DEPARTMENT OF ECONOMICS
AND BUSINESS ECONOMICS
AARHUS UNIVERSITY

EXAM ASSIGNMENT

Study Programme and level	MSc Oecon	
Term	V19-20o	
Course name and exam code(s)	4394: Financial Econometrics	461172E002
Exam form and duration	Take-home exam	4 days
Date and time	13-17 January 2020	14pm – 14pm
Other	The exam is anonymous.	
Supplementary material	data.csv	
Number of pages (incl. front page)	5	

Please read these instructions carefully:

Make sure **NOT** to state your name and student ID number in your answer.

How to contact the lecturer during a take-home exam
In case you have any questions regarding the exam, the lecturer is available to take questions during the first four hours after exam start. Questions should be posed to the lecturer by email to leopoldo.catania@econ.au.dk.

Students should not expect instant answers, as all questions received during these four hours will be answered together via Blackboard once the four hours have passed.

Handing in your paper
You must submit your exam paper within the exam deadline. The exam flow will be closed immediately when the exam deadline expires. It is therefore important that you generate a PDF file and start submitting your exam paper **approximately 10 minutes in advance** of the exam deadline.

Exam questions for 4394:Financial Econometrics
Winter 2019-20

Leopoldo Catania

Guidelines

General guidelines

The exercises must be done individually. You have to submit two files: one `PDF` file and one file with the computer code. The computer code should be submitted as "appendix". You have to report details about how you derived the solutions. The computer code should be organized such that it can reproduce all the answers of your exam. Code must be commented. The exam will be graded on the Danish 7-point grading scale. Good luck!

Guidelines for the theoretical questions

Try to be as precise as possible and write your results in a `PDF` document. If you plan to write a `Word` document and then transform it to `PDF`, please be sure that all formulas are well formatted before submitting your exam. When you write your answers, you have to be consistent with the notation used in the exam questions.

Guidelines for writing and presenting your code

You are required to comment on your code. For example, state the purpose of the functions you code, their arguments, and what they return as an output. I should be able to run your code without major modifications. You are free to employ your favorite optimization scheme. Be careful of removing all details that can reveal your identity from the code.

Guidelines for the empirical questions

For the empirical questions, you will use the Dow Jones Index returns (DJI30) and the Standard & Poor's 500 index returns (SPY), spanning the period from 2007-01-03 to 2019-01-01. You can find these two series in the file `data.csv`. Data is formatted as follow: i) the first and second columns report the returns for GSPC and DJI, respectively, ii) columns are separated

by the symbol ";", iii) decimals are indicated with the symbol ",". You are also required to download one series from Yahoo finance by yourself.

Question 1:

Methodology

VIX is an index of volatility computed by the Chicago Board Options Exchange - CBOE and freely available to the public. On Yahoo finance, it is indicated by "^VIX" note the hat symbol "^" before "VIX". It is a measure of volatility in the US equity market and is taken in much consideration by private and public investors. Since it is a measure of volatility, it can only take positive values. The modelling and the prediction of the VIX is of great importance for portfolio optimization and pricing of derivative securities. Let Y_t be the VIX at time t and consider the following model:

$$Y_t|\mathcal{F}_{t-1} \sim \mathcal{G}a(\mu_t, a),$$

where $\mathcal{G}a(\mu_t, a)$ is the Gamma distribution with mean $\mu_t > 0$ and scale $a > 0$ with probability density function given by:

$$p(y_t|\mathcal{F}_{t-1}) = \frac{1}{\Gamma(a)} a^a y_t^{a-1} \mu_t^{-a} \exp\left(-a\frac{y_t}{\mu_t}\right).$$

Here, we implement the parameterization of the Gamma distribution used by Engle and Gallo (2006), for which $E[Y_t|\mathcal{F}_{t-1}] = \mu_t$ and $Var[Y_t|\mathcal{F}_{t-1}] = \mu_t^2/a$.

