Housing and Credit Cycles

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1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$

$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = g_{yt} + \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$

$$\tau_{ht} = g_{ht} + \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

$$g_{yt} = g_{yt-1} + w_{yt}, \qquad w_{yt} \sim iidN(0, \sigma^2_{wy}) \tag{5}$$

$$g_{ht} = g_{ht-1} + w_{ht}, \qquad w_{ht} \sim iidN(0, \sigma^2_{wh}) \tag{6}$$

$$\tag{7}$$

Cycles:

$$c_{yt} = \phi^y_1 c_{yt-1} + \phi^y_x c_{ht-1} + \varepsilon_{yt}, \qquad \varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y}) \tag{8}$$

$$c_{ht} = \phi^h_1 c_{ht-1} + \phi^h_x c_{yt-1} + \varepsilon_{ht}, \qquad \varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h}) \tag{9}$$

State-Space Model

Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{10}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ g_{yt} \\ c_{yt} \\ \tau_{ht} \\ g_{ht} \\ c_{ht} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1^y & 0 & 0 & \phi_x^y \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \phi_x^h & 0 & 0 & \phi_1^h \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ g_{yt-1} \\ c_{yt-1} \\ \tau_{ht-1} \\ g_{ht-1} \\ c_{ht-1} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ w_{yt} \\ \varepsilon_{yt} \\ \eta_{ht} \\ w_{ht} \\ \varepsilon_{ht} \end{bmatrix} \tag{11}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma_{\eta y}^2 & 0 & 0 & \sigma_{\eta y \eta h} & 0 & 0 \\ 0 & \sigma_{wy}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\varepsilon y}^2 & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} \\ \sigma_{\eta y \eta h} & 0 & 0 & \sigma_{\eta h}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{wh}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma_{\varepsilon h}^2 \end{bmatrix} \tag{12}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{13}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ g_{yt} \\ c_{yt} \\ \tau_{ht} \\ g_{ht} \\ c_{ht} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

Following Kim & Nelson Chap 2, I set up the constraints on the autoregressive parameters to imply stationary $0 < \phi_1^y < 1$ and $-1 < \phi_x^y < 1$ as follow:

$$\phi_1^y = \frac{1}{1 + exp((\phi_1^y)^{-1})}$$

$$\phi_x^y = \frac{\phi_x^y}{1 + |\phi_x^y|}$$

The same applies for ϕ_1^h & ϕ_x^h.

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite matrix, in order to ensure feasible maximum likelihood estimation process.

4 Regression results

In this following section, I will apply the UC model to data from 6 countries: US, UK, Germany, France, Japan and South Korea.

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. The following estimates are selected in the manner that they would look the most stable. Perhaps a more optimal constraint on the autoregressive parameters would solve this issue.

Table 1: Correlated UC model Estimates: US data

Description	Estimate	Standard Error
ϕ_1^y	0.9316	0.0256
ϕ_x^y	0.1329	0.0486
ϕ_1^h	0.9237	0.0700
ϕ_x^h	0.0082	0.0568
σ_{ny}^2	4.0119×10^{-7}	0.0003
σ_{wy}^2	0.0043	0.0328
σ_{ey}^2	1.3677	0.1081
σ_{nh}^2	2.4002	0.2011
σ_{wh}^2	8.1835×10^{-7}	0.0568
σ_{eh}^2	5.3833	0.5091
σ_{nhnc}	-0.9716	0.0875
σ_{ehec}	-0.9388	0.0227
Log-likelihood value	-816.3580	0

Table 2: Correlated UC model Estimates: UK data

Description	Estimate	Standard Error
ϕ_1^y	0.8415	0.1157
ϕ_x^y	-0.9462	0.0152
ϕ_1^h	0.4093	0.3104
ϕ_x^h	-0.9463	0.0510
σ_{ny}^2	5.1252×10^{-7}	0.0016
σ_{wy}^2	2.1881×10^{-5}	0.0315
σ_{ey}^2	30.0763	10.0054
σ_{nh}^2	31.4934	11.4130
σ_{wh}^2	54.8623	19.0631
σ_{eh}^2	55.7822	16.7065
σ_{nhnc}	0.2034	0.1690
σ_{ehec}	-0.0528	0.1400
Log-likelihood value	-2122.9943	0

Table 3: Correlated UC model Estimates: Germany data

Description	Estimate	Standard Error
ϕ_1^y	0.5100	99.7611
ϕ_x^y	-0.3015	888.5655
ϕ_1^h	0.4470	320.4277
ϕ_x^h	-0.7364	1351.6680
σ_{ny}^2	0.9221	0.0522
σ_{wy}^2	0.1677	0.0428
σ_{ey}^2	2.7778×10^{-5}	0.0545
σ_{nh}^2	0.9664	0.0493
σ_{wh}^2	4.7600×10^{-9}	0.0018
σ_{eh}^2	9.9225×10^{-6}	0.0893
σ_{nhnc}	0.1109	0.0884
σ_{ehec}	-0.9995	8942.7588
Log-likelihood value	-547.3121	0

Table 4: Correlated UC model Estimates: France data

Description	Estimate	Standard Error
ϕ_1^y	0.5140	0.0076
ϕ_x^y	0.7498	0.0337
ϕ_1^h	0.5175	0.0222
ϕ_x^h	0.2853	0.0540
σ_{ny}^2	0.3637	0.0541
σ_{wy}^2	0.0328	0.1704
σ_{ey}^2	1.3188×10^{-7}	0.0003
σ_{nh}^2	1.6963	0.0645
σ_{wh}^2	0.7743	0.1703
σ_{eh}^2	13.2592	2.0620
σ_{nhnc}	-1.0000	0.0002
σ_{ehec}	-0.4115	0.0482
Log-likelihood value	-937.5329	0

Table 5: Correlated UC model Estimates: Japan data

Description	Estimate	Standard Error
ϕ_1^y	0.4798	0.0086
ϕ_x^y	0.3416	0.0020
ϕ_1^h	0.4739	0.0115
ϕ_x^h	0.8365	0.0165
σ_{ny}^2	0.8027	0.2201
σ_{wy}^2	1.3585×10^{-8}	0.0024
σ_{ey}^2	1.3627	0.1163
σ_{nh}^2	2.3331	0.3785
σ_{wh}^2	0.5816	0.1295
σ_{eh}^2	1.6346	0.2229
σ_{nhnc}	-1.0000	1.4650×10^{-13}
σ_{ehec}	0.7533	0.1002
Log-likelihood value	-862.6697	0

Table 6: Correlated UC model Estimates: Korea data

Description	Estimate	Standard Error
ϕ_1^y	0.5210	0.0101
ϕ_x^y	0.1118	0.0089
ϕ_1^h	0.4805	0.0197
ϕ_x^h	-0.9353	0.0796
σ_{ny}^2	0.1014	0.0341
σ_{wy}^2	0.6388	0.3411
σ_{ey}^2	0.0227	0.0151
σ_{nh}^2	19.6837	0.9131
σ_{wh}^2	0.0002	0.1603
σ_{eh}^2	17.5607	0.8514
σ_{nhnc}	0.9577	0.0574
σ_{ehec}	-0.9949	0.0133
Log-likelihood value	-1190.6402	0

5 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

For example, the model for US data shows that there is a positive relationship between a one period lag in short term house price and house hold credit. Also for the UK data, there is a positive relationship between a one period lag in short term credit and house price.

Further development for this paper should include more optimal constraints on parameters to ensure stability.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.

