Housing and Credit Cycles

Nam Nguyen

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1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$
$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$
$$\tau_{ht} = \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

Cycles:

$$c_{yt} = \phi^1_y c_{yt-1} + \phi^2_y c_{yt-2} + \phi^x_y c_{ht-1} + \varepsilon_{yt}, \qquad \varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y}) \tag{5}$$
$$c_{ht} = \phi^1_h c_{ht-1} + \phi^2_h c_{ht-2} + \phi^x_h c_{yt-1} + \varepsilon_{ht}, \qquad \varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h}) \tag{6}$$

State-Space Model

Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{7}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_y^1 & \phi_y^2 & 0 & \phi_y^x & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_h^x & 0 & 0 & \phi_h^1 & \phi_h^2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix} \tag{8}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma_{\eta y}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon y}^2 & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta h}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma_{\varepsilon h}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{10}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

Following Kim & Nelson Chap 2, I set up the constraints on the autoregressive parameters to imply stationary and $-1 < \phi_x^y < 1$ as follow:

$$\phi_x^y = \frac{\phi_x^y}{1 + |\phi_x^y|} \qquad\qquad \phi_x^h = \frac{\phi_x^h}{1 + |\phi_x^h|}$$

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite matrix, in order to ensure feasible maximum likelihood estimation process. Furthermore, I suppressed the cross trend covariance term to be zero.

4 Regression results

In this following section, I will apply the UC model to data from 6 countries: US, UK, Germany, France, Japan and South Korea.

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. The following estimates are selected in the manner that they would look the most stable. Perhaps a more optimal constraint on the autoregressive parameters would solve this issue.

Table 1: Correlated UC model Estimates: US data

Description	Estimate	Standard Error
ϕ_y^1	-0.8936	0.0347
ϕ_h^1	0.8539	0.0236
σ_{ny}	2.0069×10^{-5}	0.0321
σ_{ey}	105.7066	3.7433
σ_{nh}	4.3500	0.3811
σ_{eh}	8.9631	1.1872
σ_{eyeh}	1	3.7814×10^{-12}
μ	0.1731	0.1041
Log-likelihood value	-1617.0038	0

Table 2: Correlated UC model Estimates: UK data

Description	Estimate	Standard Error
ϕ_y^1	1.0094	NaN
ϕ_y^2	-0.1466	NaN
ϕ_h^1	1.0055	NaN
ϕ_h^2	-0.1050	NaN
σ_{ny}^2	1.3415	NaN
σ_{ey}^2	0.8083	NaN
σ_{nh}^2	0.4651	NaN
σ_{eh}^2	2.8995	NaN
σ_{eyeh}	0.5846	NaN
σ_{nynh}	0.9390	NaN
Log-likelihood value	-867.2620	0

5 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

For example, the model for US data shows that there is a positive relationship between a one period lag in short term house price and house hold credit. Also for the UK data, there is a positive relationship between a one period lag in short term credit and house price.

Further development for this paper should include more optimal constraints on parameters to ensure stability.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.

Figure 1: Appendix: US Credit components

Figure 2: US Housing Price components

Figure 3: UK Credit components

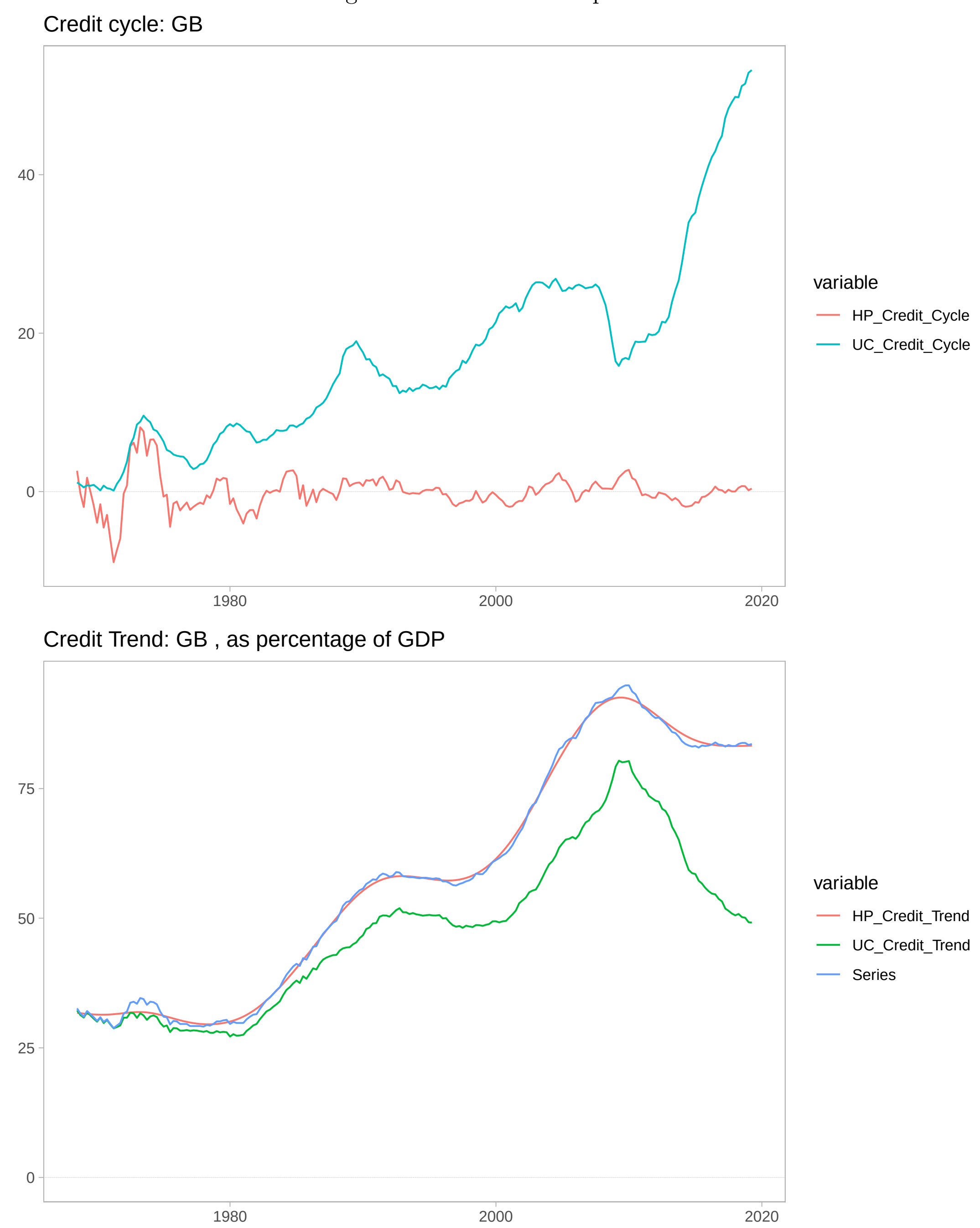

Figure 4: UK Housing Price components