Fortifying the Banks*

Allison Oldham Luedtke†

Eric Young‡

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Abstract

The 2008 financial crisis brought issues of financial stability to the forefront. We study the efficiencies associated with stabilizing the financial network. We specify a minimal set (a "fortification") of banks such that every bank has a neighbor. The government transfers resources to those banks in the fortification, similar to deposit insurance. We find that networks that are highly connected but are not concentrated around a few popular lenders are the easiest to fortify and that fortifications are more efficient than bailing out banks that are "too big to fail." Finally, we find a fortification of a historical financial network.

Keywords: Financial Networks, Financial Crisis, Interbank Lending, Bailouts

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†Assistant Professor of Economics, St. Olaf College, Northfield, Minnesota

‡Professor of Economics, University of Virginia, Charlottesville, Virginia; Federal Reserve Bank of Cleveland, Cleveland, Ohio.

1. Introduction

The 2008 financial collapse of Lehman Brothers, the fourth largest US investment bank, contributed to a global financial crisis and the worst economic downturn in the United States since the Great Depression ([Lio]). This recession was so devastating, in part due to the wave of cascading financial failures that the collapse of Lehman Brothers sent throughout the network of bank-to-bank lending. To prevent the Great Recession from becoming the next Great Depression, the US government passed the Emergency Economic Stabilization Act of 2008. This act injected $700 billion into the financial sector to bailout banks that were considered *too big to fail* ([Dav]). While these bailouts were expensive and controversial, they seem to have prevented further cascades of bank collapses, which would have led to an even worse economic crisis ([Dav]).

The lending relationships between banks form a directed network and this network can play a large role in preventing cascading bank failures like those that contributed to the recession. Depending on the structure of this network, it can either exacerbate or mitigate a crisis ([Lio]). Certain types of networks can propagate negative financial shocks and lead to these cascading bank failures, like those seen in the lead up to the 2008 financial crisis (see [AOT15a], [Jac10] and [Bar14]). However, other types of networks are protected from such cascading failures.

This paper identifies the characteristics of bank networks that protect against financial crises. The particular links that do exist and do not exist between individual banks in the network can have an significant effect on macroeconomic outcomes in the financial sector. [Lue] shows that a change of one edge - one interbank loan - can change the amount of loan dollars that are repaid to the lending banks by an order of magnitude. Despite this sensitivity of outcomes to network structure, little work has explored the connection between network structure and stabilization policy.

In this paper, we explore how the structure of financial networks helps or hurts in the face of negative financial shocks; going beyond this sensitivity analysis, we also study a policy aimed at improving stability. We contribute to the literature using financial networks to understand the strengths and weaknesses of the macroeconomy. As noted above, financial networks can create contagion, interpreted as a cascade of delinquent loans, in response to a negative economic shock (see [Has13] [CB13] [AB09]). But networks may also protect against such contagion. [AOT15a] characterize a level of network connectivity for which the network protects against shocks rather than propagating them. However, they do not propose a method for protecting those networks that are vulnerable to the effects of negative shocks. That is our main goal in this paper.[1]

The financial crisis brought increased academic attention to financial stability; while the literature is too large to adequately summarize, we highlight a few contributions here. [Bru] analyzes the role that liquidity and credit constraints played in the severity of the Great Recession. [Gor08] provides an in-depth explanation of the subprime mortgage breakdown and its role in the crisis. [SZ09] discuss the amplifying role that the erosion of trust played in the worsening of the crisis after the collapse of Lehman Brothers and the bailout of AIG. [Phi09] outlines the policy responses to the downturn. [CK] discusses the reforms of the financial sector in response to the recession. However, none of this work incorporates the role that the network of lending could have played in stabilizing the economy.

We use a network model to find a method of strengthening the banking system. We define a *fortification* as a set of nodes in a network such that every node in the network has an edge pointing to a node in the fortification. In the context of a financial network, a fortification corresponds to a set of banks such that every bank in the network is borrowing from a bank in the fortification. In the model, the government - which is not a node in the network - can transfer resources to ensure that the banks in the fortification can repay their loans, which can prevent cascades of bank failures. The fortification is designed so that, regardless of where a bank failure occurs in a fortified network, the negative shock from that failure cannot travel along more than one edge of the network before encountering a fortified bank.[2]

We show that this targeted approach to identifying banks and protecting financial networks can improve outcomes as well as decrease costs. Our policy exploits the amplification ability of networks: by saving only a small number of banks from failure, the entire network can see a large reduction in bank failures and loan defaults. Our key result is that networks that are broadly interconnected but not concentrated around a few popular lenders can be protected with the greatest success.

Using a network model of interbank lending, loan repayment, and protection, we analyze the success of fortifications. We treat the network - including banks, loans, and interest rates - as exogenous and use the model of loan repayment described in [AOT15a]. The banks in the model borrow money from one another; these loans are represented by the directed edges of the network. Banks invest in projects which have random returns, and the success or failure of these projects

[1][Jac10] [Bar14] [MLS08] [Cal+13] explore how network topology affects aggregate outcomes. See [CF13] for a summary.

[2]Preventing these cascades is important because it saves the *lenders* of the banks that cannot repay their loans from suffering the negative externalities of their borrower's outcomes.

contributes to the banks' ability to repay their loans. If a bank's project yields a low return, the bank may fail to repay its loans in full. Depending on the structure of the network, this financial failure can lead to that bank's lenders failing to make their own debt payments in turn, which we call a *cascade*.[3]

To characterize the types of networks that can be successfully fortified, we identify the smallest possible fortification for a given financial network. Suppose all banks in the fortification receive resources sufficient to ensure that they repay the entirety of their loans. In this new equilibrium, the amount that each bank is able to repay is not smaller than it was before the fortification. We therefore define the *success* of a fortification as the difference between the dollar value of loans not repaid without the fortification and the dollar value of loans not repaid with the fortification. A more successful fortification leads to a larger difference in these loan repayments. The size of the minimal fortification, the cost of the fortification, and the success of the fortification all depend on the particular network structure. Our goal is to characterize this dependence – what features make networks easy or difficult to fortify?

First, we analyze how various network characteristics affect the number of banks needed to form a fortification of a given network. Specifically, we look at the relationship between the degree distributions, average path length, clustering coefficient and fortification size. We then analyze how successful that fortification is at preventing bank failures. The *degree distributions* describe the number of lenders and borrowers of each bank in the network; in-degree refers to the number of directed edges pointing to a bank, and out-degree the number pointing away. The *path length* measures how close one bank in the network is to any other bank (roughly how spread out or disconnected the network is): if we walked along the links of the network from one randomly-chosen bank to another randomly-chosen bank, on average how many links would we be walking along?. The *clustering coefficient* captures how concentrated the banks are around a few, very popular lenders or borrowers (another measure of connectivity). The clustering coefficient measures the degree to which banks have separated into *cliques*. As an example of why these factors matters, consider this question: if we fortify a bank in a tightly grouped clique, is the benefit of that fortification going to get trapped in the clique and never make it out to other banks in the network, or can it be disseminated effectively?

[3]To keep things as simple as possible, we assume that the costs of forming new lending relationships are sufficiently high that the network does not change during the interval of time over which project returns are realized, banks are fortified, and the equilibrium payments are made. Once loans are made, thereby determining the network, it is prohibitively costly for banks to change their lending partners before their debts are settled.

We generate a number of random financial networks and fortify them. In each repetition of the simulation a random financial network is generated, including loans and loan amounts. For each network, we measure the degree distribution, average path length, and clustering coefficient and determine the unfortified repayments and liquidation decisions. Then, all possible minimal fortifications for that particular network are found. For each of these minimal fortifications, we compute the associated new repayment and liquidation decisions.

Fortifications work well on average but there is substantial heterogeneity in fortification success depending on the particular financial network in question and the individual banks that are fortified. The same network can see an improvement in loan repayment as high as $5 billion or see no improvement at all, depending on the banks included in the fortification and on the structure of the network. We find that the networks that allowed for the smallest fortifications were those that were highly interconnected but not tightly clustered around a few large banks. That is, networks that have many connections that are distributed evenly amongst the banks, without separated clusters of banks, are the easiest to fortify. Networks with a few large banks that lend to many others (known as star suppliers), are the most difficult to protect. Fortifying most networks does not require a large number banks: the average fortification size was only 3.26 banks, representing only 13% of the banks in the network.

In addition to requiring only a few banks, fortifications are cost-effective. Every fortification saved more money in repaid loans than it cost to fortify the banks in it: on average, fortifications save over $2 for every dollar spent. Furthermore, the network characteristics that allow for small fortifications also can be fortified with the greatest success, namely financial networks that are closely connected but not tightly clustered around a small number of hub lenders.

