ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

DEPARTMENT OF ECONOMICS

SECOND CYCLE DEGREE
IN

ECONOMICS AND ECONOMETRICS

Dynamic Matrix Factor Models and the EM Algorithm: A Nowcasting Framework for Mixed-Frequency Data in the Euro Area

Dissertation in Macroeconometrics

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Research Overview

Research Topic:

Nowcasting GDP with Dynamic Factor Models:

- *Vector-based (DFM)* vs *Matrix-based (DMFM)* models

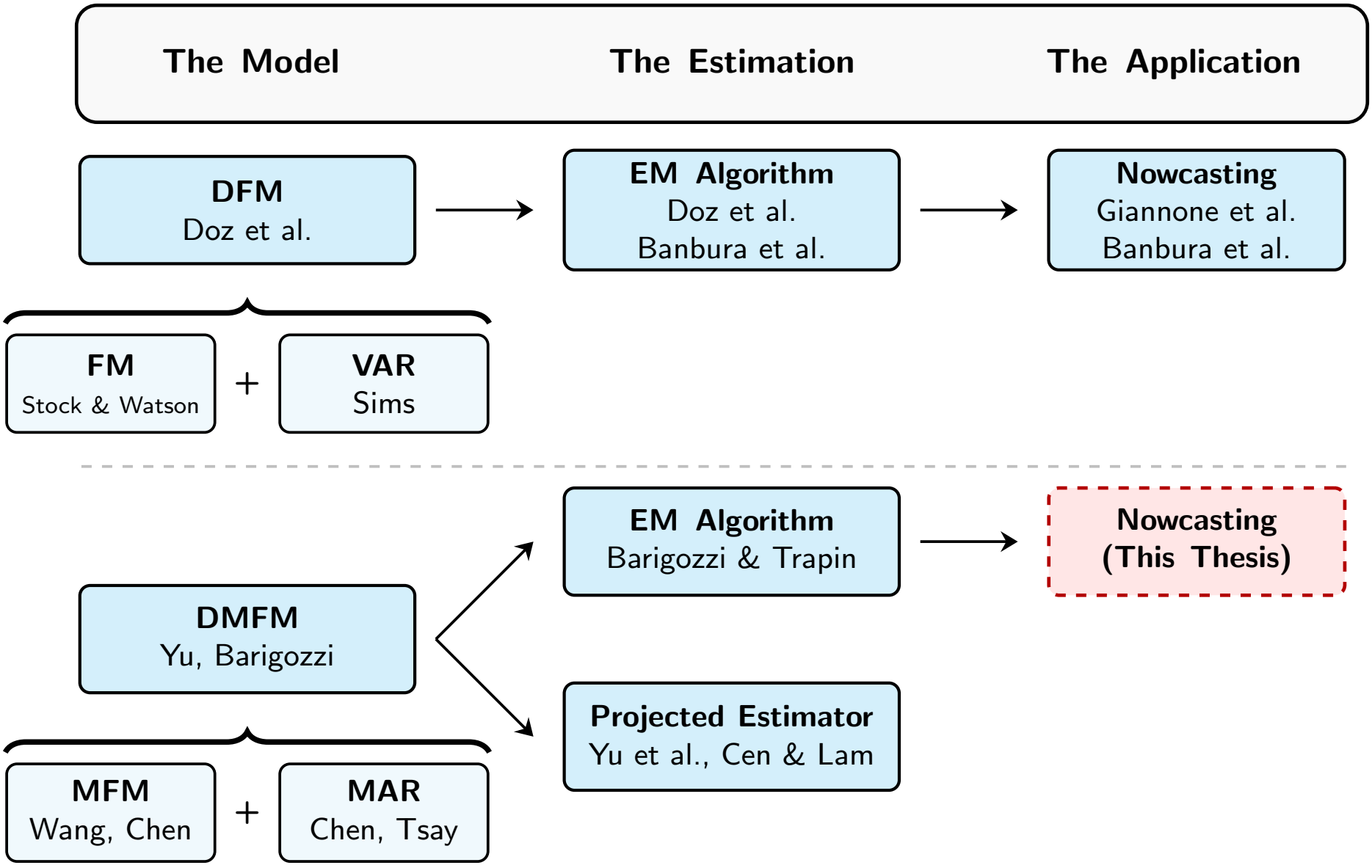
Contribution:

First empirical nowcasting application of Dynamic Matrix Factor Models (DMFMs)

Key Question:

Does the matrix formulation enhance GDP nowcasting performance when using a set mainly based on high-frequency indicators?

Factor Models: Literature Evolution



The Model: State-Space Representations

Factor Models

Dynamic Factor Model

$$y_t = \Lambda f_t + \xi_t$$
$$f_t = A f_{t-1} + v_t$$

Dynamic Matrix Factor Model

$$Y_t = R F_t C^\top + E_t$$
$$F_t = A F_{t-1} B^\top + U_t$$

DMFM Vectorized:

$$y_t = (C \otimes R) f_t + e_t$$
$$f_t = (B \otimes A) f_{t-1} + u_t$$

DMFM Key Innovation

Preserves the natural matrix structure of macroeconomic and financial datasets.

Estimation Strategy: Quasi-Maximum Likelihood

Log Likelihood Mis-Specification

- DMFM covariance decomposition:

$$\Omega_{Y_T} = (I_T \otimes C \otimes R)\,\Omega_{F_T}\,(I_T \otimes C \otimes R)^\top + \Omega_{E_T}$$

- Diagonal structure imposed on the idiosyncratic component:

$$\Omega_{E_T} \approx I_T \otimes \mathrm{diag}(K) \otimes \mathrm{diag}(H)$$

- Quasi log-likelihood (QML):

$$\ell(Y_T;\theta) = -\frac{p_1 p_2 T}{2}\log(2\pi) - \frac{1}{2}\log|\Omega_{Y_T}| - \frac{1}{2}Y_T^\top \Omega_{Y_T}^{-1} Y_T$$

No closed-form solution for QML due to unobserved latent factors.

EM Algorithm

Recursive EM algorithm with Kalman smoother to estimate latent factors and maximize the expected log-likelihood.

