Midterm Examination
Econ 835
Spring 2016

1 Analytical Questions

1. (10 points) Consider a bivariate VAR(p) model

$$A(L)y_t = \varepsilon_t, A(L) = I - A_1L - A_2L^2 - .....A_pL^p$$

$$\varepsilon_t \tilde{} iid(0, \Sigma)$$

with Wold moving average representation $y_t = \Psi(L)\varepsilon_t$,where $\Psi(L) = \sum_{k=0}^{\infty} \Psi_k L^k$ and $\Psi_0 = I_2$

   (a) Find the moving average coefficients Ψ_k for VAR(1) model.

   (b) Show that the moving average coefficients for a VAR(2) model can be found recursively by

$$\Psi_0 = I_2, \Psi_1 = A_1, and$$

$$\Psi_k = A_1\Psi_{k-1} + A_2\Psi_{k-2}, k > 1$$

2. (8 points) Consider a following vector error-correction model (VECM).

$$\Delta y_t = \phi_{11}\Delta y_{t-1} + \phi_{12}\Delta c_{t-1} + \theta_1(y_{t-1} - c_{t-1}) + \varepsilon_{1t}$$

$$\Delta c_t = \phi_{21}\Delta y_{t-1} + \phi_{22}\Delta c_{t-1} + \theta_2(y_{t-1} - c_{t-1}) + \varepsilon_{2t}$$

where Δy_t is growth rate of GDP and Δc_t is growth rate of consumption. Income and consumption tend to move together in the long-run, therefore, $y_{t-1} - c_{t-1}$ is stationary. Represent the above model in a state-space framework.

3. (8 points) Consider a UC model with AR(1) cyclical component.

$$y_t = \tau_t + c_t$$

$$\tau_t = \mu + \tau_{t-1} + v_t, v_t \tilde{} iidN(0, \sigma_v^2)$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + e_t, e_t \tilde{} iidN(0, \sigma_e^2)$$

Assume that transitory shocks and permanent shocks are correlated with each other, i.e., $\text{cov}(\text{v}_t, e_t) \neq 0$. Is the model identified?

2 Empirical Questions

1. (14 points) Consider the following reduced form bivariate VAR

$$y_{1t} = 0.8y_{1,t-1} + 0.1y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = 1.1y_{1,t-1} - 0.4y_{2,t-1} + \varepsilon_{2t}$$

   (a) Is this system covariance stationary?

   (b) What is the identification problem associated with the above reduced form VAR?

   (c) What is identification through sign restrictions? Are the structural shocks just identified? Explain.

2. (10 points) An ARMA(2,2) model was fitted to the real GDP growth in the US. The estimated model is

$$\Delta y_t = 0.81 + 1.20\Delta y_{t-1} - 0.70\Delta y_{t-2} + \widehat{\varepsilon_t} - 1.0\widehat{\varepsilon}_{t-1} + 0.5\widehat{\varepsilon}_{t-2}$$

(a) Calculate the Beveridge-Nelson Stochastic trend for this model.

(b) If $\varepsilon \tilde{} iidN(0, 4)$, then what is the long-run variance of the above estimated model?