Measuring the Velocity of Money

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ABSTRACT

The velocity of money is an important macroeconomic indicator that is conventionally measured indirectly and as an average for an economy as a whole. This measurement approach obscures heterogeneity in the underlying spending patterns. With the advent of large-scale micro-level transaction data comes the opportunity to measure the velocity of money at the level of individual spenders. In this paper we propose a new measurement methodology that accounts for currency creation and dissolution, which is commonly observed in payment systems yet not accounted for by conventional measurement approaches. For a given payment system's transaction network, our method enables a systematic comparison of the velocity of money across different spatial, temporal, and demographic subgroups of spenders. Moreover, we can observe how events such as a pandemic or targeted currency operations affect the velocity of money across these subgroups. Using data from a community currency in a developing country, we find the following: (1) transaction volume comes from fast-moving money, while much of the balance at any particular point in time is slow-moving, (2) transaction rhythms differ between rural and urban areas, in particular, money moves faster in urban communities, and (3) there was rapid circulation as COVID-19 unfolded. The approach described in this paper may improve understanding of heterogeneity in macroeconomic patterns and inform policies that affect these patterns.

JEL Classification Codes: E41, E42, C63, L14

1 INTRODUCTION

How often does a unit of currency change hands? We estimate this velocity of money using a powerful data set describing individual transactions of a digital community currency in Kenya. With a novel computational method, we build a network of transactions, compute the distribution of velocities, and find that this important macroeconomic measure varies across a number of dimensions, including when the money is spent and the spender's demographic characteristics. Good macroeconomic policy requires understanding how individuals experience the economy and when those experiences diverge. This work can help inform such policies.

The rate at which money changes hands is a key macroeconomic indicator. Previous measures indicate that money moves faster between people when the economy is doing well and that money moves slower when the economy is doing poorly. (Leão, 2005) These previous measures are typically constructed as the ratio of two large macroeconomic aggregates. For example, on the Federal Reserve Economic Data (FRED) website, the Velocity of M2 (FRED ID: M2V) is calculated as the ratio of quarterly nominal gross domestic product to M2, a measure of the aggregate money supply. (Federal Reserve Bank of St. Louis, 2022) However, such calculations obscure much of the relevant heterogeneity in the rate at which money changes hands. It is important to capture this heterogeneity because the aggregate outcomes could be hiding signs of economic trouble in a specific sector or region. Moreover, a thorough understanding of how the velocity of money changes across groups, regions, and time can better inform macroeconomic policy.

We develop new mathematical and computational techniques that allow us to make use of a novel, high-granularity data set which describes the account information, transaction records, and demographic characteristics of users of the online currency, called *Sarafu*, in Kenya (Ruddick, 2021; Mattsson et al., 2022). This data allows us to map out a large network of transactions between accounts and to therefore calculate the velocity of money with a higher level of detail than previous researchers have been able to calculate. Additionally, this data allows us to analyze how the velocity of money differs across types of transactions and across characteristics of spenders.

The transfer velocity of money is conventionally estimated as an average of total transaction amounts over a payment system over a particular period of time. Mbiti and Weil (2013) estimate the transfer velocity of e-money within *MPesa*, the dominant mobile money payment system in Kenya, over monthly intervals. They note that the average transfer velocity within MPesa includes both money that moves through accounts quickly in effecting person-to-person transfers, and money that is saved for longer periods. The rate at which dollars change hands can be more completely described by a distribution of "holding times." In an individual's mobile money account, money enters, is held there for some period of time, and then is transferred out. Crucially, the inverse of a holding time has the units of a "transfer velocity." Wang et al. (2003) use this mathematical relationship to compute the average transfer velocity of money for a toy model of transactions. The same equation can also be derived using the money-flow computations defined in Leontief and Brody (1993). We extend this method to contexts in which the total balance of money is not constant over time. This allows us to make use of the high granularity of transaction data because money enters and leaves the system throughout the period of observation. We find consistent measurements of the average velocity of money across measurement methods.

We find that on average, a unit of the Sarafu community currency changes hands 0.27 times per week. The distribution that produces this average ranges from minutes to months: much of the transaction volume comes from Sarafu that is spent the same day it is received, sometimes in a matter of minutes, while much of the balance at any particular point in time is held for weeks or months. In urban communities, money moves faster, changing hands every day or two, while in rural communities it moves more slowly, changing hands every week or two. Additionally, we find that there was a substantial uptick in transactions in the summer months of 2020, when the COVID-19 pandemic began.

