DSGE Notes

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1 Review of RBC Model

I will briefly present the RBC model rather than justifying the model by analysing the main assumptions. For more detailed introduction, please refer to Acemoglu (2012). The model assumes: only one sector of goods exists in the market, the labor is normalised to unit value, agents are homogeneous. The RBC model is also called ‘neoclassical growth model‘ or ‘Ramsey model‘.

The intuition(or question) behind this model is: how much of it’s income should a nation save or how much I should consume and save everyday to maximise my lifespan utility by enjoying life and meanwhile improving my productivity?

When we talk about business cycle, we focus on the short term and ignore long-run growth. That’s why we need apply the filter to get rid of trend. We follow the book by Miao (2014) and consider the following social planner’s problem:

$$\max_{C_t, N_t, I_t} E \sum_{t=0}^{\infty} \beta^t [\ln C_t + \chi \ln(1 - N_t)] \tag{1.1}$$

subject to

$$C_t + I_t = z_t K_t^{\alpha} N_t^{1-\alpha} \tag{1.2}$$
$$K_{t+1} = (1 - \delta) K_t + I_t \tag{1.3}$$
$$\ln z_t = \rho \ln z_{t-1} + \sigma \varepsilon_t, \varepsilon_t \sim IID\ N(0, 1) \tag{1.4}$$

where χ measures the utility weight on leisure, and the technology shock $\{z_t\}$ is modelled as an AR(1) process. The optimal allocation $\{C_t, N_t, K_{t+1}, I_t, Y_t\}$ satisfies the following system of nonlinear difference equations:

$$\frac{1}{C_t} = E_t \frac{\beta}{C_{t+1}} [\alpha z_{t+1} K_{t+1]}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta] \tag{1.5}$$
$$\frac{\chi C_t}{1 - N_t} = (1 - \alpha) z_t K_t^{\alpha} N_t^{-\alpha} \tag{1.6}$$

with conditions in (1,2) and (1.3). In a decentralized equilibirum in which households own captial and make real investment, the rental rate and the wage rate are given by

$$R_{kt} = \alpha z_t K_t^{\alpha-1} N_t^{1-\alpha} \quad and \quad w_t = (1 - \alpha) z_t K_t^{\alpha} N_t^{-\alpha}$$

In steady state, we have the following system of equations:

$$1 = \beta[\alpha K^{\alpha-1} N^{1-\alpha} + 1 - \delta]$$
$$\rightarrow \frac{K}{N} = \left(\frac{\alpha}{1/\beta - (1 - \delta)} \right)^{1/(1-\alpha)}$$
$$R_{kt} = \alpha \big(\frac{K}{N}\big)^{\alpha-1}$$
$$w_t = (1 - \alpha) \frac{Y}{N} = (1 - \alpha) \big(\frac{K}{N}\big)^{\alpha}$$

To assign the parameter values into our model, we need examine the long term time series data to derive the stylized facts.

References

Acemoglu, D. (2012). Introduction to economic growth. *Journal of economic theory*, 147(2):545–550.

Miao, J. (2014). *Economic dynamics in discrete time*. MIT press.