

Mean - Vol Relationship

VAR

A reduced form VAR of some number of variables, k , and some number of lags, p , is written as,

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (1)$$

where A_i is a $k \times k$ matrix of coefficients and,

$$y_t = \begin{bmatrix} \text{output}_t \\ \text{unemployment}_t \\ \text{inflation}_t \\ \dots \\ \text{vfc}_t \end{bmatrix}, \quad u_t = \begin{bmatrix} u_t^{\text{output}} \\ u_t^{\text{unemployment}} \\ u_t^{\text{inflation}} \\ \dots \\ u_t^{\text{vfc}} \end{bmatrix} \quad (2)$$

Each u_t are the reduced form residuals for the accompanying data series.

Max Share ID

Then, using the max share identification method, we identify one structural shock, w_t^{BC} , which drives the business cycle.

$$u_t = PQ^* w_t \quad (3)$$

where P is the choleskey of Σ_u and Q^* is the identified rotation matrix that returns w_t^{BC} in the first column.

Then we can write,

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + PQ^* \begin{bmatrix} w_t^{\text{BC}} \\ w_t^2 \\ w_t^3 \\ \dots \\ w_t^k \end{bmatrix} \quad (4)$$

Business Cycle Contribution

Using the identified business cycle shock, we can construct the time series for y_t^{BC} which are the contributions to each variable of the VAR driven by the identified shock.

$$\widehat{y_t^{\text{BC}}} = \begin{bmatrix} \widehat{\text{output}_t^{\text{BC}}} \\ \widehat{\text{unemployment}_t^{\text{BC}}} \\ \widehat{\text{inflation}_t^{\text{BC}}} \\ \dots \\ \widehat{\text{vfc}_t^{\text{BC}}} \end{bmatrix} \quad (5)$$

Business Cycle Volatility

The realized volatility for the business cycle can be defined as:

$$v_t^{\text{BC}} = \log([w_t^{\text{BC}}]^2) \quad (6)$$

This volatility can also be modeled using the variables of the VAR:

$$v_t^{\text{BC}} = \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + \epsilon_t \quad (7)$$

where α_i is a $1 \times k$ vector of coefficients.

We can then estimate $\widehat{v_t^{\text{BC}}}$ as a time series.

Mean-Vol Relationship

The mean-vol relationship is then the relationship between

(1) the predicted means:

$$\widehat{y_t^{\text{BC}}} \quad (8)$$

(2) the predicted volatility:

$$\widehat{v_t^{\text{BC}}} \quad (9)$$

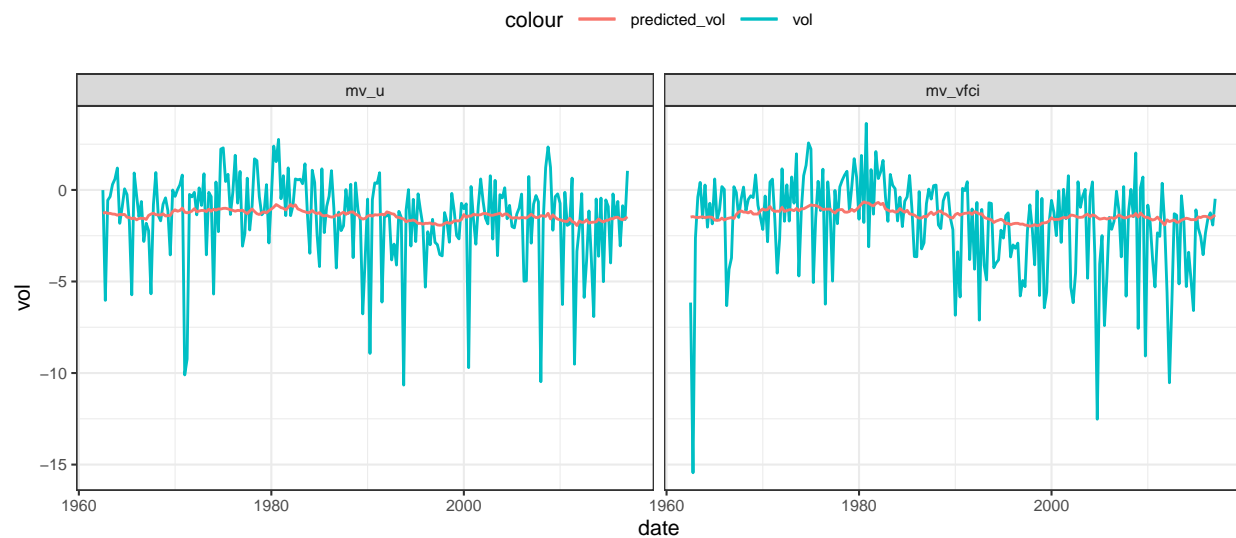
Note: There are multiple predicted means (one for each of the k variables), but only one predicted volatility of the business cycle. Therefore, there will be k mean-vol relationships which will depend upon the relationship between each variable's predicted mean and the business cycle shock.