Figure 1: Appendix: US Credit components

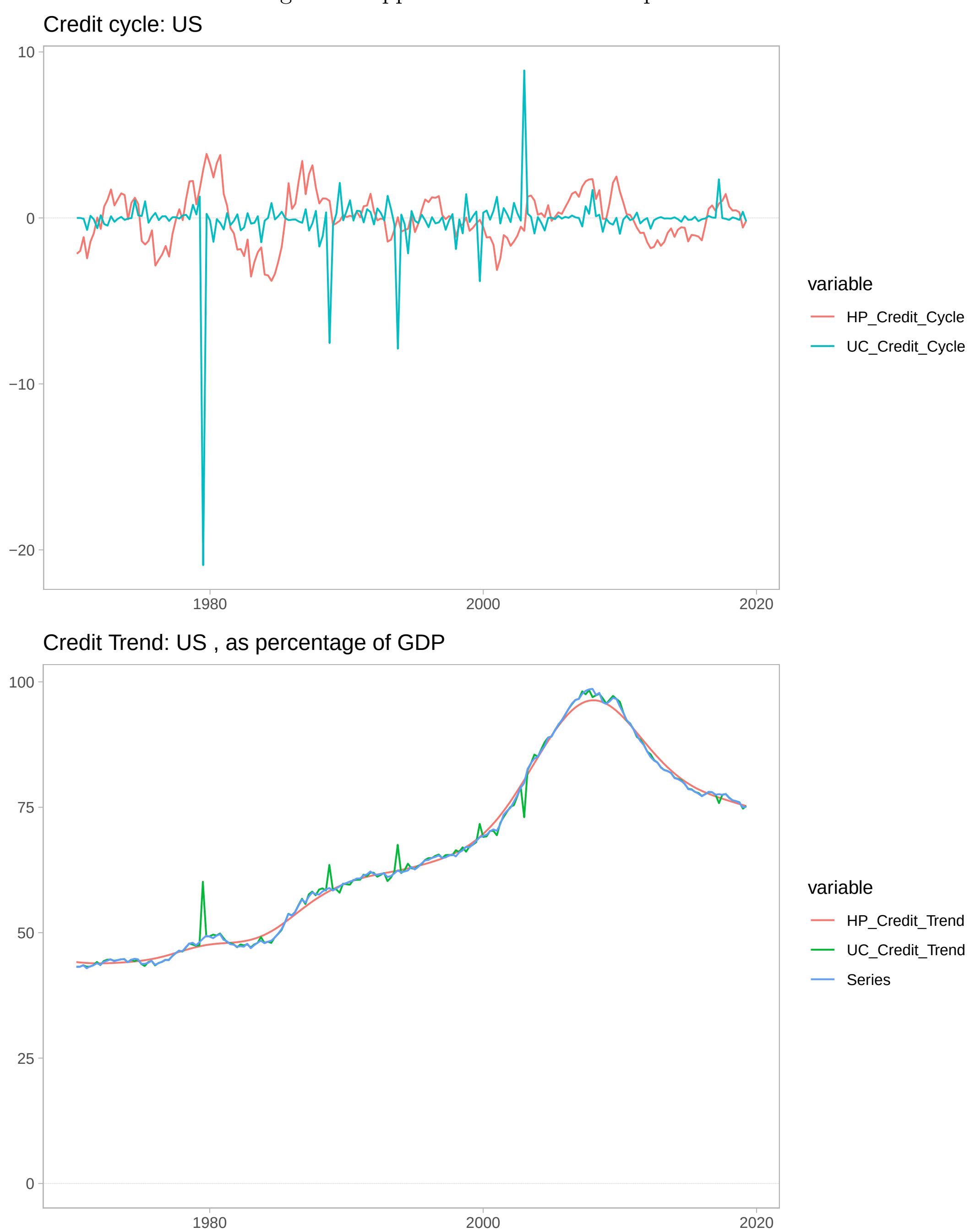


Figure 2: US Housing Price components

