Housing and Credit Cycles

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1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$

$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = g_{yt} + \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$

$$\tau_{ht} = g_{ht} + \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

$$g_{yt} = g_{yt-1} + w_{yt}, \qquad w_{yt} \sim iidN(0, \sigma^2_{wy}) \tag{5}$$

$$g_{ht} = g_{ht-1} + w_{ht}, \qquad w_{ht} \sim iidN(0, \sigma^2_{wh}) \tag{6}$$

$$\tag{7}$$

Cycles:

$$c_{yt} = \phi^y_1 c_{yt-1} + \phi^y_2 c_{yt-2} + \phi^y_x c_{ht-1} + \varepsilon_{yt} \tag{8}$$

$$c_{ht} = \phi^h_1 c_{ht-1} + \phi^h_2 c_{ht-2} + \phi^h_x c_{yt-1} + \varepsilon_{ht} \tag{9}$$

$$\varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y})$$
$$\varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h})$$

State-Space Model
Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{10}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ g_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ g_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1^y & \phi_2^y & 0 & 0 & \phi_x^y & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \phi_x^h & 0 & 0 & 0 & \phi_1^h & \phi_2^h \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ g_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ g_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ w_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ w_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix} \tag{11}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma^2_{\eta y} & 0 & 0 & 0 & \sigma_{\eta h \eta y} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & \sigma^2_{\varepsilon y} & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & \\ \sigma_{\eta y \eta h} & 0 & 0 & \sigma^2_{\eta h} & 0 & 0 & & \\ 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma^2_{\varepsilon h} & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & \end{bmatrix} \tag{12}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{13}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

Following Morley (2007), I set up the constraints on the autoregressive parameters to imply stationary as follow: (θ_{11}^{y0} and θ_{12}^{y0} are initial estimate values.)

$$
\begin{aligned}
aaa &= \frac{\theta_{11}^{y0}}{1+|\theta^{y0}|} \\
ccc &= (1-|aaa|) * \theta_{12}^{y0}/(1+|\theta_{11}^{y0}|) + |aaa| - aaa^2 \\
\theta_{11}^{y} &= 1 * aaa \\
\theta_{12}^{y} &= -1 * (aaa^2 + ccc)
\end{aligned}
$$

The same applies for the next 3 pairs: θ_{21}^{y} & θ_{22}^{y}, θ_{11}^{h} & θ_{12}^{y}, θ_{21}^{h} & θ_{22}^{h}.

The main difference in my constraint compared to Morley 2007 is that I chose $\theta_{11}^{y} = 1 * aaa$ instead of $\theta_{11}^{y} = 2 * aaa$ to account for the additional term in the transitory components. This allows the autoregressive parameters to be stationary.

However, the lower factor might take away variation in the transitory components as seen in the cases of France, Germany and Japan in the graphs section.

4 Regression results

In this following section, I will apply the UC model to data from 6 countries: US, UK, Germany, France, Japan and South Korea.

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. The following estimates are selected in the manner that they would look the most stable. Perhaps a more optimal constraint on the autoregressive parameters would solve this issue.

Table 1: Correlated UC model Estimates: US data

Description	Estimate	Standard Error
θ_1^y	-0.0283	0.0069
θ_2^y	-0.9751	0.0087
θ_h^y	-0.0469	0.0180
θ_1^h	0.6666	0.1073
θ_2^h	0.0538	0.1153
θ_y^h	0.7614	0.1653
σ_{nh}^2	0.6676	0.0396
σ_{nc}^2	1.4727×10^{-5}	3.1940×10^{-6}
σ_{eh}^2	4.3164×10^{-9}	8.4888×10^{-10}
σ_{ec}^2	0.3763	0.0849
σ_{nhnc}	3374.4131	671.2603
σ_{ehec}	1.0000	0.0050
σ_{wyy}	0.0393	0.0139
σ_{whh}	0.5783	0.1301
Log-likelihood value	-505.2641	0

Table 2: Correlated UC model Estimates: UK data

Description	Estimate	Standard Error
θ_1^y	-0.9750	0.2700
θ_2^y	-1.0000	4.6732×10^{-5}
θ_h^y	0.7299	0.1533
θ_1^h	1.2597	0.2614
θ_2^h	-0.8025	0.0627
θ_y^h	2.1172	0.9995
σ_{nh}^2	1.0671	0.0642
σ_{nc}^2	1.5677	0.1720
σ_{eh}^2	0.3398	0.0864
σ_{ec}^2	1.3590	0.2078
σ_{nhnc}	-0.0215	0.1112
σ_{ehec}	1	4.4334×10^{-17}
σ_{wyy}	0.0426	0.0209
σ_{whh}	0	0
Log-likelihood value	-797.0844	0

Table 3: Correlated UC model Estimates: Germany data

Description	Estimate	Standard Error
θ_1^y	-0.4796	0.2245
θ_2^y	0.5167	0.2238
θ_h^y	2.3642	0.5314
θ_1^h	0.0046	0.0095
θ_2^h	-0.9824	0.0064
θ_y^h	-0.0110	0.0043
σ_{nh}^2	0.4569	0.0467
σ_{nc}^2	0.6458	0.0429
σ_{eh}^2	1.5273×10^{-5}	5.1511×10^{-6}
σ_{ec}^2	0.0175	0.0066
σ_{nhnc}	-0.1112	0.0418
σ_{ehec}	0.9965	0.0853
σ_{wyy}	0.1122	0.0345
σ_{whh}	0.0613	0.0233
Log-likelihood value	-432.4172	0

Table 4: Correlated UC model Estimates: France data

Description	Estimate	Standard Error
θ_1^y	1.9364	NaN
θ_2^y	-0.9830	NaN
θ_h^y	-0.2224	NaN
θ_1^h	-0.2691	NaN
θ_2^h	-0.9956	NaN
θ_y^h	2.3783	NaN
σ_{nh}^2	0.8532	NaN
σ_{nc}^2	0.6429	NaN
σ_{eh}^2	0.0367	NaN
σ_{ec}^2	0.1004	NaN
σ_{nhnc}	-0.0859	NaN
σ_{ehec}	1	NaN
σ_{wyy}	0.0384	NaN
σ_{whh}	0.1799	NaN
Log-likelihood value	-461.7939	0

Table 5: Correlated UC model Estimates: Japan data

Description	Estimate	Standard Error
θ_1^y	0.4538	NaN
θ_2^y	-0.9992	NaN
θ_h^y	-853.3103	NaN
θ_1^h	1.1790	NaN
θ_2^h	-0.6177	NaN
θ_y^h	-0.0007	NaN
σ_{nh}^2	8.2483×10^{-142}	NaN
σ_{nc}^2	1.0244	NaN
σ_{eh}^2	0.9830	NaN
σ_{ec}^2	0.0010	NaN
σ_{nhnc}	1.0421	NaN
σ_{ehec}	-1	NaN
σ_{wyy}	0.0072	NaN
σ_{whh}	0.3989	NaN
Log-likelihood value	-704.7611	0

Table 6: Correlated UC model Estimates: Korea data

Description	Estimate	Standard Error
θ_1^y	-1.7124	0.1013
θ_2^y	-0.8386	0.0774
θ_h^y	3.1949	1.4055
θ_1^h	1.5518	0.0847
θ_2^h	-0.9895	0.0475
θ_y^h	0.1297	0.0262
σ_{nh}^2	1.9489	0.1225
σ_{nc}^2	0.9548	0.1387
σ_{eh}^2	0.0508	0.3306
σ_{ec}^2	0.0586	0.1631
σ_{nhnc}	0.2683	0.1110
σ_{ehec}	0.9925	0.0887
σ_{wyy}	0.1048	0.0742
σ_{whh}	1.5414	0.3966
Log-likelihood value	-738.8591	0

5 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

For example, the model for US data shows that there is a positive relationship between a one period lag in short term house price and house hold credit. Also for the UK data, there is a positive relationship between a one period lag in short term credit and house price.

Further development for this paper should include more optimal constraints on parameters to ensure stability.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.