Estimation Strategy: EM Algorithm

For the vectorized DMFM, iterate the EM until convergence

E-Step:

- *Kalman Smoother*
  Given observed data Y_T and current parameters $\hat{\theta}^{(n)}$, compute the smoothed estimates (means and covariances) of the latent factors over time:

$$f_{t|T}^{(n)} = \mathbb{E}_{\hat{\theta}^{(n)}}[f_t \mid Y_T], \quad \Pi_{t|T}^{(n)} = \mathrm{Var}_{\hat{\theta}^{(n)}}(f_t \mid Y_T)$$

- *Q-function Computation*
  Use the smoothed estimates to compute the expected complete log-likelihood:

$$Q(\theta, \hat{\theta}^{(n)}) = \mathbb{E}_{\hat{\theta}^{(n)}}\left[\ell(Y_T \mid F_T; \theta) \mid Y_T\right] + \mathbb{E}_{\hat{\theta}^{(n)}}\left[\ell(F_T; \theta) \mid Y_T\right]$$

M-Step:

- *Parameter Update*
  Maximize $Q(\theta, \hat{\theta}^{(n)})$ to update model parameters:

$$\hat{\theta}^{(n+1)} = \arg\max_{\theta} Q(\theta, \hat{\theta}^{(n)})$$

Recursive Nowcasting Procedure

Built a Tensor for the Entire Dataset

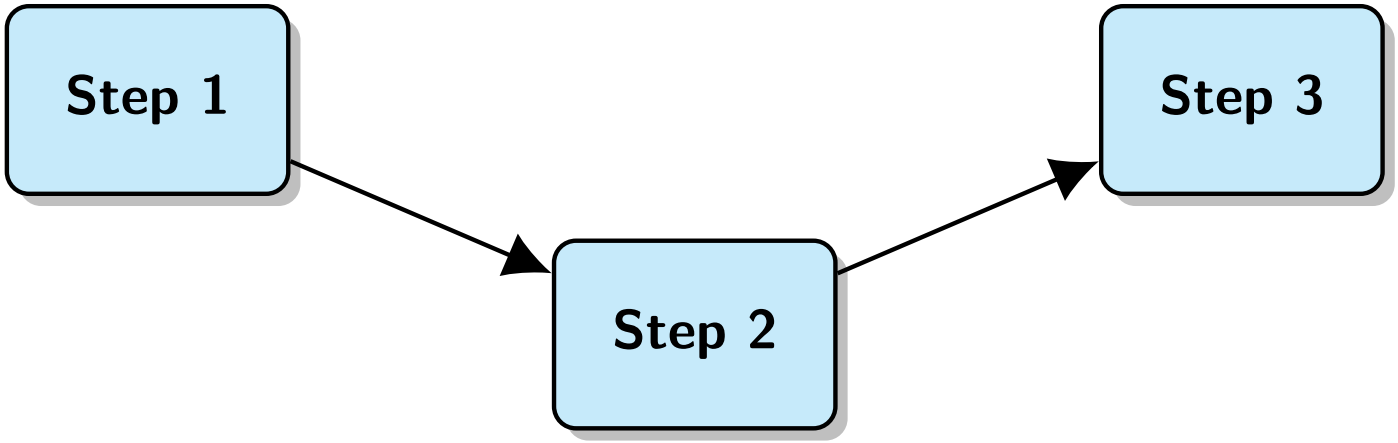
$\forall$ vintages t_v

Pseudo Real-Time Simulation

Truncate data up to t_v and apply release mask

Nowcast

Extract nowcast and compute forecast error



Parameters Estimation

Estimate DMFM through QML via EM algorithm

Empirical Application Overview

Focus:

Nowcasting GDP for Euro Area countries using both vector and matrix formulations of Dynamic Factor Models (DFMs).

Key Intuition:

Dynamic Matrix Factor Models (DMFMs) enable country-level nowcasting by leveraging cross-country information embedded in the matrix structure.

Key Questions:

- Does matrix modeling improve country-level nowcasting?
- Does including mainly high-frequency data is sufficient for GDP nowcasting?

Dataset Construction

Data Collection

Source: "EA-MD-QD" dataset (January 2000 – April 2025).

- **Country Selection:** Germany, France, Italy, Spain
- **Variable Selection:** 39 monthly and GDP quarterly macroeconomic series

Tensor Dimensions: $p_1 = 4,\ p_2 = 40,\ T = 300$

Data Preparation

COVID-19 Treatment:
Exclusion of real variables from March 2020 to July 2021.

Release Schedule:
Application of a mask according to the the release delays of the selected variables.

DMFM Factor

Factor Selection

- *Missing Values Imputation:* Cen & Lam (2025).
- *Factor Selection & Loading Estimation:* Yu et al. (2022).