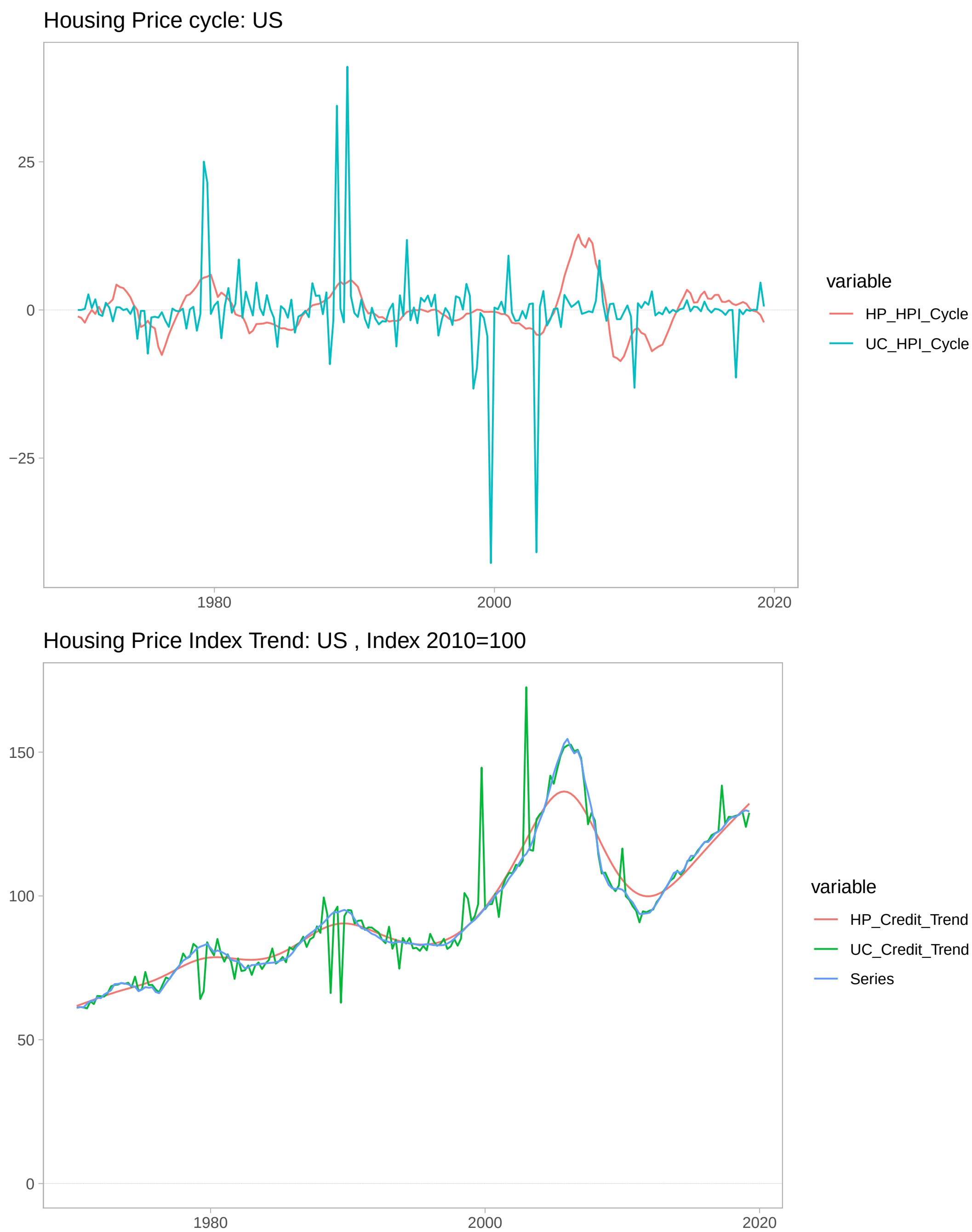


Figure 3: UK Credit components

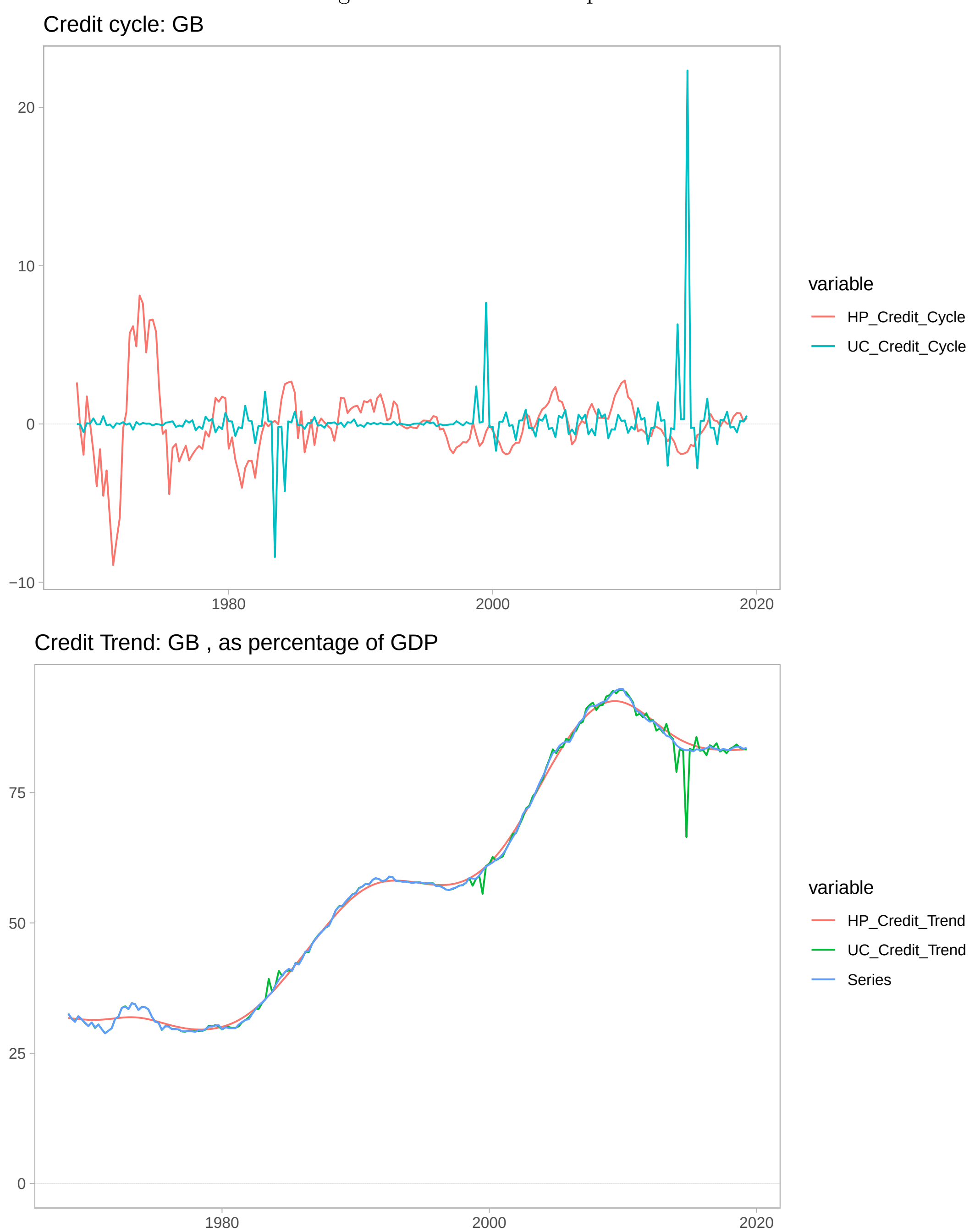
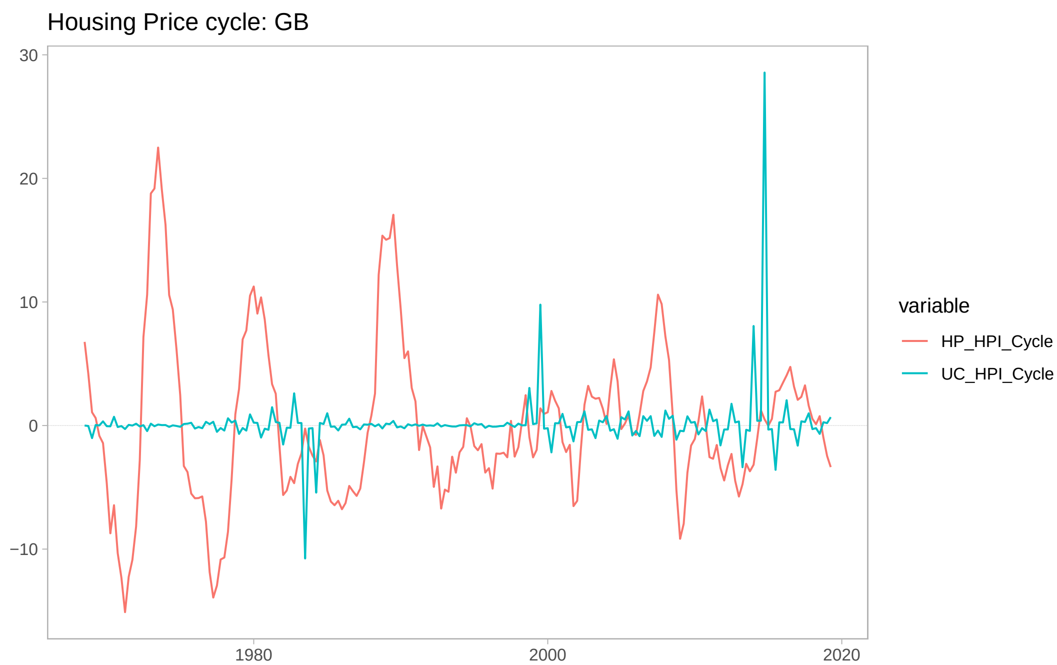


