MIE1622 Computation Finance and Risk Analysis Assignment 2

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Problem Background

We use a classical Markowitz model for portfolio optimization and try to fit different investment strategies for the portfolio. Our holding period is determined to be 2 months and we practice these strategies for 2 years. Hence, the total number of holding periods we go through will be 12. Transaction costs must be considered when trading is performed for each strategy. Covariance matrix Q, daily expected returns μ, initial portfolio value and initial weights for the assets are given to us. Three more strategies are to be implemented in addition to the strategies used in Assignment 1.

Implementation of Rounding and Validation Structure

If we implement the weights we get from our algorithm and convert into the number of stocks, we might find out that the number is not a whole number. We assume that we can only buy whole stocks and not partial stocks, so to avoid having partial stocks we round the number of stocks down to the nearest whole number. Our remaining portfolio value would be converted into cash.

Also, we must make sure that our portfolio is always non-negative. To validate our portfolio, we would allocate the negative balance to each stock value and then calculate our new stock allocation from there, which would give us a non-negative balance. If any value remains we would put it into our cash account.

Strategies Used

Equal Risk Contribution

We compute the optimal weights for a portfolio where each weight corresponds to an equal risk towards the standard deviation. The optimization problem with constraints is given as

$$
\begin{aligned}
\min_{w} \quad & \sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_i (Qw)_i - w_j (Qw)_j \right)^2 \\
\text{s.t.} \quad & \textstyle\sum_{i=1}^{n} w_i = 1 \\
& w \geq 0
\end{aligned}
$$

Leveraged Equal Risk Contribution

As with the above case we consider the risk contribution of each asset and find optimal weights for each asset. However, in this case we are allowed to borrow money to enhance our initial portfolio by a 100%. Therefore, we start at 200% of our initial value. There will be an interest accrued at each period which will need to be paid at the yearly risk-free rate, which changes depending on the year. An assumption in our problem will be that the amount borrowed will remain in the portfolio, i.e. it will not be paid off and only the interest will be paid every period.

Robust Mean Variance Optimization

It is similar to the minimum variance optimization model; it incorporates uncertainties of inputs into the optimization problem and considers estimation errors when optimizing the weights. The optimization is given as

$$
\begin{array}{lrcr}
\min & w^T Q w & & \\
\text{s. t.} & \sum_{i=1}^{n} w_i & = & 1 \\
 & \mu^T w & \geq & \varepsilon_{\text{ret}} \\
 & w^T \Theta w & \leq & \tilde{\varepsilon}_{\text{rob}} \\
 & w & \geq & 0
\end{array}
$$

In our implementation, the target portfolio return is taken to be the target of the minimum variance portfolio whereas the target risk is taken to be the risk of initial portfolio.

