Exercise set #3

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The purpose of this exercise set is to get comfortable with the SV model, and its QML estimator obtained via the Kalman Filter.
First start by opening R, create a new script and save it to your Hard Drive with the name: "Exercise3.R".

(1): Theoretical part

Consider the SV model reported in slide 31 of Lecture 7 and the SV model reported in slide 18 of Lecture 6. Note that the two models are parameterized in a differen way. In the one of Lecture 7 the log volatility follows a zero mean autoregression and you have a parameter σ in the measurement equation. In the one of Lecture 6 you have that the volatility follows a first order autoregression with mean $\omega/(1-\phi)$. Find the mapping between the two parameterizations, i.e. find a way to represent the model in Lecture 6 as the model in Lecture 7, and viceversa.

(2): Computational part

1) Write a code to perform filtering in a Gaussian Linear State Space model using the recursions in slide 22 of Lecture 7.

2) Write a code to simulate from the SV model reported in slide 31 of Lecture 7.

3) Simulate $T = 1000$ observations from the SV model with $\sigma = 1$, $\phi = 0.9$, and $\sigma_\eta^2 = 0.25$. Set the seed to 123.

4) Write a function that maximize the quasi likelihood computed via the Kalman filter for the SV model of the previous point. The likelihood to maximize is defined in slide 23 of Lecture 7.

5) Estimate the model on the simulated data. Can you recover the true parameters?

(3) Real data

Download the time series of the S&P500 index from Yahoo finance from 2005-01-01 to 2018-01-01 and compute the percentage log returns. Replace the zero returns with their empirical mean. Estimate the SV model by QML. Use the mapping between the SV specification you estimated and the one of Lecture 6 in order to compare your estimates with those obtained from the GMM estimator in Exercise Set 2.