The Data

Here are the contributions of the Business Shock, \widehat{y}_t^{BC} , to a subset of the variables, compared to the total unexplained variation in those series.

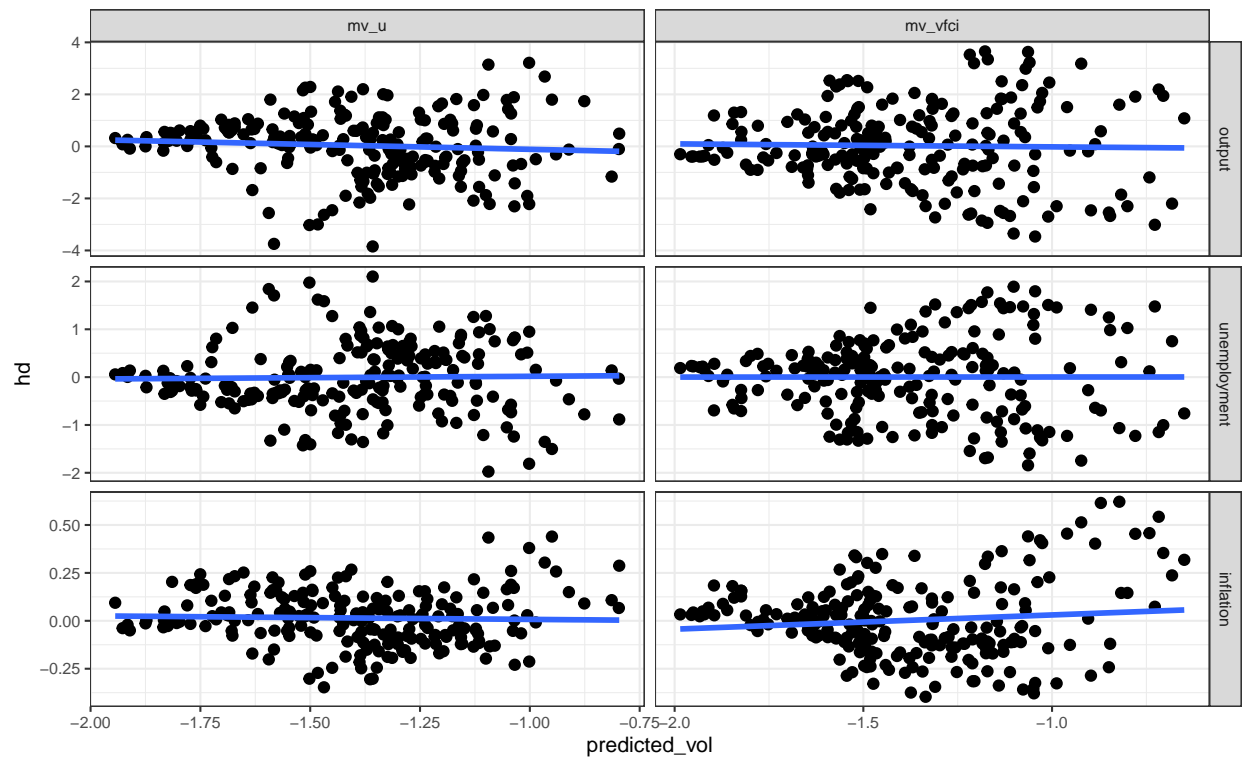


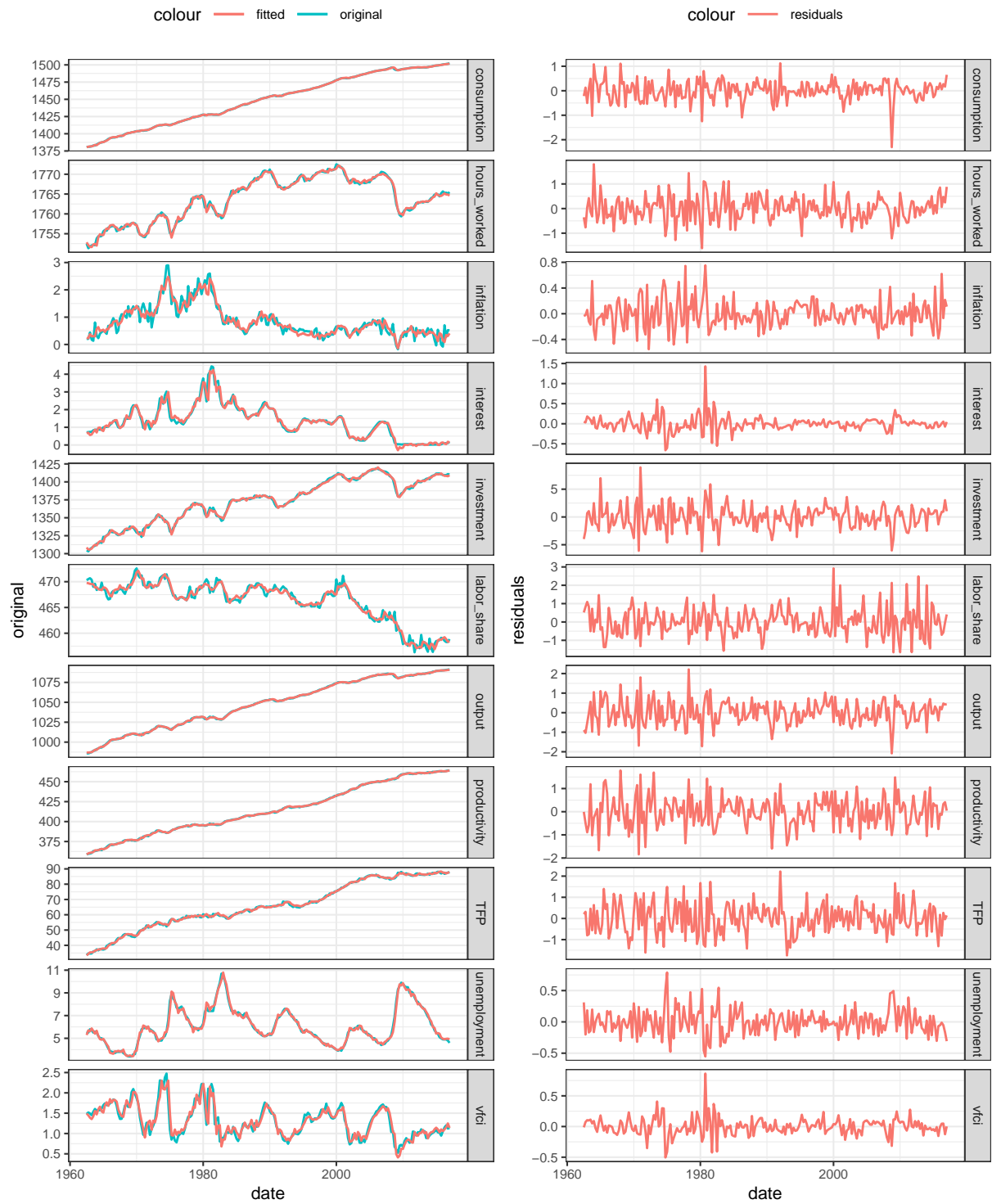
And here is the realized volatility of the business cycle shock, v_t^{BC} , and the predicted value, \widehat{v}_t^{BC} :