Results 2015-2016

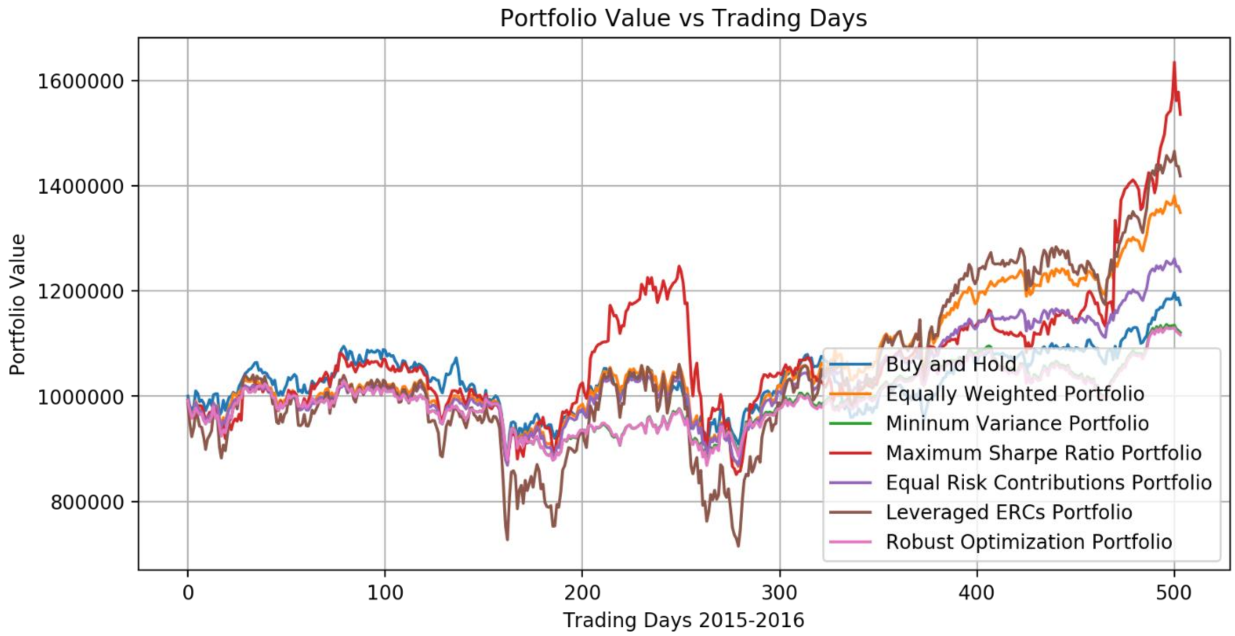

Figure 1 Portfolio Value vs Trading Days (2015-2016)

Figure 1. shows how each portfolio varies with the number of trading days for the years of 2015-2016, finishing at the 2-year mark. At the end of our analysis we see that the maximum Sharpe ratio portfolio outperforms the others, with the equal weight buy and hold strategy very close behind it in terms of the portfolio value. Our minimum variance portfolio and robust mean variance portfolio performed the poorest out of all our strategies.

The leveraged ERC strategy performed better than most strategies other than the maximum Sharpe ratio at the end of the 2-year period. The ERC strategy was average as compared to all other strategies. However, since the robust mean variance optimization strategy was closely based off of the minimum variance strategy, its expected return was quite similar to the minimum variance strategy, hence being one of the worst strategies to implement during this time period.

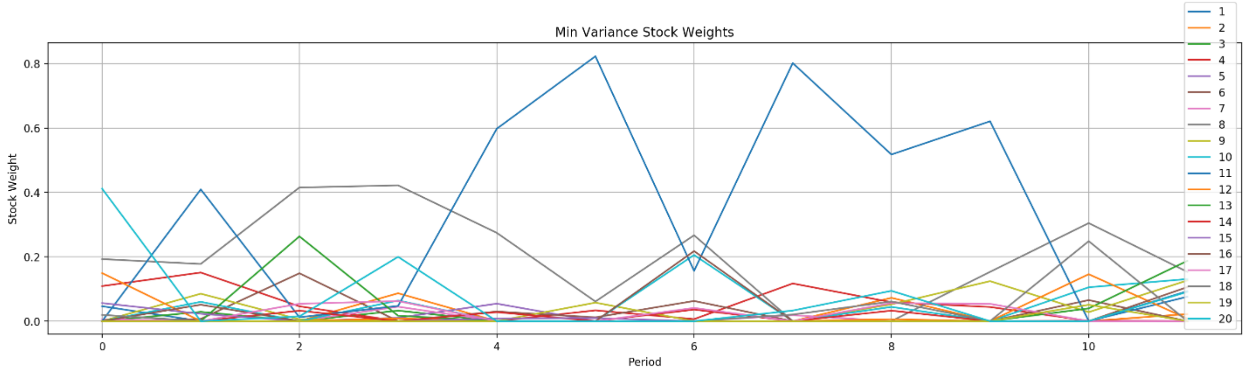

Figure 2 Minimum Variance Asset Weights (2015-2016)

Figure 2. shows us how each of our weight allocation varies when we use our minimum variance strategy throughout each period of our 2-year horizon. Asset 1 which corresponds to Microsoft stock is heavily favored by our algorithm starting from period 3 towards the end of period 10. Our optimization algorithm does not generally care about our expected returns and solely relies on minimizing the portfolio risks, which may be why we have the least expected returns from this algorithm.

