Housing and Credit Cycles

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1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$
$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$
$$\tau_{ht} = \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

Cycles:

$$c_{yt} = \phi^1_y c_{yt-1} + \phi^2_y c_{yt-2} + \phi^x_y c_{ht-1} + \varepsilon_{yt}, \qquad \varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y}) \tag{5}$$
$$c_{ht} = \phi^1_h c_{ht-1} + \phi^2_h c_{ht-2} + \phi^x_h c_{yt-1} + \varepsilon_{ht}, \qquad \varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h}) \tag{6}$$

State-Space Model
Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{7}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_y^1 & \phi_y^2 & 0 & \phi_y^x & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_h^x & 0 & 0 & \phi_h^1 & \phi_h^2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix} \tag{8}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma_{\eta y}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon y}^2 & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta h}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma_{\varepsilon h}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{10}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

I did not put stationary constraints directly on the autoregressive parameters. Since such constraints on a VAR(2) system is complex to setup. However, to achieve feasible stationary transitory measurement, I implement an additional term on the objective function:

$$l(\theta) = -w1\sum_{t=1}^{T} ln[(2\pi)^2|f_{t|t-1}|] - w2\sum_{t=1}^{T} \eta'_{t|t-1} f^{-1}_{t|t-1} \eta_{t|t-1} - w3 * \sum_{t=1}^{T}(c^2_{yt}) + w4 * \sum_{t=1}^{T}(c^2_{ht}) \tag{11}$$

The last term in the objective function acts as a penalty against too much transitory deviation from zero. Without this penalty, the trend would be linear or all the movements in the measured series would be matched by transitory movements.

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite covariance matrix.

4 Priors selection

The priors for autoregressive parameters in matrix F are taken from VAR regression of the HP filter cycle decomposition of the series.

For $\beta_{0|0}$, I set $\tau_{0|0}$ as the value of the first available row of data and omit the first observation from the regression. $c_{0|0}$ are set to be equal to their HP filter counterpart. $var(\tau_{0|0}) = 100 + 50 * random$; while other measures of the starting covariance are set to be their unconditional values.

Starting standard deviation and correlation values are randomized within reasonable range.

Table 1: Descriptive statistics

Country	Index	Mean	Max	Min	Frequency	Periods
UK	y_t	432.0829	459.1071	406.7316	Quarterly	1989:Q1-2020:Q1
	h_t	464.7302	503.8838	441.5308	Quarterly	1989:Q1-2020:Q1
US	y_t	429.0831	456.8506	395.8907	Quarterly	1989:Q1-2020:Q1
	h_t	434.0478	480.1792	378.2752	Quarterly	1989:Q1-2020:Q1

y_t is credit to household series, h_t is housing price index series. Both are log transformed.

Table 2: Correlation matrix

Country		y_t	h_t
UK	y_t	1	
	h_t	0.9359935	1
US	y_t	1	
	h_t	0.7046029	1

5 Data Description

Our quarterly data sample periods include periods from January 1989 to January 2020. Table 1 shows the description of the data used in this paper. The sample periods was chosen based on the nature of the change in regulation of credit and housing market beginning early 1990s. The main source of the data comes from the Bank of International Settlement (BIS). The housing price index is based on base index of 2010 as 100. The credit to household data is measured as percentage of GDP.

6 Regression results

In this following section, I will apply the UC model to data from 2 countries: US and UK.

Choosing priors from an estimated VAR(2) regression on HP filtered cycle and trend series. The following likelihood function weights are selected in the manner that they make the series look the most stable.

The regression tables below show Unobserved component VAR(2) regression results with and without cross-cycle parameters.

Table 3: United Kingdom regression results

Parameters	VAR(2)		VAR(2) 1-cross-lag		VAR(2) 2-cross-lags	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
ϕ_y^1	1.9725	0.0234	1.8820	0.0005	1.8895	0.0002
ϕ_y^2	-0.9827	0.0263	-0.8160	0.0022	-0.8743	0.0026
ϕ_y^{x1}			-0.0240	0.0004	0.1756	0.0008
ϕ_y^{x2}					-0.1964	0.0035
ϕ_h^1	1.5048	0.1019	1.5748	0.0056	1.5742	0.0643
ϕ_h^2	-0.5608	0.1252	-0.7094	0.0077	-0.7364	0.0586
ϕ_h^{x1}			0.3783	0.0171	0.7214	0.0492
ϕ_h^{x2}					-0.5959	0.0442
σ_{ny}	0.7063	0.0600	0.7017	0.0353	0.6040	0.0374
σ_{ey}	0.0004	0.0104	0.1127	0.0052	0.0160	0.0063
σ_{nh}	1.8676	0.1617	1.6429	0.1023	1.9038	0.1115
σ_{eh}	0.6568	0.2583	0.6323	0.0193	0.1289	0.0269
σ_{eyeh}	0.6888	13.1231	1.0000	7.0580×10^{-6}	0.9998	0.0061
σ_{nynh}	0.5680	0.1125				
Log-likelihood value	-454.6450		-464.0793		-456.5685	

