The results below are generated from an R script.

```r
linear <- function(data, s = frequency(data), seasons_to_check = 1:s) {
  require(forecast)

  # Decompose the time series using STL
  ts_data <- ts(data, frequency = s)
  decomposed_data <- stl(ts_data, s.window="periodic")

  # Extract the trend, seasonal, and random components
  trend <- as.numeric(decomposed_data$time.series[, "trend"])
  seasonal <- as.numeric(decomposed_data$time.series[, "seasonal"])
  remainder <- as.numeric(decomposed_data$time.series[, "remainder"])

  # De-trended series (seasonal + noise)
  detrended <- seasonal + remainder

  # Initialize Ui, Vi, and di
  Ui <- numeric(length(seasons_to_check))
  Vi <- numeric(length(seasons_to_check))
  di <- numeric(length(seasons_to_check))

  # Calculate Ui and Vi for each specified seasonal period
  for (index in 1:length(seasons_to_check)) {
    i <- seasons_to_check[index]
    S_j <- detrended[seq(i, length(data), by = s)]
    j <- 1:length(S_j)

    # Only proceed if S_j has a sufficient size (at least two non-NA values)
    if (sum(!is.na(S_j)) > 1) {
      b <- 0

      if (s != 1) {  # Prevent division by zero when s == 1
        Ui[index] <- (b^2 * (s * (s + 1) / 12)) + (2 * b / (s - 1)) * sum(j * S_j, na.rm = TRUE) + (1 /
      } else {
        Ui[index] <- NA
      }

      # Vi is the random component for that seasonal period
      Vi[index] <- remainder[i]

      # Calculate di
      di[index] <- Ui[index] - Vi[index]
    } else {
      Ui[index] <- NA
      Vi[index] <- NA
      di[index] <- NA
    }
  }

  return(list(Ui = Ui, Vi = Vi, di = di))
}
```

The R session information (including the OS info, R version and all packages used):

```
sessionInfo()
```

```
## R version 4.2.1 (2022-06-23)
## Platform: aarch64-apple-darwin20 (64-bit)
## Running under: macOS Ventura 13.2.1
##
## Matrix products: default
## LAPACK: /Library/Frameworks/R.framework/Versions/4.2-arm64/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
##  [1] rnoaa_1.4.0       weatherData_0.6.0 devtools_2.4.5    usethis_2.2.1
##  [5] DescTools_0.99.49 crayon_1.5.2      minpack.lm_1.2-3  forecast_8.21
##  [9] quantmod_0.4.23   TTR_0.24.3        xts_0.13.1        zoo_1.8-11
##
## loaded via a namespace (and not attached):
##   [1] nlme_3.1-157              seasonalityTest_0.0.0.9000 fs_1.5.2
##   [4] lubridate_1.9.2           httr_1.4.4                 rprojroot_2.0.3
##   [7] tools_4.2.1               profvis_0.3.8              utf8_1.2.2
##  [10] R6_2.5.1                  colorspace_2.0-3           nnet_7.3-17
##  [13] urlchecker_1.0.1          withr_2.5.0                gridExtra_2.3
##  [16] tidyselect_1.2.0          Exact_3.2                  prettyunits_1.1.1
##  [19] processx_3.8.0            curl_5.0.2                 compiler_4.2.1
##  [22] cli_3.6.0                 expm_0.999-7               xml2_1.3.3
##  [25] desc_1.4.2                tseries_0.10-54            scales_1.2.1
##  [28] lmtest_0.9-40             fracdiff_1.5-2             mvtnorm_1.1-3
##  [31] quadprog_1.5-8            callr_3.7.3                proxy_0.4-27
##  [34] rappdirs_0.3.3            stringr_1.5.0              digest_0.6.29
##  [37] pkgconfig_2.0.3           htmltools_0.5.4            sessioninfo_1.2.2
##  [40] highr_0.9                 fastmap_1.1.0              htmlwidgets_1.6.0
##  [43] rlang_1.1.0               readxl_1.4.2               httpcode_0.3.0
##  [46] rstudioapi_0.14           shiny_1.7.2                generics_0.1.3
##  [49] jsonlite_1.8.4            dplyr_1.1.2                magrittr_2.0.3
##  [52] Matrix_1.5-3              Rcpp_1.0.11                munsell_0.5.0
##  [55] fansi_1.0.3               lifecycle_1.0.3            stringi_1.7.8
##  [58] MASS_7.3-57               rootSolve_1.8.2.3          pkgbuild_1.4.1
##  [61] plyr_1.8.8                grid_4.2.1                 parallel_4.2.1
##  [64] promises_1.2.0.1          lmom_2.9                   miniUI_0.1.1.1
##  [67] lattice_0.20-45           knitr_1.41                 ps_1.7.2
##  [70] pillar_1.9.0              boot_1.3-28                gld_2.6.6
##  [73] pkgload_1.3.2             crul_1.4.0                 urca_1.3-3
##  [76] XML_3.99-0.14             glue_1.6.2                 evaluate_0.19
##  [79] hoardr_0.5.3              data.table_1.14.6          remotes_2.4.2
##  [82] vctrs_0.6.1               httpuv_1.6.6               cellranger_1.1.0
##  [85] tidyr_1.3.0               gtable_0.3.1               purrr_1.0.1
##  [88] cachem_1.0.6              ggplot2_3.4.3              xfun_0.38
##  [91] mime_0.12                 xtable_1.8-4               e1071_1.7-12
##  [94] later_1.3.0               class_7.3-20               timeDate_4022.108
##  [97] tibble_3.2.1              memoise_2.0.1              timechange_0.1.1
```

```
## [100] ellipsis_0.3.2
```

```
Sys.time()
```

```
## [1] "2023-09-04 12:37:16 BST"
```