1 $\gamma p \to J/\psi + p$

1.1 Kinematics

Let x_γ be the proton 1 longitudinal momentum fraction carried by the photon, x be the proton 2 longitudinal momentum fraction carried by the hard gluon, $Q^2 = -q^2$ is the 4-momentum transfer through the photon. We have

$$W^2 \equiv (q+p_2)^2 = x_\gamma s - Q^2 \ , \tag{1}$$

m_perp^2=m^2+pt^2

$$x_\gamma = \frac{M_{\psi_\perp}}{\sqrt{s}} e^{y_\psi} \ , \tag{2}$$

$$x = \frac{M_{\psi_\perp}}{\sqrt{s}} e^{-y_\psi} \ , \tag{3}$$

and the case where the photon couples to proton 2 is included by making the substitution $y_\psi \to -y_\psi$ (and $p_{1\perp} \to p_{2\perp}$). Neglecting the photon virtuality and J/ψ $p_\perp$, we also have the usual approximate relations

$$x_\gamma \approx \frac{W^2}{s} \qquad x \approx \frac{M_\psi^2}{W^2} \ . \tag{4}$$

In general if we write

$$q = x_\gamma (p_+)_1 + \tilde{x}_\gamma (p_-)_1 + q_\perp \ , \tag{5}$$

then imposing the mass shell limit on the outgoing proton gives $\tilde{x}_\gamma$, and we get

$$Q^2 = \frac{(x_\gamma^2 m_p^2 + \mathbf{q}_\perp^2)}{1-x_\gamma} = \frac{(x_\gamma^2 m_p^2 + \mathbf{p}_{1\perp}^2)}{1-x_\gamma} \ . \tag{6}$$

Q^2 is therefore cut-off at the minimum value

$$Q^2_{\rm min} = \frac{x_\gamma^2 m_p^2}{1-x_\gamma} \ . \tag{7}$$

1.2 Elastic cross section

The cross section for the process $pp \to p + J/\psi + p$ is given by

$$\sigma = 2 \int {\rm d}^2\mathbf{p}_{1\perp} \int_0^\infty {\rm d}k \frac{{\rm d}n}{{\rm d}k} \sigma(\gamma^* p \to J/\psi + p) \ , \tag{8}$$

where the photon energy, k, spectrum is given by

$$\frac{{\rm d}n}{{\rm d}k} = \frac{\alpha}{\pi^2 k}\left(1 - x_\gamma + \frac{x_\gamma^2}{2}\right) \frac{F_N^2(Q^2)}{Q^2} \ , \tag{9}$$

with

$$F_N(Q^2) = \frac{1}{(1+Q^2/0.71{\rm GeV}^2)^2} \ . \tag{10}$$

We can change variables from $\mathbf{p}_{1\perp}^2$ to Q^2 and perform the $p_\perp$ integration, as well as changing variables from k to the J/ψ rapidity y_ψ to finally give

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_\psi} = 2\frac{\alpha}{\pi}\left(1-x_\gamma+\frac{x_\gamma^2}{2}\right)\left[\ln A-\frac{11}{6}+\frac{3}{A}-\frac{3}{2A^2}+\frac{1}{3A^3}\right]\sigma(\gamma^* p \to J/\psi + p), \quad (11)$$

where $A = 1 + 0.71\mathrm{GeV}^2/Q^2_{\mathrm{min}}$. In fact, in the MC the $\mathbf{p}_{1\perp}$ integration (8) is instead done numerically, and we use the more precise form of the photon energy spectrum given by Eq. (4) of [1].

The $\gamma p \to J/\psi\, p$ cross section can be taken from a fit to HERA data [2]

$$\frac{\mathrm{d}\sigma(\gamma p \to J/\psi \, + \, p)}{\mathrm{d}\mathbf{p}_{2\perp}^2} = N\left(\frac{W}{1\,\mathrm{GeV}}\right)^\delta e^{-b\mathbf{p}_{2\perp}^2} \quad (12)$$

with $b = 4.5$ GeV^{-2}, $\delta = 0.72$ and $N = 3$ nb for J/ψ production [3]. The data exists (and is fitted well– see Fig. 2) for $W \lesssim 300$ GeV, i.e. $|y_\psi| < 2.7$ at the Tevatron and $|y_\psi| < 1.4$ at the LHC ($\sqrt{s} = 7$ TeV). Using this fit we also find $\mathrm{d}\sigma^{p\bar{p}\to p\bar{p}J/\psi}/\mathrm{d}y_\psi|_{y_\psi=0} \approx 3.8$ nb at the Tevatron, in excellent agreement with the data value of 3.92 ± 0.62 nb, as expected.

