Structural VAR

Structural VAR

- Perhaps the most common question that a VAR tries to figure out (though by no means the only one) is what are the effects of a monetary policy shock.
- It's no use regressing macroeconomic outcomes on shocks to interest rates, because the most obvious reason why interest rates would be tightened is that the Fed expects inflation or growth to be high in the future. Monetary policy is endogenous.
- The structural VAR though is trying to figure out the effects of exogenous monetary policy shocks—the FOMC changing monetary policy not because of differences in the outlook for growth or inflation

Different Methods to Identify Shocks

- Cholesky Restrictions
- Long-Run Restrictions (Blanchard-Quah and others)
- Identification from heteroscedasticity
- Identification from Sign Restrictions
- Identification from high frequency data

Long-run restrictions (Testing long-run neutrality)

- King and Watson (1997) survey the use of bivariate SVAR models to test some simple long-run neutrality propositions in macroeconomics.
- The key feature of long-run neutrality propositions is that changes in nominal variables have no effect on real economic variables in the long-run.
- Some examples of long-run neutrality propositions are:
  - A permanent change in the nominal money stock has no long-run effect on the level of real output

Long-run restrictions (Testing long-run neutrality)

- King and Watson (1997) survey the use of bivariate SVAR models to test some simple long-run neutrality propositions in macroeconomics.
- The key feature of long-run neutrality propositions is that changes in nominal variables have no effect on real economic variables in the long-run.
- Some examples of long-run neutrality propositions are:
  - A permanent change in the nominal money stock has no long-run effect on the level of real output
  - A permanent change in the rate of inflation has no long-run effect on unemployment (a vertical Phillips curve)

Long-run restrictions (Testing long-run neutrality)

- King and Watson (1997) survey the use of bivariate SVAR models to test some simple long-run neutrality propositions in macroeconomics.
- The key feature of long-run neutrality propositions is that changes in nominal variables have no effect on real economic variables in the long-run.
- Some examples of long-run neutrality propositions are:
  - A permanent change in the nominal money stock has no long-run effect on the level of real output
  - A permanent change in the rate of inflation has no long-run effect on unemployment (a vertical Phillips curve)
  - A permanent change in the rate of inflation has no long-run effect on real interest rates (the long-run Fisher relationship).

Long-run restrictions (Testing long-run neutrality)

- KW show that testing long-run neutrality within a SVAR framework requires the data to be I(1). They characterize long-run neutrality of money using the SMA representation for Δy_t written as:

Output: $\Delta y_t = \mu_y + \theta_{yy}(L)\eta_{yt} + \theta_{ym}(L)\eta_{mt}$
Money: $\Delta m_t = \mu_m + \theta_{my}(L)\eta_{yt} + \theta_{mm}(L)\eta_{mt}$

- where ε_{yt} represents exogenous shocks to output that are uncorrelated with exogenous shocks to nominal money ε_{mt}.

Long-run restrictions (Testing long-run neutrality)

- Long-run neutrality of money involves the answer to the question:
  - Does an unexpected and exogenous permanent change in the level of money (m) lead to a permanent change in the level of output (y)?
- If the answer is no, then money is long-run neutral towards output.
- In terms of the SMA representation, η_{mt} represents exogenous unexpected changes in money.

$\theta_{mm}(1)\eta_{mt} =$ Permanent effect of η_{mt} on m.
$\theta_{ym}(1)\eta_{mt} =$ Permanent effect of η_{mt} on y.

Long-run restrictions (Testing long-run neutrality)

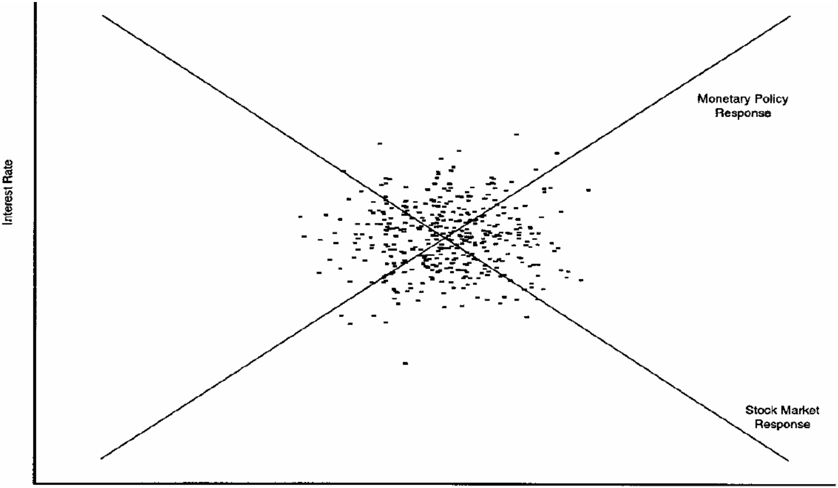
- With the data in logs, the long-run elasticity of output with respect to permanent changes in money is:

$$\gamma_{ym} = \frac{\theta_{ym}(1)}{\theta_{mm}(1)}$$

- Result: money is neutral in the long-run when $\theta_{ym}(1) = 0$, or $\gamma_{ym} = 0$
- That is, money is neutral in the long-run when the exogenous shocks that permanently alter money, η_{mt}, have no permanent effect on output.