It is well known that government insurance provisions can create incentive problems. In our case, we might be concerned that the government covering the shortfalls of banks in the fortification creates an incentive for banks to change their lending habits so as to be included in the fortification. It turns out, however, that this behavior would backfire. The more lending partners a bank has, the more likely they are to be included in the fortification, so the best way to increase the likelihood of being included in the fortification would be to lend to as many other banks as possible. But as each bank in the network increases their number of lending partners, the network becomes more connected and less tightly clustered. This change in turn allows for smaller fortifications and the probability of any *individual* bank being included decreases. If, in the limiting case, every bank in the network lends to every other bank, the network is called *complete*. A complete network allows for the minimum fortification size possible (2) and any two banks in the network constitute a fortification. Which two banks are chosen will depend on which banks have the smallest shortfall to cover, thereby inducing *fewer* lending partners and unraveling the fortification-seeking behavior. Thus, in this limited sense, fortifications seem immune to incentive problems.

We next compare our fortification method to two other methods of financial stabilization. The first alternative is to simply choose a random set of banks and cover their shortfalls. The financial process is the same as in a fortification but ignores the structure of the network in choosing the banks. Not surprisingly, fortifications outperform these randomly chosen banks: on average, fortifications save more money - $670 million more on average - and prevent more banks from failing. The second, and likely more interesting, alternative is a bailout of the most connected banks, corresponding to those banks that could be considered "too big to fail." Fortifications are also more efficient than a set containing the most connected banks: on average, fortifications are about 30% more efficient, in terms of repaid loans per dollar spent. This is because the most connected banks also tend to have the most loans to repay and therefore some of the highest total shortfalls to cover in the event of a negative financial shock. By taking advantage of the entire network structure rather than simply considering which banks have the most lending partners, fortifications are a more frugal but still successful method of protecting the financial network from cascading failures.

Finally, we study how to fortify a historical interbank lending network. We use data on interbank lending relationships in Pennsylvania and New York in 1867, as presented in [APW19]. This financial network is sparse; there are relatively few loans and the banks in the network are not very close to one another in terms of network distance. As a result, the fortification of this network is large. To fortify the 54 banks who borrow from other banks requires a fortification set that contains 37 of the 54 banks. This finding is consistent with the simulation results: networks that are spread out and not interconnected require larger fortifications.[4]

The remainder of this paper proceeds as follows. Section 2 describes the model of the interbank network, lending, repayment, and fortification. Section 3 describes simulations of this model and the results of these simulations, including comparisons to alternative methods of network stabilization. Section 4 presents a case study in which we find the fortification of a real, historical, financial network. Section 5 concludes.

[4]An interesting application would be to study the current US banking network, but we face substantial computational and data limitations.

2. Network Model

2.1. Model of Lending and Repayment

We use a model of interbank lending that describes the loans between banks using a directed network. The banks are represented by the nodes of the network and the directed edges represent loans from one bank to another.

A network G consists of a set J of nodes indexed $j = 1 \ldots n$ and a set of directed edges between them. Let ij denote a directed edge from node j to node i. Figure 1 depicts a network with 10 nodes and 31 edges. We use $N_j^+(G)$ to describe the set of edges in G that point *to* node j and $N_j^-(G)$ to describe the set of edges in G that point *away from* node j.

$$N_j^+(G) = \{ji \in G\}$$

$$N_j^-(G) = \{ij \in G\}$$

Let $N_j(G)$ be the set of all neighbors of node j regardless of edge direction, $N_j(G) = N_j^+(G) \cup N_j^-(G)$. For example, in the network depicted in Figure 1, $N_2^+(G) = \{4, 8, 10\}$, $N_2^-(G) = \{4\}$, and thus $N_2(G) = \{4, 8, 10\}$.

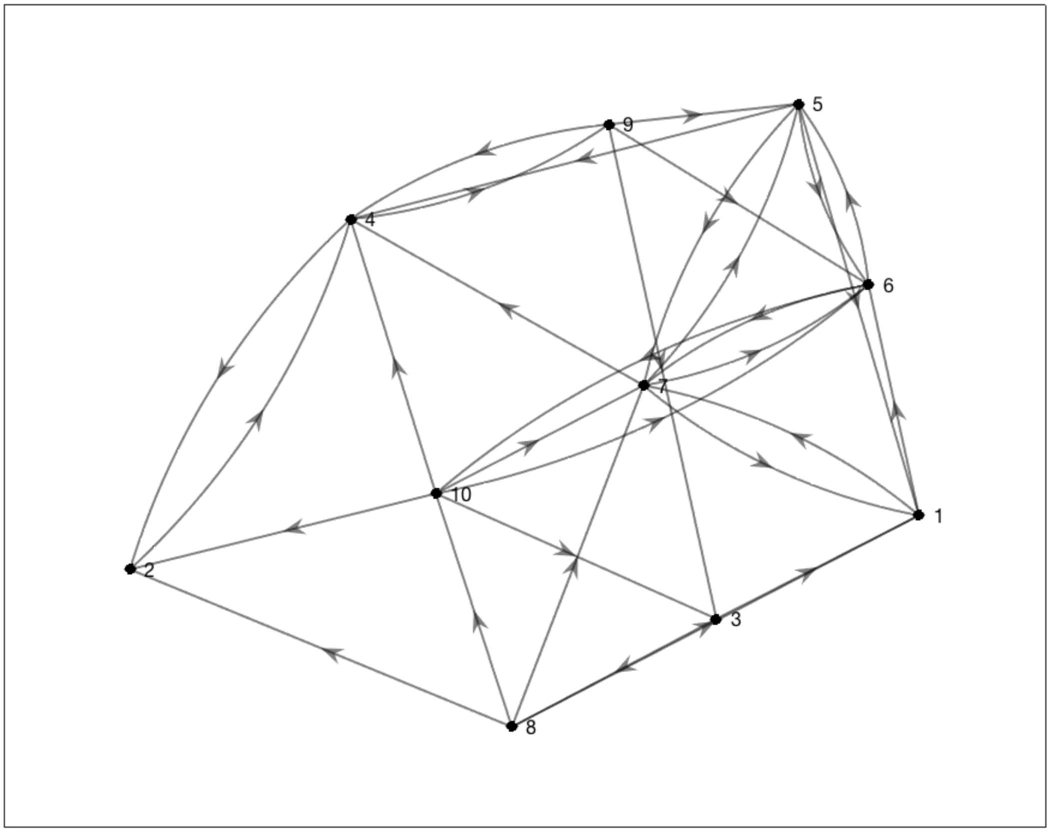


Figure 1: Network of Lending

Figure Note: *This example network contains ten nodes, labeled "1", "2", etc., with directed edges between them. The arrows on the edges indicate the directions of the edges.*

The agents of the model are n banks, represented by the n nodes, which borrow money from one another and make liquidation decisions about investment projects based on their ability to repay their loans. A directed edge from bank j to bank i indicates that bank j borrowed from bank i and now must repay the loan with interest to bank i. That is, arrows indicate the flow of repayment. We use the model of loan repayment described in [AOT15a]. Each bank in the model borrows money from at least one other bank and has invested in an outside project. Each bank j is endowed with k_j dollars of capital that are allocated between loans and investment projects. Whatever capital is not loaned to other banks or invested in an outside project is held by the bank in cash. The projects yield random returns which determine the extent to which each bank is able to pay back its loans.

The network of loans is described using a matrix, $Y_{ij} = [y_{ij}]$, containing the face values of the loans, i.e., what must be repaid. Each element, y_{ij}, of this matrix describes the amount that bank j owes to bank i in repayment after borrowing a loan amount of l_{ij}. That is $y_{ij} = (1 + \rho_{ij})\, l_{ij}$, where ρ_{ij} is the interest rate on the loan from i to j. If bank j did not borrow from bank i then $y_{ij} = 0$. We do not allow for self loops so $y_{jj} = 0\ \forall j \in J$. Let $y_j = \sum_i (y_{ij})$ be the total amount that bank j owes in repayments to its lenders.

Figure 2 depicts the same network as Figure 1, but with edge weights indicating loan amounts. Each loan is for \$100 and has an interest rate of 2.7%, thus each loan has a face value, $y_{ij} = 102.7$.

