Housing and Credit Cycles

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August 24, 2020

1 Motivation

This paper uses a multivariate extension of the model proposed by Morley et al. (2008) and Huang and Kishor (2018)

The novel contribution of this paper is the inclusion of cross transitory components effect as seen in eq(5) & eq(6). I would like to take advantage of it to explore the dynamics of the relationship between housing prices and house hold credit in the short run.

2 Model Specification

Series:
-Credit : Credit to non financial sector
-HPI : Housing Price Index

$$ln\frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$

$$lnHPI = h_t = \tau_{ht} + c_{ht} \tag{2}$$

Trends:

A random walk drift term g_t is added in the stochastic trend inspired by Clark (1987)

$$\tau_{yt} = \tau_{yt-1} + \eta_{yt}, \qquad \eta_{yt} \sim iidN(0, \sigma^2_{\eta y}) \tag{3}$$

$$\tau_{ht} = \tau_{ht-1} + \eta_{ht}, \qquad \eta_{ht} \sim iidN(0, \sigma^2_{\eta h}) \tag{4}$$

Cycles:

$$c_{yt} = \phi^1_y c_{yt-1} + \phi^2_y c_{yt-2} + \phi^{x1}_y c_{ht-1} + \phi^{x2}_y c_{ht-2} + \varepsilon_{yt}, \quad \varepsilon_{yt} \sim iidN(0, \sigma^2_{\varepsilon y}) \tag{5}$$

$$c_{ht} = \phi^1_h c_{ht-1} + \phi^2_h c_{ht-2} + \phi^{x1}_h c_{yt-1} + \phi^{x2}_h c_{yt-2} + \varepsilon_{ht}, \quad \varepsilon_{ht} \sim iidN(0, \sigma^2_{\varepsilon h}) \tag{6}$$

State-Space Model
Transition equation:

$$\beta_t = F\beta_{t-1} + \tilde{v}_t \tag{7}$$

Where the transitory components are:

$$\begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_y^1 & \phi_y^2 & 0 & \phi_y^{x1} & \phi_y^{x2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_h^{x1} & \phi_h^{x2} & 0 & \phi_h^1 & \phi_h^2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ c_{yt-1} \\ c_{yt-2} \\ \tau_{ht-1} \\ c_{ht-1} \\ c_{ht-2} \end{bmatrix} + \begin{bmatrix} \eta_{yt} \\ \varepsilon_{yt} \\ 0 \\ \eta_{ht} \\ \varepsilon_{ht} \\ 0 \end{bmatrix} \tag{8}$$

The covariance matrix for $\tilde{v}_t$, denoted Q, is:

$$Q = \begin{bmatrix} \sigma^2_{\eta y} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon y} & 0 & 0 & \sigma_{\varepsilon y \varepsilon h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta h} & 0 & 0 \\ 0 & \sigma_{\varepsilon y \varepsilon h} & 0 & 0 & \sigma^2_{\varepsilon h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Measurement Equation:

$$\tilde{y}_t = A + H\beta_t \tag{10}$$

$$\begin{bmatrix} y_t \\ h_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ c_{yt} \\ c_{yt-1} \\ \tau_{ht} \\ c_{ht} \\ c_{ht-1} \end{bmatrix}$$

3 Parameters constraints

The estimation of the unobserved component model uses a nonlinear log-likelihood function maximization. Estimating this function requires numerical optimization.

I did not put stationary constraints directly on the autoregressive parameters. Since such constraints on a VAR(2) system is complex to setup. However, to achieve feasible stationary transitory measurement, I implement an additional term on the objective function:

$$l(\theta) = -\frac{1}{2}\sum_{t=1}^{T} ln[(2\pi)^2|f_{t|t-1}|] - \frac{1}{2}\sum_{t=1}^{T}\eta'_{t|t-1}f^{-1}_{t|t-1}\eta_{t|t-1} - 0.005 * \sum_{t=1}^{T}(c^2_{yt} + c^2_{ht}) \tag{11}$$

The last term in the objective function acts as a penalty against too much transitory deviation from zero. Without this penalty, the trend would be linear or all the movements in the measured series would be matched by transitory movements.