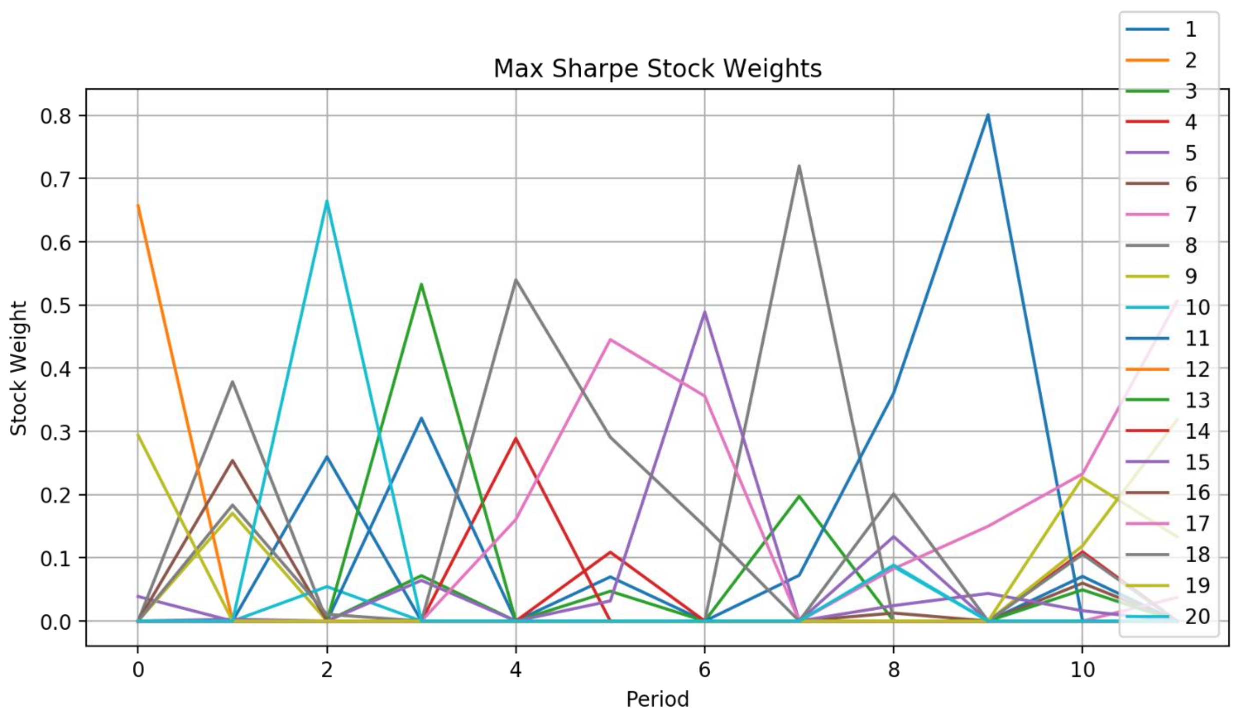

Figure 3 Max Sharpe Asset Weights (2015-2016)

Figure 3. describes how much weight we allocate to each asset when computing our maximum Sharpe ratio algorithm. Different assets are favored by our algorithm for different period, with Asset 1 which corresponds to Microsoft having the highest weights during period 9 at 80% in our portfolio. Our algorithm tries to find the maximum expected return with every unit of risk for our portfolio and may be a reason as to why it has the highest performing portfolio value as compared to other portfolios.