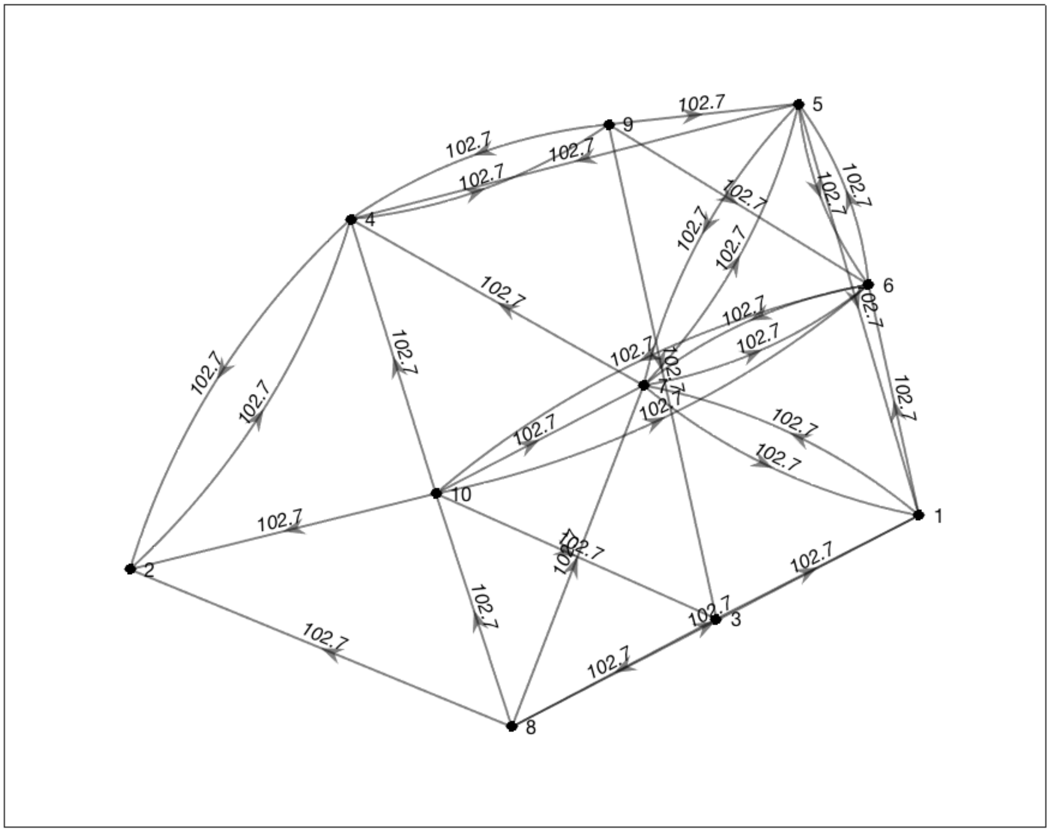


Figure 2: Network of Lending with Loan Amounts

Figure Note: *This network has the same nodes and edges as in Figure 1, but includes the loan amounts for each directed edge.*

Each bank, j, invests in an outside project and the success or failure of these projects determine the banks' abilities to repay their loans. The bank observes a preliminary random return, z_j, on the project and decides whether to liquidate or not. If they choose not to liquidate, they earn a fixed, non-pledgeable payout of A_j. If they choose to liquidate, they only receive a fraction ξ_j of this yield.

Each bank holds an amount, c_j, of their funds in cash and has an outside obligation to their senior creditors, v_j, that they must pay before they repay their loans. This primary obligation can be interpreted as the bank's costs of operation, including wages, rent, etc. Label the bank's available resources at the time of repayment if they do not liquidate as $h_j = c_j + z_j + \sum_i r_{ji}$, where r_{ij} is the amount of each loan that is repaid in equilibrium.

There is an equilibrium repayment amount, $r_{ij} \in [0, y_{ij}]$, for each loan and a liquidation decision, $L_j \in [0, A_j]$, for each bank. If a bank has the resources to meet all of its liabilities at the time of the repayment, i.e. $h_j > v_j + \sum_i (y_{ij})$, then all loans are repaid in full, $r_{ij} = y_{ij}$, for each bank i to which bank j owes a repayment. If $h_j \leq v_j + \sum_i (y_{ij})$, the bank must either partially liquidate its project to cover the difference or liquidate entirely and pay back what it can. If a bank is able to meet its senior obligation, v_j, but not its loans, the repayments are made in proportion to their face values.

First loans and investments are made. The network of loans is treated as exogenous. Then, random returns are observed, liquidation decisions are made, and loans are repaid. Finally, any projects held to maturity yield their return, A_j. All repayment and liquidation decisions by all banks are made simultaneously. Banks take their future yields into account, but make repayment decisions before they are received. Following [AOT15b], the equilibrium repayment amounts are determined by

$$r_{ij} = \frac{y_{ij}}{y_j} \max\big\{ \min\{y_j, h_j + \xi_j L_j - v_j\}, 0\big\}$$

and the liquidation decisions are determined by

$$L_j = \max\big\{ \min\{\frac{1}{\xi_j}(v_j + y_j - h_j), A_j\}, 0\big\}.$$

These equations describe how funds - or lack of funds - travel between banks. If bank j is unable to repay its loans in full, this shortfall propagates to bank j's lenders and they in turn may not be able to pay their loans in full, and so on throughout the network. Figure 3 depicts the equilibrium repayment amounts for the network of loans depicted in Figure 2 and the model parameterization used in Section 3. Many of the banks in this example are unable to repay their loans in full. For example, Bank 9 is only able to pay Bank 5 \$26.89 of the \$102.7 that they owed.

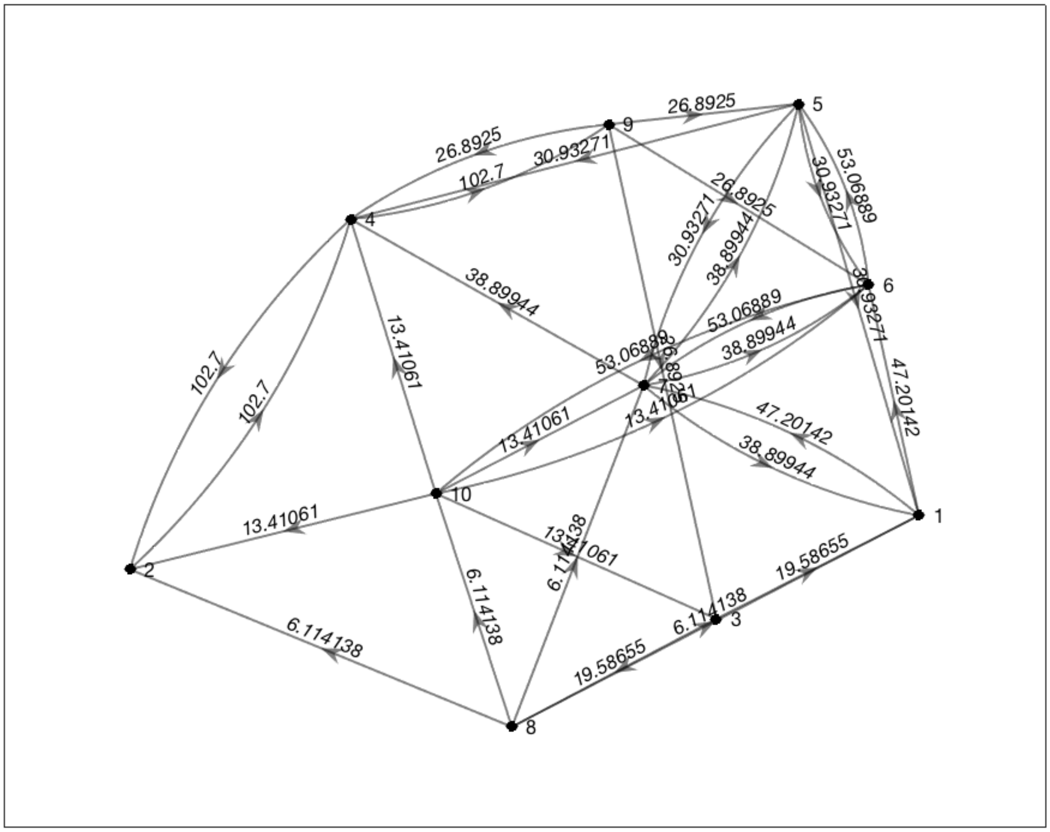


Figure 3: Repayment Equilibrium

Figure Note: *The numbers on the edges are the equilibrium repayment amounts for the example network depicted in Figure 2.*

A *bank failure* occurs when any bank is unable to repay its loans in full. A bank failure may lead to a cascade of failures as the lenders who are not repaid in full in turn are unable to repay their own loans. The repayments are determined simultaneously so these cascades do not occur sequentially in time, but rather result from the interrelated nature of the lending network. Let d denote the total dollars not repaid in the payment equilibrium for a given network, $d = \sum_{j=1}^{n} \left(\sum_{i \neq j} y_{ij} - r_{ij} \right)$. In the repayment equilibrium depicted in Figure 3, $d = \sum_{j=1}^{10} \left(\sum_{i \neq j} 102.7 - r_{ij} \right) = 2,104.4$. Of the $102.7 \times 31 = \$3,183.7$ owed in loan repayment, $\$2,104.4$ were not repaid in this example.

