Package ‘CobbDouglas’

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Type Package

Title Efficiency Analysis using the Cobb-Douglas Production Frontier

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Description
Functionalities to estimate a Cobb-Douglas production frontier from sample data through constrained least squares. It is possible to set the desired returns to scale, to predict the maximum producible output or technical efficiency, and to obtain boostrap confidence intervals for parameters.

Depends R (>= 3.5.0), quadprog, TeachingDemos

Imports graphics, stats

License GPL-2

NeedsCompilation no

R topics documented:

`CobbDouglas-package` *Efficiency analysis using the Cobb-Douglas production frontier*

Description

Functionalities to estimate a Cobb-Douglas production frontier from sample data through constrained least squares. It is possible to set the desired returns to scale, to predict the maximum producible output or technical efficiency, and to obtain boostrap confidence intervals for parameters.

Details

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License:	GPL-2

The main functions of the package are:

- CobbDouglas, to estimate the frontier from sample data;
- predict.CobbDouglas, to predict the maximum producible output or technical efficiency;
- CobbDouglas_boot, to obtain bootstrap confidence intervals for parameters.

Author(s)

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References

C. W. Cobb and P. H. Douglas (1928). A Theory of Production. *American Economic Review*, 18: 139-165.

`CobbDouglas` *Estimation of a Cobb-Douglas production frontier*

Description

Estimation of a Cobb-Douglas production frontier from sample data.

Usage

```
CobbDouglas(y.name, x.names=NULL, data, beta.sum=NULL)
```

Arguments

`y.name`	The name of the output variable.
`x.names`	The names of the input variables. If `NULL` (the default), it is set equal to the name of the variables in `data` besides `y.name`.
`data`	A data.frame containing the output and the input variables.
`beta.sum`	Constraint on the sum of beta parameters. If `NULL` (the default), beta parameters are freely estimated (subjected only to positive constraint).

Details

Consider a sample of n production units, for which the quantity of the output Y and of H input variables $X_1, \ldots, X_H$ is measured. Let y_i be the quantity of the output for unit i, and x_{hi} be the quantity of the h-th input for unit i. A Cobb-Douglas production frontier is defined as:

$$y_i^* = \tau \prod_{h=1}^{H} x_{hi}^{\beta_h}$$

where y_i^* is the maximum producible output for unit i, τ is a parameter representing the total factor productivity for a technically efficient unit, and β_h $(h = 1, \ldots, H)$ is a parameter representing the elasticity of the output with respect to the h-th input.

Constant returns to scale holds if $\sum_{h=1}^{H} \beta_h = 1$ (obtained setting the argument `beta.sum` to value 1). Instead, $\sum_{h=1}^{H} \beta_h < 1$ implies decreasing returns to scale, while $\sum_{h=1}^{H} \beta_h > 1$ implies increasing returns to scale. For $i = 1, \ldots, n$, quantities y_i^* are denoted as *fitted values*, while quantities $e_i = y_i - y_i^*$ are denoted as *residuals*.

Estimation of the Cobb-Douglas production frontier is performed through constrained least squares on the logarithmic scale:

$$(\hat{\tau}, \hat{\beta}_1, \ldots, \hat{\beta}_H) = \texttt{argmin}_{\tau,\beta_1,\ldots,\beta_H} \sum_{i=1}^{n} (\log y_i - \log y_i^*)^2$$

where:

$$\log y_i^* = \log \tau + \sum_{h=1}^{H} \beta_h \log x_{hi}$$

subjected to constraints:

$$\log y_i^* \geq \log y_i \qquad i = 1, \ldots, n$$

$$\beta_h \geq 0 \qquad h = 1, \ldots, H$$

S3 methods available for class `CobbDouglas` are:

- `print`: to get essential information.
- `summary`: to get summaries of estimation.
- `plot`: to display the scatterplot with the estimated frontier (only for frontiers with a single input).
- `predict`: to predict the maximum producible output or technical efficiency. See predict.CobbDouglas.

Also, the method CobbDouglas_boot is available to approximate confidence intervals for parameters.

Value

An object of class `CobbDouglas`, that is a list with the following components:

`parameters`	Parameter estimates.
`efficiency`	Technical efficiencies of the sample units.
`fitted`	Fitted values on both logarithmic and original scale.
`residuals`	Residuals on both logarithmic and original scale.
`beta.sum`	Value passed to argument `beta.sum`.
`y.name`	Value passed to argument `y.name`.
`x.names`	Value passed to argument `x.names`.
`data`	Data used in the estimation.

Author(s)

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References

C. W. Cobb and P. H. Douglas (1928). A Theory of Production. *American Economic Review*, 18: 139-165.

See Also

predict.CobbDouglas; CobbDouglas_boot.

