Unit Root Tests

Random Walk

- $y_t = y_{t-1} + \varepsilon_t$
- $E_t y_{t+1} = y_t$
- $\text{Var}(y_{t+k}|y_t) = k\sigma^2$
- As k$\rightarrow \infty$,$\text{Var}(y_{t+k}|y_t) \rightarrow \infty$
- Autocovariances of random walk are not defined
- Autocorrelations die out very slowly.

Motivations for Unit Root

- Stochastic trend vs. deterministic trend
- Permanent vs. transitory shocks
- Slowdown of 1970s
- Can be applied to stock returns

Least Square Regression With Unit Roots

- If we estimate the AR(1) model: $y_t = \phi y_{t-1} + \varepsilon_t$.
- We say that the above model has unit root or is non-stationary if $\phi = 1$.
- Suppose we don't know that the above model has unit root and we perform OLS, what are the properties of OLS estimator in that case
- Two properties: Superconsistency and Bias
- In the unit root case of AR coefficient $=1$, the difference between the $\widehat{\phi}_{LS}$ and 1 vanishes quickly as the sample size (T) grows, in fact, it shrinks at the rate of $1/T$. This property is called superconsistency
- It can be shown that the lease squares estimator $\widehat{\phi}_{LS}$, is biased downward so that if the true value of ϕ is ϕ^*, the expected value of $\widehat{\phi}_{LS}$ is less than ϕ^*.

- Other things the same, the larger is the true value of ϕ, the larger the bias; so the bias is worst in the unit root case. The bias is larger if an intercept is included in the regression and larger still if a trend is included.
- The bias vanishes as the sample size grows, as the estimate converges to the true population value, but the bias can be sizeable in small samples.
- Dickey and Fuller looked at the distribution of this kind of test statistics and found that OLS estimates are biased down and OLS standard errors are tighter than actual standard errors.

Inappropriate Detrending

- Suppose the real model is
- $y_t = \mu + y_{t-1} + \varepsilon_t$
- Suppose you detrend by OLS and then fit an AR(1) model
- $y_t = bt + (1 - \phi L)^{-1}\varepsilon_t$
- $(1 - \phi L)y_t = (1 - \phi L)bt + \varepsilon_t = \phi b + b(1 - \phi)t + \varepsilon_t$
- $\widehat{\phi}$ is biased downward and OLS standard errors are misleading.

Unit Root Tests in AR(1) Model

- AR(1) model: $y_t = \phi y_{t-1} + \varepsilon_t$
- $H_0 : \phi = 1$;$H_1 : |\phi| < 1$
- Dickey and Fuller showed that under the null of unit root the standard t-ratio doesn't have a t-distribution, not even asymptotically.
- The reason for this is the nonstationarity of the process invalidating standard results on the distribution of the OLS estimator.
- The appropriate critical values of t-type of tests were derived by Dickey and Fuller

Unit Root Tests in AR(1) Model

- So, two steps are involved in testing for a unit root in an AR(1) model; perform OLS and calculate the usual t-statistic.
- In the second step, obtain critical values for this statistic from Dickey-Fuller distribution
- In particular, DF distribution is skewed to the right, so the critical values are smaller than those of the standard t-distribution.

Trend Stationary Vs. Difference Stationary

- It is possible that nonstationarity is caused by the presence of a deterministic time trend in the process, rather than by the presence of a unit root. This happens when the AR(1) model is extended to
  $Y_t = \delta + \phi Y_{t-1} + \gamma t + \varepsilon_t$
- with $|\phi| < 1$ and $\gamma \neq 0$
- The nonstationarity in the above model can be removed by regressing Y_t upon a constant and t, and then considering the residuals of this regression, or by simply including t as additional variable in the model.

Trend Stationary Vs. Difference Stationary

- The process for Y_t in this case is referred to as being trend stationary. Nonstationary processes may thus be characterized by the presence of a deterministic trend, like γt a stochastic trend implied by a unit root, or both.
- It is possible to test whether Y_t follows a random walk against the alternative that it follows a trend stationary process. This can be tested by running the regression $\triangle Y_t = \delta + (\phi - 1) Y_{t-1} + \gamma t + \varepsilon_t$
- $H_0 : (\phi - 1) = 0$

Other Unit Root Tests

- The simple DF test is valid only if the series is an AR(1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances is violated.
- The augmented Dickey-Fuller (ADF) test constructs a parametric correction for higher-order correlation by assuming that Y_t series follows an AR(p) process $\triangle \mathrm{Y}_t = \delta + (\phi - 1) Y_{t-1} + \gamma t + \beta_1 \triangle Y_{t-1} + \beta_2 \triangle Y_{t-2} + \ldots\ldots + \beta_p \triangle Y_{t-p} + \varepsilon_t$
- The null hypothesis is same as the simple DF test.
- How to choose no of lags: Use AIC or BIC criteria

Other Unit Root Tests

- **Phillips-Perron test**
- Non-parametric method of controlling for serial correlation when testing for a unit root. The PP method estimates the simple DF equation and modifies the t-ratio of the ϕ coefficient so that serial correlation doesn't affect the asymptotic distribution of the test statistic
- **KPSS Test**
- Y_t is assumed to be stationary under the null. The KPSS statistic is based on the residuals from the OLS regression of y_t on exogenous variables that may include time trend t.

Problem with Unit Root Tests

- In general, ADF and PP tests have very low power against I(0) alternatives that are close to being I(1)
- Unit root test can't distinguish highly persistent stationary process from a non-stationary processes very well.
- Also, the power of the unit root tests diminish as deterministic terms are added to the test regression
- For maximum power against very persistent alternatives, the recent test proposed by Elliott, Rothenberg and Stock (1996) and Ng and Perron (2001) should be used.