Single Row and Column Factor:
$k_1 = 1$ & $k_2 = 1$

DMFM Factor Interpretation

Row Loadings: EA Membership Effect

Country	DE	FR	IT	ES
Loading	0.899	1.078	1.024	0.990

Table: Estimated Row Loadings

DMFM Factor

Column Loadings: Business Cycle Dynamics

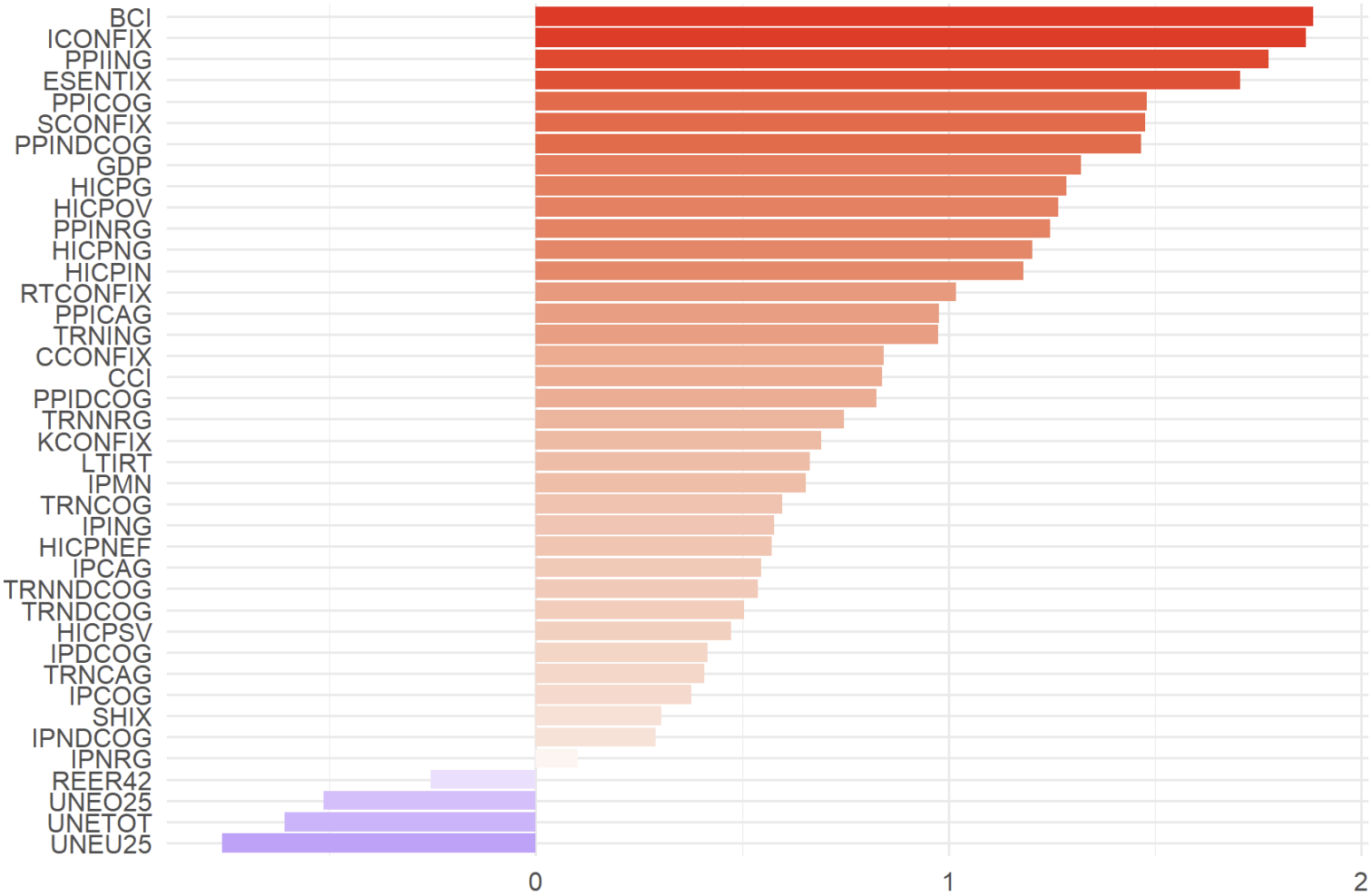
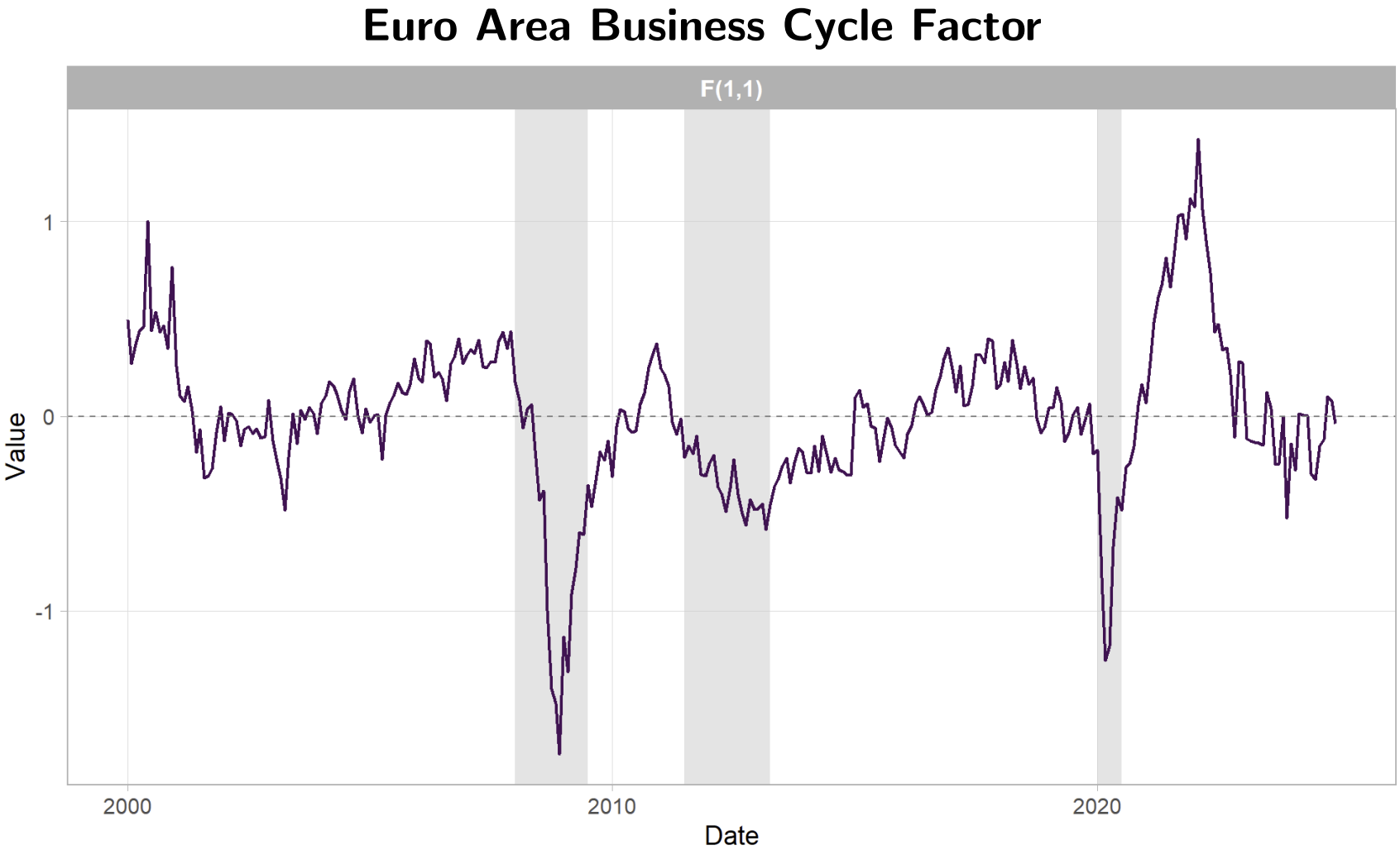


Figure: Estimated Column Loadings Ordered by Variables' Weight

DMFM Factor



The Euro Area Business Cycle Factor is aligned with Euro Area Recessions confirming the interpretation.

