(Ch. 2)

Categories of Games

(*) What is a game (& what isn't)?

→ an action situation w/ 2 or more mutually aware players and the outcome for each depends on the actions of all.

✓ • can't just be you making choices in a vacuum (decision)

• mutually aware
→ me vs. Luke on walkies

• the outcomes depend on other's actions

→ today: categories of games
↳ so you know which tools to use

(*) Is it sequential or simultaneous

→ sequential: one player goes, then the other

Example(s):

- chess: white moves, then black moves, then white moves, etc.
- bills becoming laws: congress votes, then the President signs/vetoes

→ Simultaneous: all players move at the same time

Example(s):

- the prisoner's dilemma
- goal attempts in soccer

* Is it a zero-sum game or not?

→ in a zero-sum game, one player's loss is necessarily another player's gain.

→ players' interests are in complete conflict.

Example(s):

- chess, etc.
- dividing the pie

→ most social & economic games are not zero-sum

↳ commonality of interest

Examples:

- negotiation
- trade
- travel funds

(*) Is it played only once or repeatedly

↳ w/ the same opponents or different?

→ if the game is played repeatedly, you need to think about how your choices this round affect the next round.

↳ if it's not, you don't

→ this is different from simultaneous vs. sequential.

Ex: Test giving

↳ Erin's Theory

Mechanic Shops.

→ quote from the book

(*) Do the players have $\boxed{\text{perfect information}}$?
Do they have $\boxed{\text{asymmetric information}}$?

<u>imperfect information</u>: if a game has <u>external</u> (external to the game) or <u>strategic</u> (internal to the game) uncert

Examples:
- don't know the quality of a product you might buy (external)
- don't know where your friend put his ship in battleship (strategic)

<u>perfect information</u>: otherwise
↳ there's no external or strategic uncertainty.

<u>asymmetric information</u>: one player has more or different information than another

<u>Ex</u>: Job Talk

→ can refer to people having very balanced information (playing go fish) or very unbalanced (job talk)

(*) Are rules fixed or manipulable?

→ most real life games have manipulable rules.

Example: negotiation

→ no rule that says I can't ask for money to decorate my office

→ for most real life games, there's a pregame game where the rules are set

↳ always show up to the pregame

↳ better yet, be the only one there

(*) Is it cooperative or non cooperative

→ some classic terrible econ terminology cooperative: agreements are enforceable

Example: sign a contract

noncooperative: agreements not enforceable

Example: You & your roommate divy up chores at the beginning of the semester

→ Note: every game has an answer to each of these questions.

Example: I asked the school to pay for me to go to a conference in NYC.

What We Mean When We Say...

(*) What is the goal of a player in a game?

→ We're going to say that they aim to achieve the highest payoff possible for him or herself

→ better specify what we mean by

- payoffs
- strategies
- rationality
- common knowledge
- equilibrium

all have very specific def^ns in this course

(*) Payoffs

→ every possible outcome for every player is assigned a number = payoff

↳ person & outcome specific

→ this makes more sense for some situations than others

- patent competition: profits
- bills becoming laws: ... utility?

→ the process for specifying payoffs is different for every situation

- rank them
- profit max, etc.
- sum of categories

→ 2 Characteristics:

1. They capture <u>everything</u> the player cares about
   ↳ altruism, etc. <u>included</u> in payoffs

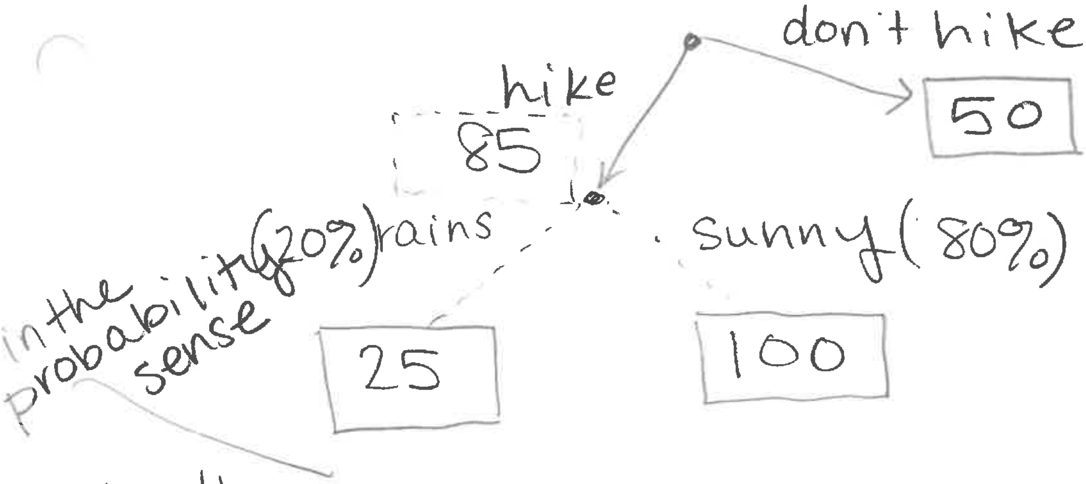
→ if someone chooses an action that you see as suboptimal

↳ they're not irrational

↳ you guessed their payoffs wrong

2. If an outcome features some randomness, the associated payoff is the expected value of the random outcomes.

Ex: To Hike or Not To Hike



→ the expected payoff of hiking is:

$0.2(25) + 0.8(100) = 85$

$5 \qquad\qquad 80$

(*) Strategies

→ a strategy is a complete plan of action

↳ easy for simultaneous, single-round games

↳ harder for repeated or sequentia stuff

→ a recipe

→ How to check if it's a strategy:

You should be able to write it down on a piece of paper, hand it to a stranger, go on vacation w/ no contact, & they can perfectly act in your stead.

→ needs to cover every possibility
→ needs to be unambiguous

* The game theory def^n is different from, e.g. financial planning, military science

* <u>Rational Behavior</u> (how we think people behave)
Assume players are:
   (i) perfect calculators of payoffs
   (ii) flawless executers of their best strategies

Assuming this requires player's have
   (i) complete knowledge of your own interests
   (i) complete understanding of how to best serve those

Obviously not true (we've all woken up hungover)

* <u>Common Knowledge of the Rules</u>
→ roughly: everyone knows the rules
→ more specifically:

* The rules of the game consist of:

(1) the list of players

(2) the strategies available to each player

(3) the payoffs for each player for all possible combinations of strategies pursued by all players (you know all the outcomes & who gets what if they happen)

(4) the assumptions that (a) every behaves rationally and (b) maximizes his payoff

→ it's not enough for you to know the rule

↳ you need to know that I know the rules

↳ & so on (like that episode of Friends)

* Equilibrium

→ an equilibrium is simply a point where everone has chosen their best reaction to everyone else's best reaction

→ each player has implemented the strategy that is the best response to the strategies of the other players

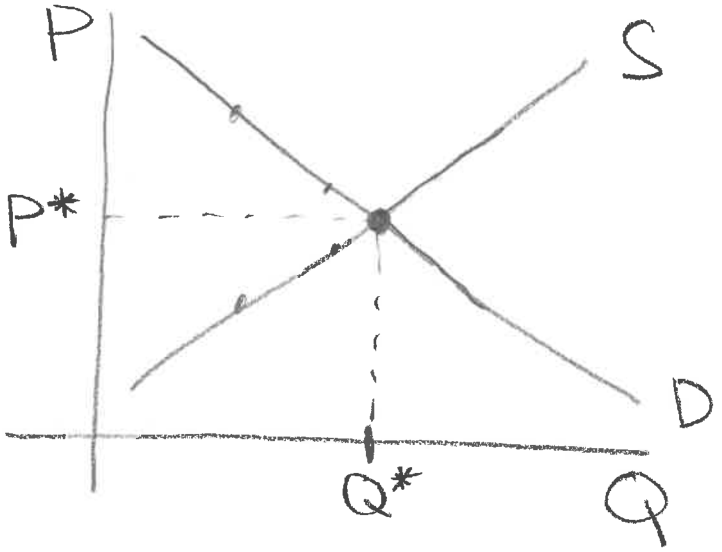
Things Equilibria are not: (necessarily

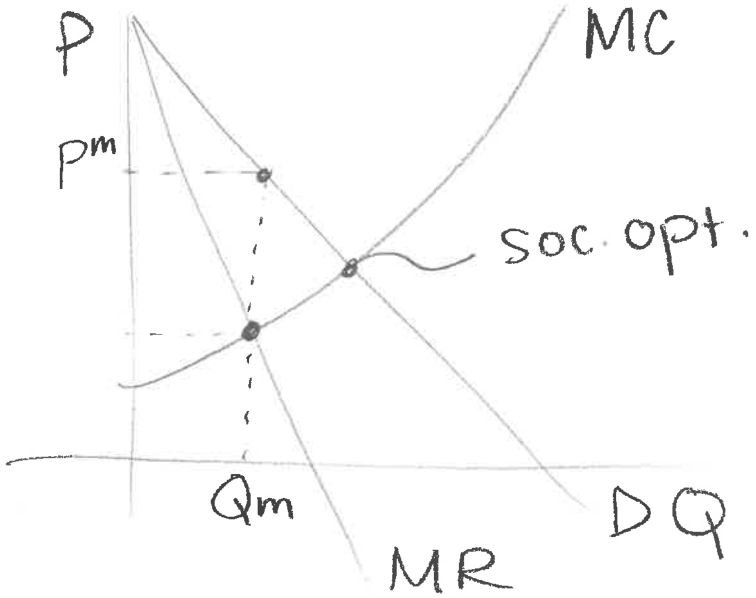
→ unchanging ("steady state")

→ best for everyone ("social optimu

↳ that's not why we're here.

→ an example we're all familiar with





Sequential Games (Ch. 3)

Representing Games Using Trees

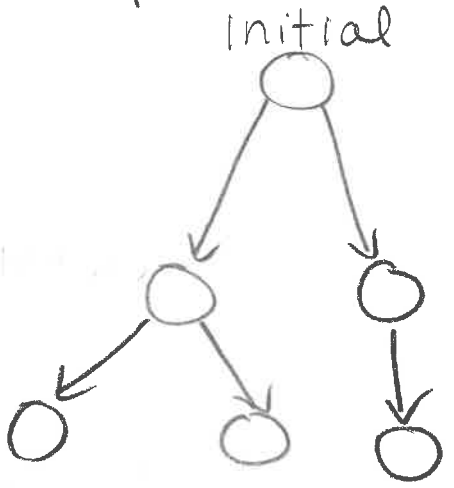
→ sequential games
→ today: how to draw the trees

→ next time: how to use them to solve the games

* The way we model (write down, represent etc.) sequential games is with game trees.

i A tree, mathematically, is a set of nodes & directed edges (a type of directed graph) such that:

1) there is one initial node
2) every other node has exactly one parent node



we're going to use this to represent sequential games

(*) What do they represent in Sequential Games

action/decision nodes

terminal nodes

- nodes: players & outcomes
- branches: possible actions

[walk through snow day example]

I'm the initial node

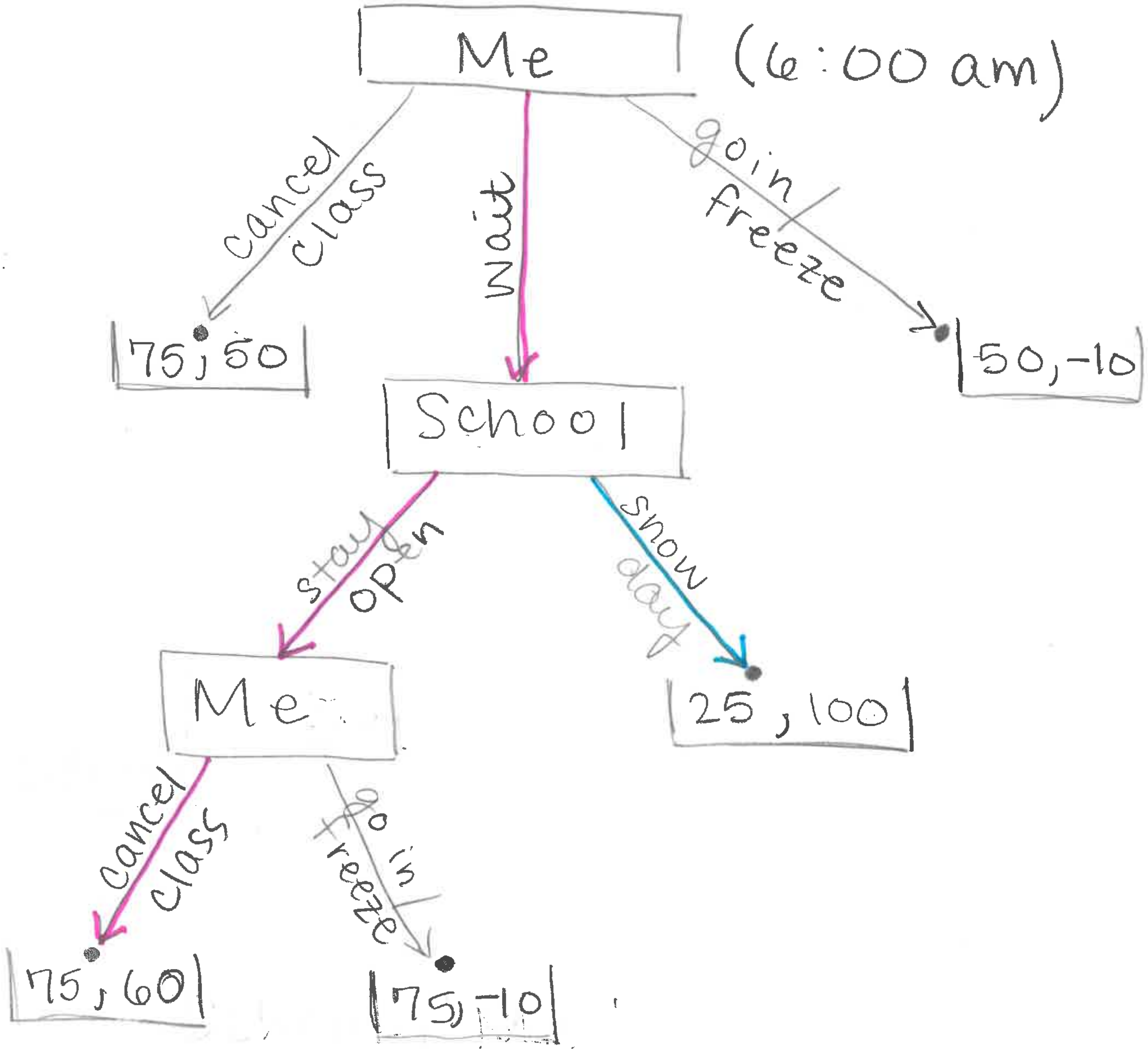
Players: Me, the School

↳ need payoffs for every player at every outcome even if they didn't make a choice to get here.

Actions: can be different for different people at different times or after different moves

Payoffs: (terminal nodes) very different from decision node

* A Simplified Version of Monday



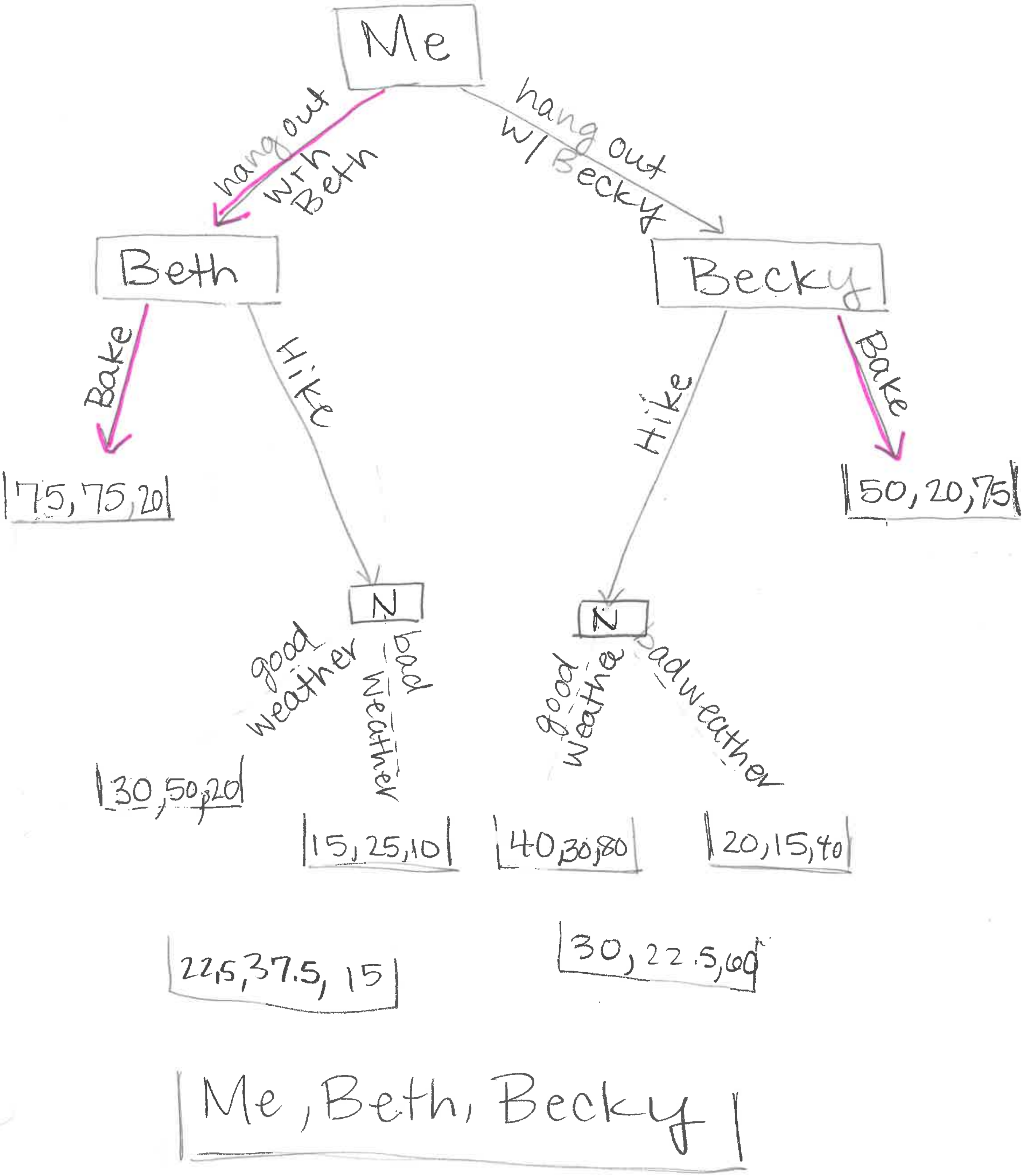
School, Me

(*) Uncertainty/Nature

→ If some choice leads to an outcom w/ randomness, we need to allow for the different outcomes (they have different payoffs)

→ add another player: Nature

→ specify probabilities


Me
hang out with Beth
hang out w/ Becky
Beth
Becky
Bake
Hike
75, 75, 20
50, 20, 75
N
N
good weather
bad weather
good weather
bad weather
30, 50, 20
15, 25, 10
40, 30, 80
20, 15, 40
22.5, 37.5, 15
30, 22.5, 60
Me, Beth, Becky

* Strategies in Sequential Games
→ needs to cover <u>all</u> eventualities

* If you only get to move once:
↳ a strategy only needs to specify one move.

