MIE1622 Computation Finance and Risk Analysis Assignment 4

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Problem Background

Using existing data, we calculate the call and put option value of the closed form Black-Scholes equation. Then we use Monti-Carlo approximations to simulate a Geometric random walk with the data to represent how the value of the stock changes as a function of time. We implement a knock-in option to the Monti-Carlo approximation as well for the same stock, with varying volatility to compare the results. Finally, we find the number of paths and scenarios which would simulate a result closest to the closed form of the Black-Scholes equation

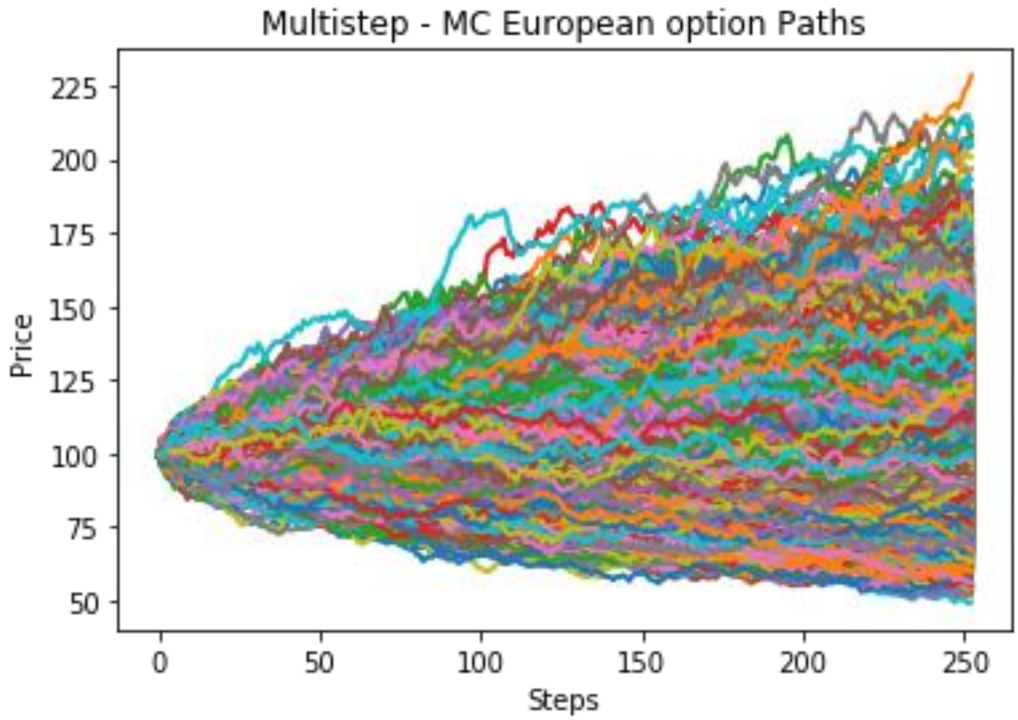
Results

The results of our simulation is given below:

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 7.925328813409936
One-step MC price of an European put option is 7.891700499207624
Multi-step MC price of an European call option is 7.981547527751872
Multi-step MC price of an European put option is 7.869439933528049
One-step MC price of an Barrier call option is 7.873464448140768
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 8.04292700403084
Multi-step MC price of an Barrier put option is 1.9705521373225081
```

For the number of paths, I chose 10,000 as the results were relatively close to the closed form solution, along with the computational time for further results was not that significant. For the multi-step Monti-Carlo approximation, the number of steps I chose is 252, which would represent the number of trading days in a year.

Visualizing the Paths



We simulate 10,000 paths for 252 steps based on a geometric random walk. The variation of price for differing steps/paths can be seen from the graph.

Comparing the three strategies

If we look at the closed form and the 1-step European option and compare those to the Multi-step European prices, we can notice that the number of steps in our iteration are not having a significant impact on the accuracy of the Monti-Carlo simulations. However, the number of paths, or scenarios will play a part in helping to converge the price of the options to the closed solution if we increase the number of paths to significantly larger number.

Difference between European and Barrier options

From the results, we can see that the Barrier and European call options for both sets are quite similar, however the put options are significantly smaller for the Barrier simulation. Let us consider the strike price K = 105 and the barrier Sb = 110. If a knock-in option occurs and the stock hits the barrier price throughout its lifetime, it becomes a European option. Since a call option would have a much higher chance of reaching the highest payoff, the put option reaches 0 during a one-step Barrier option. As we proceed to multiple-step Barrier option, after crossing the knock-in threshold, it has even a higher chance of being a call price, however consequently due to multiple-steps, the chance of being a put price also increases. Hence, we can see why the put price of the Barrier option is much lower than the European option.

Barrier Options with differing volatilities

```
One-step MC price of an Barrier call option with 10% increased volatility is
8.748044668473986
One-step MC price of an Barrier put option with 10% increased volatility is
0.0
Multi-step MC price of an Barrier call option with 10% increased volatility
is 8.704904889163371
Multi-step MC price of an Barrier put option with 10% increased volatility is
2.48833061345639
```

As we increase the volatility, we can notice that for both call and put, the value increases. This may be caused by the stock having more chance of hitting the barrier during its lifetime and achieving an even higher payoff due to the increase in volatility of the stock.

```
One-step MC price of an Barrier call option with 10% decreased volatility is
6.993944345144349
One-step MC price of an Barrier put option with 10% decreased volatility is
0.0
Multi-step MC price of an Barrier call option with 10% decreased volatility
is 7.272112473688663
Multi-step MC price of an Barrier put option with 10% decreased volatility is
1.6000205336093984
```

As we decrease the volatility, we can notice the decrease the value for both call and put prices. The stock would have less chance of hitting the barrier during its lifetime or achieving a high price from its initial state due to the decrease of its volatility.

Strategies for convergence

We use a list of differing number of paths and number of steps to find the absolute minimum values between the call and put prices of the Black-Scholes model and Monti-Carlo approximation. The optimal results for differing calculations are reported in the code.