We add to a growing body of work that demonstrates the importance of heterogeneity in macroeconomic indicators. Argente and Lee (2021) find that households experienced heterogeneous changes to the cost of living during the Great Recession. Gornemann et al. (2021) find that heterogeneous households are affected differently by monetary policy and, in fact, Bartscher et al. (2021) find that accomadative monetary policy may increase inequality between black and white households. In synthesizing much of the literature on heterogeneity in unemployment, inflation, and economic growth, Goodman-Bacon (2021) argues that without understanding the ways that different groups and individuals experience the economy, macroeconomic policy-makers cannot realize the full potential for economic growth. We expand this literature to include information on the heterogeneity in the velocity of money.

This work also contributes to a literature focused on the mechanistic modelling of simple payment systems, sometimes referred to as "money transfer models". Several papers describe theoretical agent-based models where it is possible to derive a distribution of holding times, and so compute the average transfer velocity (Wang et al., 2003, 2005; Kanazawa et al., 2018). We adapt this theoretical work to allow for empirical measurement, and expand its scope to cases where the amount of money in the system changes over time. Our methodology can be used to compare theoretical models of money transfer against empirical observation. Future work might look to develop models that better reflect features of this and other high-granularity transaction data sets.

In the next section, we describe the Sarafu Community Inclusion Currency 2020/2021 data set and the information contained therein. In Section 3, we describe our methods, both theoretical and empirical. Section 4 describes the results of our empirical investigation. Finally, Section 5 concludes.

2 DATA

Sarafu is a small and substantially re-transacted digital complementary currency in Kenya. The payment system operates via a feature-phone mobile interface and one unit of Sarafu is roughly equivalent in value to a Kenyan shilling. The dataset we use includes anonymized account information for around 55,000 users and records of all Sarafu transactions conducted from January 25, 2020 to June 15, 2021 (Ruddick, 2021). Transactions totaling around 300 million Sarafu capture various economic and financial activities such as purchases, transfers, and participation in savings and lending groups.

Figure 1 plots the total balance over time and weekly transaction volumes of transfers. The observation period includes the first year of the COVID-19 pandemic and several documented pilot projects, interventions, and currency operations by the nonprofit organization managing the currency (Ussher et al., 2021; Mattsson et al., 2022). The vast majority of transfers during this period were ordinary transactions among users, in the data denoted as type *standard*. Also included in this category are a handful of transfers by savings and lending groups that were facilitated by staff, in the data denoted as type *agent_out*. Currency creation and dissolution in the Sarafu system took place via *disbursement* and *reclamation* type transactions, respectively.

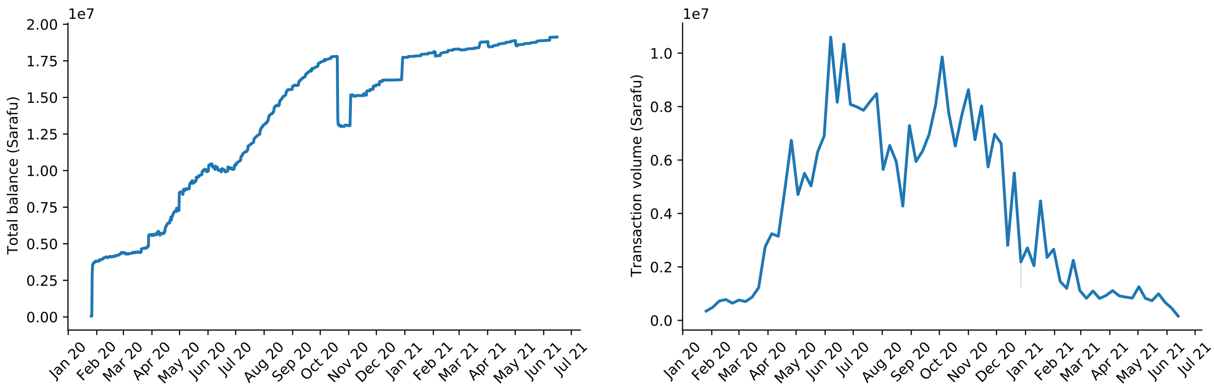


Figure 1. Total balance over time (left) and weekly transaction volumes of transfers (right).

