

PLOTS OF LOGICAL OPERATIONS ON INTEGERS

Below are 3D plots of $\text{and}(x, y)$, $\text{or}(x, y)$, and $\text{xor}(x, y)$ as functions of integers x and y . These plots were produced by Mathematica. For each function, two plots are shown, one for a word size of three bits and the other for a word size of five bits.

This material was suggested by Guy Steele.

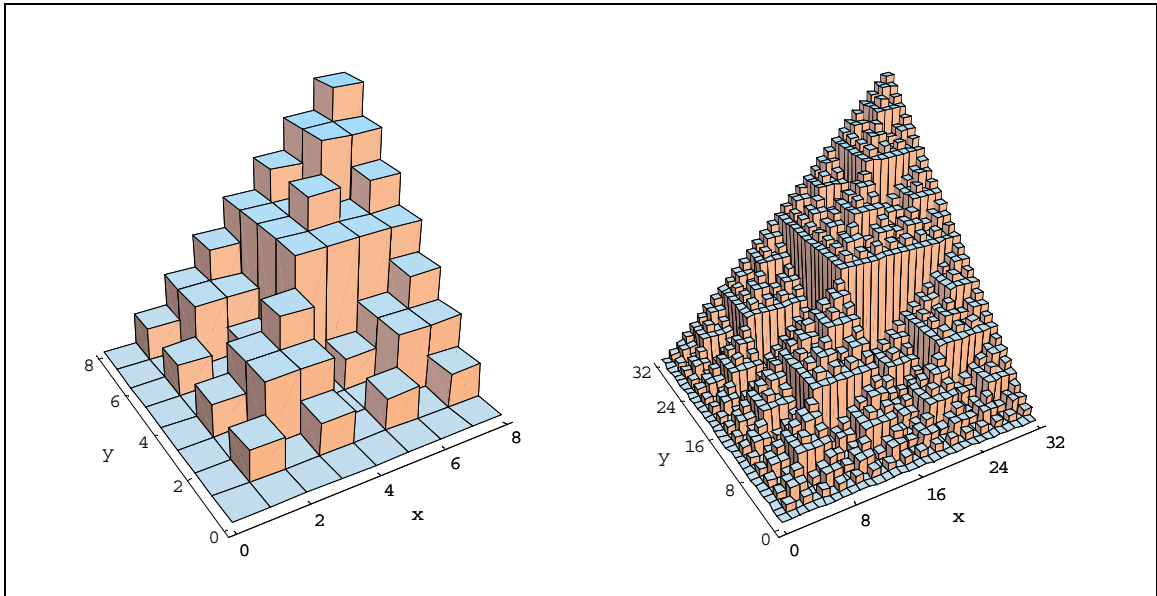


FIGURE 1. Plots of the logical *and* function.

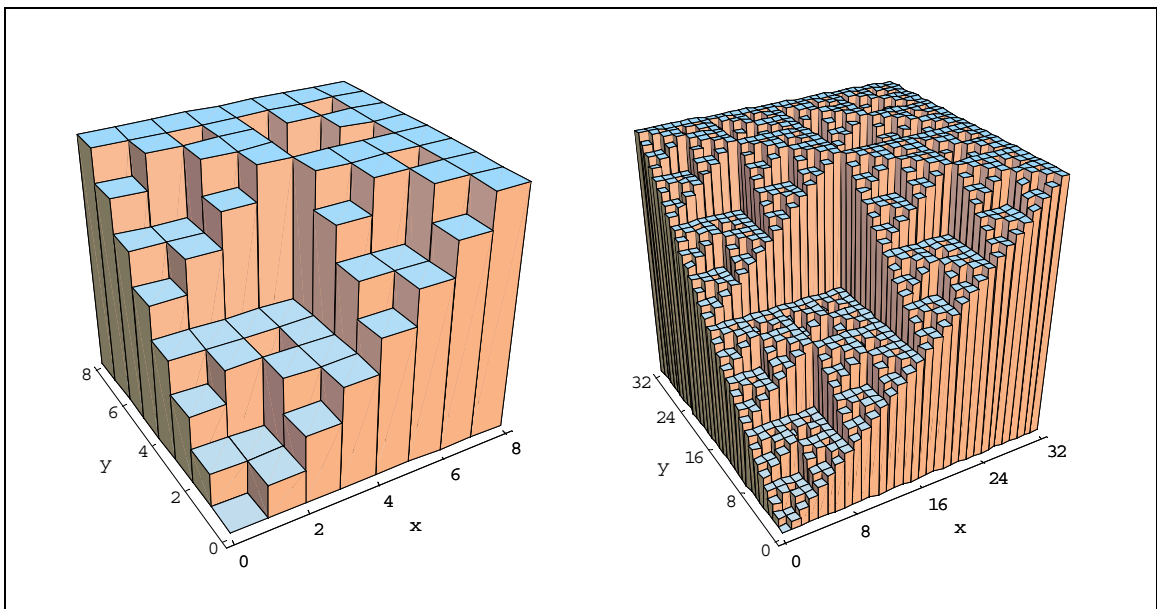


FIGURE 2. Plots of the logical *or* function.

