Econ 493 B1 - Winter 2023

Homework 4

Assignment Information

This assignment is due on Friday March 24 at 4:00 pm.

Submit the assignment on eClass. Late assignments will receive **NO MARKS**.

Answers to computing exercises must include R commands and output files when applicable. All answers must be transcribed to your written answers which must be separate from the R printout.

Total marks = 50 (5 questions).

Exercise 1

Let y_t follow an ARIMA(1,1,0) process

$$y_t' - \mu = \phi(y_{t-1}' - \mu) + \varepsilon_t,\ \varepsilon_t \sim WN(0, \sigma^2)$$

where $|\phi| < 1$. Recall the h-step ahead optimal forecast of y_{T+h} is

$$y_{T+h|T} = y_T + h\mu + (y_T' - \mu)\sum_{s=1}^{h} \phi^h.$$

a. Find the forecast errors, $\varepsilon_{T+h|T}$, for $h = 1, 2, 3$.
b. Find the forecast error variances for $h = 1, 2, 3$.

Exercise 2

Let y_t follow an ARIMA(0,1,1) process

$$y_t' - \mu = \varepsilon_t + \theta\varepsilon_{t-1},\ \varepsilon_t \sim WN(0, \sigma^2)$$

where $|\theta| < 1$.

a. Find the h-step ahead optimal forecast of y_{T+h} for $h = 1, 2, 3$.
b. Find the forecast errors, $\varepsilon_{T+h|T}$, for $h = 1, 2, 3$.
c. Find the forecast error variances for $h = 1, 2, 3$.

Exercise 3 (R)

Consider the total monthly expenditure on cafes, restaurants, and takeaway food services in Australia ($billion) for the sample April 1982 to September 2017 (data set `auscafe`).

a. Do the data need transforming? If so, find a suitable transformation.
b. Are the data stationary? If not, find an appropriate differencing which yields stationary data.
c. Identify a couple of ARIMA models that might be useful in describing the time series. Which of your models is the best according to their AIC values?
d. Estimate the parameters of your best model and do diagnostic testing on the residuals. Do the residuals resemble white noise? If not, try to find another ARIMA model which fits better.
e. Forecast the next 24 months of data using your preferred model.
f. Refit the model using `auto.arima()`. How different are the two models?
g. Compare the forecasts obtained using `auto.arima`.

Exercise 4 (R)

Consider monthly sales and advertising data for an automotive parts company (data set `advert`).

a. Plot the data using `autoplot`. Why is it useful to set `facets=TRUE`?
b. Fit a standard regression model $y_t = a + bx_t + \eta_t$ where y_t denotes sales and x_t denotes advertising using the `tslm()` function.
c. Show that the residuals have significant autocorrelation.
d. What difference does it make if you use the `Arima` function instead:
e. Refit the model using `auto.arima()`. How much difference does the error model make to the estimated parameters? What ARIMA model for the errors is selected?
f. Check the residuals of the fitted model.
g. Assuming the advertising budget for the next six months is exactly 10 units per month, produce sales forecasts with prediction intervals for the next six months.

Exercise 5 (R)

The file `NAEXKP01CAQ661S.csv` contains the series of quarterly real gross domestic product (RGDP) for Canada for the quarters 1961:Q1 to 2018:Q1, measured in millions of 2010 Canadian dollars and seasonally adjusted.

a. Use R to plot the series, the ACF, and PACF. Does the series appear to be stationary?

b. Here we will fit a *trend stationary model* for the sample 1961Q1 to 2009Q4. Using the AIC, find the AR model *with* time trend that adequately describes RGDP. Make sure your model includes a trend component. Motivate the steps that you take.

c. Using this model, compute and plot the quarterly forecasts of real GDP for the next 8 years (32 quarters) along with the prediction intervals for the forecasts and the actual value of real GDP. Interpret your results.

d. Here we will fit a *difference stationary model* for the sample 1961Q1 to 2009Q4. Using the AIC, find the AR model that adequately describes the change RGDP. Make sure your model uses $d = 1$ and includes a drift. Motivate the steps that you take.

e. Using this model, compute and plot the quarterly forecasts of real GDP for the next 8 years (32 quarters) along with the prediction intervals for the forecasts and the actual value of real GDP. Interpret your results.

f. Compare your results from (c) and (e).