Several contextual attributes are available that further describe the account holders' characteristics. The "area name" and "business type" are user-generated entries generalized into broader categories by the provider. They reflect the home location of the user and the product category of the goods or services they provide to the community. Localities are categorized into *urban*, *periurban*, and *rural* "area types." Mattsson et al. (2022) provide precise descriptions of these data fields, together with their possible values. We filter out *system* accounts, which were used for currency management and administrative operations by staff at the nonprofit organization managing the currency.

3 METHOD

With the advent of large-scale micro-level transaction data comes the opportunity to measure the velocity of money in greater detail than has been previously possible. Section 3.1 reconciles existing definitions of the transfer velocity of money and extends this definition to encompass the creation and dissolution of currency in continuous time. Empirical estimation, of course, requires measurement. Section 3.2 recounts the steps taken to estimate the average transfer velocity of money using conventional measurement methodology. Section 3.3 presents a new measurement methodology that accounts for currency creation/dissolution and enables study of the heterogeneity in underlying spending patterns.

3.1 Theoretical Definition

The transfer velocity of money is conventionally defined using an identity similar to that expressing the quantity theory of money. The total flow of money, F_T, over a period of time, $T_0 < t < T_1$, is related to the amount of money in circulation, M, and its average transfer velocity, V. Equation 1 describes the familiar relation. We include the duration $T = T_1 - T_0$ in our formulation, explicitly, so that both F_T and M have units of an amount denoted in some currency.

$$F_T = M \cdot V \cdot T \tag{1}$$

The transfer velocity can also be defined using the concept of "holding time." Denoted τ, holding times are the durations between when accounts receive and re-transact particular units of money. The average transfer velocity can be found by integrating over the probability distribution of holding times for the money in the system at a point in time, denoted $P(\tau)$ (Wang et al., 2003, Eqn. 8). Equation 2 expresses this equation in our notation.

$$V = \int_0^\infty \frac{1}{\tau} \cdot P(\tau) d\tau \text{ where } 1 = \int_0^\infty P(\tau) d\tau \tag{2}$$

These two definitions rest on remarkably similar assumptions. Formalized in the language of stochastic processes, Equation 7 holds under steady-state conditions, that is, where $P(\tau)$ is independent of t (Wang et al., 2003). This is also the condition under which M in Equation 1 can be readily approximated by its time-average (Mbiti and Weil, 2013). Equations 3 and 4 formulate a generalized version of the expression for F_T without this assumption and of the *cumulative* holding time distribution, $F_T(\tau)$ cf. Wang et al. (2003, Eqn. 6).

$$F_T = \int_{T_0}^{T_1} \int_0^\infty M(t) \cdot \frac{1}{\tau} \cdot P(\tau, t) \partial\tau \partial t \tag{3}$$

$$F_T = \int_0^\infty F_T(\tau) d\tau \text{ where } F_T(\tau) = \int_{T_0}^{T_1} M(t) \cdot \frac{1}{\tau} \cdot P(\tau, t) dt \tag{4}$$

3.2 Conventional Estimation

Conventional estimation of the velocity V uses Equation 1. In practice, F_T is a directly measurable quantity while M is not. Only rarely does the total balance of a system remain unchanged over the period $T_0 < t < T_1$ and it is common to use the time-averaged total balance (Mbiti and Weil, 2013). Equation 5 formulates an expression for V using M_{avg}, being the time-average of M. Again, $T = T_1 - T_0$.

$$V = F_T / (M_{\text{avg}} \cdot T) \text{ where } M_{\text{avg}} = \int_{T_0}^{T_1} M(t) dt / T \tag{5}$$

Values of F_T and $M(t)$ can be measured from large-scale micro-level transaction data. The total flow is the combined amount over transactions that occur in the period $T_0 < t < T_1$. The total balance at time t is the total amount of money held across all accounts in that moment.

