MEASURING THE REACTION OF MONETARY POLICY TO THE STOCK MARKET*

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Movements in the stock market can have a significant impact on the macroeconomy and are therefore likely to be an important factor in the determination of monetary policy. However, little is known about the magnitude of the Federal Reserve's reaction to the stock market, in part because the simultaneous response of equity prices to interest rates makes it difficult to estimate. This paper uses an identification technique based on the heteroskedasticity of stock market returns to measure the reaction of monetary policy to the stock market. We find a significant policy response, with a 5 percent rise (fall) in the S&P 500 index increasing the likelihood of a 25 basis point tightening (easing) by about a half.

I. INTRODUCTION

In December 1996, Federal Reserve Chairman Alan Greenspan shook global financial markets when he raised the possibility of "irrational exuberance" distorting equity prices. His concern, it appears from the text of his speech, was determining the appropriate monetary policy response in such situations. However, the central bank likely has a broader concern about equity prices, in that equity price movements, through their influence on the macroeconomy, may be an important determinant of the appropriate stance of monetary policy. Indeed, the Chairman mentioned the impact of rising stock prices on household wealth or spending in every single one of his semiannual testimonies to Congress over the subsequent four years.

This impact of the stock market on the macroeconomy comes primarily through two channels. The first, as suggested by the Chairman's testimonies, is that movements in stock prices influence aggregate consumption through the wealth channel. The total financial wealth of U. S. households stood at $35.7 trillion as of the end of 2000, of which $11.6 trillion was in the form of equity holdings.[1] Because of the magnitude of these equity holdings,

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1. The data are taken from the Flow of Funds accounts produced by the Federal Reserve Board. The figures include the equity holdings of nonprofit organizations, which are not separated from those of households in the data.

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stock price movements have a significant effect on household wealth. Second, stock price movements also affect the cost of financing to businesses. In 2000, for example, U. S. nonfinancial corporations raised a gross amount of $118 billion through equity offerings and more than $100 billion in venture capital funds.[2]

Because of their potential impact on the macroeconomy, stock market movements are likely to be an important determinant of monetary policy decisions. Despite this potential importance, there has been little empirical evidence measuring the magnitude of the Federal Reserve's reaction to the stock market. The primary reason is that it is difficult to empirically estimate the monetary policy reaction due to the simultaneous response of the stock market to policy decisions. Indeed, the policy reaction cannot be identified using traditional approaches for addressing the simultaneity problem, including exclusion restrictions or instrumental variables. For example, it is difficult to find any instruments that would affect the stock market without being correlated with interest rate movements.

This paper instead uses an identification procedure based on the heteroskedasticity of stock market returns. In particular, shifts in the variance of stock market shocks relative to monetary policy shocks affect the covariance between interest rates and stock prices in a manner that depends on the responsiveness of the interest rate to equity prices. Thus, we can compute the response of the short-term rate based on the observed shifts in that covariance matrix. The results suggest that an unexpected increase in the S&P 500 index by 5 percent increases the federal funds rate expected after the next FOMC meeting by about 14 basis points. Translating this into discrete policy moves, a 5 percent rise in the S&P 500 index increases the probability of a 25 basis point tightening by just over a half.[3] Because the model is symmetric, a 5 percent decline in stock prices has similar implications for policy easing.

It is important to note upfront that the findings do not imply that the Federal Reserve is targeting stock prices or reacting to perceived misalignments in stock prices. There is currently some

2. Data on gross equity issuance are from Securities Data Company, while those on venture capital are based on a survey conducted by Venture Economics.

3. That is, if the probability of a policy tightening were 30 percent under the existing economic situation, a 5 percent rise in stock prices would increase the probability of tightening to 80 percent. If the probability of an easing were 10 percent, the rise in stock prices would result in a 40 percent chance of tightening.

debate in the academic literature about whether such a strategy would be an effective policy approach. Cecchetti, Genberg, Lipsky, and Wadhwani [2000] argue that monetary policy-makers should react to perceived misalignments in asset prices to reduce the likelihood of asset price bubbles forming.[4] Bernanke and Gertler [1999, 2001] instead argue that monetary policy should react to asset price movements only to the extent that they affect expected inflation.[5] Along similar lines, in the context of discussing stock market bubbles, Alan Greenspan has stated that central banks should remain focused on achieving price stability and maximum sustainable growth, suggesting that policy-makers should respond to stock prices according to their influence on the outlook for output and inflation. The findings in this paper are consistent with this view. Using rough calculations, the estimated policy response is approximately of the magnitude needed to offset the expected pass-through of equity market shocks to aggregate demand. Thus, it appears that the Federal Reserve systematically responds to stock price movements only to the extent warranted by their impact on the economy.

The paper is organized as follows. Section II discusses the problem of identification and demonstrates why other widely used identification methods are inappropriate in this context. Section III develops the method for identifying the system through stock market heteroskedasticity. Section IV presents the results and evaluates whether the magnitude of the estimated policy response is sensible. Section V investigates the robustness of the results and explores whether the findings could be driven by alternative explanations. Section VI concludes.

II. The Endogeneity Problem

Although movements in the stock market may importantly affect monetary policy decisions, identifying the monetary policy response to the stock market is difficult. One problem is that the stock market endogenously responds to monetary policy decisions. The simultaneous determination of interest rates and stock

4. The basis of this argument is that preventing asset price misalignments will allow policy-makers to more successfully stabilize output and inflation (the policy-makers' objectives) by avoiding the large disruptions to the economy that might occur when those misalignments unwind.

5. Many of these issues are also addressed by Reinhart [1998]. Reinhart raises some other interesting questions as well, such as whether policy-makers should respond to the *level* of stock prices or the *change* in stock prices.

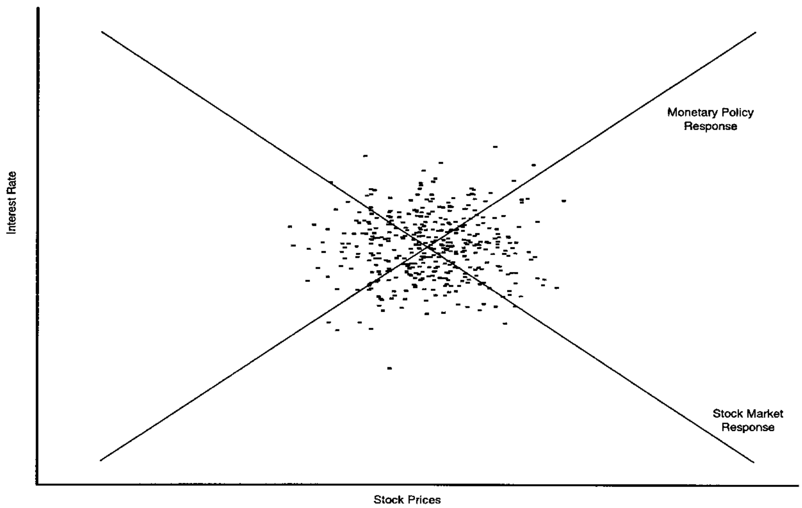

FIGURE I
Joint Determination of Interest Rates and Stock Prices

prices is depicted in Figure I. Holding everything else equal, higher interest rates are associated with lower stock market prices, given the higher discount rate for the expected stream of dividends.[6] At the same time, the Federal Reserve may react to higher stock prices by raising interest rates. Realizations of stock prices and interest rates are determined by the intersection of these two schedules and do not provide a clear reading of whether the policy reaction function is upward sloping in stock prices. In addition, comovements between interest rates changes and stock market returns are influenced by a number of other factors as well, including news about the economy.

