Systemic Risk evaluation

Leopoldo Catania
Aarhus University and CREATES

leopoldo.catania@econ.au.dk

Based on: Girardi and Ergün (2013), Mainik and Schaanning (2014), and Adrian and Brunnermeier (2016)

Introduction: Systemic Risk

- In times of financial crisis, losses spread across financial institutions, threatening the financial system as a whole.
- The spreading of distress gives rise to systemic risk: the risk that the capacity of the entire financial system is impaired, with potentially adverse consequences for the real economy.
- Spillovers across institutions can occur directly due to direct contractual links and heightened counterparty credit risk or indirectly through price effects and liquidity spirals.
- Systemic risk measures gauge the increase in tail comovement that can arise due to the spreading of financial distress across institutions.

Systemic Risk

- There is not a unique definition of Systemic Risk. However, most of the techniques used to evaluate systemic risk focus on the behaviour of a random variable, X, conditionally on a distress event affecting another random variable Y. Of course, the reaction of X to Y depends on the dependence between the two random variables, and, in general, from their joint distribution (X, Y).

- Many of the systemic risk indicators available nowadays are somehow inspired by the work of Adrian and Brunnermeier (2016) (first working paper in 2008).

- Unfortunately, the systemic risk measure proposed by Adrian and Brunnermeier (2011) and used by Adrian and Brunnermeier (2016) has an important pitfall as detailed in Mainik and Schaanning (2014). Thus, we will focus on the measure introduced by Girardi and Ergün (2013).

CoVaR$^{=}$

- Consider a random variable X with distribution $F_X(x)$, and another random variable Y, with distribution $F_Y(y)$. The joint distribution is indicated by $F_{X,Y}(x,y) = P(X \leq x, Y \leq y)$.

- The CoVaR measure introduced by Adrian and Brunnermeier (2011) is implicitly defined by:

$$P(X \leq CoVaR^{=}_{X|Y}(\alpha, \beta) | Y = VaR_Y(\beta)) = \alpha,$$

  where we see that it is the α–quantile of the conditional distribution $X|Y = VaR_Y(\beta)$.

- $CoVaR^{=}_{X|Y}(\alpha, \beta)$ can be also defined as:

$$CoVaR^{=}_{X|Y}(\alpha, \beta) = F^{-1}_{X|Y=VaR_Y(\beta)}(\alpha)$$

$\Delta CoVaR^{=}_{X|Y}(\alpha, \beta)$

- CoVaR itself is not a measure of systemic risk. The measure of systemic risk is defined as:

$$\Delta CoVaR^{=}_{X|Y}(\alpha, \beta) = CoVaR^{=}_{X|Y}(\alpha, \beta) - CoVaR^{=}_{X|Y}(\alpha, 0.5),$$

  that is the difference between the CoVaR conditionally on a distress event affecting Y, i.e. $Y = VaR_Y(\beta)$, $CoVaR^{=}_{X|Y}(\alpha, \beta)$ and the CoVaR conditionally on a "normal" (or benchmark) scenario $Y = VaR_Y(0.5)$.

- Note that $VaR_Y(0.5) = F_Y^{-1}(0.5)$ is equal to the median of Y.

$CoVaR^{=}_{X|Y}(\alpha, \beta)$ in a Gaussian world

- Assume $(X, Y) \sim N_2(\mu, \Sigma)$, where $\mu = (\mu_X, \mu_Y)$ and $\Sigma = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{pmatrix}$, and $\sigma_{XY} = \sigma_X \sigma_Y \rho$.
- In this case we know that $X|Y = y$ is Gaussian with mean $\mu_{X|Y} = \mu_X + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_x)$ and variance $\sigma^2_{X|Y} = \sigma^2_Y (1 - \rho^2)^2$.
- A bit of algebra shows that in this case $CoVaR^{=}_{X|Y}(\alpha, \beta) = \mu_X + \sigma_X(\rho \Phi^{-1}(\beta) + \Phi^{-1}(\alpha)\sqrt{1 - \rho^2})$.
- Note that $CoVaR^{=}_{X|Y}(\alpha, \beta)$ does not depend from μ_Y and σ_Y and if $\rho = 0$ we have $CoVaR^{=}_{X|Y}(\alpha, \beta) = VaR_X(\alpha)$.

