# Housing and Credit Cycles

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## 1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

## 2 Model Specification

***Series:***
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$

$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

***Trends:***

A random walk drift term $g_t$ is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma_{\eta y}^2) \tag{3}$$

$$\tau_{ht} = \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma_{\eta h}^2) \tag{4}$$

***Cycles:***

$$c_{yt} = \phi_y^1 c_{yt-1} + \phi_y^2 c_{yt-2} + \phi_y^x c_{ht-1} + \varepsilon_{yt}, \qquad \varepsilon_{yt} \sim iidN(0, \sigma_{\varepsilon y}^2) \tag{5}$$

$$c_{ht} = \phi_h^1 c_{ht-1} + \phi_h^2 c_{ht-2} + \phi_h^x c_{yt-1} + \varepsilon_{ht}, \qquad \varepsilon_{ht} \sim iidN(0, \sigma_{\varepsilon h}^2) \tag{6}$$

**State-Space Model**

*Transition equation:*

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{7}$$

Where the transitory components are:

$$
\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \phi_y^1 & \phi_y^2 & 0 & \phi_y^x & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & \phi_h^x & 0 & 0 & \phi_h^1 & \phi_h^2 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} \tau_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix}
+
\begin{bmatrix} \eta_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix}
\tag{8}
$$

*The covariance matrix for $\tilde{v}_t$, denoted Q, is:*

$$
Q =
\begin{bmatrix}
\sigma_{\eta y}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\varepsilon y}^2 & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{\eta h}^2 & 0 & 0 \\
0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma_{\varepsilon h}^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{9}
$$

*Measurement Equation:*

$$\tilde{y}_t = A + H\beta_t \tag{10}$$

$$
\begin{bmatrix} y_t \\ h_t \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \end{bmatrix}
+
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}
$$

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# 3   Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

Following Kim & Nelson Chap 2, I set up the constraints on the autoregressive parameters to imply stationary and $-1 < \phi_x^y < 1$ as follow:

$$\phi_x^y = \frac{\phi_x^y}{1 + |\phi_x^y|} \qquad\qquad \phi_x^h = \frac{\phi_x^h}{1 + |\phi_x^h|}$$

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite matrix, in order to ensure feasible maximum likelihood estimation process. Furthermore, I suppressed the cross trend covariance term to be zero.

# 4   Regression results

In this following section, I will apply the UC model to data from 6 countries: US, UK, Germany, France, Japan and South Korea.

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. The following estimates are selected in the manner that they would look the most stable. Perhaps a more optimal constraint on the autoregressive parameters would solve this issue.

Table 1: Correlated UC model Estimates: US data

| Description | Estimate | Standard Error |
|:---:|:---:|:---:|
| $\phi_y^1$ | 0.7057 | 64.8390 |
| $\phi_h^1$ | 0.6602 | 26.5767 |
| $\sigma_{ny}$ | 1.0607 | 0.0536 |
| $\sigma_{ey}$ | $9.2638 \times 10^{-6}$ | 0.0028 |
| $\sigma_{nh}$ | 1.6433 | 0.0830 |
| $\sigma_{eh}$ | 0.0018 | 0.0691 |
| $\sigma_{eyeh}$ | 0.0180 | 58.7250 |
| $\sigma_{nynh}$ | 0.2996 | 0.0650 |
| Log-likelihood value | $-655.9071$ | 0 |

Table 2: Correlated UC model Estimates: UK data

| Description | Estimate | Standard Error |
|:---:|:---:|:---:|
| $\phi_y^1$ | 1.0094 | NaN |
| $\phi_y^2$ | $-0.1466$ | NaN |
| $\phi_h^1$ | 1.0055 | NaN |
| $\phi_h^2$ | $-0.1050$ | NaN |
| $\sigma_{ny}^2$ | 1.3415 | NaN |
| $\sigma_{ey}^2$ | 0.8083 | NaN |
| $\sigma_{nh}^2$ | 0.4651 | NaN |
| $\sigma_{eh}^2$ | 2.8995 | NaN |
| $\sigma_{eyeh}$ | 0.5846 | NaN |
| $\sigma_{nynh}$ | 0.9390 | NaN |
| Log-likelihood value | $-867.2620$ | 0 |

# 5    Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

For example, the model for US data shows that there is a positive relationship between a one period lag in short term house price and house hold credit. Also for the UK data, there is a positive relationship between a one period lag in short term credit and house price.