Figure 1: Appendix: US Credit components

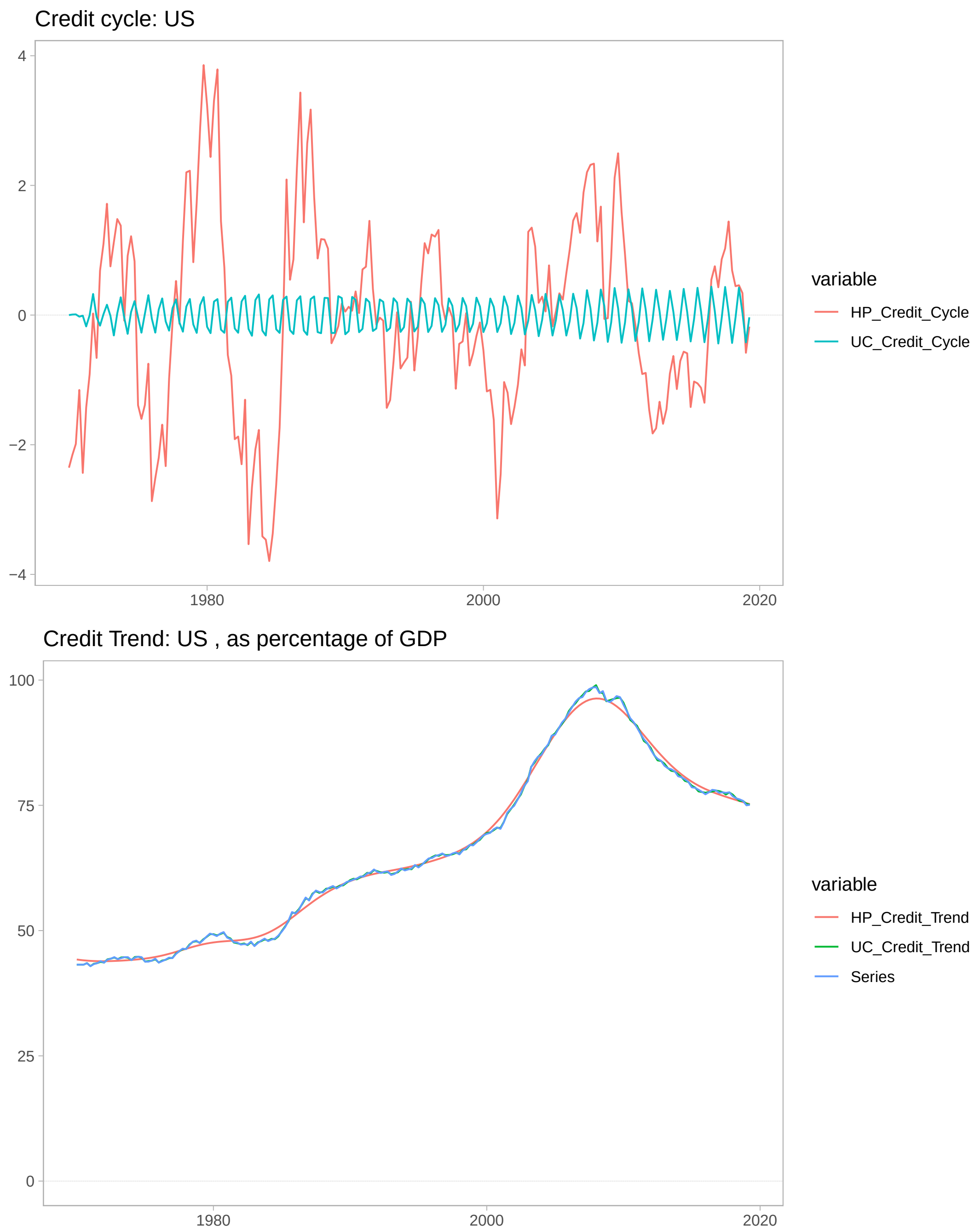
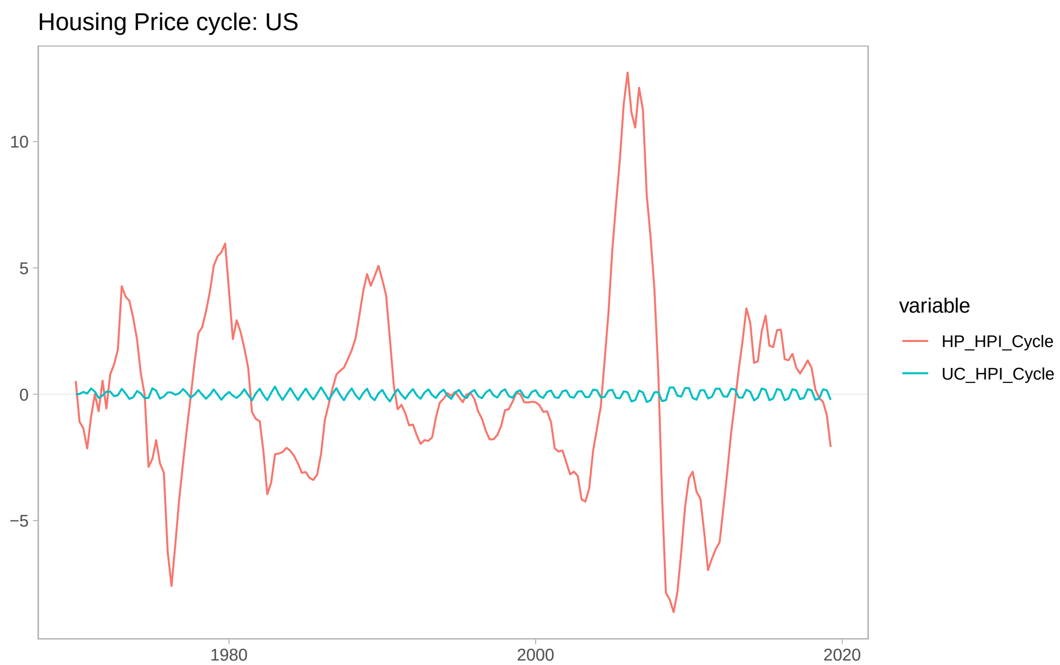


Figure 2: US Housing Price components



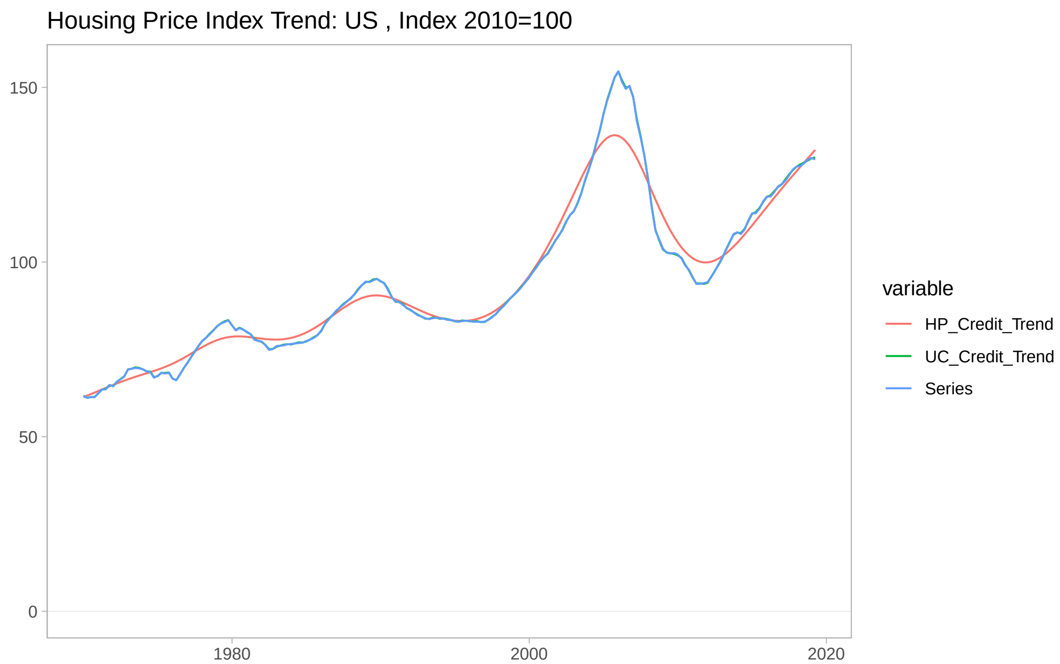
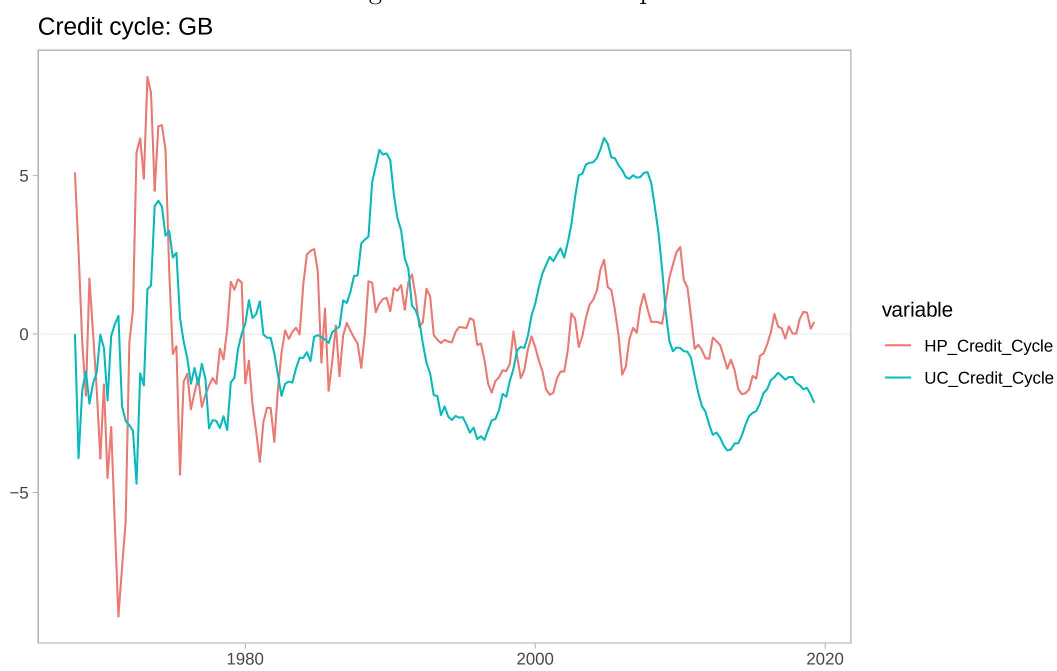