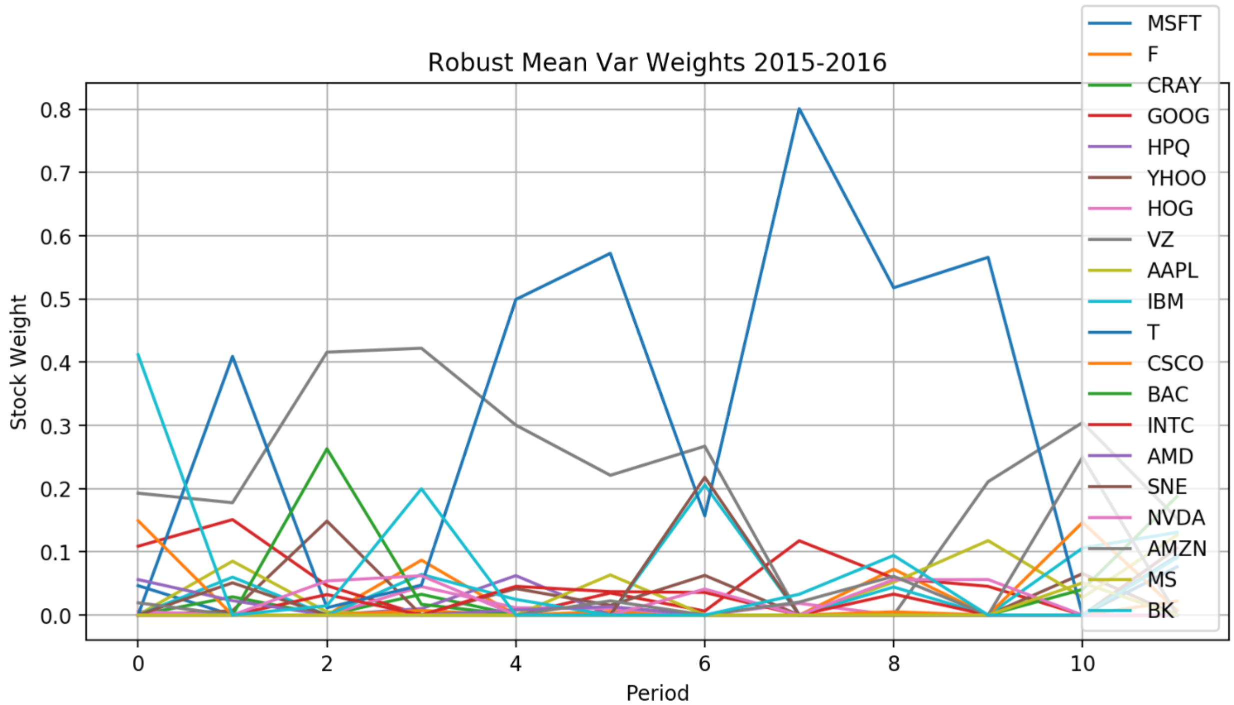

Figure 4 Robust Mean Var Weights (2015-2016)

Figure 4 shows us the asset allocation for our Robust Mean Variance Portfolio for throughout our 12 periods. Microsoft Asset is heavily favored by our algorithm after period 3 until the end of period 10. At period 7 our portfolio consists about 80% of the Microsoft shares. The trends are quite similar to the minimum variance weight allocation as we can see by comparing it to figure 2. This is because the algorithm is a modified implementation of the minimum variance optimization model.

Results 2008-2009

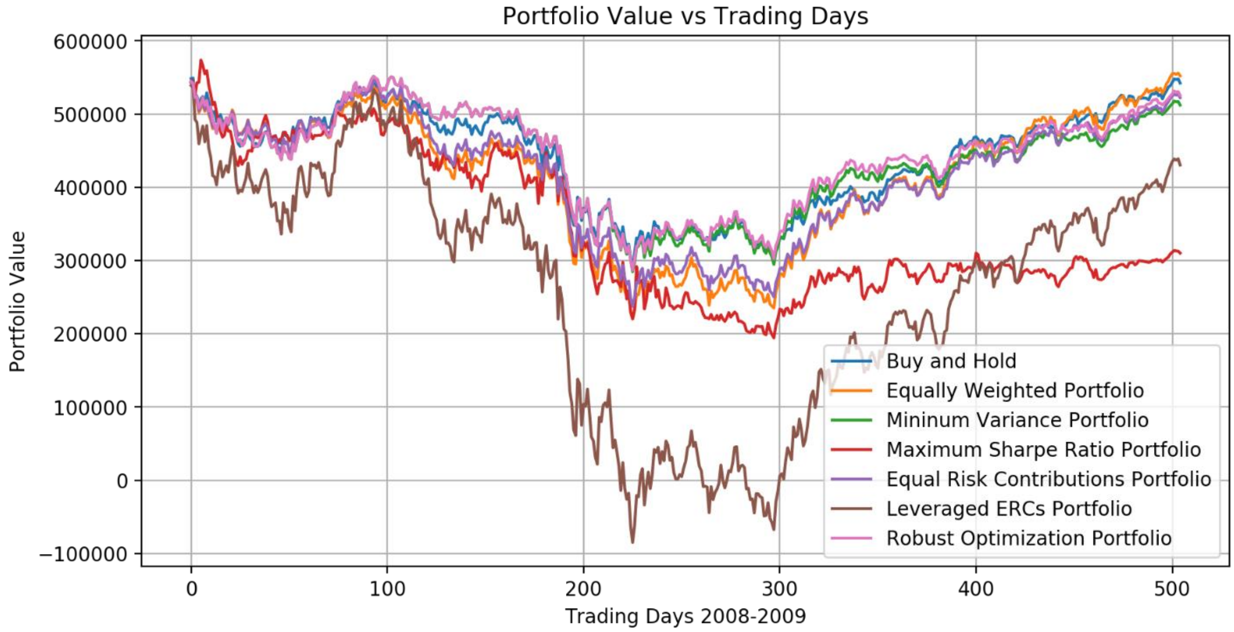

Figure 5 Portfolio Value Vs Trading Days (2008-2009)

