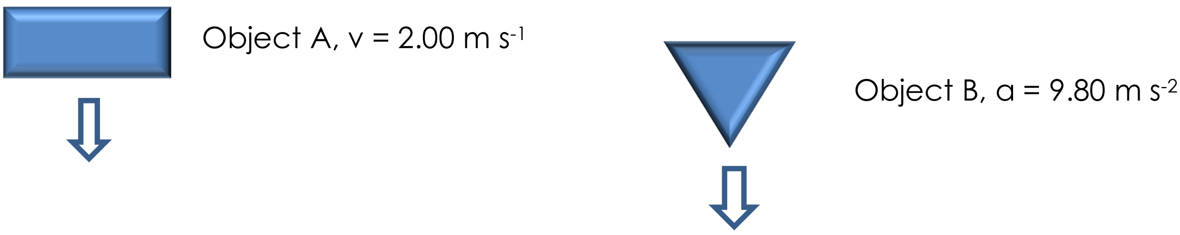
Unit 3: Gravity and Motion Solutions

Gravitation and Satellite Motion

Question 1

The diagram shows object A falling at a constant speed of 2.00 m s^{-1} towards the ground and object B accelerating at 9.80 m s^{-2} towards the ground. What is the acceleration of object B relative to object A? Explain briefly.

(3)



In the frame of reference of the ground the acceleration of A = zero and B = 9.80 m s^{-2} down. ✓

In terms of acceleration the reference frame of the ground and object A is the same. They have no acceleration in each other's reference frame. ✓

Therefore the acceleration of object B relative to A is 9.80 m s^{-2} down. ✓ 9.80 m s^{-2} = 1 explanation = 2

Question 2 (16 marks)

Kepler-186f is a planet in orbit around the red dwarf star Kepler-186. A full public announcement about the planet was made by NASA on 17 April 2014. It is the first discovery of a planet with a similar radius to that of Earth in the habitable zone of another star.

Kepler-186f is a distance of 151 ± 18 parsecs from Earth (1 parsec = 3.26 Light Years). It has an orbital radius of 0.391 AU from its host star (The Astronomical Unit (AU) = Sun-Earth distance). It has an orbital period of 129.9 days.

a) Calculate the speed of Kepler-186f around its host star

(3)

Orbital radius = $0.391 \times 1.50 \times 10^{11} = 5.87 \times 10^{10}$

Orbital period = $129.9 \times 24 \times 60 \times 60 = 11\ 223\ 360$ s ✓

$v = 2\pi r / T = (2 \times \pi \times 5.87 \times 10^{10}) / 11\ 223\ 360$ ✓

$v = 3.28 \times 10^{4}$ m s^{-1} ✓

b) Calculate the mass of the host star Kepler-186 based on the information given.

(4)

Orbital radius = $0.391 \times 1.50 \times 10^{11} = 5.87 \times 10^{10}$

Orbital period = $129.9 \times 24 \times 60 \times 60 = 11\ 223\ 360$ s

$v = 3.28 \times 10^{4}$ m s^{-1} ✓

$a_{centripetal} = v^2 / r$ = gravitational field strength = GM / r^2

$M = (v^2 \times r) / G$ ✓

$M = ((3.28 \times 10^{4})^2 \times 5.87 \times 10^{10}) / (6.67 \times 10^{-11})$ ✓

$M = 9.48 \times 10^{29}$ kg ✓

Alternatively

derive $r^3 = (G.M.T^2)/ 4\pi^2$

$M = (r^3 \times 4\pi^2) / (6.67 \times 10^{-11} \times T^2)$

c) The mass of the planet Kepler-186f is difficult to estimate and is thought to be in a range of 32% to 377% the mass of the Earth. Explain whether or not this high degree of uncertainty affects estimates for the mass of the host star.

(2)

The equations used to calculate the mass of the host star do not use the mass of the planet e.g. $M = (r^3 \times 4\pi^2) / (6.67 \times 10^{-11} \times T^2)$

or $M = (v^2 \times r) / G$ ✓

So this variation in mass for the planet has no effect on estimating the mass of the star ✓

d) The radius of the planet Kepler-186f is 1.11 ± 0.14 times that of the Earth. Use this information and the uncertainty range for the mass of Kepler-186f to calculate the possible range for the gravitational field strength on the surface of the planet compared to "g" on Earth.

(3)

Highest value of field strength when planet radius is minimum and mass is maximum. Lowest value when radius maximum and mass minimum (concept) ✓

$g = GM/r^2$

r (min) = 1.11 – 0.14 = 0.97R   m (max) = 3.77M

r (max) = 1.11 + 0.14 = 1.25R   m (min) = 0.32M

$g_{maximum} = G \times 3.77 \times M / (0.97 \times R)^2 = 4.01g$ ✓

($g_{maximum}$ = 39.2 N kg^{-1} if calculated)

$g_{minimum} = G \times 0.32 \times M / (1.25 \times R)^2 = 0.205g$ ✓

($g_{minimum}$ = 2.00 N kg^{-1} if calculated)

e) Calculate the percentage uncertainty (relative uncertainty) for the distance from Earth to Kepler-186f.

