Housing and Credit Cycles

Nam Nguyen

January 6, 2021

1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2003) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$
$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$
$$\tau_{ht} = \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

Cycles:

$$c_{yt} = \phi^1_y c_{yt-1} + \phi^2_y c_{yt-2} + \phi^x_y c_{ht-1} + \varepsilon_{yt}, \qquad \varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y}) \tag{5}$$
$$c_{ht} = \phi^1_h c_{ht-1} + \phi^2_h c_{ht-2} + \phi^x_h c_{yt-1} + \varepsilon_{ht}, \qquad \varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h}) \tag{6}$$

State-Space Model
Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{7}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_y^1 & \phi_y^2 & 0 & \phi_y^x & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_h^x & 0 & 0 & \phi_h^1 & \phi_h^2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix} \tag{8}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma_{\eta y}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon y}^2 & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta h}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma_{\varepsilon h}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{10}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

I did not put stationary constraints directly on the autoregressive parameters. Since such constraints on a VAR(2) system is complex to setup. However, to achieve feasible stationary transitory measurement, I implement an additional term on the objective function:

$$l(\theta) = -\frac{1}{2}\sum_{t=1}^{T} ln[(2\pi)^2|f_{t|t-1}|] - \frac{1}{2}\sum_{t=1}^{T}\eta'_{t|t-1}f^{-1}_{t|t-1}\eta_{t|t-1} - 0.003 * \sum_{t=1}^{T}(c^2_{yt} + c^2_{ht}) \tag{11}$$

The last term in the objective function acts as a penalty against too much transitory deviation from zero. Without this penalty, the trend would be linear or all the movements in the measured series would be matched by transitory movements.

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite matrix.

4 Priors selection

The priors for autoregressive parameters in matrix F are taken from VAR regression of the HP filter cycle decomposition of the series.

For $\beta_{0|0}$, I set $\tau_{0|0}$ as the value of the first available row of data and omit the first observation from the regression. $c_{0|0}$ are set to be equal to their HP filter counterpart. $var(\tau_{0|0}) = 100 + 50 * random$; while other measures of the starting covariance are set to be their unconditional values.

Starting standard deviation and correlation values are randomized within reasonable range.

5 Regression results

In this following section, I will apply the UC model to data from 6 countries: US, UK, Germany, France, Japan and South Korea.

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. The following estimates are selected in the manner that they would look the most stable. Perhaps a more optimal constraint on the autoregressive parameters would solve this issue.

The regression table below shows Unobserved component VAR(2) regression results with and without cross-cycle parameters.

Table 1: VAR(2) cross-lag covariance UC model Estimates: US data

Description	VAR(2)		VAR(2) 1st-cross-lag		VAR(2) 2-cross-lags	
Parameters	1	2	3	4	5	6
ϕ_y^1	1.5217	0.3236	1.8903	0.0363	1.8866	0.0003
ϕ_y^2	-0.5922	0.2828	-0.7732	0.0217	-0.8942	0.0032
ϕ_y^{x1}			-0.0127	0.0012	0.0428	0.0005
ϕ_y^{x2}					-0.0403	0.0009
ϕ_h^1	1.8040	0.0394	1.4655	0.0646	1.8647	0.0387
ϕ_h^2	-0.8210	0.0393	-0.7369	0.0478	-0.8980	0.0391
ϕ_h^{x1}			2.5769	1.6420	0.0897	0.1145
ϕ_h^{x2}					-0.0320	0.1136
σ_{ny}	0.9681	0.0646	0.9758	0.0667	0.8590	0.0554
σ_{ey}	0.1366	0.0739	0.0004	0.0087	0.0306	0.0167
σ_{nh}	0.9643	0.1072	1.2720	0.1280	1.1356	0.1060
σ_{eh}	0.4711	0.0790	0.2960	0.1616	0.3638	0.0775
σ_{eyeh}	-0.9994	0.0302	-0.8812	0.3118	-1	5.1900×10^{-7}
σ_{nynh}	0.4642	0.0944				
Log-likelihood value	-369.9163		-384.7974		-363.3991	