3.3 Distributional Estimation

Distributional estimation of the velocity V can be done by drawing on Equations 4 and 2. The probability that money has some holding time at any particular point in time, $P(\tau, t)$, is neither directly observable nor necessarily stable over time. Instead, we consider the cumulative distribution—$F_T(\tau)$—that can be directly observed. Equation 6 normalizes this distribution into a proper probability, and Equation 7 expresses the inverse relationship between the average duration held and the average transfer velocity of money V.

$$P_T(\tau) = F_T(\tau) / F_T \tag{6}$$

$$V^{-1} = \int_0^\infty \tau \cdot P_T(\tau) d\tau \tag{7}$$

The distribution $F_T(\tau)$ can be measured from large-scale micro-level transaction data. Section 3.3.1 describes how "trajectory extraction" can be used to obtain empirical holding times. Specifically, we obtain the duration τ for which money was held prior to each of the transactions that occur in the period $T_0 < t < T_1$. Section 3.3.2 introduces kernel density estimation, which can be used to smooth the empirical distribution of held durations. This makes it possible to evaluate Equation 7 numerically.

3.3.1 Holding Times

Empirical holding times can be found by tracing the flow of money through a payment system. This is done using a methodology called "trajectory extraction" based in the theory of walk processes on networks (Mattsson and Takes, 2021). Specifically, each transaction out of an account is allocated funds from prior transactions into that account. In this way, an amount is assigned to every possible pair of incoming to outgoing transactions. We use the so-called "well-mixed" heuristic to determine the amounts, ensuring that they reflect the proportional assignment of incoming funds to outgoing transactions without imposing any possible arbitrary distinctions regarding how people might see these funds.

From the extracted trajectories, we consider all directly subsequent pairs of transactions. Each of these pairs of incoming to outgoing transactions corresponds to an empirical holding time. The timestamp of the outgoing transaction is the point at which the holding time is observed, and so we favor the past tense: these are "held durations." Notably, there is a particular account where the money was held for this duration of time.

In the considered data set, the account where the money was held has its own characteristics, including the "area type" and "business type." Considering different subsets of holding times makes it possible to study heterogeneity along demographic dimensions. From the transaction pair, it is also possible to determine what was happening to the funds. In the considered data set, newly-created Sarafu could enter circulation via a transfer transaction or be dissolved without ever entering circulation. Sarafu received via a transfer could remain in circulation via a subsequent transfer or be dissolved and so removed from circulation.

3.3.2 Cumulative Distribution of Held Durations

Holding times that end in a given period are collected into a cumulative distribution of held durations; this is $F_T(\tau)$. The observations are weighted and the total weight is used to normalize the distribution into a probability; this is $P_T(\tau)$. We obtain a smooth empirical distribution over τ using kernel density estimation (KDE) (Scott, 1992; Wand et al., 1991; Jones et al., 2018). KDE is performed in the space of $log(\tau)$, since the held durations range over several orders of magnitude—from seconds to months. Selecting a weighted gaussian kernel of suitable bandwidth in $log(\tau)$ is the equivalent of using a log-normal kernel in τ. Charpentier and Flachaire (2015) demonstrate the effectiveness of log-transformed KDE for heavy-tailed distributions of income, particularly at low values. The estimated probability density is back-transformed into the space of τ prior to numerical integration (Charpentier and Flachaire, 2015, Eqn. 9).

3.4 Implementation

We compute the velocity of money for the Sarafu currency over the 71 full weeks from February 2nd, 2020 through June 12th, 2021. Monthly and overall estimates are produced for this same period, using coarser dis-aggregation or forgoing dis-aggregation entirely. We measure the total balance at every hour to estimate the time-averaged balances.

Kernel density estimation and numerical integration are implemented in Python using the `gaussian_kde` and `quad` modules from `scipy.integrate` and `scipy.stats`, respectively (Virtanen et al., 2020). The bandwidth for KDE is selected using the so-called Scott's Rule, as is the default (Scott, 1992). Analysis is performed in `pandas` (Reback et al., 2020), and figures are produced using `seaborn` (Waskom, 2021) and `matplotlib` (Caswell et al., 2019).

Empirical holding times are found using open-source software available at https://github.com/carolinamattsson/follow-the-money (Mattsson, 2020). The `--pairwise` option is used to limit trajectory extraction to pairs of sequential transactions. Currency creation and dissolution defines the boundary of the system, and is captured by the *disbursement* and *reclamation* transaction labels. Since we are studying currency management we select the *well-mixed* allocation heuristic (see Section 3.3.1).