- Derive a GAS model for μ_t and scale the score by the inverse of the square root of the Fisher information quantity for μ_t, i.e. set $d = 1/2$ in slide 19 of Lecture 10. Note that $E[Y_t^2|\mathcal{F}_{t-1}] = \frac{\mu_t^2(1+a)}{a}$. If you cannot derive the information quantity, use identity scaling, i.e. $d = 0$.
- Write the log Likelihood of the model.

Computational part

- Write a function to estimate the GAMMA-GAS model of the previous point using the Maximum Likelihood estimator. The function should accept a vector of observations and return the estimated parameters, the filtered means μ_t, for $t = 1, \ldots, T$, and the log likelihood evaluated at its maximum value. Assume that ω, α, and β are the intercept, score coefficient and autoregressive coefficient of the GAS process. Impose the following constraints during the optimization: $\omega \in [-0.5, 0.5]$, $\alpha \in [0.001, 1.5]$, $\beta \in [0.01, 0.999]$, $a \in [0.1, 300]$.

Empirical analysis

- Download the VIX index from Yahoo finance using the `quantmod` package. Consider the period from "2010-01-01" to "2019-01-01". Your series is the one reported in the column named "Adjusted".
- Estimate the GAS model you derived in the previous exercise.
- Consider the constraint versions of the model with $\mu_t = \mu$. Using BIC, choose the best specification between the static and time–varying specifications.

Engle and Gallo (2006) introduced a series of Multiplicative Error Models to model positive valued series like the VIX. In their simpler specification, they assume that:

$$Y_t|\mathcal{F}_{t-1} \sim \mathcal{G}a(\mu_t, a),$$

and

$$\mu_t = \kappa + \eta y_{t-1} + \phi\mu_{t-1}.$$

- Write a function to estimate the MEM model of Engle and Gallo (2006). Impose these constraints on the parameters of the MEM model: $\kappa \in [0.1, 10]$, $\eta \in [0.01, 0.99]$, $\phi \in [0.01, 0.99]$, and $a \in [0.1, 300]$.
- Compare the GAS and the MEM models: which one reports the highest likelihood?
- Create a 3×1 figure containing: i) the original series, ii) the filtered conditional mean, and iii) the conditional variance of the selected model.

Question 2:

Methodology

Consider the bivariate random vector $\mathbf{Y}_t = (Y_{1,t}, Y_{2,t})'$, where $Y_{1,t}$ and $Y_{2,t}$ are the GSPC and DJI returns at time t, respectively.

- Assume that $\mathbf{Y}_t|\mathcal{F}_{t-1}$ is bivariate Gaussian with mean $\mathbf{0}$ and variance covariance matrix $\mathbf{\Sigma}_t$.
  - Derive a DCC model for $\mathbf{\Sigma}_t$ assuming that each marginal process is GARCH(1,1).
  - Clearly state the constraints of the model.
  - Detail how the likelihood factorizes and what this implies for the estimation of the model parameters.

  - Obtain the Constant correlation model (CCC) as a special case of the DCC

- Estimate the DCC and CCC models on the series of GSPC and DJI returns. You **cannot** use the **rugarch** package for GARCH estimation. For both the DCC and CCC:
  - Compute the weight ω_t associated to the Minimum Variance Portfolio (MVP) constructed using the GSPC and DJI returns at each point in time, i.e. $y_t = \omega_t y_t^{GSPC} + (1-\omega_t) y_t^{DJI}$. Compare the portfolio weights of the two models ω_t in a figure.
  - Compute the CoVaR of GSPC given DJI at level α at time t defined as $P(Y_{1,t} \leq CoVaR_t(\alpha) | Y_{2,t} \leq VaR_t(\alpha), \mathcal{F}_{t-1}) = \alpha$. In a Figure, report results for $\alpha = 1\%$ and $\alpha = 5\%$.

References

Engle, R. F. and Gallo, G. M. (2006). A multiple indicators model for volatility using intra-daily data. *Journal of Econometrics*, 131(1-2):3–27.