Figure 5. shows how each portfolio varies with the number of trading days for the years of 2008-2009, finishing at the 2-year mark. We see a downward decline in all the portfolio values in general, with the values starting to climb up after the 1st quarter of 2009. This analysis is done during a crisis year, i.e. the time during which the market trend in general was downward for most stocks, which reflects in our figure. Our Leveraged ERC portfolio reaches a negative value during this period, as it shows the value of our portfolio subtracted by the amount borrowed, so although our overall portfolio is positive, the amount remaining is less than what we owe. The best performing models during this time period are the buy and hold and equally weighted portfolios, with the robust mean var portfolio and minimum variance portfolios coming close behind.

During some periods, some algorithms were infeasible in coming up with a solution. For those periods we have set the asset weights to carry over from the previous period towards the next period. A reason for the infeasibility, in the case of the Sharpe Ratio Algorithm, is that the rate of return of our risk-free asset is higher than all other assets, since our constraints for that algorithm do not hold for that period, our solution would be infeasible. A similar case for the constraints is observed for our robust mean variance optimization algorithm.

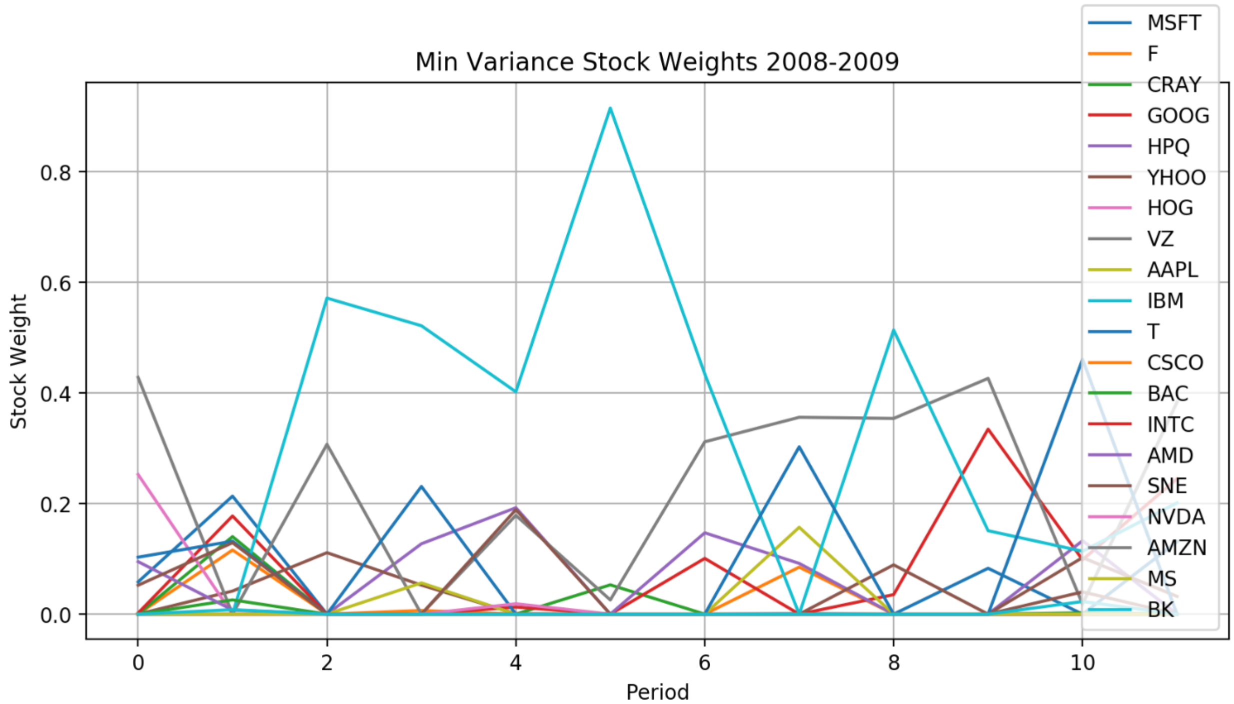

Figure 6 Minimum Variance Stock Weights (2008-2009)

Figure 6. shows us how each of our weight allocation varies when we use our minimum variance strategy throughout each period of our 2-year horizon between 2008 and 2009.IBM stock is heavily favored by our algorithm starting from period 3 towards the end of period 6. This might imply the IBM stock delivering the least risk of all the assets during these time periods.