Further development for this paper should include more optimal constraints on parameters to ensure stability.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.
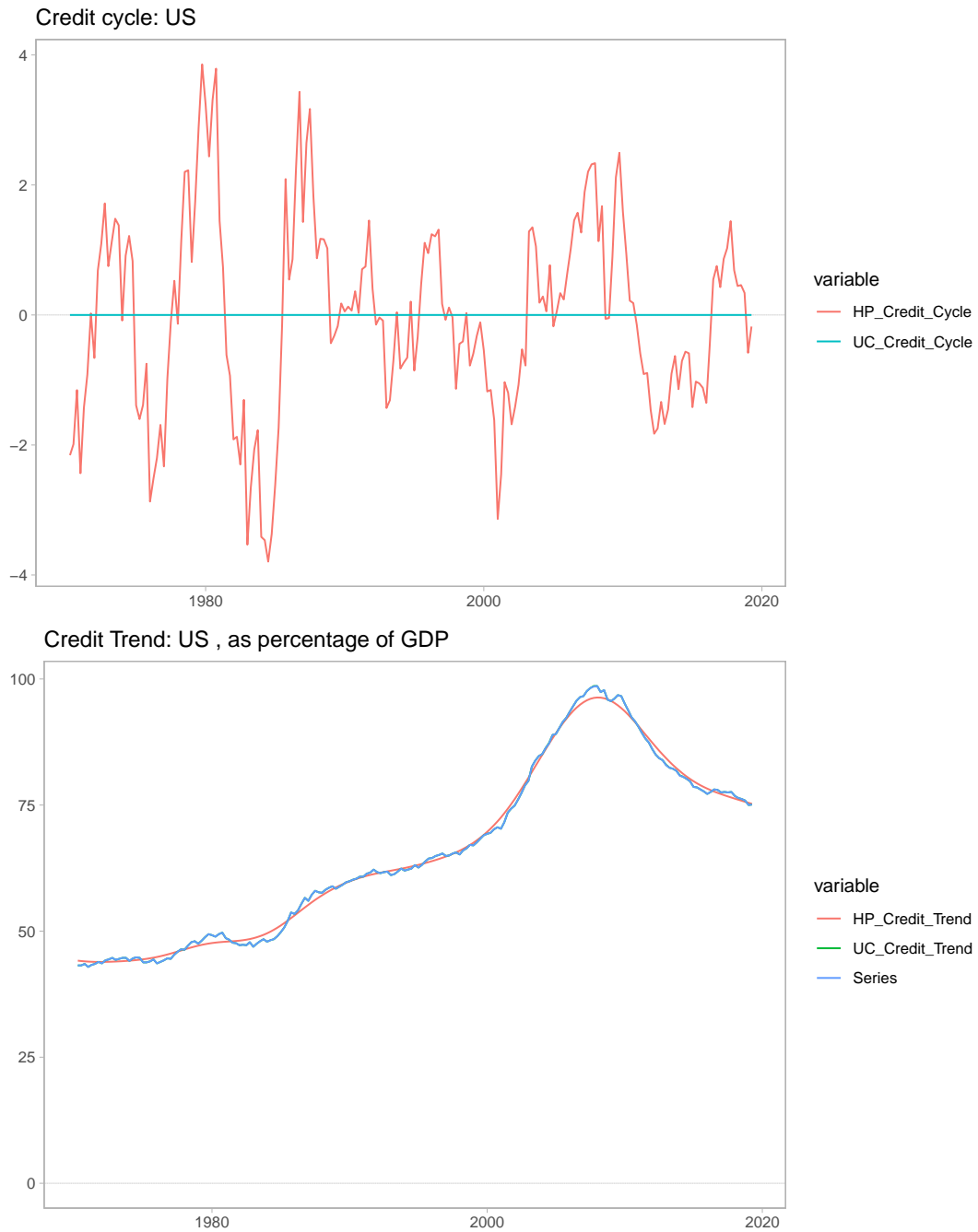
Figure 1: Appendix: US Credit components