2.2. Fortifications

To limit these cascading bank failures, we identify a set of nodes to bolster in the face of financial difficulty. We define a *fortification* to be a set, F, of nodes in a financial network such that every node in the network has an edge pointing from it to a node in F. This set is similar to the open-locating dominating (OLD) sets used in the graph-covering literature in mathematics (see [KOY15]). Fortifications differ from OLD sets in that (1) they are defined for directed networks rather than undirected networks, and (2) they specify a direction (in-pointing) for the neighbor in the covering set. In the network depicted in Figure 1 the nodes 2 and 3 constitute a fortification.

The government transfers resources to the banks in the fortification to ensure that they pay their loans in full. Therefore, regardless of where a financial failure occurs, it can travel no further than one lending relationship before it encounters a fortified bank, thereby stemming the flow of financial failures from bank to bank throughout the network. Note that this does not ensure that a failure stops after one link of the network. The definition of a fortification merely ensures that the financial failure will encounter at least one fortified bank for every link it travels. Every bank has a fortified lending partner, so for every additional link along which the failure cascades, it is guaranteed to encounter at least one more fortified bank.

The *minimal fortification* is the smallest possible set of nodes that satisfies the definition of a fortification. There may be multiple minimal fortifications of the same size for any given network. For example, the network depicted in Figure 1 has four minimal fortifications, each containing four nodes: $\{4, 7, 8, 9\}$, $\{2, 4, 7, 8\}$, $\{1, 4, 7, 9\}$, and $\{1, 2, 4, 7\}$. The smallest possible fortification for any network must contain at least two nodes, because no bank lends to itself in the model. So if a single bank (node) lends to every other bank - and therefore satisfies the definition of a fortification for (or *covers*) every other node in the network - the fortification will still need one more node to cover the original node.

The *cost* of a fortification is the total difference between what the fortified banks owe and their equilibrium repayment amounts.

$$cost^F = \sum_{f \in F} \Big(\sum_{j \neq f} (y_{jf} - r_{jf}) \Big)$$

The total cost of a fortification depends on the number of banks in the fortification as well as how far those banks are from repaying their loans in full. For a given network, two fortifications of the same size may have widely different costs. For example, the cost of the first fortification of the network depicted in Figure 1, $\{4, 7, 8, 9\}$, is \$944.78. Additionally, the cost of a fortification may be 0, which occurs when the particular banks in the fortification already pay their loans in full, regardless of whether other banks in the network are able to repay their loans.

There is a new payment equilibrium associated with any fortification. Denote the repayment and liquidation decisions associated with a particular fortification, F, be $\widetilde{r_{ij}}^F$ and $\widetilde{L_j}^F$, respectively. Figure 4 depicts the fortified repayment equilibrium using the first fortification, $\{4, 7, 8, 9\}$. The nodes in the fortification are designated with larger node markers and in red. Following the same notation, define $\widetilde{d}^F$ to be the total dollars not repaid in the fortified equilibrium. Before the fortification, \$2,104.4 were not repaid. After the fortification, only \$459.24 are not repaid. In the fortified payment equilibrium, no bank repays *less* than they did in the unfortified network, so that $\widetilde{d}^F \leq d$.

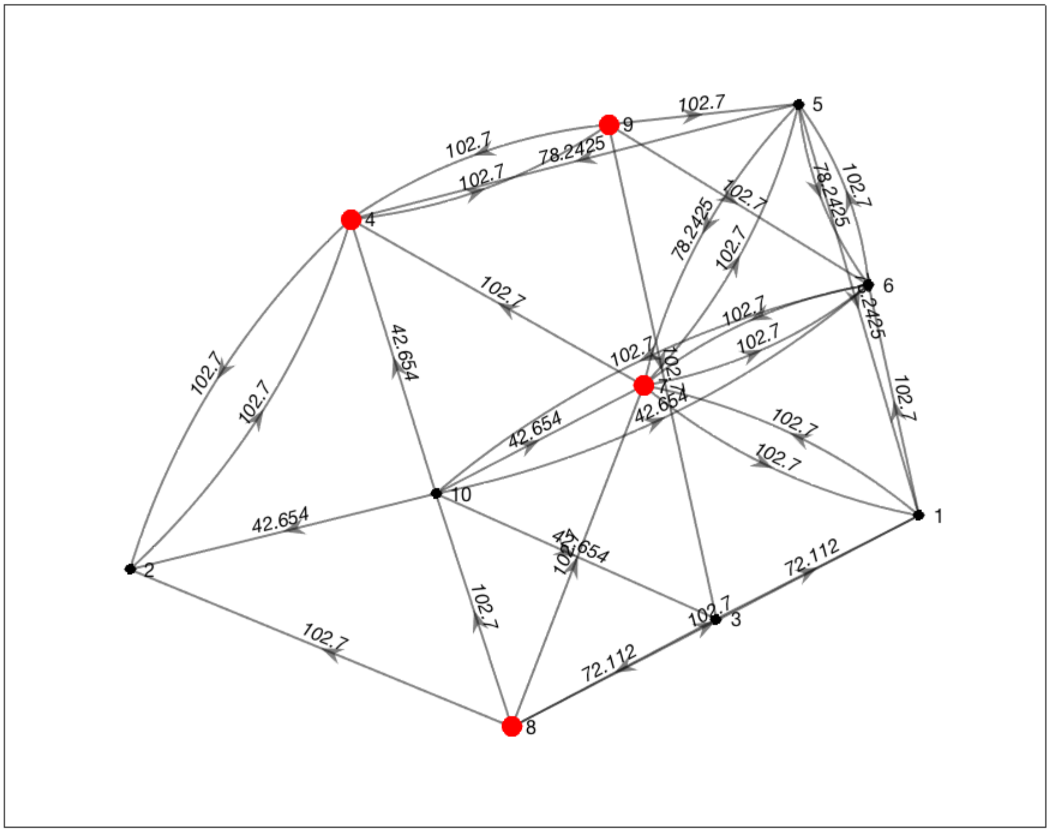


Figure 4: Fortified Repayment Equilibrium

Figure Note: *The nodes highlighted in red and with a larger marker size represent one of the minimal fortifications of this example network.*

How successful a fortification is at preventing financial failures varies between the different minimal fortifications of a given network. Define the *success* of a fortification, s^F, to be the difference between the total dollars not repaid in the original payment equilibrium and the total dollars not repaid in the fortified payment equilibrium: $s^F = d - \widetilde{d}^F$. The success of the fortification used in the previous example is $s^F = d - \widetilde{d}^F = 2104.4 - 459.24 = 1,645.16$.

The ratio of fortification success to fortification cost describes the dollars saved per dollar spent on a fortification. Define the *efficiency* of a fortification to be the ratio: $e^F = \frac{s^F}{cost^F}$. The most efficient fortifications are those that lead to a large amount of money being repaid that would not have been repaid without the fortification while simultaneously costing the government as little as possible. The efficiency of the fortification used in the previous example is $e^F = \frac{s^F}{cost^F} = \frac{1645.16}{944.78} = 1.74$, so this fortification saved \$1.74 for each dollar spent.

2.3. Network Characteristics

The number of fortifications, the cost of each fortification, and the success of each fortification all depend on the structure of the financial network in question. We investigate how these outcomes depend on several different network characteristics: in-degree and out-degree distributions, path distance, and clustering coefficient.

The *in-degree* of a node is the number of edges pointing to that node: $in - deg_i = |N_i^+(G)|$.; similarly, the *out-degree* of a node is the number of edges pointing from that node: $out - deg_i = |N_i^-(G)|$. The in-degree and out-degree distributions are the sets of in-degrees and out-degrees for every node in a network G, respectively. In a financial network, the mean of the in-degree distribution is the average number of borrowers each bank has, while the mean of the out-degree distribution is the average number of lenders each bank has.