Table 2: VAR(2) cross-lag covariance UC model Estimates: GB data

Description	VAR(2)		VAR(2) 1st-cross-lag		VAR(2) 2-cross-lags	
Parameters	1	2	3	4	5	6
ϕ_y^1	1.2542	0.1910	1.2572	0.0608	1.1506	0.1764
ϕ_y^2	-0.2631	0.2052	-0.1698	0.0949	-0.2922	0.1860
ϕ_y^{x1}			-0.0323	0.0120	-9.2500×10^{-5}	0.0406
ϕ_y^{x2}					0.0013	0.0499
ϕ_h^1	1.6249	0.1069	0.6803	0.1111	0.3790	0.1344
ϕ_h^2	-0.7357	0.1164	-0.0581	0.1194	-0.5132	0.1730
ϕ_h^{x1}			1.0953	0.6445	1.0422	0.9597
ϕ_h^{x2}					-0.2495	1.4444
σ_{ny}	0.8608	0.0821	1.1443	0.1162	0.7399	0.0402
σ_{ey}	0.2894	0.1205	0.1927	0.0763	0.4752	0.1062
σ_{nh}	2.3401	0.1723	2.1626	0.1875	1.4238	0.0591
σ_{eh}	0.7506	0.2205	1.8984	0.3425	2.0796	0.4701
σ_{eyeh}	0.9571	0.2528	0.5021	0.1633	0.4853	0.1539
σ_{nynh}	0.5919	0.0720				
Log-likelihood value	-410.0147		-448.3113		-512.4697	

Given the regression results from the above table. To avoid the problem of perfect collinearity as shown in US data regression, and also to have a more significant estimate of the cross cycle component; I select the second model - VAR(2) with 1 cross lag in the cycle component as the one to focus on.

The novel contribution of this paper is to introduce this parameter ϕ_h^{x1} in which it measure the effect of a change in last period credit transitory component on the current housing price transitory component. In both regressions in the US and GB, I can observe that there is a nearly significant positive effect of last period credit cycle deviation on current housing cycle component.

The following graphs shows the UC forecast series against the actual data series.

