Midterm Examination
Econ 835
Spring 2017

1. (20 points) Consider the following bivariate structural VAR

$$y_{1t} = \gamma_{10} - b_{12}y_{2t} + \gamma_{11}y_{1,t-1} + \gamma_{12}y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = \gamma_{20} - b_{21}y_{1t} + \gamma_{21}y_{1,t-1} + \gamma_{22}y_{2,t-1} + \varepsilon_{2t}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tilde{} iid \left[\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right]$

(a) Can you estimate above two equations by OLS separately? Explain.

(b) In matrix form, the above model can be written as $BY_t = \Gamma_0 + \Gamma_1 Y_{t-1} + \varepsilon_t$, where $E(\varepsilon_t \varepsilon_t') = \Sigma$ is a diagonal matrix. The reduced form representation of the above VAR is $Y_t = A_0 + A_1 Y_{t-1} + u_t$, where $A_0 = B^{-1}\Gamma_0, A_1 = B^{-1}\Gamma_1, u_t = B^{-1}\varepsilon_t$. $E(u_t u_t') = \Omega$ is a non-diagonal matrix. Solve the reduced form errors in terms of structural errors. Is the model identified?

(c) Why do we care about identification in the above model?

(d) Suppose b_{12}=0, how would you use the instrumental variable method to estimate the above VAR model?

(e) Now consider Rigobon and Sack (2003) model of identification through heteroscedasticity where they assume two regimes in which the structural errors have variance Σ_{ε_1} and Σ_{ε_2}. In addition they make the assumption that B matrix (loading on Y) does not change across regime. Show how this assumption leads to the identification of the above structural VAR model.

2. (10 points) Consider the following AR(1) model:

$$\triangle y_t = c + \phi \triangle y_{t-1} + +\varepsilon_t$$

Calculate the formula for the Beveridge-Nelson cycle for this AR(1) model. How would you generalize this formula for AR(p) model?

3. (30 points) Suppose we want to decompose the movements in GDP of France, Germany and Italy into common component and idiosyncratic components. To perform such a decomposition, we write down the GDP growth of these countries as

$$\Delta y_{it} = \theta_i \tau_t + \eta_{it}, i = 1, 2, 3$$

where τ_t is the common component in these three countries, and η_{it} (i=1,2,3) are idiosyncratic components and θ_i are the loadings on the common component. The common component is assumed to follow an AR(1) process

$$\tau_t = \mu + \delta \tau_{t-1} + \varepsilon_t, \varepsilon_t \tilde{\ } iid(0, \sigma_\varepsilon^2)$$

and the idiosyncratic components follow an AR(1) process:

$$\eta_{it} = \phi_i \eta_{it-1} + \upsilon_{it}, \upsilon_{it} \tilde{\ } iid(0, \sigma_i^2), i = 1, 2, 3$$

(a) Write the above model in a state space system with a measurement equation and a transition equation. Assume cov$(\varepsilon_t, \upsilon_{it}) = 0$.

(b) Can we identify θ_i and σ_ε^2 for each country separately?

(c) Write the prediction and filtering equations for the above system (Kalman filter algorithm).

(d) How much of the variation in GDP of each country is due to the common factor and how much of the variation is due to the idiosyncratic factor? Write the formula for this variance decomposition.

(e) It has been argued that business cycles show time-varying sensitivity to interest rates. If you want to examine this hypothesis in the present example, what kind of model would you estimate? Assume that the common component τ_t is a proxy for business cycles in the EMU.

(f) Write the state space representation of the above model without an intercept in the transiton equation. That is, reformulate the state space representation above such that the transition equation takes the form: $\beta_t = F\beta_{t-1} + v_t$.