→ in the hiking game everyone's strategy is just one move

Me: "choose to hang out with Beth"

Beth: "Choose to Bake"

Becky: "choose to Bake"

* If you ever could <u>possibly</u> move more than once, you need to specify a move <u>for</u> <u>every</u> <u>turn</u>.

↳ even if it's incredibly unlikely it happens

Me: "choose wait, if the school chooses to stay open, choose to cancel class."

School: "choose to stay open"

→ But if the school's ~~best~~ strategy was to have a snow day, we would still need to specify a choice for my 2nd turn..

School: "Choose a snow day"

Me: "Choose to wait, if the school chooses to stay open, choose to cancel class."

Solving Sequential Games

* Solving: find the equilibrium (or equilibria)
  → not the social optimum

* A Return to Strategies
  → Why do you need to specify moves that would never happen if you & the other players are rational?
    ↳ You never want the person you gave your strategy to (while you go on vacation) to be standing there w/ no guidance
      ↳ Other players might hit the wrong button
      ↳ You/Your strategy person might hit the wrong button
  → as we go through these examples, we'll practice writing down <u>all</u> their strategies.

(*) How to find the equilibrium in a sequential game: start at the end, move backwards.

(Do w/ Carmen Game) (Set up side by side)

Step By Step:

1. Start at terminal nodes
(do all terminal nodes at the same lev before going to the next level up)

2. For every player that makes a decision that leads to a terminal node:
   - Choose the branch that leads to the highest payoff <u>for that player</u>.
   - rule out
   (Prune any branches that don't lead to that outcome.)

→ now, for every player that leads to a terminal node, you have only one possibility.

3. Go up one level to the next player. For each of their possible choices, follow the path you

picked in the previous step

- choose the branch that leads to the highest payoff for that player.
- (Prune the branches that don't lead to that outcome.)

(do all players at the same level before moving to the next level up.)

4. Repeat until you've done the initial node / first player.

→ the path you've highlighted is the rollback equilibrium

↳ the outcome it leads to is the rollback equilibrium outcome.

(The textbook outlines this process on p. 56)

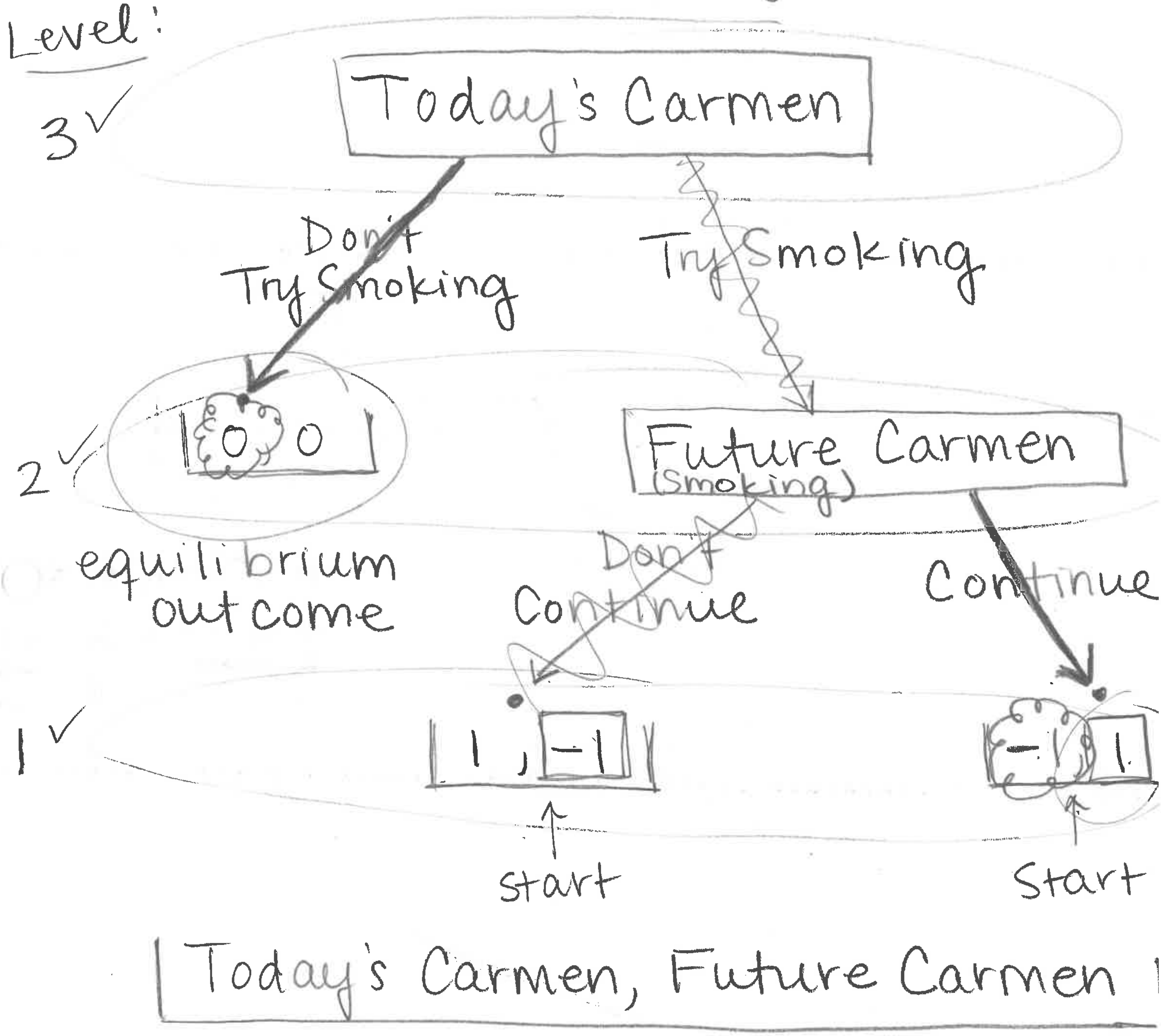
(*) An equilibrium is a set of strategies for each player.

↳ write them all out

↳ NOT the outcome.

equilibrium: today's Carmen chooses don't try smoking, Future Carmen chooses to Continu smoki

Carmen's Smoking Game:



All Possible Strategies:

Future Carmen:

Choose to continue smoking

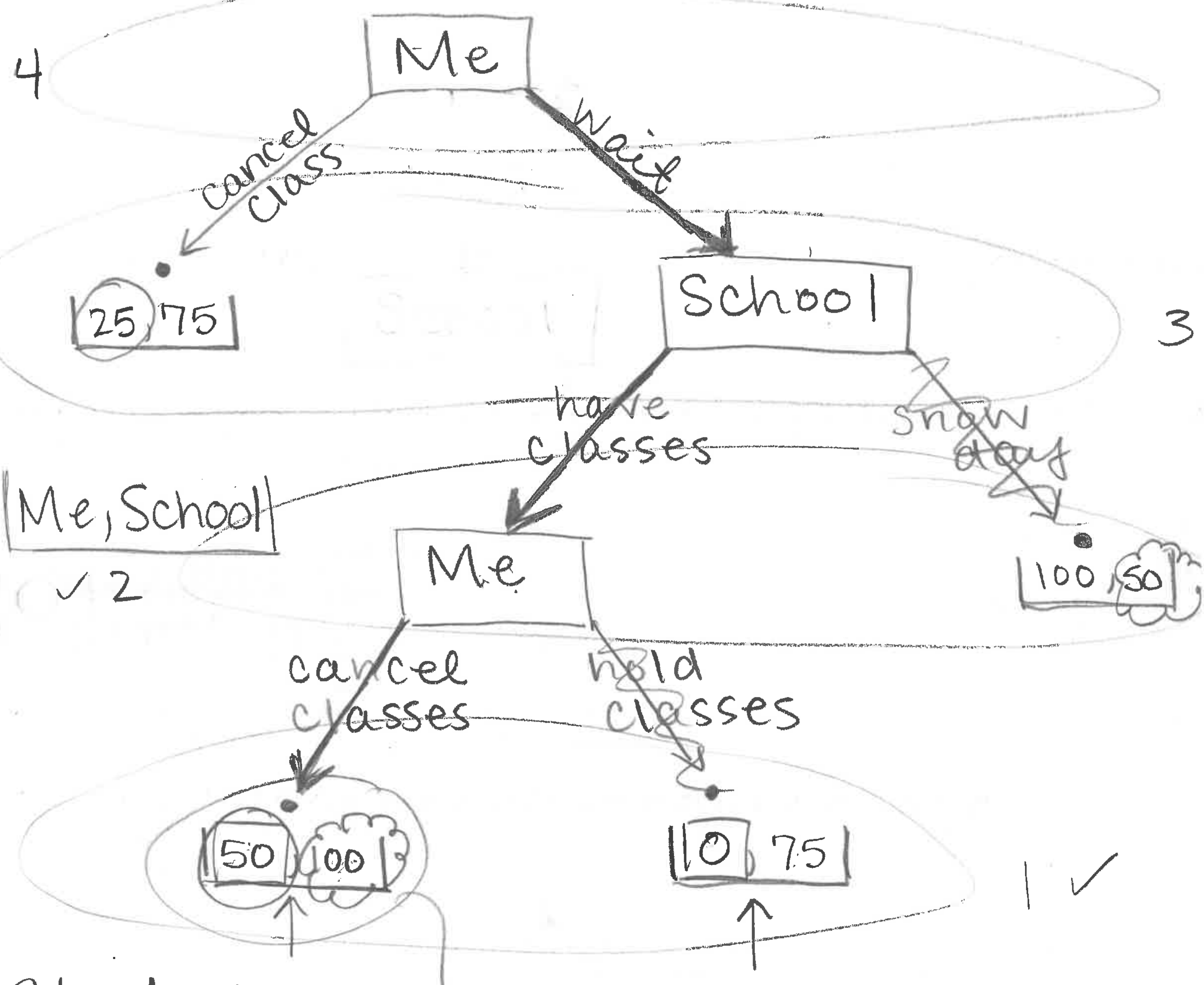
Choose don't continue

Today's Carmen:

choose try smoking

choose don't try smoking

Snow Day Game:
(no delays allowed)



eq. outcome

Strategies:

School: (2)
- Choose to have classes
- Choose to have a snow day

Me: (4)
- Choose wait, if the school chooses have classes, choose to hold class
- Choose wait, if the school chooses to have class, cancel class

- choose to cancel class, if the school chooses to have classes, choose to hold class
- choose to cancel class, if the school chooses to have classes, choose to cancel class.

More Players, More Moves

→ revisit Dean Game

→ the step by step process we outlined for solving sequential games was designed for any number of players or moves

↳ make the tree more complicated

↳ process is the same

① Study Lounge Game

→ there are 3 of you that share a suite

↳ you'd like to have a Keurig in there

→ if 2 or more of you contribute, it's enough

→ if 1 or fewer of you contr., no Keurig

→ for each person, 4 possible ~~outcomes~~ scenarios

There's a Keurig

→ She contributes, at least one other does = 3

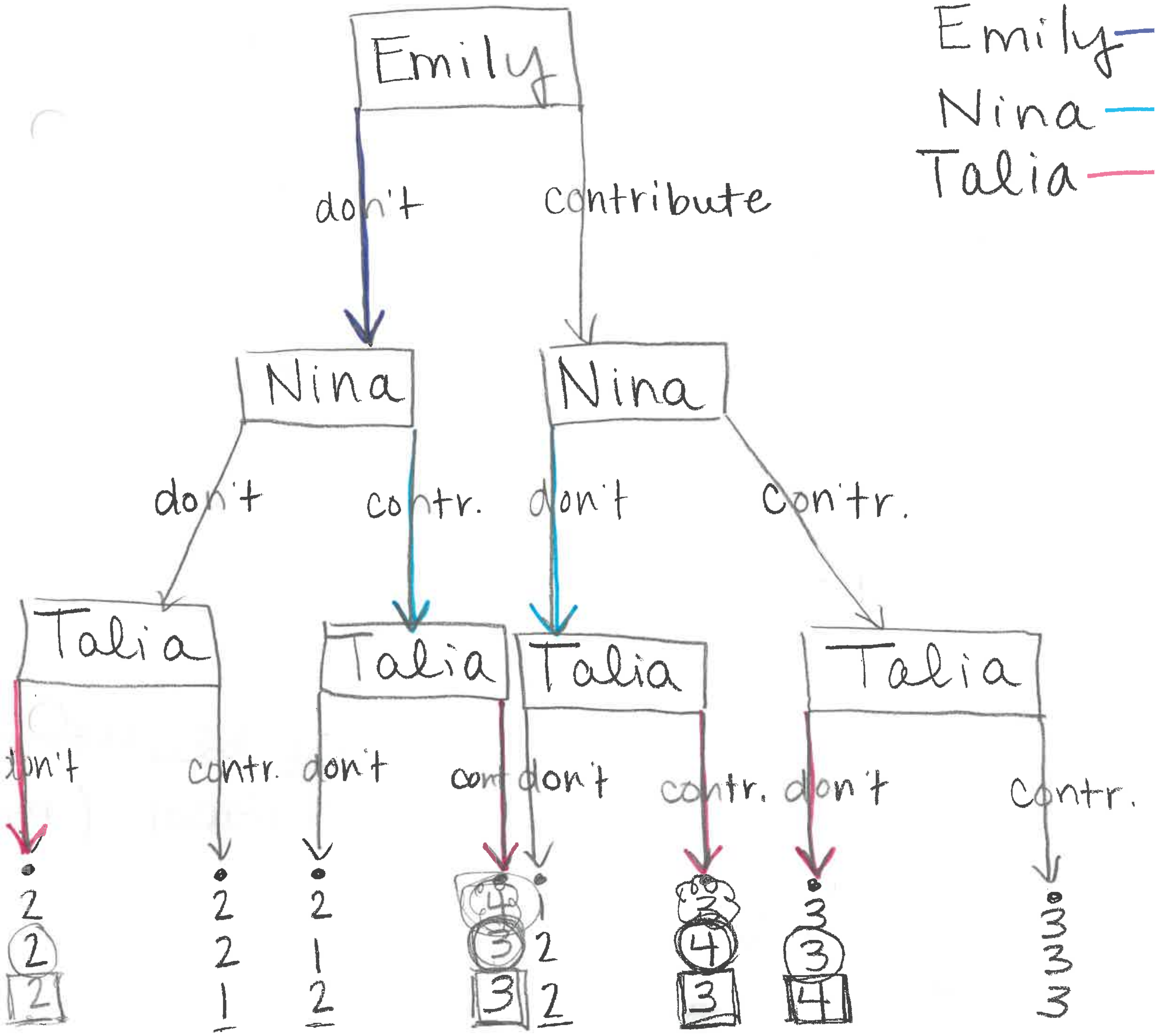
→ She doesn't, both others do = 4
* best

There's No Keurig

* worst
→ she contributes, no one else does = 1

→ she doesn't, & only one or none of the others do = 2

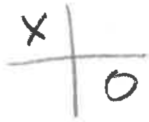
(build a few payoffs together)
(talk about ~~each~~ a few)


Emily
Nina
Talia
Emily
don't
contribute
Nina
Nina
don't
contr.
don't
contr.
Talia
Talia
Talia
Talia
don't
contr.
don't
contr.
don't
contr.
don't
contr.
2 2 2
2 2 1
2 1 2
3 3
2 2
3 4 3
3 3 4
3 3 3

→ so more players make things more complicated

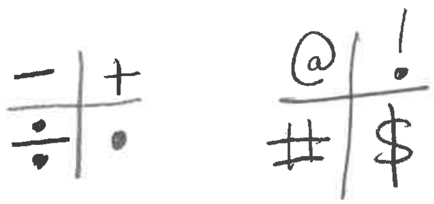
↳ but more moves is a whole new ballgame