The first hint that the endogeneity of the stock market response may be an important consideration comes from the simple correlation between movements in short-term interest rates and the stock market. The correlation between daily changes in the three-month Treasury bill rate and daily changes in the S&P 500 index, shown later in Figure III, is typically negative over the

6. Of course, the impact on stock prices likely depends on the source of the interest rate movement. On average, however, it appears from the evidence below that higher interest rates cause stock prices to fall.

period since 1985—the opposite of the sign that would have been expected from the reaction of monetary policy.[7]

II.A. A Simple Setup

In the analysis that follows, we characterize the dynamic interaction between the stock market and interest rates using a Vector Autoregression (VAR). In particular, the dynamics of the short-term interest rate and stock market returns are written as follows:

$$i_t = \beta s_t + \theta x_t + \gamma z_t + \epsilon_t, \tag{1}$$

$$s_t = \alpha i_t + \phi x_t + z_t + \eta_t, \tag{2}$$

where i_t is the three-month Treasury bill rate and s_t is the daily return on the S&P 500 index. The regressions also include five lags of the stock market return and the interest rate, which are not shown for notational simplicity.[8] The data are daily, and the sample runs from March 1985 to December 1999.

The variables x_t and z_t represent macroeconomic shocks that might influence stock prices and interest rates (for reasons discussed further below). Ideally, the VAR would account for as much of the impact of macroeconomic shocks as possible. To that end, the variable x_t includes the monthly releases of major macroeconomic variables, including the core consumer price index (CPI), the National Association of Purchasing Managers survey (NAPM), nonfarm payrolls (NFPAY), the core producer price index (PPI), and retail sales (RETL). Each of these variables is measured by the difference between the released value and the expected value, where those expectations are taken from the Money Market Services survey about a week before the release. (The surprises are set to zero on days other than release dates.) Of course, those releases do not encompass all macroeconomic news. To capture the influence of other sources of macroeconomic news that we do not observe, the specification also includes a common shock to both equations, z_t, which enters in the same manner as x_t.

7. Note also that the correlation exhibits rich patterns, often becoming positive during periods when the volatility of the stock market increases. These patterns are the basis for the identification procedure used, as discussed shortly.

8. Similar results are obtained if the interest rate equation is specified in differences, rather than levels. One reason is that the VAR has a rich lag structure, and the interest rate equation estimated in levels has a large coefficient on the first lag of the dependent variable.

The VAR equations have straightforward interpretations. Equation (2) captures influences over daily movements in stock prices. Going as far back as the "Gordon equation," equity prices are typically assumed to incorporate the discounted value of future dividends, where the discount rate embodies risk preferences. Using the approximation suggested by Campbell and Shiller [1988] for a dynamic setting, the log level of equity prices can be written as

$$(3) \qquad S_t = \frac{k}{1-\delta} + \sum_{j=0}^{\infty} \delta^j (1-\delta) \cdot E_t(d_{t+1+j}) - \sum_{j=0}^{\infty} \delta^j E_t(h_{t+j}),$$

where δ and k are constant terms, d_{t+1+j} is the log dividend, and h_{t+j} is the return to holding equities between $t + j$ and $t + j + 1$. By definition, the expected holding return for equities can be broken down into the short-term interest rate plus a risk premium, or $E_t(h_{t+j}) = i_{t+j} + \rho_{t+j}$. To arrive at the VAR equation (2) from the asset-price condition (3), we assume that expectations of future dividends and short-term interest rates are shaped (linearly) by current and lagged values of macroeconomic news x and z and the interest rate i.[9] Under this interpretation, the shocks η, which we will refer to as "stock market shocks," are driven by changes to investor risk preferences $(\Sigma_{j=0}^{\infty} \delta^j E_t \rho_{t+j})$.[10]

Equation (1) from the VAR can be interpreted as a high frequency monetary policy reaction function. Of course, it is more common to estimate such reaction functions using lower frequency data. But the use of daily (or weekly) data is important in this paper because it allows us to more accurately define the heteroskedasticity of the shocks, as will become apparent below. Note that the three-month Treasury bill rate is used in the daily reaction function rather than the federal funds rate. While the federal funds rate is adjusted only every six weeks or so, the

9. Another difference is that equation (3) determines the log level of equity prices S_t, while equation (2) uses the change in equity prices s_t. However, this difference is inconsequential given the lags included in the VAR. Indeed, nearly identical results are reached if the VAR instead uses the log level of equity prices.

10. More generally, η represents any movements in stock prices not attributable to macroeconomic news and interest rates. Thus, it would also capture the "nonfundamental" movements discussed by Bernanke and Gertler [1999].

Treasury bill rate will adjust daily according to changes in expectations of monetary policy over the coming three months.[11]

The near-term path of monetary policy will be influenced by the Federal Reserve's best estimates of the strength of the economy going forward and potential inflationary pressures. Those estimates will obviously be shaped by the macroeconomic variables x_t and z_t. In addition, movements in equity prices s_t will affect the strength of final demand (primarily) through the wealth channel, and therefore they also influence the short-term interest rate. Shocks to the policy reaction function, ϵ_t, represent deviations from the typical response of the short-term interest rate, which are widely interpreted in the monetary policy literature as "monetary policy shocks."[12]

According to the interpretations offered, the policy response to equity prices—measured by the parameter β in equation (1)—arises strictly from the impact of equity prices on economic activity. The short-term interest rate does not react to shifts in investors' willingness to bear risk itself (that is, to η) beyond the impact on equity prices, which is consistent with how policymakers describe their actions. The interpretation of the VAR could be generalized slightly, however, to allow for another role for equity prices in determining monetary policy. In particular, the central bank could look at equity price movements as a signal of future economic activity, reflecting either private information received by investors or simply revisions to investors' opinions about economic prospects. In that case, the near-term path of policy might respond to equity price movements not just because of the direct wealth channel, but also because of the information content of equity price movements as indicators of future economic activity.

II.B. Identification Problems

It is well-known that equations (1) and (2) cannot be directly estimated due to the endogeneity of the regressors. The parameters of the structural form can sometimes be recovered by imposing restrictions on equations (1) and (2) that allow one to solve for the relevant structural parameters from the covariance matrix of

11. Because the liquidity of the Treasury bill has declined over the sample, we have also performed the exercise using eurodollar futures rates. The results, which are discussed in Section V, are similar.

12. In our case, those shocks could also partly reflect any factors driving a wedge between the Treasury bill rate and policy expectations.

TABLE I
EQUATION FOR DAILY CHANGES IN SHORT-TERM RATE (IGNORING ENDOGENEITY)

Sample: 1985:3 to 1999:12	Number of obs.: 2733	
Std. dev. of dep. var.: 1.43	Std. error of estimate: 0.05	
R^2: 0.99	Durbin-Watson stat.: 2.08	
Variable	*Impact of one std. dev. change (bp)*	*T-statistic of coefficient*
NAPM	0.10	0.53
NFPAY	3.88	8.75
CPI	1.09	1.74
PPI	0.04	−0.65
RETL	1.65	3.53
GDP	0.08	0.13
S&P500	−0.21	−1.90

Regressions include a constant and five lags of the interest rate and stock returns.

the reduced-form residuals. (To simplify the identification, we will ignore the common errors z_t for the moment.) In the macroeconomic literature the identification of the VAR often takes the form of exclusion restrictions—that either β or α is zero.

However, neither of these restrictions is appropriate in the current context. Obviously, we do not want to set β to zero because we are interested in estimating the interest rate response to the stock market. If we instead assume, inappropriately, that the stock market has no contemporaneous response to the interest rate ($\alpha = 0$), the policy reaction function (1) can be estimated directly. The results from that estimation are shown in Table I. The three-month interest rate reacts significantly to several of the macroeconomic news releases. More importantly, however, the estimated response to the stock market, β, is negative. The most likely explanation for the perverse sign of this coefficient is the endogeneity of the stock market response, as highlighted in Figure I. In that case, the estimate of the policy reaction is strongly biased.

A more general approach to addressing the endogeneity problem is through instrumental variables. In fact, one could think of the exclusion restriction behind Table I as allowing the stock market to instrument for itself. But because the stock prices are likely influenced by the interest rate shock, it is not a valid instrument. Moreover, it is hard to conceive of any instrument that would affect the stock market without affecting the path of interest rates. Any instrument related to the macroeconomic outlook certainly would not meet this criterion. Even variables that are more closely related to corporate profits, such as earnings surprises, would likely contain information about the macroeconomic outlook as well, and thus be correlated with interest rate changes.[13] Thus, instrumental variables is unlikely to be an effective approach for addressing the endogeneity problem in this context.