Identification from Heteroscedasticity

- Suppose we want to identify the response of monetary policy to stock market
- Problem: identification



Identification

- Consider what happens if the variance of the stock market shocks rises while the variance of the monetary policy shocks remains unchanged. Such a shift causes the realizations of stock market returns and interest rates to more closely trace out the policy reaction function than before.
- The disturbances are distributed around an ellipse that enlarges along the policy reaction function when the shocks to the stock market are more volatile.
- Thus, we are able to identify the slope of the policy reaction function based on changes in the covariance of interest rate and stock market movements across periods when the variance of their shocks shifts.

Identification

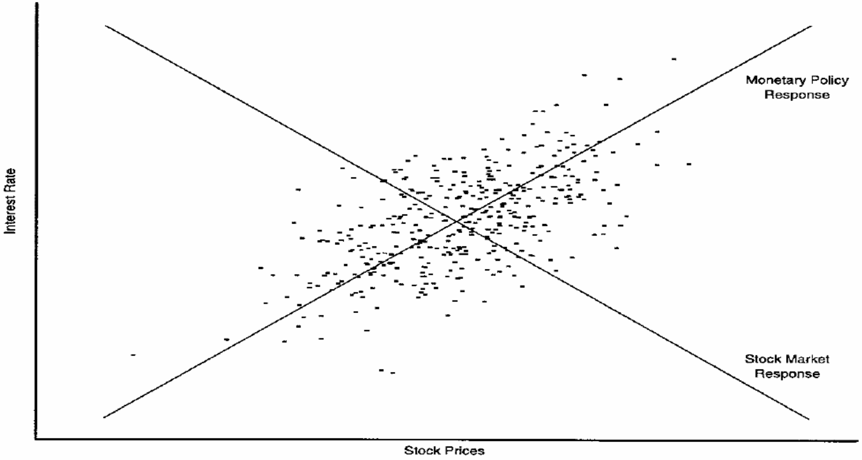
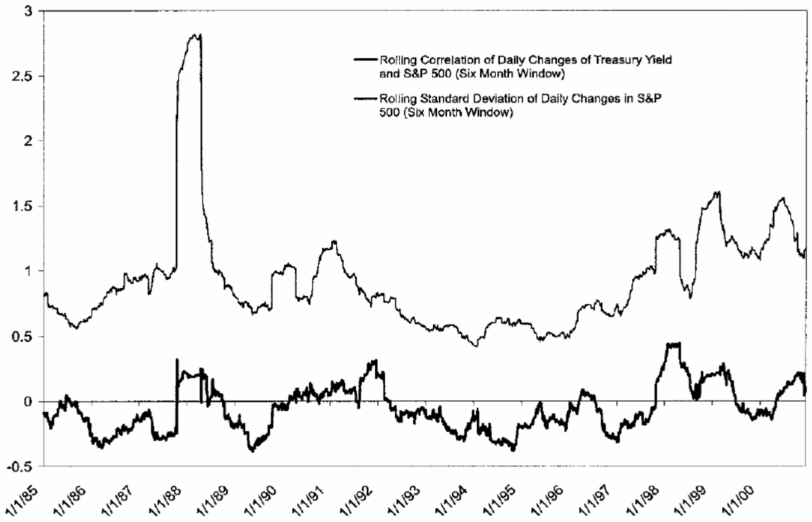


FIGURE II
Periods of High Stock Market Volatility

Evidence



Rolling Correlation of Daily Changes of Treasury Yield and S&P 500 (Six Month Window)
Rolling Standard Deviation of Daily Changes in S&P 500 (Six Month Window)
3
2.5
2
1.5
1
0.5
0
-0.5
1/1/85
1/1/86
1/1/87
1/1/88
1/1/89
1/1/90
1/1/91
1/1/92
1/1/93
1/1/94
1/1/95
1/1/96
1/1/97
1/1/98
1/1/99
1/1/00

Identification from Heteroscedasticity

- Suppose that there are two regimes in which the structural errors have variance Σ_{η_1} and Σ_{η_2}
- Let R=B^{-1}, and normalize R to have 1s on the diagonal.
- Then the variance-covariance matrix of reduced form errors is $R\Sigma_{\eta_1}R'$ in the first regime and $R\Sigma_{\eta_2}R'$ in the second regime
- This scheme gives us one way to solve the identification problem
- Two problems:
    - What if R changes across regimes?

Identification from Heteroscedasticity

- Suppose that there are two regimes in which the structural errors have variance Σ_{η_1} and Σ_{η_2}
- Let R=B^{-1}, and normalize R to have 1s on the diagonal.
- Then the variance-covariance matrix of reduced form errors is $R\Sigma_{\eta_1}R'$ in the first regime and $R\Sigma_{\eta_2}R'$ in the second regime
- This scheme gives us one way to solve the identification problem
- Two problems:
    - What if R changes across regimes?
    - What if the variance-covariance matrix of the structural errors does not change (much) across regimes?

