BCA Replication: Classical VAR IRFs, Bootsstrapped

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July 25, 2023

Replication of the Bootstrapped VARs

Using a resampled bootstrap method (drawing from VAR errors with replacement), the bootstrapped IRFs look very close to the original BCA IRFs.

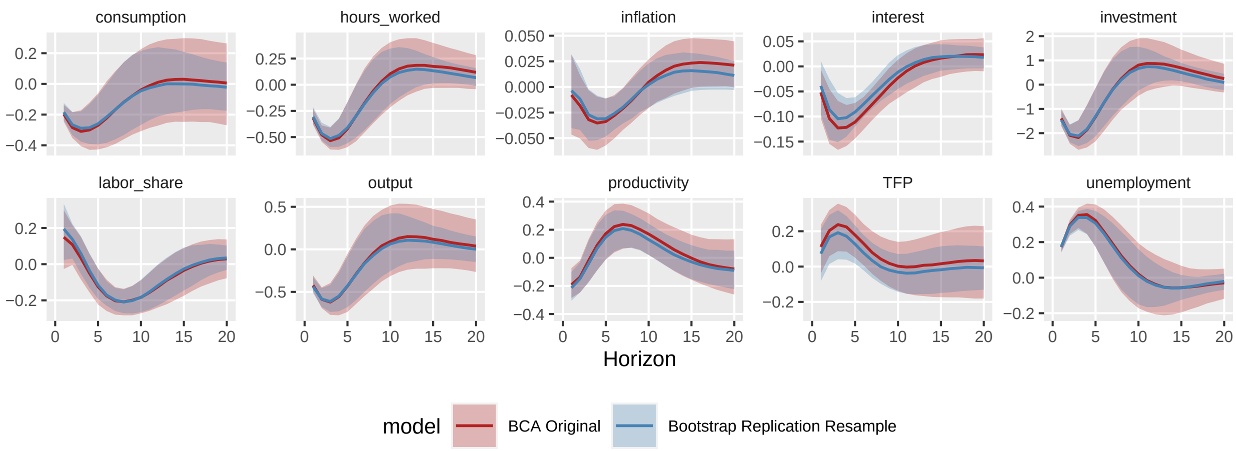


Figure 1: Comparison of Replicated IRF to Original, Resample Bootstrap

The original BCA paper also bias-adjusts their bootstraps, through a method called bootstrap-after-bootstrap. Implementing this makes the replicated IRFs even close to the original BCA IRFs.

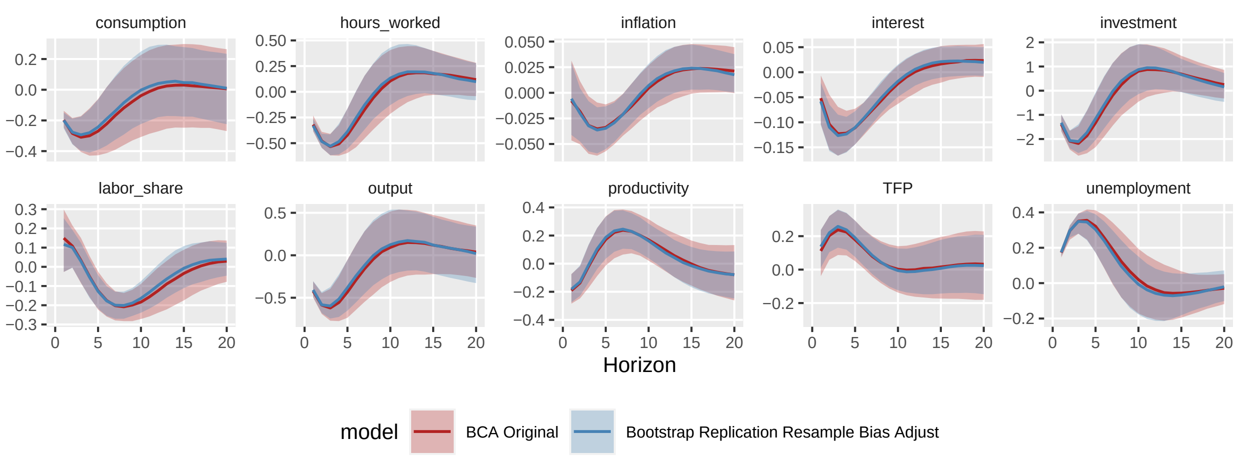


Figure 2: Comparison of Replicated IRF to Original, Resample Bootstrap, Bias adjusted

Comparison to Wild Bootsstrap

Using the Wild bootstrap with the gaussian distribution (multiply the errors by a random vector drawn from N(0,1)), the impulse responses are very different from the original BCA results.

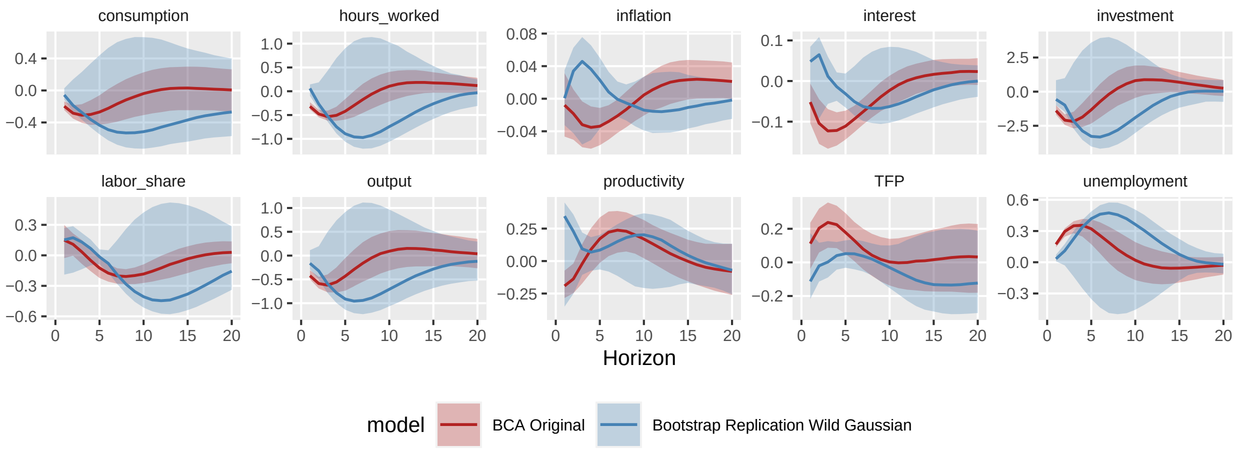


Figure 3: Comparison of Replicated IRF to Original, Wild Bootstrap, Gaussian, nboot = 1000