Figure 4: UK Housing Price components



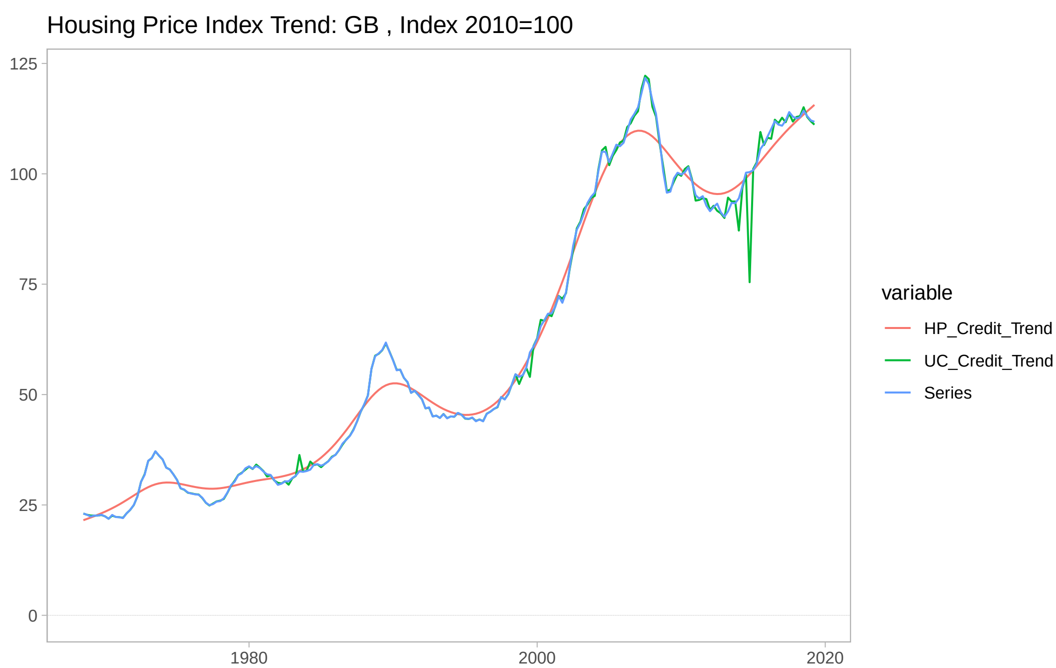


Figure 5: Germany Credit components

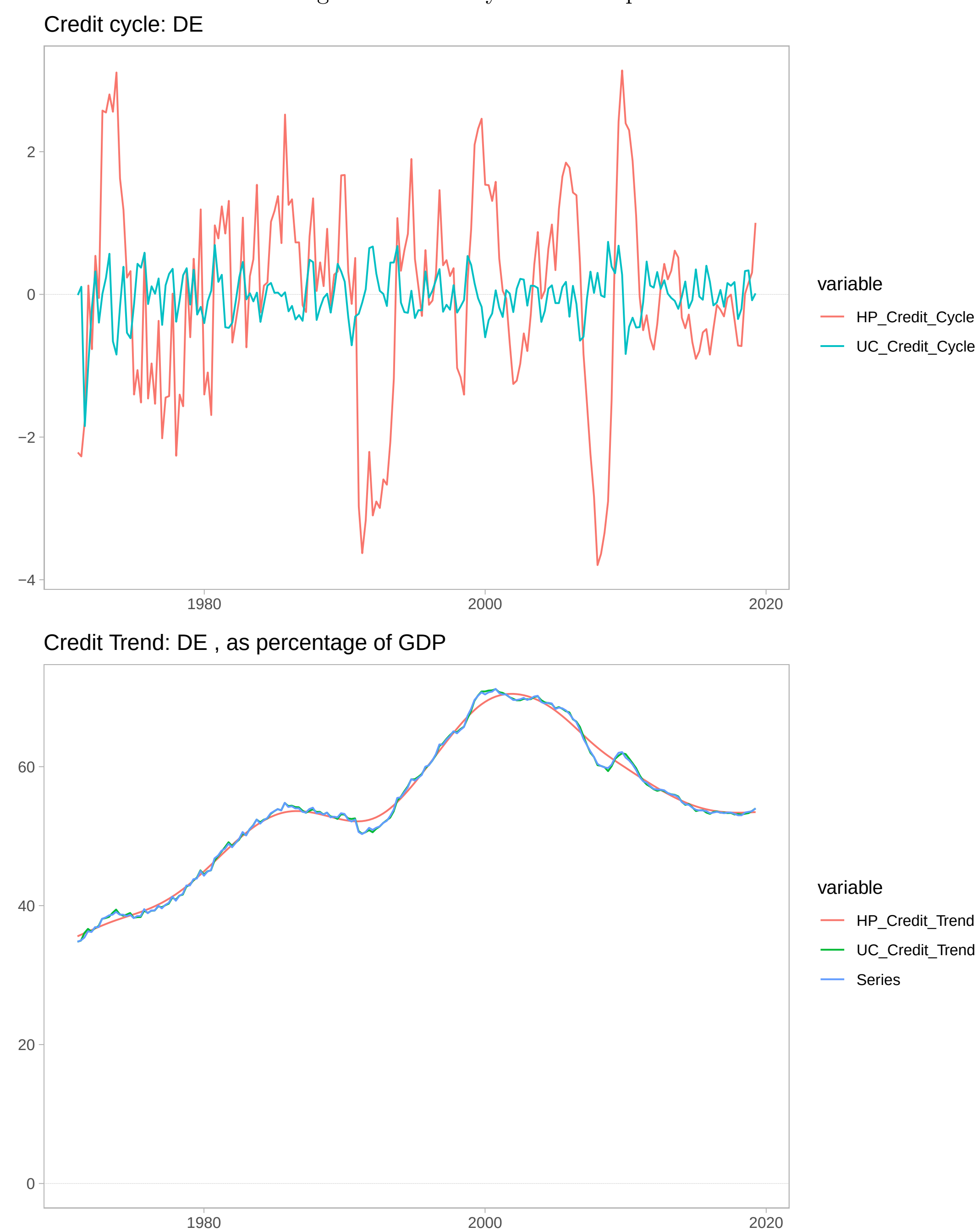


Figure 6: Germany Housing Price components

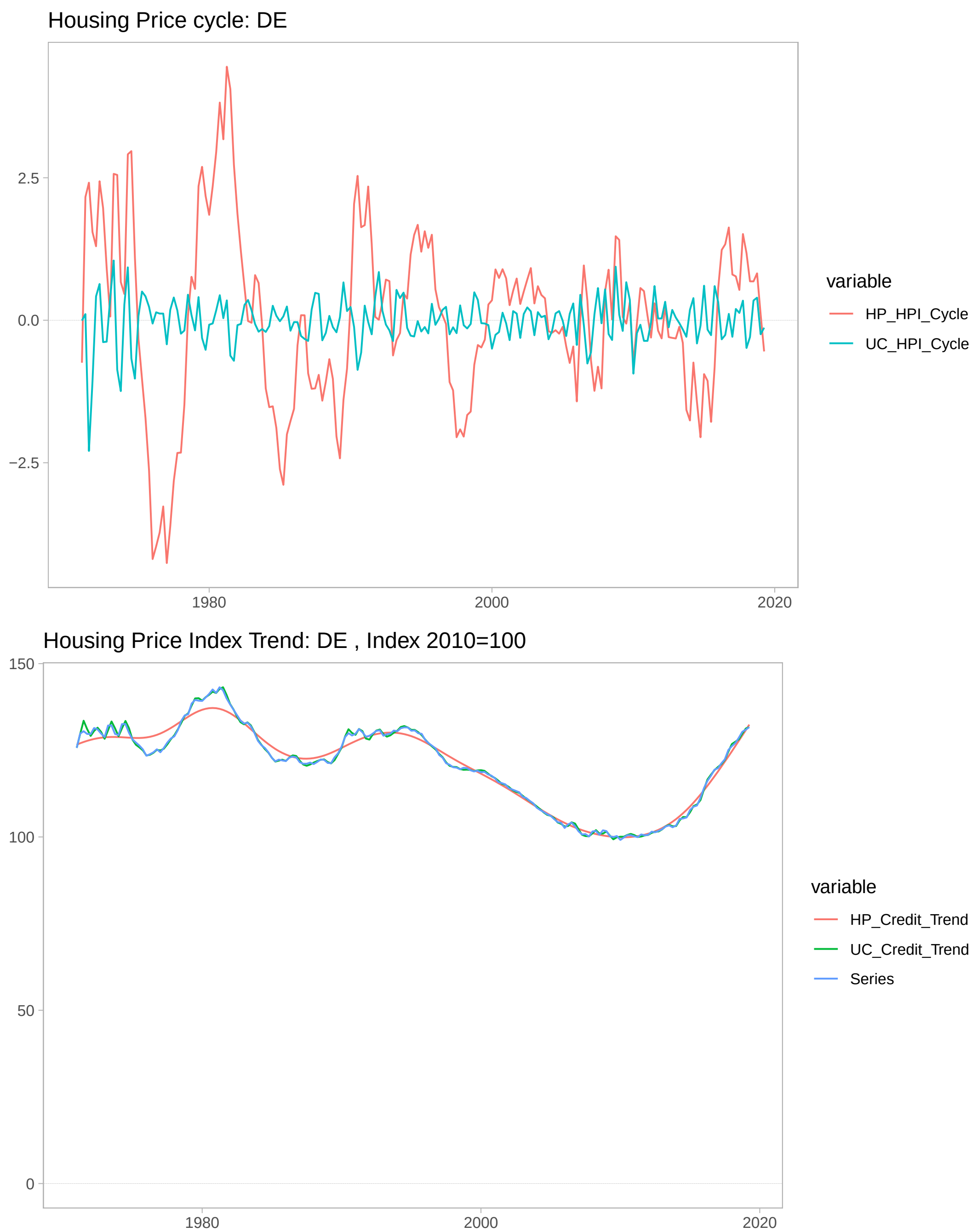
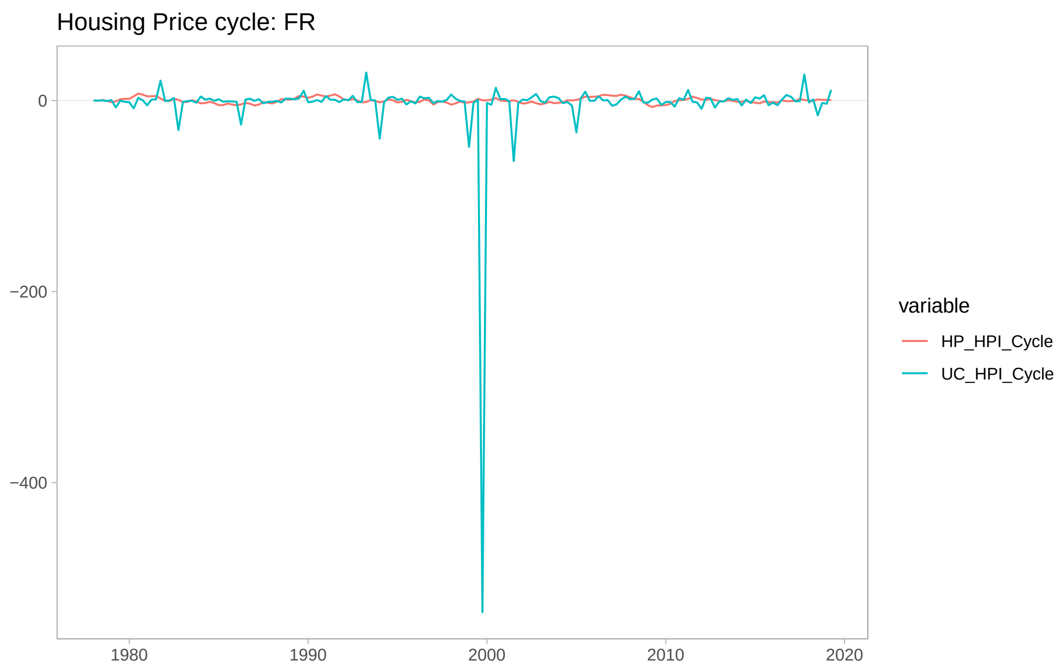