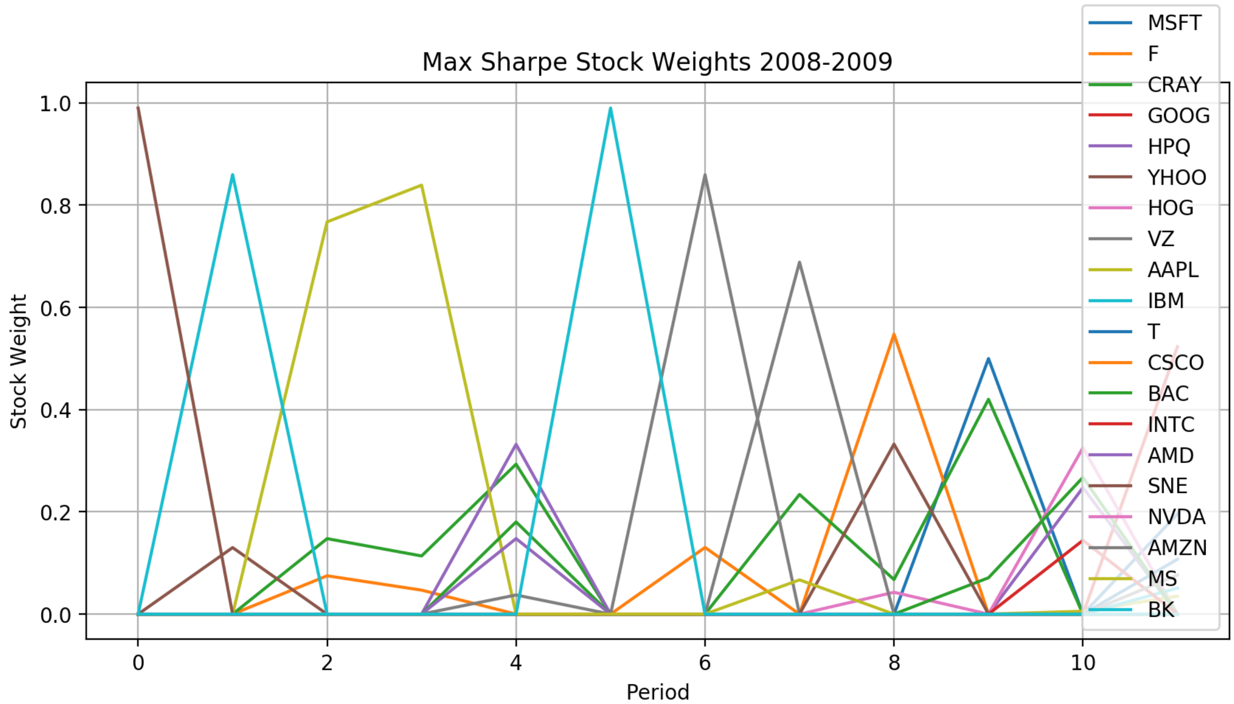

Figure 7 Max Sharpe Ratio Stock Weights (2008-2009)

Figure 7. describes how much weight we allocate to each asset when computing our maximum Sharpe ratio algorithm for the years 2008-2009. Different assets are favored by our algorithm for different period, with the IBM asset having the highest weights during period 5 at 100% in our portfolio. This asset had the highest gain per relative risk out of all assets during this period.

