Suggested Solution to the Empirical Part of Assignment 1

1 Empirical Part

Note: Please take a look at R script for this solution. The results and the figures are appended at the end of this document.

1. (a) See the plot of log of real GDP in the attachment. The plot shows that in the long-run real GDP is increasing. The level though declines for some time which are the recessionary periods in the U.S. For example, we see a big decline during the recent financial crisis. Similarly for other recessions, we observe a drop in the log of level of real GDP. The ACF and PACF of the log of real GDP has the classic AR property. The PACF is significant until lag 1 and the ACF is declining. It seems like this time-series can be represented by AR(1) process. One problem, however, is that the ACF is declining not exponentially which should be the case for a stationary time-series process. The very slow decline of the ACF suggests a process that may be closer to a non-stationary process.

   (b) The estimated model has the following representation:

   $$\text{lrgdp}_t = 8.7260 + 0.999 * lrgdp_{t-1} + e_t$$
   $$\qquad\qquad (0.00) \qquad (0.00)$$

   The p-values are in parentheses. The estimated AR coefficient is 0.99 which is very close to one. Therefore, our intuition in part (b) is correct, that this series looks like a non-stationary process. Since we still don't know how to test for non-stationarity for a series, we still can't explicitly test for it.

   (c) We detrend the series, by running a regression of log of real GDP on a constant and time trend. The detrended series which is the residual from the linear time trend model

is shown in Figure 3 along with the ACF and PACF. The plot of the detrended series has the mean zero since by definition it is the residual from the linear time trend model. The graph shows high degree of persistence implying that if there is a positive shock and it goes up this period, it tends to stay up for a long period of time and vice-versa. The ACF and PACF have classic features of an AR model. The PACF is significant till lag 2 and ACF is declining though not exponentially. This implies that this may be represented by AR(2) model. The slow decline of ACF implies high AR coefficient. Also, it should be kept in mind that ACF and PACF provide us a guess about the dynamics of the time-series process. For model selection, we need to look at different model selection criteria.

(d) The first difference of the log of GDP is rate of growth of real GDP. For this variable, ACF is significant until lag 2 and PACF is significant until lag 1. It implies that it may be represented by an ARMA(1,2) model.

(e) For detrended GDP, AIC and BIC values provide us contradictory results. The AIC and the BIC values from this model are

values for detrended GDP

	MA			
AR	0	1	2	3
0	-746.92	-1071.93	-1320.61	-1454.06
1	-1762.49	-1791.30	-1792.01	-1803.58
2	-1802.33	-1802.30	-1803.24	-1798.18
3	-1803.36	-1802.54	-1803.39	-1794.39

values for detrended GDP

	MA			
AR	0	1	2	3
0	-739.69	-1061.09	-1306.16	-1436.01
1	-1751.66	-1776.85	-1773.95	-1781.90
2	-1787.88	-1784.23	-1781.56	-1772.89
3	-1785.29	-1780.86	-1778.09	-1765.48

AIC values show that for detrended model ARMA(3,1) has the lowest value, though the difference with other model is not very large. BIC on the other hand suggests AR(2)

model as it has the lowest BIC value. This is consistent with the idea that BIC tends to choose model that are parsimonious.

values for GDP growth

	MA			
AR	0	1	2	3
0	-1761.40	-1789.70	-1800.71	-1800.80
1	-1799.83	-1799.31	-1800.38	-1798.83
2	-1800.19	-1799.58	-1801.48	-1797.83
3	-1800.70	-1800.33	-1804.84	-1806.00

