MIE1622 Computation Finance and Risk Analysis Assignment 1

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Problem Background

We use a classical Markowitz model for portfolio optimization and try to fit different investment strategies for the portfolio. Our holding period is determined to be 2 months and we practice these strategies for 2 years. Hence, the total number of holding periods we go through will be 12. Transaction costs must be considered when trading is performed for each strategy. Covariance matrix Q, daily expected returns μ, initial portfolio value and initial weights for the assets are given to us and borrowing money is not allowed for any portfolio.

Strategies Used

Buy and Hold

Buy our initial weights and hold them till the end of each period throughout our investment horizon of 2 years. Transaction costs and cash account flow are disregarded, since no change is made for our portfolio.

Equally Weighted

We allocate money to each asset equally in our portfolio. In this case we need to consider transaction costs and make sure that our cash account is non-negative, i.e. we are not spending more than what we have in our portfolio.

Equally Weighted Buy and Hold

We allocate money to each asset equally in our portfolio only at the beginning of period 1. Transaction costs and rebalancing only needs to be considered during period 1 when we reshuffle our portfolio, after which the asset allocation remains the same. Then throughout each period we hold our assets without changing the weights, until the end of our 2-year investment horizon.

Minimum Variance

We compute the minimum variance, or low volatility portfolio using the CPLEX algorithm subjecting it to constraints given as:

$$\begin{aligned} \min_{w} \quad & w^T Q w \\ \text{s.t.} \quad & \textstyle\sum_i w_i = 1 \\ & w \geq 0 \end{aligned}$$

Transaction costs and rebalancing the cash account need to be considered here when optimizing our portfolio

Maximum Sharpe Ratio

I could not properly implement the CPLEX algorithm for max Sharpe ratio, so I have used CVXPY to show the results. My CPLEX algorithm is commented out in the code, any help with letting me know what is wrong with it would be appreciated.

We compute the highest measure of return per unit risk for each asset in our portfolio using the CPLEX algorithm. The higher the return per risk the better the asset performs. We choose our weights in accordance to that and the constraints we subject our optimization problem are given as:

$$\begin{aligned} \min_{y,k} \quad & y^T Q y \\ \text{s.t.} \quad & \textstyle\sum_i y_i - k = 0 \\ & \textstyle\sum_i (mu_i - r_f) y_i = 1 \\ & k \geq 0 \\ & y_i \geq 0 \end{aligned}$$

Transaction costs and rebalancing the cash account need to be considered.