$\Delta CoVaR^{=}_{X|Y}(\alpha, \beta)$ in a Gaussian world

- Exploiting the definition of $\Delta CoVaR^{=}_{X|Y}(\alpha, \beta)$ we obtain

$$\Delta CoVaR^{=}_{X|Y}(\alpha, \beta) = \sigma_X \rho \Phi^{-1}(\beta)$$

  Which does not depend on α.

- It is also evident that $\Delta CoVaR^{=}_{X|Y}(\alpha, \beta)$ in the Gaussian case does not bring any additional information then the correlation ρ.

- Similar results can be derived in the Student's t case. Mainik and Schaanning (2014) provides general results in the general elliptical case.

An important pitfall of $CoVaR^{=}_{X|Y}(\alpha, \beta)$

- Mainik and Schaanning (2014) show that $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is not an increasing function of the correlation between X and Y.
- This is counterintuitive since we expect the measure to increase for a higher level as association.
- It follows that $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is not monotone in ρ.

Monotonicity of $CoVaR^{=}_{X|Y}(\alpha, \beta)$

- To show this, consider the derivative of $CoVaR^{=}_{X|Y}(\alpha, \beta)$ with respect to ρ in the Gaussian:

$$\frac{\partial CoVaR^{=}_{X|Y}(\alpha, \beta)}{\partial \rho} = \sigma_X \left(\Phi^{-1}(\beta) - \frac{\rho \Phi^{-1}(\alpha)}{\sqrt{1-\rho^2}} \right),$$

- We see that this function is positive if $\Phi^{-1}(\beta)\sqrt{1-\rho^2} > \rho\Phi^{-1}(\alpha)$ and negative if $\Phi^{-1}(\beta)\sqrt{1-\rho^2} < \rho\Phi^{-1}(\alpha)$.
- Furthermore, besides the degenerate case $\alpha = \beta = 1/2$, there are four cases depending on the sign of $\Phi^{-1}(\alpha)$ and $\Phi^{-1}(\beta)$:
  - i) If $\alpha \geq 1/2$ and $\beta \geq 1/2$, then $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is increasing in ρ for $\rho < \rho_0$ and decreasing for $\rho > \rho_0$, where $\rho_0 = \frac{|\Phi^{-1}(\beta)|}{\sqrt{(\Phi^{-1}(\alpha))^2 + (\Phi^{-1}(\beta))^2}}$.
  - ii) If $\alpha < 1/2$ and $\beta \geq 1/2$, then $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is increasing in ρ for $\rho > -\rho_0$ and decreasing for $\rho < -\rho_0$.
  - iii) If $\alpha \geq 1/2$ and $\beta < 1/2$, then $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is increasing in ρ for $\rho < -\rho_0$ and decreasing for $\rho > -\rho_0$.
  - iv) If $\alpha < 1/2$ and $\beta < 1/2$, then $CoVaR^{=}_{X|Y}(\alpha, \beta)$ is increasing in ρ for $\rho > \rho_0$ and decreasing for $\rho < \rho_0$.

Monotonicity of $CoVaR^{=}_{X|Y}(\alpha, \beta)$

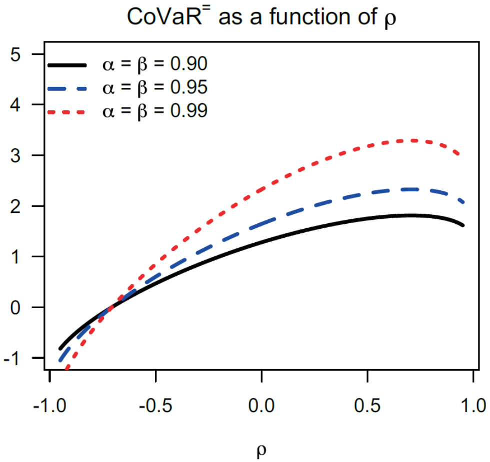

Figure: From Mainik and Schaanning (2014): $CoVaR^{=}_{X|Y}(\alpha, \beta)$ for a bivariate Gaussian distribution with correlation ρ.

Another CoVaR

- To solve the non monotonicity issue of $CoVaR^{=}_{X|Y}(\alpha, \beta)$, Girardi and Ergün (2013) have introduced another measure defined as:

$$P(X \leq CoVaR_{X|Y}(\alpha, \beta) | Y \leq VaR_Y(\beta)) = \alpha,$$

where we see that it is the α–quantile of the conditional distribution $X|Y \leq VaR_Y(\beta)$.