In addition to the mean, we consider the variance and skewness of the degree distributions. The variance describes how widely the number of lenders per bank and the number of borrowers per bank varies. A higher variance indicates that some banks have very few lenders/borrowers and some banks have many. The skewness describes how skewed the degree distribution is. A positive skewness indicates that the distribution is skewed left - many banks have a low number of lenders or borrowers, while a negative skewness indicates that the distribution is skewed right - many banks have a high number of lenders or borrowers. In particular, networks with an out-degree distribution with high positive skewness have a "hub" or "star supplier" bank that lends to many other relatively-isolated banks.

The undirected shortest path between two nodes in a network is the smallest sequential set of nodes and edges between those two nodes, regardless of the direction of the edges.[5] We use the average of all of these shortest paths to measure how widely spread the nodes of the network are. The path distance of the network depicted in Figure 1 is 1.4286, so on average it takes between 1 and 2 links to get from one node in the network to another. A longer average path distance

[5]We use the undirected path distance and clustering coefficients because they are always defined for a connected, directed network, while their directed counterparts may be infinite or undefined.

indicates that the network is more widely spread; it takes more edges, on average, to get from one node to another. In a financial context, a longer average path distance indicates that the network of loans is less interconnected in general.

The undirected local clustering coefficient describes how tightly grouped, or *cliqueish*, the nodes of a network are. Specifically, for a given node in a network, the individual clustering coefficient computes the fraction of possible triangles that are actually present in the network. For a node i, suppose that there is an edge between i and j as well as between i and k (ignoring direction). The clustering coefficient of i computes the fraction of the time that there is an edge between j and k, as well. Let $\{ij\}$ be the undirected counterpart of an edge either from i to j or from j to i.

$$cc_i(G) = \frac{\#\{\{jk\} \in G : k \neq j, j \in N_i(G), k \in N_i(G)\}}{\#\{\{jk\} : k \neq j, j \in N_i(G), k \in N_i(G)\}}$$

The local clustering coefficient is then the average of these individual clustering coefficients across all nodes in the network, G.

$$cc(G) = \frac{\sum_{i \in J} cc_i(G)}{n}$$

The clustering coefficient for the network depicted in Figure 1 is 0.5921, so the third edge of a possible triangle occurs a little more than half of the time. A larger clustering coefficient in a financial network indicates that if two banks have a lender or borrower in common, those two banks are more likely to lend or borrow from one another.

In the next section, we explore the connection between these network characteristics and the size, cost, and success of fortifications.

3. Simulation Results

We simulate the model of loan repayment and financial network fortification described in the previous section. We generate negative financial shocks and compute the network characteristics described in Section 2.3 to understand what role these characteristics play in the ease of fortifying the financial networks.

In each simulation repetition, we generate a random financial network with 25 banks. With no self-loops, a network of 25 nodes may have up to $25 \times 24 = 600$ edges. In our simulation, three different numbers of edges were used: in one third of the repetitions 25% of the possible edges were used, in one third 50% of the possible edges were used, and in one third 75% of the possible edges were used. We simulated 100 repetitions for each number of edges, for a total of 300 repetitions. Of the 600 possible edges, the appropriate number were selected randomly and uniformly without replacement. In 2 of the repetitions, the network that was generated was not fully connected and therefore those repetitions was dropped from the sample, for a total of 298 random financial network observations.

Each edge in the network represents a loan from one bank to another. Each loan was for $100 million and the interest rate on each was 2.7%. Because each value of $[y_{ij}]$ is the same, the financial network is said to be *regular*. We use regular networks so that changes in outcomes are driven solely by differences in network structure rather than by the loan values or the interest rates. The details of the model parameterization can be found in the Appendix.

In each of the repetitions, we found all of the minimal fortifications for the network and the cost and success of each fortification. We recorded the in-degree and out-degree distributions - including the mean, variance, and skewness of each distribution - as well as the average path length and clustering coefficient of each network. We investigate the relationship between these characteristics and the size and success of the fortifications.

3.1. Fortification Size

The average size of the minimal fortification across the 298 repetitions was 3.26, which represents 13% of the 25 banks in the network. The smallest possible size of a fortification, 2, was achieved in 35.91% of the repetitions. The percentage of fortifications containing 2 banks increases with the number of edges present in the network. None of the fortifications of the networks with 25% of the edges contained only 2 banks, 2.35% of the networks with 50% of the edges contained 2 banks, and 33.56% of the networks with 75% of the edges contained only 2 banks.

The cost of fortifications - the amount by which the fortified banks were short on their loans - varied widely across simulation repetitions as well as within individual networks. The mean cost across all fortifications in all repetitions was $652.39 million, or about six times the loan size. The average cost of the *most* expensive fortification available for a given network was $1.25 billion, or about ten times the loan amount, while the average cost of the *least* expensive fortification available for a given network was only $269.30 million. It is common for the least expensive fortification to be zero-cost; the modal cost of the least expensive fortification available for a given network was 0. Naturally, a zero-cost fortification also has no effect, since the member banks are already able to pay.

On average, there were 31.88 different minimal fortifications for each network, with a range between 1 and 276 fortifications.

Below are the results of a linear regression of fortification size on the network characteristics described in the previous section and indicator variables for the number of edges used. The 50% case is omitted as the base case, so the coefficient on the 25% edge indicator variable can be interpreted as the effect of having relatively few edges present in the network and the coefficient on the 75% edge indicator can be interpreted as the effect of having relatively many edges present. The means of the degree distributions are omitted because they always take on the same value for a given number of edges and are therefore perfectly collinear with the edge indicator variables.

Table 1: Fortification Size Regression Results

Network Characteristic	Coefficient
Edge % = 0.25	1.53
Edge % = 0.75	−1.00
In-Degree Distribution Standard Deviation	−0.26
In-Degree Distribution Skewness < 0	−0.10
Out-Degree Distribution Standard Deviation	0.06
Out-Degree Distribution Skewness < 0	−0.11
Average Path Length	2.13
Clustering Coefficient	2.02
Constant	−0.71

As the number of edges in the financial network increases, the number of banks in the fortification falls. Compared to the baseline of half of the possible edges, a relatively small number of edges, 25 percent, leads to larger fortifications: on average, a fortification contains 1.53 more banks. In the other direction, when there are 75 percent of the possible edges, the fortification contains one fewer bank on average. If there are more loans throughout the network, it is easier to fortify with fewer banks.

The presence of many hub lenders and borrowers decreases the size of the minimal fortification. A negative skewness indicates that the degree distribution is skewed right, corresponding to a larger number of banks lending to or borrowing from many other banks. A negative skew in the in-degree distribution decreases the fortification size by 0.10 banks, on average. Similarly, a negative skewness in the out-degree distribution decreases the number of banks in the fortification by 0.11 on average.

A financial network that is more connected but not tightly clustered allows for smaller fortifications. When the average path length increases by one link, the fortification grows by 2.13 banks on average. When it takes longer to get from one node in the network to another - when the network is more spread out - the fortification is larger and when the network is less widely spread, it allows for a smaller fortification. When the network is more tightly clustered around a small number of lenders or borrowers, the fortification is larger. A one unit increase in the clustering coefficient is associated with an increase in fortification size of 2.02 banks. These results indicate that a network that is relatively well connected but in a uniform way, without hubs, will allow for smaller fortification.

3.2. Fortification Success

The amount of money that a fortification saves in loans that are now able to be repaid that were not repaid before also varies widely across and within networks. A given network can be fortified very successfully or very unsuccessfully depending on the particular banks used in the fortification. On average, the most successful fortification for a given network leads to $3.05 billion being repaid that were not repaid without the fortification. The least successful fortifications, however, only saved $798 million on average and only saved a few cents in some cases. The standard deviation of the fortification success is quite large: $615 million.

Successful fortifications are often costly. The most successful fortifications for a given network cost an average of $1.24 billion. In 247 of the 298 repetitions, the most successful fortification was also the most expensive fortification; in contrast, in only 27 repetitions was it the least expensive fortification. However, every fortification saved more money than it cost. Every fortification in the sample had an efficiency greater than 1 and, excluding zero-cost fortifications, the average efficiency was 2.50, meaning that on average a fortification saved over \$2 in newly repaid loans for every dollar spent to fortify the banks in it.

Below are the results of a linear regression of the amount of money saved by the most successful fortification on the fortification size, the indicators for the number of edges, and the network characteristics. Because the dependent variable, fortification success, is measured in millions of dollars, coefficients can be interpreted as the increase or decrease in money saved in millions of dollars.