Results

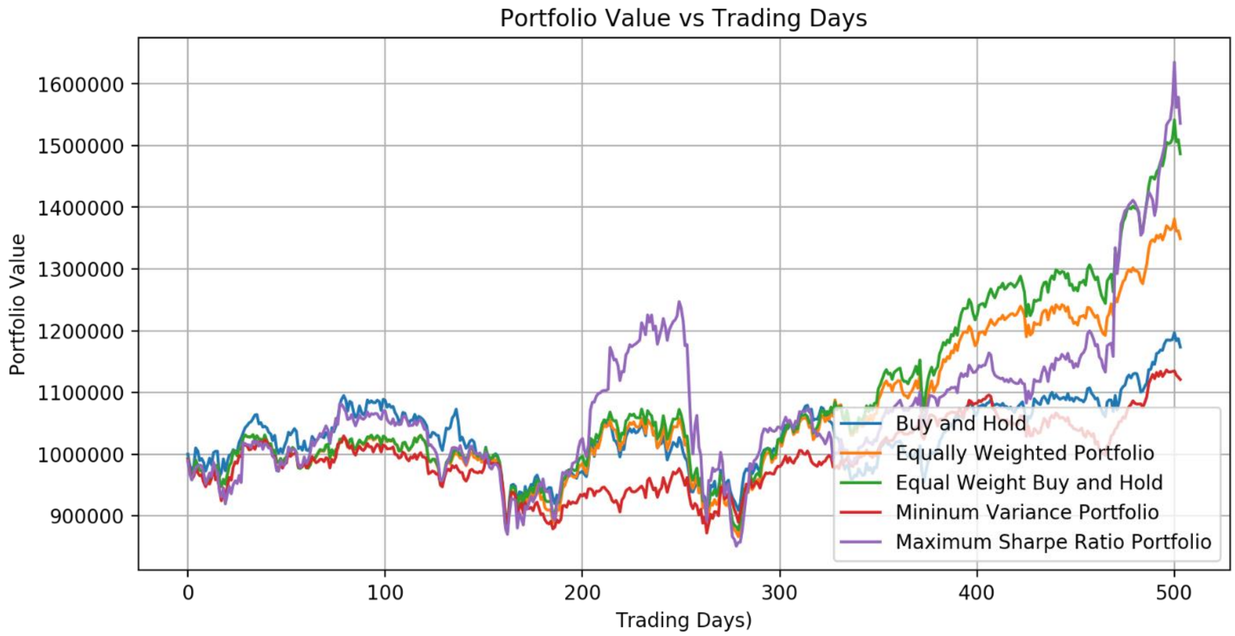

Figure 1 Portfolio Value vs Trading Days

Figure 1. shows how each portfolio varies with the number of trading days, finishing at the 2-year mark. At the end of our analysis we see that the maximum Sharpe ratio portfolio outperforms the others, with the equal weight buy and hold strategy very close behind it in terms of the portfolio value. Our minimum variance portfolio performed the poorest out of all our strategies.

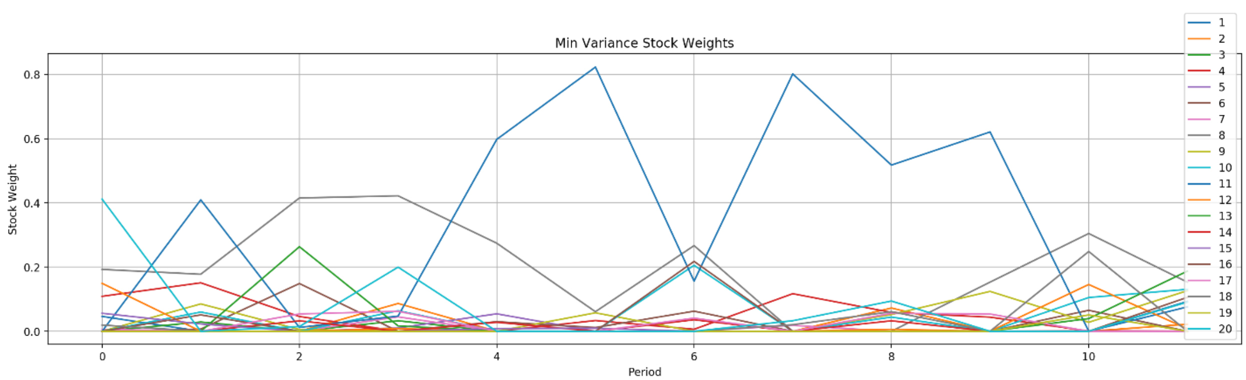

Figure 2 Minimum Variance Asset Weights

Figure 2. shows us how each of our weight allocation varies when we use our minimum variance strategy throughout each period of our 2-year horizon. Asset 1 which corresponds to Microsoft stock is heavily favored by our algorithm starting from period 3 towards the end of period 10.

Our optimization algorithm does not generally care about our expected returns and solely relies on minimizing the portfolio risks, which may be why we have the least expected returns from this algorithm.