A very small amount of Sarafu is mis-recorded in the data (Mattsson et al., 2022). We employ the functionality provided in `follow-the-money` to infer the existence of missing funds. In extracting the empirical holding times, we also do not continue to track fragments of received transactions with a size below 0.1 Sarafu. While some ambiguity arises from the cases where the entry or exit time is unobserved, we find this to be an insignificant source of noise. To quantify, we compute our estimates twice. In one formulation all unobserved funds are treated as absent from the system, and in the other, as present within the system. These give the same estimates at the precision with which we report our results.

4 RESULTS

The average transfer velocity of one unit Sarafu was 0.27 transactions per week. However, this average value obscures underlying heterogeneity in the velocity of money as well as its inverse: how long spenders hold their money in their accounts. Section 4.1 presents the distribution of held durations that produces this estimate, and compares the distributional estimate to the conventional one over time. Section 4.2 reveals substantial differences in spending patterns between rural and urban areas, which grew pronounced as the COVID-19 pandemic unfolded. Section 4.3 describes a case in which fine-grained data gives insight about the effect, or lack thereof, of currency operations on the velocity of money.

4.1 Descriptive distributions

The conventional and distributional estimates for the average transfer velocity of Sarafu are 0.31 transactions per week and 0.27 transactions per week, respectively. Figure 2 (left) plots the average transfer velocity of Sarafu from February 2020 to June 2021, as calculated using the methods described in Section 3. The two approaches reveal broadly similar trends over time; the distributional estimate lags the conventional estimate. Transfer velocity rose in the spring/summer of 2020, a period in which the user base was growing substantially. Then, circulation slows. The conventional and distributional estimates diverge in October 2020 due to a large currency dissolution operation that caused a sharp decrease in the total balance of the system; this case is considered in Section 4.3. There is higher volatility in the velocity early on and this volatility tapers down over time.

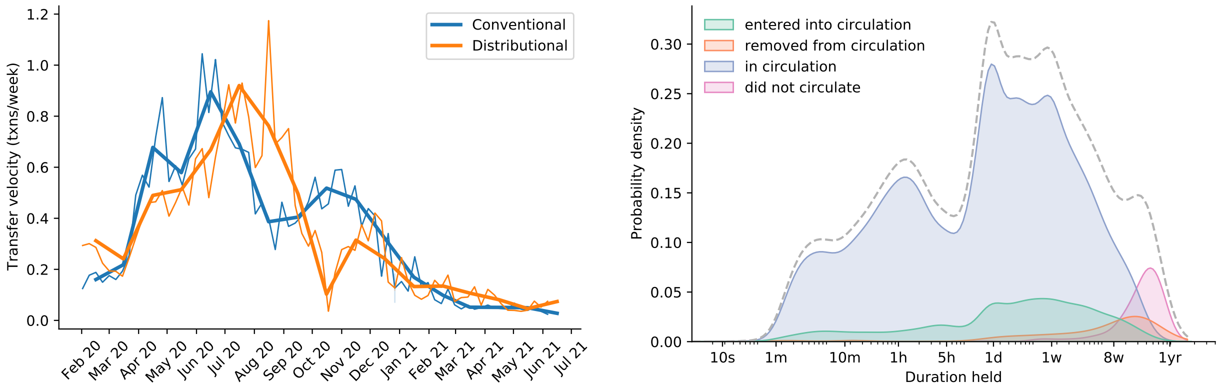


Figure 2. Weekly and monthly estimates of the average transfer velocity of money, using conventional and distributional normalization (left). Probability distribution over the durations for which funds were held prior to an observed transaction between February 2020 and June 2021 (right). Contributing to the overall distribution (dashed line) are funds that entered circulation by being transferred for the first time (green), remained in circulation (blue), were removed from circulation (orange), and were removed without having entered circulation (red).