Figure 7: France Credit components

Figure 8: France Housing Price components



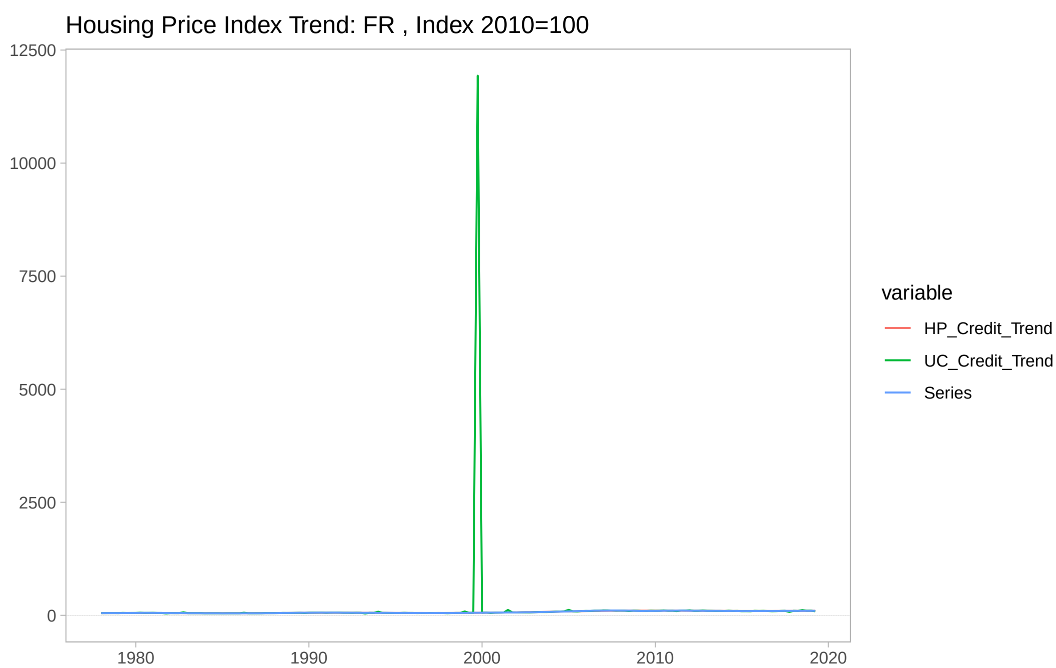


Figure 9: Japan Credit components

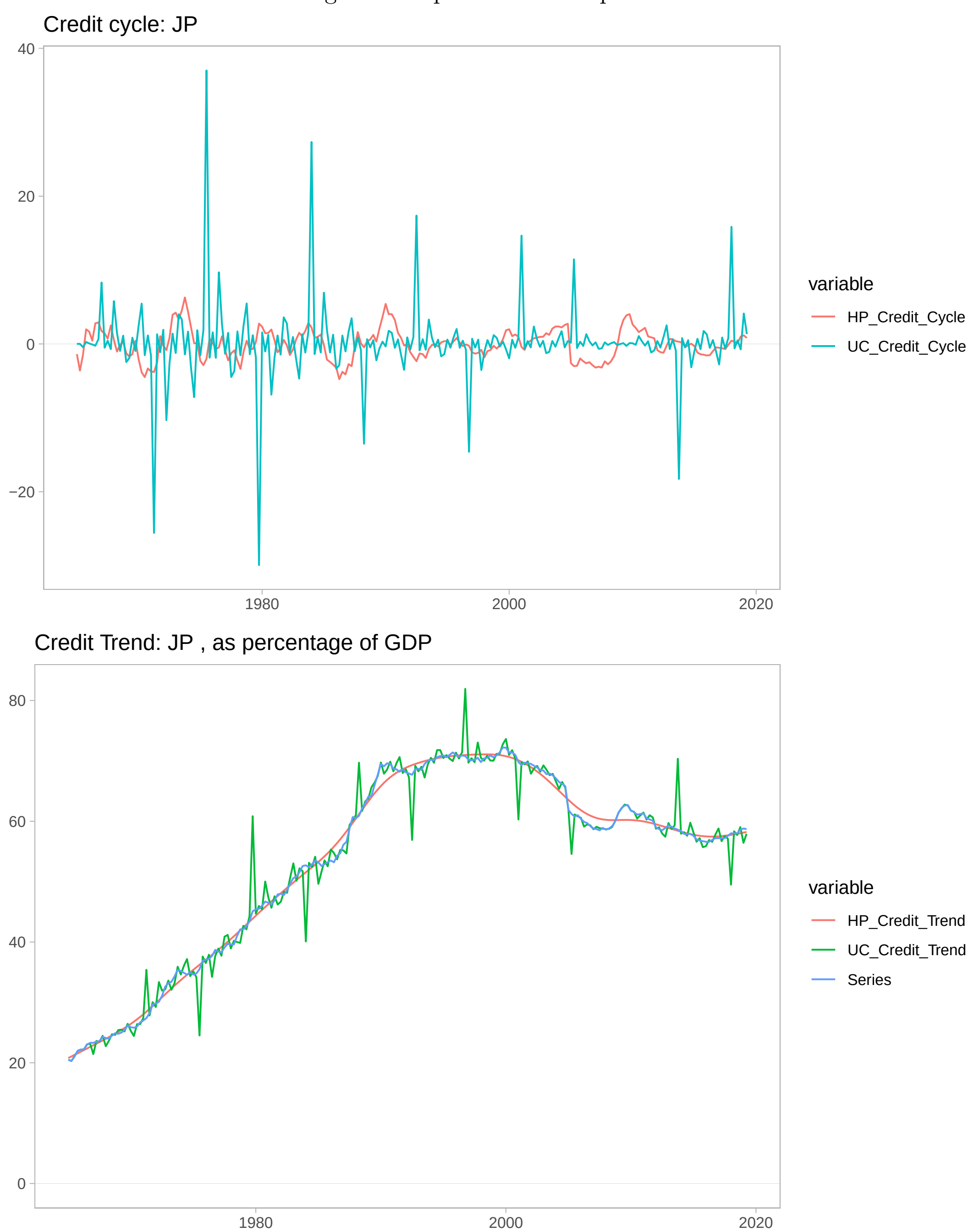
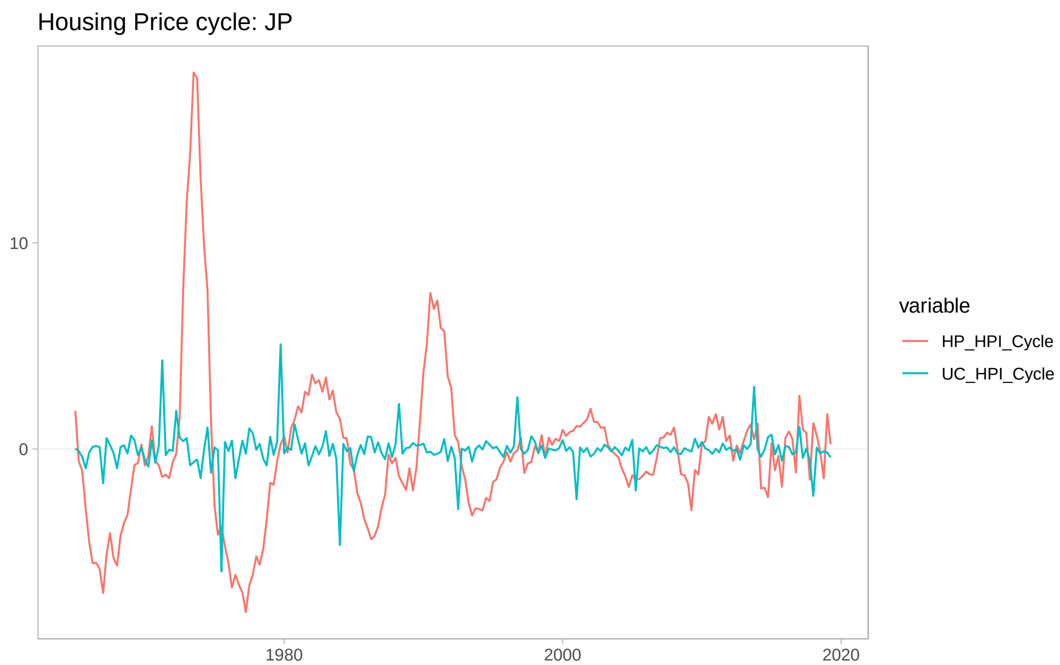