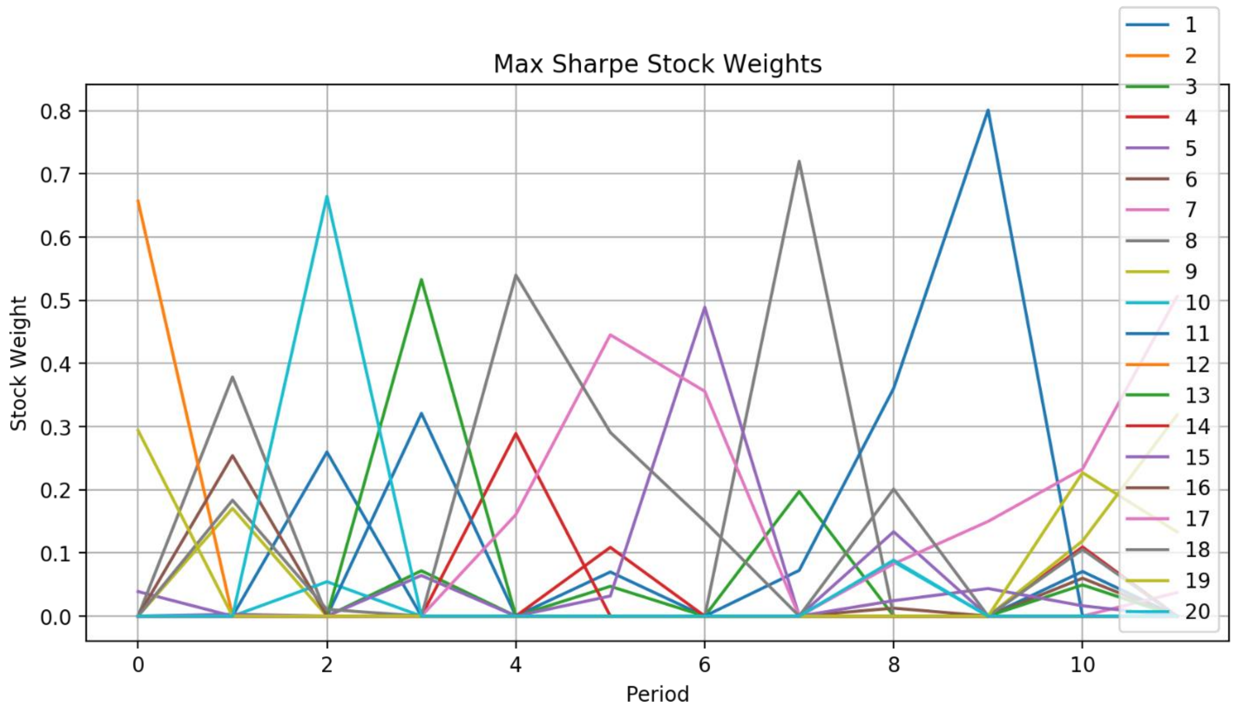

Figure 3 Max Sharpe Asset Weights

Figure 3. describes how much weight we allocate to each asset when computing our maximum Sharpe ratio algorithm. Different assets are favored by our algorithm for different period, with Asset 1 which corresponds to Microsoft having the highest weights during period 9 at 80% in our portfolio. Our algorithm tries to find the maximum expected return with every unit of risk for our portfolio and may be a reason as to why it has the highest performing portfolio value as compared to other portfolios.

From figure 1 we can also see that the equally weighted buy and hold strategy performs significantly better than every strategy other than the maximum Sharpe ratio strategy. This may be the case that performing the equally weighted strategy each time may incur various transaction costs resulting in more losses then gains, which may be why our equally weighted strategy could not perform this well.

We can also improve our portfolio optimization problem by introducing more algorithms and checking the portfolio value. Algorithms such as maximum return portfolio selection, equal risk portfolio selection etc. would give the investor more options at selecting the optimal portfolio. We might also consider adding more constraints to our existing algorithms to minimize transaction costs and having less money in the cash account. Constraints such as the cardinality constraint to select a maximum diverse asset number or the minimal holding constraint to avoid having small weights in our portfolio would help in eliminating high transaction costs for our portfolio.