This remains true even if the number of bootstrap iterations are doubled. There is almost no different between 1000 and 2000 iterations.

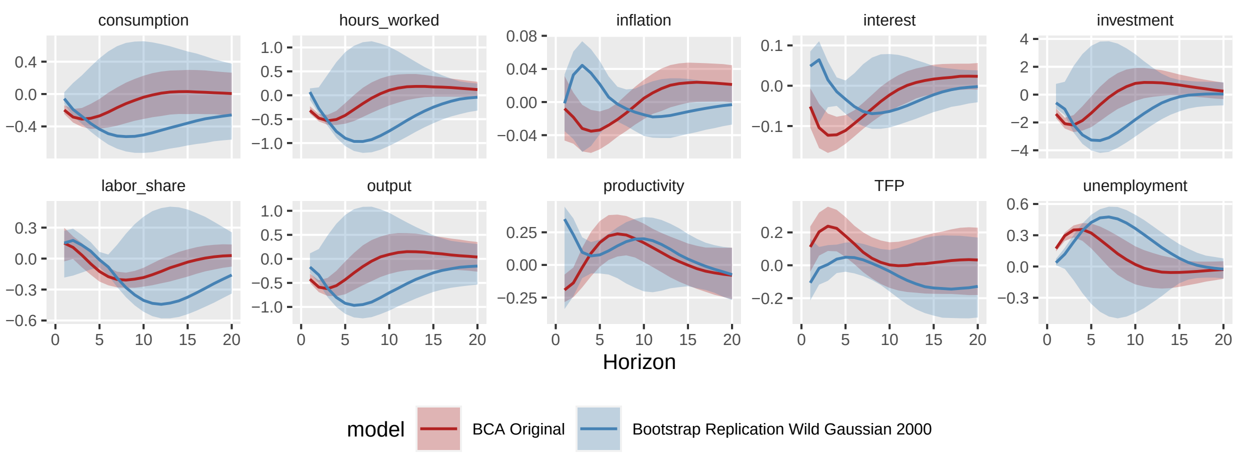


Figure 4: Comparison of Replicated IRF to Original, Wild Bootstrap, Gaussian, nboot = 2000

Another distribution can be used in the Wild bootstrap method, “Rademacher” simply multiplies the VAR errors by a random vector of -1s and 1s.

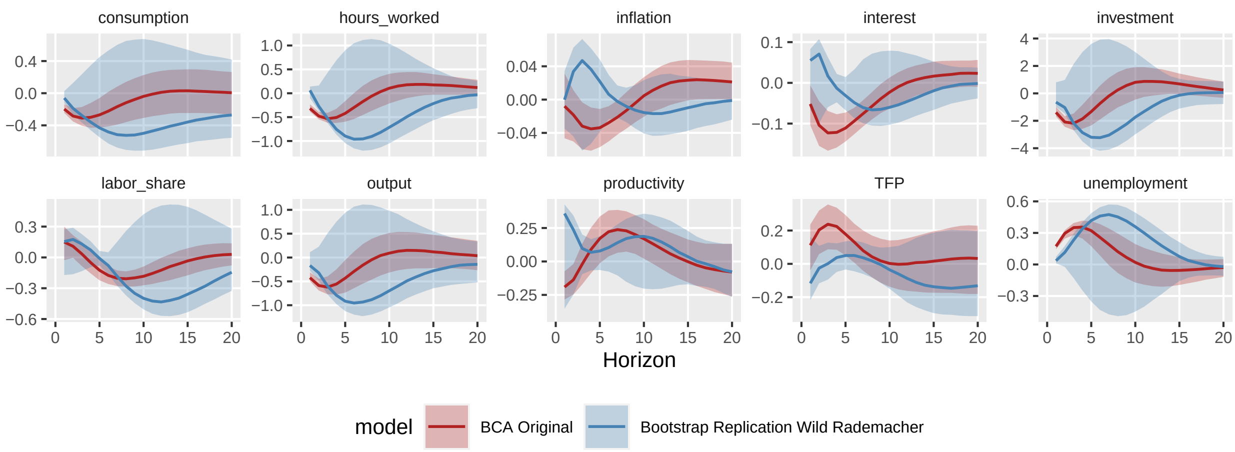


Figure 5: Comparison of Replicated IRF to Original, Wild Bootstrap, Rademacher

In order to deal with this we can choose to set the sign of the identified VARs by the sign of the cummulative IRF of the target variable out to `n` quarters. Effectively, by setting the sign based off of impact, we do this out to `n = 1`. If instead we set it out to `n = 10`, then we allow for negative IRFs on impact but that become strongly positive shortly thereafter. These bootstraps look much better.