Figure 10: Japan Housing Price components



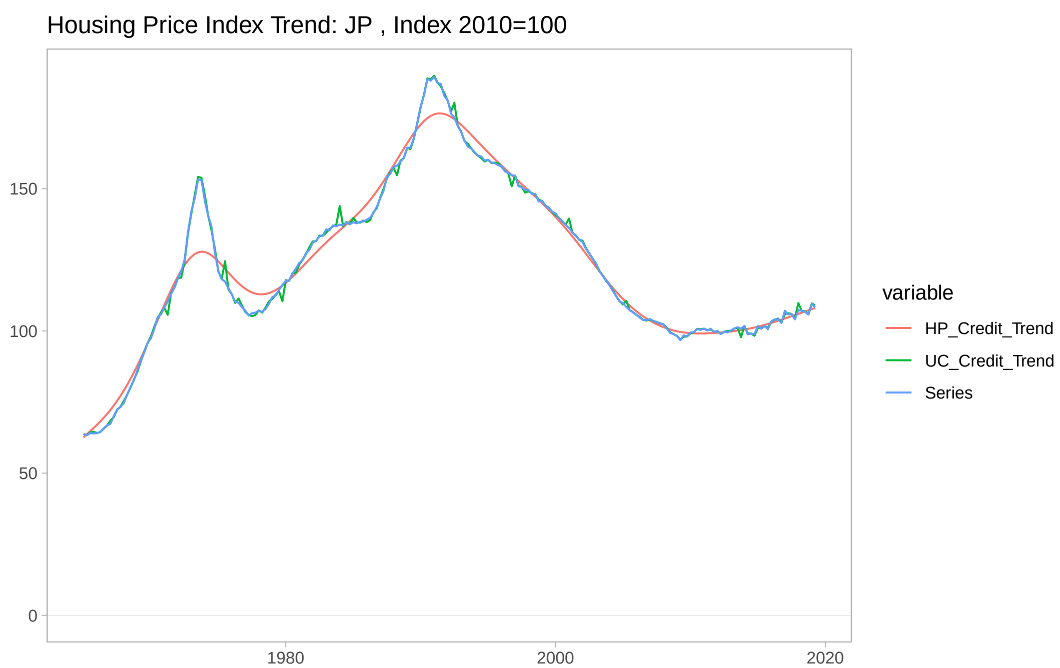
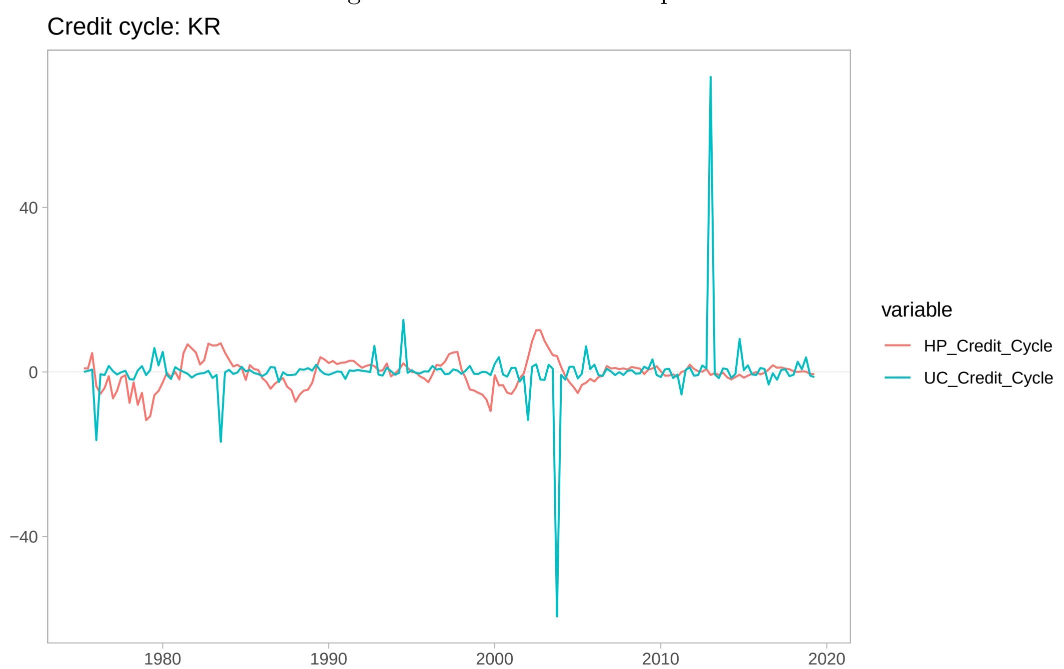