In Figure 3, almost half of the points are hidden behind the diagonal $x = \bar{y}$.

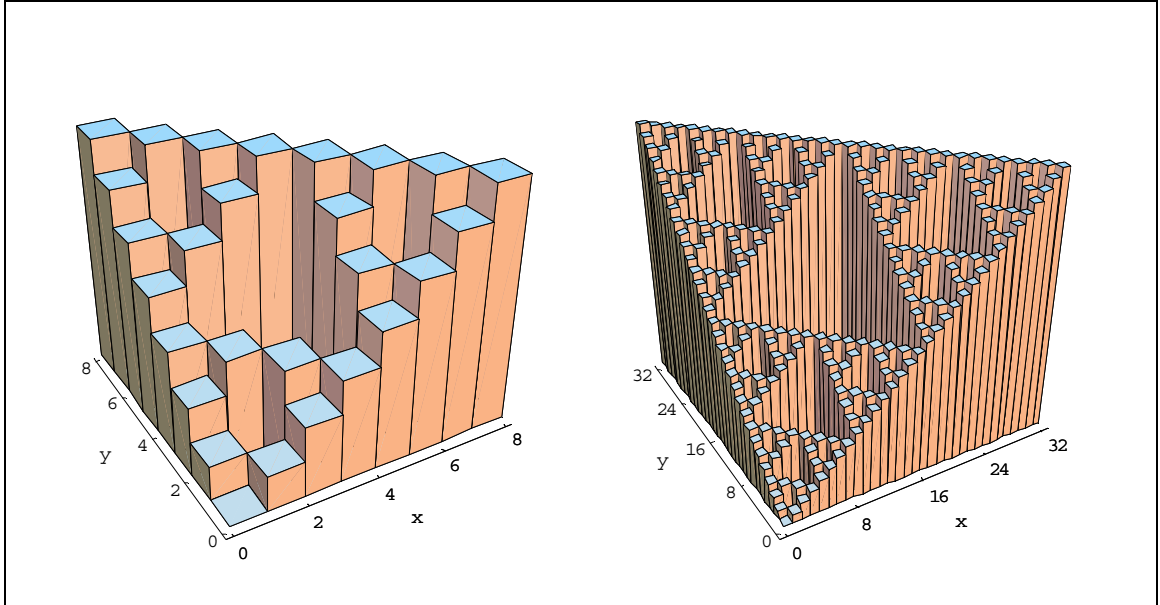


FIGURE 3. Plots of the logical *exclusive or* function.

For $\text{and}(x, y)$ (Figure 1), a certain self-similar, or fractal, pattern of triangles is apparent. If the figure is viewed straight on parallel to the y -axis, and taken to the limit for large integers, the appearance would be as shown in Figure 4 below.

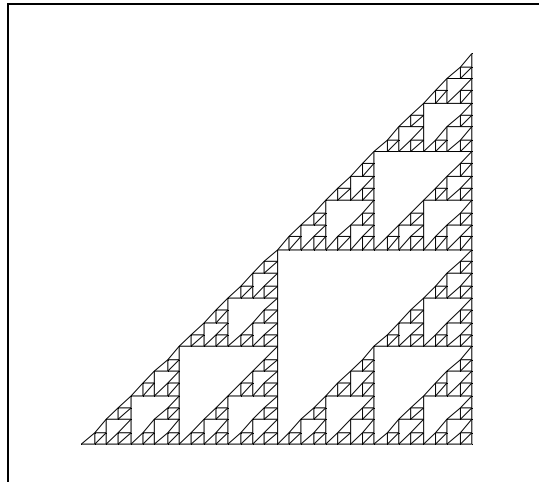


FIGURE 4. Self-similar pattern made by $\text{and}(x, y)$.

This is much like the Sierpinski triangle [Sagan], except Figure 4 uses right triangles whereas Sierpinski used equilateral triangles. In Figure 3, a pattern along the slanted plane is evident that is precisely the Sierpinski triangle if carried to the limit.

Reference

[Sagan] Sagan, Hans. *Space-Filling Curves*. Springer-Verlag, 1994.