Figure 3: UK Credit components



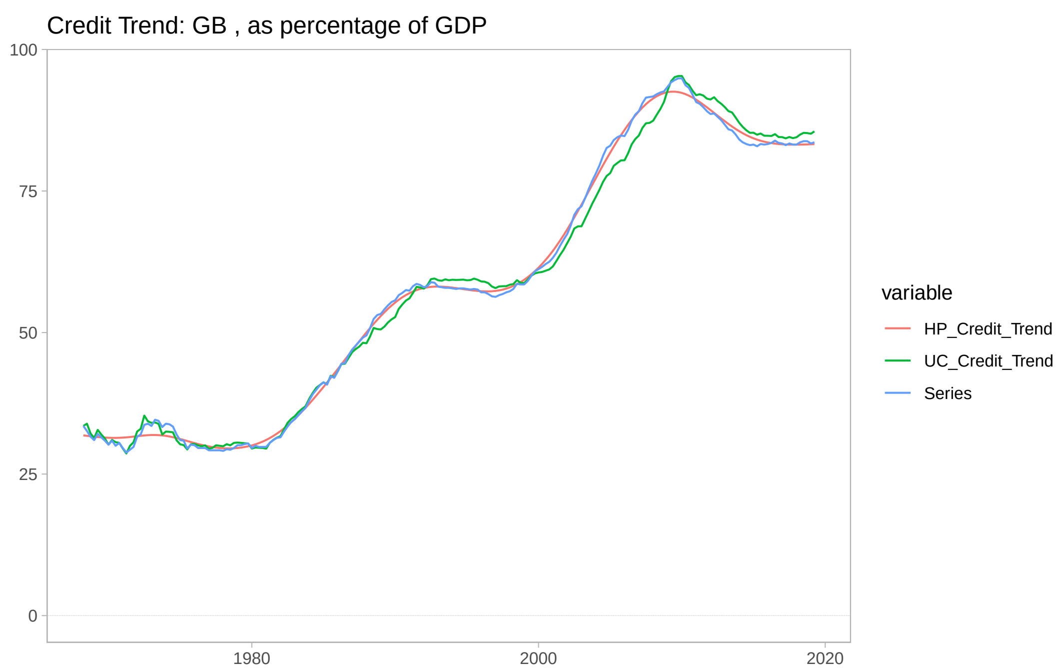
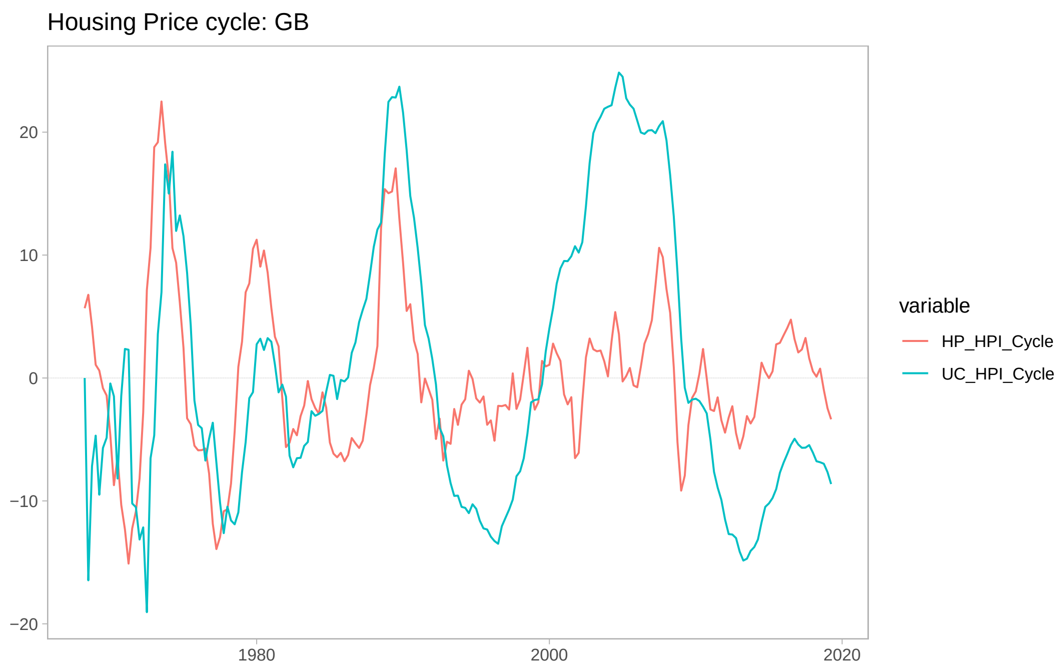


Figure 4: UK Housing Price components



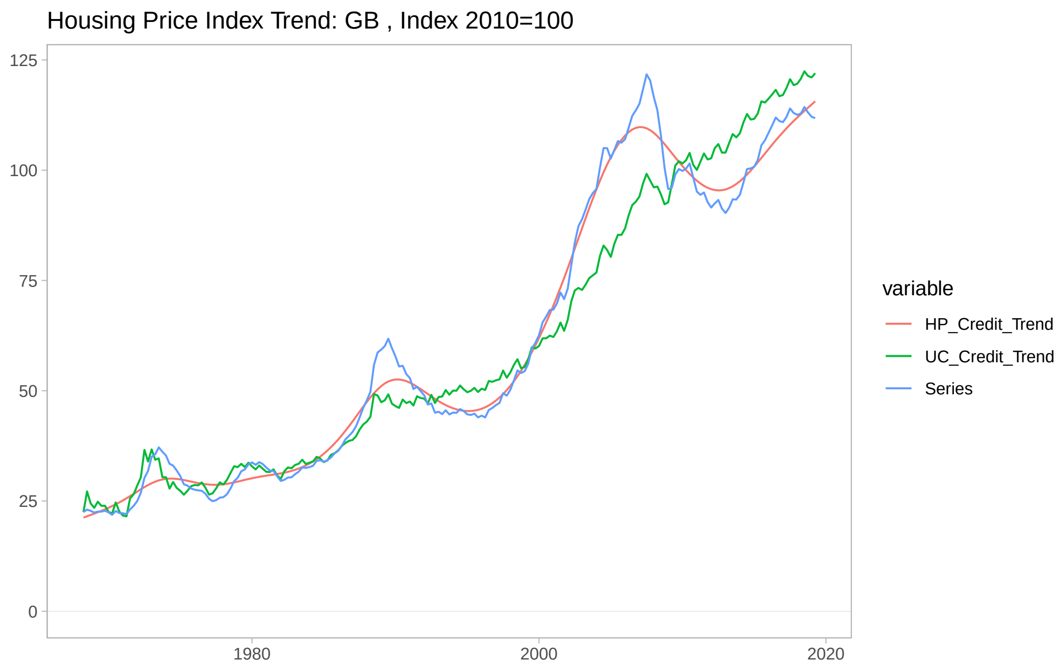
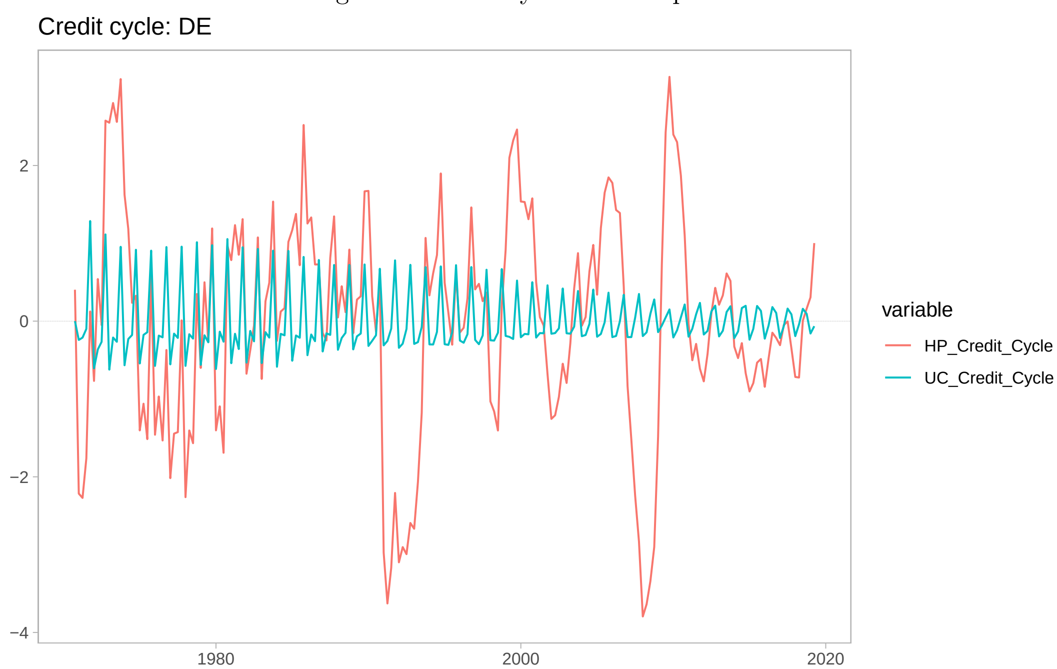


