Assignment 3
Due 03/29/19

1 Analytical Exercise

1. Let $\mathrm{y}_t = (X_t, Y_t)'$ be given by

$$X_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

$$Y_t = h_1 X_{t-1} + u_t$$

where $(\varepsilon_t, u_t)'$ is a vector white noise with contemporaneous variance-covariance matrix given by

$$\begin{bmatrix} E(\varepsilon_t^2) & E(\varepsilon_t u_t) \\ E(u_t\varepsilon_t) & E(u_t^2) \end{bmatrix} = \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix}$$

Calculate the autocovariance matrices $\{\Gamma_k\}_{k=-\infty}^{\infty}$ for this process.

2. Consider the following bivariate VAR

$$y_{1t} = 0.3y_{1,t-1} + 0.8y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = 0.9y_{1,t-1} + 0.4y_{2,t-1} + \varepsilon_{2t}$$

Is this system covariance stationary?

3. Consider the bivariate VAR(p) model

$$A(L)y_t = \varepsilon_t, \varepsilon_t\tilde{\ } iid(0, \Sigma)$$

$$A(L) = I - A_1L - A_2L^2 - .... - A_pL^p$$

with Wold (moving average) representation

$$y_t = \Psi(L)\varepsilon_t$$

where $\Psi(L) = \sum_{k=0}^{\infty} \psi_k L^k$ and $\psi_0 = I_2$.

(a) Find the moving average coefficients ψ_k for a VAR(1) model.

(b) Show that the moving average coefficients for a VAR(2) model can be found recursively by

$\psi_0 = I_2, \psi_1 = A_1$

and $\psi_k = A_1 \psi_{k-1} + A_2 \psi_{k-2}, k > 1$

2 Empirical Exercise

1. **VAR Models**

   This problem considers a bivariate VAR model for $Y_t=(\Delta s_t; fp_t)'$, where s_t is the logarithm of the monthly exchange rate between the US and the UK, $fp_t = f_t - s_t = i_t^{US} - i_t^{UK}$ is the forward premium or interest rate differential, and f_t is the natural logarithm of the 30-day forward exchange rate. The file usuk.txt has monthly data over the period February 1976 through June 1996 as analyzed in Zivot (2000).

   - Column 1: year / month

   - Column 2: f_t

   - Column 3: s_t

   - Column 4: fp_t

   (a) Plot the monthly return Δs_t and the forward premium fp_t over the period March 1976 through June 1996.

   (b) Plot the lag correlation matrix and comment.

   (c) Determine the lag length of the VAR using the AIC. Use a maximum lag of 4. Estimate the VAR and comment on the fit.

   (d) Test for Granger-causality. Does s_t Granger-cause fp_t? Does fp_t Granger-cause s_t? Comment.

   (e) Plot the impulse response functions and comment on the results.

   (f) Perform forecast error variance decomposition for maximum lag of 12. Comment on your results.

   (g) Now change the ordering of the variables and plot the impulse response functions. Do the results change as compared to part (e).

2. **Forecasting jobs growth using excess bond premium, GZ spread, loan officer survey and real house price growth**

   In this exercise we will examine the predictability of jobs growth using three credit condition measures and house price growth. The details of these variables have been discussed in the class. You can also refer to Kishor (2018) for details. Here is the information on data files associated with this exercise:

   demp_rt.csv: real-time jobs growth from 1985:Q1 with data vintage starting from 1995:01. The latest vintage data is for 2018:Q1.

   demp_var contains data for jobs growth data as it appears today. Column 2 contains jobs growth data, the third column is the financing condition from loan officer's survey (SLO hereafter), 4th column is real house price growth. Use jobs growth data from this file as the actual value.

   gz_ebpq3.txt has GZ spread and excess bond premium as the first and the second column. The starting point for all data series is 1985:Q1.

   Forecast sample and Estimation sample: we perform the following forecasting experiment with all the VAR models outlined below. Our first forecasts cover the period 1995:Q1-1995:Q4 and would have been prepared in 1995:Q1 using 1995:Q1 vintage data. The estimation sample for the first forecasts is 1985:Q1-1994:Q4. We then move ahead one quarter, re-estimate the VAR model and forecast 1995:Q2-1996:Q1, etc. Our final set of forecasts, for 2016:Q4 2017:Q3, would have been prepared in 2016:Q3. Note that in this case, we are simply following the conventional real-time VAR estimation where we move along the columns of the real-time data base for each iteration. We consider 1-Q ahead through 4-Q ahead forecasts. In addition to these quarterly forecasts, we also examine the average over next four quarters.

   (a) Create four bivariate VAR models: jobs growth and GZ spread; jobs growth and excess bond premium; jobs growth and SLO; and job growth and real house price growth. Perform the forecasting exercise for the sample period described above. Make sure to choose the optimal lag length for each iteration of the model estimation. Compare the results with the forecasts generated from AR(1) model by reporting the results of the ratio of RMSE from the VAR model to that of the RMSE from the AR model.

(b) Explain your results in part (a).

(c) Now create three trivariate models: jobs growth, GZ spread and real house price growth; jobs growth, excess bond premium and real house price growth; and jobs growth, SLO and real house price growth. Perform the same exercise as in part (a) and report the ratio of RMSE as compared to an AR(1) model.

(d) Does inclusion of housing price growth model lead to an improvement in forecast of the bivariate VAR models?

(e) Now combine the forecasts from all the models using simple average, Bates-Granger, Bayesian Model Averaging and Akaike Model Averaging. Report the results for this exercise. Does forecast combination yield superior forecasts? Explain.