- $CoVaR_{X|Y}(\alpha, \beta)$ can be also defined as:

$$CoVaR_{X|Y}(\alpha, \beta) = F^{-1}_{X|Y \leq VaR_Y(\beta)}(\alpha)$$

CoVaR with inequality conditioning event

Unfortunately, we generally have no closed form for $CoVaR_{X|Y}(\alpha, \beta)$. However, we note that:

$$\begin{aligned} P(X \leq CoVaR_{X|Y}(\alpha, \beta) | Y \leq VaR_Y(\beta)) &= \frac{P(X \leq CoVaR_{X|Y}(\alpha, \beta), Y \leq VaR_Y(\beta))}{P(Y \leq VaR_Y(\beta))} \\ &= \frac{P(X \leq CoVaR_{X|Y}(\alpha, \beta), Y \leq VaR_Y(\beta))}{\beta}, \end{aligned}$$

That is: $CoVaR_{X|Y}(\alpha, \beta)$ is found as the solution of:

$$P(X \leq CoVaR_{X|Y}(\alpha, \beta), Y \leq VaR_Y(\beta)) = \alpha\beta$$

or

$$F_{X,Y}(CoVaR_{X|Y}(\alpha, \beta), VaR_Y(\beta)) = \alpha\beta$$

This equality can be solved using a rootfinder like the bisection method (see the `uniroot()` function in R).

Monotonicity of $CoVaR_{X|Y}(\alpha, \beta)$

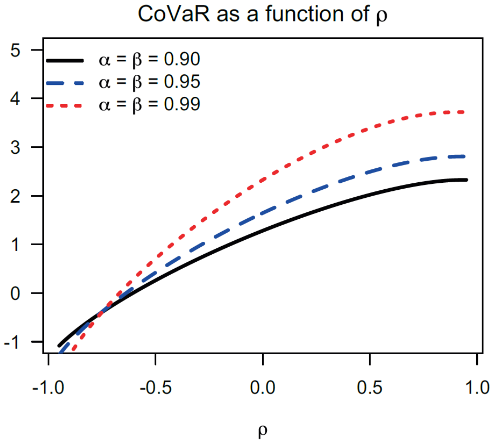

Figure: From Mainik and Schaanning (2014): $CoVaR_{X|Y}(\alpha, \beta)$ for a bivariate Gaussian distribution with correlation ρ.

$\Delta CoVaR_{X|Y}(\alpha, \beta)$

Girardi and Ergün (2013) also proposed another definition of $\Delta CoVaR_{X|Y}(\alpha, \beta)$ which is given by:

$$\Delta CoVaR_{X|Y}(\alpha, \beta) = 100 \times \frac{CoVaR_{X|Y}(\alpha, \beta) - CoVaR_{X|b(Y)}(\alpha)}{CoVaR_{X|b(Y)}(\alpha)},$$

where $CoVaR_{X|b(Y)}(\alpha, \beta)$ is implicitly defined as:

$$P(X \leq CoVaR_{X|b(Y)}(\alpha) | \mu_Y - \sigma_Y \leq Y \leq \mu_Y + \sigma_Y) = \alpha,$$

i.e., the benchmark state is $\mu_Y - \sigma_Y \leq Y \leq \mu_Y + \sigma_Y$.

Evaluating systemic risk

- It is important to see that we only need the joint distribution of the pair of Random Variables X and Y to evaluate the CoVaR.
- Of course, if the joint distribution is time–varying, we can compute CoVaR at each point in time. Furthermore, CoVaR can be predicted.
- Girardi and Ergün (2013) have estimated CoVaR assuming a bivariate DCC model for different pairs of assets.
- Their results show that during the sample period June 2000 to February 2008, depository institutions were the largest contributors to systemic risk, followed by broker-dealers, insurance companies, and non-depository institutions.

References I

Adrian, T. and Brunnermeier, M. K. (2011). CoVaR. *NBER Working Paper No. 17454*.

Adrian, T. and Brunnermeier, M. K. (2016). Covar. *American Economic Review*, 106(7):1705–41.

Girardi, G. and Ergün, A. (2013). Systemic risk measurement: Multivariate GARCH estimation of CoVaR. *Journal of Banking & Finance*, 37(8):3169 – 3180.

Mainik, G. and Schaanning, E. (2014). On dependence consistency of covar and some other systemic risk measures. *Statistics & Risk Modeling*, 31(1):49–77.