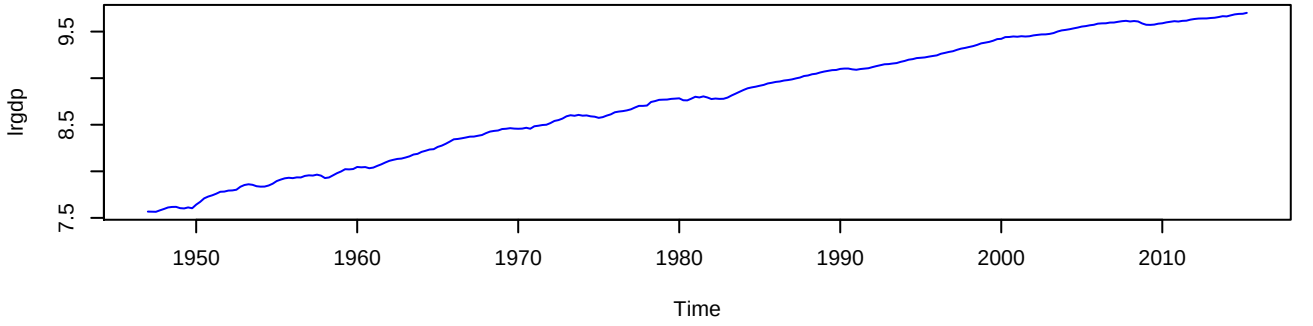
values for GDP growth

	MA			
AR	0	1	2	3
0	-1754.19	-1778.87	-1786.27	-1782.75
1	-1789.01	-1784.87	-1782.34	-1777.17
2	-1785.76	-1781.54	-1779.82	-1772.56
3	-1782.66	-1778.67	-1779.57	-1777.13

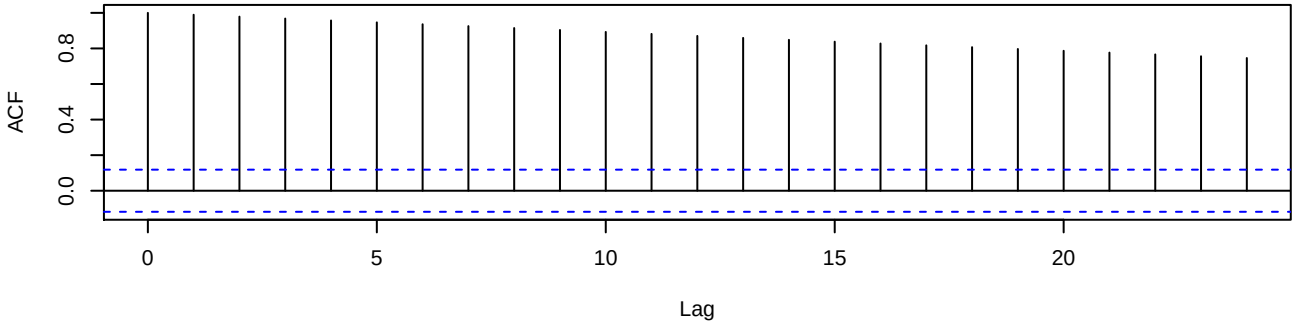
For first difference of log of GDP or the rate of growth of real GDP, we again observe contradictory results for AIC and BIC model selection criteria. AIC suggests ARMA(3,3), whereas BIC suggests AR(1) model. To summarize, ACF and PACF are good tools to observe and perform preliminary analysis, but are not the substitutes for different model selection criteria.

(f) The ACF and the PACF for the first sub-sample (1947:02-1981:04) are shown in Figure 3, and Figure 4 shows the corresponding plots for the second sub-sample (1982:01-2015:02). The ACF and the PACF for the first sample suggests either AR(1) or ARMA (1,2) model. For the second sub-sample, the ACF and the PACF plot suggests AR(2) model, as the ACF is declining exponentially and the PACF is significant until lag 2. If we use auto.arima function in R, the it selects AR(1) model for the first sample period and ARIMA(0,1,1) for the second sample period. ARIMA(0,1,1) model suggests MA(1) model in the first difference of growth rate of GDP.