Nowcast Comparison

DMFM Outperforms DFM

Accuracy
in Pre- and Post-COVID Periods

Responsiveness to News
During COVID and Recovery Phase

RMSFE Comparison:

RMSFE

Time Series Comparison:

Spain & France

Germany & Italy

Interpretation:
DMFM proves most effective for countries highly linked to the EMU, especially during periods of strong cross-country economic interdependence.

Future Research Directions

Extensions to Model Specification

- Introduce higher-order dynamics in the DMFM transition equation.
- Expand the dataset to cover additional countries and variables.

Interpreting Nowcasts with Neural Networks

- Break down nowcast revisions not just by *variable type*, but also for *country* to uncover the main cross-country drivers of GDP nowcasts.

Long-Term Goal:
Investigate cross-country information spillovers during critical economic periods to support more targeted and effective monetary policy decisions.

Conclusion

Main Contribution:
First application of Dynamic Matrix Factor Models (DMFMs) to GDP nowcasting.

Key Result:
Including cross-country dynamics through DMFM improves nowcasting accuracy and responsiveness to news, especially in times of heightened interconnectedness.

Dynamic Matrix Factor Models and the EM Algorithm:
A Nowcasting Framework for Mixed-Frequency Data in the Euro Area

Appendix

Supplementary Tables & Figures

Matrix-Variate Time Series: sequences of two-dimensional arrays evolving over time.

- $p_1 \rightarrow$ rows (e.g., countries, assets)
- $p_2 \rightarrow$ columns (e.g., macroeconomic indicators, integrated volatility)
- $T \rightarrow$ slices over time (e.g., daily, monthly, quarterly observations)

Country	GDP Growth	Unemployment Rate	...	Industrial Production
Germany	$X_{t,11}$	$X_{t,12}$	...	$X_{t,1p}$
France	$X_{t,21}$	$X_{t,22}$	...	$X_{t,2p}$
Italy	$X_{t,31}$	$X_{t,32}$	...	$X_{t,3p}$
Spain	$X_{t,41}$	$X_{t,42}$	...	$X_{t,4p}$

Matrix-valued time series dataset for a specific time t.

Back to Nowcasting Diagram

N	ID	Series Name	Class	Category	Freq.	Delay
(1) National Accounts / Real Economy						
1	GDP	Real Gross Domestic Product	R	H	Q	45
(2) Labor Market						
2	UNETOT	Unemployment: Total	R	H	M	45
(3) Exchange and Interest Rates						
3	REER42	Real Exchange Rate (42 countries)	F	H	M	35
(4) Industrial Production and Turnover						
4	IPMN	Industrial Production Index: Manufacturing	R	H	M	45
(5) Prices						
5	HICPOV	HICP: Overall Index	N	H	M	40
(6) Confidence Indicators						
6	ESENTIX	Economic Sentiment Indicator	C	S	M	5
(7) Others						
7	SHIX	Share Price Index	F	S	M	1

Table: A Sample of Selected Macroeconomic Variables and Their Release Delays

Back to Data

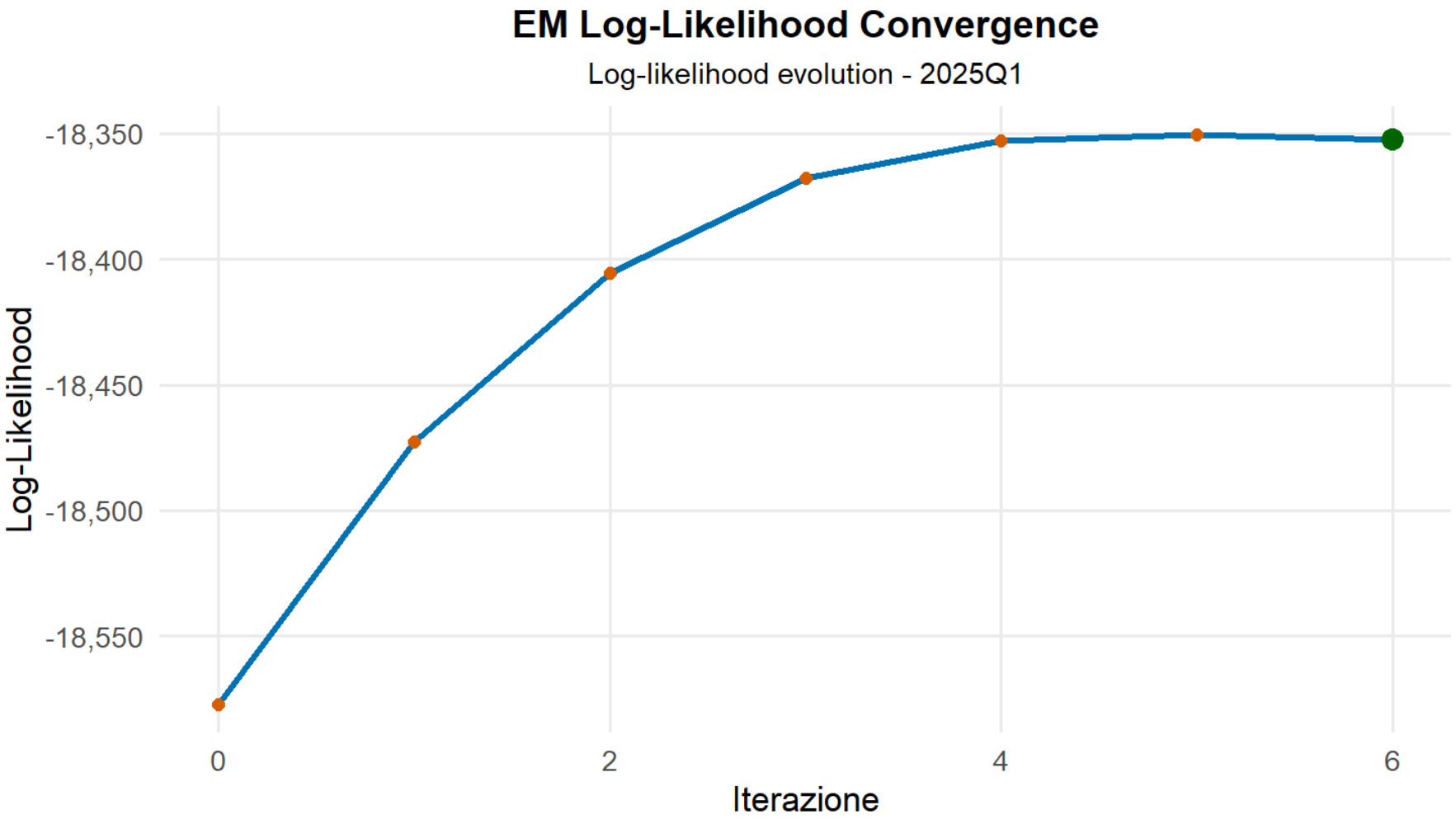
Country	Month	DMFM		DFM	
		Pre-COVID	*Post-COVID*	*Pre-COVID*	*Post-COVID*
Germany	M1	**0.6469**	**0.7361**	0.6493	1.0490
	M2	**0.6373**	**0.6833**	0.6512	0.9078
	M3	**0.6289**	**0.6959**	*0.6294*	*0.8488*
France	M1	**0.3747**	**0.6481**	0.4505	0.6992
	M2	**0.3585**	**0.6098**	0.4468	*0.6861*
	M3	**0.3498**	**0.5916**	*0.4405*	0.6895
Italy	M1	0.3222	**0.8192**	**0.3031**	1.0025
	M2	*0.3084*	**0.8045**	**0.2964**	1.4274
	M3	0.3186	**0.8185**	**0.2953**	*1.3826*
Spain	M1	**0.2723**	**0.4434**	0.3882	0.6004
	M2	**0.2438**	**0.4753**	*0.3782*	0.5828
	M3	**0.2399**	**0.4775**	0.3796	*0.5761*