Figure 11: Korea Credit components



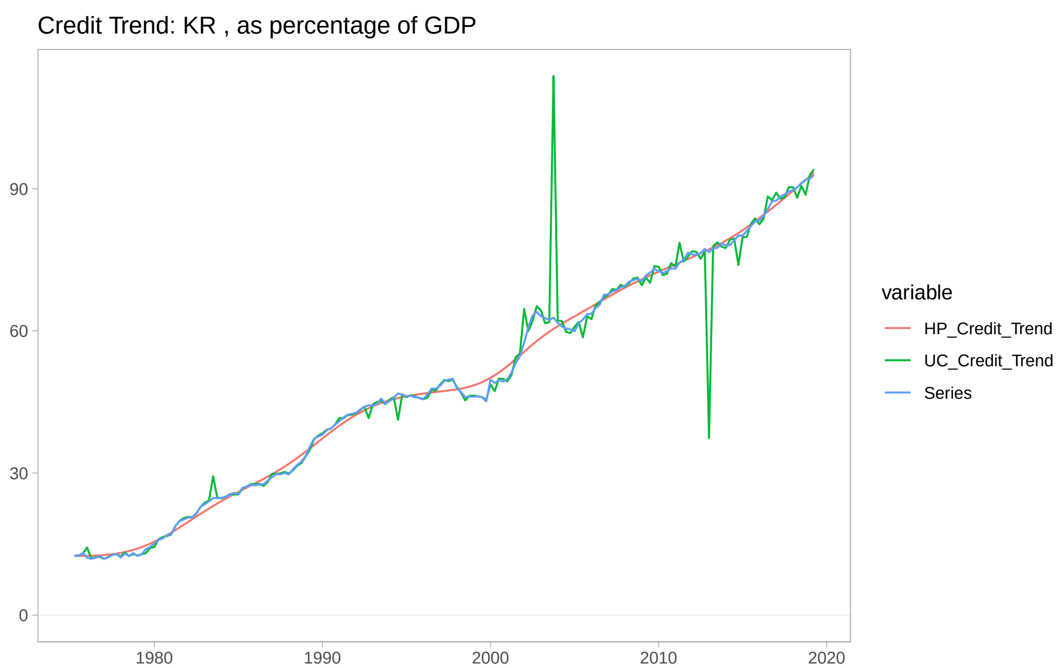
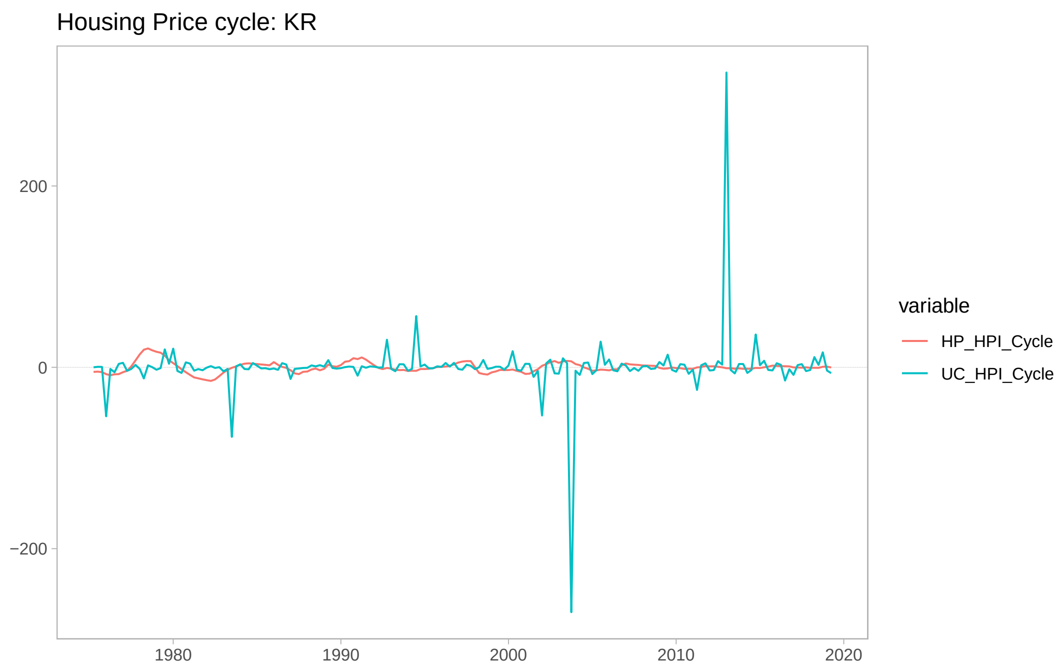
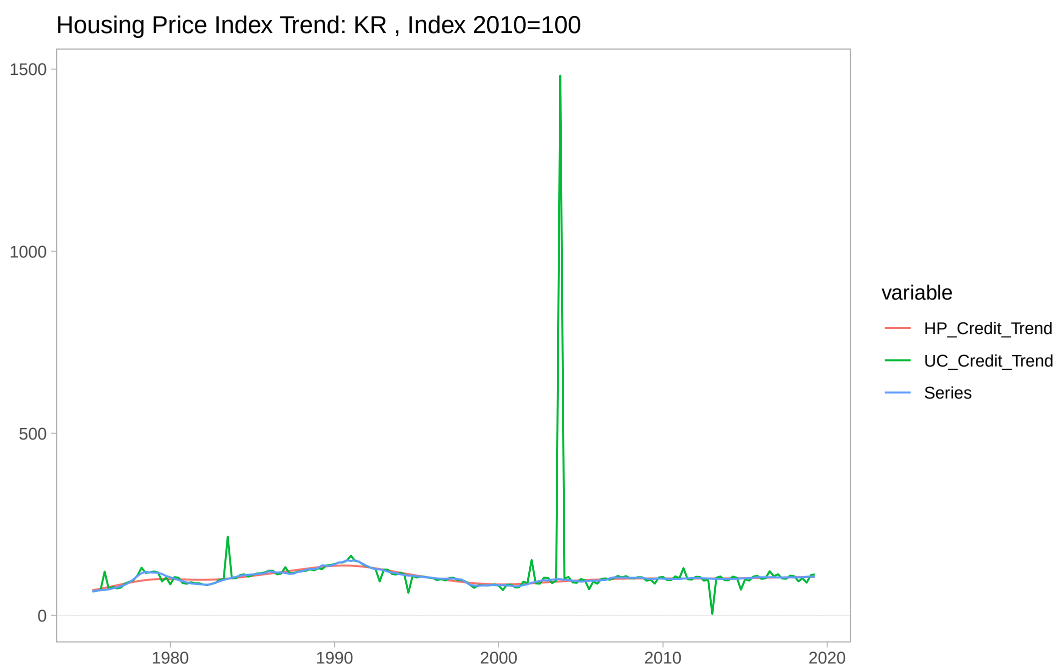


Figure 12: Korea Housing Price components





6 Impulse Response Function

This section show IRFs that are really unstable. I am guessing that is because the way I specify the function:

Instead of normally having: $\psi_t = \phi_{11}^y * \psi_l + \phi_{12}^y * \psi_{ll}$

I specify the IRF as: $\psi_t = \phi_{11}^y * \psi_l + \phi_{12}^y * \psi_{ll} + \phi_{21}^y * \psi_l + \phi_{22}^y * \psi_{ll}$

This potentially causes the unstability in the following IRF graphs. Also the fact that the constraints for autoregressive parameters have not been optimally setup could cause the issue.