Figure 1: VAR(2) cross-cycle 1st lag US: Cycle components

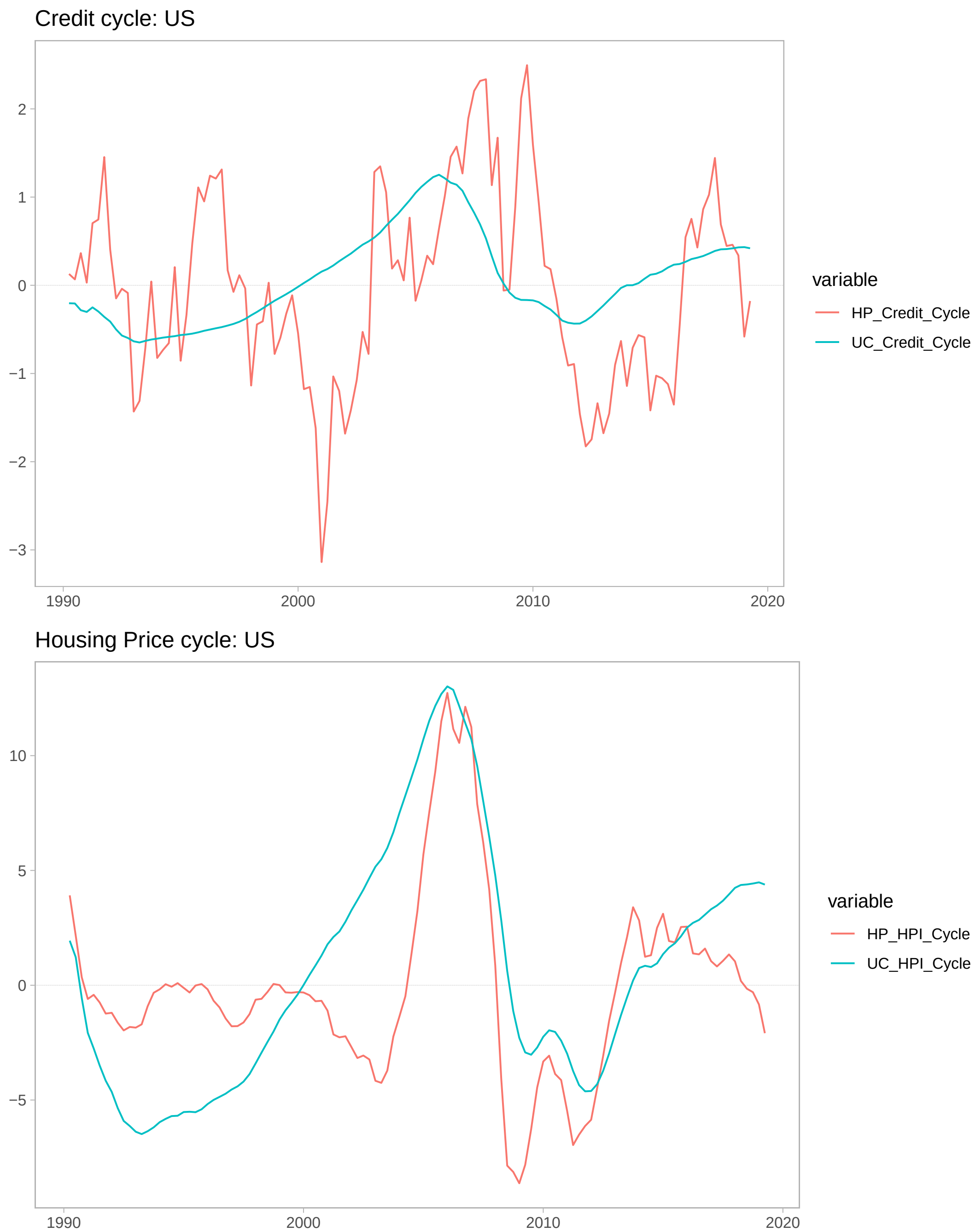
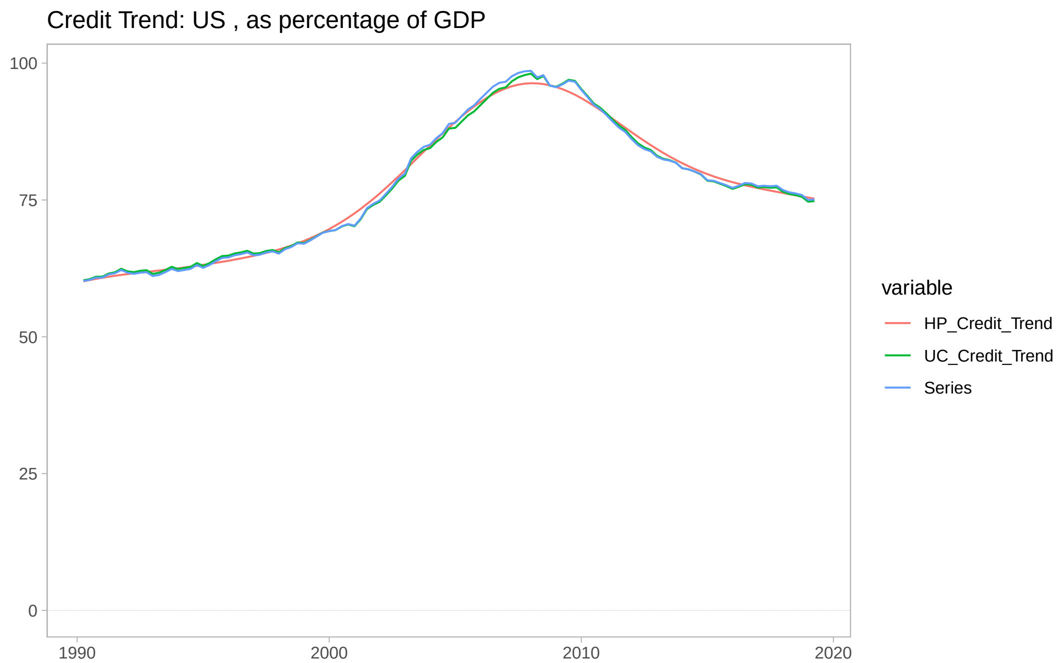


