Assignment 5
Due 04/12/19

1 Analytical Exercise

1. Consider an AR(2) model: $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$. Write the state-space representation of this model. Assuming stationarity of the model, calculate the initial value of the state vector $(\beta_{\frac{0}{0}})$. Also outline the steps involved in the calculation of the initial value for the variance of the state vector $(P_{\frac{0}{0}})$.

2. Consider a following vector error-correction model (VECM).

$$\Delta y_t = \phi_{11} \Delta y_{t-1} + \phi_{12} \Delta c_{t-1} + \theta_1 (y_{t-1} - c_{t-1}) + \varepsilon_{1t}$$

$$\Delta c_t = \phi_{21} \Delta y_{t-1} + \phi_{22} \Delta c_{t-1} + \theta_2 (y_{t-1} - c_{t-1}) + \varepsilon_{2t}$$

where Δy_t is growth rate of GDP and Δc_t is growth rate of consumption. Income and consumption tend to move together in the long-run, therefore, $y_{t-1} - c_{t-1}$ is stationary. Represent the above model in a state-space framework.

3. Consider a UC model with AR(1) cyclical component.

$$y_t = \tau_t + c_t$$

$$\tau_t = \mu + \tau_{t-1} + v_t, v_t \tilde{} iidN(0, \sigma_v^2)$$

$$c_t = \phi c_{t-1} + e_t, e_t \tilde{} iidN(0, \sigma_e^2)$$

Assume that transitory shocks and permanent shocks are correlated with each other, i.e., $\text{cov}(v_t, e_t) \neq 0$. Is the model identified?

2 Empirical Exercise

1. Time-Varying Parameter Model (TVP Model)

   Re-estimate the TVP model presented in chapter 3 of Kim and Nelson (page 44) using R. The data file is tvp.txt. The sample period is 1959.3–1987.4

   column 1: Quarter Index

   column 2: m1=growth rate of quarterly average M1

   column 3: dint=change in the lagged interest rate (3-month T-bill)

   column 4: inf=lagged inflation

   column 5: surpl=lagged full employment budget surplus

   column 6: m1lag=lag of m1

   The model is

   $$\triangle M_t = \beta_{0t} + \beta_{1t}\triangle i_{t-1} + \beta_{2t}INF_{t-1} + \beta_{3t}SURP_{t-1} + \beta_{4t}\triangle M_{t-1} + e_t$$

   $$\beta_{it} = \beta_{it-1} + v_{it}, i = 0, 1, 2, 3, 4$$

   (a) Estimate this model using the Kalman filter. Report the estimated parameters and plot the time-varying coefficients.

   (b) Estimate this model using the Kalman smoother. Report the estimated parameters and plot the time-varying coefficients.

   (c) Why is the filtered estimates different than the smoothed estimates?

2. Multivariate Unobserved Component Model

   Consider the multivariate version of the UC model as outlined in Kim and Nelson's book (pages 38-40). Data file rgdp_us.txt is real GDP and ur_us.txt is US unemployment rate from 1948:01-2010:03. This model uses Okun's law to jointly model the cycle of GDP and unemployment. Use R to estimate GDP trend and cycle for this bivariate model. Does the estimated cycle capture the different recessions in the U.S.?