Table: RMSFE pre- and post-COVID for DMFM and DFM Across Euro Area Countries

Back to Results

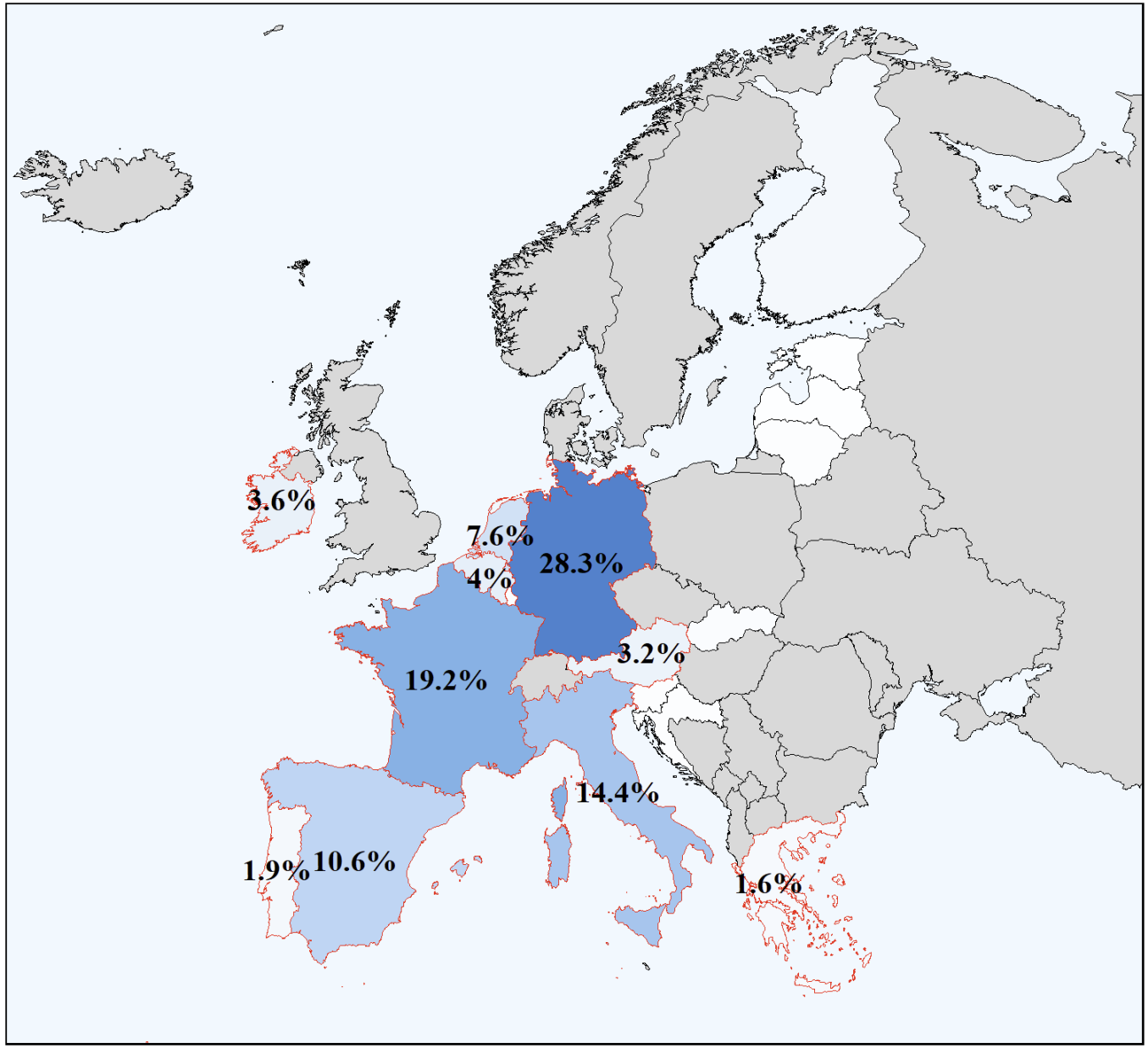
Smoothed GDP - 2025Q1

Back to EM Algorithm



Back to EM algorithm

Euro Area GDP Composition



