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Time-Varying Structural Vector Autoregressions and Monetary Policy: A Corrigendum

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Abstract

This note corrects a mistake in the estimation algorithm of the time-varying structural vector autoregression model of Primiceri (2005) and proposes a new algorithm that correctly applies the procedure proposed by Kim, Shephard, and Chib (1998) to the estimation of VAR or DSGE models with stochastic volatility. Relative to Primiceri (2005), the correct algorithm involves a different ordering of the various Markov Chain Monte Carlo steps.

Key words: Bayesian methods, time-varying volatility

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1. THE MODEL IN SHORT

Consider the time-varying VAR model of Primiceri (2005)

$$(1.1) \quad y_t = c_t + B_{1,t}y_{t-1} + \dots + B_{k,t}y_{t-k} + A_t^{-1}\Sigma_t\varepsilon_t,$$

where y_t is an $n \times 1$ vector of observed endogenous variables; c_t is a vector of time-varying intercepts; $B_{i,t}$, $i = 1, \dots, k$, are matrices of time-varying coefficients; A_t is a lower triangular matrix with ones on the main diagonal and time-varying coefficients below it; Σ_t is a diagonal matrix of time-varying standard deviations; ε_t is an $n \times 1$ vector of unobservable shocks with variance equal to the identity matrix. All the time-varying coefficients evolve as random walks, except for the diagonal elements of Σ_t , which behave as geometric random walks. All the innovations in the model (shocks to coefficients, log-volatilities and ε_t) are jointly normally distributed, with mean equal to zero and covariance matrix equal to V . The block structure of the matrix V is described in detail in Primiceri (2005).

2. ORIGINAL ALGORITHM

The unknown objects of the model are the history of the volatilities (Σ^T), the history of the coefficients (B^T and A^T), and the covariance matrix of the innovations (V). To simplify the notation, define $\theta \equiv [B^T, A^T, V]$. The posterior $p(\Sigma^T, \theta | y^T)$ can be simulated using an extension of the Gibbs sampling algorithm presented in Cogley and Sargent (2005). However, this algorithm involves drawing the history of volatilities Σ^T using the single-move technique of Jaquier, Polson and Rossi (1994). A more efficient alternative is to draw the history of volatilities using the multi-move algorithm proposed by Kim, Shephard and Chib (1998), which is based on data augmentation techniques and the approximation of each element of $\log \varepsilon_t^2$ with a mixture of n Gaussian random variables. In our case, this strategy involves simulating $p(\Sigma^T, \theta, s^T | y^T)$ instead of $p(\Sigma^T, \theta | y^T)$, where $s^T \equiv \{s_t\}_{t=1}^T$ denotes the history of the discrete random vectors selecting the component of the mixture for each variable at each point in time. To simulate $p(\Sigma^T, \theta, s^T | y^T)$, one possibility would be to use a Gibbs sampling algorithm based on the following three steps:

- (1) Draw Σ^T from $p(\Sigma^T | y^T, \theta, s^T)$
- (2) Draw s^T from $p(s^T | y^T, \Sigma^T, \theta)$
- (3) Draw θ from $p(\theta | y^T, \Sigma^T, s^T)$.

The problem with this algorithm is that step (3) is typically hard to implement, due to the presence of s^T in the conditioning set. In fact, conditional on specific components of the mixture, the innovations in (1) are non-Gaussian, which essentially precludes the possibility to draw θ easily. In Primiceri(2005), and other papers that closely followed his approach, step (3) was mistakenly replaced by “Draw θ from $p(\theta|y^T, \Sigma^T)$.” This is generally much easier, but cannot be justified based on the algorithm presented above.

3. MODIFIED ALGORITHM

The previous problem can be easily fixed by dividing the parameter space into two blocks, Σ^T and (s^T, θ) , and using a Gibbs sampling algorithm that iterates over these two blocks:

- (1) Draw Σ^T from $p(\Sigma^T|y^T, \theta, s^T)$
- (2) Draw (θ, s^T) from $p(\theta, s^T|y^T, \Sigma^T)$, which can be implemented using the following procedure:
 - (a) Draw θ from $p(\theta|y^T, \Sigma^T)$
 - (b) Draw s^T from $p(s^T|y^T, \Sigma^T, \theta)$.

Step (2a) can be implemented by drawing from $p(B^T|y^T, A^T, V, \Sigma^T)$, $p(A^T|y^T, B^T, V, \Sigma^T)$ and $p(V|y^T, A^T, B^T, \Sigma^T)$, as in Primiceri (2005). This algorithm is therefore identical to the one used in Primiceri (2005), except for the fact that the indicators s^T are sampled *before* Σ^T . In other words, the complete, correct version of the algorithm can be obtain by switching steps (d) and (e) in the algorithm summarized in Appendix A.5 of Primiceri (2005).

Finally, the the observation that the indicators s^T must be sampled before the history of volatilities is relevant for several papers that, following Primiceri (2005), estimate VARs with time-varying volatility using the Kim, Shephard and Chib (1998) approach, e.g., Canova and Gambetti (2009), D’Agostino et al. (2013), D’Agostino and Surico (2012), and Koop and Potter (2011). It also applies to other models whose estimation involves the Kim, Shephard and Chib (1998) mixture of normal approximation, such as DSGE models (e.g., Justiniano and Primiceri, 2008¹ and Curdia, Del Negro and Greenwald, 2012), and factor models (e.g., Del Negro and Otrok, 2008) with time-varying volatilities.

¹The estimation algorithm of Justiniano and Primiceri (2008) is correct, although their appendix describes an algorithm with the wrong order.

4. CONSEQUENCES FOR THE RESULTS

The correct version of the algorithm applied to the same data used in Primiceri (2005) generates qualitatively similar results. The main difference is that some estimates of the time-varying objects are now smoother. The full set of results generated with the correct algorithm can be found in an online appendix.

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