Examples

```
data(prod)


### 1 input variable: 'labour'

m1 <- CobbDouglas("output", "labour", data=prod)
summary(m1)

# plot the estimated frontier
plot(m1, cex.axis=1.1, cex.lab=1.2)

# technical efficiencies
m1_eff <- m1$efficiency
## NOT RUN:
# m1_eff

# efficient units
m1_eff[which(m1_eff$y.side==1),]


### 1 input variable: 'labour' + constraint on beta

# beta=1 (constant returns to scale)
m1c <- CobbDouglas("output", "labour", data=prod, beta.sum=1)
m1c$parameters
m1c$efficiency[which(m1c$efficiency$y.side==1),]
plot(m1c, cex.axis=1.1, cex.lab=1.2, main="beta = 1", cex.main=1.6)

# beta=1.25 (increasing returns to scale)
m1i <- CobbDouglas("output", "labour", data=prod, beta.sum=1.25)
m1i$parameters
m1i$efficiency[which(m1i$efficiency$y.side==1),]
plot(m1i, cex.axis=1.1, cex.lab=1.2, main="beta = 1.25", cex.main=1.6)

# beta=0.3 (decreasing returns to scale)
m1d <- CobbDouglas("output", "labour", data=prod, beta.sum=0.3)
m1d$parameters
m1d$efficiency[which(m1d$efficiency$y.side==1),]
plot(m1d, cex.axis=1.1, cex.lab=1.2, main="beta = 0.3", cex.main=1.6)


### 2 input variables: 'labour' and 'capital'

# no constraints on the sum of beta parameters
m2 <- CobbDouglas("output", c("labour","capital"), data=prod)
summary(m2)
```

```
m2$efficiency[which(m2$efficiency$y.side==1),]

# beta.sum=1 (constant returns to scale)
m2c <- CobbDouglas("output", c("labour","capital"), data=prod, beta.sum=1)
summary(m2c)
m2c$efficiency[which(m2c$efficiency$y.side==1),]

# beta.sum=0.7 (decreasing returns to scale)
m2d <- CobbDouglas("output", c("labour","capital"), data=prod, beta.sum=0.7)
summary(m2d)
m2d$efficiency[which(m2d$efficiency$y.side==1),]
```

CobbDouglas_boot *Bootstrap confidence intervals for a Cobb-Douglas frontier*

Description

Boostrap resampling to approximate confidence intervals for the parameters of a Cobb-Douglas production frontier.

Usage

```
CobbDouglas_boot(x, nboot=500, conf=0.95)
```

Arguments

x	An object of class CobbDouglas.
nboot	The number of bootstrap replications. It must be at least 50.
conf	The confidence level. Default is 0.95.

Details

Non-parameteric bootstrap resampling (Efron and Tibshirani, 1993) is performed. Confidence intervals are computed from the bootstrap simulation using Highest Posterior Density (HPD) intervals (Box and Tiao, 1992).

Value

A data.frame containing the estimated value and the bootstrap confidence interval at level conf for each parameter.

Author(s)

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References

G. Box and G. Tiao (1992). Bayesian Inference in Statistical Analysis. John Wiley \& Sons, New York (US-NY).

B. Efron and R. Tibshirani (1993). An Introduction to the Bootstrap. Chapman \& Hall/CRC, Boca Raton (US-FL).

See Also

CobbDouglas.

Examples

```
data(prod)

m2 <- CobbDouglas("output", c("labour","capital"), data=prod)
set.seed(123)
CobbDouglas_boot(m2, nboot=150)

m2c <- CobbDouglas("output", c("labour","capital"), data=prod, beta.sum=1)
set.seed(123)
CobbDouglas_boot(m2c, nboot=150)

m2d <- CobbDouglas("output", c("labour","capital"), data=prod, beta.sum=0.7)
set.seed(123)
CobbDouglas_boot(m2d, nboot=150)
```

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predict.CobbDouglas *Prediction using a Cobb-Douglas production frontier*

Description

Prediction of the maximum producible output or of technical efficiency using a Cobb-Douglas production frontier.

Usage

```
## S3 method for class 'CobbDouglas'
predict(object, newdata=NULL, type="output", ...)
```

Arguments

`object`	An object of class `CobbDouglas`.
`newdata`	A data.frame in which to look for variables with which to predict the maximum producible output (if `type="output"`) or technical efficiency (if `type="efficiency"`). If `NULL` (the default), fitted values or technical efficiencies of the sample units are returned.
`type`	The type of prediction: `"output"` (maximum producible output)) or `"efficiency"` (technical efficiency). It can be abbreviated.
`...`	Further arguments passed to the generic `predict` method.

Value

An object of class `data.frame`.

Author(s)

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References

C. W. Cobb and P. H. Douglas (1928). A Theory of Production. *American Economic Review*, 18: 139-165.

See Also

CobbDouglas.

Examples

```
data(prod)
m2 <- CobbDouglas("output", c("labour","capital"), data=prod)

# prediction of the maximum producible output
predict(m2, newdata=data.frame(labour=20,capital=5))

# prediction of technical efficiency
predict(m2, newdata=data.frame(output=15,labour=20,capital=5), type="eff")
```

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prod *Simulated production data*

Description

Simulated data for production processes with one output variable and two input variables.

Usage

```
data(prod)
```

Format

A data.frame with a total of 60 observations on the following 3 variables:

`output` The amount of output produced.

`capital` The amount of capital utilized.

`labour` The amount of labour employed.

References

T. Coelli (1996). A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation. CEPA Working Paper 96/08, University of New England.

`rice` *Rice production data*

Description

Data on fictitious rice production processes.

Usage

```
data(rice)
```

Format

A data.frame with a total of 100 observations on the following 5 variables:

`rice` The amount of rice produced (tonnes).

`land` The amount of land planted (hectares).

`labour` The amount of labour employed (man-days).

`fertil` The amount of fertilizer used (kilograms).

`machinery` The amount of machinery utilized (index, firm 41=100).

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