Figure 2: VAR(2) cross-cycle 1st lag US: Trend components



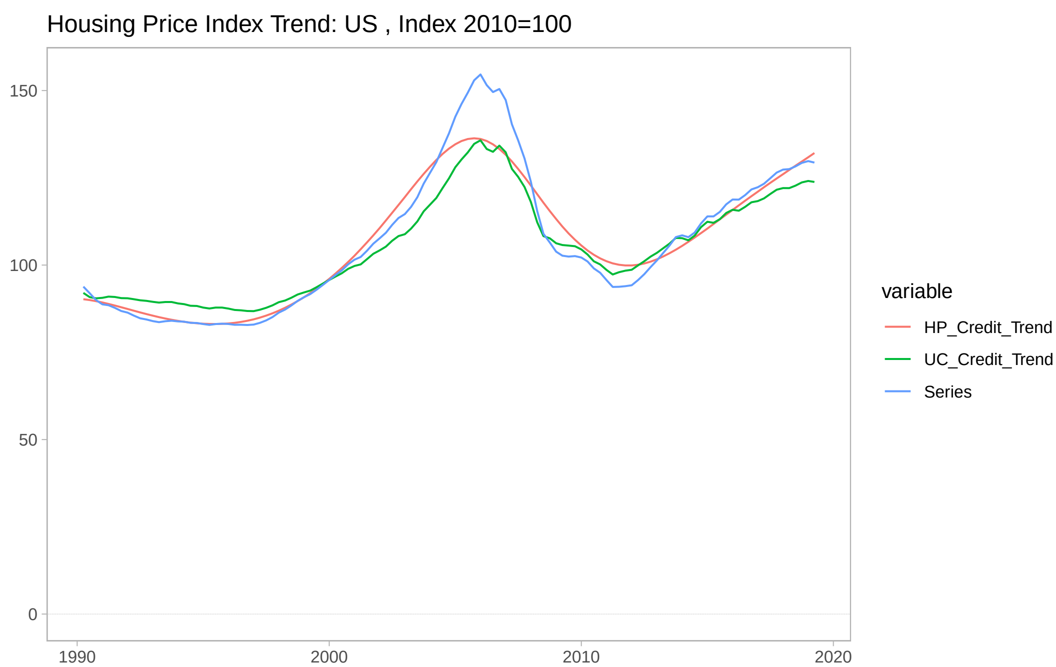


Figure 3: VAR(2) cross-cycle 1st lag GB: Cycle components

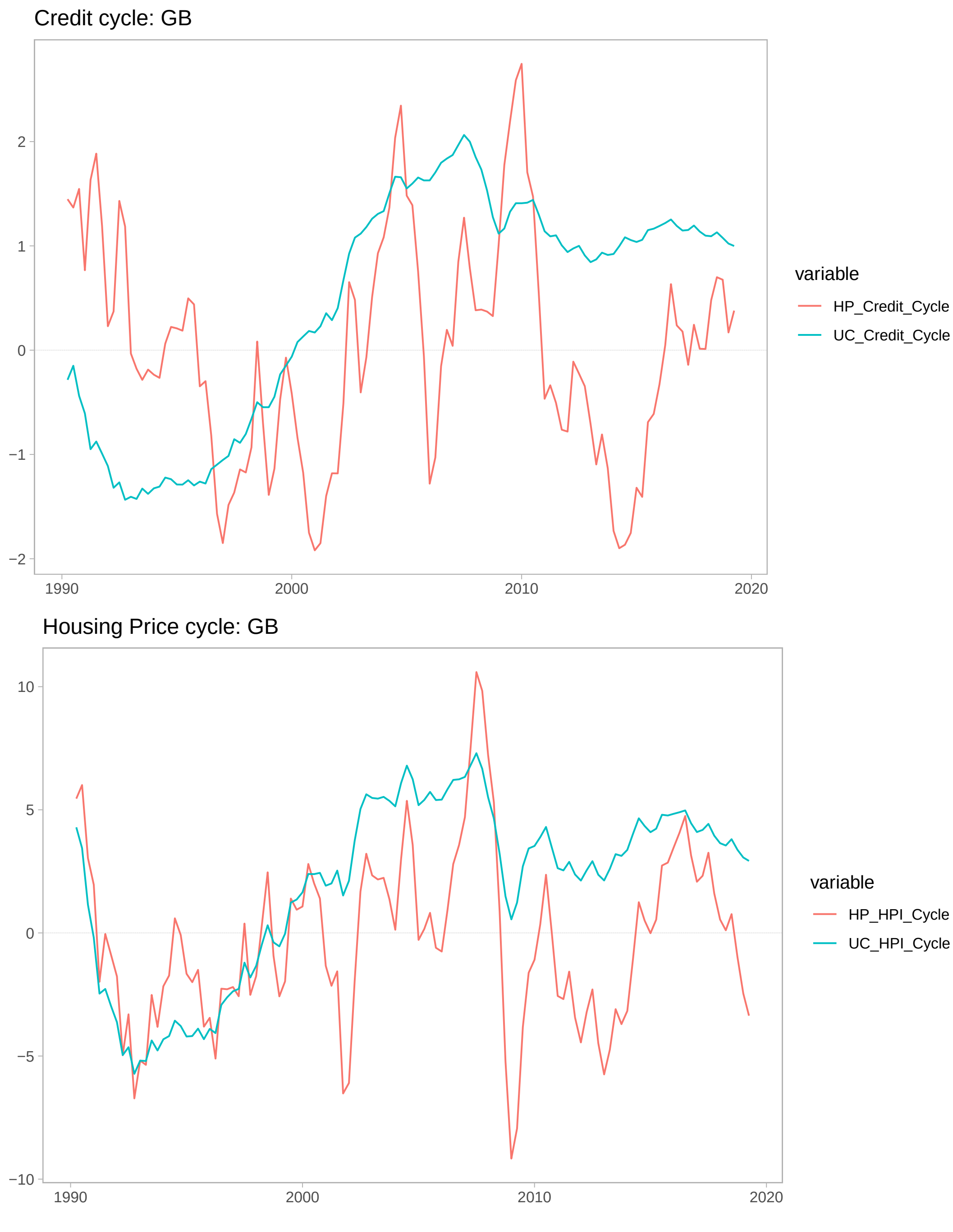
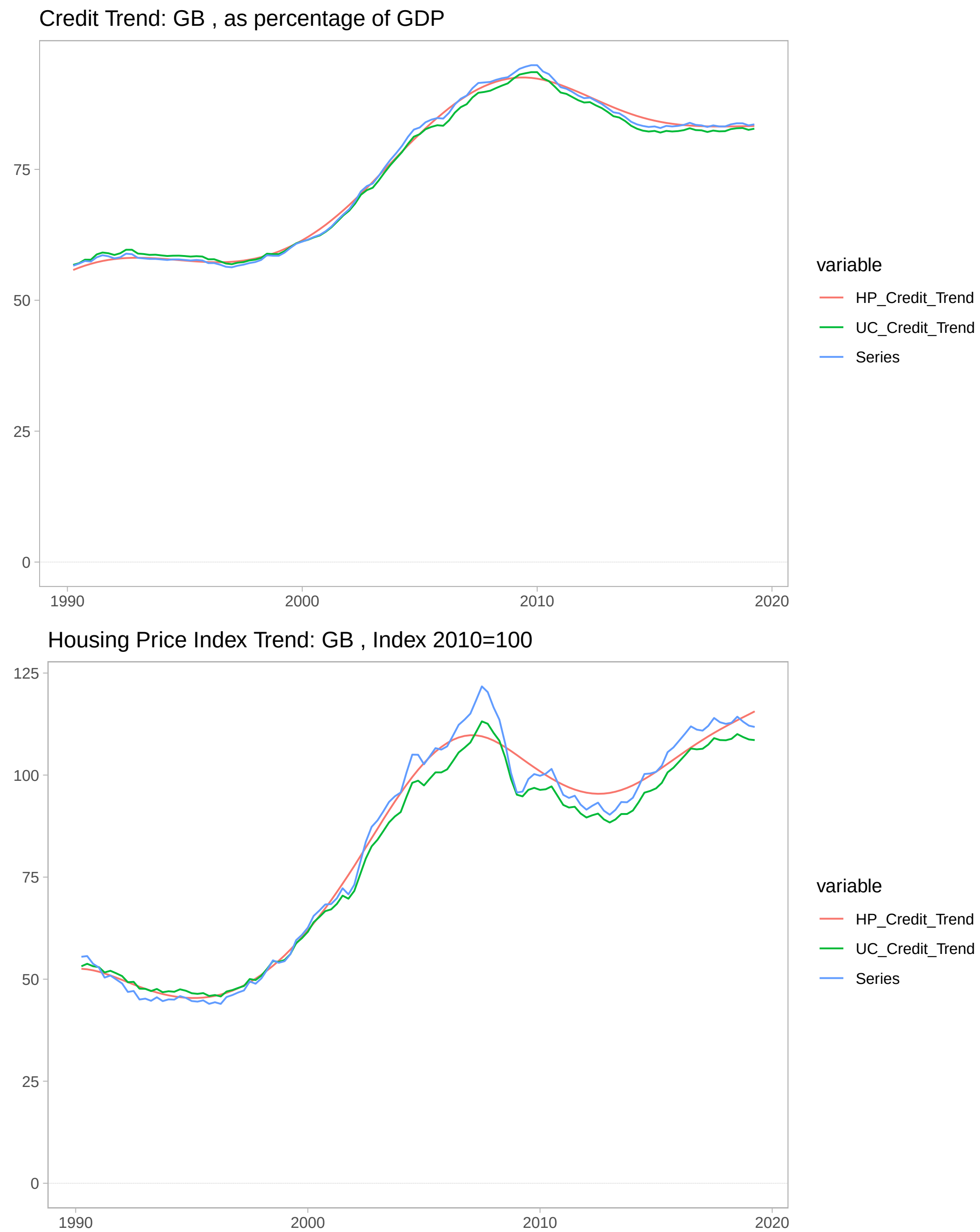