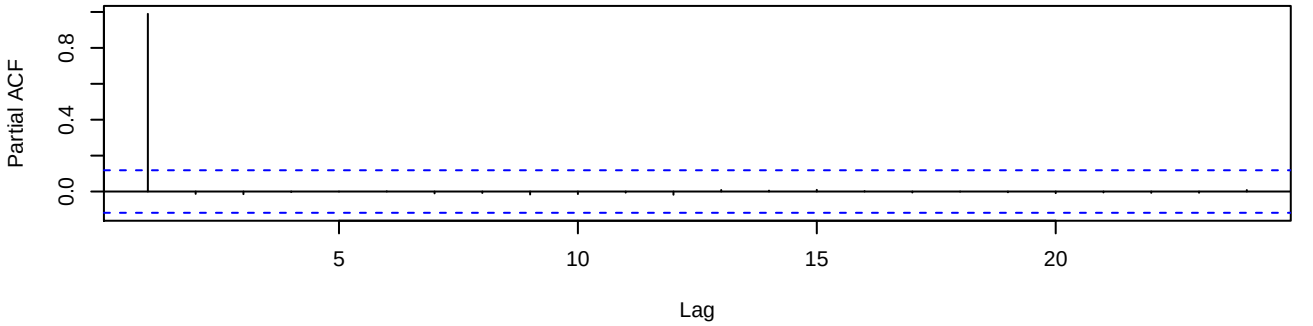
(g) We can see that the volatility of GDP growth has declined in the second period. We also find that the dynamics of the DGP has changed over time.