Regarding constraints on covariance matrix, I applied the same constraints as in Morley 2007 to imply for positive definite matrix.

4 Priors selection

The priors for autoregressive parameters in matrix F are taken from VAR regression of the HP filter cycle decomposition of the series.

For $\beta_{0|0}$, I set $\tau_{0|0}$ as the value of the first available row of data and omit the first observation from the regression. $c_{0|0}$ are set to be equal to their HP filter counterpart. $var(\tau_{0|0}) = 100$ while other measures of the starting covariance are set to be their unconditional values.

Starting standard deviation and correlation values are randomized within reasonable range.

5 Regression results

In this following section, I will apply the UC model to data from 2 countries: US, UK

The estimated parameters vary greatly as I uniformly randomized priors for the MLE process. Applying the HP filter estimates as priors helped stabilizing regression results.

Table 1: Correlated UC model Estimates: US data

Description	Estimate	Standard Error
ϕ_y^1	-0.0243	0.0483
ϕ_y^2	0.9590	0.0397
ϕ_y^{x1}	-0.0875	0.1870
ϕ_y^{x2}	0.1457	0.1839
ϕ_h^1	1.7464	0.0565
ϕ_h^2	-0.7737	0.0631
ϕ_h^{x1}	0.0210	0.5189
ϕ_h^{x2}	-0.0294	0.5042
σ_{ny}	0.9855	0.0510
σ_{ey}	0.0238	0.0420
σ_{nh}	1.2023	0.0850
σ_{eh}	0.3802	0.0729
σ_{eyeh}	-1.0000	6.6422×10^{-7}
σ_{nynh}	0.4431	0.0743
Log-likelihood value	-618.1682	0

Table 2: Correlated UC model Estimates: UK data

Description	Estimate	Standard Error
ϕ_y^1	2.0909	0.0693
ϕ_y^2	-1.0460	0.1109
ϕ_y^{x1}	-0.0648	0.0243
ϕ_y^{x2}	0.0394	0.0107
ϕ_h^1	1.4457	0.1013
ϕ_h^2	-0.7067	0.1567
ϕ_h^{x1}	1.6725	1.1159
ϕ_h^{x2}	-0.9142	1.1773
σ_{ny}	1.4961	0.0772
σ_{ey}	0.1461	0.0454
σ_{nh}	2.3667	0.1668
σ_{eh}	0.4747	0.2581
σ_{eyeh}	1.0000	4.6808×10^{-8}
σ_{nynh}	0.3043	0.0785
Log-likelihood value	-847.3206	0

One major problem of this current regression model is that it produces collinearity or perfect correlation as the estimated σ_{eyeh} is either 1 or -1. This collinearity complicates the estimated results and will require further improvement on the model to overcome.

6 Conclusion

Employing cross effects on the transitory components of the two series allows me to measure the effect of short term shock from house hold credit on housing price and vice versa.

For example, the model for US data shows that there is a negative relationship between a one period lag in short term house price and current period house hold credit, but this relationship turns positive in the second lag. More significantly, there is a positive relationship between one period lag in short term household credit and current house price, but this is countered by a negative relationship in the second lag.

These relationships hold true for UK estimates also.

Further development for this paper should include non-adhoc penalty in the objective function and a fix to the perfect collinearity issue.

Additionally, comparison in term of fitness and prediction error with other model such as a conventional multivariate UC model, univariate UC model, AR(2) model could be done in order gauge the benefit of including extra variables in the transitory component.