Table 2: Fortification Success Regression Results

Network Characteristic	Coefficient
Fortification Size	1966.50
Edge % = 0.25	−4521.80
Edge % = 0.75	3424.10
In-Degree Distribution Standard Deviation	209.02
In-Degree Distribution Skewness < 0	52.63
Out-Degree Distribution Standard Deviation	965.22
Out-Degree Distribution Skewness < 0	−266.64
Average Path Length	−7919.50
Clustering Coefficient	−17942.00
Constant	15769.00

As the number of edges present in the financial network increases, so does the success of the fortifications of the network. A network with only 25 percent of the possible edges saves \$4.52 billion less relative to the baseline case of 50 percent of the edges, while a network with 75 percent of the possible edges saves \$3.42 billion more than the baseline. When there are more edges in the network to connect the banks, the fortification can save more money.

The presence of many hub lenders increases the success of fortifications while the presence of hub borrowers decreases it. If a network features a negative skewed in-degree distribution - many banks that lend to a large number of other banks - fortifications save $52 million more relative to non-negative skewed networks. In contrast, if a network features a negatively skewed out-degree distribution - many banks that borrow from a large number of other banks - fortifications save $267 million less.

The effect of the average path length and clustering coefficient on fortification success reflect the fortification size results. A more closely connected network that is not tightly clustered will be more successfully fortified. An increase in the average path length of one link is associated with an decrease in new repayment of $7.92 billion, so a longer path length is associated with lower savings and a shorter path length is associated with higher savings. A one unit increase in the clustering coefficient is associated with a decrease in new repayment of $17.94 billion. A network that is not widely spread out and not disproportionately clustered around a few key banks will have more successful fortifications. Such a network is likely to have a small fortification that saves a great deal of money in previously unpaid loans.

In general, if a network characteristic leads to smaller fortifications, it also leads to more successful fortifications. That is, if the coefficient is negative in the fortification size regression, it is positive in the fortification success regression. However, the out-degree distribution is an exception to this general rule: more hub borrowers lead to smaller fortifications but they also lead to less successful fortifications.

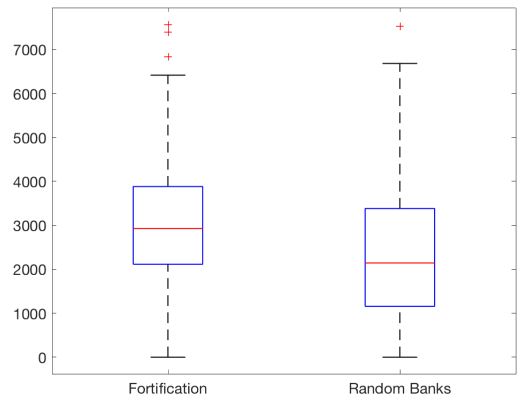
A fortification with more banks generally saves more money across the entire network. An increase in the fortification size by one bank leads to an increase in repayment of $1.97 billion on average. In this paper we only consider fortifications of the smallest number of banks allowed by the network. This result suggests that more research is needed to investigate whether the cost of fortifying more banks may be offset by increased savings.

3.3. Performance Comparisons

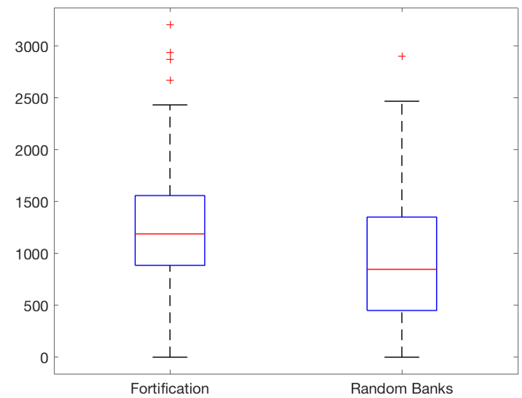
We compare the performance of fortifications to two alternatives. The first is a random set of banks of the same size. For each repetition of the simulation, a set of banks is chosen randomly and uniformly from the universe of banks and this set contains the same number of banks as the most successful fortification in the repetition. If any of these banks are unable to repay their loans in full, and if this shortfall is covered by the government as in a fortification, how does the performance of these two methods compare in terms of cost, efficiency, money saved in newly repaid loans, and banks saved from failure?

We also compare the performance of fortifications to that of a set containing the most connected banks in the network. Banks are ranked by their out-degree and banks are added to a set in order of highest out-degree until that set contains as many banks as the most successful fortification in the repetition. For example, if the fortification contains three banks, then this alternative set contains the top three most connected - in terms of out-degree - banks. As with fortifications and the set of random banks, any shortfall in loan repayment on the part of these most connected banks is covered and the ensuing increase in financial stability is compared to that of fortifications.

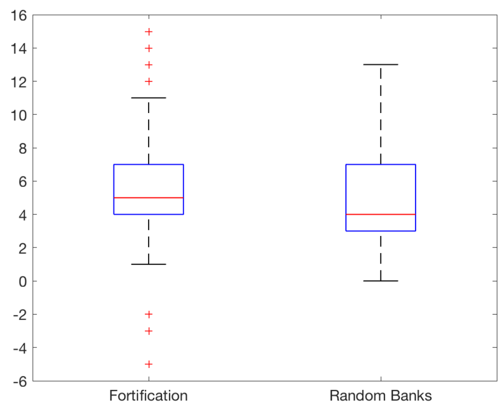
Figures 5 and 6 compare the measures of success for these two alternatives to those for the corresponding fortification. Figure 5 contains box plots that compare the money saved, cost, banks saved, and efficiency of fortifications and the random set of banks. Figure 6 contains box plots that compare the same measures between fortifications and the set of most connected banks. Tables 3 and 4 summarize the average money saved, cost, banks saved, and efficiency for each of these methods. All of the differences listed are statistically significantly different from zero. That is, the differences between fortification and the alternatives are statistically significant.



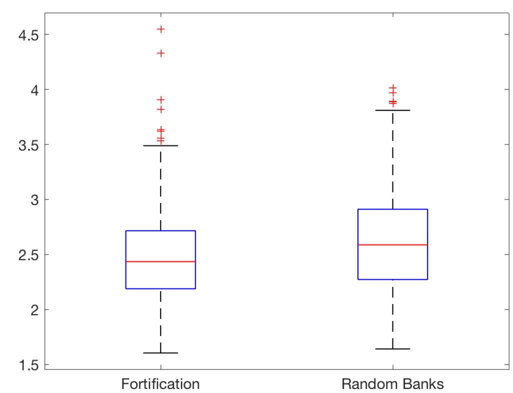
(a) Money Saved



(b) Cost



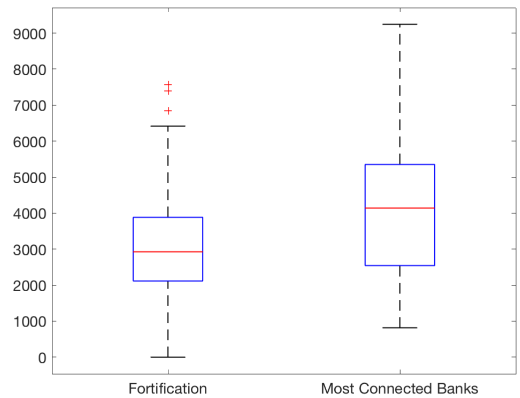
(c) Banks Saved



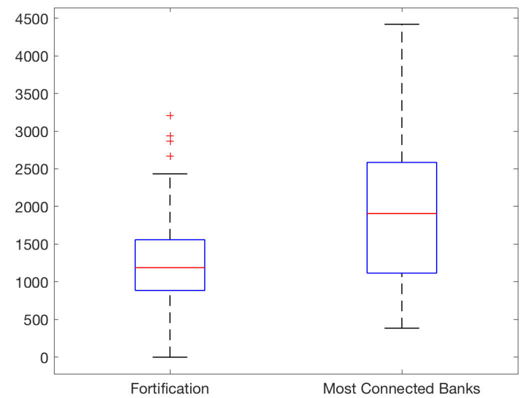
(d) Efficiency

Figure 5: Comparison of Fortifications vs. Random Banks

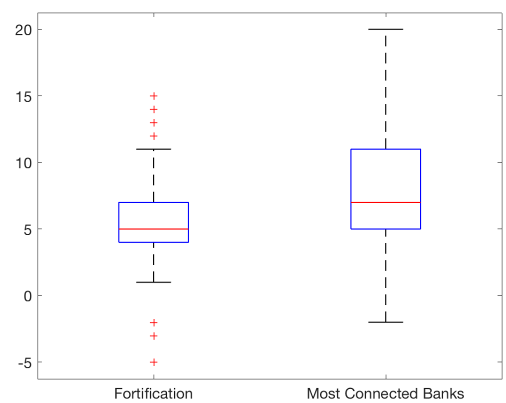
Figure Note: *These four box-and-whisker plots depict the differences in performance between fortifications and a random set of banks.*