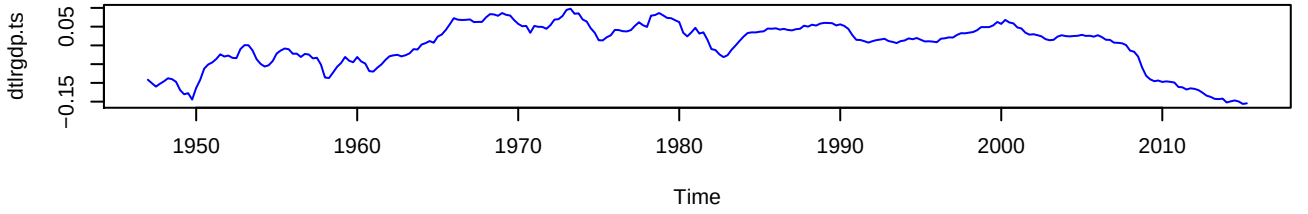


acf of level of real GDP

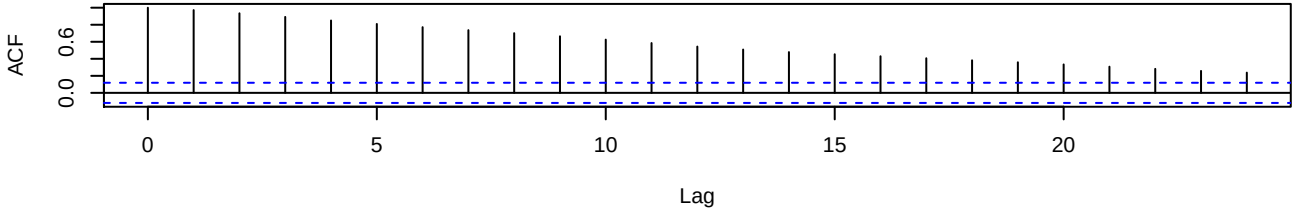


pacf of level of real GDP

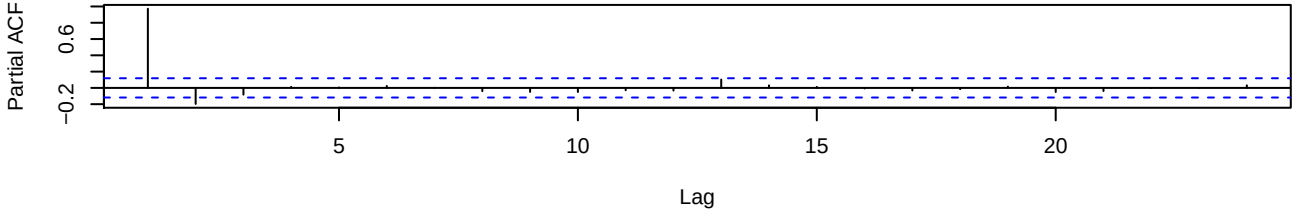


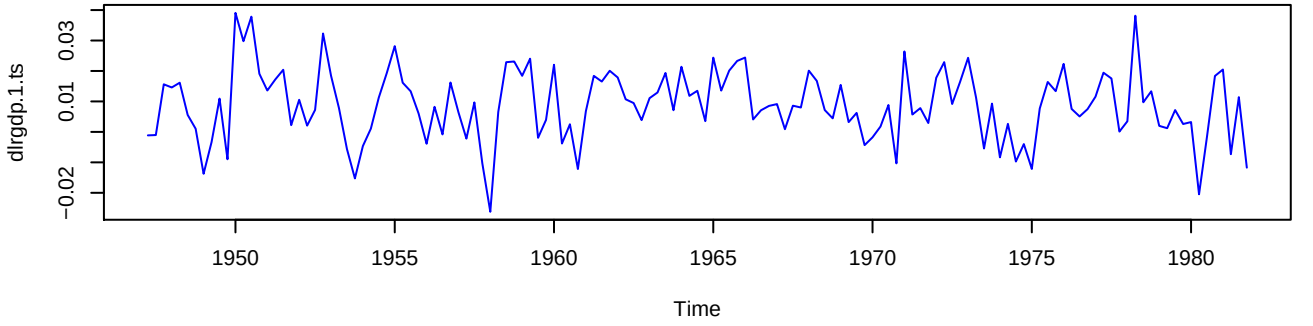


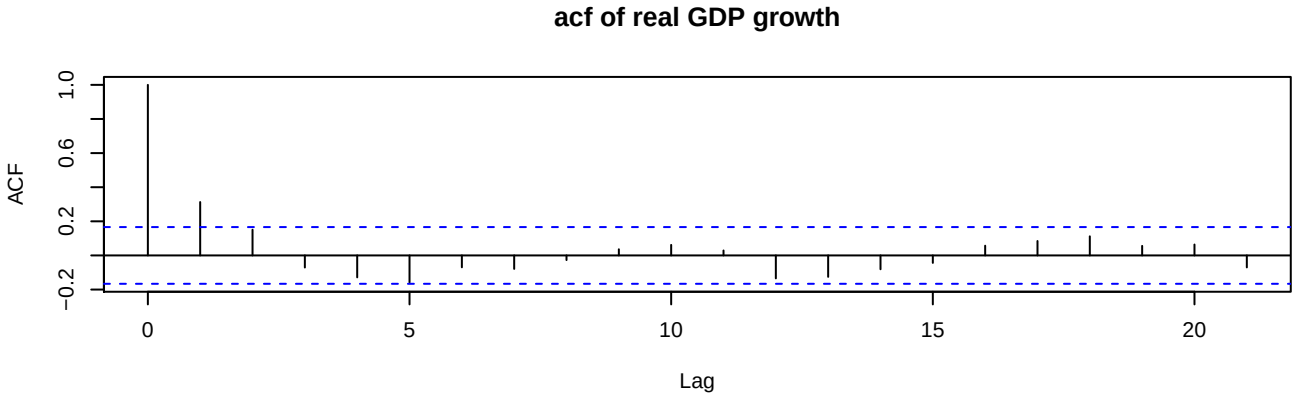
acf of detrended real GDP

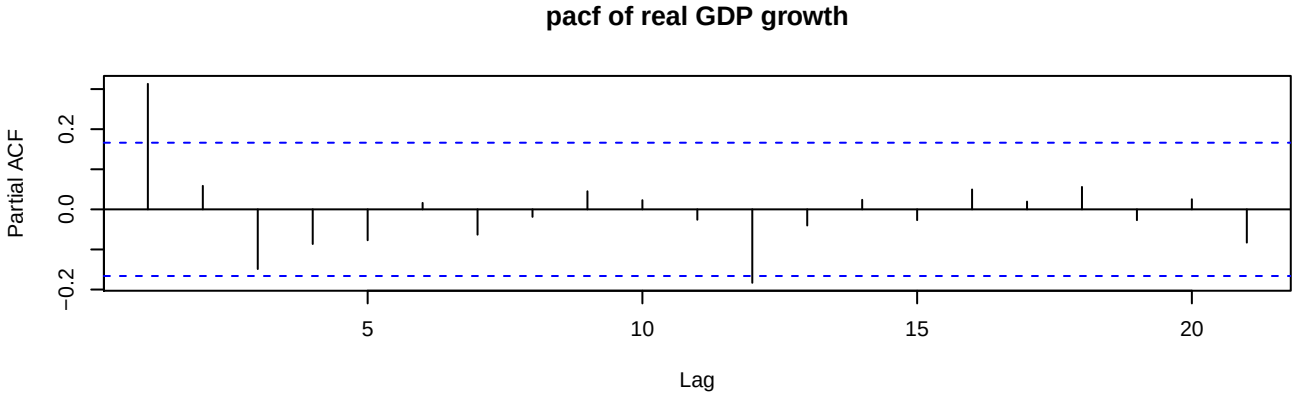