Credit cycle: US



Credit Trend: US , as percentage of GDP

Figure 2: US Housing Price components

Housing Price cycle: US



Housing Price Index Trend: US , Index 2010=100

Figure 3: UK Credit components

Figure 4: UK Housing Price components

Housing Price cycle: GB



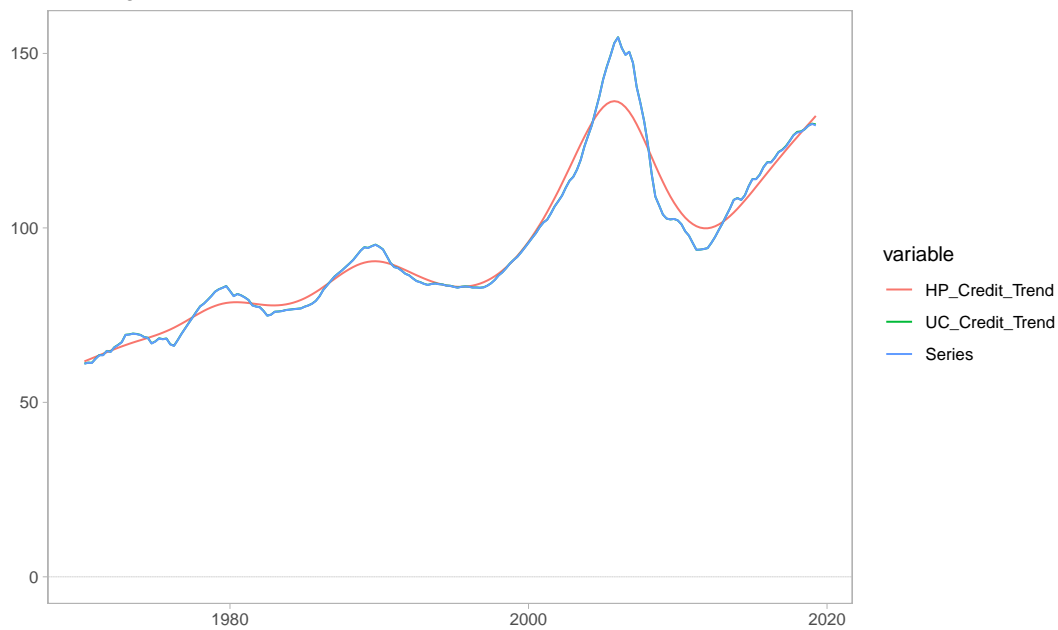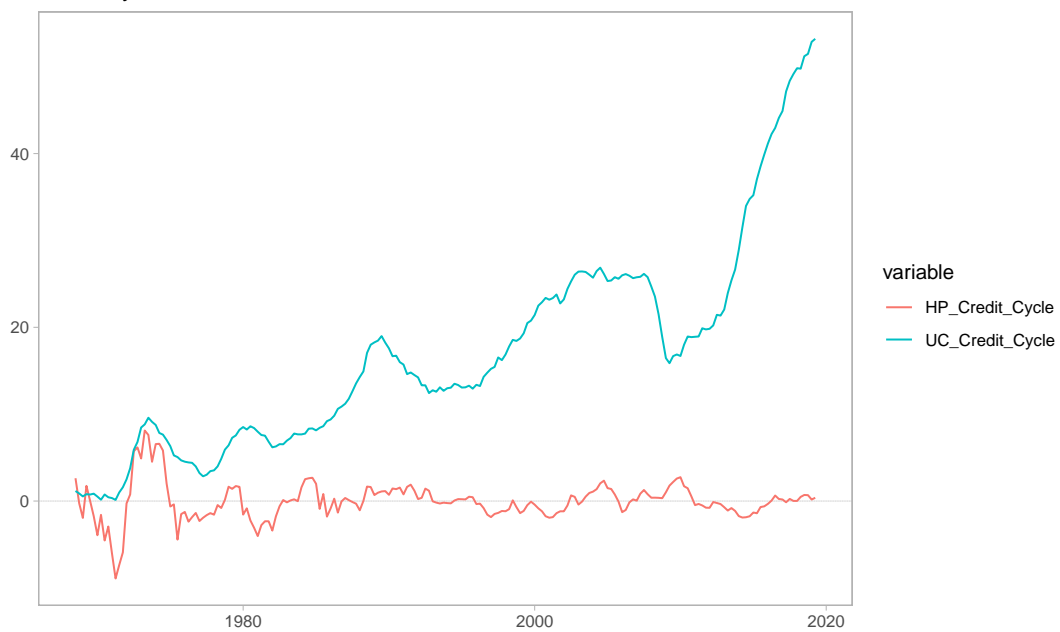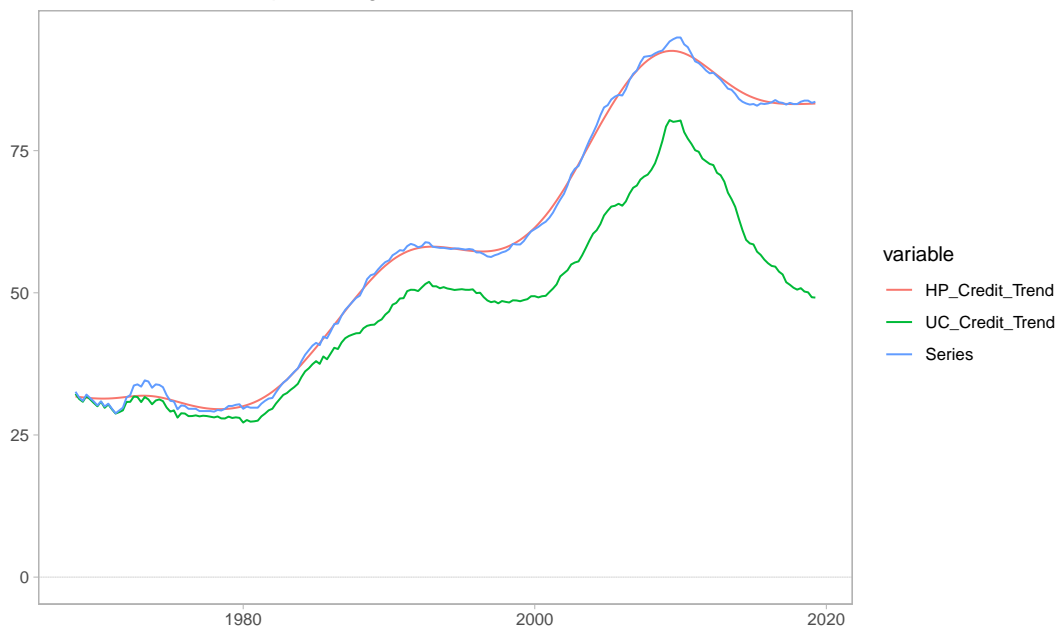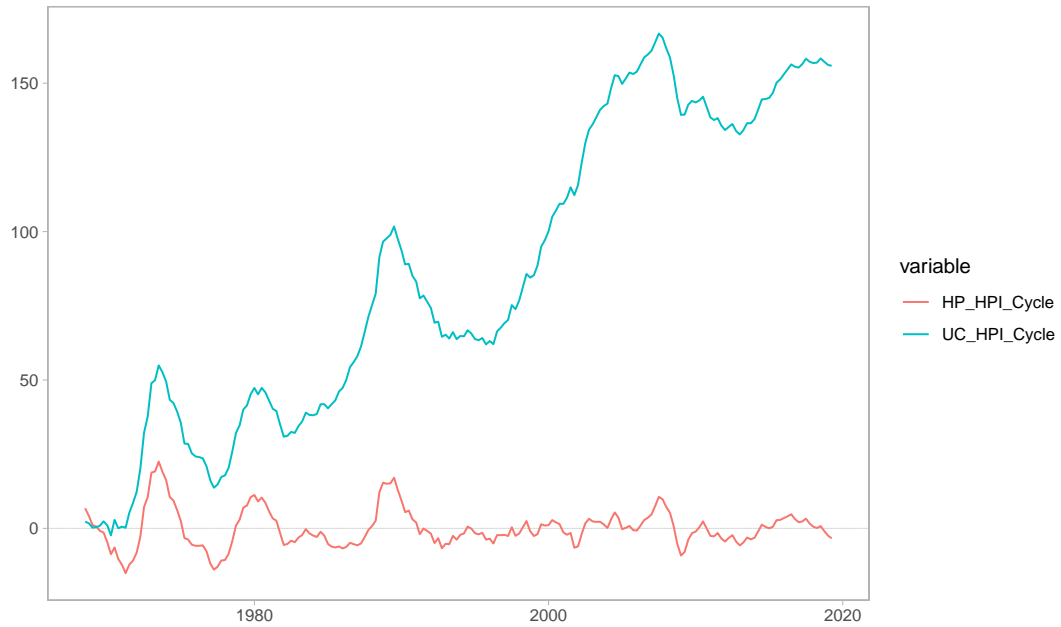Housing Price Index Trend: GB , Index 2010=100