Figure 5: Germany Credit components



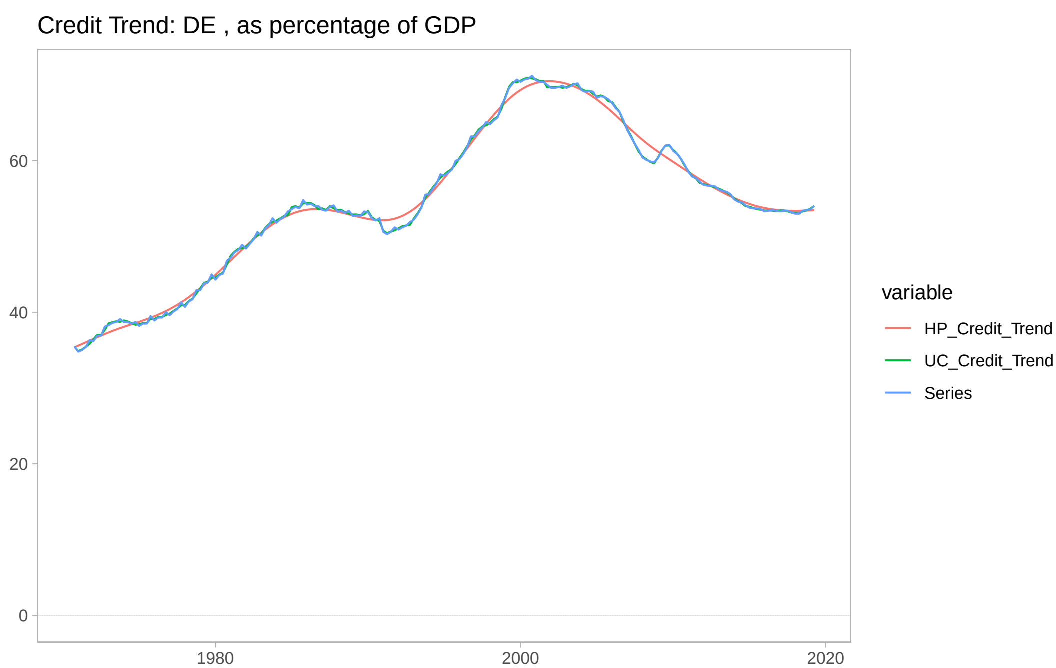
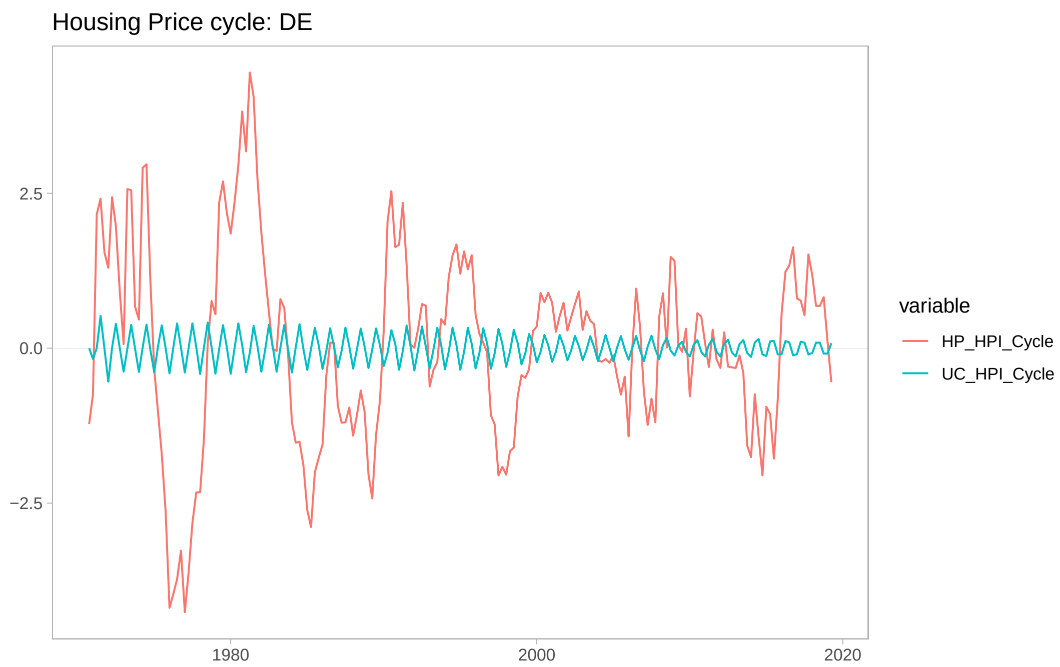


Figure 6: Germany Housing Price components



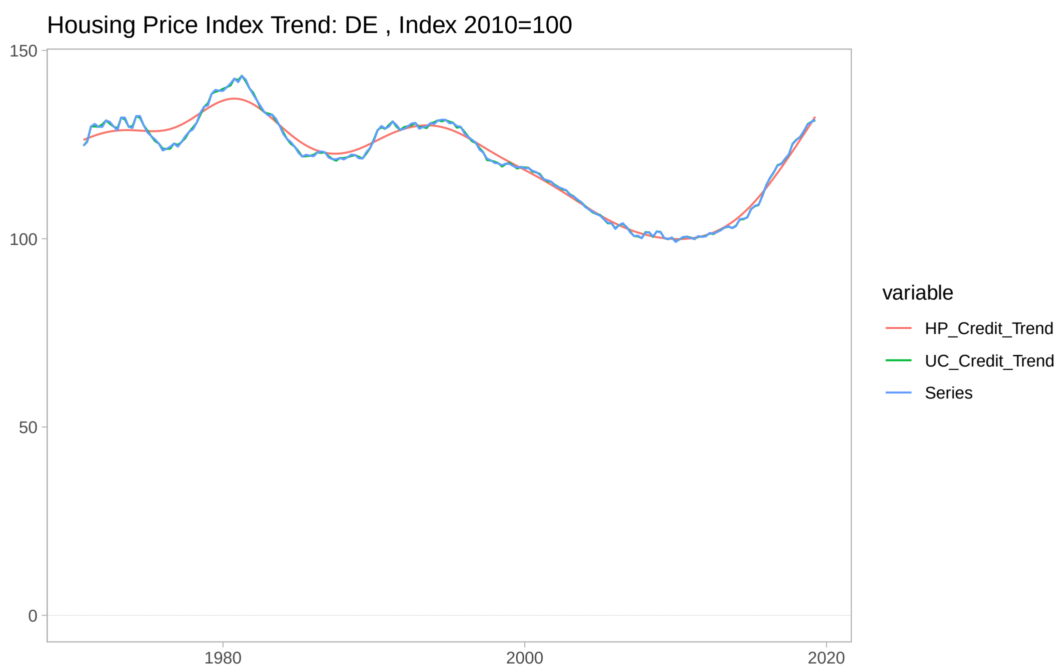
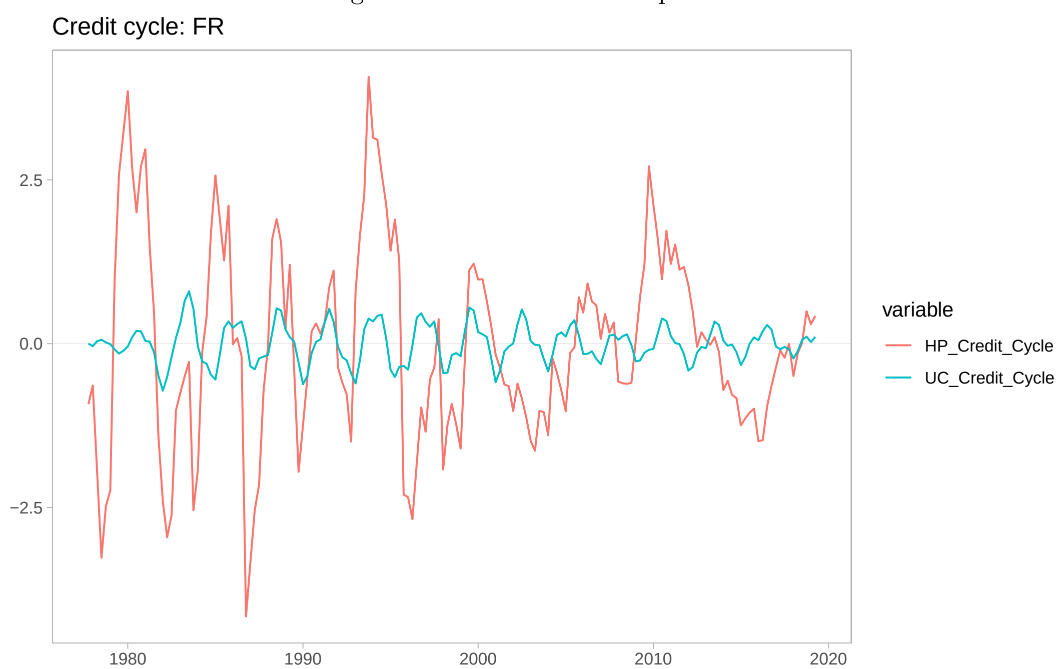