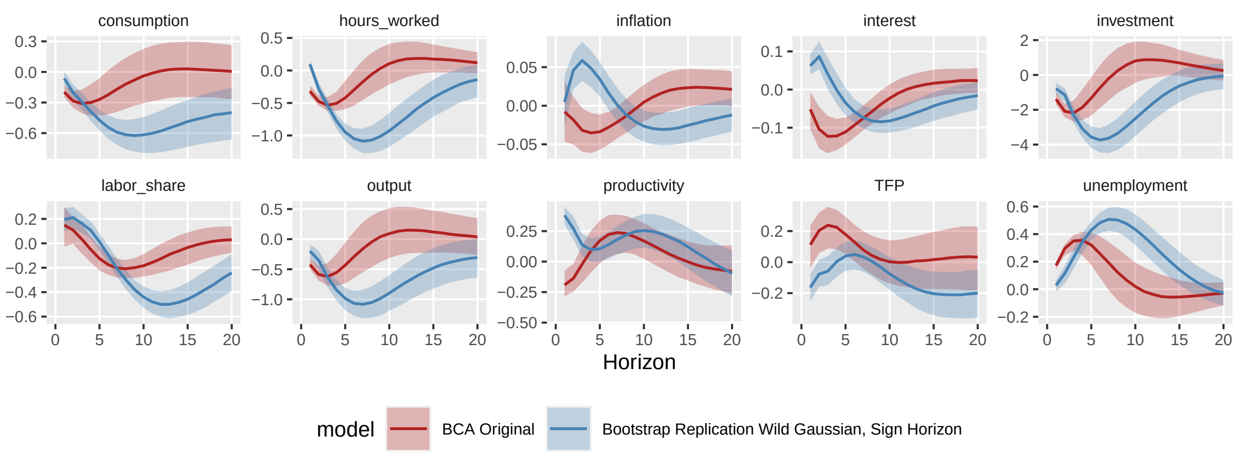
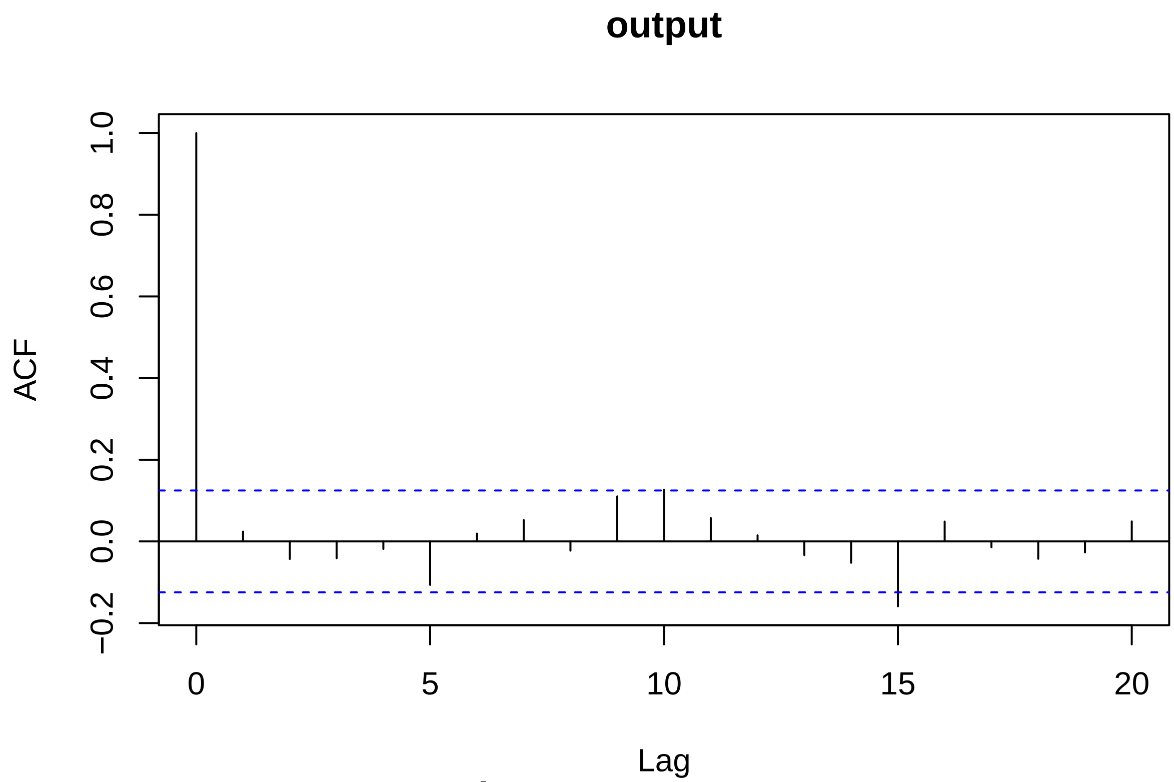
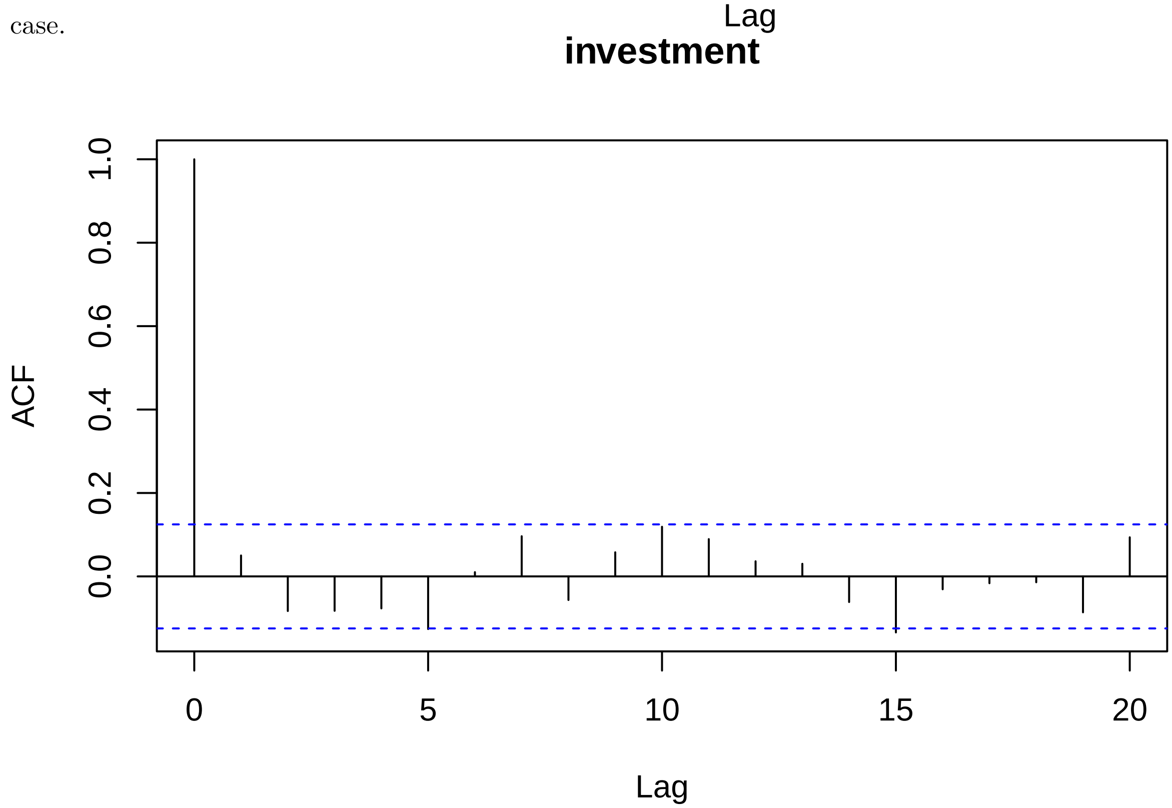


