Assignment 1
Due 02/15/19

1 Analytical Exercise

1. Is the following MA(2) process invertible?

$$Y_t = (1 + 2.4L + 0.8L^2)\varepsilon_t$$

Find the invertible representation of the process. Calculate the autocovariances of the invertible representation. (Hint: Read pages 67-68 in Hamilton).

2. Consider the AR(2) process

$$y_t = 2.5 + 1.1y_{t-1} - 0.18y_{t-2} + \varepsilon_t, \varepsilon_t \tilde{} i.i.d.(0, 1)$$

   (a) Show that the AR(2) is stable/stationary and calculate its autocovariance and autocorrelation function. Also, calculate the unconditional mean of the process. Indicate what the ACF and PACF of this series looks like (you do not have to compute it exactly just show me a rough plot).

   (b) Determine the Wold representation of the series (MA representation) and thus the sequence of dynamic multipliers required for the impulse response function. Compute the first four MA weights (dynamic multipliers) and plot the impulse response function.

2 Empirical Exercise

For the computer exercises, answer all questions on the computer output. Also, please feel free to comment liberally on the computer output. Note any unusual results or simply make comments to yourself about what the results tell you.

2.1 Exercise 1: Identify ARMA models for log real GDP

1. Use data lrgdp.txt or lrgdp.csv for this exercise.

   (a) Plot the log level of real GDP over the period 1947:Q1 to 2015.Q2. Comment on any unusual features of the data. Compute, plot and interpret the correlogram for the level of real GDP.

   (b) Estimate an AR(1) model for the level of log real GDP. What does your estimate of ϕ tell you about the stationarity of the model? What is the unconditional mean of log real GDP?

   (c) Detrend log real GDP by forming the residuals of a regression of log real GDP on a constant and a time trend. Compute, plot and interpret the correlogram (ACF) and partial correlogram (PACF) for the detrended series. What ARMA models seem most appropriate?

   (d) Compute, plot and interpret the ACF and PACF for the first difference of log real GDP. What ARMA models seem most appropriate?

   (e) For detrended log real GDP and the first difference of real GDP, compute the AIC and BIC information criteria for all ARMA(p,q) models with p=0,1,2,3 and q=0,1,2,3. Which models are suggested by these model selection statistics?

   (f) Now break the sample into two parts: 1947:1-1981:4 and 1982:1-2015:2 What ARMA models are most appropriate for both the sample periods?

   (g) Comment on the difference in the dynamics of the first difference of log real GDP for the full sample as compared to the dynamics of the different sub-samples.