Journal of Statistical Software

MMMMMM YYYY, Volume VV, Issue II. doi: 10.18637/jss.v000.i00

The Fractionally Cointegrated Vector Autoregression Model in R

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Abstract

This article illustrates how to estimate the fractionally cointegrated vector autoregression model in R.

Keywords: cofractional process, cointegration rank, fractional autoregressive model, fractional cointegration, fractional unit root, VAR model, Matlab, R.

1. Introduction: Cointegration and fractional integration in R

The fractionally cointegrated vector autoregression model is an excellent model...

In R (R Core Team 2017), contegration is performed by the `function1()` and `function2()` in the **pscl** package (Jackman 2015).

This R packages is based on the Matlab package **FCVARmodel.m** with documentation in Nielsen and Popiel (2016) and Nielsen and Morin (2014).

The next section describes the FCVAR model and the restricted models that can be estimated with this program. Section 3 describes the functioning of the main program, which is a replication of one of the tables of results in Jones, Nielsen, and Popiel (2014). Section 4 describes another example program, which demonstrates some additional functionality of the software. Importantly, these are the only two files that would need to be changed to apply the program for other empirical analyses.

2. The fractionally cointegrated VAR model

The fractionally cointegrated vector autoregressive (FCVAR) model was proposed in Johansen (2008) and analyzed by, e.g., Johansen and Nielsen (2010, 2012). For a time series X_t of dimension p, the fractionally cointegrated VAR model is given in error correction form as

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b}L_b X_t + \sum_{i=1}^{k}\Gamma_i\Delta^d\ L_b^i X_t + \varepsilon_t, \tag{1}$$

where ε_t is p-dimensional $i.i.d.(0,\Omega)$, Δ^d is the fractional difference operator, and $L_b = 1-\Delta^b$ is the fractional lag operator.[1] Johansen and Nielsen (2012) imposed two restrictions on the parameter space, $d \geq b$ and $d - b < 1/2$, in their asymptotic analysis. However, these restrictions were relaxed in Johansen and Nielsen (2018a,b).

Model (1) includes the Johansen (1995) CVAR model as the special case $d = b = 1$; see Johansen and Nielsen (2018b). Some of the parameters are well-known from the CVAR model and these have the usual interpretations also in the FCVAR model. The most important of these are the long-run parameters α and β, which are $p \times r$ matrices with $0 \leq r \leq p$. The rank r is termed the cointegration, or cofractional, rank. The columns of β constitute the r cointegration (cofractional) vectors such that $\beta' X_t$ are the cointegrating combinations of the variables in the system, i.e. the long-run equilibrium relations. The parameters in α are the adjustment or loading coefficients which represent the speed of adjustment towards equilibrium for each of the variables. The short-run dynamics of the variables are governed by the parameters $\Gamma = (\Gamma_1, \ldots, \Gamma_k)$ in the autoregressive augmentation.

The FCVAR model has two additional parameters compared with the CVAR model, namely the fractional parameters d and b. Here, d denotes the fractional integration order of the observable time series and b determines the degree of fractional cointegration, i.e. the reduction in fractional integration order of $\beta' X_t$ compared to X_t itself. These parameters are estimated jointly with the remaining parameters. This model thus has the same main structure as in the standard CVAR model in that it allows for modeling of both cointegration and adjustment towards equilibrium, but is more general since it accommodates fractional integration and cointegration.

In the next four subsections we briefly describe the accommodation of deterministic terms as well as estimation and testing in the FCVAR model.

2.1. Deterministic terms

3. Main Program

Estimating the fractionally cointegrated vector autoregression model works like this...

Here is an example of code:

```
glm(formula, data, subset, na.action, weights, offset,
```

[1] Both the fractional difference and fractional lag operators are defined in terms of their binomial expansion in the lag operator, L. Note that the expansion of L_b has no term in L^0 and thus only lagged disequilibrium errors appear in (1).

