Suggested Solutions to Analytical Problem

(1) (a) We can't estimate them by OLS since there is endogeneity problem, or they are simultaneously determined

(b) $B = \begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix}$

$\Rightarrow B^{-1} = \begin{pmatrix} \frac{1}{1-b_{12}b_{21}} & \frac{b_{12}}{b_{12}b_{21}-1} \\ \frac{b_{21}}{b_{12}b_{21}-1} & \frac{1}{1-b_{12}b_{21}} \end{pmatrix}$

$u_t = B^{-1}\varepsilon_t = \begin{pmatrix} \frac{\varepsilon_{1t}}{1-b_{12}b_{21}} - \frac{\varepsilon_{2t}b_{12}}{1-b_{12}b_{21}} \\ \frac{\varepsilon_{2t}}{1-b_{12}b_{21}} - \frac{\varepsilon_{1t}b_{21}}{1-b_{12}b_{21}} \end{pmatrix}$

If we estimate the reduced form parameters & try to identify structural parameters, then we have identification problem.

(c) Suppose $\Sigma_{\varepsilon_1} = \begin{pmatrix} \sigma_{11}^1 & 0 \\ 0 & \sigma_{22}^1 \end{pmatrix}$ $\Sigma_{\varepsilon_2} = \begin{pmatrix} \sigma_{11}^2 & 0 \\ 0 & \sigma_{22}^2 \end{pmatrix}$

& we have reduced form Variance-Covariance matrix

$\Omega_1 = \begin{pmatrix} \omega_{11}^1 & \omega_{12}^1 \\ \omega_{12}^1 & \omega_{22}^1 \end{pmatrix}$, $\Omega_2 = \begin{pmatrix} \omega_{11}^2 & \omega_{12}^2 \\ \omega_{12}^2 & \omega_{22}^2 \end{pmatrix}$

$$\Omega = B^{-1} \Sigma \cdot B^{-1'}$$

$$\Rightarrow \Omega_1 = B^{-1} \Sigma_{\varepsilon_1} B^{-1'}$$

$$\Omega_2 = B^{-1} \Sigma_{\varepsilon_2} B^{-1'}$$

Since, we get 2 extra parameters from estimation of Ω_1 & Ω_2, we can estimate b_{12} & b_{21} & the model is just identified in this case.