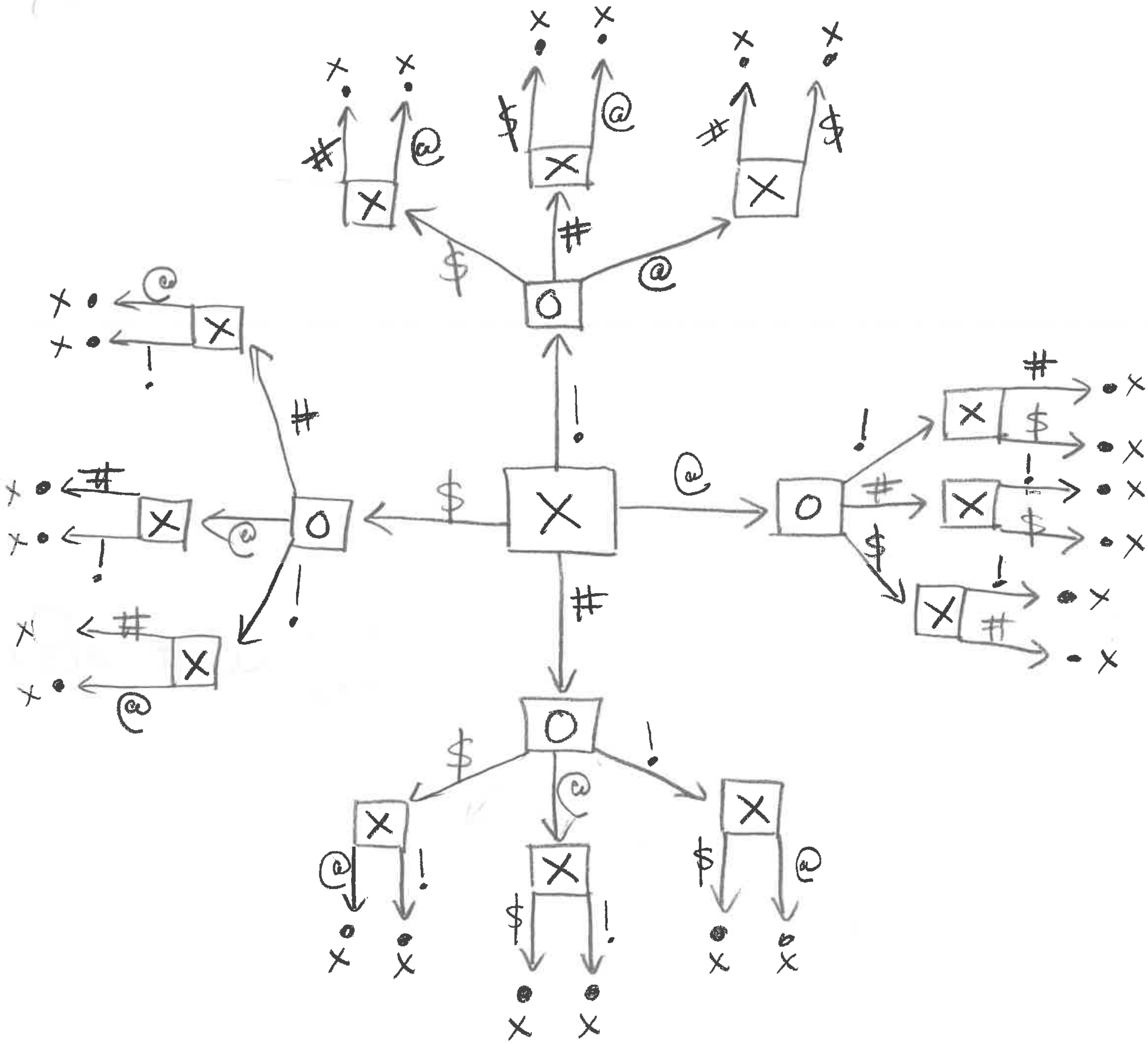
(*) Tic-Tac-Toe



→ Let's look at a simplified version: 2×2 (not 3×3)

→ every move the player before you makes defines the choices available to you

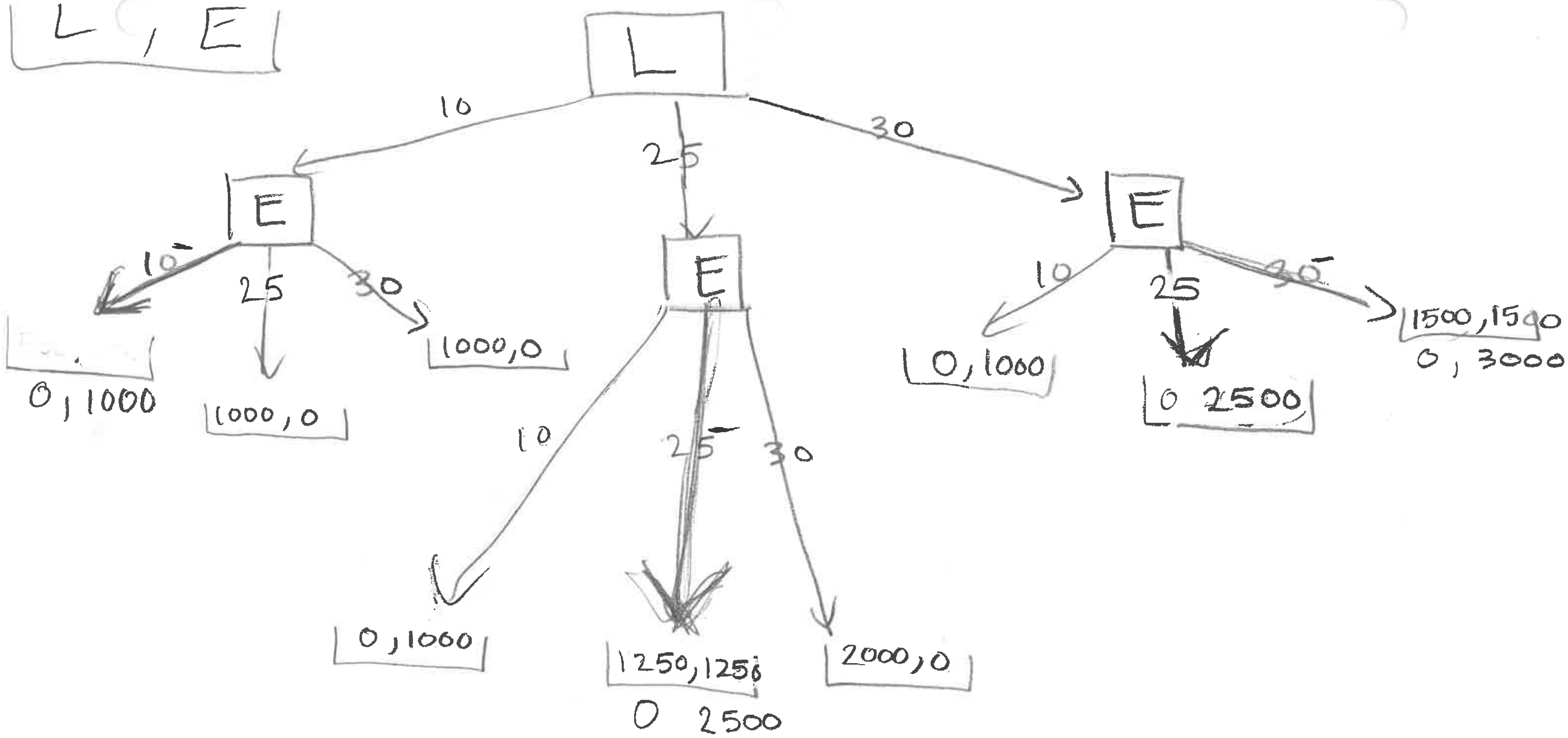




Mover Advantage

* **first-mover advantage**: if given a choice between going first or second, you would choose to go first.
  → forcing opponents onto a useful path
* **second-mover advantage**: if you would choose to go second
  → adapting to the situation to your benefit

Ex: Keurig in the coffee lounge?
First mover advantage


L, E
L
10
25
30
E
E
E
10
25
30
0, 1000
1000, 0
1000, 0
10
25
30
0, 1000
1250, 1250
0 2500
2000, 0
10
25
30
0, 1000
0 2500
1500, 1500
0, 3000

Bill Passage Problem

1. Two different tax bills:

Alexandria Ocasio-Cortez: A

(Much) Higher Income tax

75,000 | 150,000

75,000(0.05) ⟷ 75,000(0.05)

(100 - 75,000)(0.10

(150,000 - 100,000)(0.

⟶ wants to make the "tippy top" tax rate 70%

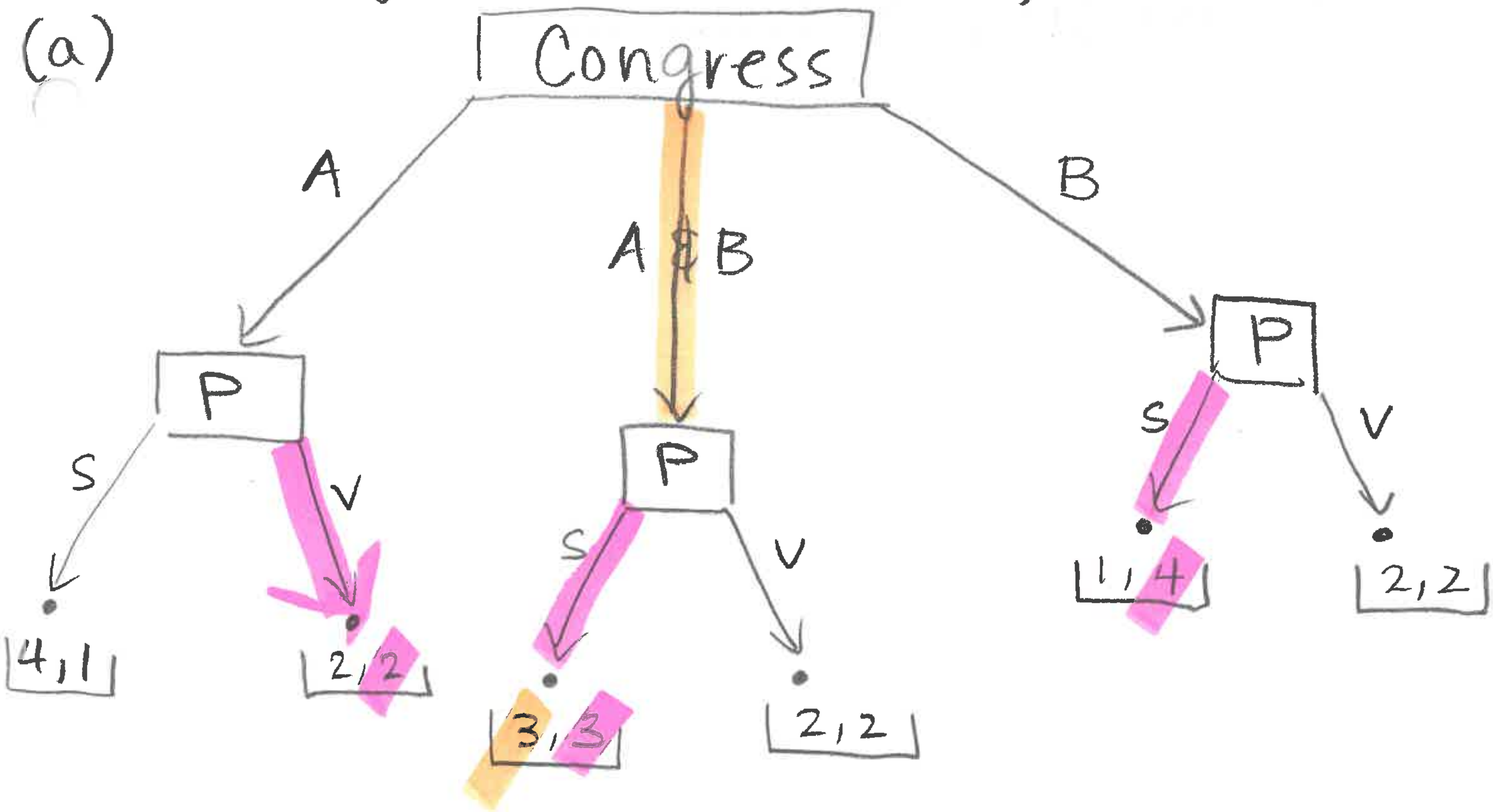
Elizabeth Warren: B

Wealth Tax

⟶ a 2-3% tax on your accumulated wealth (savings, capital gains, 401k, etc)

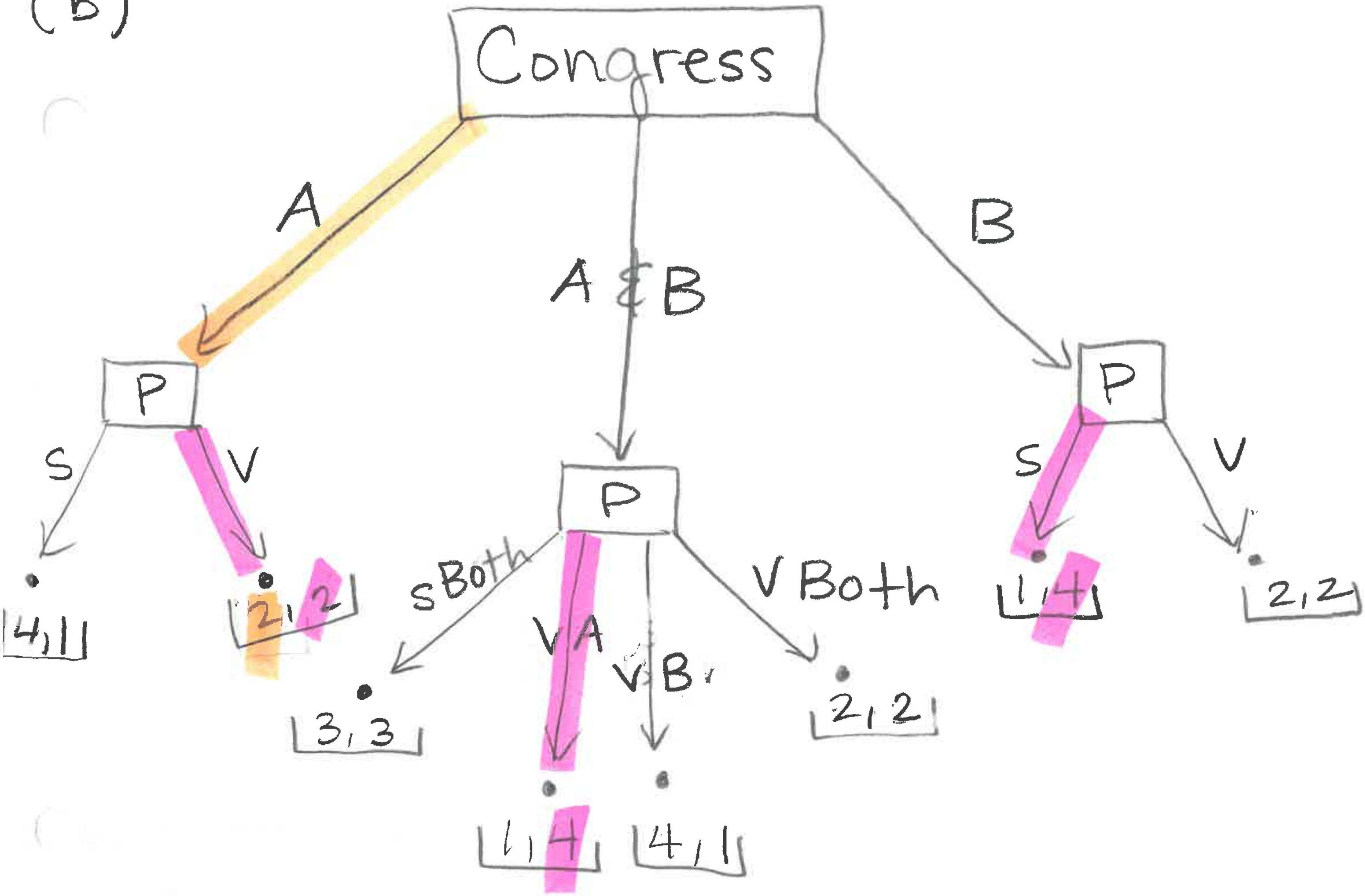
(not doing the no bill option)

(a)



⟶ both bills are passed by Congress & signed by the President

(b)



→ start w/ what they could do

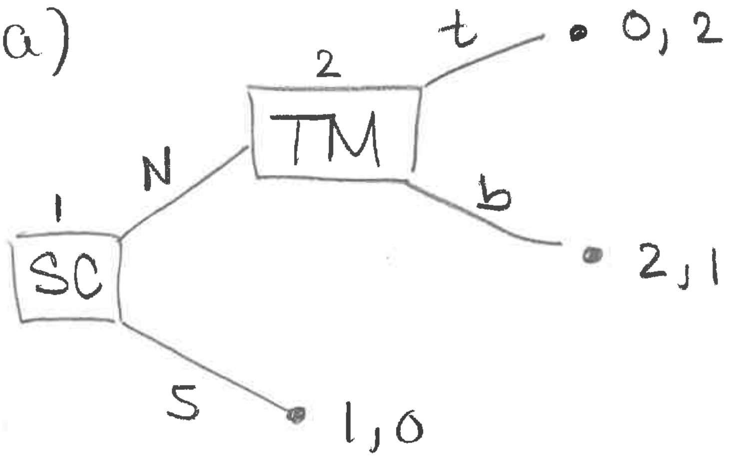
↳ then let the payoffs reflect what they want to do

⊛ Tin Man Question Strategies

→ a strategy requires specifying a decision at every decision node

↳ number the nodes
list decisions for each

(a)