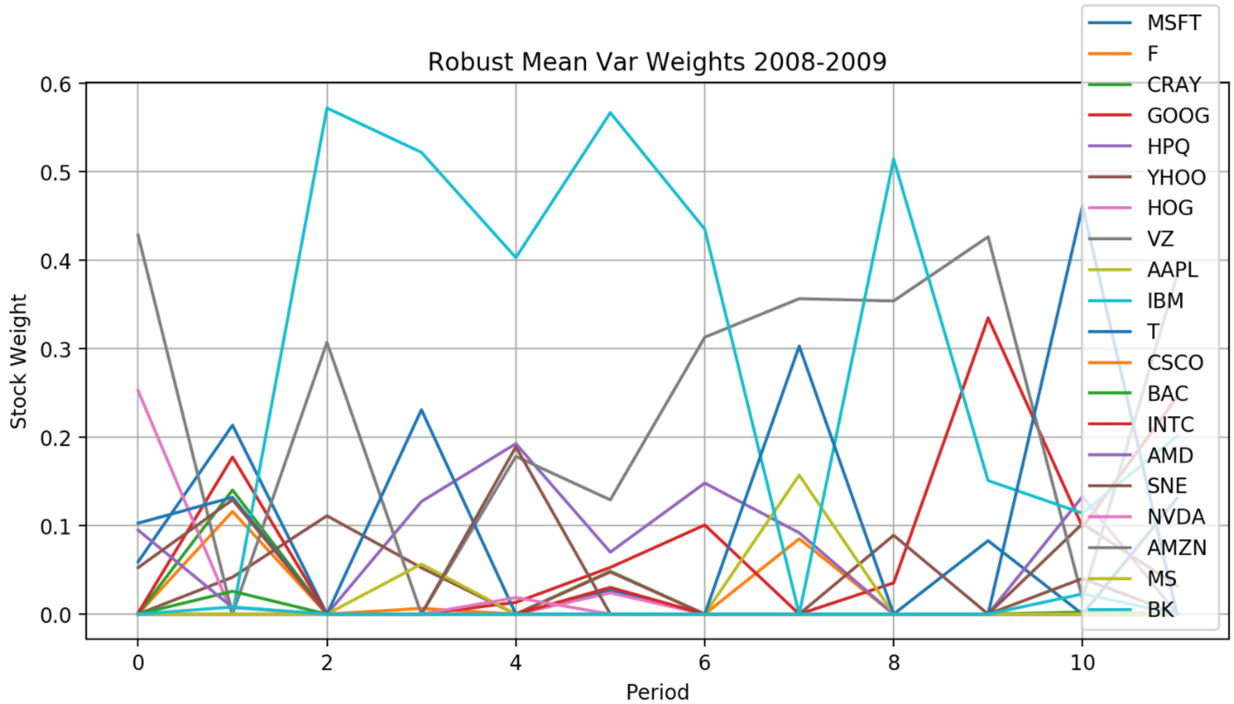

Figure 8 Robust Mean Var Weights (2008-2009)

Figure 8 shows us the asset allocation for our Robust Mean Variance Portfolio for throughout our 12 periods for the years 2008-2009. The IBM asset is favored throughout different time periods by our algorithm, staying above 50% portfolio weight for at least 3 different periods.

Conclusion

During the years 2015-2016, I would suggest implementing the maximum Sharpe ratio strategy, as it yields the highest rate of return at the end of the 2-year period. However, during periods 8-10 we see that Leveraged ERC and Equally Weighted portfolios are much better in terms of rate of returns, with leveraged ERC coming just behind the max Sharpe ratio strategy at the end of the 2-year period, which should also be considered.

During the years 2008-2009, we should avoid using the leveraged ERC strategy or max Sharpe Ratio strategy, as it puts the investor into a very high deficit during the crisis years. I would suggest implementing the minimum variance strategy or robust mean variance strategy as they offer the least amount of risk to the highest amount of return during this period. We can also observe that the IBM stock during this period performs relatively better compared to other stocks for our 3 strategies observed in the figures above.

From the dynamic changes in above figures, we can also see that there is a high amount of trading between assets when using the maximum Sharpe ratio as compared to the minimum variance or robust mean variance portfolios. In general, during years where the market is down, we should tend to avoid strategies like maximum Sharpe ratio which prioritize on the return of the portfolio and avoid borrowing money as in the case of leveraged ERC portfolio and favor strategies like minimum variance to avoid as much loss as we can. However, in years when the market is stable or appreciating, max Sharpe Ratio and leveraged ERC strategies should be heavily favored for maximum returns.

To improve our portfolio, we might also consider adding more constraints to our existing algorithms to minimize transaction costs and having less money in the cash account. Constraints such as the cardinality constraint to select a maximum diverse asset number or the minimal holding constraint to avoid having small weights in our portfolio would help in eliminating high transaction costs for our portfolio. The transaction costs might be why our Leveraged ERC portfolio performs poorly.