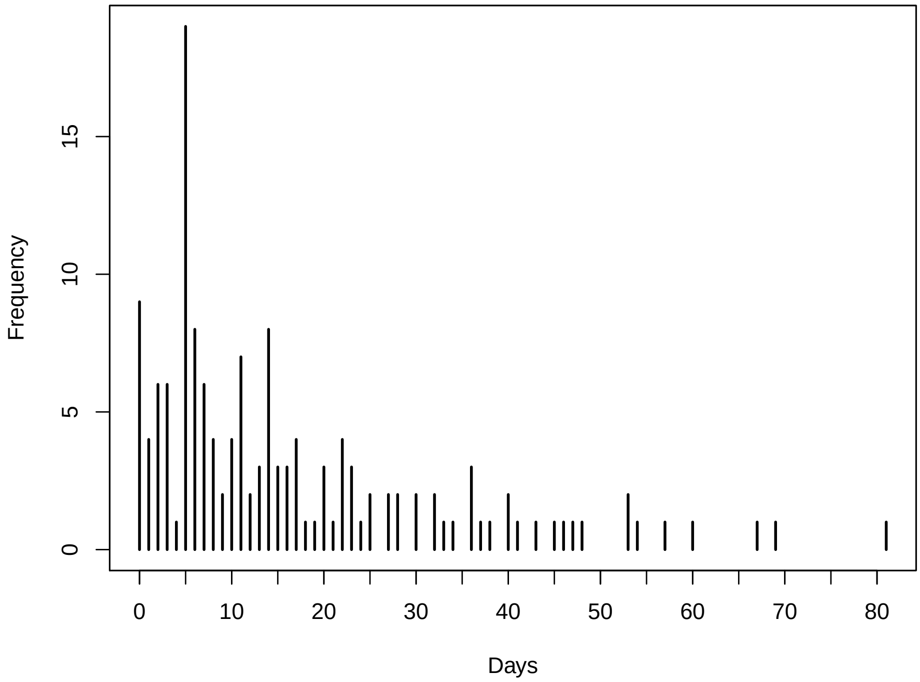

Figure 1: Frequency distribution for number of days absent from school.

```
  family = gaussian, start = NULL, control = glm.control(...),
  model = TRUE, y = TRUE, x = FALSE, ...)
```

4. Illustrations

For a simple illustration of the FCVAR... The data can be loaded by

```
R> data("quine", package = "MASS")
```

and a basic frequency distribution of the response variable is displayed in Figure 1.

As a first model for the `quine` data, we fit the basic Poisson regression model. (Note that JSS prefers when the second line of code is indented by two spaces.)

```
R> m_pois <- glm(Days ~ (Eth + Sex + Age + Lrn)^2, data = quine,
+    family = poisson)
```

Hence, the full summary of that model is shown below.

```
R> summary(m_nbin)

Call:
glm.nb(formula = Days ~ (Eth + Sex + Age + Lrn)^2, data = quine,
    init.theta = 1.60364105, link = log)
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.0857  -0.8306  -0.2620   0.4282   2.0898

Coefficients: (1 not defined because of singularities)
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  3.00155    0.33709   8.904  < 2e-16 ***
SexM        -0.77181    0.38021  -2.030  0.04236 *
EthN:AgeF2  -1.23283    0.42962  -2.870  0.00411 **
SexM:AgeF2   1.55330    0.51325   3.026  0.00247 **
SexM:AgeF3   1.25227    0.45539   2.750  0.00596 **
AgeF3:LrnSL       NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(1.6036) family taken to be 1)

    Null deviance: 235.23  on 145  degrees of freedom
Residual deviance: 167.53  on 128  degrees of freedom
AIC: 1100.5

Number of Fisher Scoring iterations: 1


              Theta:  1.604
          Std. Err.:  0.214

 2 x log-likelihood:  -1062.546
```