Weights of likelihood function: w1 = 0.6, w2 = 0.4, w3 = 0.004, w4 = 0.003

$l(\theta) = -w1\sum_{t=1}^{T} ln[(2\pi)^2|f_{t|t-1}|] - w2\sum_{t=1}^{T} \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1} - w3 * \sum_{t=1}^{T}(c_{yt}^2) + w4 * \sum_{t=1}^{T}(c_{ht}^2)$

Table 4: United States regression results

Parameters	VAR(2)		VAR(2) 1-cross-lag		VAR(2) 2-cross-lags	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
ϕ_y^1	1.8497	0.0645	1.3050	0.1048	1.5502	0.0622
ϕ_y^2	−0.8917	0.0639	−0.5099	0.0696	−0.5754	0.0642
ϕ_y^{x1}			0.0332	0.0027	0.0141	0.0083
ϕ_y^{x2}					0.0037	0.0114
ϕ_h^1	1.7847	0.0345	2.0529	0.0421	1.8338	0.0658
ϕ_h^2	−0.8034	0.0345	−1.2469	0.0731	−0.9358	0.0611
ϕ_h^{x1}			1.0795	0.2918	1.7429	0.4406
ϕ_h^{x2}					−1.6544	0.4175
σ_{ny}	0.4793	0.0244	0.4718	0.0241	0.4195	0.0206
σ_{ey}	0.0281	0.0154	0.0256	0.0136	0.0375	0.0132
σ_{nh}	0.4549	0.0440	0.4742	0.0383	0.4937	0.0367
σ_{eh}	0.2566	0.0323	0.0876	0.0756	0.0966	0.0478
σ_{eyeh}	−1.0000	0.0001	1.0000	8.5939×10^{-5}	1.0000	2.5743×10^{-6}
σ_{nynh}	0.3974	0.0721				
Log-likelihood value	−339.7258		−346.9160		−332.0706	

Weights of likelihood function: w1 = 0.8, w2 = 0.2, w3 = 0.003, w4 = 0.004

$l(\theta) = -w1\sum_{t=1}^{T} ln[(2\pi)^2|f_{t|t-1}|] - w2\sum_{t=1}^{T} \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1} - w3 * \sum_{t=1}^{T}(c_{yt}^2) + w4 * \sum_{t=1}^{T}(c_{ht}^2)$

Regarding results for the UK, the model selection criteria (likelihood function value) indicates that a simple VAR(2) fit the data the best. However, with the introduction of cross-cycle terms, at a slight cost of lower likelihood value I can better estimate the correlation value of short run credit and house price index (σ_{eyeh}) at a more significant value. Additionally, the cross-cycle results shows a better

Regression results for the US are less obvious. This could be attributed by the low correlation between the two series as shown in Table 2 and potentially a identification problem. All of the correlation value of short run credit and house price index (σ_{eyeh}) in all three models show a multi-collinearity problem. The model selection criteria shows that VAR(2) with 2-cross-lag coefficients have the highest likelihood value.

The novel contribution of this paper is to introduce this parameter ϕ_h^{xt} in which it measure the effect of a change in last periods credit transitory component on the current housing price transitory component. In both cross-cycle regressions in the UK and US, I can observe that there is a significant positive effect (ϕ_h^{x1}) of last period credit cycle deviation on current housing cycle component. While the coefficients of transitory housing index deviation on household credit (ϕ_y^{x1}) are much smaller.

The following graphs shows the UC forecast series against the actual data series.