Alternatively, we can use the simplest LO pQCD theory expression [4, 5]

$$\frac{\mathrm{d}\sigma(\gamma^* p \to J/\psi \, + \, p)}{\mathrm{d}\mathbf{p}_{2\perp}^2} = K\frac{\Gamma_{\mathrm{ee}}\pi^3 M_\psi^3}{48\alpha}\frac{\alpha_S^2(\overline{Q}^2)}{\overline{Q}^8}[xg(x,\overline{Q}^2)]^2 e^{-b\mathbf{p}_{2\perp}^2} \quad (13)$$

with b taken as before from data, the $J/\psi \to e^+e^-$ width $\Gamma_{\mathrm{ee}} = 5.52$ keV, and $\overline{Q}^2 = (M_\psi^2 + Q^2)/4$. In this case the cross sections tend to be a little larger than the fit (i.e. need $K \sim 1/2$ to fit the data), which is consistent with [6] (see Fig. 3 in particular). From the literature (e.g. [7]), we would expect that extra corrections to this simple approach (allowing for a skewdness $x' \neq 0$, allowing for a real part of the amplitude, NLO corrections) will increase this LO result, although in [5] it is suggested the inclusion of a non-zero gluon $k_\perp$ and $c\bar{c}$ rescattering may decrease the cross section.

In Figure 1, the results of using (12) and (13) for $\sigma(\gamma p \to J/\psi)$ as a function of the $p\gamma$ cms energy W are shown (for this data $\langle Q^2\rangle = 0.05\,\mathrm{GeV}^2$, and so $\overline{Q}^2 \approx M_\psi^2/4$). The CTEQ6L and MSTW08NLO (with a suitable choice of scale and K-factor) pdfs describe the shape of the data well. In all cases the results are consistent with the previous analysis of [6]. Different choices of pdf will also have an effect on the rapidity y_ψ dependence, see Figure 1.2 for some examples of this.

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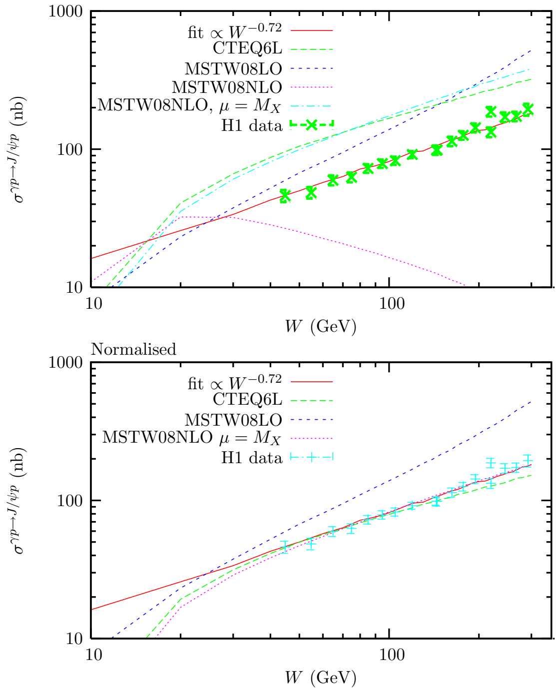

Figure 1: Fit to HERA data, and LO pQCD theory predictions (using different PDFs) for $\sigma(\gamma p \to J/\psi)$ as a function of the $p\gamma$ cms energy W. Also shown is the comparison with the pQCD theory expressions normalised to the data at $W = 80$ GeV

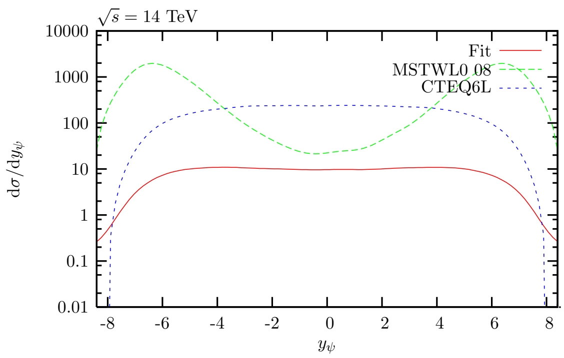

Figure 2: Fit to HERA data, and LO pQCD theory predictions (using different PDFs) for $\mathrm{d}\sigma/\mathrm{d}y_\psi$ at $\sqrt{s} = 14$ TeV. Similar results apply for $\sqrt{s} = 7(8)$ TeV.

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