Figure 7: France Credit components



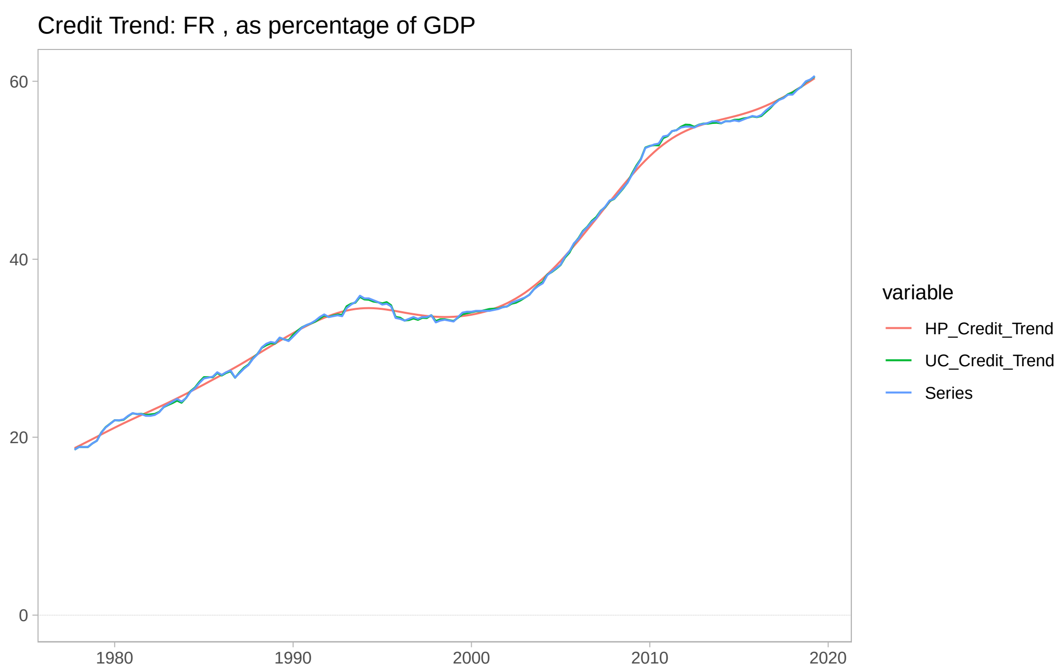
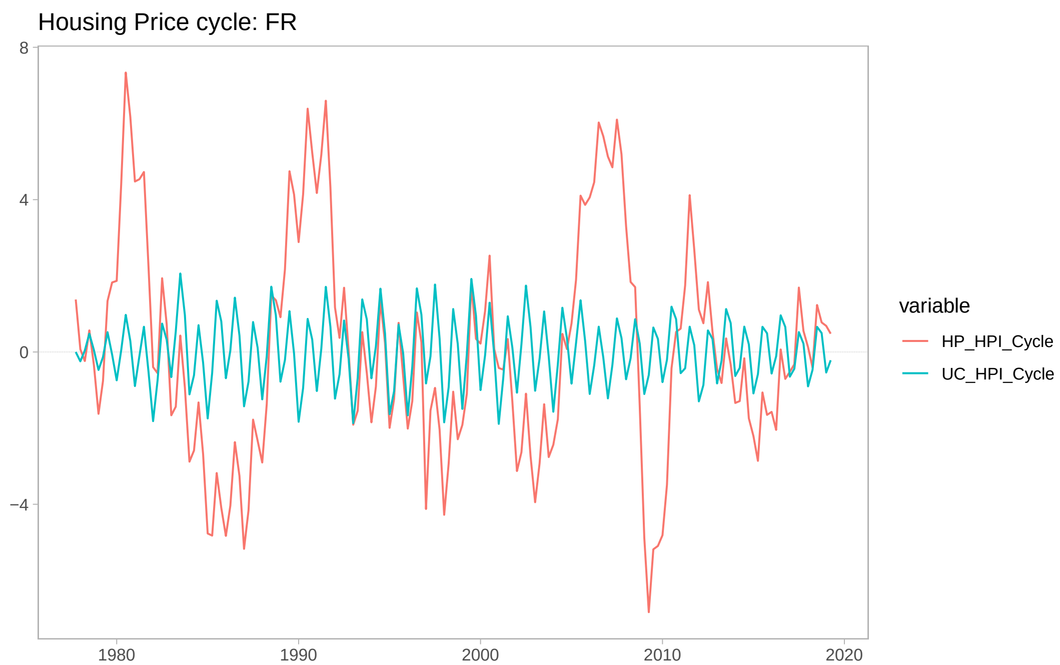