Figure 1: VAR(2) UK:

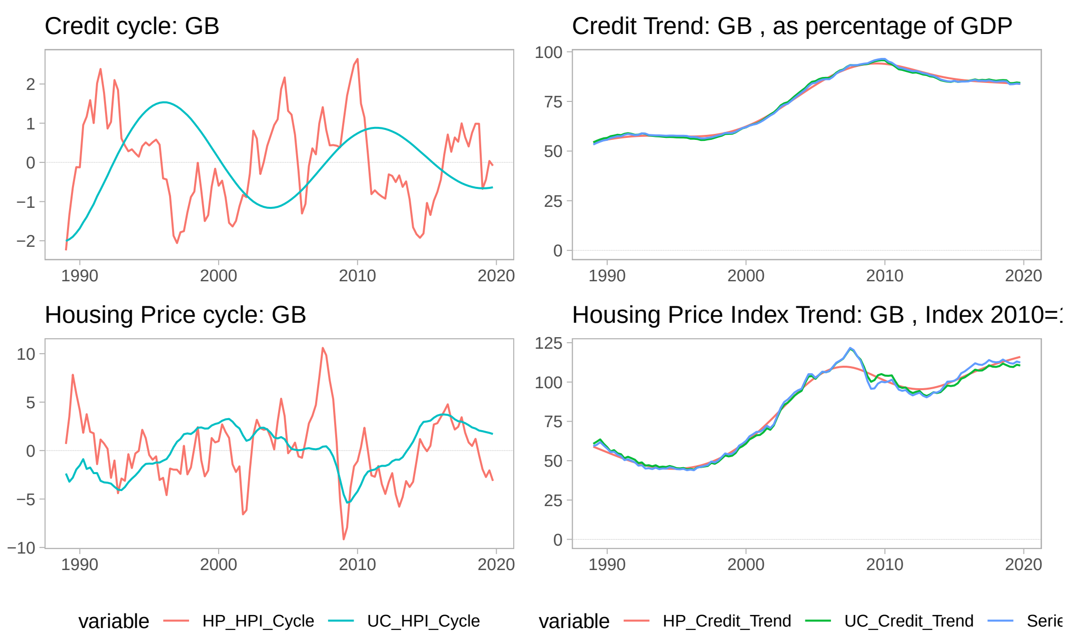


Figure 2: VAR(2) Crosscycle UK:

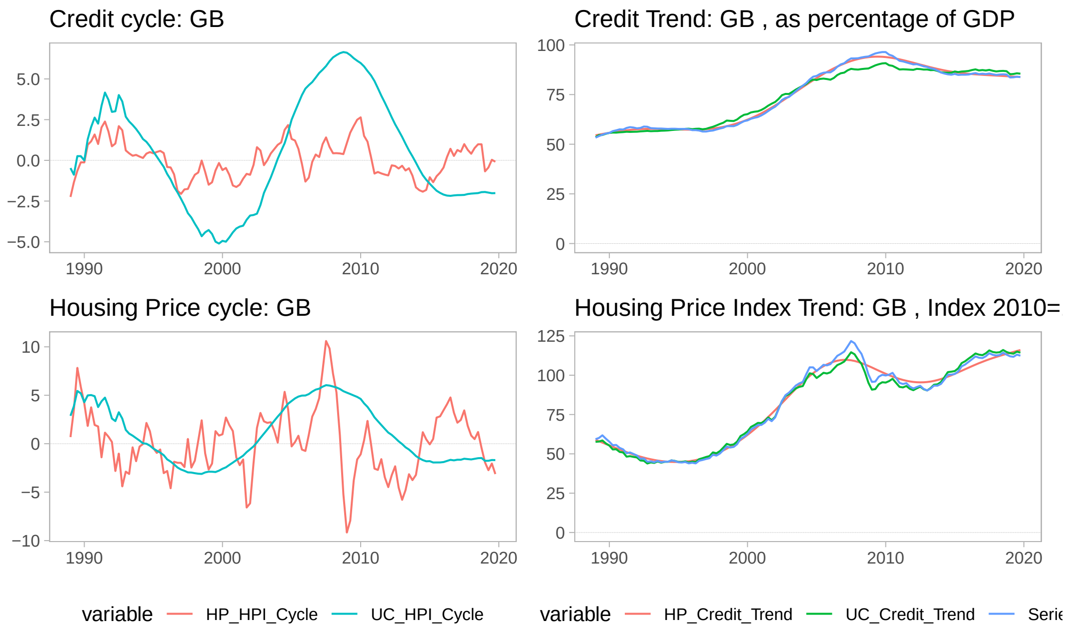


Figure 3: VAR(2) Crosscycle 1st lag only UK:

