure Translation

- A transformation is defined as making a movement in space.
- A pure translation.
- A pure rotation about an axis.
- A combination of translation or rotations

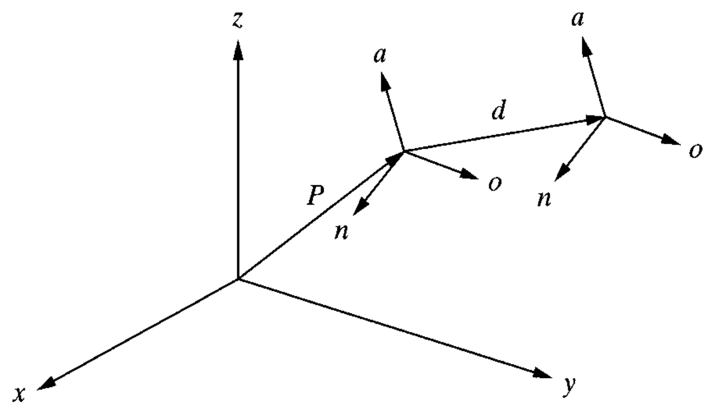


Fig. 2.10 Representation of a pure translation in space

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Rotation about an Axis

Assumption: The frame is at the origin of the reference frame and parallel to it.

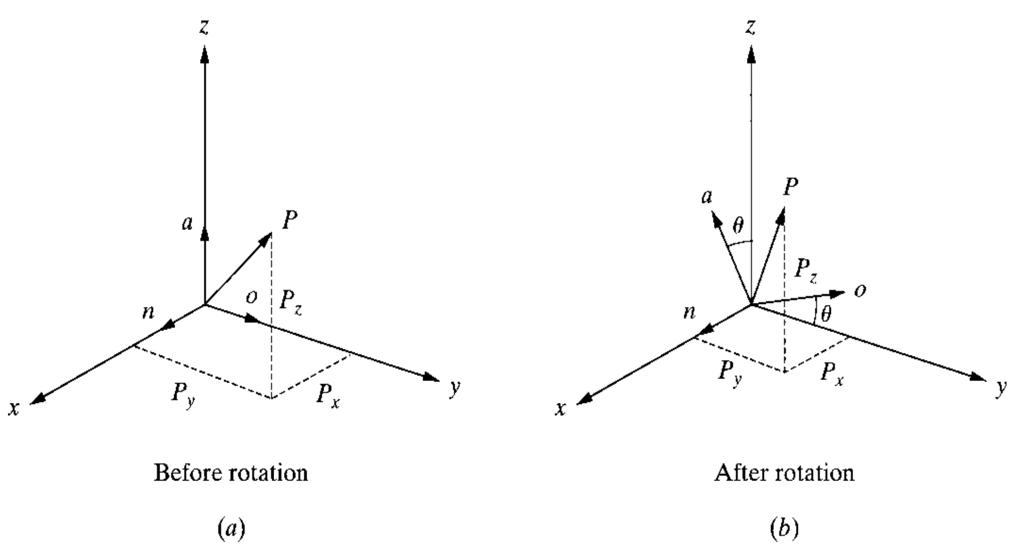


Fig.2.11 Coordinates of a point in a rotating frame before and after rotation