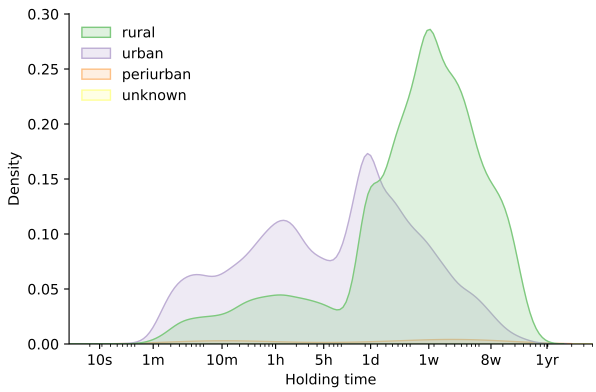
The held durations that produce the distributional estimate vary over orders of magnitude. Figure 2 (right) shows the probability density of observing a Sarafu transaction that had been held, prior to then, for a particular length of time. Sarafu is often spent the same day it is received, sometimes in a matter of minutes. It is also common to leave funds overnight to spend later in the same week, sometimes longer. Long-held Sarafu contributes little to the total spent. This wide distribution of held durations invites a different intuition than does an average velocity: much of the total balance at any point in time is slow moving—held for weeks or months—while a small amount of fast-moving money produces a high volume of transactions.

Obtaining the full distribution of held durations also lets us consider its constituent pieces. Currency creation and dissolution were routine features of currency management for the Sarafu system, as detailed in Section 2. In Figure 2 (right), we see that much of the newly-created Sarafu was transferred before long (green) and began circulating (blue). Still, some Sarafu was created and never transferred. A substantial amount of such funds were targeted for removal, dissolved without ever entering circulation (red). Long-held Sarafu was also targeted for removal from circulation (orange). By dis-aggregating in this way, we see that much of the distribution's long-duration tail reflects currency dissolution operations acting on inactive balances. The distributional estimate for the average transfer velocity of money *remaining in or entering circulation* is higher, at 0.35 transactions per week.

4.2 Geographic heterogeneity

The speed of transaction activity also varies over spender characteristics. In particular, we find that transaction rhythms differ substantially between rural and urban areas. Figure 3 (left) plots the probability density of durations for which Sarafu was held by accounts registered to urban, rural, and periurban areas prior to transfers made between February 2020 to June 2021. Spenders in urban areas tend to hold their money for shorter periods—spend faster—than those in rural areas. The transaction volume in urban areas emerges from regular, daily interaction while the rural rhythm appears to be weekly.

Sarafu is a community currency and circulation is highly localized, geographically. For this reason, the urban and rural areas contributing to the same total transaction volume are nearly independent in practice. Figure 3 (right) plots the average transfer velocity of money in or entering circulation, separately for urban and rural areas; these values are effectively decoupled. If we merely looked at the average we would miss the main story: there was a dramatic increase in use centered around an urban slum in Nairobi as the COVID-19 pandemic unfolded. For some months, use in this area outpaced currency creation and circulation became quite rapid. Rural areas also saw a pandemic-driven increase in use, particularly in the sub-county of Kinango Kwale. However, Sarafu was more established in this area and currency management operations appear to have kept pace; the transfer velocity remained relatively steady. Eventually, in both areas, demand receded and the velocity began a gradual decrease.



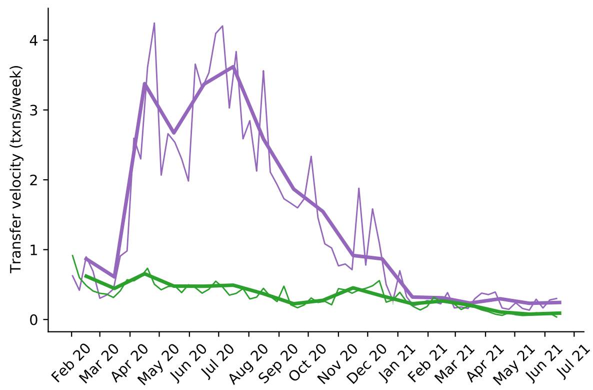


Figure 3. Contribution to the distribution of durations held by users in urban, rural, periurban, and unknown areas (left) and the weekly and monthly average across urban and rural users (right). Calculations based on funds in or entering circulation.

4.3 Currency dissolution operation

On October 19, 2020, a fee was assessed on around 31,000 accounts via a batch of RECLAMATION transactions (Mattsson et al., 2022). This major currency dissolution operation served dual purposes. Large-value accounts were warned of upcoming regular demurrage charges; these began in November. Long-inactive accounts, on the other hand, received a substantial penalty. In many cases, this charge brought the balance of long-inactive accounts to zero. The total reclaimed was 4.8 million Sarafu, or 26.7% of the balance in the system at that point in time. This money was dissolved as the batch was processed over October 19th and 20th, resulting in a nearly discontinuous drop in the total balance (see Figure 1, left). The operation affected almost entirely long-held funds. The distribution of the durations held just prior to reclamation corresponds closely to that in Figure 2 (right, red), which this operation dominates.