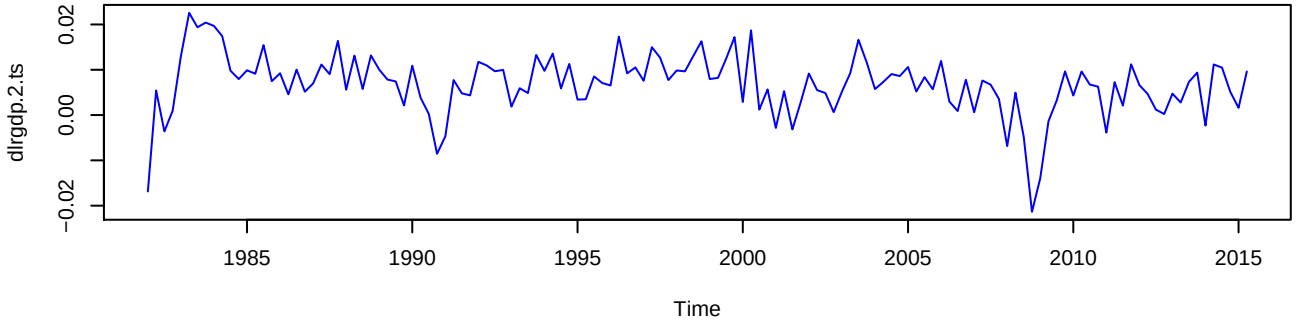
pacf of detrended real GDP



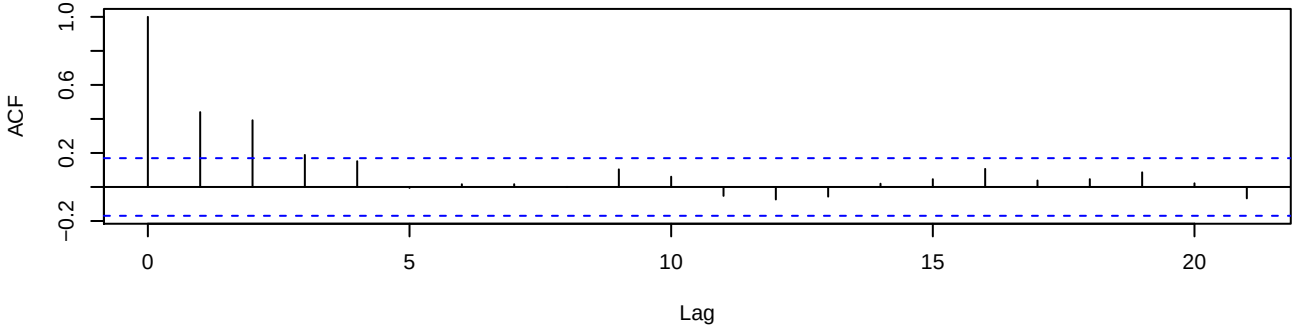

dlrgdp.1.ts
−0.02
0.01
0.03
1950
1955
1960
1965
1970
1975
1980
Time


acf of real GDP growth
ACF
−0.2
0.2
0.6
1.0
0
5
10
15
20
Lag


pacf of real GDP growth
Partial ACF
−0.2
0.0
0.2
5
10
15
20
Lag



acf of real GDP growth



pacf of real GDP growth