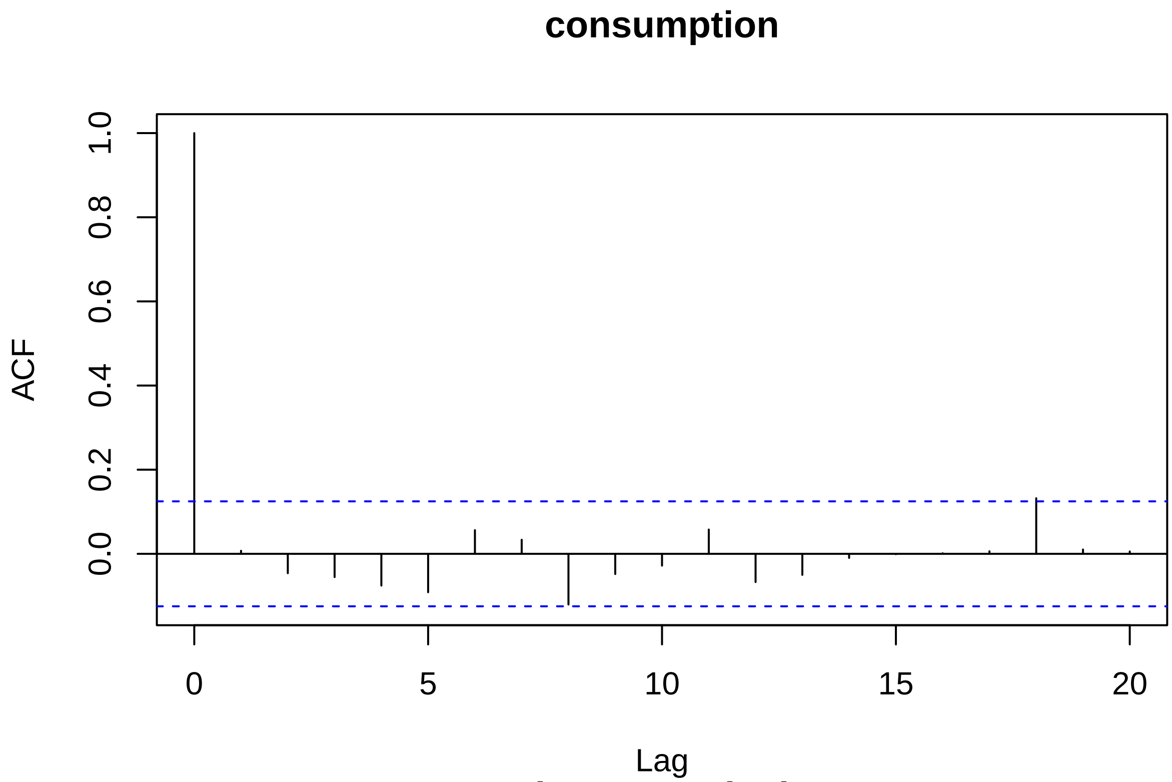
Figure 6: Comparison of Replicated IRF to Original, Wild Bootstrap, Gaussian, Sign Horizon