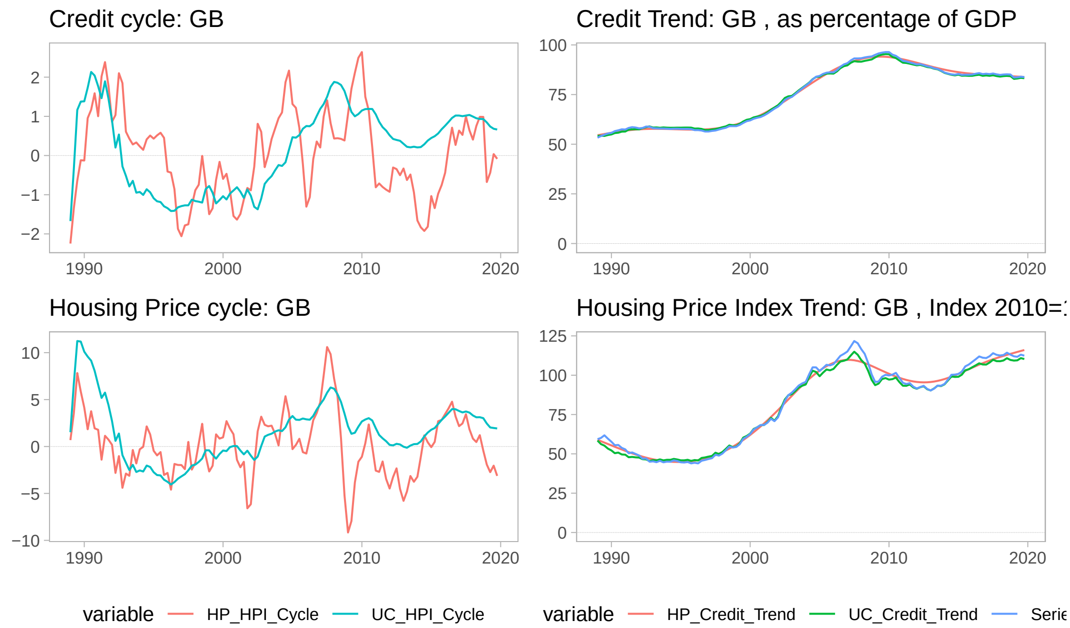


Figure 4: VAR(2) US:

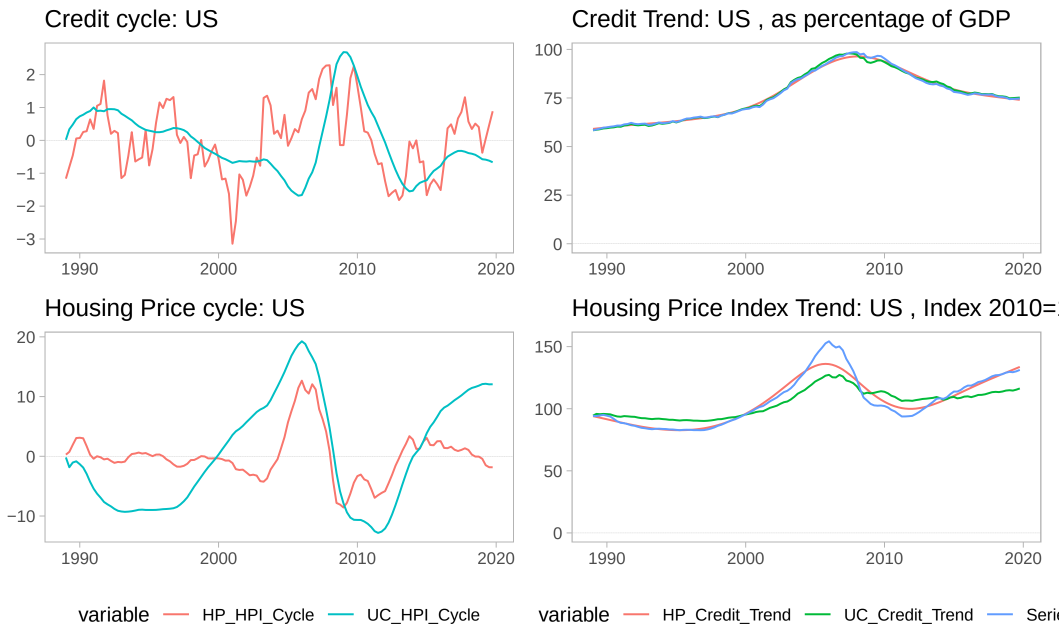


Figure 5: VAR(2) Crosscycle US:

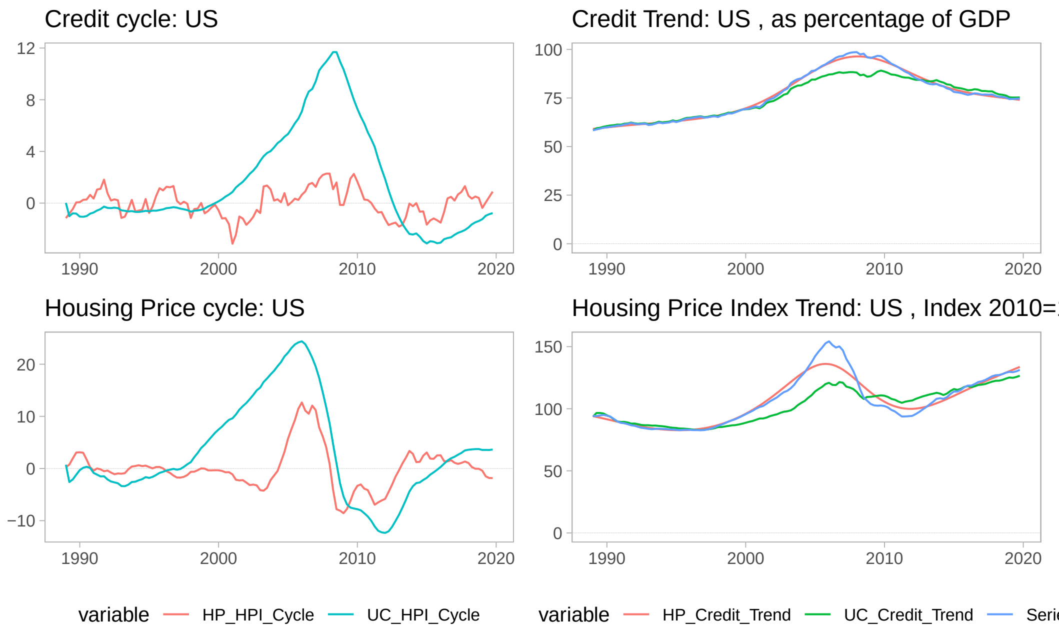
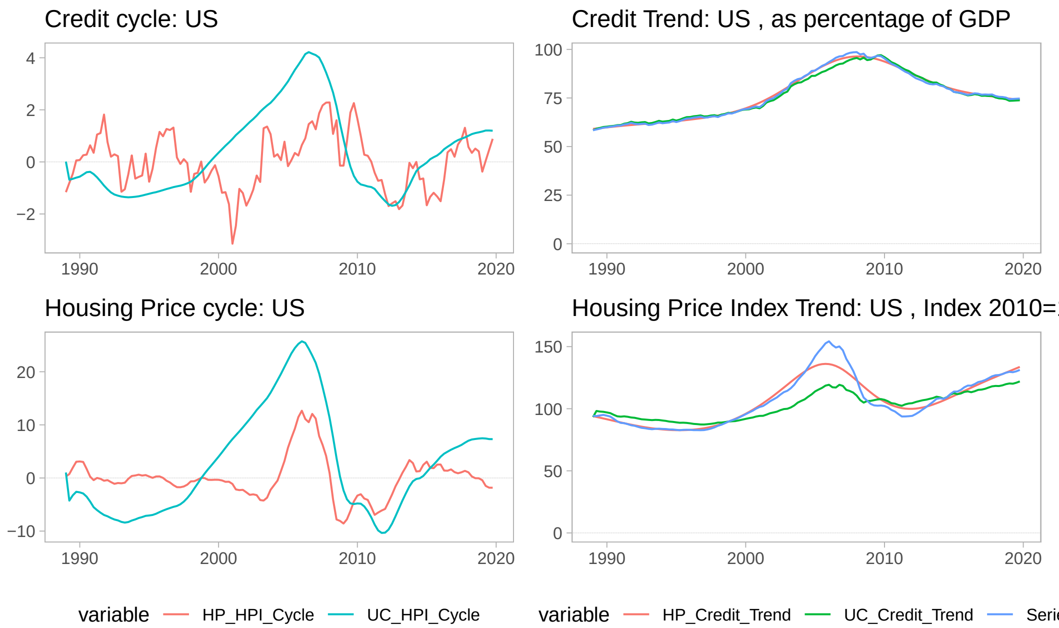


Figure 6: VAR(2) Crosscycle 1st lag only US:



7 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of past short term shock from house hold credit on current housing price and vice versa.

In this paper, the models for US and GB data shows that there is a positive relationship between a one period lag in short term house hold credit and current house price.

Further development for this paper should include more robust optimal constraints on parameters to ensure stability rather than an adhod approach to selecting. Additional examination of the multicolliearity / identification issue also needs to be addressed.