Figure 8: France Housing Price components



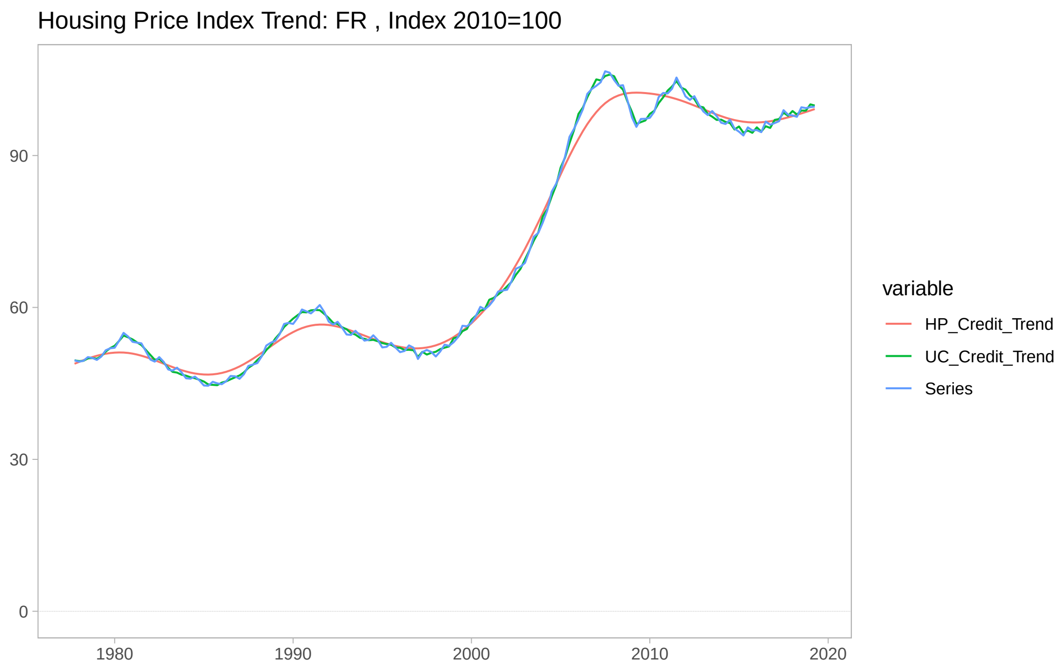
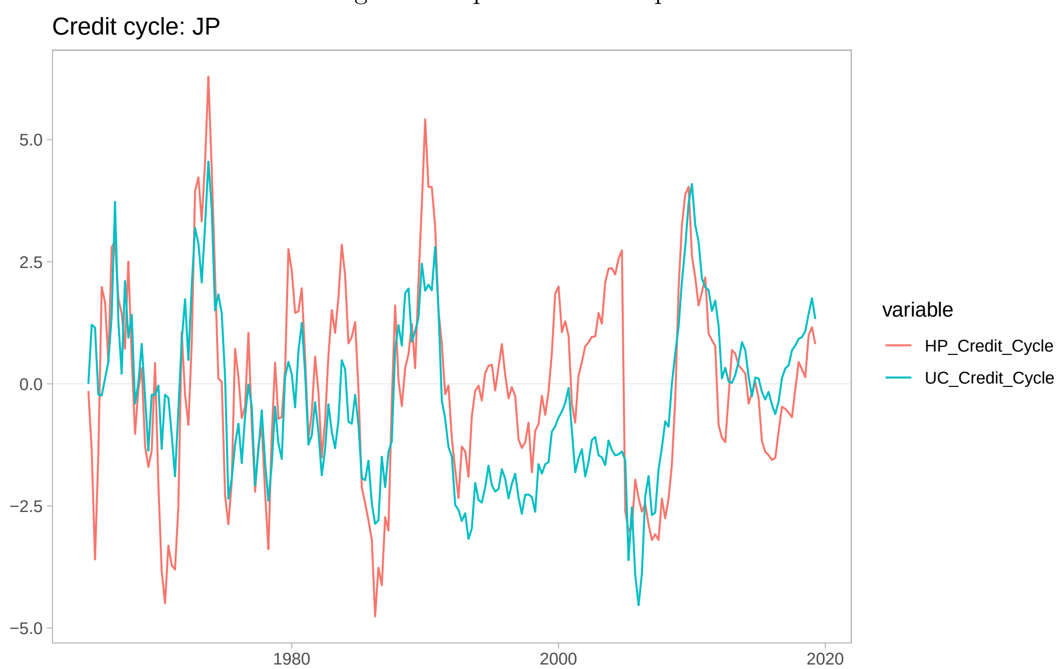


Figure 9: Japan Credit components



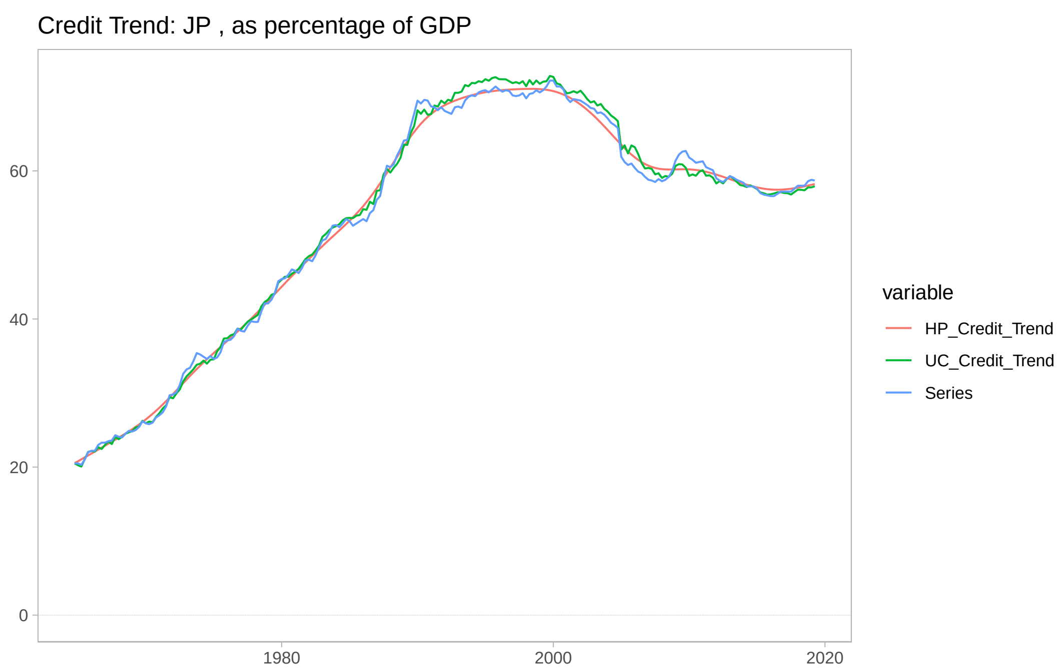
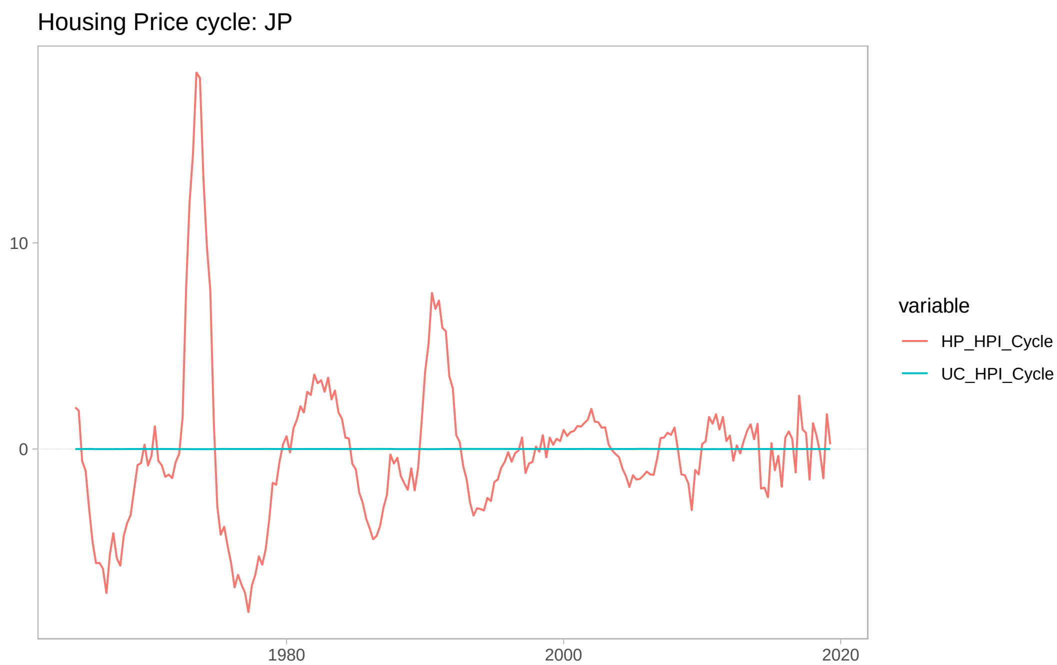