We would expect such problems to affect a wider class of estimated policy rules as well. For example, a large literature has developed on estimating simple policy rules that describe quarterly movements in the federal funds rate as a function of the output gap, inflation, and the lagged interest rate. If stock prices are added to such a rule, the estimated response coefficient is typically around zero.[14] Moreover, Bernanke and Gertler [1999] find a similar result using a forward-looking rule. Their finding could reflect the forward-looking nature of the rule, as the impact of stock price changes could already be incorporated into the forecasts of output and inflation. However, it may also reflect the endogeneity problem and the lack of effective instruments for identifying the policy response.

Overall, the simultaneous equations problem that arises in identifying the VAR (1) and (2) cannot be effectively addressed using exclusion restrictions or instrumental variables. Alternative identification approaches commonly used in the macroeconomics literature, including long-run restrictions or sign restrictions, also do not help with the identification of this parameter. There are no obvious long-run restrictions that could be imposed to separate policy shocks from stock market shocks, as both stock market returns and the interest rate likely revert to some equilibrium values. Sign restrictions also do not pin down the magnitude of the parameters, as the observed correlation could be explained under larger (or smaller) policy responses as long as the endogenous response to the stock market were also larger (or smaller). Given the shortcomings of commonly used identification techniques, we instead use a methodology based on the het-

13. More exogenous events such as changes in tax rates on capital gains or corporate profits would also have consequences for monetary policy, for example through their impact on after-tax income or firms' investment decisions.

14. The working paper version of this paper presents a set of estimates for a backward-looking quarterly policy rule that includes stock prices.

eroskedasticity of the error terms to identify the monetary policy reaction to the stock market, as described in the next section.

III. Identification through Heteroskedasticity

The above discussion indicates that identification approaches widely used in the macroeconomics literature cannot appropriately separate the response of monetary policy to the stock market from the endogenous reaction of the stock market to interest rates. In this section we use the heteroskedasticity found in interest rates and stock market returns to identify the reaction of monetary policy to the stock market.

The identification relies on the following observation: the responsiveness of monetary policy will become a stronger determinant of the covariance between interest rates and stock market returns during periods when equity market shocks are more variable. To see this, consider what happens if the variance of the stock market shocks rises while the variance of the monetary policy shocks remains unchanged. Such a shift causes the realizations of stock market returns and interest rates to more closely trace out the policy reaction function than before, as shown in Figure II. In other words, the disturbances are distributed around an ellipse that enlarges along the policy reaction function when the shocks to the stock market are more volatile. Thus, we are able to identify the slope of the policy reaction function based on changes in the covariance of interest rate and stock market movements across periods when the variance of their shocks shifts.

This intuition was first introduced by Wright [1928]. He discusses how the bias from simultaneous equations goes to zero when the variance of one of the shocks goes to infinity, in which case one of the equations is identified. The intuition described above is in the same spirit, but it requires only a shift in the relative magnitudes of the variances of the shocks.[15]

A rough glance at the data suggests that shifts in the volatility of shocks do in fact affect the correlation between changes in interest rates and stock prices. Figure III shows the six-month rolling correlation between daily changes in the three-month Treasury bill rate and daily changes in the S&P 500 index. As

15. Recently, there have been several papers extending the intuition of Philip Wright in several dimensions. Sentana and Fiorentini [2001] study the case in which the data are characterized by conditional heteroskedasticity, and Rigobon [1999] examines the case with unconditional heteroskedasticity.

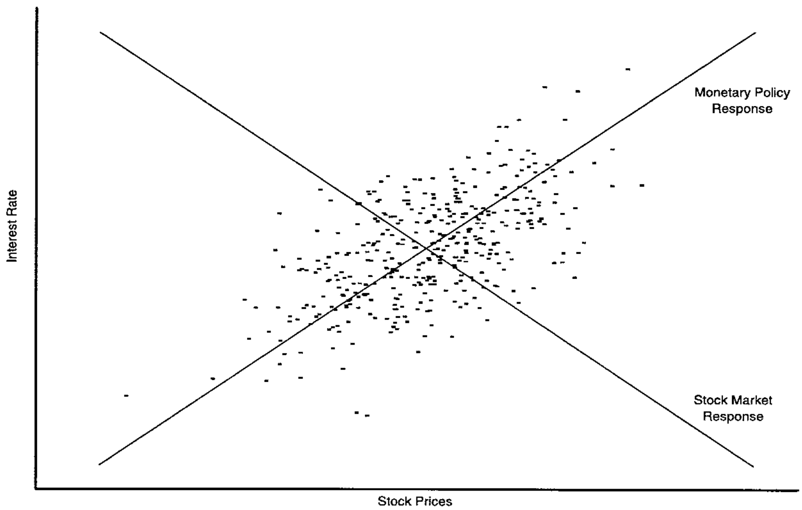

FIGURE II
Periods of High Stock Market Volatility

noted earlier, this correlation is typically negative, likely reflecting the endogenous response of stock prices to the interest rate (the downward-sloping schedule in Figure II). However, the correlation varies fairly extensively over time, often becoming positive during periods in which the rolling standard deviation of stock price changes is elevated. Such shifts in sign are likely explained by a shift in the relative importance of different shocks, as depicted in Figures I and II.

In the empirical exercise that follows, we again use the VAR described in equations (1) and (2) of the previous section. The specification is very general, in that it allows for the common shock z_t in addition to the structural shocks η_t and ϵ_t. However, having such a general specification comes at some cost. In particular, instead of having full identification of the system, we will be able to achieve only partial identification. But the partial identification will be sufficient to measure the response of monetary policy to the stock market—the focus of this paper. Moreover, as we discuss below, the presence of z_t is crucial for properly measuring the policy reaction, and we can reject the specification in which the common shock is not included.

The parameter of interest is β—the reaction of the short-term interest rate to the stock market. However, equation (1) cannot be

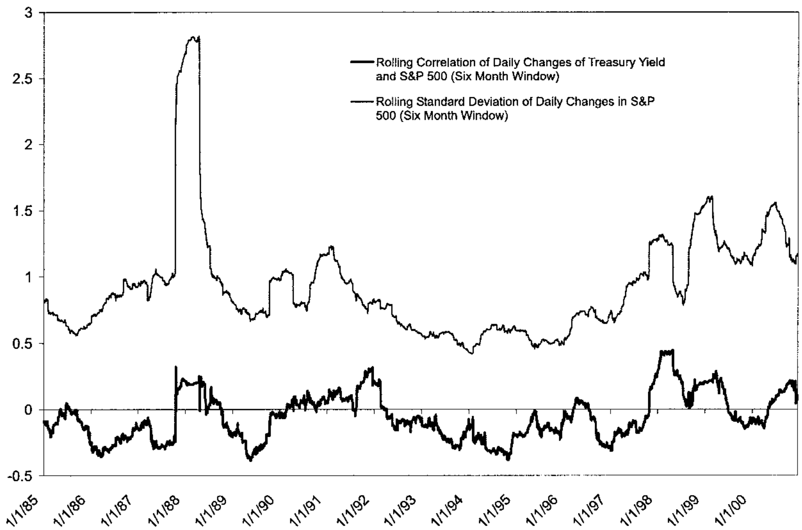

FIGURE III
Comovements in Equity Prices and Interest Rates

estimated directly, because of the simultaneity problem discussed above, and because z_t is unobserved. In fact, only the following reduced form of this system can be estimated:

$$(4) \qquad \begin{pmatrix} i_t \\ s_t \end{pmatrix} = \Phi x_t + \begin{pmatrix} \nu_t^i \\ \nu_t^s \end{pmatrix},$$

where the reduced-form residuals (ν_t^i and ν_t^s) are given by

$$\nu_t^i = \frac{1}{1-\alpha\beta}\left[(\beta+\gamma)z_t + \beta\eta_t + \epsilon_t\right],$$

$$\nu_t^s = \frac{1}{1-\alpha\beta}\left[(1+\alpha\gamma)z_t + \eta_t + \alpha\epsilon_t\right].$$

Given the interpretation of the structural shocks, they are assumed to be uncorrelated at all leads and lags. In that case, the reduced-form innovations have the following covariance matrix:

$$(5) \quad \Omega = \frac{1}{(1-\alpha\beta)^2} \times \begin{bmatrix} (\beta+\gamma)^2\sigma_z^2 + \beta^2\sigma_\eta^2 + \sigma_\epsilon^2 & (1+\alpha\gamma)(\beta+\gamma)\sigma_z^2 + \beta\sigma_\eta^2 + \alpha\sigma_\epsilon^2 \\ \cdot & (1+\alpha\gamma)^2\sigma_z^2 + \sigma_\eta^2 + \alpha^2\sigma_\epsilon^2 \end{bmatrix}.$$

The problem of identification is that the covariance matrix only provides three equations—two variances and a covariance—while there are six unknowns: α, β, γ, σ_z^2, σ_η^2, and σ_ϵ^2.

