Midterm Examination
Econ 835
Fall 2017

1. (30 points) Consider the following bivariate structural VAR model of Blanchard and Quah

$$\Delta y_t = \gamma_{10} - b_{12} y_{2t} + \gamma_{11} y_{1,t-1} + \gamma_{12} y_{2,t-1} + \varepsilon_{1t}$$

$$u_t = \gamma_{20} - b_{21} y_{1t} + \gamma_{21} y_{1,t-1} + \gamma_{22} y_{2,t-1} + \varepsilon_{2t}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tilde{\ } iid \left[\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right]$

(a) y_t is log of GDP and u$_t$ is unemployment rate. Can you estimate the above two equations by OLS separately? Explain.

(b) In matrix form, the above model can be written as $BY_t = \Gamma_0 + \Gamma_1 Y_{t-1} + \varepsilon_t$, where $E(\varepsilon_t \varepsilon_t') = \Sigma$ is a diagonal matrix. The reduced form representation of the above VAR is $Y_t = A_0 + A_1 Y_{t-1} + u_t$, where $A_0 = B^{-1}\Gamma_0, A_1 = B^{-1}\Gamma_1, u_t = B^{-1}\varepsilon_t$. $E(u_t u_t') = \Omega$ is a non-diagonal matrix. Solve the reduced form errors in terms of structural errors.

(c) Is the model identified?

(d) Why do we care about identification in the above model?

(e) Blanchard-Quah assumed that the long-run impact of transitory shock on the level of GDP is zero implying that the long-run impact matrix becomes lower triangular. In particular, the long-run matrix takes the form

$$\Theta(1) = \begin{pmatrix} \Theta_{11}(1) & 0 \\ \Theta_{21}(1) & \Theta_{22}(1) \end{pmatrix}$$

where $\Theta_{ij}(1)$ refers to the long-run cumulative impact of jth shock on ith variable. Show that the restriction $\Theta_{12}(1) = 0$ solves the identification problem in the above model.

(f) In addition to the long-run restriction $\Theta_{12}(1) = 0$ imposed by Blanchard-Quah, if we also impose $b_{12} = 0$ we have overidentified model. How would we test whether our overidentification restrictions are valid?

2. (20 points) Consider a UC model with AR(1) cyclical component.

$$y_t = \tau_t + c_t$$

$$\tau_t = \mu + \tau_{t-1} + v_t, v_t\tilde{\ }iidN(0, \sigma_v^2)$$

$$c_t = \phi c_{t-1} + e_t, e_t\tilde{\ }iidN(0, \sigma_e^2)$$

(a) Assume that transitory shocks and permanent shocks are correlated with each other, i.e., $\text{cov}(\text{v}_t, e_t) \neq 0$. Is the model identified?

(b) Now assume that the trend has time-varying mean and the model becomes

$$y_t = \tau_t + c_t$$

$$\tau_t = \mu_{t-1} + \tau_{t-1} + v_t, v_t\tilde{\ }iidN(0, \sigma_v^2)$$

$$\mu_t = \mu_{t-1} + w_t, w_t\tilde{\ }iidN(0, \sigma_w^2)$$

$$c_t = \phi c_{t-1} + e_t, e_t\tilde{\ }iidN(0, \sigma_e^2)$$

Write the measurement and the transition equation for this model.

(c) How would you test whether the mean of the trend, τ_t is time-varying?

(d) There was a break in the volatility of GDP in the US in 1982. Assume that y_t is GDP in above model. However, the model above assumes iid error terms. How would you modify the above model to capture the break in volatility. Write the measurement equation and transition equation for the modified model.