Assignment 3
Due 4/5/19

1 Analytical Exercise

1. Consider the following bivariate structural VAR

$$y_{1t} = \gamma_{10} - b_{12}y_{2t} + \gamma_{11}y_{1,t-1} + \gamma_{12}y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = \gamma_{20} - b_{21}y_{1t} + \gamma_{21}y_{1,t-1} + \gamma_{22}y_{2,t-1} + \varepsilon_{2t}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tilde{\ } iid \left[\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right]$

(a) Can you estimate above two equations by OLS separately? Explain.

(b) In matrix form, the above model can be written as $BY_t = \Gamma_0 + \Gamma_1 Y_{t-1} + \varepsilon_t$, where $E(\varepsilon_t \varepsilon_t') = \Sigma$ is a diagonal matrix. The reduced form representation of the above VAR is $Y_t = A_0 + A_1 Y_{t-1} + u_t$, where $A_0 = B^{-1}\Gamma_0, A_1 = B^{-1}\Gamma_1, u_t = B^{-1}\varepsilon_t$. $E(u_t u_t') = \Omega$ is a non-diagonal matrix. Solve the reduced form errors in terms of structural errors. Is the model identified?

(c) Now consider Rigobon and Sack (2003) model of identification through heteroscedasticity where they assume two regimes in which the structural errors have variance Σ_{ε_1} and Σ_{ε_2}. In addition they make the assumption that B matrix (loading on Y) does not change across regime. Show how this assumption leads to the identification of the above structural VAR model.

2 Empirical Exercise

1. (SVAR with Short-run Restrictions) Use the VAR model data from assignment 3 (first question in empirical exercise). The relevant data set is usuk.txt.

   (a) The impulse response for the baseline model assumed Cholesky decomposition. Use SVAR function in the vars package to impose the Cholesky restriction and plot the impulse response functions. Are your results the same as you obtained when you plotted IRFs for the reduced form VAR?

   (b) Suppose the reduced form errors are orthogonal to each other. How would you impose this restriction? Plot the impulse response functions for this restriction.

   (c) Do you reject the null of orthogonality of the reduced form errors in part (b)?

2. (Blanchard-Quah Decomposition) Using the output and unemployment data in the excel file BQ.csv on the class webpage, specify and estimate a SVAR model of the form

$$By_t = \gamma_0 + \sum_{j=1}^{p} \Gamma_j y_{t-j} + \varepsilon_t$$

$$E(\varepsilon_t \varepsilon_t') = D = diagonal$$

$$y_t = (\Delta y_{1t}, y_{2t})'$$

   where y_1 represents log output and y_2 represents unemployment. Specify ε_1 as a permanent supply shock and ε_2 as a transitory demand shock. To identify the parameters of the SVAR impose the Blanchard-Quah restriction

$$\lim_{s \to \infty} \frac{\partial y_{1t+s}}{\partial \varepsilon_{2t}} = \sum_{s=0}^{\infty} \theta_{12}^{(s)} = \theta_{12}(1) = 0$$

   that transitory shocks have no long-run effect on the level of output.

   (a) Determine the lag length of the reduced form VAR using the AIC information criteria. Use a maximum lag of 8. Report the VAR estimates and briefly comment on the fit of the VAR.

(b) After imposing the restriction $\theta_{12}(1) = 0$, estimate the SVAR and report the resulting estimates.

(c) Compute the impulse response functions and forecast error variance decompositions from the SVAR model using a maximum horizon of 40 quarters. Briefly comment on what you find.

(d) Now detrend the unemployment data and perform (a)-(b) steps outlined above.

(e) Compute the impulse response functions and forecast error variance decompositions from the SVAR model with output and detrended unemployment using a maximum horizon of 40 quarters.