Figure 4: VAR(2) cross-cycle 1st lag GB: Trend components



6 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

In this paper, the models for US and GB data shows that there is a positive relationship between a one period lag in short term house hold credit and house price.

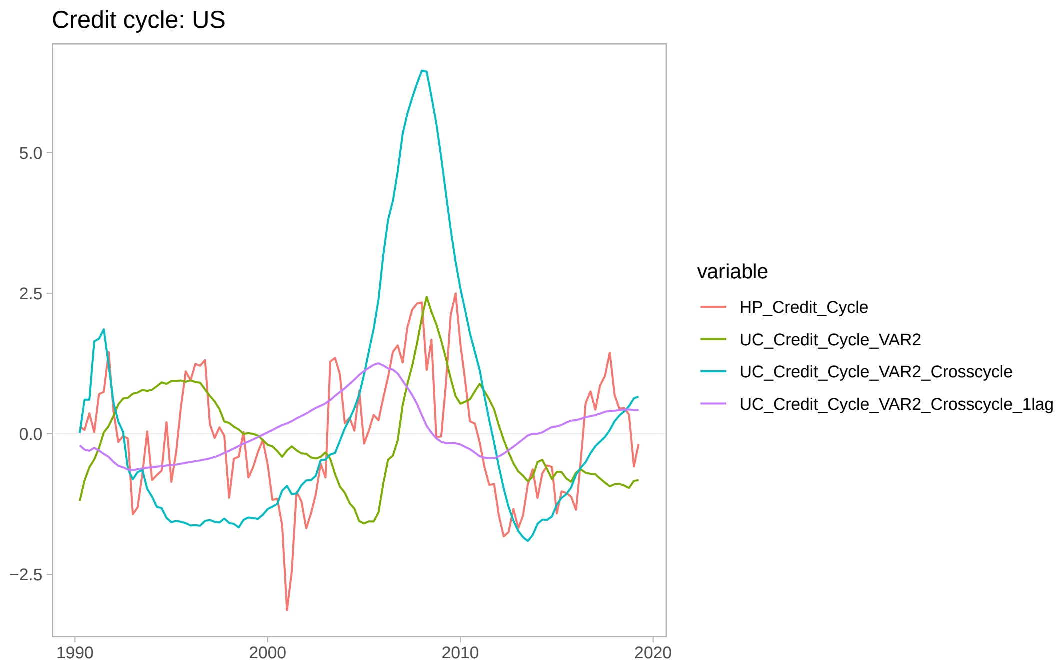
Further development for this paper should include more robust optimal constraints on parameters to ensure stability.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.

Appendix

I will also include graphs that compare the 3 regression models forecast against HP filter cycle components.

Figure 5: VAR(2) cross-cycle US: Cycle components



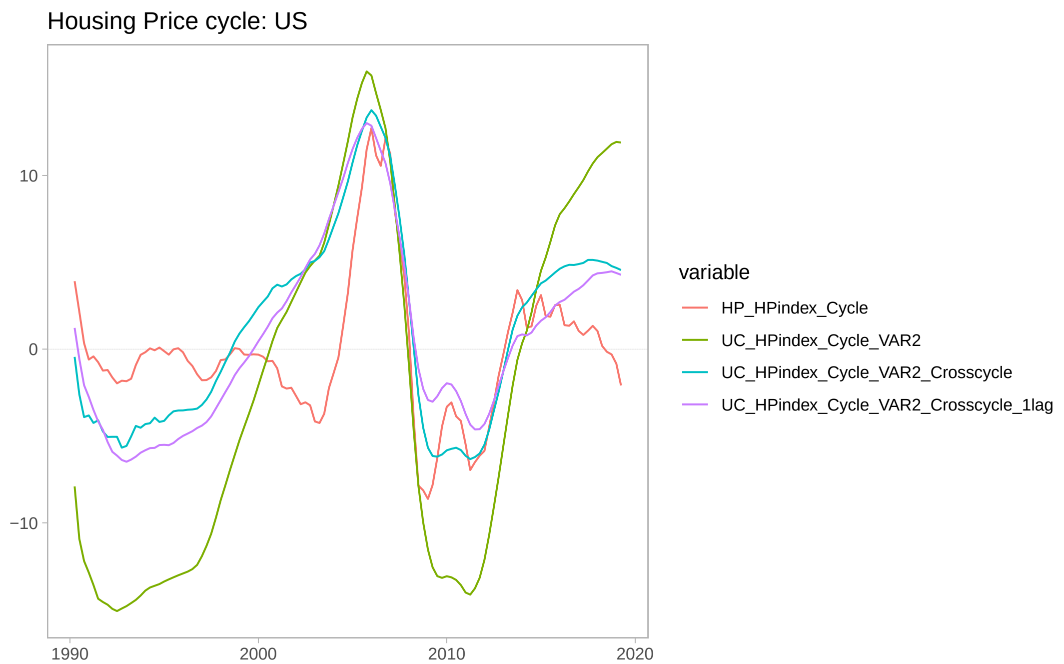


Figure 6: VAR(2) cross-cycle GB: Cycle components