Midterm Examination
Econ 835
Spring 2015

1 Analytical Questions

1. (10 points) Consider the following bivariate structural VAR

$$y_{1t} = \gamma_{10} - b_{12}y_{2t} + \gamma_{11}y_{1,t-1} + \gamma_{12}y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = \gamma_{20} - b_{21}y_{1t} + \gamma_{21}y_{1,t-1} + \gamma_{22}y_{2,t-1} + \varepsilon_{2t}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tilde{\ } iid \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right]$

   (a) Can you estimate above two equations by OLS separately? Explain.

   (b) Show how the identification problem arises if you estimate the above model using a reduced form VAR that has following form

$$Y_t = a_0 + A_1 Y_{t-1} + v_t$$

   where $Y_t = (y_{1t}, y_{2t})', a_0 = \begin{pmatrix} a_{01} \\ a_{02} \end{pmatrix}, A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$

   The error terms are serially uncorrelated, but there is a contemporaneous correlation between cross-equation errors. How does this identification problem get resolved?

2. (20 points) Stock and Watson suggested to combine the state-space approach with the cointegration technique to model the long-run relationship between consumption and GDP. According to them, the level of GDP and consumption can be written as follows:

$$C_t = \tau_t + \eta_{1t}$$

$$Y_t = \theta\tau_t + \eta_{2t}$$

where C_t is log of level of consumption, Y_t is log of level of GDP. τ_t is common trend, η_{it} (i=1,2) are respective cycles of consumption and real GDP. The trend can be written as random walk with a drift:

$$\tau_t = \mu + \tau_{t-1} + \varepsilon_t, \varepsilon_t \tilde{\ } iid(0, \sigma_\varepsilon^2)$$

and the cycles follow AR(2) process:

$$\eta_{it} = \phi_{i1}\eta_{it-1} + \phi_{i2}\eta_{it-2} + \upsilon_{it}, \upsilon_{it} \tilde{\ } iid(0, \sigma_i^2), i = 1, 2$$

(a) Write the above model in a state space system with a measurement equation and transition equation.

(b) Write the prediction and filtering equations for the above system (Kalman filter algorithm).

(c) The relationship between consumption and income may have changed over time. How would you modify the above model to take into account a break in the long-run relationship between consumption and income.

(d) How would the measurement and the transition equation change in the presence of a break in the long-run relationship between consumption and income.

3. (5 points) Consider the following ARMA(2,1) model:

$$\triangle y_t = 0.3 \triangle y_{t-1} + 0.5 \triangle y_{t-1} + \varepsilon_t + 0.2\varepsilon_{t-1}$$

Calculate the Beveridge-Nelson stochastic trend for y_t.

2 Empirical Questions

1. (15 points) This question analyzes the dynamic behavior of pound-dollar exchange rate.

   (a) Suppose we run a regression of the level of pound-dollar exchange rate on its lag using OLS and get the following results

   $$EX = 0.001 + 0.99EX(-1) + error_t$$

   What are the problems associated with the OLS estimate of the above regression? What will you do to test for a problem if there is one?

   (b) Table 1 shows the estimated GARCH model for the changes in exchange rate (DEX) with an AR(1) specification for the conditional mean equation. Is the change in exchange rate a white noise process? Calculate the unconditional mean and unconditional variance of changes in exchange rate.

   (c) Why is this process called conditionally heteroskedastic? Explain.

   (d) Table 2 shows the estimated result for the EGARCH model of changes in exchange rate. Is there an evidence of leverage effect in the above regression? How would you test for the presence of the leverage effect?