Exercise set #6

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The purpose of this exercise set is to get comfortable with multivariate volatility modelling with the DCC model.

(1): Theoretical part

- We know that the log likelihood of a DCC model with Gaussian shocks nicely factorizes in two parts: one for the volatilities and one for the correlation, see lecture 10. However, we also know that the assumption of joint Gaussianity for financial returns is too restrictive in practice. Can you state three reasons why this is true? Use formal derivations to support your arguments.

- A possibility to depart from the Gaussian assumption is to assume that returns, conditionally on past observations, are multivariate Student's t distributed. The probability density function of a p–dimensional Student's t distribution with location vector $\boldsymbol{\mu}$, scale matrix $\boldsymbol{\Psi}$, and ν degrees of freedom is given by:

$$p(\mathbf{y};\boldsymbol{\mu},\boldsymbol{\Psi}) = \frac{\Gamma\left(\frac{\nu+p}{2}\right)}{\Gamma\left(\frac{\nu}{p}\right)\pi^{p/2}\nu^{p/2}|\boldsymbol{\Psi}|^{1/2}}\left(1+\frac{(\mathbf{y}-\boldsymbol{\mu})'\boldsymbol{\Psi}^{-1}(\mathbf{y}-\boldsymbol{\mu})}{\nu}\right)^{-\frac{\nu+p}{2}},$$

  where $\Gamma(\cdot)$ is the gamma function. The covariance matrix $\boldsymbol{\Sigma}$ is related to the scale by $\boldsymbol{\Sigma} = \boldsymbol{\Psi}\frac{\nu}{\nu-2}$ and can be factorized as $\mathbf{D}^{1/2}\mathbf{R}\mathbf{D}^{1/2}$, where $\mathbf{D}$ is a diagonal matrix with variances at its main diagonal and $\mathbf{R}$ is the correlation matrix.

  i) Can you derive a DCC model with multivariate Student's t shocks? Write down the log–likelihood of this model.

  ii) Show that the log–likelihood of this model does not factorize in two parts as in the Gaussian case.

  iii) How can we still estimate the Student's t DCC model in two steps? At which costs?

(2): Computational part

- Write a function to estimate a Gaussian DCC model with constant location. For univariate GARCH models you can use the `rugarch` package. The function should returns the total likelihood of the model, the filtered correlations and variances and estimated parameters.

- Write a function to estimate a multivatiate Student's t DCC model with constant locations where variances are estimated by QML in a first step and the degrees of freedom parameter is estimated in a second step. For univariate GARCH models you can use the `rugarch` package. The function should returns the total likelihood of the model, the filtered correlations and variances and estimated parameters.

(3): Estimation part

- Consider a couple of assets of your choice from the `dji30ret` dataset which is available in the `rugarch` package.

- Estimate a bivariate Gaussian DCC model.

- Estimate a bivariate Student's t DCC model.

- Compare the filtered correlation of the Gaussian and Student's t DCC models

- Compare the two DCC models using BIC. Which model is selected?