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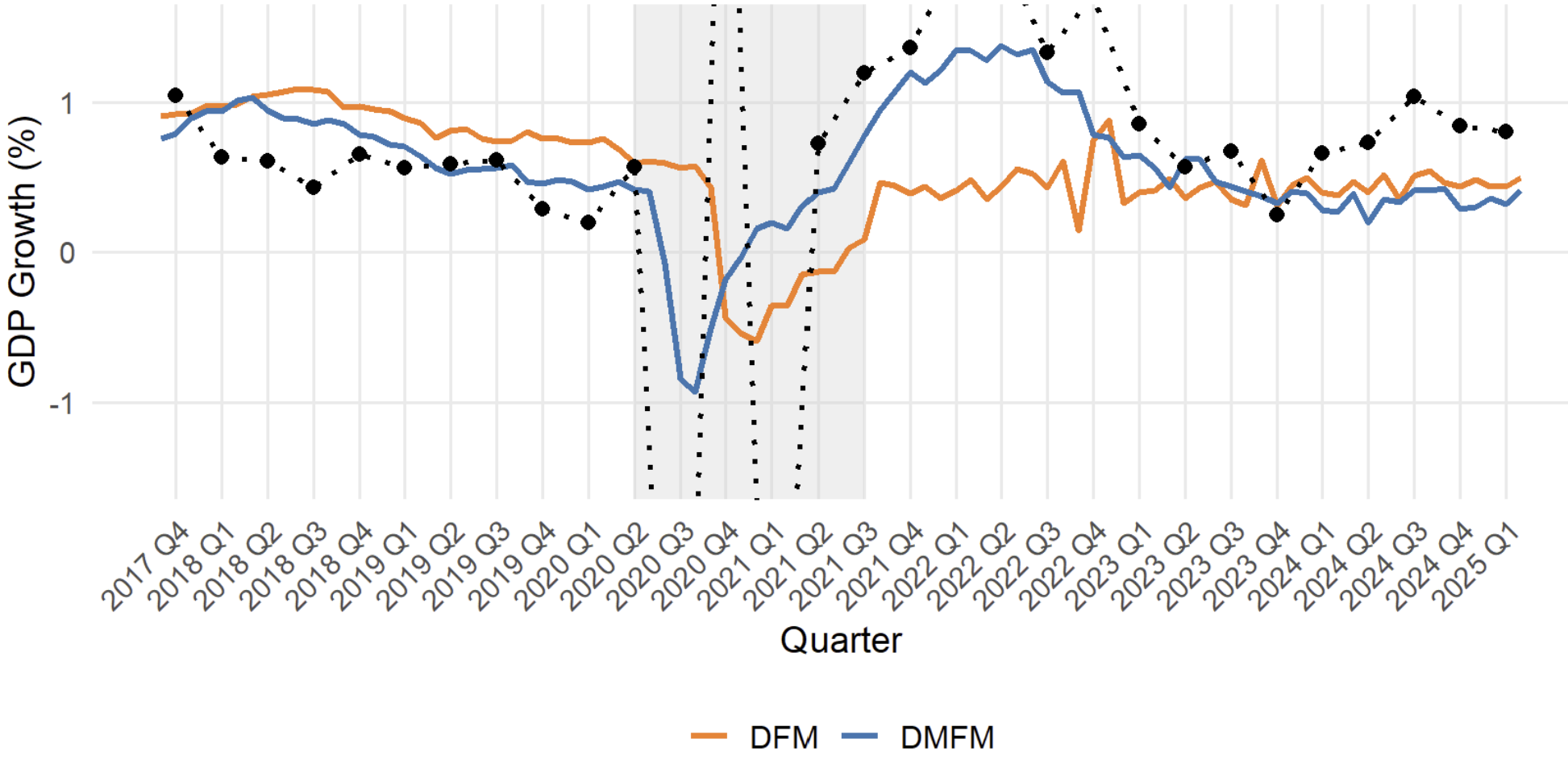
Nowcasts Comparison - Spain



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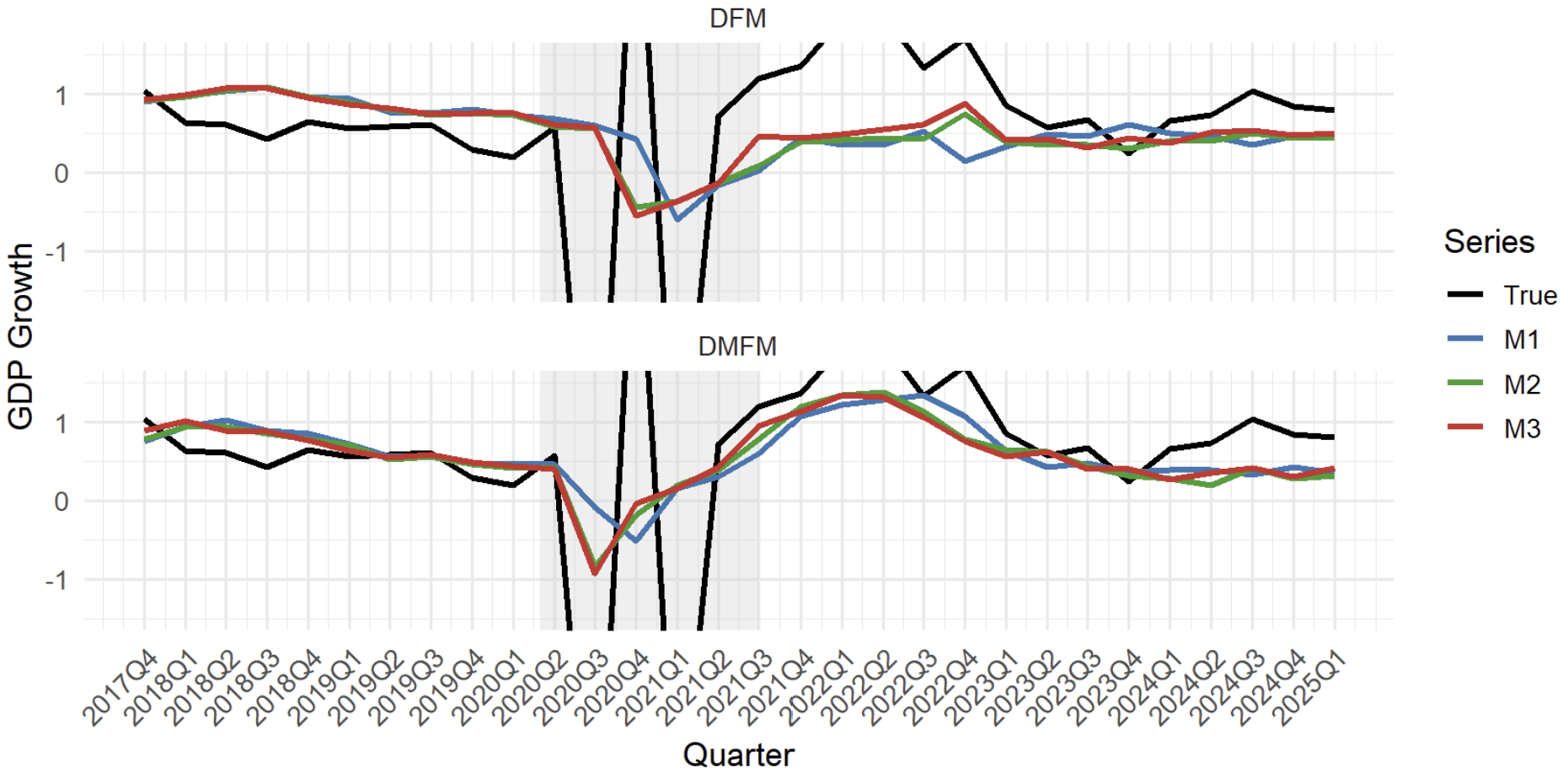
Nowcasts Monthly Revision - Spain