Figure 10: Japan Housing Price components



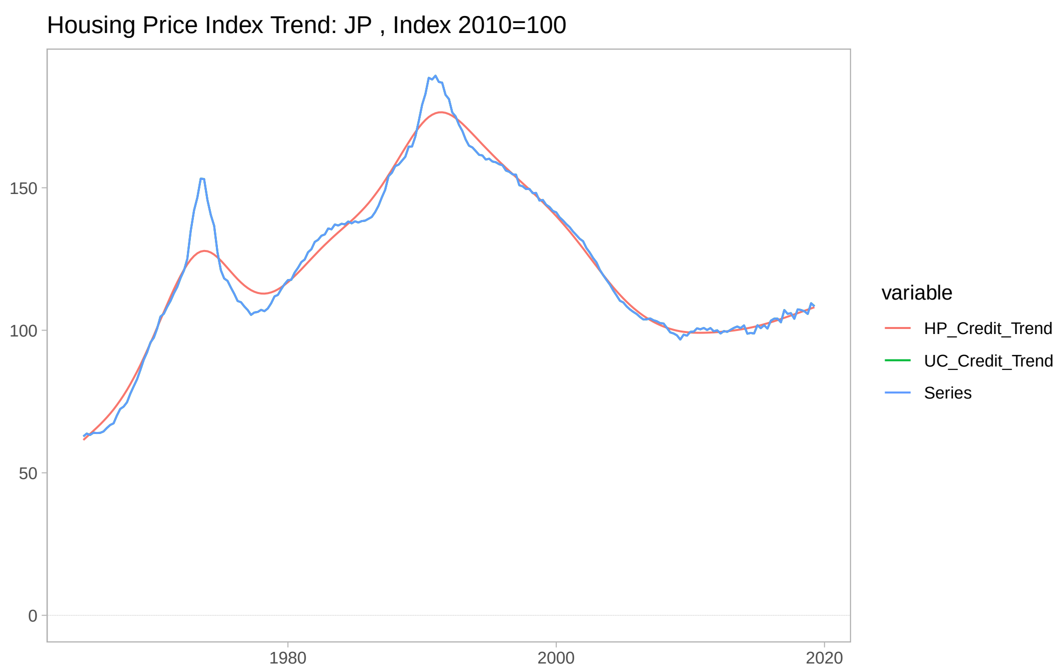
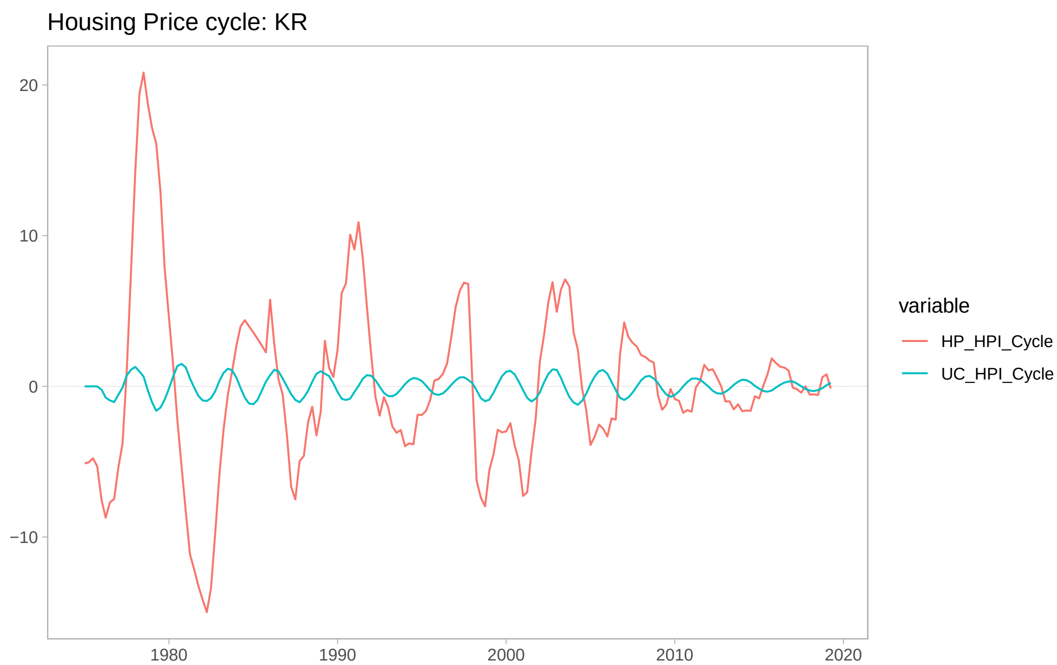
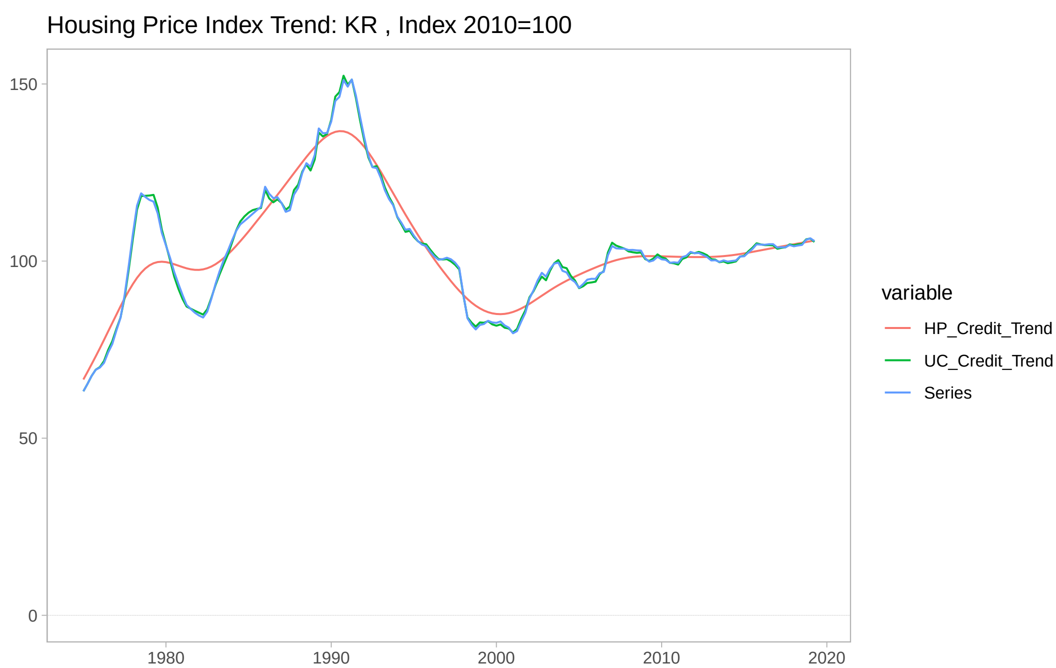


Figure 11: Korea Credit components

Figure 12: Korea Housing Price components





6 Impulse Response Function

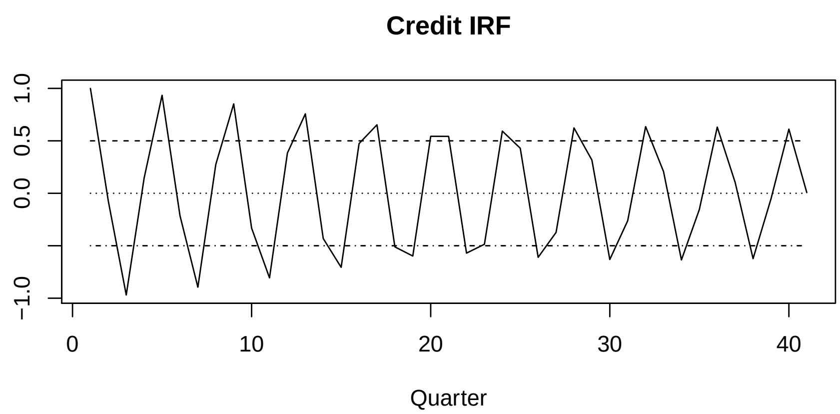
This section show IRFs that are really unstable. I am guessing that is because the way I specify the function:

Instead of normally having: $\psi_t = \theta^y_{11} * \psi_l + \theta^y_{12} * \psi_{ll}$

I specify the IRF as: $\psi_t = \theta^y_{11} * \psi_l + \theta^y_{12} * \psi_{ll} + \theta^y_{21} * \psi_l + \theta^y_{22} * \psi_{ll}$

This potentially causes the unstability in the following IRF graphs. Also the fact that the constraints for autoregressive parameters have not been optimally setup could cause the issue.