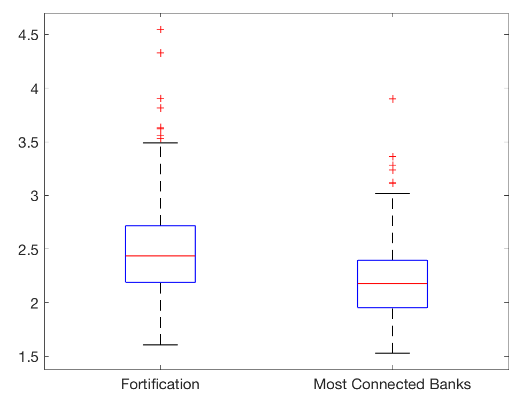
(a) Money Saved



(b) Cost



(c) Banks Saved



(d) Efficiency

Figure 6: Comparison of Fortifications vs. the Most Connected Banks

Figure Note: *These four box-and-whisker plots depict the differences in performance between fortifications and the most connected banks.*

Compared to a set of the same size but containing randomly selected banks, fortifications save more money. On average, fortifications save \$670.62 million more than a random set of banks. A random set of banks will not be connected by loan contracts to as many other banks as those in a fortification. As a result, funds that help a random set of banks will not have the same stabilizing influence as those used in a fortification because they will not propagate as far.

In addition to saving more money, fortifications save more banks from failure compared to a random set of banks. On average, fortifications save 0.69 more banks. As with money saved, this result is driven by fortification-banks' centrality and influence on other banks' ability to repay loans.

Naturally, fortifications also cost more than a random set of banks; on average, fortifications cost $305.43 million more. The reason fortifications are more successful in saving money and banks is the same reason that they tend to be more costly: banks in a fortification tend to be more central and connected to other banks both in lending and in borrowing. They tend to borrow money from more banks and therefore they have correspondingly larger liabilities; however, covering those liabilities benefits more banks than covering the liabilities of randomly selected banks.

Perhaps counter-intuitively, on average, a random set of banks saves more money per dollar spent than a fortification. That is, the random set has a higher efficiency. However, this apparent advantage is simply the result of the low cost of covering the loan shortfall of random banks. They save a small amount of money and cost almost nothing. Fortifications save significantly more money and banks, while costing more to do so.

Table 3: Fortifications vs. Random Banks

	Fortification	Random Set	Difference
Money Saved (millions of dollars)	3,046.93	2,376.31	670.62
Banks Saved	5.68	4.99	0.69
Cost (millions of dollars)	1,235.35	929.92	305.43
Efficiency	2.50	2.63	−0.13

Table 4: Fortifications vs. Most Connected Banks

	Fortification	Most Connected	Difference
Money Saved (millions of dollars)	3,046.93	4,137.78	−1,090.85
Banks Saved	5.68	7.60	−1.92
Cost (millions of dollars)	1,235.35	1,919.55	−684.20
Efficiency	2.50	2.21	0.29

Compared to a set of the most connected banks, fortifications save less money and fewer banks. On average, a set of the most connected banks saves \$1.09 billion more than the corresponding fortification. Additionally, they save 1.92 more banks from failure. Because these banks are the most connected in the network, they have a direct and strong effect on the other banks in the network and on those banks' ability to repay their loans. However, these most connected banks are also more costly to stabilize. To cover the shortfall of these highly connected banks costs on average \$684.20 million more than to pay for the fortification. They are costly for the same reason they save so much money and so many banks: they are very highly connected. These most connected banks have a large number of lenders to whom they owe repayments. As a result, they have a larger number of loans on which they may fall short and so it takes more money stabilize these banks, on average.

Finally, fortifications are more efficient than a set of the most connected banks. On average, fortifications save \$0.29 more for every dollar spent. Fortifications, while not saving as much money in repaid loans in total as compared to the most connected banks, still save \$3.05 billion and they do it at a lower cost. A dollar spent on a fortification goes farther in stabilizing the financial network.

4. Case Study: US Financial Network in 1867

We use a historical data set describing interbank debts to analyze the fortification of a real financial network. [APW19] describes the network of lending between banks in Pennsylvania and New York City in 1867, following the National Banking Acts in 1863 and 1864. We use network linkages constructed from the *Reports of the Several Banks and Savings Institutions of Pennsylvania (1863, 1868)* and the National Banks Examination Reports that resulted from the National Banking Acts. Figure 7 depicts this network consisting of 202 banks. We use this network to find a fortification of those banks engaged in interbank lending.

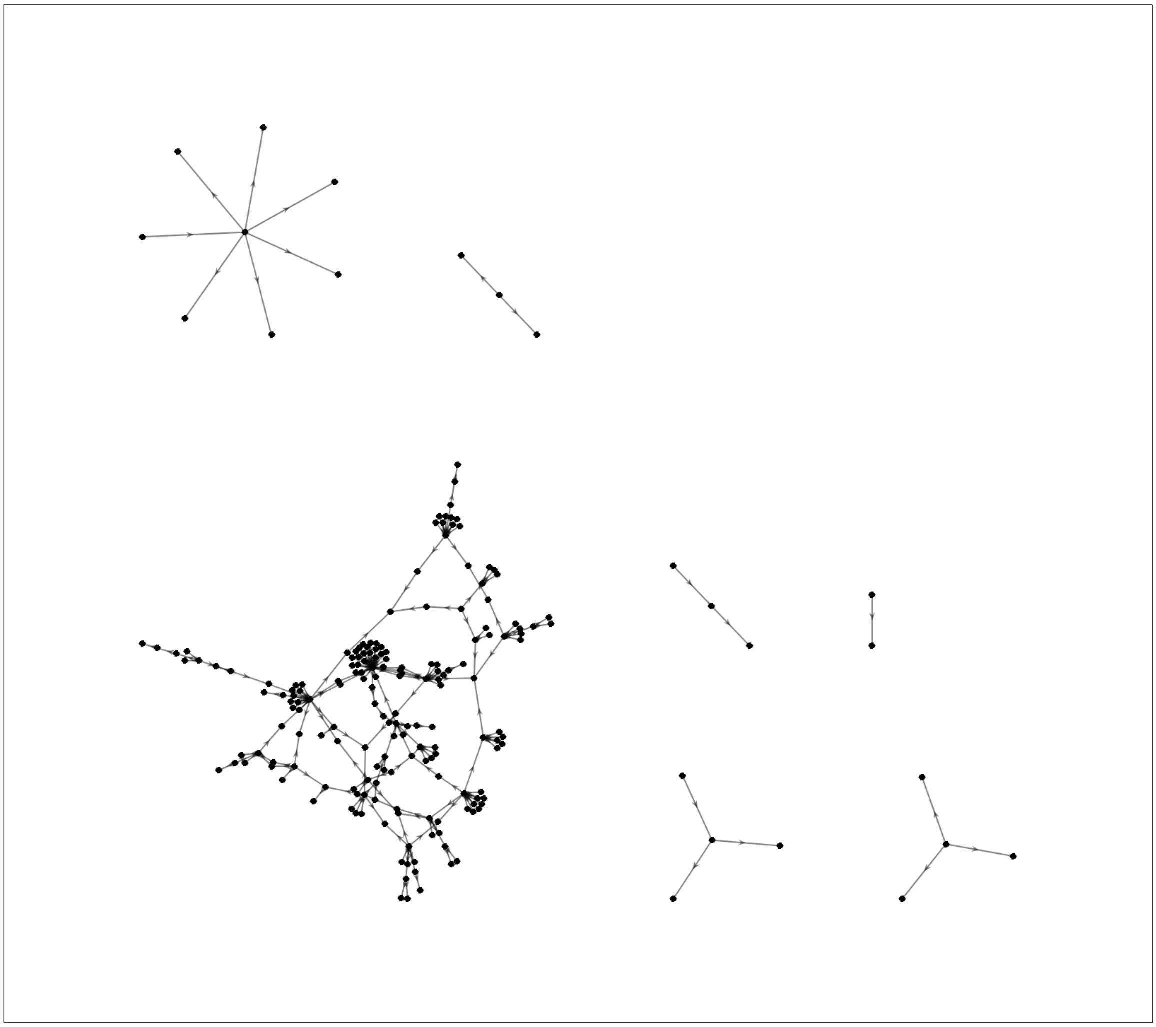


Figure 7: Pennsylvania and New York Financial Network (1867)

Figure Note: *This is the financial network comprised of a set of banks in Pennsylvania and New York in 1867.*

There is little data available describing real financial networks. While the data from [APW19] is from 154 years ago, the availability of any data represents a significant improvement in financial network research. Additionally, the banks included in this sample cover a wide variety of bank types, ranging from local banks to large financial center banks. This data allows us to demonstrate how the fortifications of real world networks can be constructed and that our simulation results are reasonable.