It is unlikely that this covariance matrix remains constant over the sample, though. Indeed, a prevalent characteristic of macroeconomic and financial data is heteroskedasticity. Both interest rates and stock market returns are known to exhibit patterns of volatility that are somewhat predictable, including periods during which their variances are elevated. While such heteroskedasticity is typically ignored in VAR studies, here we use it to our advantage to appropriately identify the parameter β.[16]

More specifically, the presence of unconditional heteroskedasticity in the reduced-form innovations provides additional equations to the system represented by (5). Suppose, for example, that we could identify a shift to a regime with a different covariance matrix of the reduced-form residuals. In this case, the new regime provides three new equations (the elements of the covariance matrix). Of course, without any restrictions the new regime also adds three new unknown parameters—the parameters σ_z^2, σ_η^2, and σ_ϵ^2 under the new regime. But if additional assumptions can be imposed on the variances of the shock processes, then allowing for heteroskedasticity will help identify the system. Assumptions of this type may be more appropriate than the exclusion restrictions considered above.

In the current context, we assume that the monetary policy shocks ϵ_t are homoskedastic. This assumption is motivated by the interpretation of such shocks.[17] As discussed above, the monetary policy reaction function controls for the systematic response of the interest rate to changes in the outlook for output and inflation, as captured by the macroeconomic news variables x_t and z_t and equity prices s_t. The remaining movements in the interest rate, defined to be the policy shocks, are driven by other factors. One of the primary interpretations of these shocks, as discussed in Christiano, Eichenbaum, and Evans [1999], is shifts in the preferences of individual FOMC members or in the manner in which their views are aggregated. In addition, other institutional

16. In the present paper we concentrate only on identification under unconditional heteroskedasticity. Identification under conditional heteroskedasticity can also be solved using similar arguments.

17. We can also empirically investigate the validity of this assumption using the test of overidentifying assumptions described below. The results indicate that the heteroskedasticity of the policy shock, if present, is weak enough that the overidentifying restrictions are not rejected.

details, such as the timing of FOMC meetings, might also play a role. One would not expect the variance of these institutional factors to vary over time, which is the basis for our identification assumption. Note, importantly, that this identification assumption does not impose that the interest rate itself is homoskedastic. On the contrary, heteroskedasticity in the macroeconomic news or equity prices will pass through to the interest rate. The identification assumption only requires that all of the heteroskedasticity in the interest rate comes through the systematic response of policy.

If the policy shocks are homoskedastic, then a shift in the covariance matrix adds three equations but only two unknown parameters. In that case, the parameter β is identified as long as there are at least three different regimes for the covariance matrix. (The results below suggest that there are at least three regimes.) For each regime i the covariance matrix can be written as

$$(6)\quad \Omega_i = \frac{1}{(1-\alpha\beta)^2} \times \begin{bmatrix} (\beta+\gamma)^2\sigma_{i,z}^2 + \beta^2\sigma_{i,\eta}^2 + \sigma_\epsilon^2 & (1+\alpha\gamma)(\beta+\gamma)\sigma_{i,z}^2 + \beta\sigma_{i,\eta}^2 + \alpha\sigma_\epsilon^2 \\ \cdot & (1+\alpha\gamma)^2\sigma_{i,z}^2 + \sigma_{i,\eta}^2 + \alpha^2\sigma_\epsilon^2 \end{bmatrix}.$$

Two important assumptions have been made in equation (6). First, as indicated above, the variance of the monetary policy shocks remains constant across regimes. Second, we have assumed that the parameters (α, β, and γ) are stable across the covariance regimes. Note that this second assumption is implicitly made in the vast majority of related empirical studies. In the macroeconomics literature, VARs are often estimated across samples that surely exhibit heteroskedasticity, without allowing shifts in parameters. Similarly, in the finance literature many studies that even explicitly allow for variation in volatility, including GARCH models, often impose that the parameters of the underlying equation are fixed.

Under these assumptions, the parameter β can be identified by the impact of regime changes on the variance-covariance matrix. For example, an increase in the variance of stock market shocks, σ_η^2, increases the variances of both reduced-form innovations and makes the covariance between them more positive (assuming β is positive)—the intuition shown earlier in Figures I and II. Because the specification allows heteroskedastic unobserved shocks, the algebra is more complicated. However, it can be shown (see Appendix 1) that the parameter β must solve the following quadratic equation:

$$a\beta^2 - b\beta + c = 0, \tag{7}$$

where

$$\begin{aligned} a &= \Delta\Omega_{31,22}\Delta\Omega_{21,12} - \Delta\Omega_{21,22}\Delta\Omega_{31,12} \\ b &= \Delta\Omega_{31,22}\Delta\Omega_{21,11} - \Delta\Omega_{21,22}\Delta\Omega_{31,11} \\ c &= \Delta\Omega_{31,12}\Delta\Omega_{21,11} - \Delta\Omega_{21,12}\Delta\Omega_{31,11}, \end{aligned}$$

and $\Delta\Omega_{ij,km}$ denotes element (k,m) of the matrix $\Delta\Omega_{ij}$, which is the difference between the covariance matrix in regime i and regime j.[18]

Note that when there are more than three regimes for the covariance matrix, any three can be used to arrive at a solution to equation (7). If all of the assumptions of the model hold, the estimate of β should be the same under any subset of three regimes. In the empirical implementation, we use this as a test of overidentifying restrictions. A rejection that the coefficient estimates are the same could indicate that the parameters are unstable across regimes, that the homoskedasticity assumption on the policy equation is violated, or that there are nonlinearities that are not captured in our specification. The test is described in more detail below.

IV. Results

The initial step in the estimation procedure is to determine the different regimes for the variance-covariance matrix of the reduced-form shocks to monetary policy and the stock market. To do so, we begin by estimating the reduced form (4) and computing the residuals. As expected, there are rich patterns to the volatility of the shocks to stock market returns and the short-term interest rate. Because the periods of elevated volatility for interest rates and the stock market at times coincide with one another and at other times do not, we define four regimes: one in which both interest rates and stock returns have low variance, one in which they both have high variance, and two in which one has high and the other low variance. Periods of high variance are defined as

18. See Appendix 1 for interpreting the other root to equation (7).

TABLE II
REGIMES FOR VARIANCE-COVARIANCE MATRIX OF REDUCED-FORM SHOCKS

	Variance of int. rate shocks	*Variance of stk. mkt. shocks*	*Covariance*	*Frequency of obs.*
Regime 1	0.00226	0.5238	−0.00262	90.2%
Regime 2	0.00374	**2.4732**	0.02757	3.1%
Regime 3	**0.02326**	**4.5422**	0.03907	2.6%
Regime 4	**0.01059**	0.4659	−0.02462	4.1%

High variance regimes are in bold. All variables are measured in percentage points.

when the 30-day rolling variance of the residual is more than one standard deviation above its average. The four covariance regimes that result are described in Table II. As shown in the table, the covariance between these variables tends to fluctuate with the movements in their variances, often becoming positive as the variance of the stock market increases.

