Mean - Vol Relationship

VAR

A reduced form VAR of some number of variables, k, and some number of lags, p, is written as,

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \tag{1}$$

where A_i is a kxk matrix of coefficients and,

$$y_t = \begin{bmatrix} \text{output}_t \\ \text{unemployment}_t \\ \text{inflation}_t \\ ... \\ \text{vfci}_t \end{bmatrix}, \qquad u_t = \begin{bmatrix} u_t^{\text{output}} \\ u_t^{\text{unemployment}} \\ u_t^{\text{inflation}} \\ ... \\ u_t^{\text{vfci}} \end{bmatrix} \tag{2}$$

Each u_t are the reduced form residuals for the accompanying data series.

With this, we can estimate the predicted values, $\widehat{y}_t$, and the residuals, $\widehat{u}_t$.

Volatility

We can then define the log-variance as

$$\text{Var}_t \equiv \log(\widehat{u}_t^2) \tag{3}$$

This can be estimated with the regression,

$$\text{Var}_t = \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \epsilon_t \tag{4}$$

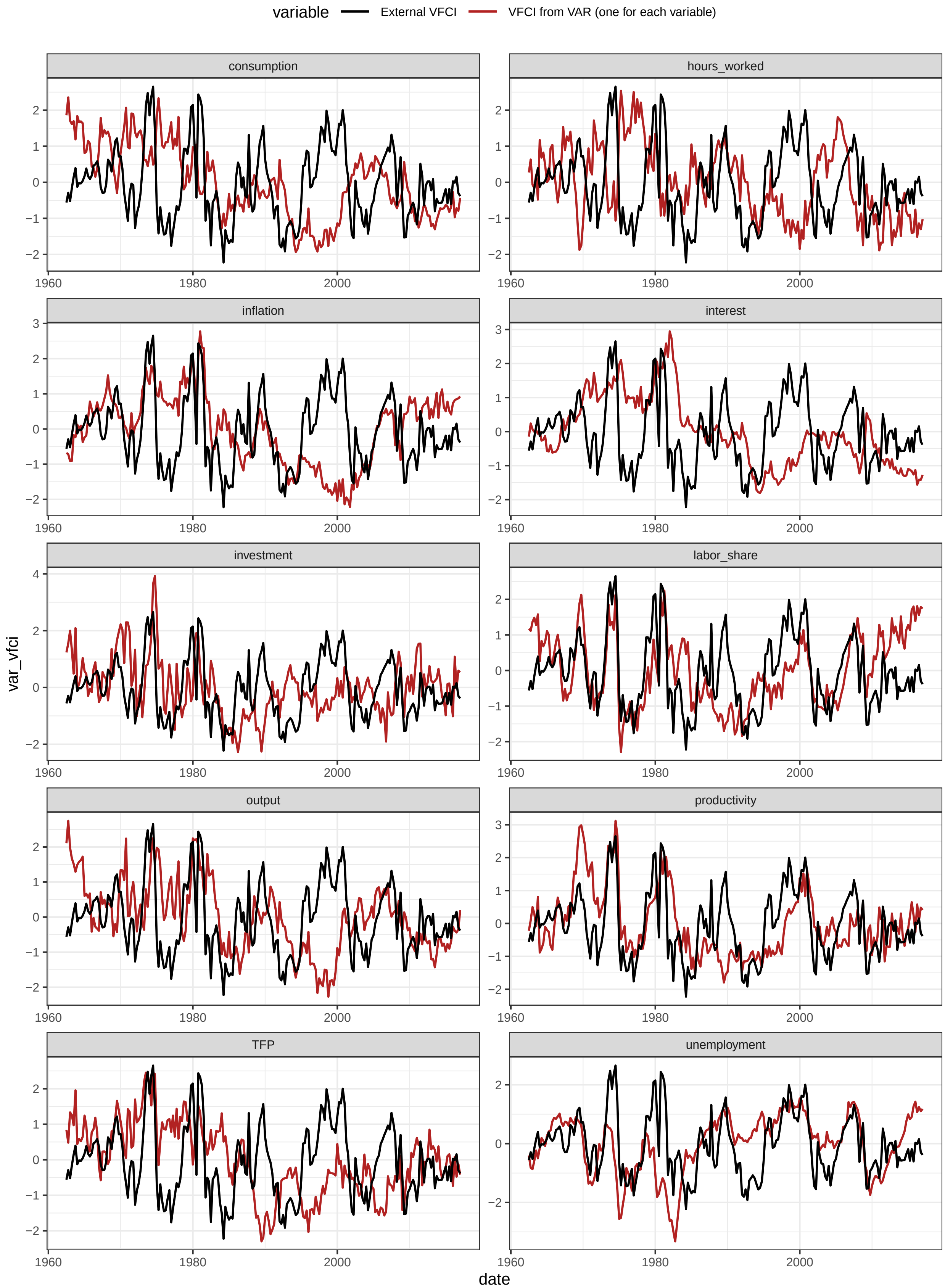
Then the VFCI is defined as,

$$\text{VFCI}_t \equiv \widehat{\text{Var}_t} \tag{5}$$

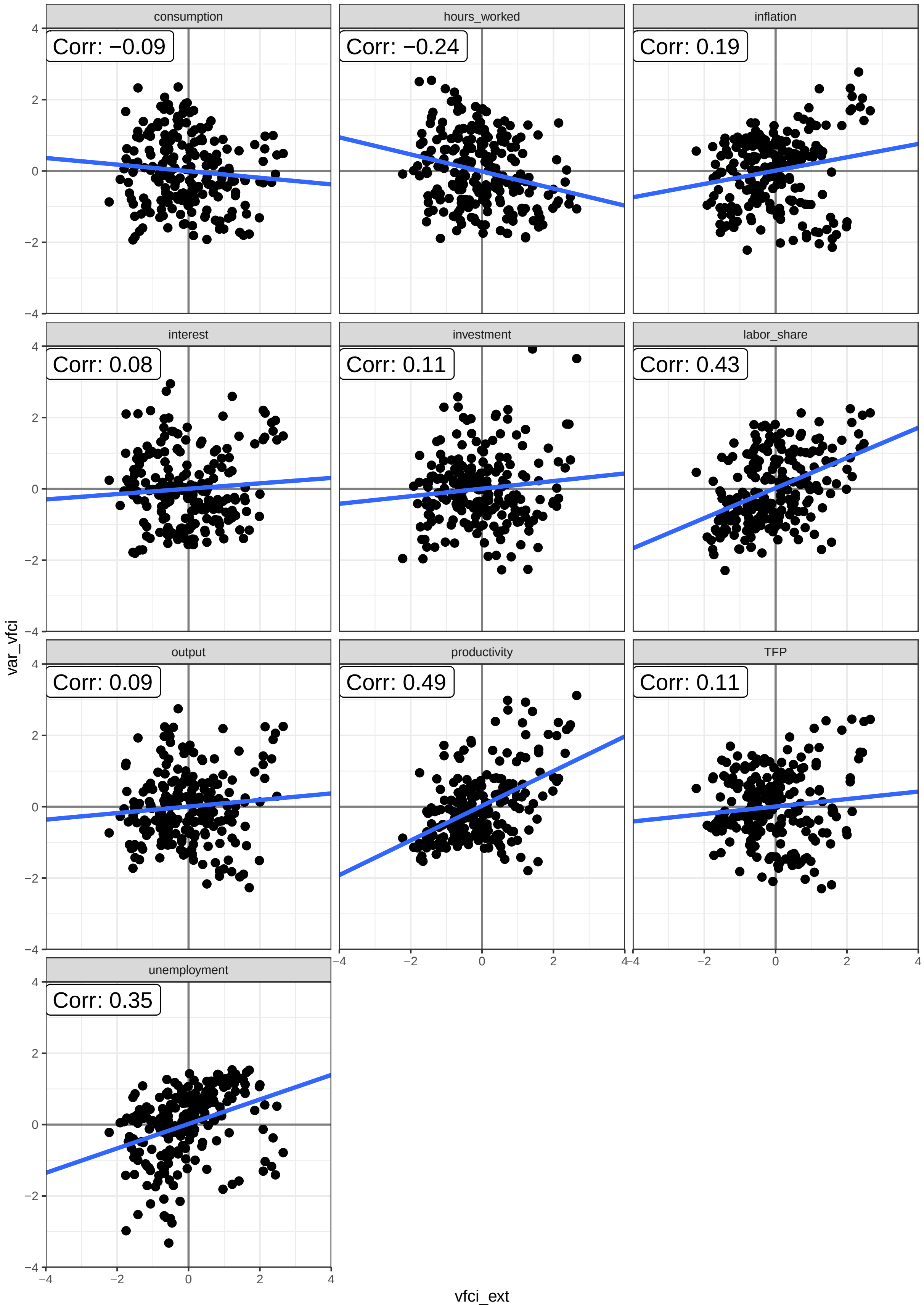
The VFCI series is then rescaled to N(0,1).

External VFCI

We will compare to the externally estimated VFCI on forward GDP growth (forwarded 1 quarter) and financial principal components.



variable
External VFCI
VFCI from VAR (one for each variable)
consumption
hours_worked
inflation
interest
investment
labor_share
output
productivity
TFP
unemployment
var_vfci
date
1960
1980
2000


consumption
Corr: −0.09
hours_worked
Corr: −0.24
inflation
Corr: 0.19
interest
Corr: 0.08
investment
Corr: 0.11
labor_share
Corr: 0.43
output
Corr: 0.09
productivity
Corr: 0.49
TFP
Corr: 0.11
unemployment
Corr: 0.35
var_vfci
vfci_ext