5. Extensions

5.1. Extension for P-values

Although the Matlab program can run standalone, one of the functions, `RankTests.m`, makes an external system call to a separately installed program, `fdpval`. This external program is the C++ implementation of a Fortran program used to obtain simulated P-values from MacKinnon and Nielsen (2014). If the user would like P-values for the cointegration rank tests to be automatically calculated, we recommend obtaining this companion program, which is made available by Jason Rhinelander and can be downloaded from:

https://github.com/jagerman/fracdist/releases

It can be either installed or downloaded in a compressed folder. It is important to note where the program is stored or installed, because the Matlab program requires the program location as an input in the estimation options. For example, if the program is stored in the folder `/usr/bin/` on a Linux system, the location variable is defined as follows, `progLoc = '"/usr/bin/fdpval"'`. For details see Sections **??** and **??**.

5.2. Badly behaved objective function

We also make use of the excellent `extrema.m` and `extrema2.m` functions, which are written by Carlos Adrián Vargas Aguilera and are freely available from the Mathworks website. For simplicity these are included in the Auxiliary subfolder.

6. Summary and discussion

Summary goes here.

Computational details

The results in this paper were obtained using R 3.4.1 with the **MASS** 7.3.47 package. R itself and all packages used are available from the Comprehensive R Archive Network (CRAN) at `https://CRAN.R-project.org/`.

Acknowledgments

We are grateful to Federico Carlini, Andreas Noack Jensen, Søren Johansen, Maggie Jones, James MacKinnon, Jason Rhinelander, and Daniela Osterrieder for comments, and to the Canada Research Chairs program, the Social Sciences and Humanities Research Council of Canada (SSHRC), and the Center for Research in Econometric Analysis of Time Series (CREATES, funded by the Danish National Research Foundation DNRF78) for financial support.

References

Jackman S (2015). ***pscl**: Classes and Methods for R Developed in the Political Science Computational Laboratory, Stanford University.* R package version 1.4.9, URL `https://CRAN.R-project.org/package=pscl`.

Johansen S (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models.* Oxford University Press, New York.

Johansen S (2008). "A representation theory for a class of vector autoregressive models for fractional processes." *Econometric Theory*, **24**, 651–676.

Johansen S, Nielsen MØ (2010). "Likelihood inference for a nonstationary fractional autoregressive model." *Journal of Econometrics*, **158**, 51–66.

Johansen S, Nielsen MØ (2012). "Likelihood inference for a fractionally cointegrated vector autoregressive model." *Econometrica*, **80**, 2667–2732.

Johansen S, Nielsen MØ (2018a). "Nonstationary cointegration in the fractionally cointegrated VAR model." QED working paper 1405, Queen's University.

Johansen S, Nielsen MØ (2018b). "Testing the CVAR in the fractional CVAR model." Forthcoming in *Journal of Time Series Analysis*.

Jones M, Nielsen MØ, Popiel MK (2014). "A fractionally cointegrated VAR analysis of economic voting and political support." *Canadian Journal of Economics*, **47**, 1078–1130.

MacKinnon JG, Nielsen MØ (2014). "Numerical distribution functions of fractional unit root and cointegration tests." *Journal of Applied Econometrics*, **29**, 161–171.

Nielsen MØ, Morin L (2014). "FCVARmodel.m: a Matlab software package for estimation and testing in the fractionally cointegrated VAR model." QED working paper 1273, Queen's University.

Nielsen MØ, Popiel MK (2016). "A Matlab program and user's guide for the fractionally cointegrated VAR model." QED working paper 1330, Queen's University.

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

A. More technical details

Technical details go here.

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Journal of Statistical Software http://www.jstatsoft.org/
published by the Foundation for Open Access Statistics http://www.foastat.org/
MMMMMM YYYY, Volume VV, Issue II *Submitted:* yyyy-mm-dd
doi:10.18637/jss.v000.i00 *Accepted:* yyyy-mm-dd