This approach for defining the different regimes of the variance-covariance matrix is admittedly arbitrary. However, the estimates are consistent even if the heteroskedasticity is misspecified, as long as the true data exhibit heteroskedasticity and the regimes are not misspecified too badly. As demonstrated in Appendix 2, because the covariance matrices of misspecified regimes are linear combinations of the true covariance matrices, the system of equations obtained from the misspecified system has the same solution as those derived from a more well-specified set of regimes. The only case in which the estimates are not consistent is if the regimes are specified so poorly that each regime contains the same weightings of the true underlying regimes. Some direct evidence on the impact of misspecifying the heteroskedasticity is presented in Section V.

The analysis from the previous section indicated that the monetary policy reaction to the stock market—β in equation (1)—could be identified as long as there were three regimes for the covariance matrix. We first estimate the reaction coefficient using the first three regimes listed in Table II. The results indicate a positive policy response to the stock market, with an estimated coefficient β of 0.0214. The distribution of the coefficient can be calculated by bootstrap and is shown in Figure IV (top panel). In implementing the bootstrap, we use the asymptotic

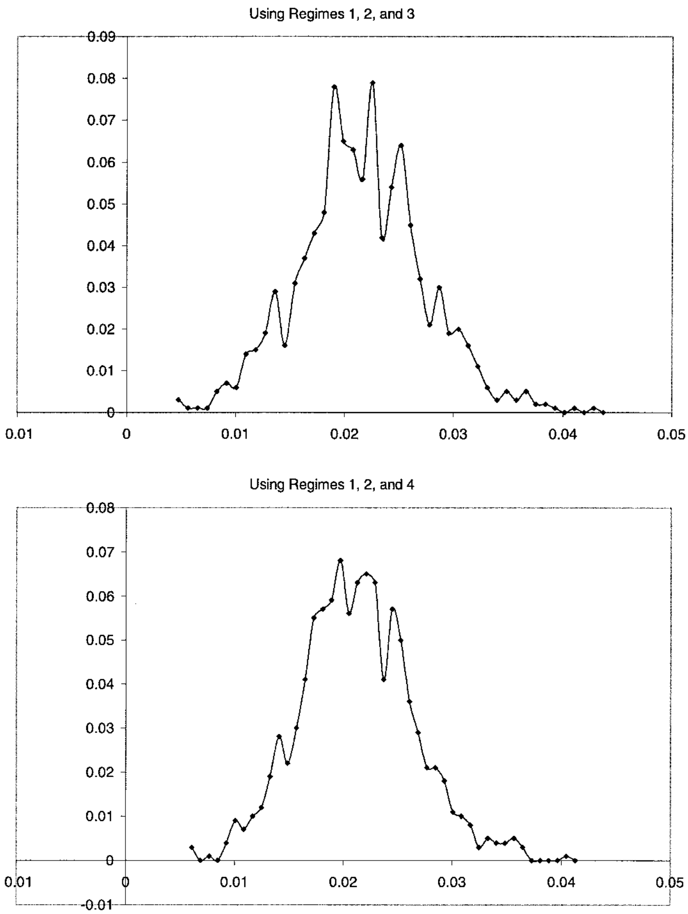

FIGURE IV
Distributions of Estimated Policy Response Coefficient

distribution of the covariance matrices in each regime and solve for the estimated coefficient in 1000 draws. As is apparent from the figure, the estimated parameter is strongly significant, with none of the calculated distribution falling below zero.

This is the major result of the paper: by employing a more appropriate identification approach based on heteroskedasticity, we find a significant positive reaction of monetary policy to the stock market. The point estimate for the response coefficient β indicates that a 5 percent rise in the S&P 500 index tends to

TABLE III
ESTIMATES UNDER ALTERNATIVE SUBSETS OF REGIMES

	Regimes 1, 2, 3	*Regimes* 1, 2, 4	*Regimes* 1, 3, 4	*Regimes* 2, 3, 4	*Regimes* *All*
Mean of distribution	0.0214	0.0211	0.0273	0.1402	0.0210
Std. dev. of distribution	0.0058	0.0052	0.3137	3.8123	0.0052
Median of distribution	0.0212	0.0208	0.0169	0.0191	0.0208
Mass below zero	0.0%	0.0%	1.4%	1.4%	0.0%

increase the three-month interest rate by 10.7 basis points. It may be useful to translate this estimate into the probability of a policy tightening. On average, the next FOMC meeting will be about three weeks away, and the three-month rate reflects the expected rate to prevail over the next twelve weeks. Thus, only three-quarters of the expected impact on the federal funds rate shows up in the three-month interest rate on impact. The estimated coefficient therefore corresponds roughly to an increase in the expected federal funds rate of 14.3 basis points (equal to 10.7 times 4/3). Translating this into discrete policy actions, a 5 percent rise in the S&P 500 index increases the probability of a 25 basis point tightening by about 57 percent. A similar-sized fall in stock prices would have the same implications for a 25 basis point easing.

Similar results are obtained if the parameter is estimated from other subsets of regimes. As shown above, the parameter β is just identified when there are three regimes and is overidentified when there are four. Indeed, we can instead estimate β using any other subset of three regimes, such as regimes 1, 2, and 4. The point estimate resulting from that subset of regimes is 0.0211, and the distribution of the estimate is shown in Figure IV (bottom panel). The estimate is again significantly positive, with no realizations falling below zero, and its magnitude is very similar to the previous estimate.

The estimates resulting from all four of the possible subsets of regimes are summarized in Table III. As can be seen, the estimates from those subsets shown in Figure IV are very close to one another. The other two subsets of regimes yield estimates that are much less precise. The standard deviation of these estimates blows up considerably, primarily because of some realizations with very high values. However, even in those cases, only a

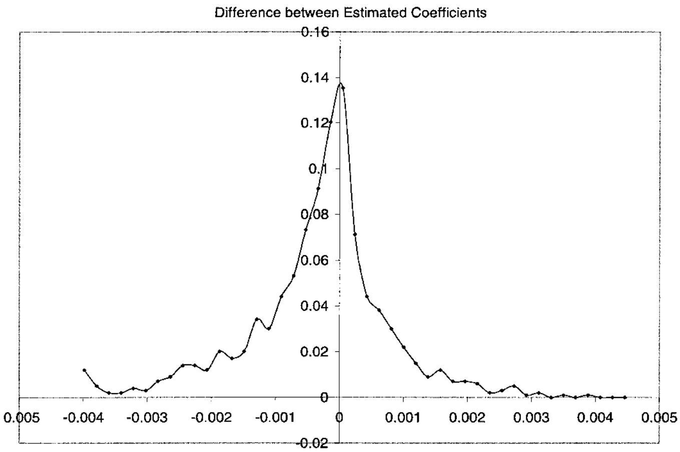

FIGURE V
Test of Overidentifying Restrictions (Regimes 1-2-3 versus Regimes 1-2-4)

small mass of observations falls below zero, and the medians of the distributions are very close to the other estimates. Table III also shows the results obtained if all four regimes are used to estimate β. In this case, we treat equation (6) as moment conditions and solve for the parameters using GMM. The resulting estimates are very similar to those from the subsets of regimes shown in Figure IV.

As previously mentioned, the difference between estimates under the various subsets of regimes can be used as a test of overidentifying restrictions. Such a test would indicate whether the restrictions imposed on the model, most notably the assumption that the parameters are stable across different regimes and the homoskedasticity assumption of the policy shocks, can be rejected. Note, however, that this approach does not test the assumption that the three shocks are uncorrelated. Because there are five estimates available, many different overidentification tests could be performed. We focus first on comparing the estimates from using regimes 1, 2, and 3 and regimes 1, 2, and 4. The distribution of the difference in the estimates, which is also computed using the same bootstrap as before, is shown in Figure V. According to that distribution, 61.8 percent of the observations are negative, and 38.2 percent positive, which implies that we cannot reject the hypothesis that the estimates are the same. In other words, the overidentifying restrictions are easily passed.

Moreover, additional tests indicate that the overidentifying restrictions could not be rejected using any subset of the five estimates.