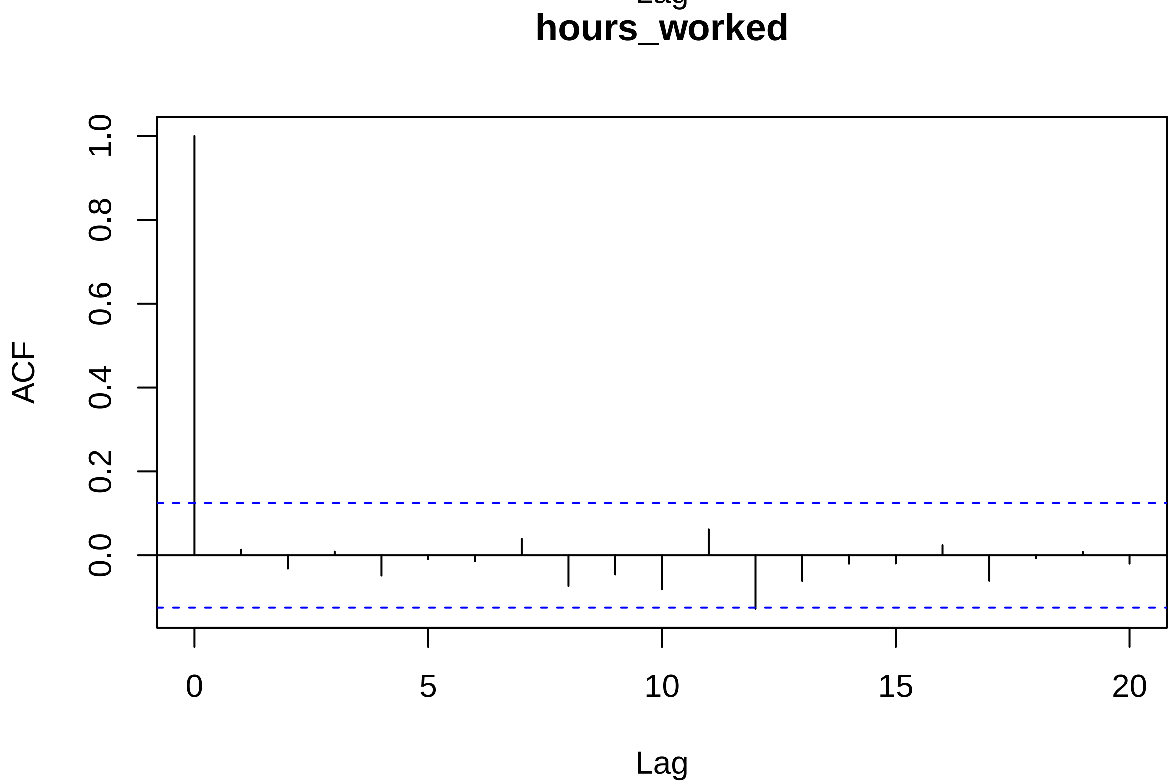
A question is if there is autocorrelation in the erorrs of the VAR that is causing this difference between the bootstrap methods. However, looking at the autocorrelation plots (below), this does not seem to be the