Figure 1: Appendix: US transitory components

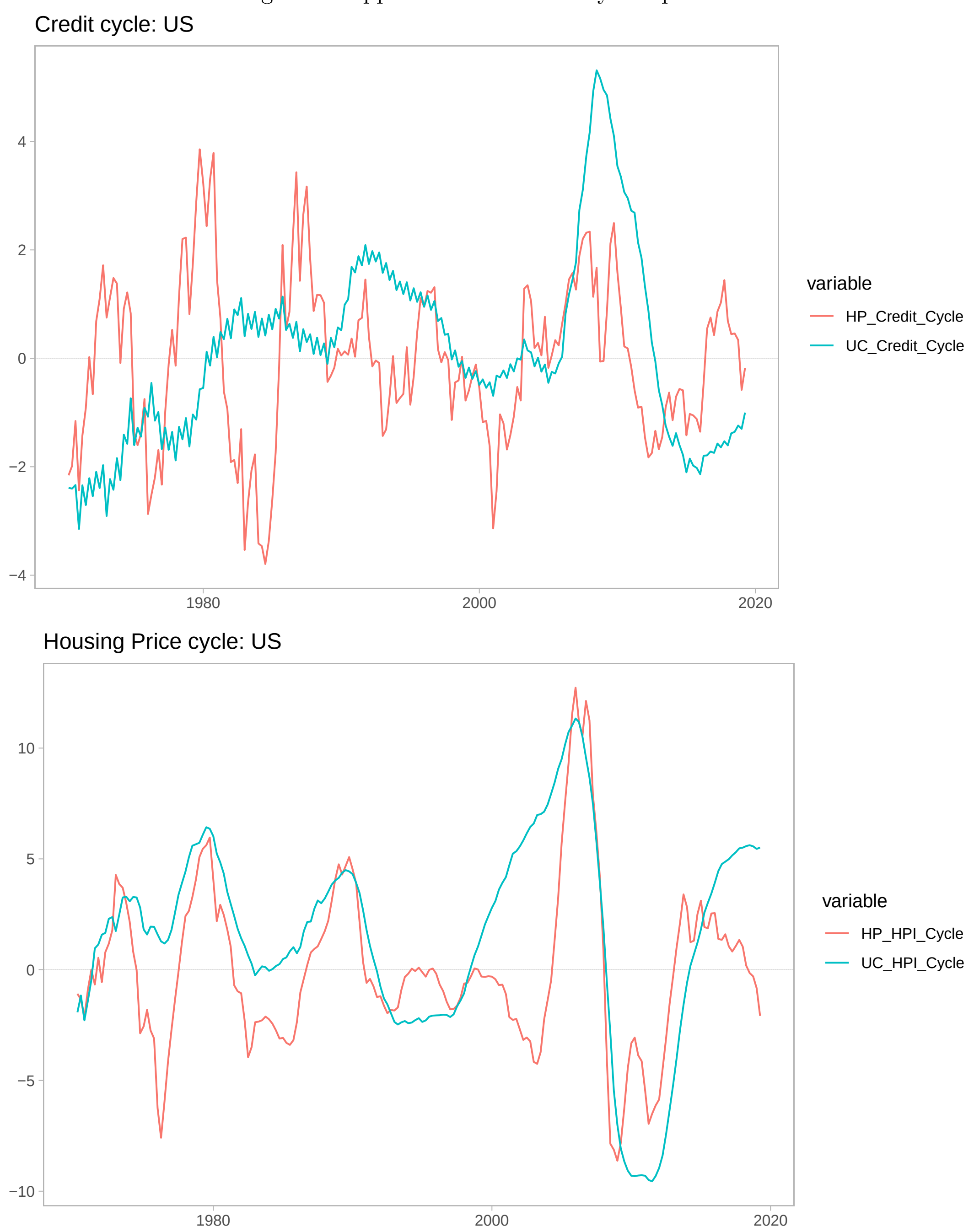
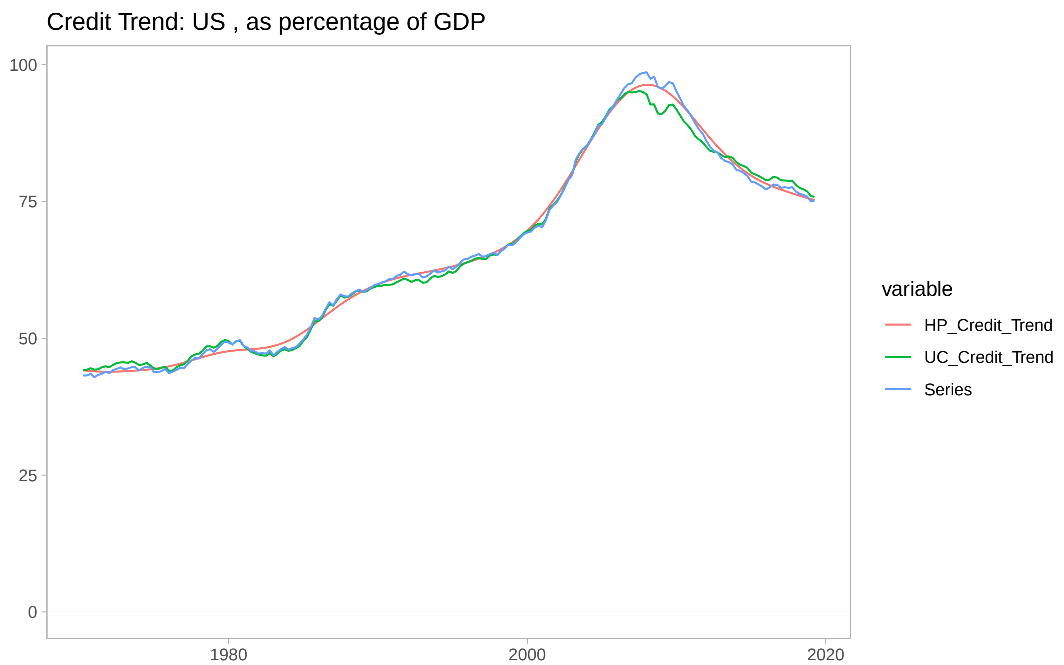


