DEPARTMENT OF ECONOMICS
AND BUSINESS ECONOMICS
AARHUS UNIVERSITY

EXAM ASSIGNMENT

Study Programme and level	MSc Oecon	
Term	W18-19o	
Course name and exam code(s)	4394: Financial Econometrics	461172E002
Exam form and duration	Take-home	5 days
Date and time	17-22 January 2019	9am – 9am
Other	The exam is anonymous.	
Supplementary material	data.csv	
Number of pages (incl. front page)	5	

Please read these instructions carefully:

Make sure **NOT** to state your name and student ID number in your answer.

How to contact the lecturer during a take-home exam
In case you have any questions regarding the exam, the lecturer is available to take questions during the first four hours after exam start. Questions should be posed to the lecturer through Blackboard.

Students should not expect instant answers, as all questions received during these four hours will be answered together via Blackboard once the four hours have passed.

In Blackboard, you should go to **My Institution → Tools** and choose **Send Email** (choose specific instructor/ teacher)

Handing in your paper
You must submit your exam paper within the exam deadline. The exam flow will be closed immediately when the exam deadline expires. It is therefore important that you generate a PDF file and start submitting your exam paper **approximately 10 minutes in advance** of the exam deadline.

Exam questions for 4394:Financial Econometrics
Winter 2018-19

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Guidelines

General guidelines

The exercises must be done individually. You have to submit two files: one `PDF` file and one file with the computer code. The computer code should be submitted as "appendix". The `PDF` file should include the answers to the theoretical part as well as any answer to the computational and empirical part which requires a written comment. The computer code should be organized such that it can reproduce all the computational and empirical answers of your exam. Code must be commented. The exam will be graded on the Danish 7-point grading scale. Good luck!

Guidelines for the theoretical part

Try to be as precise as possible and write your results in a `PDF` document. If you plan to write a `Word` document and then transform it to `PDF`, please be sure that all formulas are well formatted before submitting your exam. When you write your answers, you have to be consistent with the notation used in the exam questions.

Guidelines for the computational part

You are required to comment on your code. For example, state the purpose of the functions you code, their arguments, and what they return as an output. I should be able to run your code without major modifications. You are free to employ your favorite optimization scheme.

Guidelines for the empirical part

For the empirical exercises, you will use the General Motor (GM) and Dow Jones Index (DJI30) returns in percentage points spanning the period from 2007-01-03 to 2009-02-03. You can find these two series in the file `data.csv`. Data is formatted as follow: i) the first column reports the dates in the format "yyyy-mm-dd", ii) the second and third columns report the returns for GM and DJI30, respectively, iii) columns are separated by the symbol ";", iv) decimals are indicated with the symbol ",".

Theoretical part

During the course, we have widely employed the Gaussian and the Student's t distributions to model the conditional distribution of financial returns. However, many other distributions are available, and it is not always easy to choose the best one. For example, Nelson (1991) proposed to model the conditional distribution of financial returns using the Generalized Error Distribution (GED). Specifically, for a generic random variable $Y \in \Re$ with mean $\mu \in \Re$ and scale $\varphi > 0$, the probability density function (pdf) evaluated in y is:

$$p(y; \mu, \varphi, \upsilon) = \left[2^{1+1/\upsilon}\varphi\Gamma(1 + 1/\upsilon)\right]^{-1} \exp\left(-\frac{|(y-\mu)/\varphi|^{\upsilon}}{2}\right), \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function, and $\upsilon > 0$ is a tail-thickness parameter. The standard deviation σ is related to the scale by the formula:

$$\sigma = 2^{1/\upsilon}\sqrt{\frac{\Gamma(3/\upsilon)}{\Gamma(1/\upsilon)}}\varphi. \tag{2}$$

Consider a Generalised Autoregressive Score (GAS) model, where the conditional distribution of financial returns $Y_t|\mathcal{F}_{t-1}$ is GED with location 0 ($\mu = 0$) and time varying scale φ_t:

$$Y_t|\mathcal{F}_{t-1} \sim GED(0, \varphi_t, \upsilon),$$

where $\mathcal{F}_t$ represents the filtration generated by the process $\{Y_s, s \leq t\}$.