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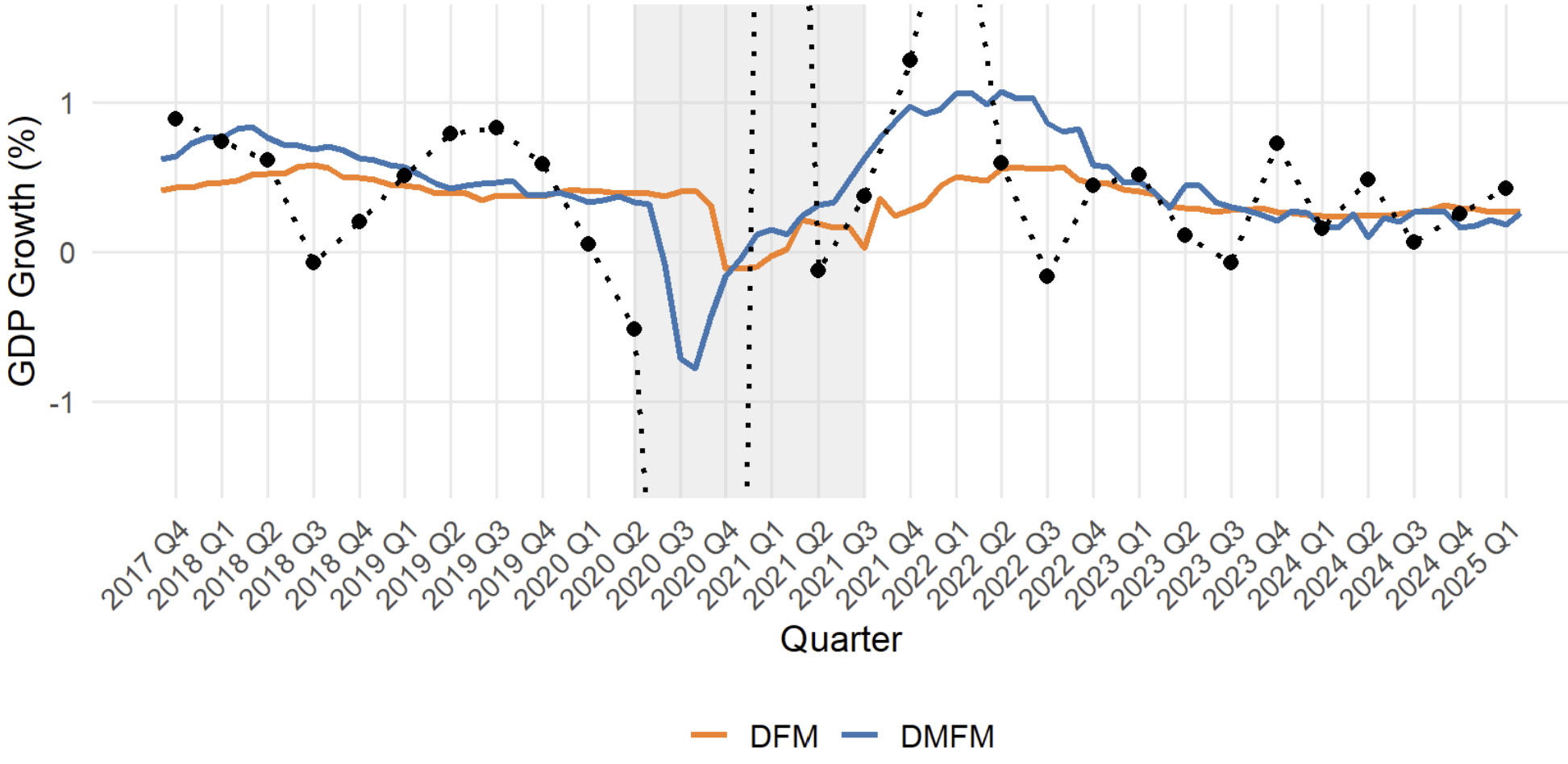
Nowcasts Comparison - Franch