Not every bank in the network described by the data borrows money from another bank. As a result, not every bank in the network will have a neighbor in the fortification.[6] Only banks who borrow from at least one other bank will have such a neighbor; of course, those banks who do not use the interbank market are also not exposed to contagion along network links. Of the 202 banks in the network, only 54 of them borrow from at least one other bank. The average bank has only 1.07 debtor-banks. Figure 8 depicts the out-degree distribution of the network. That is, it depicts a histogram of the number of banks to which other banks owe loan repayments. The modal out-degree is 0, meaning the most common number of debtor-banks is none.

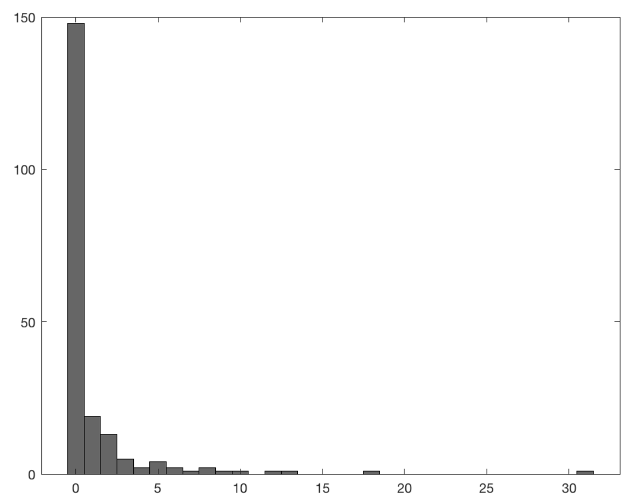


Figure 8: 1867 Financial Network Out-Degree Distribution

Figure Note: *The out-degree distribution describes the frequency of a given number of edges pointing away from a node in the network.*

The minimal fortification of the 54 banks engaged in borrowing contains 38 nodes. In Figure 9, nodes in the fortification are highlighted with a larger node size and colored red. This large

[6] Banks are of course connected through other mechanisms, such as correlated default risk by household borrowers. We leave extensions of our model to include additional connections to future work.

fortification is the result of the lack of connectivity in the network. Not only are there relatively few banks borrowing from one another but furthermore, very few borrow from the same bank. Of the 38 banks in the fortification, only 9 cover more than a single bank.

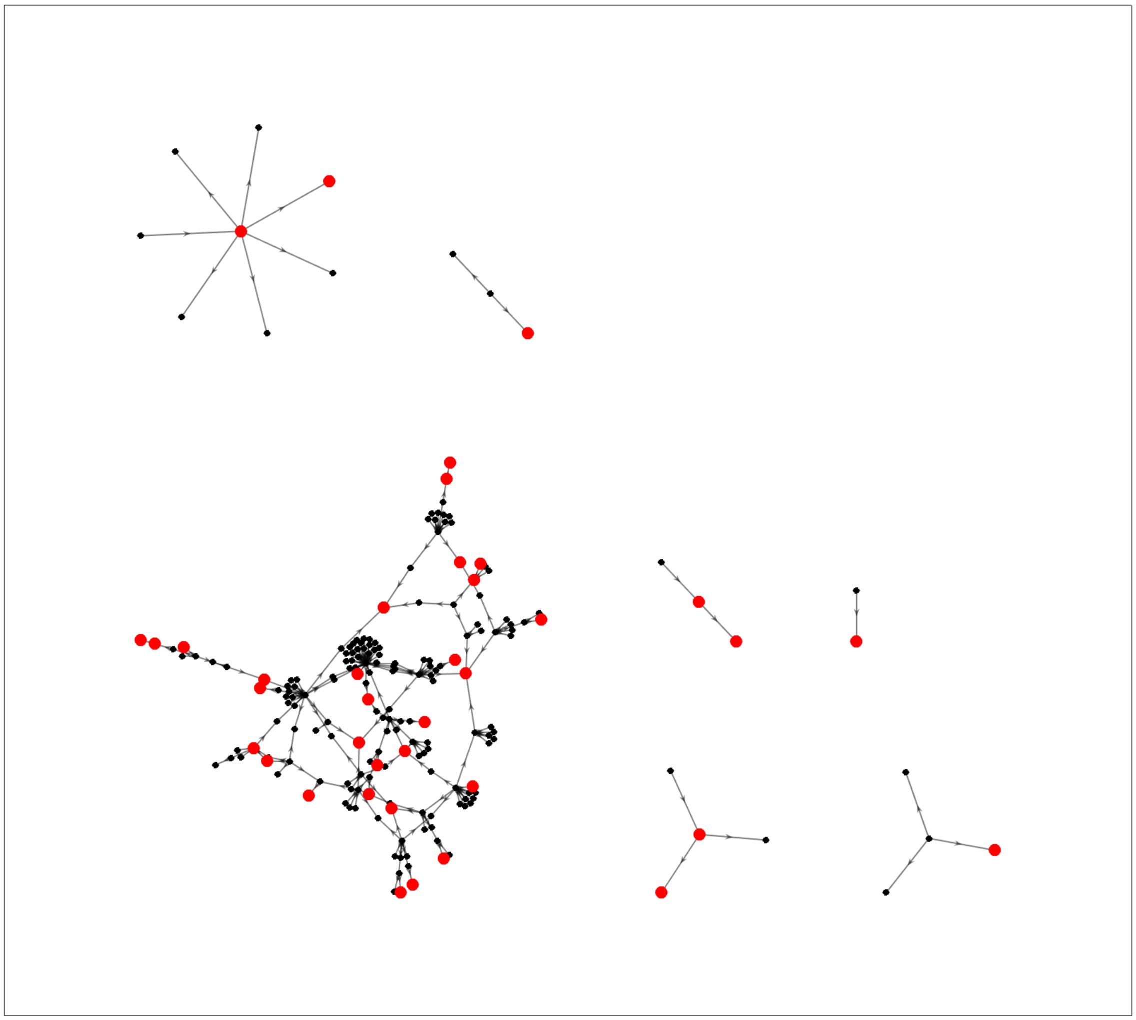


Figure 9: Fortification of 1867 Financial Network

Figure Note: *The nodes highlighted in red and with a larger marker size represent the minimal fortification of the financial network depicted in Figure 7.*

These results are consistent with the simulation results discussed in the previous section. The number of edges (loans) present and the distance between banks in the network both play a large role in determining the fortification size. A network of 202 nodes with no self-loops can have up to $202 \times 201 = 40,602$ edges. This network has only 216, representing only 0.5% of the possible edges. Additionally, because this network is not made up of a single connected component, a path does not exist from every node to every other node, and as a result the path distance is defined to be infinite. Both of these characteristics contribute to the large fortification size relative to the number of borrowers. As discussed in Section 3.1, as the number of edges decreases, the size of the fortification increases. Similarly, as the path distance of a network grows, so does the fortification size.

5. Conclusion

In the face of recent research that identifies the financial networks that are most vulnerable to cascading financial failures, this paper identifies those networks that can be most successfully protected from such cascades. Networks that are globally interconnected but not overly cliqueish allow for the least costly and most successful stabilization. There is ongoing works in many fields of economics that suggest similar results: in a network of economic actors, those that are connected but not tightly clustered are the most efficient to target.

The method of stabilization presented in this paper performs well in comparison to alternative methods. Fortifications save more money and banks than a random selection of protected banks. Furthermore, fortifications are more efficient than protecting the banks with the most lending partners, that is, those that would be "too big to fail."

Our study is just the first step in exploring how to exploit network characteristics to prevent financial contagion. A natural next step would be to study a modern banking network.

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6. Appendix

6.1. Model Parameterization in Simulations

Table 5: Model Parameterization

Parameter	Value
Number of Banks, n	20
Loans size, l_{ij}, in millions of dollars	100
Interest rate, $\rho_{ij} = \rho$	0.027
Negative shock, z_j	0.01
Mature project yield, A_j, in millions of dollars	7
Fraction recoverable, ξ_j	0.4
Cash held, c_j, in millions of dollars	3
Senior creditor obligation, v_j, in millions	10

The data generated in the simulations are available from the author upon reasonable request.

Declarations of interest: none