Figure 13: US IRF

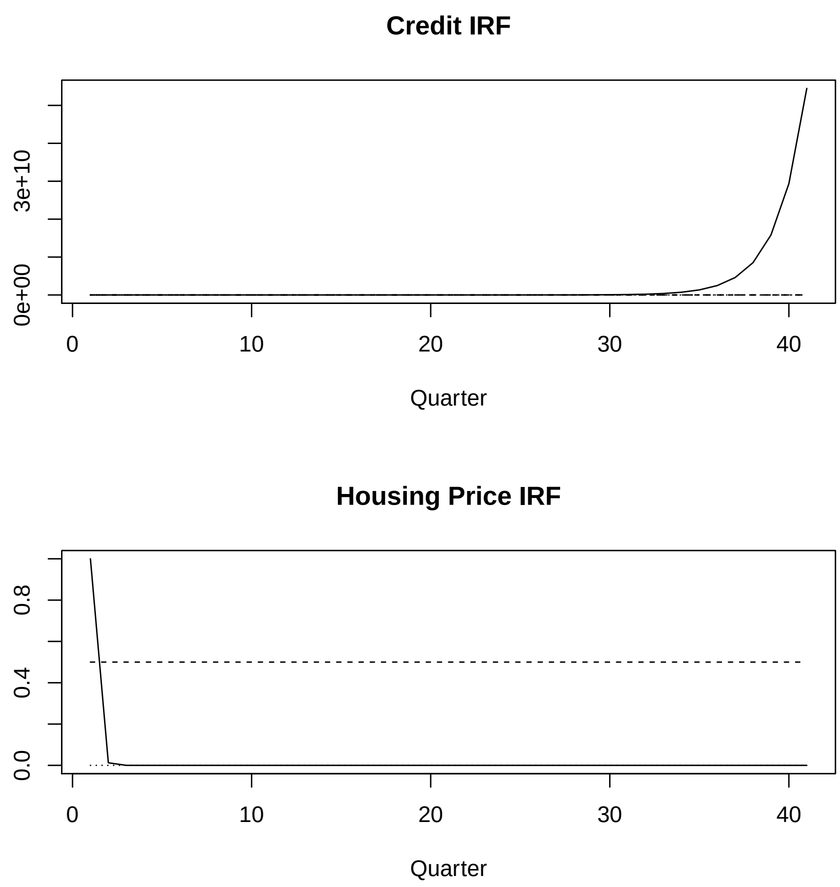


Figure 14: UK IRF

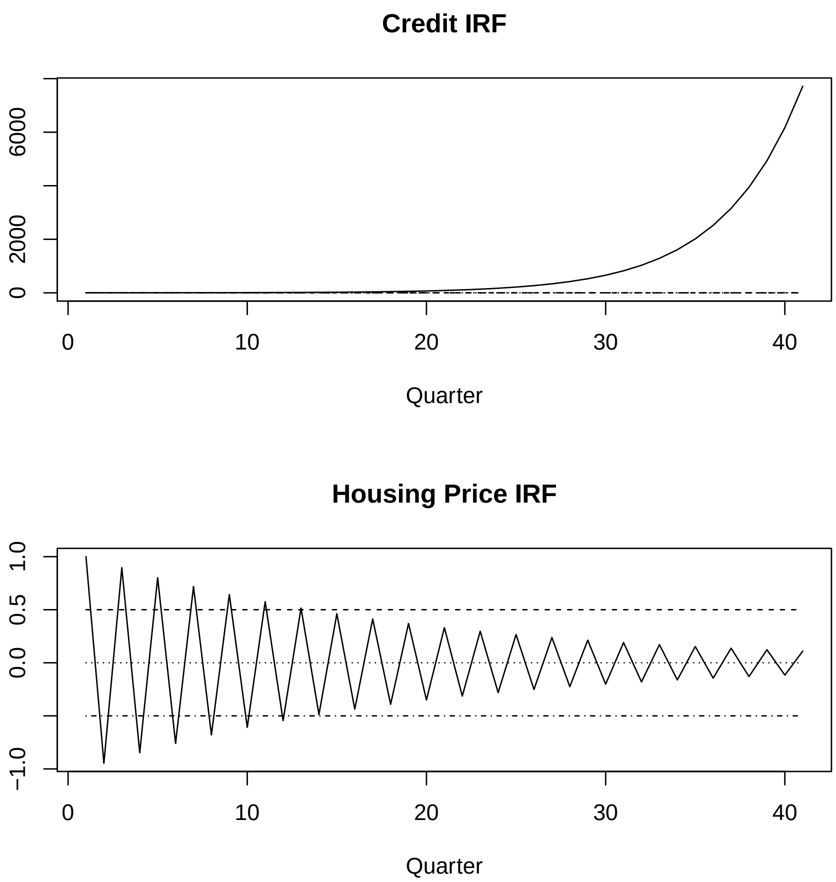


Figure 15: Germany IRF

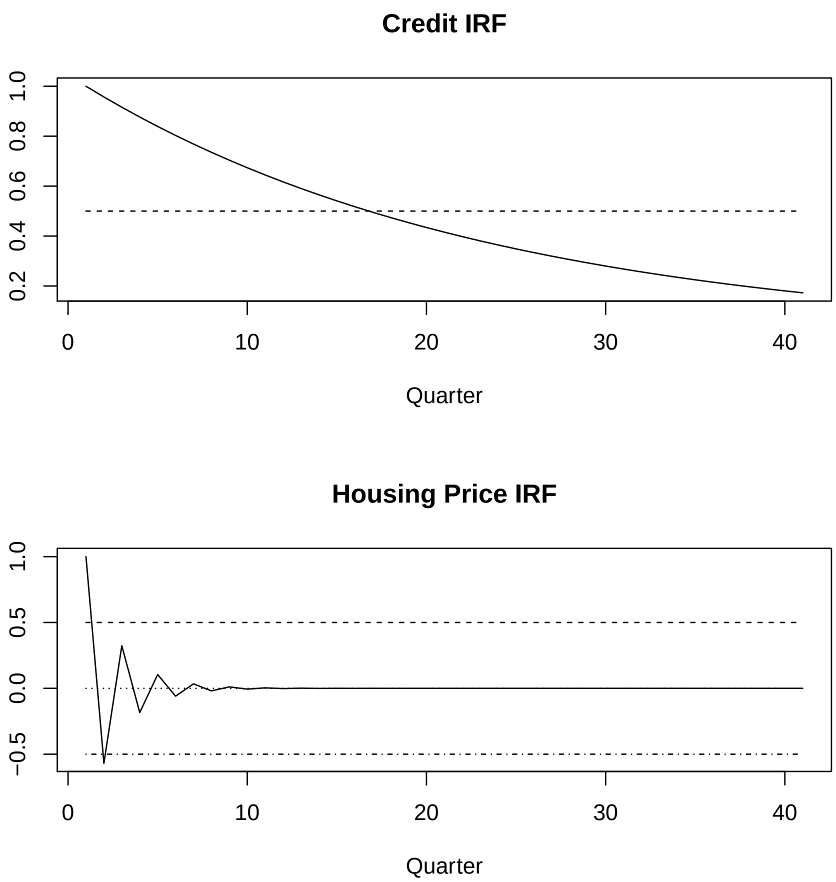


Figure 16: France IRF

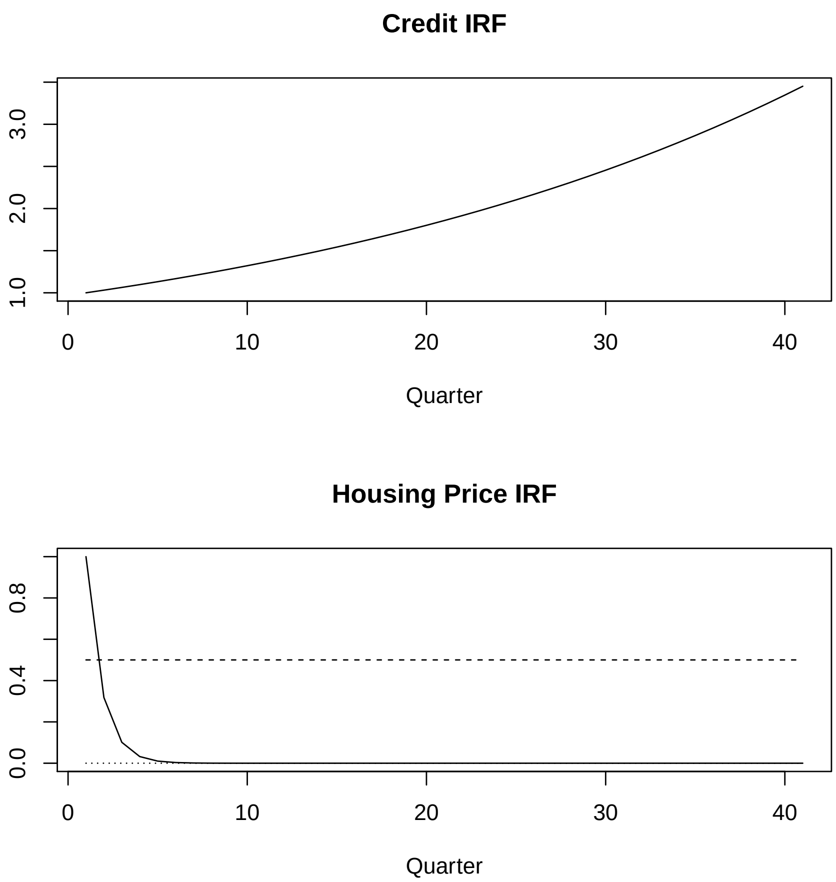


Figure 17: Japan IRF

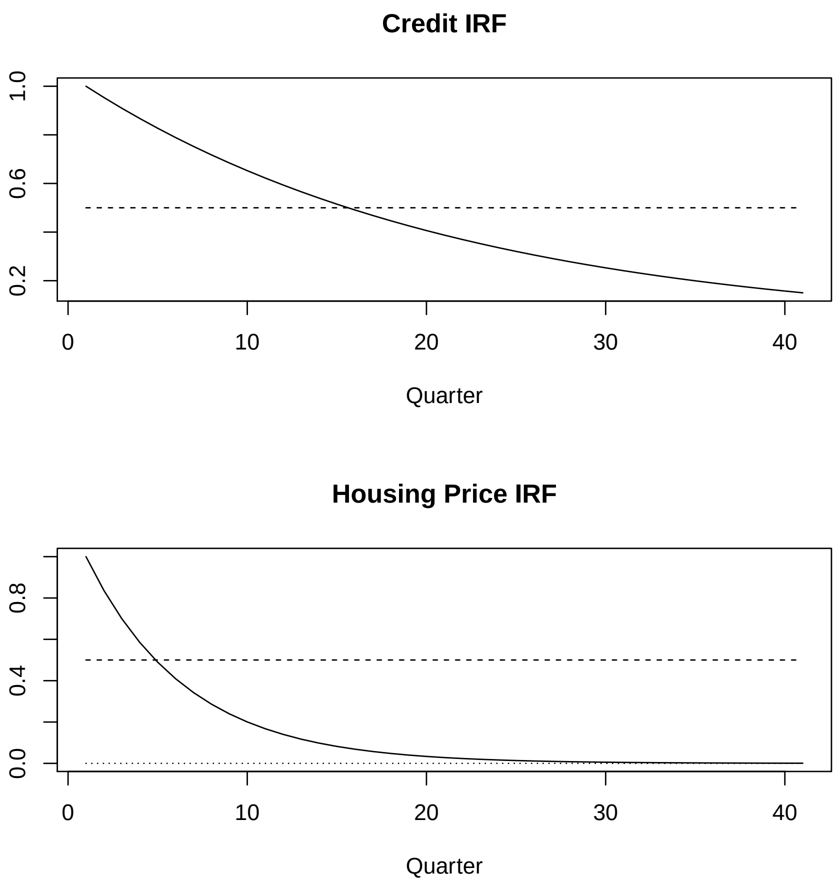


Figure 18: Korea IRF