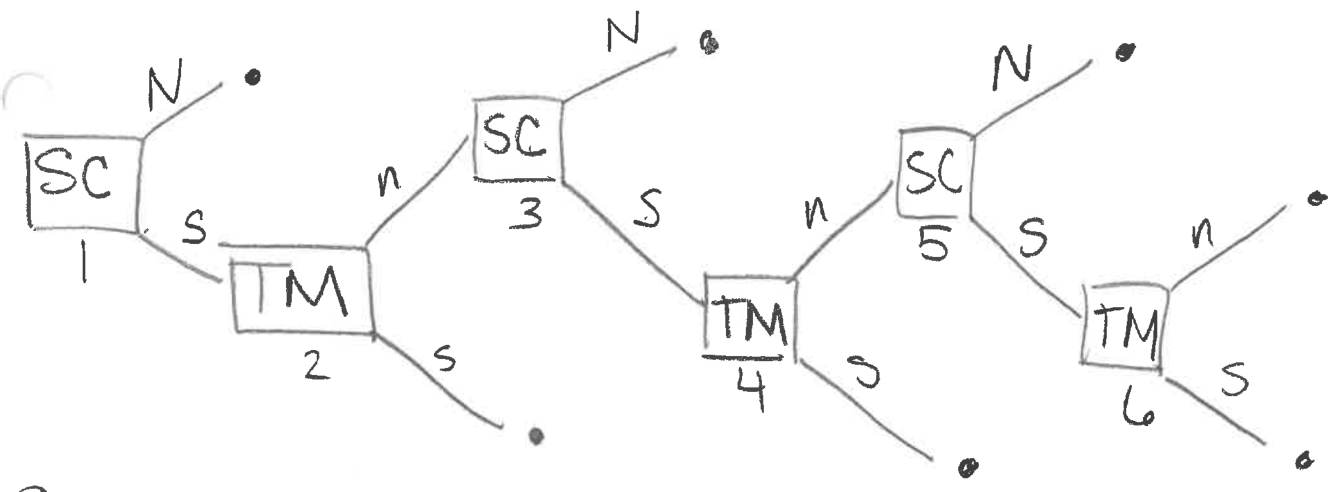


For each person, for each node

Scarecrow:
- 1: N
- 1: N

Tinman:
- 2: t
- 2: b

(b)



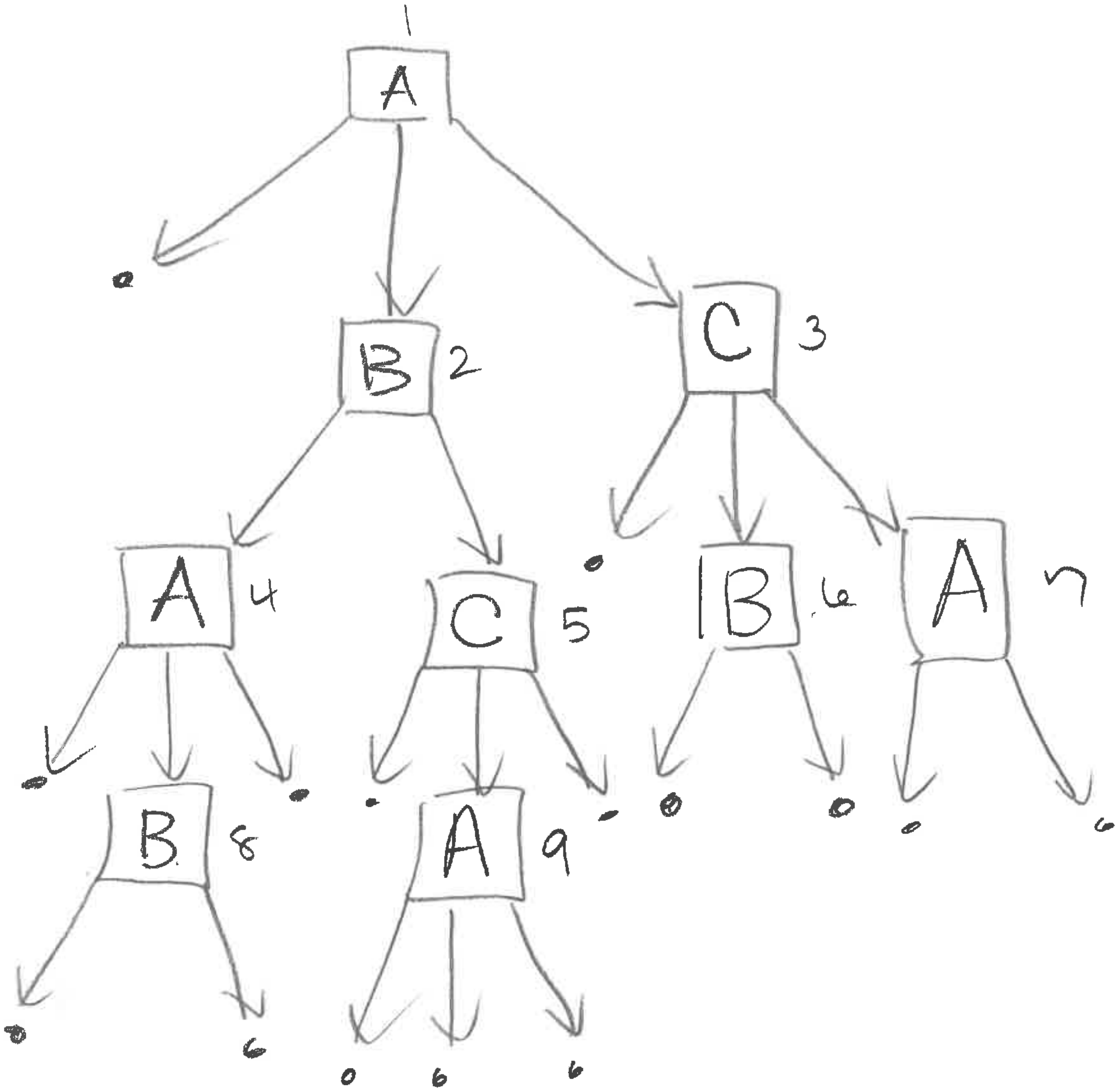
Scarecrow:

- 1: N 3: N 5: N
- 1: N 3: S 5: N
- 1: N 3: N 5: S
- 1: N 3: S 5: S
- 1: S 3: N 5: N
- 1: S 3: S 5: N
- 1: S 3: N 5: S
- 1: S 3: S 5: S

→ number of nodes

$\underline{2} \cdot \underline{2} \cdot \underline{2} = 8$ total strategies

number of decisions at each



Strategies for A:

$\underline{3}\ \ \underline{3}\ \ \underline{2}\ \ \underline{3} = 54$

B:

$\underline{2}\ \ \underline{2}\ \ \underline{2} = 8$

C:

$\underline{3}\ \ \underline{3} = 9$

(c)

Scarecrow: $\underline{2}\ \underline{2}\ \underline{2} = 8$

Tinman: $\underline{2} = 2$

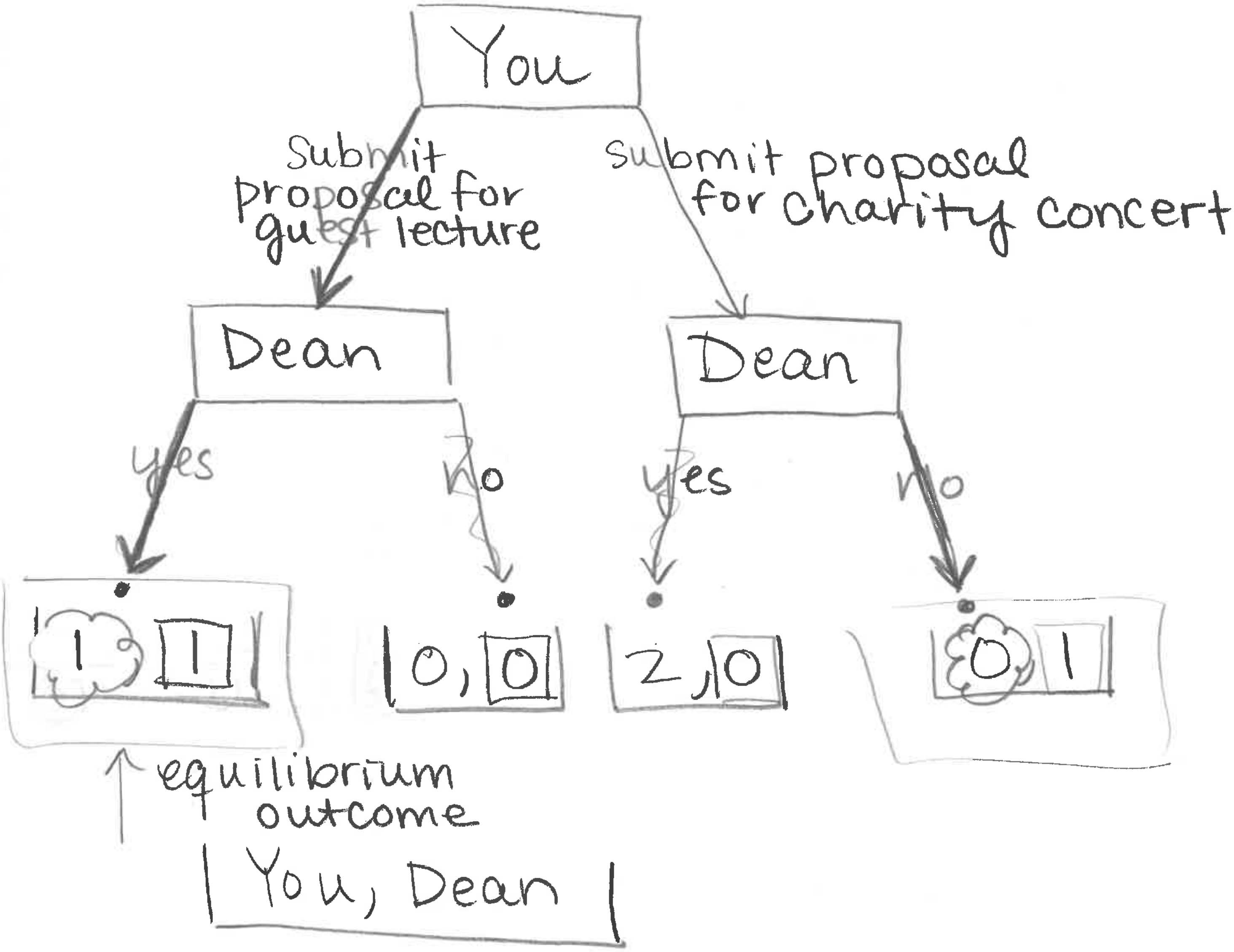
Lion: $\underline{2} = 2$

- N N N
- N N S
- N S N
- N S S

1 3 5

- S N N
- S N S
- S S N
- S S S

1 3 5



Strategies:

You:

- choose charity concert
- choose guest lecture

Dean:

- if you choose concert choose yes
- " no
- if you choose guest lecture choose yes
- if " no

equilibrium

Simultaneous Games w/ Pure Strategies I: (Discrete Strategies)

(*) Simultaneous-Move Games:

→ players move w/out knowing what others chose.

- exactly the same moment
- before they get the information

(*) Pure Strategies: a strategy w/o any ~~ambiguity or~~ randomness

(Mixed Strategy: a strategy that specifies randomness)

(*) Discrete Strategies: there are a finite number of strategies available to each player.

↳ you can list the options

up, down

attack, defend

(Continuous Strategies: there is a continuum of options

e.g. price choice)

* Examples:

- voting: you make your choice without knowing the other players' choices
- goalies: choose a direction before they've taken their shot

* We depict simultaneous games with discrete strategies using a game table

= game matrix
= payoff table
= normal form
= strategic form

know em all

→ describes the payoff associated with each outcome (like a game tree)

* the dimension of the table is the number of players
  - 2 players: 2D
  - 3 players: 3D
  - 1072 players: 1072 Dim.

(*) the specific rows & columns are associated w/ action choices

(Strategies are much more straightf. now.)

Rows: Player 1
Columns: Player 2

→ So a 2 dim table w/ 4 rows & 7 columns: 2 players, one of whom has 4 options & one of whom ha 7.

Chicken
2 Players: Paul Walker
Vin Deisel

Each has 2 options:
Swerve
Go Straight

Vin

Paul

	Swerve	Straight
Swerve	0, 0	−1, 1
Straight	1, −1	−2, −2

Paul, Vin

Nash Equilibrium

* Broad definition of equilibrium:
everyone is happy with what they've done, based on what other people did or would do.

* A <u>Nash Equilibrium</u>:
a list of strategies, one for each player, such that no player can get a better payoff by switching to some other strategy while every other player adheres to theirs specified strategy.

→ given everybody else stays the same, you're happy where you are.

→ a list of strategies such that for each player, it's the best strategy given the other strategies in the list

↳ ends up being a list of best strategies

(*) the "given other player's choices is the hardest & most important part.

		Column		
		L	M	R
Row	T	~~3,1~~	2,3	10,2
	M	4,5	3,0	6,4
	B	2,2	5,4	~~12,3~~

(*) Cell-by-Cell Inspection

→ for each cell (in the game table)

either order

- hold column fixed
  → is row happy?
  No? Not an eq.
  Yes? check column?

only need to check one No for it to be ruled out.

- hold row fixed
  → is column happy?
  No? Not an eq.
  Yes? An eq!

		Column		
		L	M	R
Row	T	3,1	2,3	10,2
	M	4,5	3,0	6,4
	B	2,2	5,4	12,3
	U	5,6	5,5	9,7

16!

(*) Some important things to note about N.E

1. A payoff has to be better to rule out a cell as a nash equil. (can't just be equal)
   ↳ B, M still an equilibrium

2. Adding options can remove equilibria
   ↳ M, L isn't one anymore

3. Removing options can add equilibria.
   → if we remove column's middle option
      ↳ B, R is an eq now.

4. Not necessarily what's jointly best (the social optimum).
   ↳ U, R gives the highest combined payoffs
   ↳ not an eq
   ↳ ruled out by 12 (B, R)

* Sometimes there are multiple nash equilibria, sometimes there's one, & sometimes there's none.

→ in our game of chicken there's 2

Vin

Paul	0, 0	−1, 1
	1, −1	−2, −2

→ here's an example with none

0, 1	1, 0
1, −1	−1, 1

Dominant Strategies

(*) Cell-by-Cell inspection will find you any Nash Equilibria if there are any to find.

↳ but ugh it can get tedious

↳ today: a way that can help w/ <u>some</u> games.

(*) Some games have the following neat feature:

there is a strategy for one of the players that is better than any other strategy no matter what his belief about the other player's choice is.

↳ a <u>dominant strategy</u>

↳ Better how? His payoffs are higher for all other player's options

→ a strategy that is always worse than another strategy is a <u>dominated strategy</u>.

(*) Made Up Game From Last Time:

		Column		
		L	M	R
Row	T	~~3,1~~	2,3	10,2
	~~H~~	~~4,5~~	~~3,0~~	~~6,4~~
	L	~~2,2~~	5,4	12,3
	B	~~5,6~~	4,5	9,7

(*) Row's strategy High is dominated by Bottom (no dominant strategies)

→ if a strategy is dominated, you can remove it from consideration (even if it's only dominated by one other.)

(*) Column's strategy Left is now dominated by right.

(*) Now Low is a dominant strategy.
↳ given that Row will play Low, column chooses middle.

→ rather than inspecting $3 \times 4 = 12$ possible cells, basically narrowed it down to 2.

(*) If you can find the Nash Eq. by successively eliminating strategies the game is <u>dominance</u> <u>solvable</u>.

(*) Finding dominant strategies can be very helpful in real life
→ professors are apparently very bad at recycling.

	Me	
	Recycl.	Trash
? ? people who deal w/ recycl ? ?		

w/ something I'm unsure about

→ Sustainability Comittee
↳ payoffs always higher if I choose to throw it out

(*) If everyone has a dominant Strategy, then the Nash Eq. is everyone playing their dominant strategies

↳ outcome defined by their intersection.

(*) What makes something a prisoner's dilemma:

1. There are two strategies for each player: cooperate w/ the other player, defect from cooperati

2. Defecting is a dominant strategy for each player

3. Everybody (Both Players) would be made better off if they could just both cooperate

↳ but they can't

→ I don't need to know the payoffs, I just need to know they're always higher if I choose to throw it in the trash if I'm unsure

"If in doubt, throw it out."

→ Also: "should I ask for more money"?

(*) The Prisoner's Dilemma

→ the Good Wife clip

dominant strategy

		Jon	
		Confess	Deny
Alexis	conf	10y, 10y	3mo, 25y
	Deny	25y, 3mo	8mo, 8mo

(*) smaller numbers better

dominant strategy

Best Response Analysis

→ we can check em all (Cell-by-Cell Inspection)

→ we can (sometimes) eliminate dominated strategies.

→ but we can do better: Best Response Analysis

(*) Really just a smarter way of doing cell by cell

↳ in order for it to be a Nash equilibrium, it needs to be one player's best choice given what the other player's do.

↳ so why don't we just take all the things someone else might do, & find all the best choices?

↳ do that for every player & you've got an equilibrium!

(Another) Algorithm:

1. For each strategy available to the column player, find the highest payoff for the row player.

2. For each strategy available to row player, find the highest payoff to the column player.

3. Any cell that has both: a Nash Equil
   ↳ many, one, or none

		Column		
		L	M	R
Row	T	3,1	2,3	10,2
	H	4,5	3,0	6,4
	L	2,2	5,4	12,3
	B	5,6	4,5	9,7

(*) Best Response Analysis will find any Nash equilibria in pure strategies.

↳ the alternative is mixed strategies (when they randomize

(*) The Fed & Congress:

↳ each has 2 strategies

Congress { Budget Balance
Budget Deficit (fiscal policy)
↳ can lead to inflation

The Fed { Low interest rates
↳ mortgages
High interest rates (monetary policy)
↳ fight inflation

→ usually happy to set low interest rates so long as inflation isn't a concern.

		The Fed	
		Low r	High r
Congress	Bal.	3, 4	1, 3
	Def.	4, 1	2, 2

if they all line up like that: dominant strategy.

More Than Two Players

→ when we have 2 players we just use a table with rows & columns.

↳ when we add more players we just create collections of tables

→ most of the time we just deal with three players

↳ add pages of tables, one "page" for each strategy available to the third player

Page 1:
3nd Player: Left

		Column	
		L	R
R O W	U	_ _ _	_ _ _
	D	_ _ _	

Page 2:
3rd Player: Right

		L	R
R O W	U		
	D		

→ can also thing of as Excel Spreadsheets

→ a set of matrices, whatever speaks to you 1

→ makes checking for Nash equilibria dominant strategies etc. more difficult

↳ always start w/ the definition (figure out what you need to hold constant, etc.)