- Derive the GAS updating equation for φ_t using an exponential link function, $\varphi_t = \exp(\tilde{\varphi}_t)$, and imposing unit scaling, i.e. do not scale the score ($d = 0$). Use s_t to label the score of $Y_t|\mathcal{F}_{t-1}$ with respect to $\tilde{\varphi}_t$. Evidently, s_t is going to be a function of the realization y_t, the tail-thickness parameter υ, and the reparametrized scale $\tilde{\varphi}_t$, i.e. $s_t = s(y_t, \upsilon, \tilde{\varphi})$. Compute $E[s_t]$ and $E[s_t|\mathcal{F}_{t-1}]$.
- Write the log likelihood of the model and state the model parameter constraints. Which constraint do we need to impose to ensure that the process of $\{\tilde{\varphi}_t, t > 0\}$ is covariance stationary (assume that $E[s_t^2] < \infty$ for all values of $\tilde{\varphi}_t$ and υ)?
- When $\upsilon = 2$, the GED distribution collapses to the Gaussian distribution. Compare the functions $f(y_t) = s(y_t, 1, 0)$ and $f(y_t) = s(y_t, 2, 0)$. Which conclusion can you draw about the response of the scale parameter to different values of y_t in the case $\upsilon = 1$ and $\upsilon = 2$?

Computational part

- Write a function to estimate the GAS–GED model you have derived in the previous exercise. The function should accept the data and return: i) The filtered scale and volatility, i.e. the vectors $(\sigma_1, \sigma_2, \ldots, \sigma_T)$ and $(\varphi_1, \varphi_2, \ldots, \varphi_T)$, respectively, ii) the estimated model parameters, iii) the value of the likelihood evaluated at its optimum, and iv) the average BIC (the BIC divided by the number of observations).

- Write a function to compute the Value–at–Risk (VaR) at level $\alpha \in (0, 1)$ for the conditional distribution $Y_t|\mathcal{F}_{t-1}$. You have two options: i) write a function that computes the cumulative density function $P(Y_t \leq y|\mathcal{F}_{t-1})$ using the `integrate` function and numerically solve $P(Y_t \leq VaR_t(\alpha)|\mathcal{F}_{t-1}) - \alpha = 0$ (please always use this definition of VaR) by $VaR_t(\alpha)$ using the `uniroot` function, or ii) use the `qdist` function in the **rugarch** package with argument `distribution = "ged"`. If you go for the second option, note that `qdist` is parameterized in terms of the volatility σ_t, and the tail-thickness parameter is labelled as `shape`. The function should accept: i) a vector of volatilities or scales, ii) the tail-thickness parameter, and iii) the α confidence level. The function should return a vector with the VaR computed for each time t at the desired confidence level α.

Empirical part

a) Consider this general dynamic volatility model:

$$y_t = \sigma_t \varepsilon_t,$$

where ε_t is independently and identically distributed as a standardized ($E[\varepsilon_t] = 0$, $E[\varepsilon_t^2] = 1$) GED with tail-thickness parameter υ, and σ_t is a deterministic function of past observations y_{t-s} for $s \geq 1$. Write down the GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models for σ_t^2. For both the GM and DJI30 series:

   i) Estimate these three models under the GED assumption for the innovations using the **rugarch** package.

   ii) Compare the filtered volatilities according to these three specifications in a figure.

   iii) Select the best model using the BIC criteria.

b) Estimate the GAS-GED model on the GM and DJI30 series. For each series:

   i) Compare the filtered volatility of the GAS-GED model with that of the GARCH(1,1) model in a figure.

ii) Compare VaR at levels $\alpha = 1\%$ and $\alpha = 5\%$ estimated with the GAS-GED and GARCH(1,1) models in a figure.

iii) Select the best model between GAS-GED and GARCH(1,1) using BIC.

Try to draw some general conclusion about the difference between the GAS-GED and GARCH(1,1) models.

c) Assume that the correlation matrix (R) between GM and DJI30 is constant over time. Let Σ_t be the covariance matrix between GM and DJI30 at time t. Use the following decomposition of Σ_t:

$$\Sigma_t = D_t R D_t, \tag{3}$$

where D_t is a 2−dimensional diagonal matrix with element $(1, 1)$ and $(2, 2)$ equal to the standard deviations of GM and DJI30 at time t, respectively.

i) Compute Σ_t for each t, when GM and DJI30 both follow the GAS-GED model.

ii) Compute Σ_t for each t, when GM and DJI30 both follow the GARCH(1,1) model.

iii) Under the two model specifications of previous points i) and ii), compute the weight ω_t associated to the Minimum Variance Portfolio (MVP) constructed using the GM and DJI30 returns at each point in time, i.e. $y_t = \omega_t y_t^{GM} + (1 - \omega_t) y_t^{DJI30}$.

iv) Compare the portfolio weights of the two models ω_t in a figure.

References

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, pages 347–370.