The dramatic decrease in the total balance of the system has a direct impact on the average transfer velocity, as conventionally estimated. The week of October 12-18 and of October 26-31 saw similar transaction volumes, around 7.7 million Sarafu, while the conventional estimate of the velocity jumps from 0.44 to 0.59 transactions per week. The sudden movement of long-held money also has a direct impact on the distributional estimate for the week of October 18-25, which drops to 0.03 transactions per week. However, the operation was restricted to long-held funds and touched predominantly long-inactive accounts. It is unclear that this would have a dramatic impact on organic use of the system.

To check this, we consider the impact on funds remaining in or entering circulation. Figure 4 shows the distribution of durations held prior to regular transfers occurring in the week prior, week of, and week after the currency dissolution operation of October 19th, 2020. Part of the intention behind the operation may have been to encourage renewed activity among lapsed users or those with large balances. Rural users seem to have responded in line with this expectation in that the distribution shifts towards the right, somewhat, as long-held funds were brought back into circulation. Urban users, on the other hand, showed no discernible change in their propensity to transact with long-held funds.

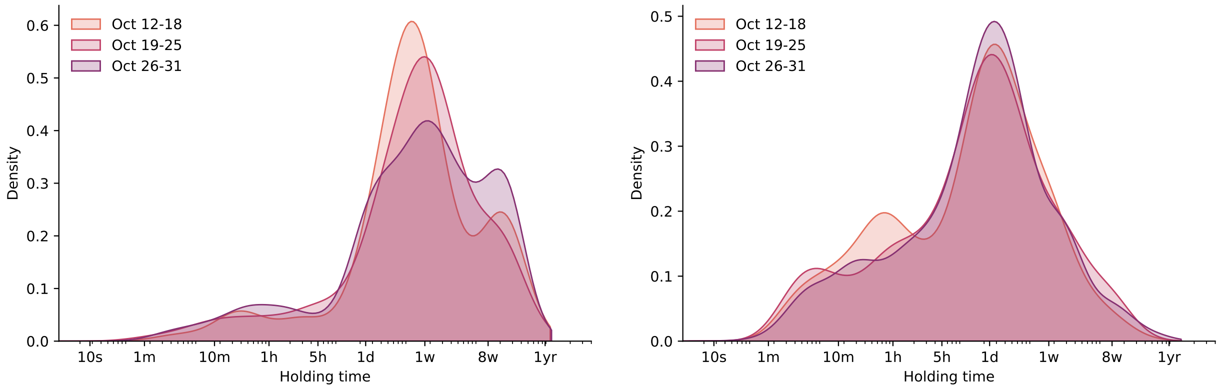


Figure 4. Distribution of durations held prior to transfers occurring in the week prior, week of, and week after the currency dissolution operation of October 19th & 20th, 2020. Left, rural users. Right, urban users.

This illustrates a scenario where the conventional and distributional estimates of the velocity of money respond differently, even without a change in total balance. A behavioral shift towards transacting with older funds, that is, spending down savings, would produce somewhat larger transaction volumes for a time. The conventional estimate for the velocity of money would rise. In contrast, the distributional estimate would fall—the effective balance of the system increases as long-held funds re-enter circulation. The distributional estimate for the transfer velocity of money ensures that it provides information about the system that is entirely orthogonal to the transaction volume. With further study, this measure could improve monitoring and understanding of transitory monetary phenomena.

5 CONCLUSION

We measure the velocity of money using new mathematical and computational methods. These methods allow us to describe the transaction velocity, as well as the money-holding times, across a number of dimensions including time, geographical location, and spending patterns. We find that most of the transaction volume is driven by a small number of spenders who spend their money quickly, while most spenders spend their money more slowly. Additionally, we find that spenders in urban areas complete more transactions in a short period of time, while more rural spenders complete fewer transactions. Finally, we see an uptick in transaction velocity during the beginning of the COVID-19 pandemic. A better understanding of spending patterns, and the rate at which money changes hands, as well as a better understanding of how these differ across groups and locales, can inform better macroeconomic policy.

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