(*) Roommates Game Revisited

↳ want to buy a Keurig for their common area.

↳ but now they move simultaneously (also a few other changes)

- if all 3 contribute: the nicest Keurig around
- if only 2: a fine Keurig, standard
- if only 1: the worst Keurig
- if none: no Keurig

(*) From the first roommate's (Emily) perspective

→ if the other 2 both contribut

- Contribute? 5
- don't? 6

→ if only 1 other contributes

- contribute? 3
- don't? 4

→ if none do

- contribute? 1
- don't? 2

I got confused b/c I looked at the wrong payoffs

Talia: C

		Nina	
		C	D
Emily	C	5,5,5	3,6,3
	D	6,3,3	4,4,1

Talia: D

		Nina	
		C	D
Emily	C	3,3,6	1,4,4
	D	4,1,4	$\boxed{2,2,2}$

(*) What's a Nash Equilibrium Again?

→ your best response to anything someone else might do. (for every player)

(*) What's a Dominant Strategy?

→ a strategy that's your best option nomatter what someone else does

→ they each have a dominant strategy to not contribute

↳ that's the Nash Equilibrium

(*) Remember: start by specifying everything done by someone else.

↳ then once you're down to a single player's choices, ask what's best

---

Homework worktime!

Multiple Equilibria & No Equilibrium

* Sometimes there are multiple N.E & sometimes there are none
  - ↳ we've don examples with multiple before
  - ↳ let's start with none

* Sports! (In this case: tennis)
  → Player 1 is about to return a volley
  - Down the line (DL)
  - Cross court (CC)

  → Player 2 needs to pick a defense
  ↳ does she prepare for a straight down the line shot or a crosscourt
  DL CC

  → Lets say the payoffs to player 1 are the probability she is successful
  ↳ to player 2, probability player 1 isn't successful
  ↳ zero-sum (constant-sum) game

(*) Coordination Games

① Pure Coordination:

	B	W
B	1,1	0,0
W	0,0	1,1

PC

Mac Users

~~2 Students are doing a group project & one needs to bring the Mac & one needs to bring the Dongle™~~

Assur.

② AssuranceS

H	B	W
B	1,1	0,0
W	0,0	2,2

focal point

2 students agree to meet up at Scout to study

↳ But which Scout! B or W?

③ Battle of the Sexes

H		
	2,1	0,0
	0,0	1,2

Planning a Birthday celebr.

B of the S Drinks-n-apps

~~Romantic Comedy~~

~~Action Adv.~~ sit down

④ Chicken

	Sw	St
Sw	0,0	-1,1
St	1,-1	-2,-2

Chicken

Simultaneous Game Assignment

→ Like the sequential game but a few changes:
↳ everyone has their own
↳ 2 stages

1. Today:
   - think up a game
   - 2 players
   - 2 versions
     → one smaller, can be fairly easy to write down & solve on a piece of paper
     → one larger, more strategie
       ↳ solve w/ a computer
       ↳ still need to be written down

2. Next Week:
   → program a computer to solve it

Programming Games

(*) Goal this week: program a computer to solve a simultaneous matrix game.
↳ using Cell-by-Cell Inspection.

(*) Why that way?
⟶ computers are very good at tedious tasks
⟶ very bad at abstract ones

(*) Using Python
⟶ very popular, versatile programming language
⟶ it's also free

(*) <u>Very</u> applied foray into computer science
↳ not going into <u>how</u> it works
↳ as far as we're concerned, it's magic.

⊛ Cell-By-Cell Inspection

→ Check each cell of the matrix to see if it satisfies the conditions of a Nash Equilibrium

⊛ Tasks Today:

1. Download & Install Thonny
2. Hello World
3. Revisit our Games from Wed.
4. How would I instruct a computer to do c-b-c insp.?

⊛ Hello World

→ the traditional first program you write w/ any new language

→ make the computer output "Hello World!"

→ learn how to make the computer print out what you want

```
print('Hello World!')
```

make an identifier : uncl. NE Not

* Cell-By-Cell inspection:

- For each cell:
  → Check each cell for being a NE
  - for the row player:
    → is there a higher payoff avail. in this column
      yes : not a NE
      no : check col.

  if still unclassif.
  - for the column player
    → is there a higher payoff avail in this row?
      yes : not an NE
      no : NE

  if class = NE
  add to list
- at the end of each cell, add it to the NE List if its an NE
  ↳ store row & col
  ↳ print out at the end

Print(M) Print(M[0]) M[0][1] = 5

(*) Make a Matrix in Python

4	5	12
9	3	10
4	9	15
11	9	16

```
M=[[4,5,12],
   [9,3,10],
   [4,9,15],
   [11,9,16]]
```

= comment

EC: if you can tell me where these numbers come from, I'll give you 3 points

→ in Python the row & column numbers start at 0.

Print(M[0])

[1]

:

(*) for loops

```
for row in range(4):
    print(M[row])
```

row = 0	print(M[0])	[4, 5, 12]
row = 1	print(M[1])	[9, 3, 10]
row = 2	print(M[2])	[4, 9, 15]
row = 3	print(M[3])	[11, 9, 16]

* Visit Each cell of the matrix:

→ each cell is named by a row & column

↳ so that's how we make sure we visit each one

↳ do all the rows & do all the columns.

start at the first row
    start at the first col
        then the 2nd col
        then the 3rd
go to the 2nd row
    start at the first col
    then the 2nd col
    then the 3rd
go to the 3rd row
    start at the 1st col
        ⋮

got to here

→ it's a for-loop in a for-loop
[draw matrix]
[demonstrate for loop movement]

variable special code words

```
for row in range(0,4):
    for col in range(0,3):
        print(M[row][col])
```

→ now that we can get to the cells, we cando things to them.

(*) Check a condition:

→ Check if the number in the cell is >= 10

↳ if it is, print yes

↳ if it isn't, print no

```
for row in range(0,4):
    for col in range(0,3):        conditi
        if M[row][col] >= 10:
            print('yes')
        else
            print('no')
```

(*) Now can't do much more until we figure out how to input a game

(*) Check if a given cell is an NE matrx

⊛ How to store a game matrix:

Game_Matrix = [[[3, 1], [2, 3], [10, 2]],
[[4, 5], [3, 0], [6, 4]],
[[2, 2], [5, 4], [12, 3]],
[[5, 6], [4, 5], [9, 7]]]

3, 1	2, 3	10, 2
4, 5	3, 0	6, 4
2, 2	5, 4	12, 3
5, 6	4, 5	9, 7

* Check a given cell at row, col

current_row-payoff = M[row][col][0

~~ col ~~ = M[row][col][1]

→ check row

for r in range(4):

if flag ~~unset NE~~ unset

if new payoff > old

set flag to 2

→ Check col

if flag not 2

for c in range(3)

if flag not 2

if new > old

set fl = 2

matrix of flags all set to 0 (unset)

~~flag~~ label = { 0 unset
1 NE
2 Not an NE

→ if flag = 0

flag = 1

Game Matrix

3,1	2,3	10,2
4,5	3,0	6,4
2,2	5,4	12,3
5,6	4,5	9,7

2	0	0
0	0	0
0	0	0
2	0	0

Simultaneous-Move Games w/ Pure Strategies Part II: Continuous Strategies

→ In the last section, the strategies available to each player we discrete
↳ a small, finite number

→ there are many games where players have LOTS of strategies
↳ either they're actually continuous variables (uncountably infinite)

↳ or they're discrete but there's so many options that a game table is not practical

Ex: I'm applying to go to a mentoring workshop for junior faculty at Liberal Arts Schools. The application window is open from now until Aug. 1. I can choose any day between now & then

→ there are 141 days between now & Aug. 1.

↳ I could make a 141 × 141 game table but should I?

↳ no. Just treat it as a continuou variable

(*) So now strategies can just be treated as variables!

(*) Start w/ an example, then write down general methods.

→ two businesses: X & Y

↳ they sell the same type of good

↳ compete through prices

→ each business has the same goal: maximize profits by setting their price.

↓

their strategy

‖

P_x, P_Y

* Gotta figure out how price affects profit : profit equation

$$\pi = \underbrace{TR}\, - \underbrace{TC}$$

so let's figure out these

→ these 2 businesses are competitor so the price X sets affects how much Y sells & vice versa.

$Q_X(P_X) = 44 - 2P_X + P_Y$ (let's say in thousand

↑ a function of price

$Q_Y(P_Y) = 44 - 2P_Y + P_X$

→ let's say each unit costs \$8 to make

$\pi_X = P_X Q_X - 8Q_X = (P_X - 8)Q_X$

$\pi_Y = \underbrace{P_Y Q_Y}_{TR} - \underbrace{8Q_Y}_{TC} = (P_Y - 8)Q_Y$

→ excellent! but we're choosing P not Q → get those π's as a function of P

3

$$\Pi_x(P_x) = P_x(\overset{Q_x}{44 - 2P_x + P_Y}) - 8(\overset{Q_x}{44 - 2P_x + P_Y})$$

$$= 44P_x - 2P_x^2 + P_Y P_x - 352 + 16P_x - 8P_Y$$

$$= 60P_x - 2P_x^2 + P_x \underline{P_Y} - 8\underline{P_Y} - 352$$

$$\Pi_Y(P_Y) = P_Y(44 - 2P_Y + P_x) - 8(44 - 2P_Y + P_x)$$

$$= 44P_Y - 2P_Y^2 + P_Y P_x - 352 + 16P_Y - 8P_x$$

$$= 60P_Y - 2P_Y^2 + P_Y \underline{P_x} - 8\underline{P_x} - 352$$

* Each business is going to choose <u>their</u> price to maximize <u>their</u> profits

↳ derivatives!

↳ how do we optimize things?
take the derivative
set it = 0.

(*) Some Quick Derivative Reminders:

1) $f(x) = x^2$, $\frac{\partial f}{\partial x} = 2x$

x^3 $= 3x^2$

$\vdots$

2) Find partial derivatives by holding any other variables constant.

↳ just pretend they're a plane jane number

$$f(x,y,z) = 2xy^2 + 2y + z^2y^4$$

$$\frac{\partial f}{\partial y} = 2x(2y) + 2 + z^2(4y^3)$$

$$\frac{\partial \Pi_X}{\partial P_X} = 60 - 4P_X + P_Y - 0 - 0$$

$$60 - 4P_X^* + P_Y = 0$$

$$60 + P_Y = 4P_X^*$$

$$\boxed{15 + \frac{1}{4}P_Y = P_X^*}$$

these are called **Best-Respon[se] Functions.**

$$\frac{\partial \Pi_Y}{\partial P_Y} = 60 - 4P_Y + P_X - 0$$

$$60 - 4P_Y^* + P_X = 0$$

$$60 + P_X = 4P_Y^*$$

$$\boxed{15 + 1/4\, P_X = P_Y^*}$$

→ but hmmm there's a P_Y in my P_X^* formula & a P_X in my P_Y^* formula...

Simultaneous Continuous Games (Day 2)

Last Time: 2 Businesses: X & Y

$$\pi_X = 60P_X - 2P_X^2 + P_XP_Y - 8P_Y - 352$$

$$\pi_Y = 60P_Y - 2P_Y^2 + P_YP_X - 8P_X - 352$$

(*) To maximize our profits by picking our price we:

- take the derivative
- set it = 0.

$$\frac{\partial \pi_X}{\partial P_X} = 60 - 4P_X^* + P_Y - 0 - 0 = 0$$

$$60 + P_Y = 4P_X^*$$

$$15 + \frac{1}{4}P_Y = P_X^*$$

Best Response Function

* A player's best response function describes his best response (duh) given the other player(s)' choices.

→ all of the players' BRFs describe a system of equations

$$\begin{bmatrix} P_x^* = 15 + \frac{1}{4} P_Y \\ P_Y^* = 15 + \frac{1}{4} P_x \end{bmatrix}$$

$0 = 4P_x - 60$

$P_x = 15 + \frac{1}{4} P_y \quad 15 = P_x$

$P_x - 15 = \frac{1}{4} P_Y$

$4P_x - 60 = P_Y$

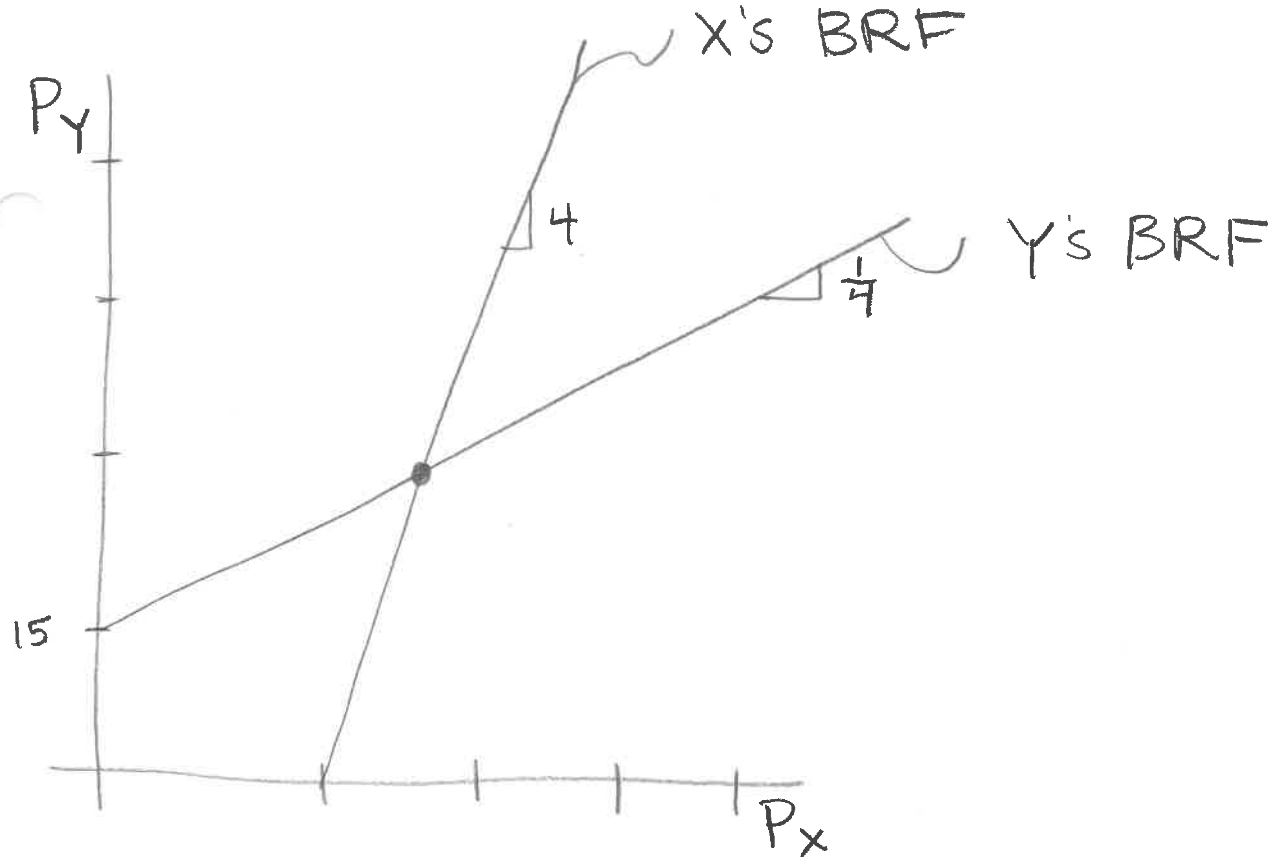
$\sim \quad P_Y = 15 + \frac{1}{4} P_x$

$\cancel{\frac{1}{4} P_x = 15 - P_Y}$

$\cancel{P_x = 60 - 4P_Y}$

→ we have lots of ways to solve systems:

- plug one into another
- computationally
- linear algebra
- graphically | set em equal



$$15 + \tfrac{1}{4}P_x = \overset{\frac{16}{4}}{4}P_x - 60$$

$$75 = \tfrac{15}{4} P_x$$

$$20 = P_x \longrightarrow P_Y = 15 + \tfrac{1}{4}(20)$$

$$= 15 + 5 = 20$$

(*) How much profit do they make?

$$\pi_x = 60(20) - 2(20)^2 + (20)(20) - 8(20) - 352$$

$$1200 \quad -800 \quad +400 \quad -160 - 352$$

$$= 288$$

$\longrightarrow$ in 100's $\longrightarrow$ \$28,800 per month

(3

(*) General Method for Solving these Continuous simultaneous games:

1. Construct everybody's payoffs as a function of everybody's choices

$$\Pi_X = F(x, y, z, \ldots)$$
$$\Pi_Y = G(x, y, z, \ldots)$$
$$\Pi_Z = H(x, y, z, \ldots)$$

2. Take the partial derivative of each player's payoff function w/r/t <u>that</u> <u>player's</u> choice.

$$\frac{\partial \Pi_X}{\partial x} = 0$$
$$\frac{\partial \Pi_Y}{\partial y} = 0$$
$$\frac{\partial \Pi_Z}{\partial z} = 0$$