The overidentifying restrictions of the model can be rejected, however, if we exclude the common shocks z_t. In that case, the estimated β under some subsets of regimes tends to be much larger than the estimates obtained above.[19] This finding likely reflects that unobserved macroeconomic developments have tended to push the interest rate and stock prices in the same direction. Consider market developments in the fall of 1998, for example. The strains that emerged in financial markets at that time threatened to disrupt the U. S. economy, leading to a decline in equity prices and an easing of monetary policy by the Federal Reserve. But while the economic *outlook* shifted considerably, that shift was not apparent in the *current* data releases x_t. Without the common shock z_t, the VAR would inappropriately attribute all of the movement in the interest rate at that time to its response to equity prices. Thus, the inclusion of the common shock is important for arriving at an accurate estimate of the policy response β. More importantly, the rejection of the model in this case suggests that the test of overidentifying restrictions does have some power.

The empirical exercise in this paper is concerned only with measuring the policy reaction to the stock market, and not with determining whether such a reaction is optimal. Nevertheless, it may be useful to assess whether the magnitude of the estimated policy reaction (from Table III) is reasonable from a macroeconomic perspective. To do so, we compare the estimated reaction with two benchmarks. The first is based on a rough calculation of the impact of stock price movements on aggregate spending. As previously indicated, there are several channels through which the stock market affects the economy, including the wealth effect on aggregate consumption and the cost of financing channel for investment. For simplicity, we will focus only on the first of these, realizing that we could be underestimating the expected impact.

If equity prices were to broadly rise by 5 percent, for example, household wealth would increase by about \$578 billion, based on household equity holdings as of the end of 2000. If the marginal propensity to consume out of stock market wealth is 4 percent,

19. See the working paper version of this paper for detailed estimates and a lengthier discussion.

then aggregate consumption would rise by $23 billion, pushing up GDP by about 0.23 percent.[20] To map this change into a movement in the federal funds rate, we rely on the literature on estimated monetary policy rules. A vast number of papers, initiated by Taylor [1993] and others, have estimated policy rules in which the federal funds rate reacts to output and inflation. While Taylor wrote down a monetary policy reaction coefficient of 0.5 to the output gap, other papers have estimated this coefficient to be higher, perhaps around 1.[21] Under that range of estimates, a 5 percent rise in the equity market, through its expected impact on the output gap, would be expected to result in a 12 to 23 basis point rise in the federal funds rate.

Of course, there are a number of reasons to take this back-of-the-envelope calculation with caution. We ignored other channels through which the stock market impacts the economy as well as any multiplier effects. Moreover, the calculation does not account for dynamics. Stock market wealth has an impact on spending only after a considerable lag, although policy-makers may still choose to react immediately given the lags in the impact of monetary policy. Despite these possible complications, the calculation suggests that the magnitude of the estimated policy reaction to the stock market is in the ballpark of the reaction that would be expected if the Federal Reserve were concerned only with the ultimate impact of the stock market on output and inflation.

A second benchmark for gauging the magnitude of the policy reaction relies on the Federal Reserve's model of the U. S. economy. The model, which is described in Reifschneider, Tetlow, and Williams [1999], provides an estimate of the extent to which changes in stock market wealth feed through into aggregate demand. In that paper the authors present the simulated response of the economy to a permanent shock to the equity risk premium, which affects stock prices without having a direct effect on other macroeconomic variables. Under a fixed real federal funds rate, the shock would stimulate aggregate demand and lead to a rise in inflation. Because of these effects, though, the central bank would likely tighten in response to the shock. Indeed, the authors consider the response of the

20. For estimates of the impact of stock market wealth on aggregate consumption, see Maki and Palumbo [2001], among others.

21. See, for example, Clarida, Gali, and Gertler [2000].

economy under a "stabilizing policy"—one that effectively reduces the impact of the shock on output and inflation. Judging from the path of the policy rate under the stabilizing policy, it appears that a permanent 5 percent shock to the value of the stock market warrants a persistent response of the federal funds rate of about 12.5 basis points.

These two benchmarks suggest that one would expect a positive reaction of monetary policy to the stock market, consistent with our finding of a significant, positive response coefficient. Moreover, both of these benchmarks indicate that the estimated policy response coefficient is within a reasonable range of the magnitude that one would have expected for a central bank concerned only with macroeconomic activity.

V. Robustness

In this section several sensitivity exercises are run to evaluate the robustness of the specification we have used. In short, the findings are that the results are quite robust to misspecification of the heteroskedasticity of the shocks and to changes in the choice of the interest rate variable and the frequency of the data.

V.A. Heteroskedasticity Misspecification

As was mentioned briefly in Section IV, the estimates are consistent even under some misspecification of the heteroskedasticity. This result is shown formally in Appendix 2, but the intuition for it is straightforward. If the regimes for the shifts in variance are misspecified, then the computed covariance matrices are linear combinations of the true covariance matrices. Thus, the estimates of the shifts in the variances of the shocks (σ_z^2, σ_ϵ^2, and σ_η^2) across the regimes will be mismeasured. However, the parameter β is determined by how the covariance between the reduced-form innovations changes as the variances shift. Because the misspecified covariance matrices are just linear combinations of the true matrices, the system of equations is equally identified; the shift in the covariance will still be proportional to the measured shift in the variances. Thus, the mismeasurement in the shift in the variances does not bias the estimates of the parameter β.

This argument holds so long as the regimes are not misspecified too badly. In that case, the misspecification of the regimes only reduces the efficiency of the estimates, as it averages out the

TABLE IV
ESTIMATES UNDER ALTERNATIVE DEFINITIONS OF REGIMES

	Random 3-month regimes	*Random 6-month regimes*
Mean of distribution	0.0333	0.0360
Std. dev. of distribution	0.1896	0.2141
Median of distribution	0.0449	0.0248
Mass below zero	31.8%	45.1%

Estimates based on all regimes.

differences across regimes that are the basis for identification. However, if the misspecification of the regimes is sufficiently poor, a necessary rank condition will be violated. Indeed, if the regimes are misspecified so much as to have equal weights of the true regimes, the Ω_i matrices from equation (6) will be proportional to one another, and thus the system of equations based on the shifts in those matrices is underidentified.

To illustrate these points, consider a case in which the regimes are very poorly specified. In particular, we split the sample every three and six months and treat each as a separate regime. The results are shown in Table IV, which for brevity reports only the estimates obtained using all regimes. As is evident from the table, the estimates of the contemporaneous effect of stock prices on the interest rate are extremely noisy. This imprecision should be expected, as it is a direct consequence of the washing out of the regimes due to the misspecification.

If we stick to reasonable changes to the definitions of the regimes, though, the results found in Section IV are not strongly affected. To see this, we compute the estimates reducing the criteria for determining high volatility periods to half of a standard deviation (from the one standard deviation threshold used above) and then increasing it to two standard deviations. The point estimates, reported in Table V, vary some but are largely similar to the results from Section IV. Indeed, one cannot statistically reject the hypothesis that they are equal to the previous results, with a significance level of 19.0 percent for the two standard deviation regimes and 9.7 percent for the one-half standard deviation regimes.

Finally, the regimes could also be defined based on a regime-switching model estimated by maximum likelihood. We experi-

TABLE V
ESTIMATES UNDER ALTERNATIVE DEFINITIONS OF REGIMES

	One-half std. dev. regimes	*Two std. dev. regimes*
Mean of distribution	0.0095	0.0318
Std. dev. of distribution	0.0029	0.0099
Median of distribution	0.0096	0.0313
Mass below zero	0.0%	0.0%

Estimates based on all regimes.

mented with such a model and found that the estimated parameter was very similar to the ones reported earlier.

V.B. Changes in Frequency

Another important issue is whether the central bank's reaction to stock price movements depends on the frequency of the data. Indeed, one might expect that a rise in equity prices over the course of a day would prompt a different policy response than a same-sized rise that is sustained over a longer period. If daily movements in the stock market are more volatile (on an annualized basis) than weekly or monthly movements, as is the case in our sample, one might expect a more tempered response to higher frequency changes.[22]

To address this issue, we analyze the impact of changing the frequency of the exercise above from daily to weekly. The results, shown in Table VI, indicate that under the one-standard deviation threshold, the estimated response of monetary policy is very imprecise. One cannot even reject the hypothesis that the estimate equals zero. A possible explanation for this finding is that relying on lower frequency data reduces the heteroskedasticity in the data. Because there are fewer observations under the high variance regimes, it is more difficult to get precise estimates under this identification method.

