Exercise set #4

Leopoldo Catania

November 4, 2021

The purpose of this exercise set is to get comfortable with the Bootstrap filter for the SV model.
First start by opening R, create a new script and save it to your Hard Drive with the name: "Exercise4.R".

(1): Computational part

1) Write a function to perform filtering of the volatility in the Stochastic Volatility ($E[\exp(\alpha_t/2)|y_{1:t}]$) using the Bootstrap filter reported in slide 25 of Lecture 8.

2) Write a function like the one in point 1) but with resampling that occurs only when the Effective Sample Size (slide 26 of Lec. 8) is below the threshold gN. Let g be an argument of this function. Note that, when $g = 1$ resampling occurs at each iteration of the algorithm.

3) Simulate $T = 1000$ observations from the SV model reported in slide 12 of Lecture 8 with $\omega = 0$, $\phi = 0.9$, and $\tau = 0.5$. Set the seed to 123.

4) Perform filtering of the volatility using the Bootstrap filter you derived in point 2) using $N = 10000$ particles and $g = 1$. Repeat also for $g = 0.75$ and $g = 0.5$. Is the quality of the estimate affected? Also play with N and see how the number of particles affect the precision of the estimate.

5) Estimate the parameters ω, ϕ and τ using the QML estimator you derived in Exercise Set 3. Perform filtering via the Bootstrap filter using the estimated parameters. Is the precision of the filtered volatility affected?

(2) Real data

1) Download the time series of the S&P500 index from Yahoo finance from 2005-01-01 to 2018-01-01 and compute the percentage log returns. Replace the zero returns with their empirical mean.

2) Estimate the SV model by QML.

3) Perform filtering using the Bootstrap filter with $g = 1$ and $N = 10000$.

4) Estimate a GARCH(1,1) with Gaussian shocks using the rugarch package.

5) Compare in a figure the series of filtered volatility from the SV model and the one obtained by the GARCH model.