Econ 493 B1 - Winter 2023

Homework 3

Assignment Information

This assignment is due on Friday March 10 at 4:00 pm.

Submit the assignment on eClass. Late assignments will receive **NO MARKS**.

Answers to computing exercises must include R commands and output files when applicable. All answers must be transcribed to your written answers which must be separate from the R printout.

Total marks = 50 (5 questions).

Exercise 1

Consider the stationary AR(2) process:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2).$$

a. Derive the autocorrelation function (ACF) for the AR(2) process.
b. Find the optimal forecasts made at time T, $y_{T+h|T}$, for $h = 1, 2, 3$.
c. Find the corresponding forecast errors $\varepsilon_{T+h|T}$ for $h = 1, 2, 3$.
d. Find the forecast error variances for $h = 1, 2, 3$.

Exercise 2

Consider the simple trend stationary process:

$$y_t = \delta_0 + \delta_1 t + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2).$$

Its first difference can be written as

$$y'_t = \delta_1 + \varepsilon'_t,$$

with $y'_t = y_t - y_{t-1}$ and $\varepsilon'_t = \varepsilon_t - \varepsilon_{t-1}$.

a. What process does ε'_t follow? Is this process invertible?
b. Derive the mean, variance, and autocorrelation function of ε'_t.
c. Plot the autocorrelation function.

Exercise 3 (R)

The annual bituminous coal production in the US from 1920 to 1968 is in data set `bicoal`.

a. You decide to fit the following model to the series:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \phi_4 y_{t-4} + \varepsilon_t$$

where y_t is the coal production in year t and ε_t is a white noise series. What sort of ARIMA model is this (that is, what are p, d, q)?

b. Explain why this model was chosen using the ACF and PACF.

c. The estimated parameters are $c = 162.00$, $\phi_1 = 0.83$, $\phi_2 = -0.34$, $\phi_3 = 0.55$, and $\phi_4 = -0.38$. The last five values of the series are given below. Without using the `forecast` function, calculate forecasts for the next three years (1969–1971).

Year	1964	1965	1966	1967	1968
Millions of tons	467	512	534	552	545

d. Now fit the model in R and obtain the forecasts from the same model. How are they different from yours? Why?

Exercise 4 (R)

A classic example of a non-stationary series is the daily closing IBM stock price series (data set `ibmclose`).

a. Use R to plot the series, the ACF, and PACF. Explain how each plot shows that the series is non-stationary and should be differenced.

b. Fit a stationary AR(1) process to the series. What is the estimated coefficient? How do the results relate to your answers in a?

c. Use R to plot the first difference of the series, the ACF and PACF. Explain how each plot shows that the differenced series is stationary.

d. Fit a stationary AR(1) process to the differenced series. What is the estimated coefficient? How do the results relate to your answers to c?

Exercise 5 (R)

The file `NAEXKP01CAQ661S.csv` contains the series of quarterly real gross domestic product (RGDP) for Canada for the quarters 1961:Q1 to 2018:Q1, measured in millions of 2010 Canadian dollars and seasonally adjusted.

a. Use R to plot the series, the ACF, and PACF. Does the series appear to be stationary?

b. Use R to plot the change in log RGDP (that is, growth rate of the series), the ACF, and PACF. Does the differenced series appear to be stationary?

c. What model do the plots in (b) suggest?

d. Using the BIC, find the AR model that adequately describes the change in log RGDP. Motivate the steps that you take. Make sure your model includes a drift to capture the time trend observed in the series.

e. Using the AIC, find the AR model that adequately describes the change in log RGDP. Motivate the steps that you take. Make sure your model includes a drift to capture the time trend observed in the series.

f. Use the models selected in parts (d) and (e) to forecast the quarterly log RGDP in 2018:Q2, 2018:Q3, and 2018:Q4. Compare your results.