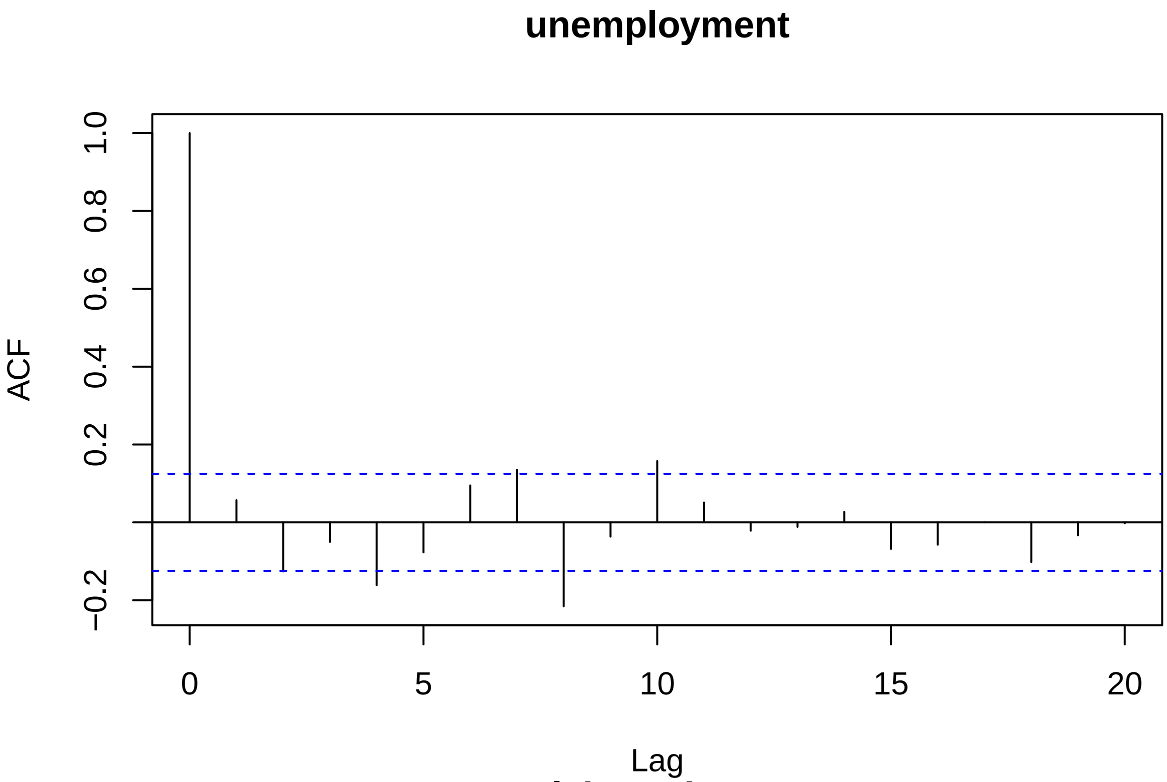

output
ACF
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
5
10
15
20
Lag

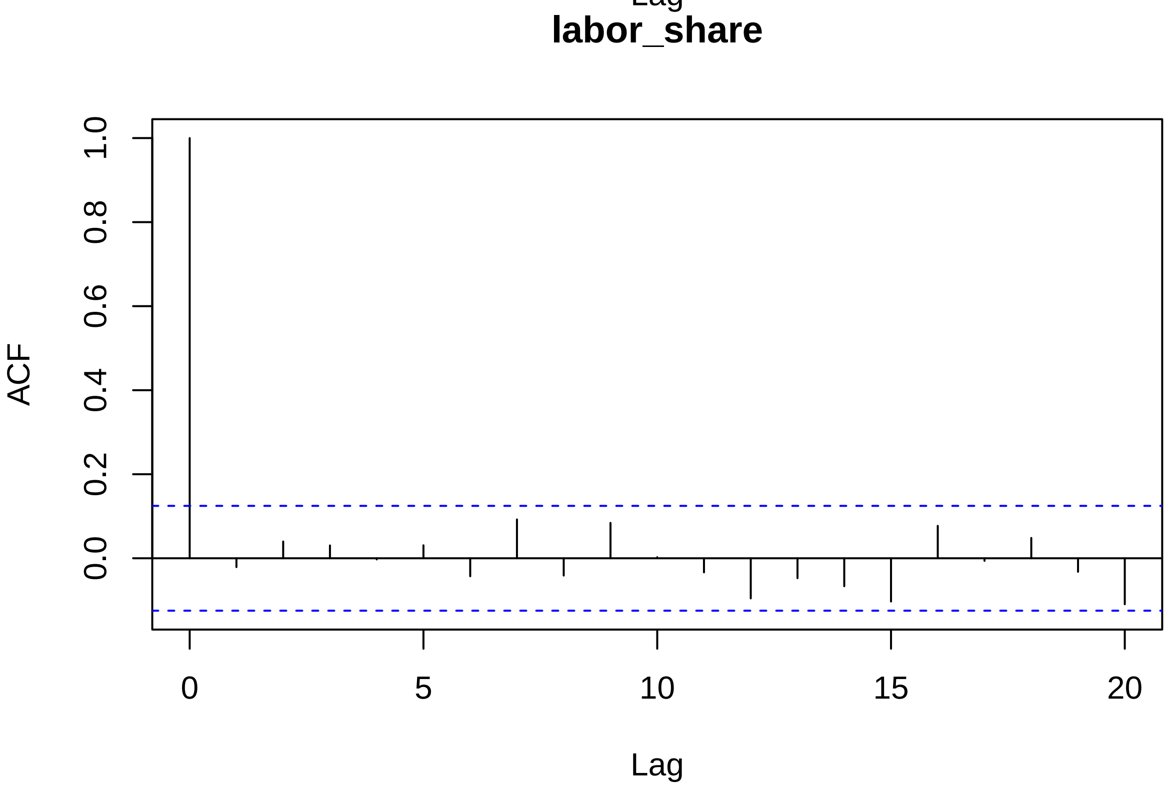
case.