Identification from Heteroscedasticity

- Rigobon and Sack (2003) estimate the effects of monetary policy shocks on asset prices in this way in a VAR with stock prices and short-term interest rates.
- The two regimes are (i) days of FOMC announcements and monetary policy testimonies and (ii) all other days.
- It seems reasonable to suppose that the variance of structural monetary policy shocks is greater in the first regime than the second.
- They estimate the effects of a surprise tightening in monetary policy on stock price; a 25 basis point monetary policy surprise lowers stock prices by about 2 percentage points.

Identification with Sign Restrictions (Uhlig, 2005)

- The standard approach of using a Cholesky decomposition or applying short-run or long-run restrictions to recover the structural shocks as suggested by e.g. Sims (1980), Blanchard and Quah (1989) and Gali (1992) are hard to reconcile with standard theoretical models.
- Instead of imposing hard restrictions on the model coefficients, sign restrictions only impose relatively weak prior beliefs on the responses of the form: "x does not increase y for a certain period of time".
- While it is difficult to impose sign restrictions directly on the coefficient matrix of the model, it is easy to impose them ex-post on a set of orthogonalised impulse response functions.
- Thus, sign restrictions essentially explore the space of orthogonal decompositions of the shocks to see whether the responses conform with the imposed restrictions
- In addition to making a choice about the signs of the responses, one has to specify for how long these restrictions apply after the impact of the shock.

Identification with Sign Restrictions (Uhlig, 2005)

- The steps involved in recovering the structural shocks, given a set of sign restrictions, can be summarised as follows:
- Run an unrestricted VAR in order to get reduced form parameters and reduced form covariance matrix
- Extract the orthogonal innovations from the model using a Cholesky decomposition. The Cholesky decomposition here is just a way to orthogonalise shocks rather than an identification strategy.
- Calculate the resulting impulse responses .
- Randomly draw an orthogonal impulse vector U
- Multiply the responses from Step 3 times U and check if they match the imposed signs.
- If yes, keep the response. If not, drop the draw.
- Repeat the above steps

Orthogonal Impulse Vector U

- Any matrix R that satisfies $\Sigma = RR'$ can be written as $R = PU$ where the Cholesky factorization of Σ is $\Sigma = PP'$ and U is an orthonormal matrix.
- The standard prior for U is uniform on the space of orthonormal matrices (and independent of the other priors).
- Rubio-Ramirez, Waggoner and Zha (2010) propose a simple algorithm for this.
- Let X be a matrix of independent standard normal random numbers, and let $X = QR$ be its QR decomposition (this is a decomposition into an orthonormal matrix Q and an upper triangular matrix R).
- Normalize the diagonal elements of R to be positive. Now Q is uniformly distributed on the space of orthonormal matrices.

- In simulating this posterior distribution, care must be taken that the posterior is approximated using a sufficiently large number of reduced form draws as well as a sufficiently large number of rotations for each posterior draw from the reduced form.
- Given the posterior distribution of the structural impulse responses we can make probability statements about the structural impulse responses.
- The standard approach in the literature for many years has been to report the vector of pointwise posterior medians of the structural impulse responses as a measure of the central tendency of the impulse response functions.

Interpretation in SVAR with Sign Restrictions

- A fundamental problem in interpreting VAR models identified based on sign restrictions is that there is not a unique point estimate of the structural impulse response functions.
- Unlike conventional structural VAR models based on short-run restrictions, sign- identified VAR models are only set identified.
- This problem arises because sign restrictions represent inequality restrictions.

Interpretation in SVAR with Sign Restrictions

- The cost of remaining agnostic about the precise values of the structural model parameters is that the data are potentially consistent with a wide range of structural models that are all admissible in that they satisfy the identifying restrictions. Without further assumptions there is no way of knowing which of these models is most likely.
- A likely outcome in practice is that the structural impulse responses implied by the admissible models will disagree on the substantive economic questions of interest.

Identification from high frequency data

- Suppose that we have federal funds futures, and we treat these as expectation of the future federal funds rate.
- Around the time of an FOMC announcement, we can run the regression

$$\Delta ff_{t,h} = \beta_h MPS_t + \varepsilon_t$$

- where $\Delta ff_{t,h}$ denotes the change in the h -month ahead federal funds futures contract from just before the FOMC announcement to just after and MPS_t is the difference between the FOMC's actual policy decision and the ex-ante expectation.
- If we assume that the unexpected component of the FOMC decision is a monetary policy shock, then β_h measures the impulse response of a monetary policy shock on interest rates h months later. .

Identification from high frequency data

- This gives us restrictions on the impulse responses estimated at lower frequency.
- Given the combination of our estimate of $\Theta(L)$ and elements of $\Psi(L)$ we can in principle back out R and hence other impulse responses.
- This was done in Faust, Swanson and Wright (2003).
- The confidence intervals for the impulse responses are wide, but avoid the "price puzzle".