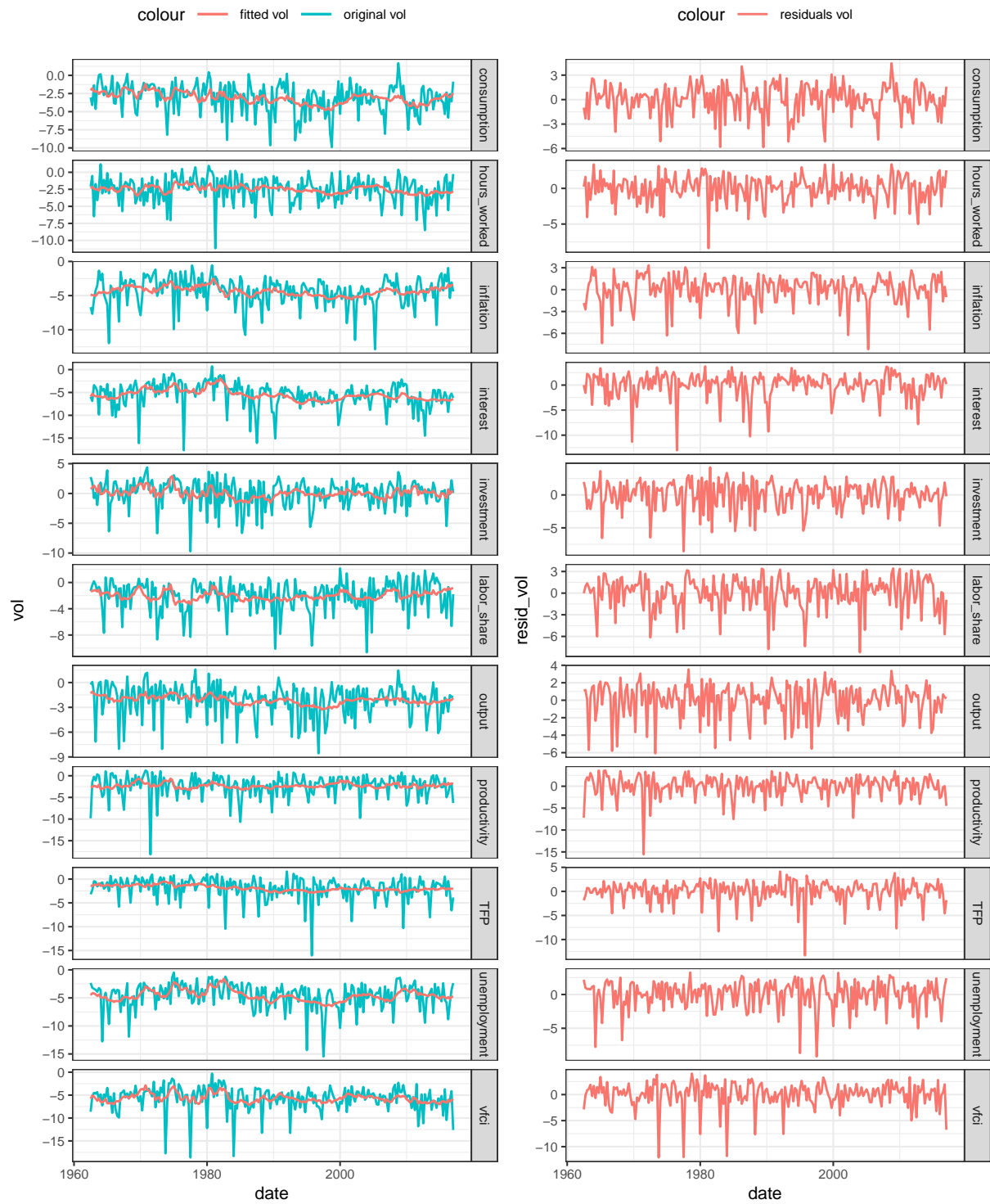


The correlation between the two sets of predicted values are not particularly high:

```
## `geom_smooth()` using formula = 'y ~ x'
```







```
## `geom_smooth()` using formula = 'y ~ x'
```

