Section One: Calculator-free (52 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (6 marks)

The points A, B and C have coordinates (4, 6), (10, -2) and (7, 10) respectively.

(a) Find the vector $\overrightarrow{BC}$. (1 mark)

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$= \begin{bmatrix} 7 \\ 10 \end{bmatrix} - \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$
$$= \begin{bmatrix} -3 \\ 12 \end{bmatrix}$$

(b) Find $|\overrightarrow{AB}|$ (2 marks)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= \begin{bmatrix} 10 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$
$$= \begin{bmatrix} 6 \\ -8 \end{bmatrix}$$
$$|\overrightarrow{AB}| = 10$$

(c) The point D divides the line segment CB internally in the ratio 2:3.

Find the position vector of the point D. (3 marks)

$$\overrightarrow{CB} = \begin{bmatrix} 3 \\ -12 \end{bmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OC} + \frac{2}{5}\overrightarrow{CB}$$
$$= \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 3 \\ -12 \end{bmatrix}$$
$$= \begin{bmatrix} 8.2 \\ 5.2 \end{bmatrix}$$

Question 2 **(8 marks)**

A simple type of robot can be programmed to travel in a straight line with constant velocity.

Relative to an origin O, robot A leaves position $-13\mathbf{i}+22\mathbf{j}$ m and travels with velocity $3\mathbf{i}-2\mathbf{j}$ m/s.

One second later, robot B starts from position $5\mathbf{i}+15\mathbf{j}$ m and travels with velocity $-4\mathbf{i}-\mathbf{j}$ m/s.

(a) Calculate the position and velocity of robot A relative to robot B at the instant robot B starts and hence explain why the robots will not collide. (4 marks)

When B starts A is at $\begin{bmatrix} -13+3 \\ 22-2 \end{bmatrix} = \begin{bmatrix} -10 \\ 20 \end{bmatrix}$

$${}_A\mathbf{r}_B = \begin{bmatrix} -10 \\ 20 \end{bmatrix} - \begin{bmatrix} 5 \\ 15 \end{bmatrix} = \begin{bmatrix} -15 \\ 5 \end{bmatrix}$$

$${}_A\mathbf{v}_B = \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} -4 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

No collision because ${}_A\mathbf{v}_B$ is clearly not a multiple of ${}_A\mathbf{r}_B$.

(b) Robot C, travelling with velocity $8\mathbf{i}-7\mathbf{j}$ m/s, leaves its initial position five seconds after A starts and collides with B, three seconds later. Determine the initial position of robot C. (4 marks)

At time of collision B is at $\begin{bmatrix} 5 \\ 15 \end{bmatrix} + 7\begin{bmatrix} -4 \\ -1 \end{bmatrix} = \begin{bmatrix} -23 \\ 8 \end{bmatrix}$

Position of C $\begin{bmatrix} a \\ b \end{bmatrix} + 3\begin{bmatrix} 8 \\ -7 \end{bmatrix} = \begin{bmatrix} -23 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -47 \\ 29 \end{bmatrix}$

Question 3 **(6 marks)**

A true statement is 'if a hexagon is regular then it has six sides of equal length'.

(a) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

If a hexagon does not have six sides of equal length then it is not regular.

True – contrapositive statements are always true.

(b) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

If a hexagon is not regular then it does not have six sides of equal length.

False – sides can be equal so long as at least two of its angles are not the same.

(c) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

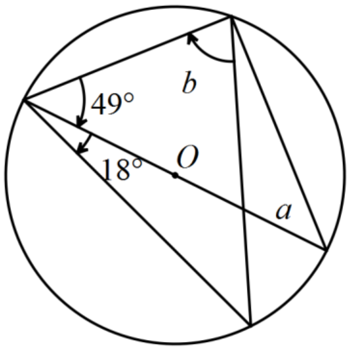
If a hexagon has six sides of equal length then it is regular.

False – the angles must also be equal for a polygon to be regular.

Question 4 **(10 marks)**

(a) Determine the values of the pronumerals a, b and c in the diagrams below.

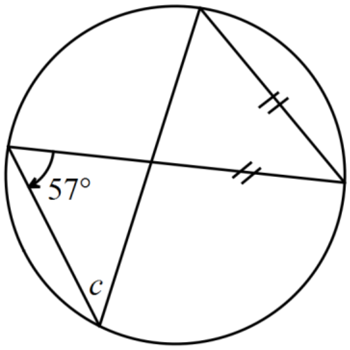
(i) (2 marks)



$$a = 90 - 49$$
$$= 41°$$

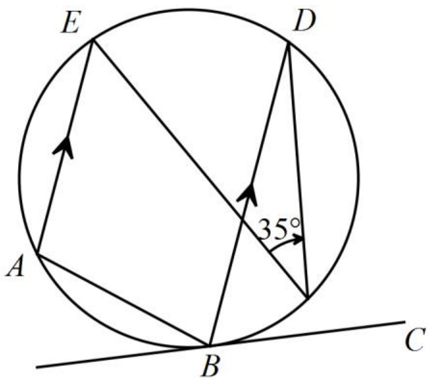
$$b = 90 - 18$$
$$= 72°$$

(ii) (2 marks)



$$c = 180 - 57 - 57$$
$$= 66°$$

(b) Determine the size of angle ABC. (2 marks)