Figure 2: US permanent components



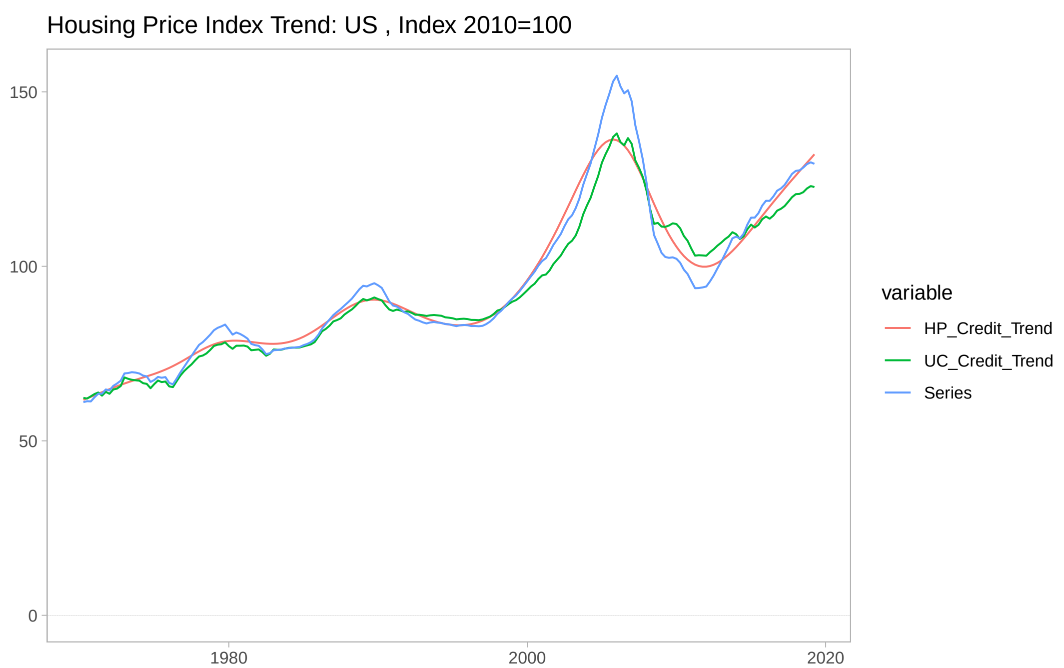


Figure 3: UK