Figure 13: US IRF



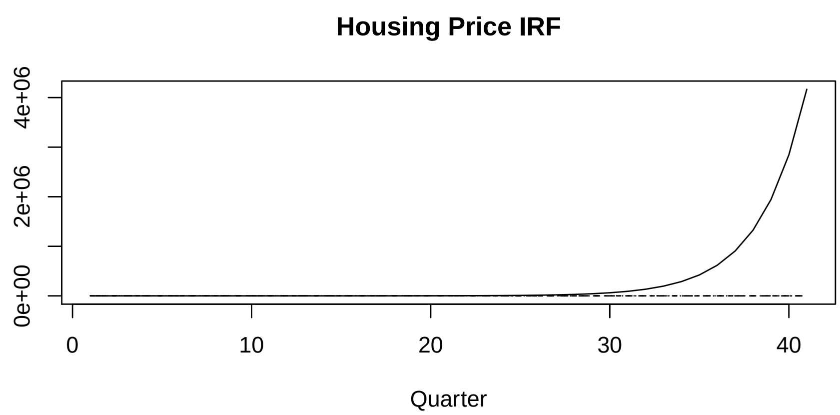


Figure 14: UK IRF

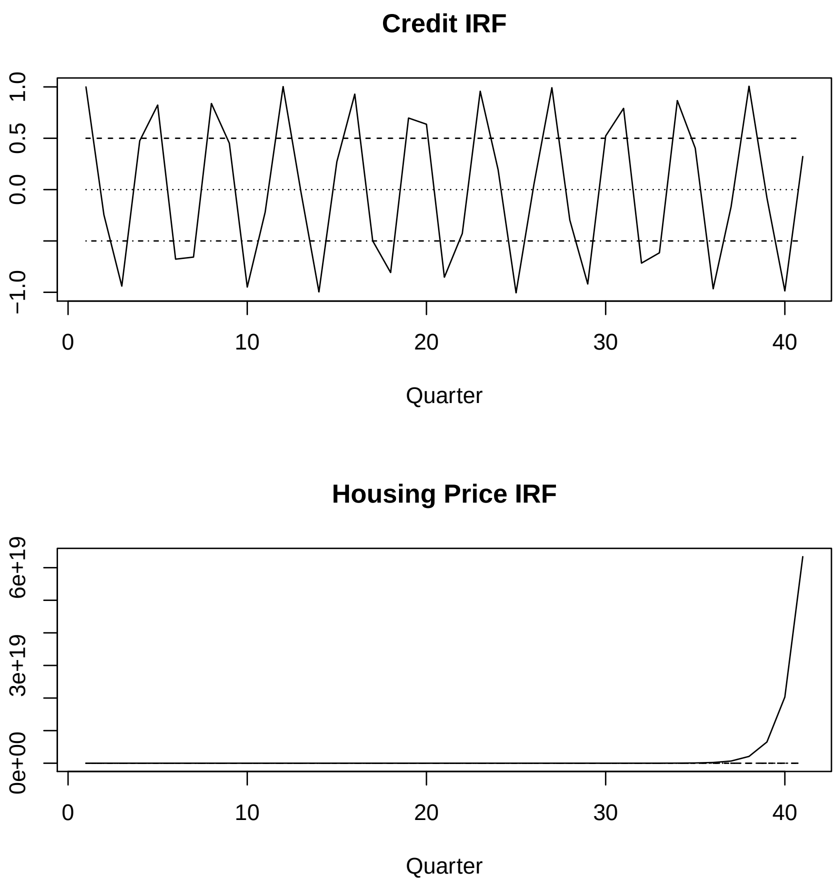


Figure 15: Germany IRF

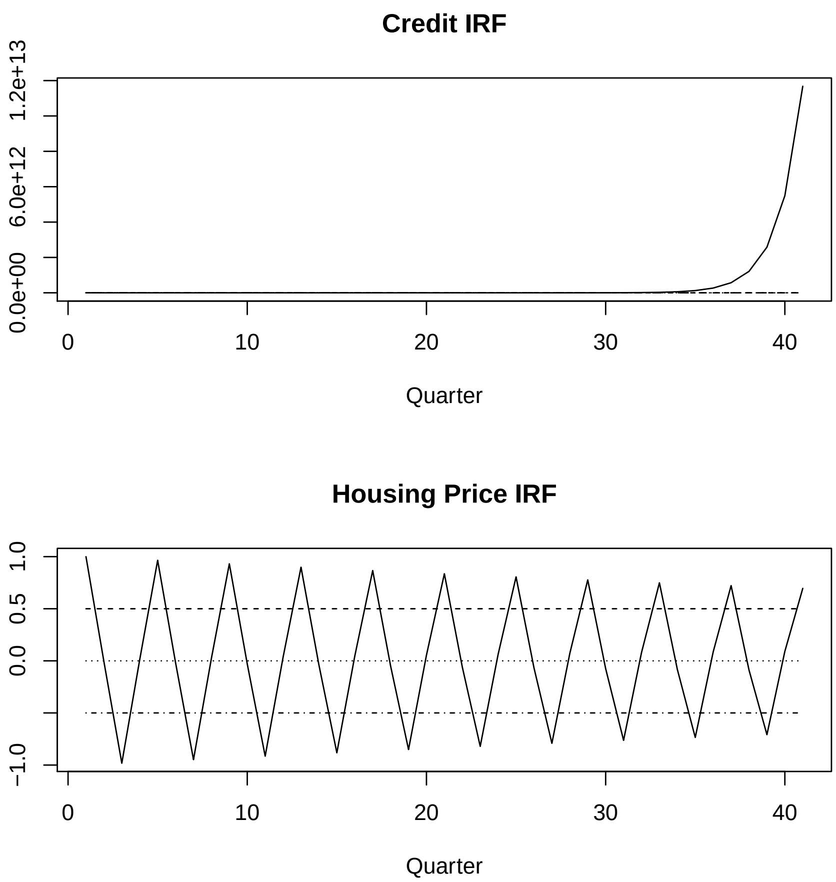
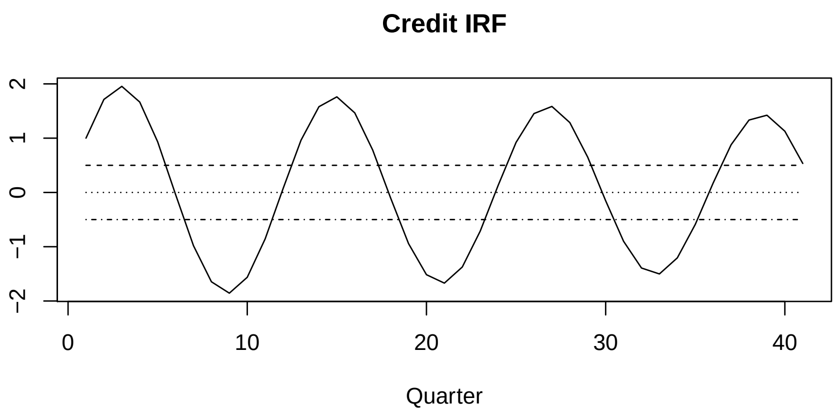


Figure 16: France IRF



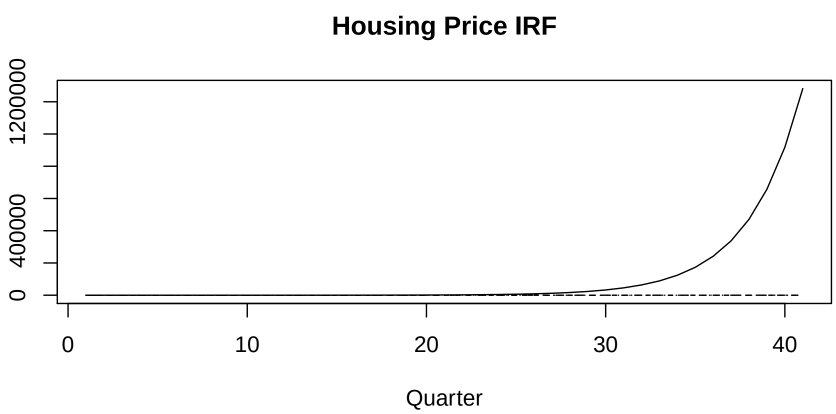


Figure 17: Japan IRF

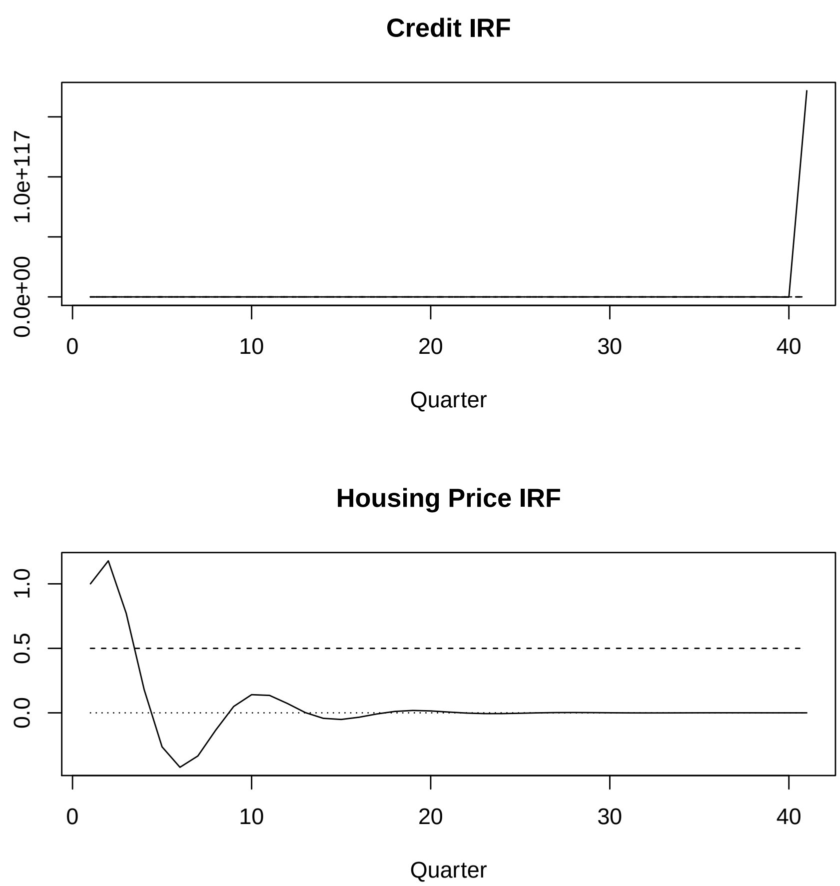


Figure 18: Korea IRF