Exercise set #8

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The goal of this exercise set is to get comfortable with applications in risk management and portfolio optimization using univariate and multivariate volatility models.

(1): Theoretical part

Assume $\mathbf{y}_t = (y_{1,t}, \dots, y_{N,t})'$ follows a Gaussian DCC model with AR(1)–GARCH(1,1) marginals. Let $\boldsymbol{\omega}_t = (\omega_{1,t}, \dots, \omega_{N,t})'$ with $\sum_{i=1}^N \omega_{i,t} = 1$ be a vector of portfolio weights such that $\omega_{i,t}$ is the proportion of wealth invested in asset i at time t. Let $\mathcal{F}_t$ be the filtration generated by the process up to time t.

a) What is the distribution of $\mathbf{y}_{t+1}|\mathcal{F}_t$?

b) What about the distribution of $\mathbf{y}_{t+h}|\mathcal{F}_t$ for $h > 1$? How would you compute $P(\mathbf{y}_{t+h} \leq \mathbf{y}|\mathcal{F}_t)$ for a generic $\mathbf{y} \in \Re^N$?

c) Assume that $\boldsymbol{\omega}_{t+1}$ is measurable with respect to $\mathcal{F}_t$. What is the distribution of $\boldsymbol{\omega}_{t+1}'\mathbf{y}_{t+1}|\mathcal{F}_t$?

d) Assume that $\boldsymbol{\omega}_{t+h}$ for $h > 1$ is measurable with respect to $\mathcal{F}_t$. What about the distribution of $\boldsymbol{\omega}_{t+h}'\mathbf{y}_{t+h}|\mathcal{F}_t$? How would you compute $P(\boldsymbol{\omega}_{t+h}'\mathbf{y}_{t+h} \leq y|\mathcal{F}_t)$ for a generic $y \in \Re$?

e) Derive the Value at Risk at level α for the portfolio $y_{t+1}^p = \boldsymbol{\omega}_{t+1}'\mathbf{y}_{t+1}$ distribution conditional on $\mathcal{F}_t$. (Use your favorite definition of VaR, i.e. defined on the returns or on the losses)

f) Describe a procedure to compute the Value at Risk at level α for the h–step ahead distribution of the portfolio.

(2): Computational part

Consider two assets $\mathbf{y}_t = (y_{1,t}, y_{2,t})'$ and the portfolio $y_t^p = \omega_t y_{1,t} + (1 - \omega_t)y_{1,t}$. Assume that the distribution of $\mathbf{y}_{t+1}|\mathcal{F}_t$ is Gaussian and let $\widehat{\boldsymbol{\mu}}_{t+1|t} = E[\mathbf{y}_{t+1}|\mathcal{F}_t]$ and $\widehat{\boldsymbol{\Sigma}}_{t+1|t} = Cov[\mathbf{y}_{t+1}|\mathcal{F}_t]$

be the one step ahead predictive mean and covariance matrix, respectively. Assume short selling and:

i) Write a function that select ω_{t+1} by minimizing the one step ahead portfolio variance. (The minimum variance portfolio).

ii) Write a function that select ω_{t+1} by minimizing the one step ahead portfolio variance subject to a minimum expected return of $k\%$.

iii) Write a function to estimate the CCC model in the Gaussian bivariate case assuming that the univariate models are AR(1)–GARCH(1,1). The function should also return the one step ahead prediction of the conditional mean and covariance matrix. You can use the rugarch package to estimate the univariate models.

iv) Write a function to estimate the DCC model in the Gaussian bivariate case assuming that the univariate models are AR(1)–GARCH(1,1). The function should also return the one step ahead prediction of the conditional mean and covariance matrix. You can use the rugarch package to estimate the univariate models.

(3): Empirical part

Consider the last 2000 financial returns of Hewlett–Packard (HPQ) and Procter & Gamble (PG) in the dji30ret dataset available in the rugarch package.

i) Compute the percentage returns.

ii) For each t in $(1000, 1001, \dots, 1999)$:

   a) Estimate the models of point 2iii) and 2iv) (the CCC and DCC models) using the observations from $t - 999$ until t.

   b) For both models compute ω_{t+1} using the functions of point 2i) and 2ii) using the one step ahead predictions of the conditional mean and covariance function.

   c) Store the four resulting ω_{t+1} in an array of proper dimensions.

iii) For each combination of model and investment strategy compute the series of realized portfolio returns and then compute:

   a) The average portfolio return.

   b) The portfolio standard deviation.

   c) The portfolio Sharpe ratio.

   d) The portfolio kurtosis and skewness coefficients.

Discuss your results.

iv) Report a graphical representation of the portfolio composition according to each combination of model and investment strategy. Discuss your results.