3. Set it $= 0$

4. Solve that system of equations.

$\longrightarrow$ the solution to the system of equations is the values of the choice vars that make the system true

$$15 + \frac{1}{4} P_Y - P_X = 0$$

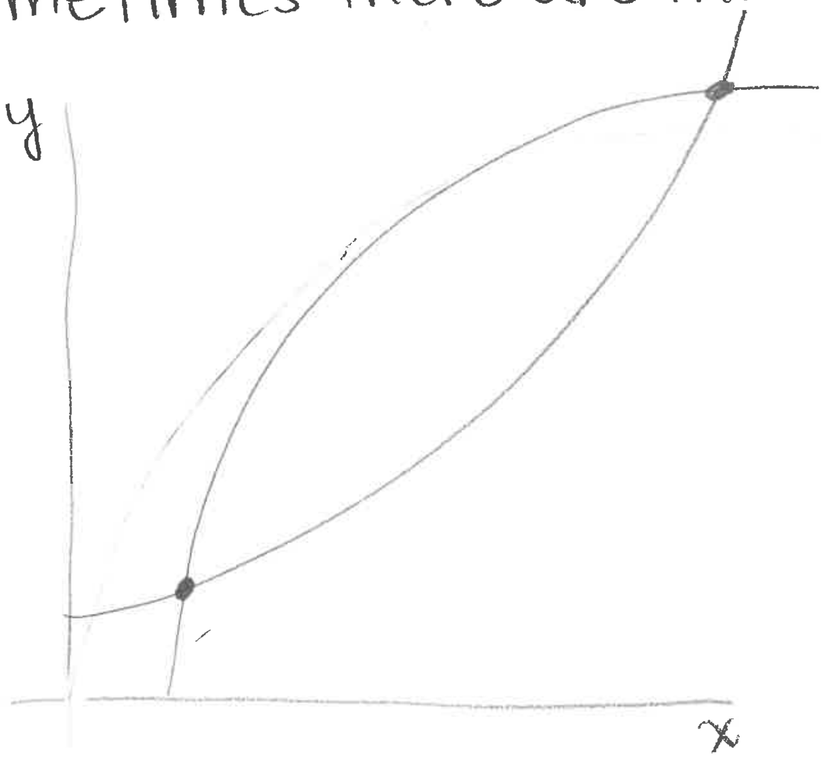
$$15 + \frac{1}{4} P_X - P_Y = 0$$

Solution: $P_X = 20$ $P_Y = 20$

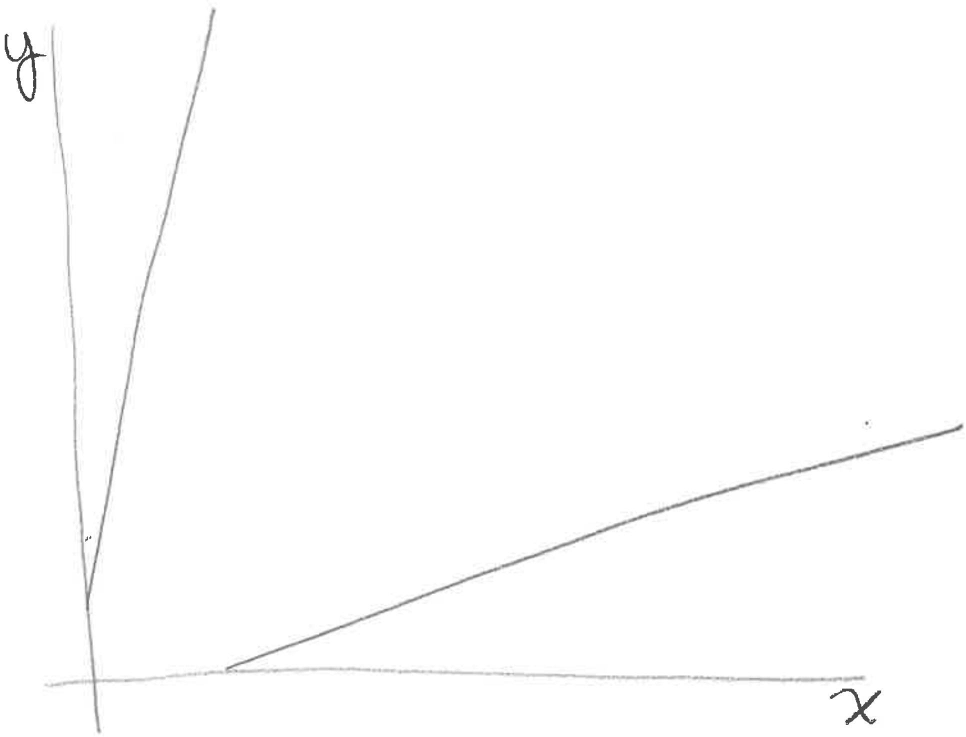
$\longrightarrow$ that solution is the Nash Equilibrium.

(*) For these games, the NE is a set of variable values

(*) Sometimes there are multiple



(:) Sometimes there are none



Political Advertising Game

Two Political Parties Campaigning for office: L & R

→ they need to choose how much money to spend on advertising

→ their objective is to win the election

↳ by maximizing their vote share

→ for today: assume that the share of the votes they get is the share of total campaign advertising that is done in the race.

→ Party L spends $\$\ell$ million

→ Party R spends $\$r$ million

Total ad spending $= \ell + r$

L's spending share = vote share

$$= \frac{\ell}{\ell + r}$$

R's vote share = spending share

$$= \frac{r}{l+r}$$

→ In addition to spending money on ads, they need to pay campaign workers (who work to raise the ad money).

↳ assume linearly proportional to the amount raised/spent : l, r

What are the Nash equilibrium spending amounts?

<u>Method:</u>

1. Construct Payoffs ✓
2. Take Partials
3. Set $= 0$
4. Solve the System of Equations

(*) Payoffs:

L's Payoff:

$$100 \cdot \underbrace{\frac{l}{l+r}}_{\text{vote share}} - \underbrace{l}_{\text{cost}} = \pi_R$$

R's Payoff:

$$100 \frac{r}{l+r} - r = \pi_L$$

(*) Partials:

$$\frac{\partial \pi_L}{\partial l} = 100 \frac{(l+r)(1) - l(1)}{(l+r)^2} - 1$$

$$= 100 \frac{l+r-l}{(l+r)^2} - 1 = 100 \frac{r}{(l+r)^2} - 1$$

$$\frac{\partial \pi_R}{\partial r} = 100 \frac{(l+r)(1) - r(1)}{(l+r)^2} - 1$$

$$100 \cdot \frac{l+r-r}{(l+r)^2} - 1 = 100 \cdot \frac{l}{(l+r)^2} - 1$$

⊛ Set them $= 0$

$$100 \cdot \frac{r}{(l^*+r)^2} - 1 = 0$$

$$\sqrt{\frac{100\,r}{(l^*+r)^2}} = \sqrt{1}$$

$$\frac{10\sqrt{r}}{l^*+r} = 1$$

$$10\sqrt{r} = l^*+r$$

$$10\sqrt{r} - r = l^*$$

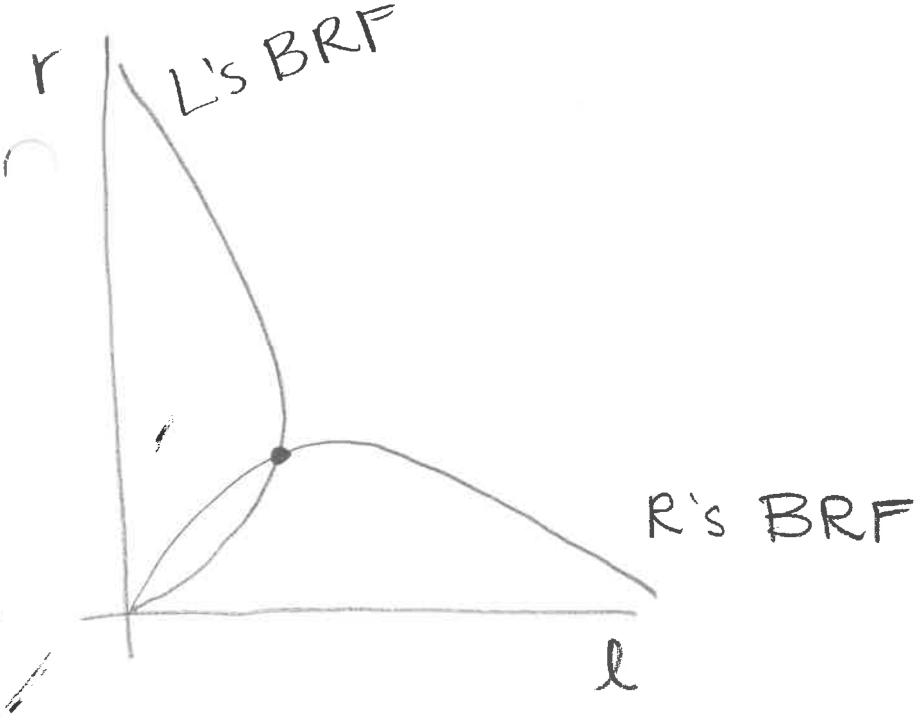
$$\frac{100\,l}{(l+r^*)^2} - 1 = 0$$

$$\sqrt{\frac{100\,l}{(l+r^*)^2}} = \sqrt{1}$$

$$\frac{10\sqrt{l}}{l+r^*} = 1$$

$$10\sqrt{l} = l + r^*$$

$$10\sqrt{l} - l = r^*$$



→ for small amounts of spending, it's best to increase your spending when they do

→ for larger amounts, it's best scale back when they spend more

→ because of that denominator $(l+r)$

4. Solve the System

$10\sqrt{r} - r = \ell$

$10\sqrt{\ell} - \ell = r$

$10\sqrt{10\sqrt{\ell} - \ell} - (10\sqrt{\ell} - \ell) = \ell$

$10\sqrt{10\sqrt{\ell} - \ell} - 10\sqrt{\ell} + \ell = \ell$

$\qquad\qquad\qquad\qquad -\ell \quad -\ell$

$(\cancel{10}\sqrt{10\sqrt{\ell} - \ell})^2 = (\cancel{10}\sqrt{\ell})^2$

$\cancel{100(10\sqrt{\ell} - \ell) = 100\ell}$

$\cancel{100(10\sqrt{\ell}) - 100\ell = 100\ell}$

$\sqrt{10\sqrt{\ell} - \ell} = \sqrt{\ell}$

$10\sqrt{\ell} - \ell = \ell$ $\qquad$ $0 = \ell$

$\quad +\ell \quad +\ell$ $\qquad$ $0 = 4\ell - 100$

$(10\sqrt{\ell})^2 = (2\ell)^2$ $\qquad$ $100 = 4\ell$

$100\ell = 4\ell^2$ $\qquad$ $25 = \ell$

$0 = 4\ell^2 - 100\ell$

$0 = \ell(4\ell - 100)$

6

Oligopoly

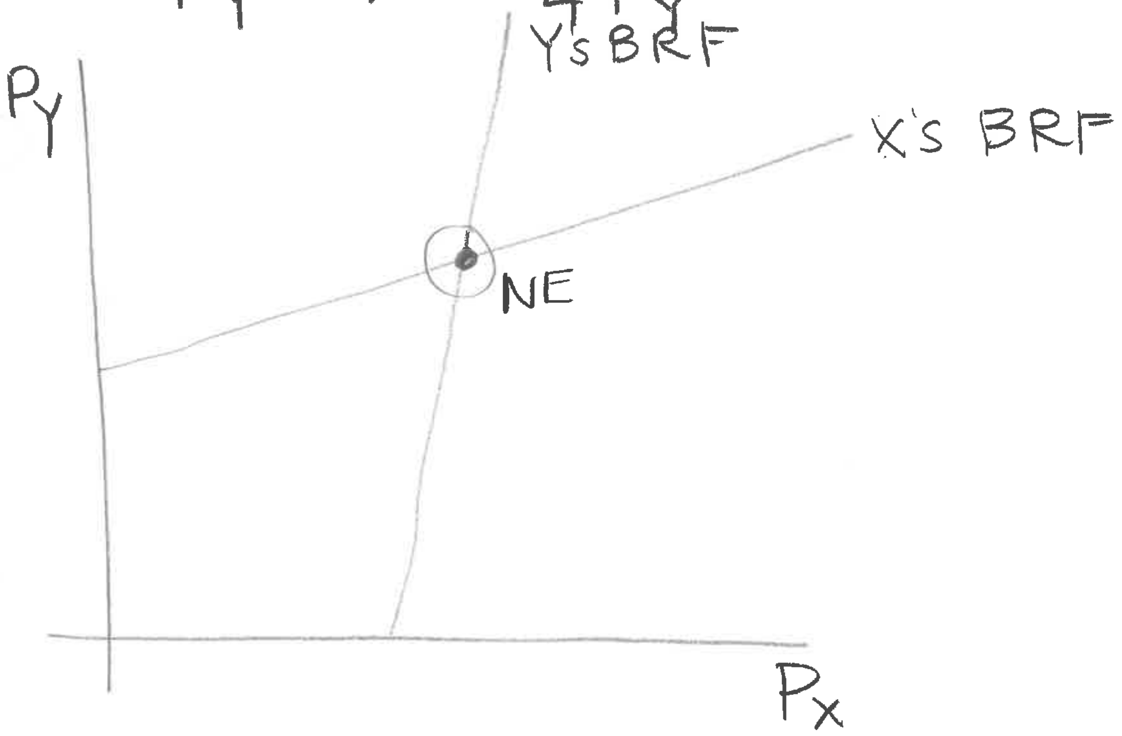
→ back to the competing firms.

BRF for X:

$$P_X = 15 + \frac{1}{4} P_Y$$

BRF for Y:

$$P_Y = 15 + \frac{1}{4} P_Y$$



→ hey look! they're upward sloping!
↳ this is because they're substit.

→ when business X raises their price by $1, business Y should raise their price by $0.25

→ what's the economics?

- When business X increases their price, their quantity demanded goes down. (Law of Demand)
  - Some of those customers go to business Y, increasing their ~~quantity~~ demanded.
  - So they have the ability to raise their profits.

→ So these 2 businesses <u>help</u> each othe when they raise their prices

→ in fact: if they both raised their prices from 20 to 24:

$Q_x = 44 - 2(24) + 24 = 20$ (hundred

$= 2000$

$TR = 24 \cdot 2000 - 8 \cdot 2000$

$= 48000 - 16000 = 32,000$

→ bigger than the \$28,800 they make in the NE.

(*) So why don't they do this?

→ if business Y knows that business x is going to charge \$24, what is their B-R?

$$P_Y = 15 + 1/4(24) = 15 + 6 = 21$$

→ and if they charge THAT what happens?

$$Q_Y = 44 - 2(\overset{P_Y}{21}) + (\overset{P_X}{24})$$

$$= 44 - 42 + 24 = 26$$

→ 2600

→ for a profit of

$$TR = 21(2600) = 54,600$$

$$TC = 8(2600) = 20,800$$

$$33,800 > 32,000$$

→ can be made better off by undercutting.

(*) What if they could (somehow) prevent that?

↳ somehow agree to work together

= collude

= a cartel (illegal)

→ so they charge the same price and share the profits equally.

$$P_X = P_Y = P$$

$$\pi_c^J = \pi_X + \pi_Y = (P-8)(\overbrace{44-2P+P}^{44-P}) + (P-8)(44-2P+P)$$

$$= 2(P-8)(44-P)$$

$$= 2(44P - P^2 - 352 + 8P)$$

$$= 2(-352 + 52P - P^2)$$

→ if they split profits evenly than each gets

$$\pi_c^i = -352 + 52P - P^2$$

→ What price should they each charge?

$$\max_{P} \left(-352 + 52P - P^2\right)$$

$$\frac{\partial \Pi}{\partial P} = 0 + 52 - 2P = 0$$

$$52 = 2P$$

$$\boxed{26 = P}$$

→ how much profit is that?

$$\Pi = -352 + 52(26) - (26)^2$$

$$= -352 + 1352 - 676$$

$$= 32,400$$

(*) These 2 goods were substitutes. What if they were complements?

Test 3 Expectations

(*) Vocab:

- best-response curves
- best-response rule
- continuous strategy
- refinement

(*) Put together payoff functions

( ) Find BRFs

↳ talk about them

(*) Find NE prices or whatever

(*) Find joint payoffs, prices, etc...

↳ compare to NE's.

Name:

1. You and a partner are working on a project for class. The payoff to you is the points that you get on the project minus the cost you incur by working on this project. If you put in x hours of work and your partner puts in y hours of work, your grade will be $10xy$. The cost you incur is $x^2 + 10x$ and your partner's is $y^2 + 10y$.

   (a) Construct the payoff functions for yourself and your partner.

$$\pi_x = 10xy - x^2 - 10x$$
$$\pi_y = 10xy - y^2 - 10y$$

   (b) Find your best response function and their best response function.

$$\frac{\partial \pi_x}{\partial x}: 10y - 2x - 10 = 0 \qquad 10y - 10 = 2x \qquad \boxed{5y - 5 = x}$$

$$\frac{\partial \pi_y}{\partial y}: 10x - 2y - 10 = 0 \qquad 10x - 10 = 2y \qquad \boxed{5x - 5 = y}$$

   (c) Find the Nash equilibrium hours-spent for you and your partner.

$$5(5x - 5) - 5 = x$$
$$25x - 25 - \cancel{5x}\ 5 = x$$
$$25x - 30 = x \qquad 1.25$$
$$24x = 30 \qquad x = \frac{30}{24}$$

$$5\left(\frac{30}{24}\right) - 5 = y$$
$$1.25 = y$$

Practice WS

(a) Build the Π's

(b) BRFs

(c) Solve the system

---

1. A non P–Q one

You & a partner are working on a project at work

the grade you each get is

$\Pi_x = x^2 \cdot y^2 - xy$

$\Pi_y \; x^2 \cdot y^2 - xy$

$\frac{\partial \Pi_x}{\partial x} = 2xy^2 - y =$

$\frac{\partial \Pi}{\partial y} = 2x^2 y - x = 0$

$y = 2xy^2$

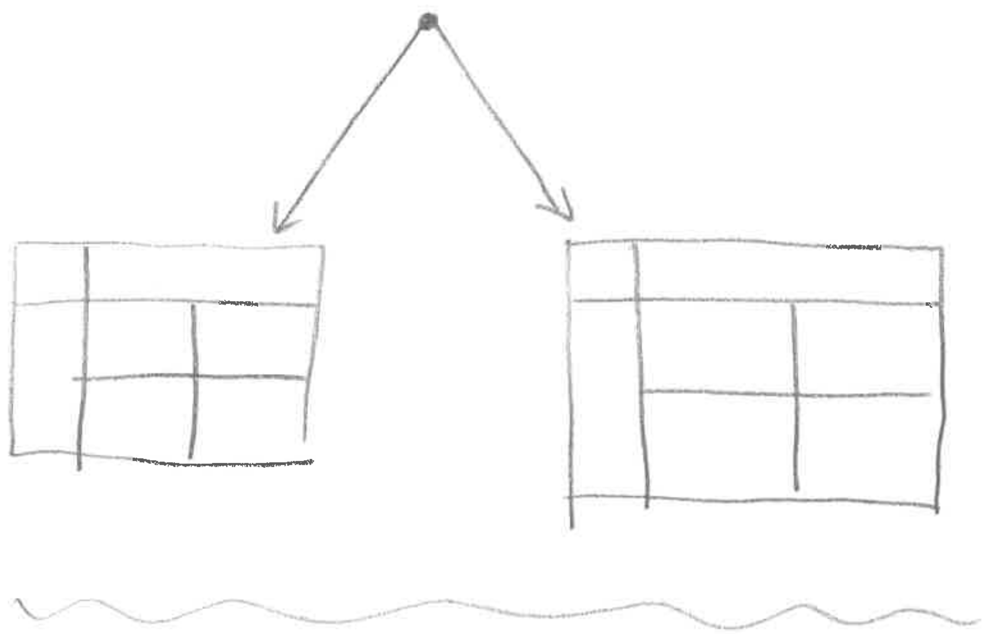
$x = 2x^2 y$

$x = 2x^2(2xy^2) = 24x^3y^2$

Simultaneous & Sequential Games

Ch. 3: Sequential Games
Ch's 4 & 5: Simultaneous Games
Ch. 6: Both!