Indeed, the precision of the estimate improves considerably if we reduce the threshold for the high variance regime to

22. In our sample, daily stock price changes are slightly more volatile relative to weekly changes than would be expected if stock prices followed a random walk. The relative variance of stock market returns over different horizons has been explored in a number of papers, some of which find that changes over longer horizons are too volatile relative to daily changes to be consistent with a random walk. See, for example, Lo and MacKinlay [1988].

TABLE VI
ESTIMATES USING WEEKLY DATA

	One std. dev. regimes	*One-half std. dev. regimes*
Mean of distribution	0.0148	0.0239
Standard deviation	0.4748	0.0111
Median of distribution	0.0247	0.0226
Mass below zero	27.2%	0.3%

Estimates based on all regimes.

half a standard deviation. In that case, the coefficient is precisely estimated, with a point estimate of 0.0239—somewhat larger than that found in the daily results. One can marginally reject the hypothesis that the weekly estimate is the same as the daily one, but the economic interpretation of the estimate is quite similar, with a 5 percent rise in the S&P 500 over a week increasing the probability of a 25 basis point monetary policy tightening by 64 percent. Moreover, as in the daily results, one cannot reject that the estimates are the same across other combinations of regimes, but the overidentification restrictions were rejected in the specification that excluded the common shocks.

V.C. Changes in the Interest Rate Variable

In all of the above results, inferences about future monetary policy decisions were made based on the behavior of the three-month Treasury bill. While the Treasury bill was once the most liquid security at short maturities, its liquidity has deteriorated notably over the sample. Much of the market activity at shorter maturities has shifted to other securities, including eurodollar deposits and eurodollar futures contracts. Thus, one could argue that the eurodollar rate would be a more appropriate instrument for measuring policy expectations in the latter part of the sample. We ran an alternative set of daily results replacing the three-month Treasury yield with the rate on the near-term eurodollar futures contract (the next contract to expire). The value of this contract is based on the three-month eurodollar rate at the time of expiration, which is always within three months (the contracts are quarterly). The results were very similar, indicating that the conclusions reached above are not importantly influenced by the choice of the interest rate variable.

VI. Conclusions

This paper attempts to decompose daily and weekly movements in interest rates and stock prices into the endogenous responses to different types of shocks. By relying on heteroskedasticity to identify the system of equations, we are able to effectively measure the reaction of the short-term interest rate to the stock market, even when the stock market is endogenously reacting to the interest rate at the same time. The results suggest that stock market movements have a significant impact on short-term interest rates, driving them in the same direction as the change in stock prices. According to the estimates, a 5 percent rise in stock prices over a day causes the probability of a 25 basis point interest rate hike to increase by a half, while a similar-sized movement that takes place over a week has a slightly larger effect on anticipated policy actions. The estimated response is consistent with some rough calculations of the impact of stock price movements on aggregate demand, suggesting that policy-makers are reacting to stock price movements to the extent warranted by their implications for the economy.

The results presented should be regarded as a first step in addressing a very difficult question. One issue is that policymakers might want to react differently to equity price movements that are driven by different types of shocks. Our analysis attempts to measure the policy response to an exogenous movement in equity prices—one driven by a change in investors' willingness to bear risk. To this end, the empirical exercise controls for the influence of macroeconomic shocks by including the major economic data releases in the specification. However, it is still possible that the remaining equity price movements are systematically driven by factors that also affect the outlook for the economy, and thus the measured response to equity prices could in part reflect the impact of those factors on the policy path. Nevertheless, the exercise provides an estimate of the magnitude of the response of short-term interest rates to a typical movement in stock prices—an important step in understanding the complicated interactions between interest rates and equity prices. A useful topic for future research may be to measure the policy response to stock price movements that are more effectively conditioned on the type of underlying shock.

Appendix 1: Solution to the Identification Problem

This appendix demonstrates the solution to the identification problem described in Section III. The parameter β is identified as long as there are at least three different regimes for the covariance matrix. The covariance matrix under each regime, $i = 1, 2, 3$, can be written as follows:

$$(8)\quad \Omega_i = \frac{1}{(1-\alpha\beta)^2} \times \begin{bmatrix} (\beta+\gamma)^2\sigma_{i,z}^2 + \beta^2\sigma_{i,\eta}^2 + \sigma_\epsilon^2 & (1+\alpha\gamma)(\beta+\gamma)\sigma_{i,z}^2 + \beta\sigma_{i,\eta}^2 + \alpha\sigma_\epsilon^2 \\ \cdot & (1+\alpha\gamma)^2\sigma_{i,z}^2 + \sigma_{i,\eta}^2 + \alpha^2\sigma_\epsilon^2 \end{bmatrix}.$$

Define $\Delta\Omega_{21} = \Omega_2 - \Omega_1$ and $\Delta\Omega_{31} = \Omega_3 - \Omega_1$. Equation (8) implies that

$$\Delta\Omega_{j1} = \frac{1}{(1-\alpha\beta)^2} \times \begin{bmatrix} (\beta+\gamma)^2\Delta\sigma_{j1,z}^2 + \beta^2\Delta\sigma_{j1,\eta}^2 & (1+\alpha\gamma)(\beta+\gamma)\Delta\sigma_{j1,z}^2 + \beta\Delta\sigma_{j1,\eta}^2 \\ \cdot & (1+\alpha\gamma)^2\Delta\sigma_{j1,z}^2 + \Delta\sigma_{j1,\eta}^2 \end{bmatrix},$$

where $\Delta\sigma_{j1,z}^2 = \sigma_{j,z}^2 - \sigma_{1,z}^2$ and $\Delta\sigma_{j1,\eta}^2 = \sigma_{j,\eta}^2 - \sigma_{1,\eta}^2$ for $j = \{2,3\}$. Because the parameters are stable and σ_ϵ^2 is homoskedastic, the change in the covariance matrix does not depends on the variance of the monetary policy shocks.

These two changes in the covariance matrices, Ω_{21} and Ω_{31}, form a system of six nonlinear equations with seven unknowns, but in which β is just identified. To see this, rewrite the covariance matrix as

$$\Delta\Omega_{j1} = \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \omega_{z,j} + \beta^2\Delta\sigma_{j1,\eta}^2 & \theta\omega_{z,2} + \beta\Delta\sigma_{j1,\eta}^2 \\ \cdot & \theta^2\omega_{z,2} + \Delta\sigma_{j1,\eta}^2 \end{bmatrix},$$

where

$$\theta = (1+\alpha\gamma)/(\beta+\gamma)$$
$$\omega_{z,j} = (\beta+\gamma)^2\Delta\sigma_{j1,z}^2.$$

The six equations that result can be written as follows:

$$\begin{aligned}
\omega_{z,2} + \beta^2 \Delta\sigma^2_{21,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{21,11} \\
\theta\omega_{z,2} + \beta \Delta\sigma^2_{21,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{21,12} \\
\theta^2\omega_{z,2} + \Delta\sigma^2_{21,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{21,22} \\
\omega_{z,3} + \beta^2 \Delta\sigma^2_{31,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{31,11} \\
\theta\omega_{z,3} + \beta \Delta\sigma^2_{31,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{31,12} \\
\theta^2\omega_{z,3} + \Delta\sigma^2_{31,\eta} &= (1 - \alpha\beta)^2 \cdot \Delta\Omega_{31,22},
\end{aligned}$$

where $\Delta\Omega_{j1,kl}$ is the k and l element of the j matrix. If $\theta\beta \neq 1$, which assures finite variance, then the three equations for each covariance matrix collapse to

$$\theta = \frac{\Delta\Omega_{21,12} - \beta\Delta\Omega_{21,22}}{\Delta\Omega_{21,11} - \beta\Delta\Omega_{21,12}} \tag{9}$$

$$\theta = \frac{\Delta\Omega_{31,12} - \beta\Delta\Omega_{31,22}}{\Delta\Omega_{31,11} - \beta\Delta\Omega_{31,12}}, \tag{10}$$

which is a system of two equations with two unknowns. Finally, equations (9) and (10) imply a quadratic equation for β:

$$a\beta^2 - b\beta + c = 0,$$

where

$$\begin{aligned}
a &= \Delta\Omega_{31,22}\Delta\Omega_{21,12} - \Delta\Omega_{21,22}\Delta\Omega_{31,12} \\
b &= \Delta\Omega_{31,22}\Delta\Omega_{21,11} - \Delta\Omega_{21,22}\Delta\Omega_{31,11} \\
c &= \Delta\Omega_{31,12}\Delta\Omega_{21,11} - \Delta\Omega_{21,12}\Delta\Omega_{31,11}.
\end{aligned}$$