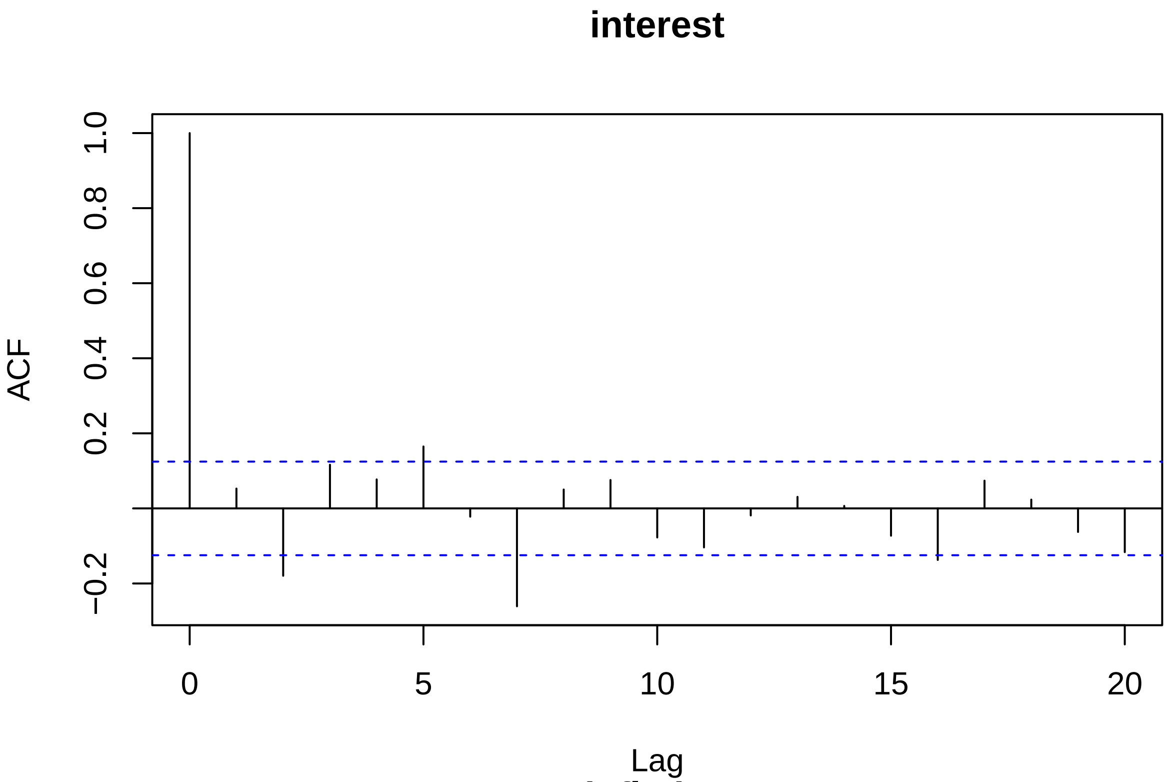

investment
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag

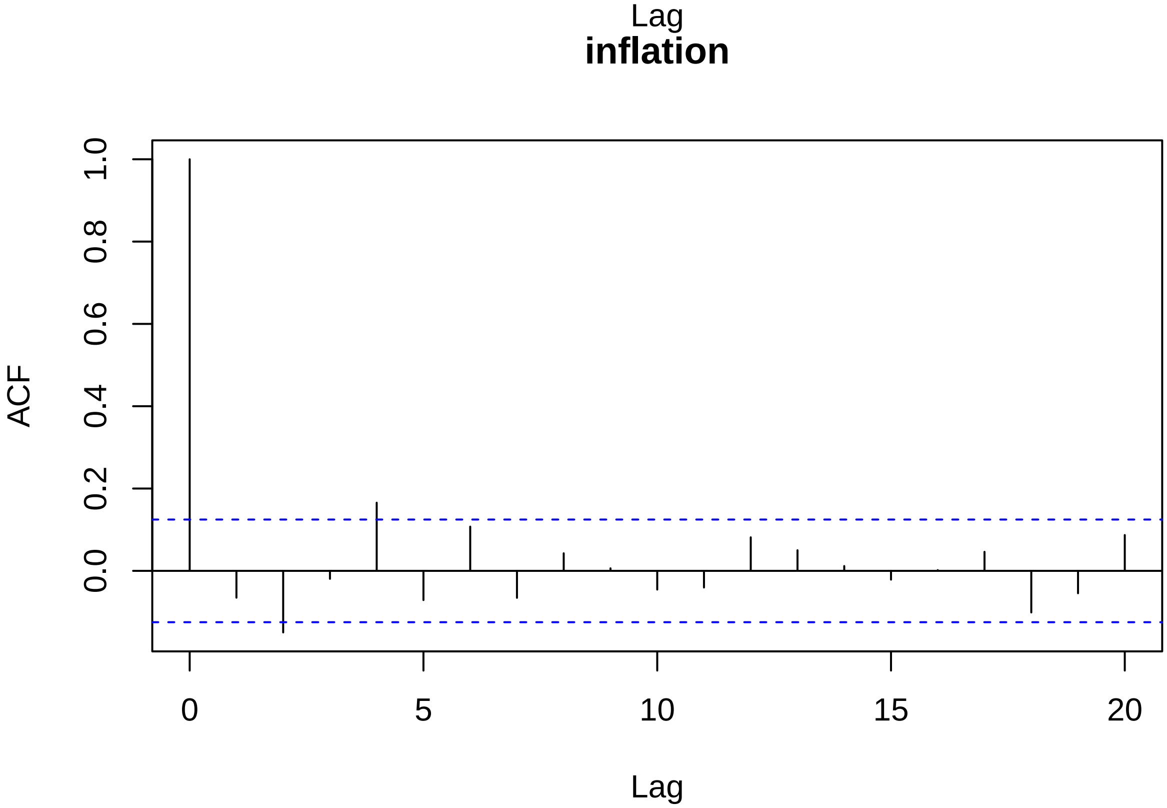

consumption
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag

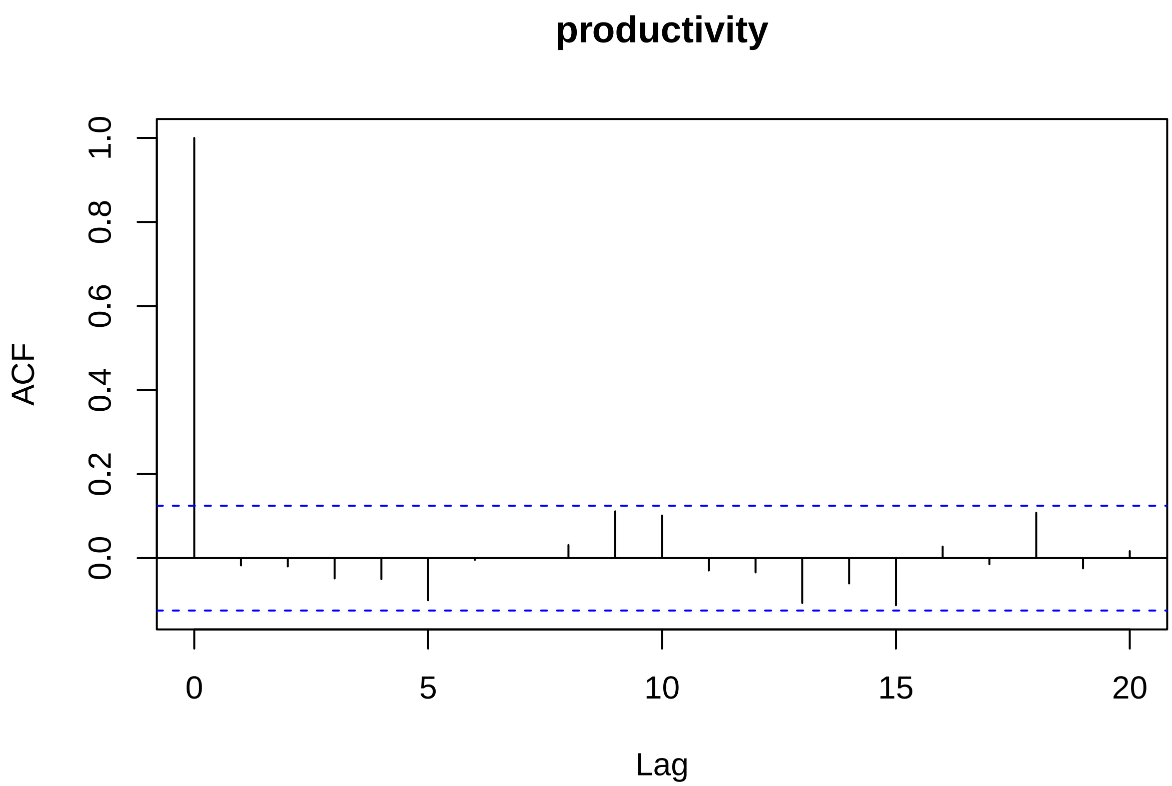

hours_worked
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag

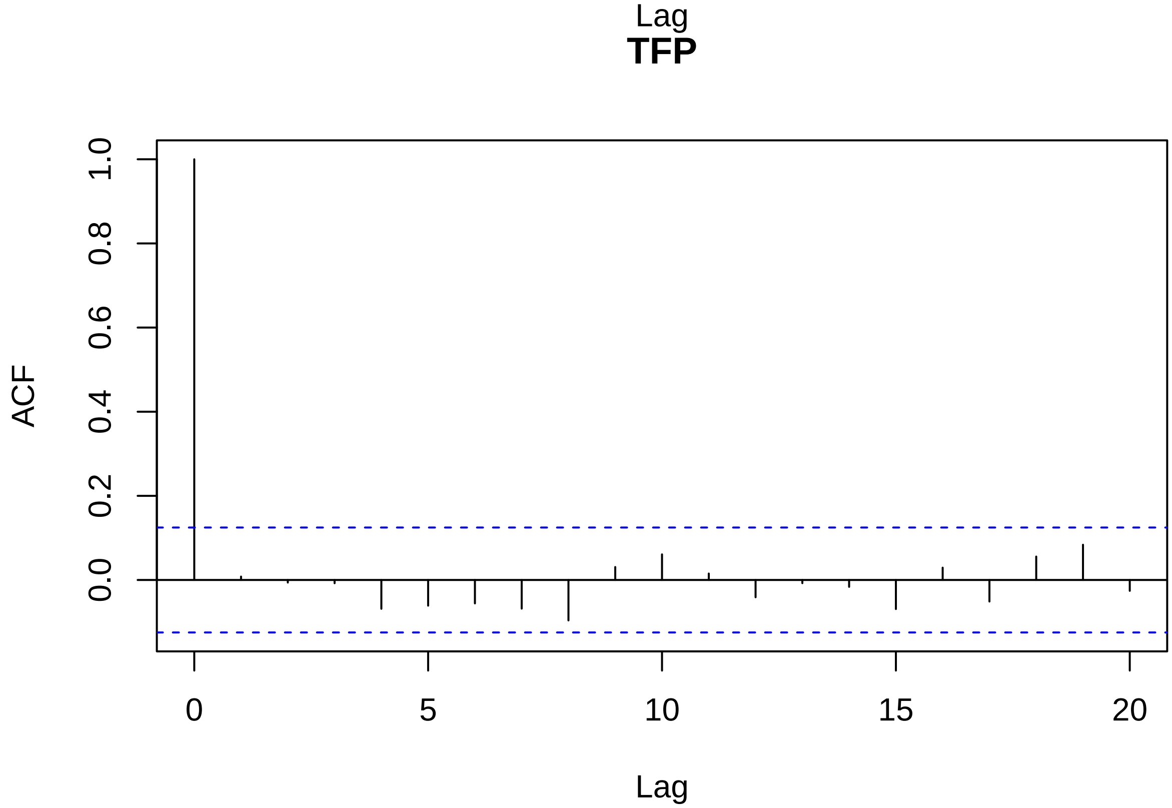

unemployment
ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0
5
10
15
20
Lag


labor_share
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag


interest
ACF
−0.2
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
Lag


inflation
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
Lag






```
## [[1]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
```

```
##  1.000  0.024 -0.043 -0.041 -0.018 -0.106  0.019  0.052 -0.023  0.110  0.127
##     11     12     13     14     15     16     17     18     19     20
##  0.057  0.015 -0.034 -0.052 -0.159  0.049 -0.014 -0.043 -0.027  0.049
##
## [[2]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.050 -0.083 -0.083 -0.077 -0.126  0.010  0.096 -0.057  0.058  0.119
##     11     12     13     14     15     16     17     18     19     20
##  0.089  0.036  0.030 -0.061 -0.134 -0.031 -0.016 -0.014 -0.086  0.094
##
## [[3]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.007 -0.046 -0.055 -0.076 -0.091  0.056  0.033 -0.121 -0.048 -0.028
##     11     12     13     14     15     16     17     18     19     20
##  0.058 -0.067 -0.050 -0.010 -0.001  0.001  0.006  0.132  0.010  0.006
##
## [[4]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.014 -0.032  0.009 -0.048 -0.010 -0.014  0.040 -0.073 -0.046 -0.081
##     11     12     13     14     15     16     17     18     19     20
##  0.062 -0.128 -0.061 -0.020 -0.020  