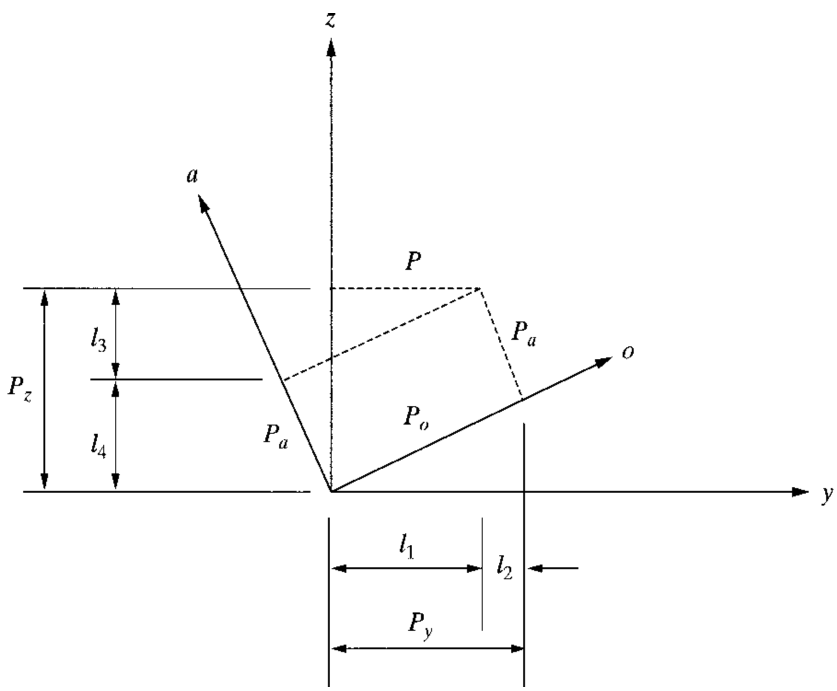


Fig. 2.12 Coordinates of a point relative to the reference

Representation of Combined Transformations

A number of successive translations and rotations

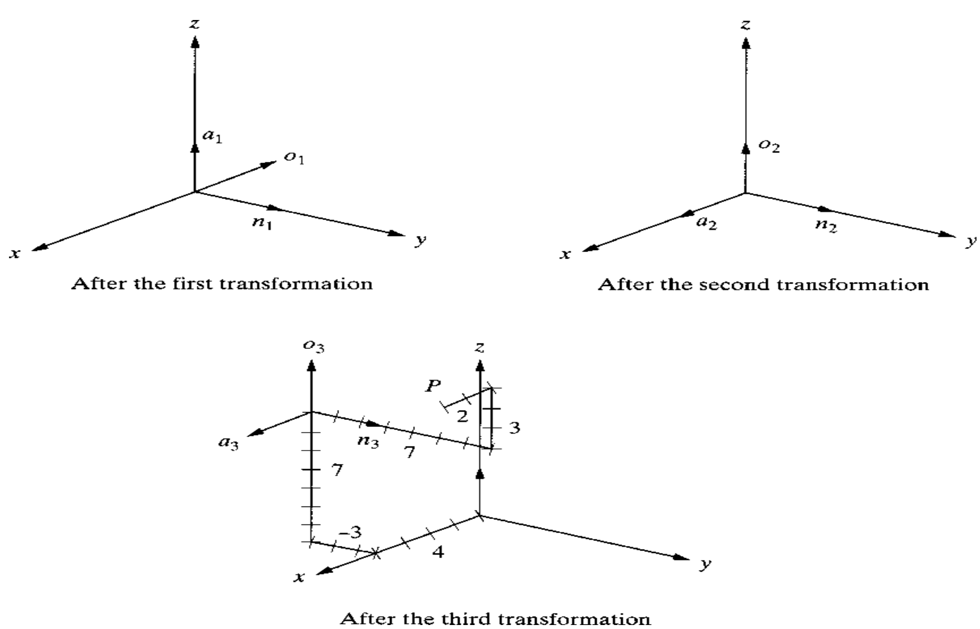


Fig. 2.13 Effects of three successive transformations

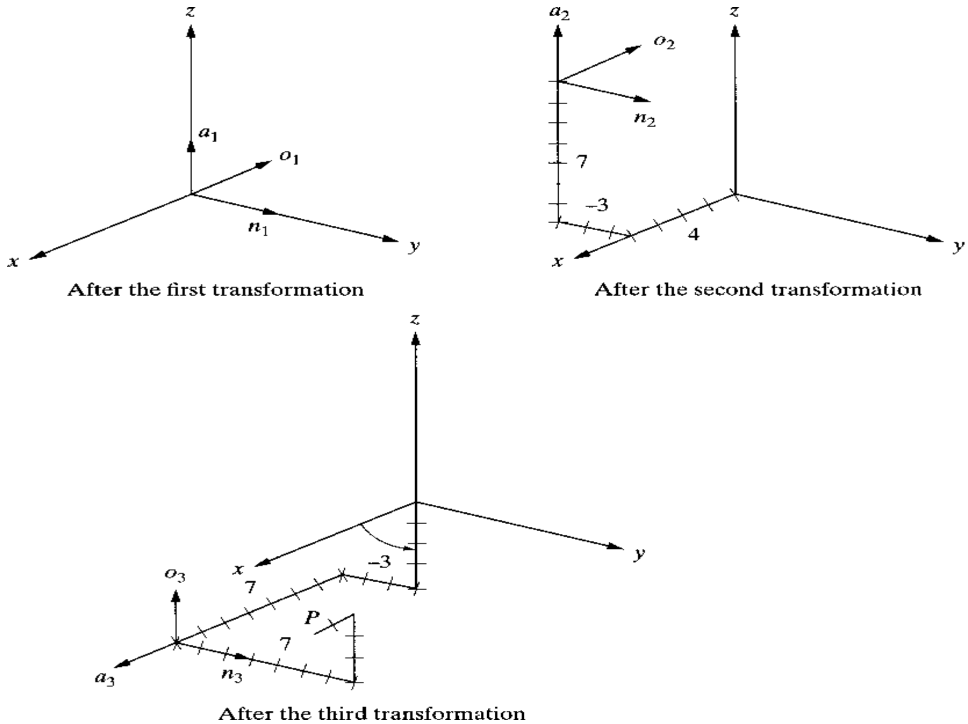


Fig 2.14 Changing the order of transformations will change the final result

Transformations Relative to the Rotating Frame

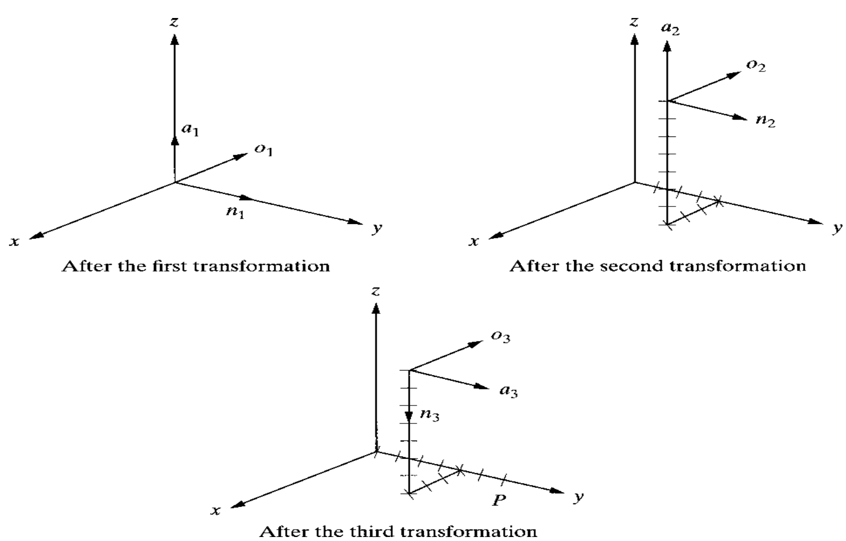


Fig.2.15 Transformations relative to the current frames