Credit components

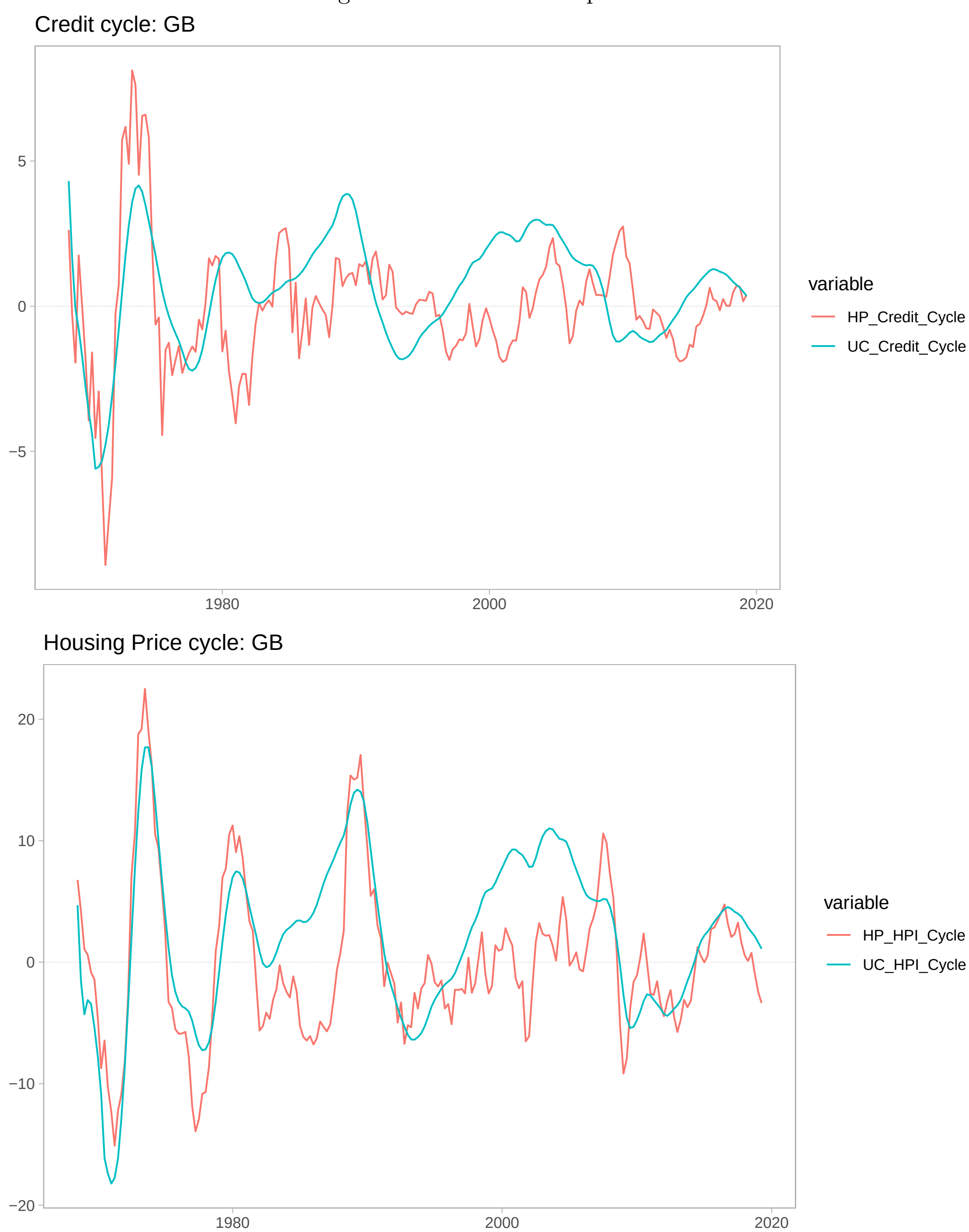


Figure 4: UK Housing Price components