0.024 -0.061 -0.007  0.009 -0.020
##
## [[5]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.057 -0.126 -0.050 -0.161 -0.077  0.095  0.135 -0.216 -0.037  0.158
##     11     12     13     14     15     16     17     18     19     20
##  0.051 -0.021 -0.011  0.027 -0.068 -0.057  0.000 -0.102 -0.033 -0.002
##
## [[6]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.021  0.040  0.031 -0.003  0.031 -0.043  0.092 -0.041  0.084  0.002
##     11     12     13     14     15     16     17     18     19     20
## -0.034 -0.095 -0.047 -0.067 -0.103  0.077 -0.006  0.048 -0.032 -0.109
##
## [[7]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
```

```
##  1.000  0.053 -0.179  0.116  0.077  0.165 -0.022 -0.261  0.050  0.076 -0.077
##     11     12     13     14     15     16     17     18     19     20
## -0.104 -0.019  0.031  0.007 -0.073 -0.137  0.074  0.024 -0.063 -0.116
##
## [[8]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.065 -0.150 -0.019  0.166 -0.071  0.108 -0.066  0.043  0.006 -0.046
##     11     12     13     14     15     16     17     18     19     20
## -0.041  0.082  0.050  0.012 -0.021  0.002  0.046 -0.101 -0.054  0.087
##
## [[9]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.017 -0.020 -0.048 -0.050 -0.100 -0.004  0.000  0.031  0.111  0.101
##     11     12     13     14     15     16     17     18     19     20
## -0.029 -0.034 -0.107 -0.060 -0.112  0.028 -0.015  0.108 -0.024  0.017
##
## [[10]]
##
## Autocorrelations of series 'u[i, ]', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000  0.008 -0.006 -0.008 -0.068 -0.061 -0.056 -0.068 -0.096  0.031  0.061
##     11     12     13     14     15     16     17     18     19     20
##  0.015 -0.041 -0.008 -0.016 -0.069  0.029 -0.051  0.056  0.084 -0.026
```