Exercise set #2

Leopoldo Catania

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The purpose of this exercise set is to get comfortable with the SV model, and its GMM estimator.
First start by opening R, create a new script and save it to your Hard Drive with the name: "Exercise2.R".

(1): Theoretical part

a) Consider the SV reported at slide 18 of Lecture 6. Show that:

$$
\begin{aligned}
E[|y_t|] &= (2/\pi)^{1/2} E[\sigma_t] \\
E[y_t^2] &= E[\sigma_t^2] \\
E[|y_t^3|] &= 2\sqrt{2/\pi} E[\sigma_t^3] \\
E[y_t^4] &= 3E[\sigma_t^4] \\
E[|y_t y_{t-j}|] &= (2/\pi) E[\sigma_t \sigma_{t-j}], \quad j = 1, \ldots, 10 \\
E[y_t^2 y_{t-j}^2] &= E[\sigma_t^2 \sigma_{t-j}^2], \quad j = 1, \ldots, 10 \\
E[\sigma_t^p] &= \exp\left(\frac{p\alpha}{2} + \frac{p^2\beta^2}{8}\right) \\
E[\sigma_t^2 \sigma_{t-j}^2] &= E[\sigma_t^2]^2 \exp\left(\phi^j \beta^2\right)
\end{aligned}
$$

b) Show that $E[\sigma_t^2 y_{t-j}] = 0$ when $j > 0$. What this result implies from an empirical point of view?

c) Derive the following densities $p(\sigma_t^2)$, $p(y_t)$, $p(\sigma_t^2, y_t)$ and $p(y_t|\sigma_t^2)$.

(2): Computational Part

Write a code to estimate the SV model of slide 18 of Lecture 6 using the 24 moments you have derived in the previous point.

Simulate $T = 1000$ observations from the SV model with $\omega = 0$, $\phi = 0.9$, and $\sigma_\eta^2 = 0.25$. Set the seed to 123. Estimate the model parameters with the GMM estimator you derived.

(4) Real data

Download the time series of the S&P500 index from Yahoo finance from 2005-01-01 to 2018-01-01 and compute the percentage log returns. Replace the zero returns with their empirical mean. Estimate the SV model by GMM.