→ the game trees & game tables can get pretty complicated



not too bad, but what if the choices in the matrices lead to new branches??

(*) Let's do an example:

→ two Telecom companies, Verizon & Comcast, are deciding whether to invest $10 billion dollars in a Fiber Optic network in a particular area.

First: they simultaneously choose to invest or not.

Then: they have a pricing decision to make: high or low

↳ but the outcome of the first round affects which pricing game they face.

If only one company invests:

- pricing high will get them
  - 60 million customers
  - Profit of $400 per customer

- pricing low will get them
  - 80 million customers
  - $200

If they both invest, it's another simultaneous game

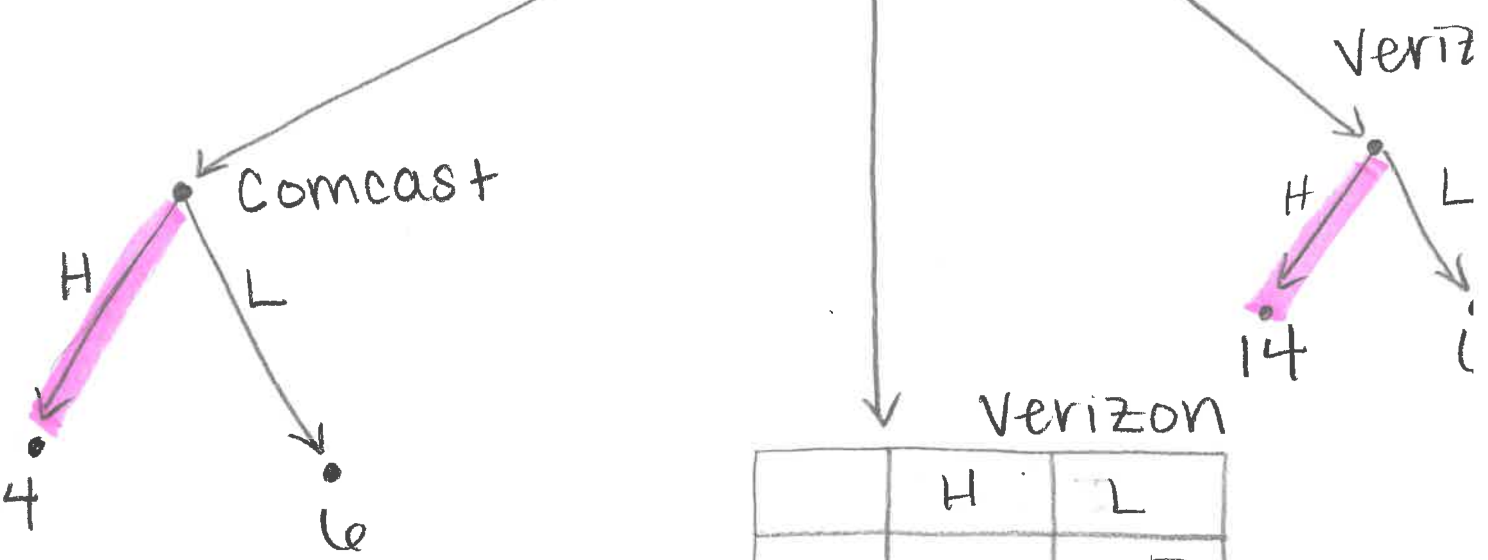
→ if they choose thesame price decision, the split the market, make the same profit

- high: 30 mill., $400
- low: 40mill., $200

→ if one chooses high & one chooses low, the one that chooses low gets all 80 mill. customers at $200 each.

(*) Payoffs. Profits - investment

Comcast \ Verizon	Don't	Invest
Don't	0, 0	0, 14
Inv.	14, 0	−2, −2



Comcast \ Verizon	H	L
H	2, 2	−10, $\boxed{6}$
L	$\boxed{6}$, −10	$\boxed{-2}$, $\boxed{-2}$

prisoner dilemm

Just One Company:

$\Pi_H = (60 \text{ mill.} \times 400) - 10 \text{ bill.}$

$= 24 \text{ bill} - 10 \text{ bill} = 14 \text{ bill}$

$\Pi_L = (80 \text{ mill} \times 200) - 10 \text{ bill.}$

$= 16 \text{ bill} - 10 \text{ bill} = 6 \text{ bill.}$

Both:

- Both choose high, split market:

  each get:

  $(30\text{ mill} \times 400) - 10\text{ bill}$

  12 bill

  $= 2\text{ bill.}$

- Both choose low, split market

  $(40\text{ mill} \times 200) - 10\text{ bill.} = -2\text{ bill.}$

  8 bill.

- One chooses high, one chooses low.

  → low gets the total low payoff: 6 bill

  → high gets nothing, pays $10 bill.

Continuous Comcast & Verizon

	Don't	Invest
D	0, 0	0, •
I	•, 0	2038 • 2038

Micro Theory Problem
$\pi = 1048$

Game Theory Problem

Micro Theory Problem
$\pi = 104$

	D	I
D	0, 0	0, 1048
I	1048, 0	2,038, 2038

(*) Let's say they have a contiuum of price options.

↳ 3 problems to solve.

(*) When there are both firms in the market

$Q_c = 100 - 2P_c + P_v$

$Q_v = 100 - 2P_v + P_c$

Cost per unit for both = 4

(*) When there is only one firm in the market

$Q = 100 - 2P$

Cost per unit = 4

⟶ ok, solve the game.

(*) Both in the market

↳ only new thing is the -10

$$\pi_c = P_c Q_c - 4Q_c - 10$$

$$= P_c(100 - 2P_c + P_v) - 4(100 - 2P_c + P_v) - 10$$

$$= 100P_c - 2P_c^2 + P_v P_c - 400 + 8P_c - 4P_v - 10$$

$$= 108P_c - 2P_c^2 + P_v P_c - 4P_v - 410$$

BRF for Comcast:

$$\frac{\partial \pi_c}{\partial P_c}: 108 - 4P_c + P_v = 0$$

$$108 + P_v = 4P_c$$

$$\boxed{27 + \frac{1}{4}P_v = P_c} \qquad \boxed{27 + \frac{1}{4}P_c = P_v}$$

⟶ solve the system of equations

$$27 + \frac{1}{4}\left(27 + \frac{1}{4}P_c\right) = P_c$$

$$27 + \frac{27}{4} + \frac{1}{16}P_c = P_c$$

$$27 + \frac{27}{4} = \frac{15}{16}P_c$$

$$\frac{108}{4} + \frac{27}{4} = \frac{15}{16}P_c$$

$$\frac{135}{4} = \frac{15}{16}P_c$$

$$\boxed{\begin{aligned} 36 &= P_c \\ 36 &= P_v \end{aligned}}$$

⊛ What's the payoff to each

$\pi_C = \pi_V = 108(36) - 2(36^2) + (36)(36) - 4(36) - 1$

$= \boxed{2{,}038}$

⊛ Only one in the market

$\pi = PQ - 4Q - 10$

$= P(100 - 2P) - 4(100 - 2P) - 10$

$= 100P - 2P^2 - 400 + 8P - 10$

$= 108P - 2P^2 - 410$

$\frac{\partial \pi}{\partial P} : 108 - 4P = 0$

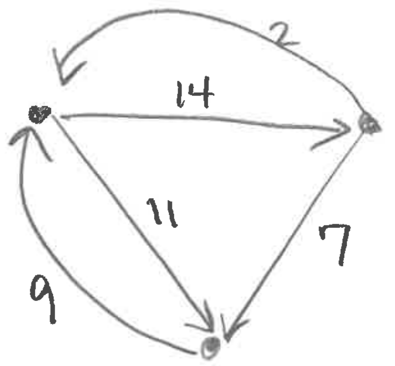
$\boxed{27 = P^*}$

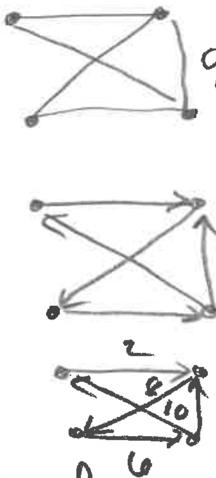
$\pi = 108(27) - 2(27^2) - 410 = \boxed{1{,}048}$

Network Economics

(*) What the heck's a network?

→ it's just dots & arrows & numbers on the arrows





graph

direc grap

→ netw

→ it's a purely mathematical concept (just like derivatives) until you give it an economic meaning

→ the dots can be:

- firms (Production Network)
- banks (Financial Network)
- countries (Trade Network)
- people (Social Network)

→ after computer scientists & biologists had a corner on the network market for decades, Economists were like hey maybe this could be useful (duh).

↳ Started doing really cool stuff

↳ but almost always treating the network (whatever it was) as exogenous (just plugging it in instead of building it)

→ so that's what I set out to do in my thesis.

(*) This is Economics, what we need is a network equilibrium.

↳ all the dots - whatever they are - to be happy given what all the other dots are doing.

→ to do this we need payoffs.

↳ which requires picking a context: Production Networks

⊛ Production Networks

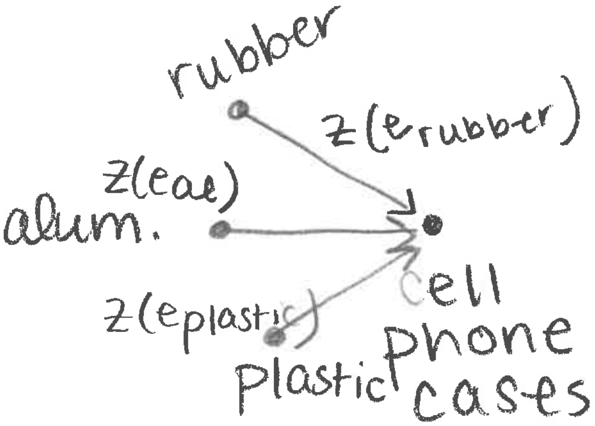
→ n firms who each make one thing

↳ to make their one thing they use:

- one (1) intermediate input (produced by another firm in the network)
- labor (supplied by one big representative consumer)

→ the rep. consumer supplies labor & consumes the good

→ they have options about which input to use



each of these edges, e, has a number that measures how good a match as an input it is

→ Why not pick max z?

prices

↳ which one they choose will determine the edges of the equil. network.

(*) But figuring out which one they choose is hard. Why?

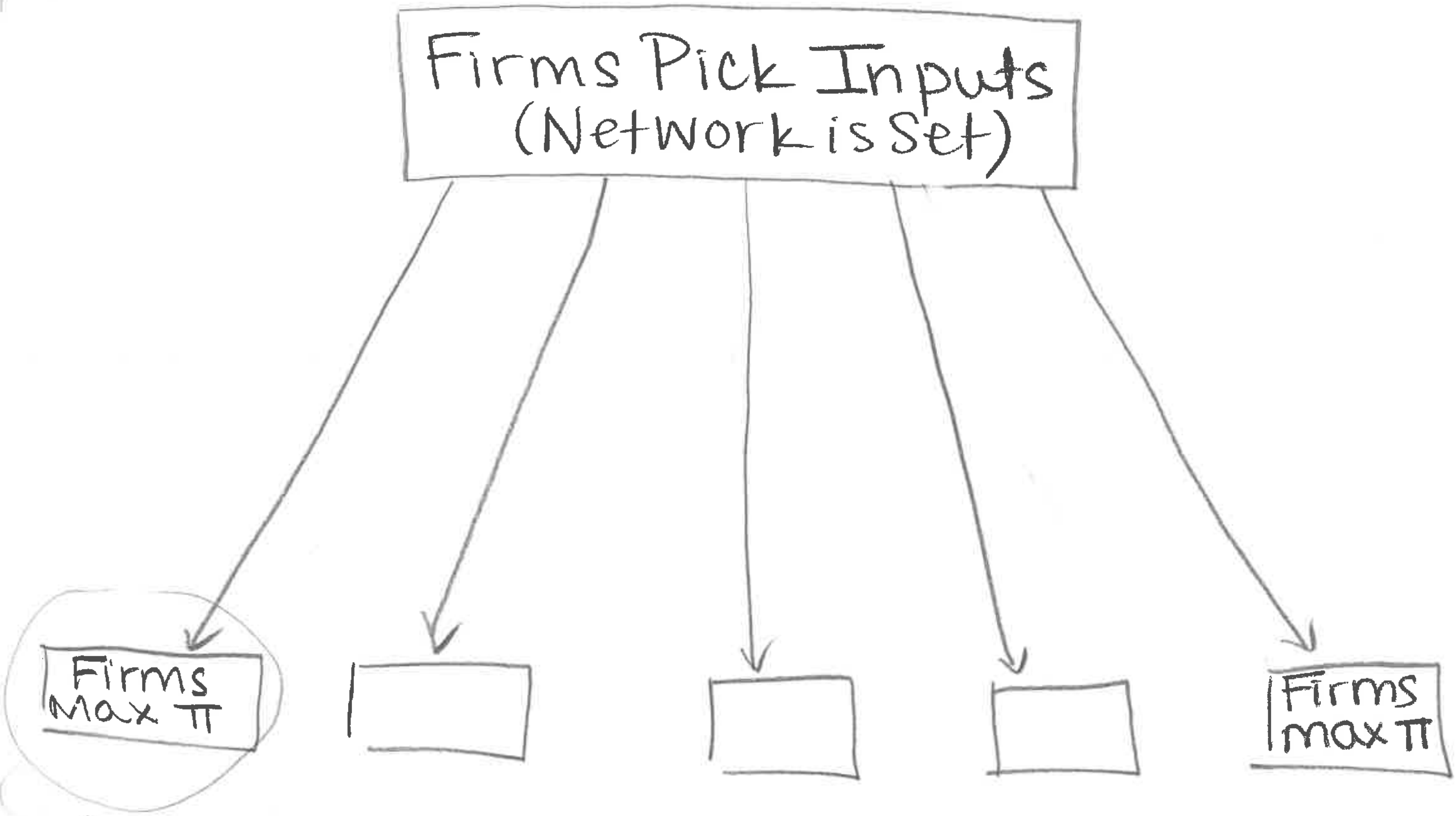
↳ b/c they're going to choose based on the prices that they're options charge

↳ But the prices that their options charge depend on which inputs THEY pick.

↳ which depends on their options prices and THEIR options prices and so on.

→ So everyone's decisions depend on the whole network of inputs.

↳ that sounds like a multistage game.