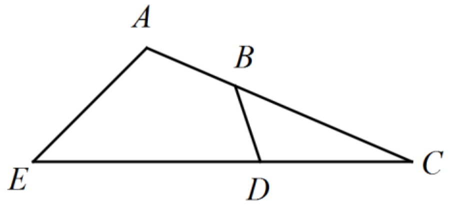
$\angle BEA = \angle EBD$ (alternate)

$= 35$ (angles on same chord)

$180 - \angle ABC = 35$ (alt seg theorem)

$\angle ABC = 145°$

(b) In the figure below, $AB = 2,\ BC = 4,\ CD = 3$ and $DE = 5$ cm. Prove that $ABDE$ is a cyclic quadrilateral. (4 marks)



$AC = 2 + 4 = 6$

$EC = 3 + 5 = 8$

$$\begin{aligned} AC \times BC &= 6 \times 4 \\ &= 24 \\ &= 8 \times 3 \\ &= EC \times CD \end{aligned}$$

Hence, by converse of intersecting chord theorem $ABDE$ is a cyclic quadrilateral.

Question 5 **(7 marks)**

$A(2, 3)$, $B(1, -2)$ and $C(-3, 1)$ are the vertices of a triangle.

(a) State the vector $\overrightarrow{AC}$. (1 mark)

$$\overrightarrow{AC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

(b) Determine the exact value of $|\overrightarrow{BC}|$. (2 marks)

$$\overrightarrow{BC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{4^2 + 3^2} \\ &= 5 \end{aligned}$$

(c) Determine all vectors of magnitude 10 that are

(i) parallel to $\overrightarrow{BC}$. (2 marks)

$$\pm 2\begin{bmatrix} -4 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -8 \\ 6 \end{bmatrix} \text{ and } \begin{bmatrix} 8 \\ -6 \end{bmatrix}$$

(ii) perpendicular to $\overrightarrow{AC}$. (2 marks)

$$\left\| \begin{bmatrix} -5 \\ -2 \end{bmatrix} \right\| = \sqrt{29}$$

$$\pm \frac{10}{\sqrt{29}} \begin{bmatrix} 2 \\ -5 \end{bmatrix} \Rightarrow \frac{10}{\sqrt{29}} \begin{bmatrix} 2 \\ -5 \end{bmatrix} \text{ and } \frac{10}{\sqrt{29}} \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

Question 6 **(7 marks)**

(a) Use the method of contradiction to prove that a triangle with sides of 5 cm, 5 cm and 7 cm is not right angled. (4 marks)

Assume that the triangle is right angled, so that Pythagoras' Theorem can be applied and we can deduce that:

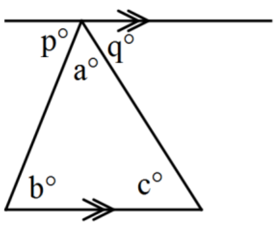
$$5^2+5^2=7^2$$

But

$$\begin{aligned}5^2+5^2&=25+25\\&=50\\&\neq 7^2\end{aligned}$$

This result contradicts our original assumption and so the triangle cannot be right angled.

(b) Use the fact the angles on a straight line are supplementary to prove that the angle sum of a triangle is 180º. (3 marks)



The diagram shows a line drawn through the vertex of a triangle parallel to the base.

$\angle p+\angle a+\angle q=180$ (given)

$\angle b=\angle p$ (alternate angles)

$\angle c=\angle q$ (alternate angles)

Hence $\angle b+\angle a+\angle c=180°$ as required.

Question 7 **(8 marks)**

Two vectors are $\mathbf{c} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{d} = -12\mathbf{i} + 5\mathbf{j}$.

(a) Find

(i) $5\mathbf{c} + \mathbf{d}$ (1 mark)

$$\boxed{5\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -15 \end{bmatrix}}$$

(ii) $|\mathbf{d}|$ (1 mark)

$$\boxed{13}$$

(iii) $-|\mathbf{c}|\mathbf{d}$ (2 marks)

$$\boxed{-5\begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 60 \\ -25 \end{bmatrix}}$$

(b) Find $\mathbf{e}$ and $\mathbf{f}$ if $2\mathbf{e} + \mathbf{f} = 2\mathbf{c}$ and $\mathbf{e} - \mathbf{f} = \mathbf{d}$. (4 marks)

$$2\mathbf{e} + \mathbf{f} + \mathbf{e} - \mathbf{f} = 2\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$3\mathbf{e} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \mathbf{f} = \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} 10 \\ -6 \end{bmatrix}$$

End of questions