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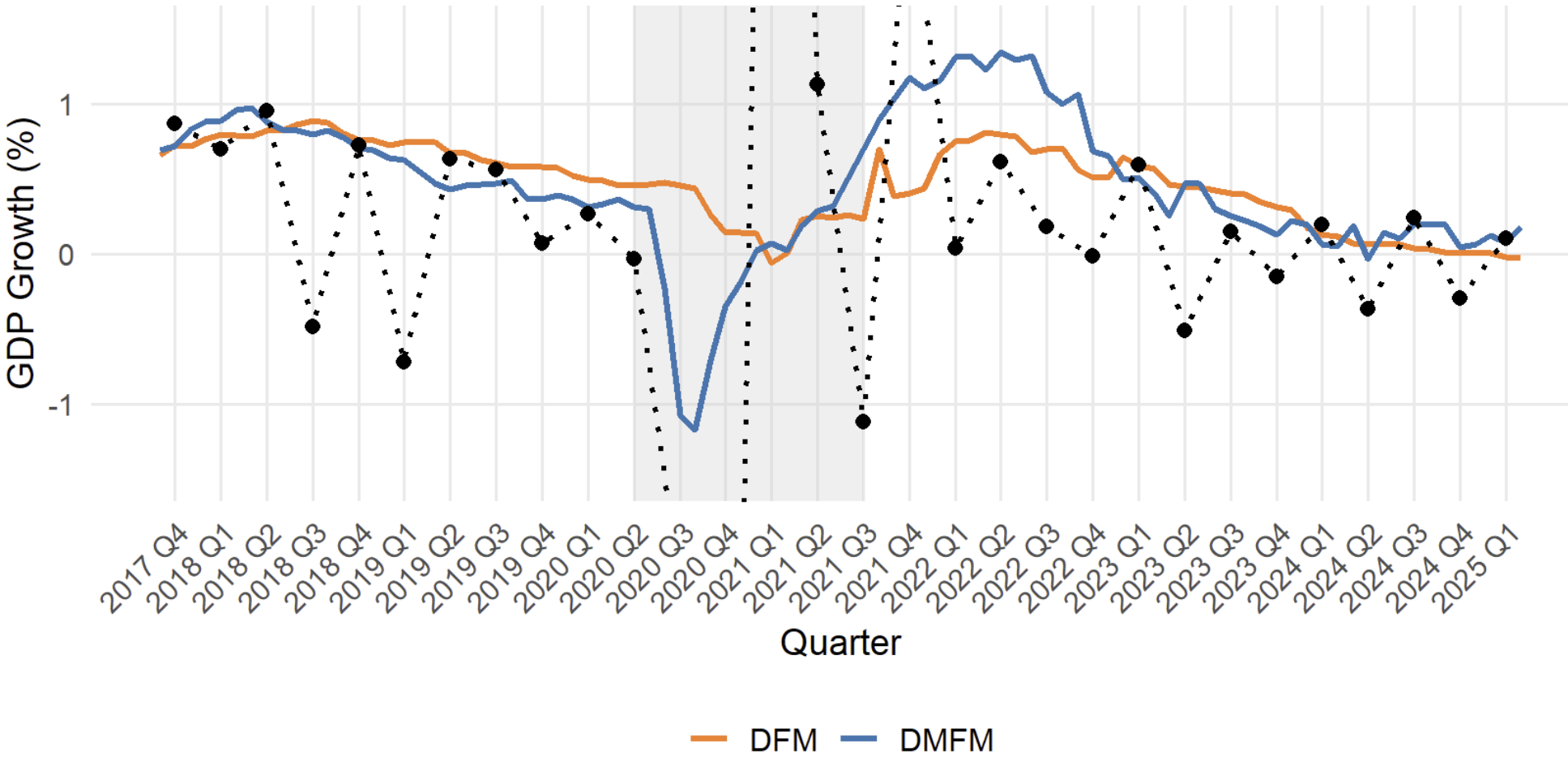
Nowcasts Comparison - Germany



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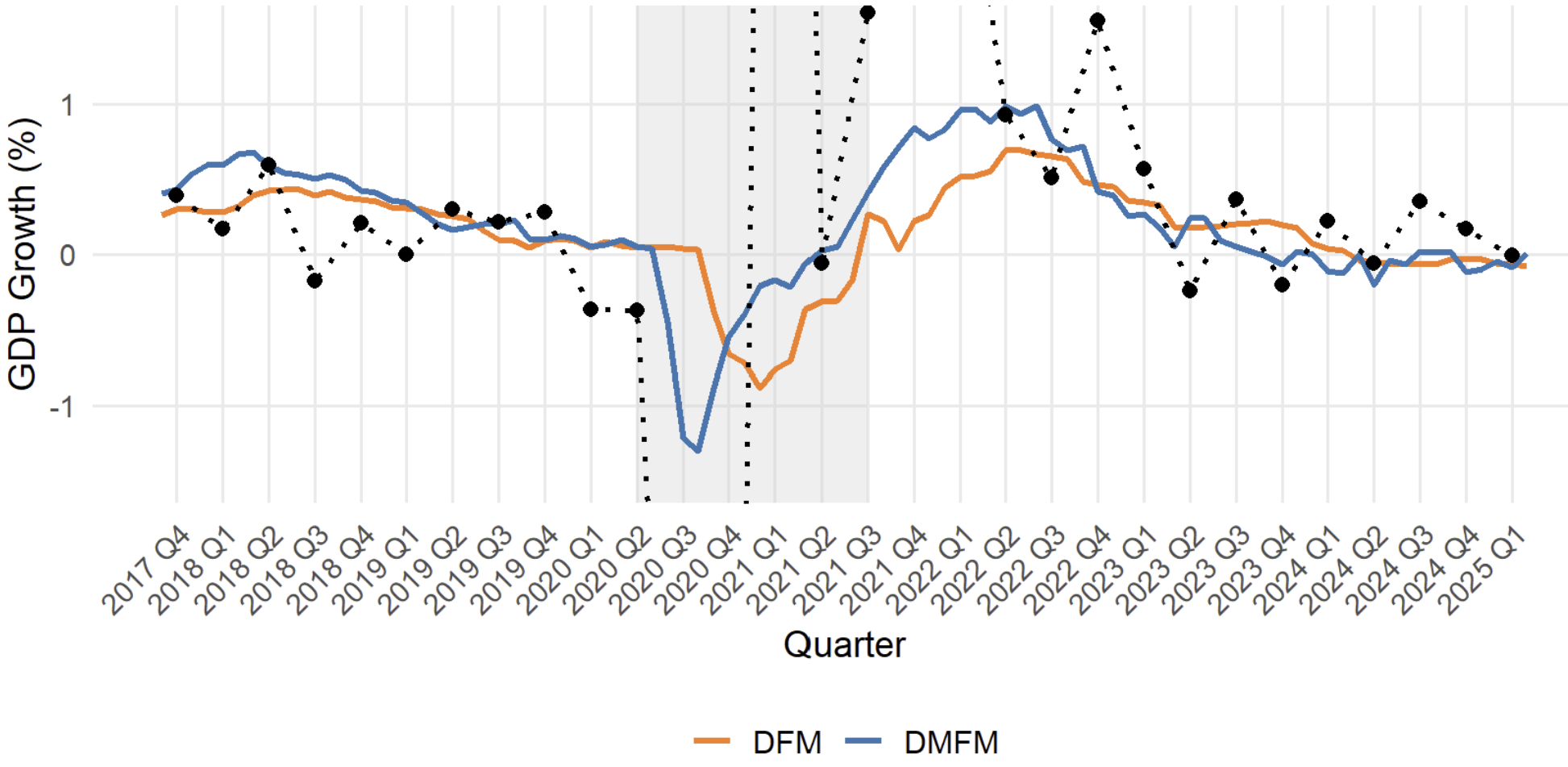
Nowcasts Comparison - Italy



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