→ each of this is like the problems we've been doing there's just

- n players instead of 2
- 4 choice variables insead of 1

(*) Each firm maximizes their profit by picking

- x_j: the amount of their one input they use
- l_j: the amount of labor that they use

- y_j: the amount of their good they sell to the consumer

- P_j: the price they charge the consumer

- $p(e)$: the price they charge the other firms (I did some math & showed this is just their MC, so I don't need to mess w/ it)

(*) Each firm solves the following profit maximization problem:

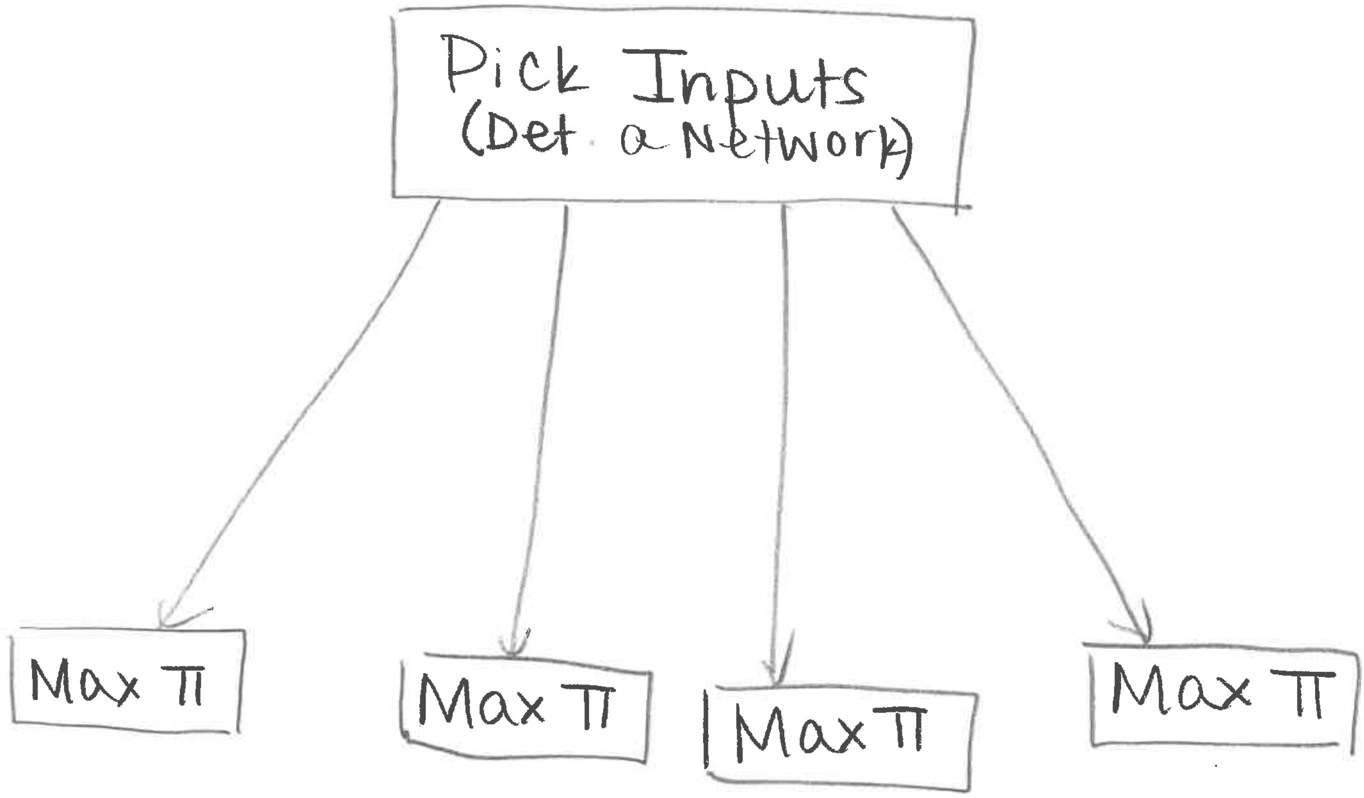
$$\max_{\{x_j, \ell_j, y_j, P_j\}} \underbrace{y_j P_j}_{\text{rev. from rep. cons.}} + \underbrace{\sum_{e \in D_j} [p(e)x(e)]}_{\text{rev. from other firms buying their thing}} - \underbrace{[p(e_j)x_j]}_{\text{cost of their one input}} - \underbrace{w\ell_j}_{\text{cost of labor}}$$

s.t.

$$\underbrace{y_j + \sum_{e \in D_j} x(e)}_{\text{total sold}} \leq \underbrace{\frac{z(e_j)\, x_j^{\alpha}\, \ell_j^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}}_{\text{total produced (production function)}}$$

Network Equilibrium

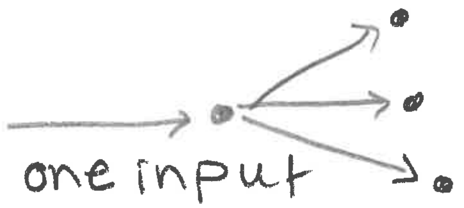
→ Last time: two stage game for businesses selecting inputs



→ Wrote down the profit maximization problem that each firm solves.

↳ weird b/c more variables to choose than we're used to

↳ not weird b/c of any network stuff



some customers profit maximize

→ So after that happens, every firm has a profit for every possible network

↳ THAT's how we find the equilibr network.

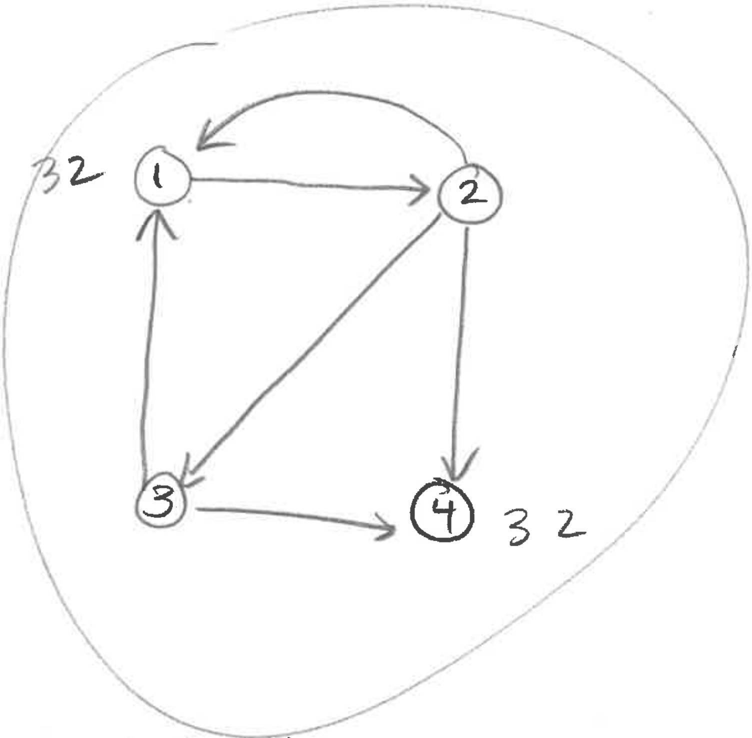
→ We want a network where all the firms are happy w/ their input choice given other firms input choices.

(*) Pairwise-Stable Equilibrium Network: no buyer-supplier relationship that could exist would prefer to exist.

↳ ~~everyone is happy w/ the supply relationship that they've chosen relative to changing a single~~

→ Would any buyer-supplier pair see bigger profits if they switched to a network where the buyer chose that supplier?

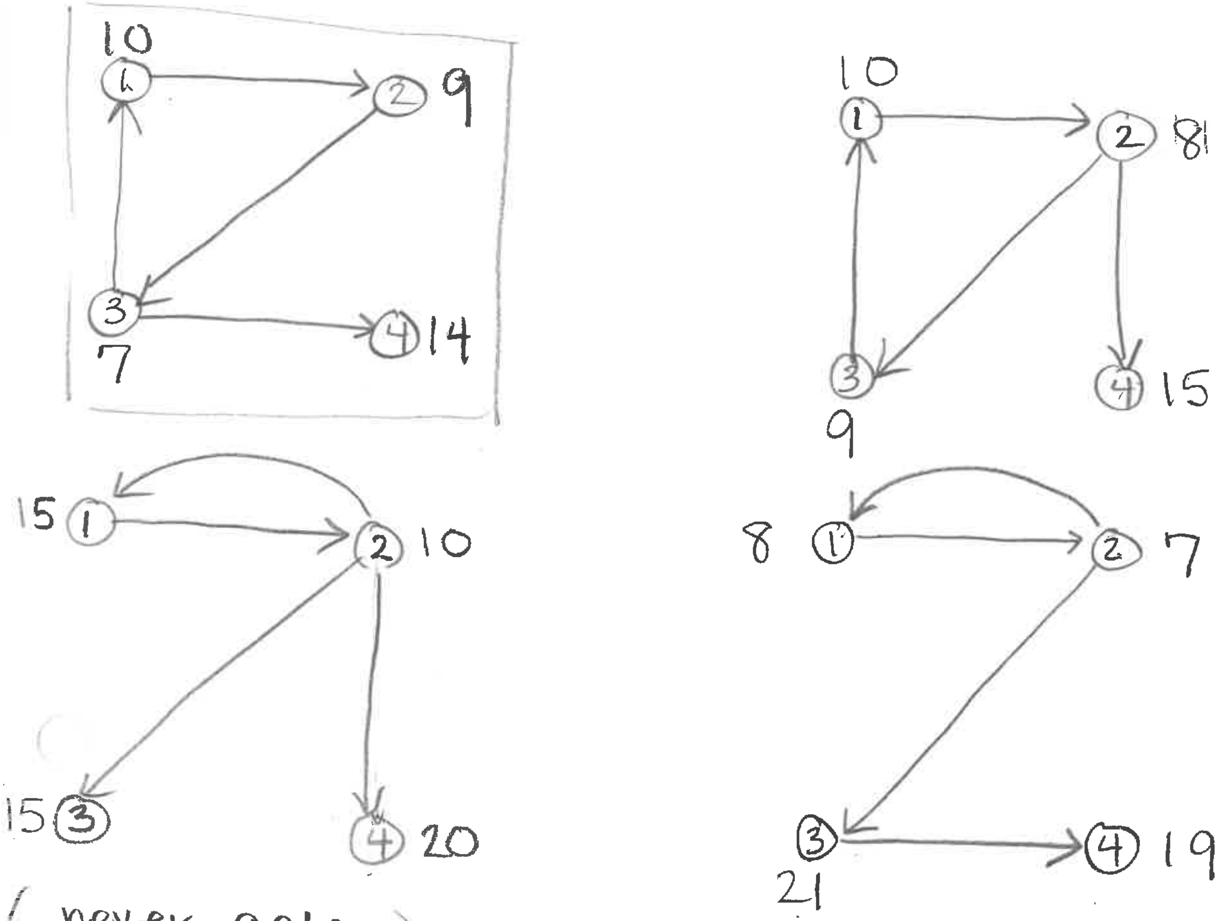
No? For everyone? It's pairwise stable.



→ want it to be pw but not cp.

1: 2, 3
2: 1
3: 2
4: 2, 3

(*) How do you find the pairwise-stable networks? You check them all.



(never gets compared to this one)

→ the first network is pairwise-stable

↳ only check changing one edge at a time.

→ what if BOTH 2 and 4 switch

↳ in this case, everyone is made better off

(*) Coordination-Proof Eqbm. Network:
no Combination of buyer-supplier pairs that could exist would prefer to.
↳ check every possible group of firms switching suppliers.

→ check switching all single edges
↳ then all pairs of edges
↳ then all triplets
⋮
↳ everyone switches at the same time.

→ if at no point in there, the associated buyers & suppliers would be made better off (see higher profits), then the network is coordination proof.

→ all c-p networks are pw stable but not all pw stable networks are cp.

(*) So what did I actually do with this?
I was interested in what it does to agg. output if a firm suddenly loses access to it's eq supplier (regulation, natural disaster, etc.)

→ Randomly generate firms and supplier options

↳ find the equilibrium network

→ delete an edge at random

↳ find the new equilibrium network

→ compare the agg. output produced by the 2 networks

(*) Sometimes output goes up.

↳ Why? Because if you remove strategies it can create new eq.

↳ sometimes those new eq. have higher output.

(*) What am I working on now?

1. Financial Network Protection
   → How can we protect the network of banks from cascading financial failures that ruin everything?

2. Tiny Changes, Big Problems
   → As I mentioned last time, very small changes in the network
   (like who's borrowing from whom)
   (or what supplier's firm's can use)
   can lead to bonkers changes in agg. outcomes.
   → I want to write a paper showing this.
   ↳ for a general interest journal.

(Ch. 14)

Mechanism Design: Price Discriminatio

→ the idea behind mechanism design is to get people to reveal inform. about themseslves that is useful to you

↳ usually so you can make money but not always.

(*) Some Terminology:

1. Principal-Agent Problem:

A situation wherein a less informed player (principal) wants to design a mechanism that creates incentives for the more-informed player (agent) to take actions beneficial to the principal.

→ mechanism design is the study of such problems.

2. Moral Hazard: (not the book's def^n)

Lack of incentive to guard against risk where one is protected from its consequences

Somewhat Bummer Example:
Sub-prime mortgage lenders who sold the mortgages to other companies.

→ When those borrowers aren't able to pay back their mortgages those lenders aren't the ones who take the hit

Major Bummer Example:
Sexual Harassment in the workplace.
Your supervisor's incentives (sadly) don't necessarily match up with yours or with the right thing to do.
↳ & mandatory reporting creates a new moral hazard problem.
→ Don't have a good mechanism design answer yet. (Maybe that's what my Nobel will be for.)

ANYWAY: that's what mechanism design is:

- getting people to reveal info about themselves that you can profit off of
- incentivizing problems away.

(*) Price Discrimination (Example)

→ An airline would like to sell expensive tickets to people who are willing to pay for expensive tickets and cheap tickets to people who are not.

"Economy": cheap, for people who aren't willing to pay a lot.

Tourists

"First Class": expensive, for people who are.

Business

Max you're willing to pay.

Type of Service	Airline's Costs	Reservation Pr.		Potential Airline Profit	
		Tourist	Business	Tourist	Business
Economy	100	140	225	40	125
First	150	175	300	25	150

per ticket.

→ A ideal situation would be for the airline to sell only FC to business people & ~~Economy~~ only tickets to tourists.

↳ But the problem is there's no way to tell who is who.

↳ need to design a mechanism to get them to do it themselves.

↳ need to give them incentives to do it.

→ Let's start with them charging $140 for Economy tickets.

↳ won't be less

↳ won't be more. (participation constraint).

→ This gives business folk a <u>Consumer surplus</u> of

$$\underbrace{225}_{\text{what they were willing to pay}} - \underbrace{140}_{\text{What they paid}} = 85$$

(*) What ever they charge the busine
people for FC tickets can't give
them any less consumer surplus.

↳ $300 - 85 = 215$

what they're willing to pay

minimal amount of CS

maximum price they can charge

= incentive-compatibility constraint

(*) More formally, if X is the price of Economy tickets & Y FC tickets

$X < 140$ participation

$225 - X < 300 - Y$ incentive-compatibili

EC 327: Game Theory • Final Homework Assignment Options

1. **Problem Set:** Selected problems from Chapters 6 and 14
   - Chapter 6: S4, S5 part (a), S7
   - Chapter 14: S1, S2, S4, S8

2. **Business Merger Analysis:** Choose two real companies and analyze the effect of a merger between the two on consumer prices.
   - Choose two (real) companies.
   - Choose one unique product that each company makes/provides.
   - Create a demand function for each company that, when used to create a profit function and find profit maximizing prices, produces an equilibrium price that matches the price that the company actually charges.
   - Using the techniques that we have developed in class, analyze the effect on the prices for the products if the two companies merge. Predict the new prices.
   - $\sim$ 3 pages typed.

3. **Government Policy Analysis:** Choose a policy that has been proposed at the federal level and analyze the likelihood of it being implemented.
   - Choose a (national) policy that has been proposed.
   - Summarize the policy and discuss the popularity of the policy in the two political parties, in Congress, and in the White House.
   - Using a game with both simultaneous and sequential components, predict the outcome of this policy proposal. Will it become a law? Why or why not?
   - $\sim$ 3 pages typed.

Incentives for Effort

→ Last time: defined moral hazard

→ Another example: you own a company & the company's success depends on your workers putting in a lot of effort

↳ but workers don't like effort.

(*) Can we design a mechanism to fix this?

Example: You own a company that requires good managerial supervisio to succeed.

↳ you want to hire a manager and write his contract to incentivize high managerial effort.

You, the business owner's outcomes:

Business suceeds: $1 million

Business fails : $0

if themager putsin high effort 50% chance of succ.
low 25%

Manager's Outcomes: $u = \begin{cases} \sqrt{y} \\ \sqrt{y} - 0.1 \end{cases}$

utility from income (y) (in millions) $= \sqrt{y}$

disutility of putting in high effort $= 0.1$ (mill.)

→ he has an outside option that doesn't require high effort will give him a salary of \$90,000 or 0.09 million

↳ this gives him utility $\sqrt{0.09} = 0.3$

Two Cases:

1. Effort is observable (by a court)
2. Effort is not observable

Case 1: Effort is observable & enforceable

→ how much do you offer the manager for his high effort work ~~and his low effort~~ work to ensure he takes the job & puts in high effort?

→ he must be at least as well off as he would be if he took the outside option:

utility from working for you = utility from outside option

$\sqrt{y} - 0.1 = 0.3$

$\sqrt{y} = 0.4$

$y = 0.16$ } $160,000

Case 2: Effort isn't observable

→ now you need to tie the monagers salary to the success of the project.

x if the project fails

y if the project succeeds

→ want to make it so he'll choose to exert high effort.

↳ need to make his utility from exerting high effort larger than from exerting low effort

→ make it incentive-compatible

(expected) utility from high effort > (expected) utility from low effort

↓ 50% chance of success (50% of failure)

↓ 25% chance of success (75% of failure)

$$0.5\sqrt{y} + 0.5\sqrt{x} - 0.1 > 0.25\sqrt{y} + 0.75\sqrt{x}$$

$$+0.1 \quad -0.25\sqrt{y} \quad -0.75\sqrt{x}$$

$$\underline{0.25\sqrt{y}} - 0.25\sqrt{x} > 0.1$$

$$\boxed{\sqrt{y} - \sqrt{x} > 0.4}$$ incentive comp. constr.

=

→ make him participate in the way you want

(exp.) utility from high effort > utility from outside option

$$0.5\sqrt{y} + 0.5\sqrt{x} - 0.1 > 0.3$$

$$0.5(\sqrt{y} + \sqrt{x}) > 0.4$$

$$\boxed{\sqrt{y} + \sqrt{x} > 0.8}$$ particip. constr.

=

→ if both of these constraints hold with equality:

$$\sqrt{y} - \sqrt{x} = 0.4$$

$$\sqrt{y} + \sqrt{x} = 0.8$$

$$2\sqrt{y} = 1.2$$

$$\sqrt{y} = 0.6 \longrightarrow 0.6 + \sqrt{x} = 0.8$$

$$y = 0.36 \qquad \sqrt{x} = 0.2$$

$$x = 0.04$$

Contract:

the business succeeds, you get $360,000

the business fails, you get $40,000 ~ lower than outside option

Your Expected Profit =

$$\tfrac{1}{2}(1-y) + \tfrac{1}{2}(0-x) = \tfrac{1}{2}(1-0.36-0.4)$$

$$= 0.3$$