Exercise set # 5

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The purpose of this exercise set is to get comfortable with GAS models in R.
First start by opening R, create a new script and save it to your Hard Drive with the name: "Exercise5.R".

(1): Theoretical part

- Assume that $Y_t|\mathcal{F}_{t-1} \sim \mathcal{T}(0, \phi_t, \nu)$, where $\mathcal{T}$ indicates a Student's t distribution with mean 0, scale ϕ_t, and ν degrees of freedom. Note that the distribution is parameterized in terms of the scale, ϕ_t, and not the variance, σ_t^2. However, the following relation holds $\sigma_t^2 = \phi_t^2 \nu/(\nu-2)$ which exists for $\nu > 2$. The conditional density of $Y_t|\mathcal{F}_{t-1}$ is given by:

$$p(y_t|\mathcal{F}_{t-1}) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)\phi_t}\left[1+\frac{y_t^2}{\nu\phi_t^2}\right]^{-\frac{\nu+1}{2}}.$$

  Derive a GAS model with identity scaling $d = 0$ for the scale parameter ϕ_t. Use an exponential link function to ensure $\phi_t > 0$ for all t.

(2): Estimation

- Create a function to evaluate the likelihood of the GAS model of the previous exercise.

(3) Real data

1) Download the time series of the S&P500 index from Yahoo finance from 2005-01-01 to 2018-01-01 and compute the percentage log returns. Replace the zero returns with their empirical mean.

2) Estimate the GAS model you developed in point 1 using the code of point 2.

3) Compare the volatility implied by GAS and the ones from the GARCH and SV models you have estimated in the "Real Data" part of Exercise Set 4.