If the quadratic equation has a real solution, the system of equations is identified and can be solved for β and θ. It is easy to show that one of the solutions of the system of equations gives the true values, while the second solution gives their inverse.[23] Moreover, it is easy to show that, because the covariance matrices are positive definite, there should always be a real solution to the quadratic equation.

23. The quadratic equation has two solutions. One corresponds to the values of θ and β from the system of equations (1) and (2). The other corresponds to a system of equations in which the stock market reaction equation is written in terms of the interest rate and the policy reaction equation in terms of stock prices. In that case, the solution gives the values for $\theta^* = 1/\beta$ and $\beta^* = 1/\theta$.

Appendix 2: Consistency under Misspecification of Heteroskedasticity

Section IV argues that the parameter estimates are consistent even under some misspecification of the heteroskedasticity. This proposition is demonstrated in this appendix. Taking the reduced-form innovations from our specification, equations (1) and (2) can be expressed as

$$A\begin{pmatrix} \nu_t^i \\ \nu_t^s \end{pmatrix} = \Gamma z_t + \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix}, \tag{11}$$

where A includes the parameters measuring the contemporaneous reactions of the variables and $\Gamma = \binom{\gamma}{1}$.[24] In the identification procedure with three regimes for the variance-covariance matrix of reduced-form innovations, we solve the following system of equations:

$$\begin{aligned} \Omega_{i,s}^{reg1} &= A^{-1}\Gamma \cdot \sigma_z^{reg1} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg1} \cdot A'^{-1}, \\ \Omega_{i,s}^{reg2} &= A^{-1}\Gamma \cdot \sigma_z^{reg2} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg2} \cdot A'^{-1}, \\ \Omega_{i,s}^{reg3} &= A^{-1}\Gamma \cdot \sigma_z^{reg3} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg3} \cdot A'^{-1}, \end{aligned} \tag{12}$$

where $\Omega_{i,s}^{regj}$ is the covariance matrix between the reduced-form shocks in regime j, $\Omega_{\epsilon,\eta}^{regj}$ is the covariance matrix of the structural shocks, and σ_z^{regj} is the variance of the common shock. We assume that the system of equations has a solution, or implicitly that it satisfies a rank condition.

If the regimes are misspecified, then the computed covariance matrices are linear combinations of the true covariance matrices. Formally, the misspecified covariance matrix of regime j can be written as a linear combination of the true covariance matrices, as follows:

$$\begin{aligned} \Omega_{i,s}^{mis1} &= \lambda_{11}\Omega_{i,s}^{reg1} + \lambda_{12}\Omega_{i,s}^{reg2} + \lambda_{13}\Omega_{i,s}^{reg3} \\ \Omega_{i,s}^{mis2} &= \lambda_{21}\Omega_{i,s}^{reg1} + \lambda_{22}\Omega_{i,s}^{reg2} + \lambda_{23}\Omega_{i,s}^{reg3} \\ \Omega_{i,s}^{mis3} &= \lambda_{31}\Omega_{i,s}^{reg1} + \lambda_{32}\Omega_{i,s}^{reg2} + \lambda_{33}\Omega_{i,s}^{reg3}, \end{aligned}$$

where the matrix of λ's determines the extent of the misspecification. If these misspecified regimes are used in (12) to solve for the parameters, the resulting system of equations will be a linear

24. To arrive at (11), first solve equations (1) and (2) to get reduced-form equations for i_t and s_t as functions of x_t and z_t. Next, project both sides of the equation on x_t. The left-hand side becomes the reduced-form innovations ν_t^i and ν_t^s (see equation (4)), and the x_t falls out of the right-hand side.

combination of the original system. Indeed, the following equations will hold:

$$\begin{aligned}\Omega_{i,s}^{mis1} = {} & \lambda_{11}(A^{-1}\Gamma \cdot \sigma_z^{reg1} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg1} \cdot A'^{-1}) \\ & + \lambda_{12}(A^{-1}\Gamma \cdot \sigma_z^{reg1} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg1} \cdot A'^{-1}) \\ & + \lambda_{13}(A^{-1}\Gamma \cdot \sigma_z^{reg1} \cdot \Gamma' A'^{-1} + A^{-1} \cdot \Omega_{\epsilon,\eta}^{reg1} \cdot A'^{-1}),\end{aligned}$$

which for each regime can be rewritten as

$$\begin{aligned}\Omega_{i,s}^{mis1} &= A^{-1}\Gamma(\lambda_{11}\sigma_z^{reg1} + \lambda_{12}\sigma_z^{reg2} + \lambda_{13}\sigma_z^{reg3})\Gamma' A'^{-1} \\ &\quad + A^{-1}(\lambda_{11}\Omega_{\epsilon,\eta}^{reg1} + \lambda_{12}\Omega_{\epsilon,\eta}^{reg2} + \lambda_{13}\Omega_{\epsilon,\eta}^{reg3})A'^{-1} \\ \Omega_{i,s}^{mis2} &= A^{-1}\Gamma(\lambda_{21}\sigma_z^{reg1} + \lambda_{22}\sigma_z^{reg2} + \lambda_{23}\sigma_z^{reg3})\Gamma' A'^{-1} \\ &\quad + A^{-1}(\lambda_{21}\Omega_{\epsilon,\eta}^{reg1} + \lambda_{22}\Omega_{\epsilon,\eta}^{reg2} + \lambda_{23}\Omega_{\epsilon,\eta}^{reg3})A'^{-1} \\ \Omega_{i,s}^{mis3} &= A^{-1}\Gamma(\lambda_{31}\sigma_z^{reg1} + \lambda_{32}\sigma_z^{reg2} + \lambda_{33}\sigma_z^{reg3})\Gamma' A'^{-1} \\ &\quad + A^{-1}(\lambda_{31}\Omega_{\epsilon,\eta}^{reg1} + \lambda_{32}\Omega_{\epsilon,\eta}^{reg2} + \lambda_{33}\Omega_{\epsilon,\eta}^{reg3})A'^{-1}.\end{aligned} \tag{13}$$

Note that using equation (13) rather than equation (12) generates the same solution for the parameters of the matrices A and Γ. The misspecification of the heteroskedasticity regimes instead passes through entirely into the estimates of σ_z and $\Omega_{\epsilon,\eta}$, which will not equal their values under the true regimes. Thus, the misspecification of the regimes does not bias our estimates (A and Γ) of the monetary policy response to the stock market.

This analysis holds as long as the misspecified system of equations continues to satisfy the equivalent rank condition that the original system satisfied. In that case, the misspecification of the regimes only reduces the efficiency of the estimates, as it averages out the differences across regimes that are the basis for identification. However, if the misspecification of the regimes is bad enough, the rank condition will be violated. Indeed, if the regimes are misspecified so much as to have equal weights of the true regimes, the $\Omega_{i,s}^{misj}$ may be proportional to one another. In that situation, equation (13) is a system of equations that is underidentified.[25]

25. This is easily testable. The equivalent approach in a linear system is to test the rank condition conditional on the order condition. In this case the heteroskedasticity provides the order condition, and the linear independence of the covariance matrices is the rank condition.

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