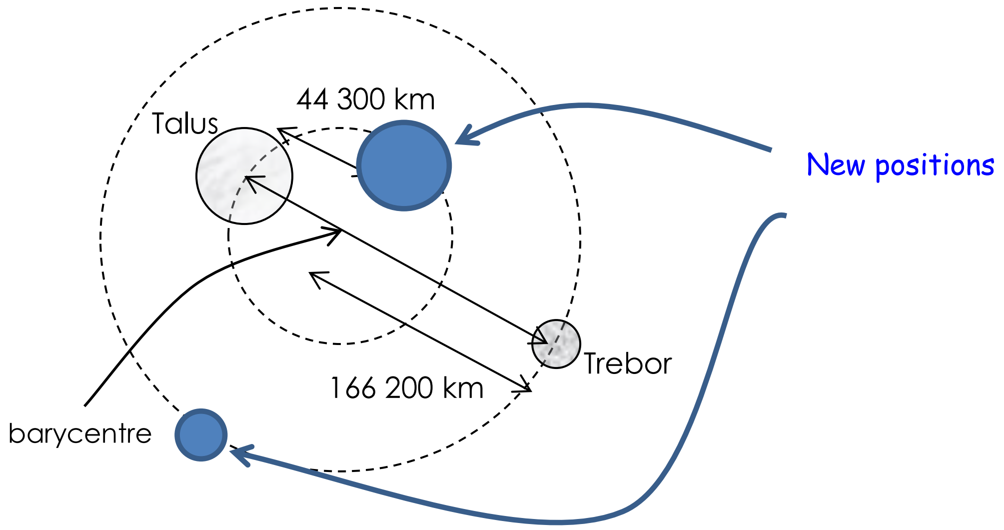
(1)

151±18 parsecs

Relative uncertainty = 18/151 = 0.119 = 11.9% ✓   (151 ±11.9%)

Question 3 (13 marks)

A **binary** planet system consists of two planets orbiting around their common centre of mass. This location is known as the *barycentre*. A binary planet system is shown below. Planet Talus has a mass of 2.04 x 10^{25} kg, Planet Trebor has a mass of 5.44 x 10^{24} Kg. The total separation between the 2 planets is **always** 210 500 km and the *barycentre* **always** lies on a straight line between Talus and Trebor, The distance between each planet and the barycentre is detailed in the diagram below (not to scale).



a) Calculate the gravitational force of attraction between Talus and Trebor.

(3)

r = 44 300 000 + 166 200 000 = 210 500 000 m

M_1 = 2.04 x 10^{25} kg M_2 = 5.44 x 10^{24} Kg

$$F = \frac{GM_1M_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 2.04 \times 10^{25} \times 5.44 \times 10^{24}}{(210500000)^2}$$

$$F = 1.67 \times 10^{23} N$$

b) Calculate the speed of Talus around the *barycentre*.

(3)

$$F = \frac{mv^2}{r} \qquad v^2 = \frac{rF}{M}$$

$$v^2 = \frac{44\,300\,000 \times 1.67 \times 10^{23}}{2.04 \times 10^{25}}$$

v = 602.2989 = 6.02 x 10^2 m/s✓

c) Calculate how many Earth hours it takes for Talus to orbit the *barycentre*.

(2)

T = 2πr/v = 2×π×44 300 000 / 602.2989 ✓

T = 462 137.77 s = 462 137.77/(60×60) = 128 hours ✓

d) Estimate the position of the planets after 32 hours of time from the initial position shown. Sketch them on the diagram and label them. Talus orbits in a clockwise direction about the *barycentre*. (If you could not determine the previous answer use 128 hours)

(2)

32 / 128 = $\frac{1}{4}$ of a revolution. ✓

Show on diagram ✓

e) Show by algebraic proof that the following relationship must be true for any binary planet system that rotates around a *barycentre* in the pattern described in this question.

$$m_1 = \frac{m_2 \times r_2}{r_1}$$

m_1 = mass of planet 1 (kg) m_2 = mass of planet 2 (kg)

r_1 = distance of planet 1 to *barycentre* (m)

Common centripetal force F

$$F = \frac{m_1 {v_1}^2}{r_1} = \frac{m_2 {v_2}^2}{r_2}$$ substitute $v = \frac{2\pi r}{T}$ ✓

$$\frac{m_1 4\pi^2 r_1}{T^2} = \frac{m_2 4\pi^2 r_2}{T^2}$$ Common period T✓

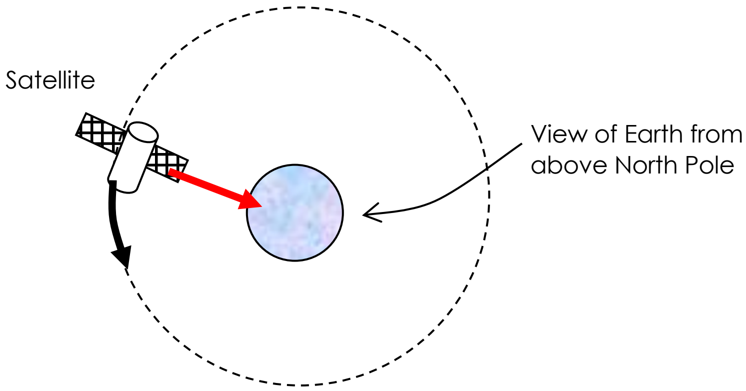
Divide out common terms

$$m_1 = \frac{m_2 r_2}{r_1}$$ ✓

(3)

Question 4 (13 marks)

A satellite is in orbit around the equator of the Earth. It has a mass of 1495 kg and is at an altitude of 1.91×10^4 km above the Earth's surface.



a) Calculate the **period** of this satellite and state your answer in hours.

(4)

$M = 5.98 \times 10^{24}$ kg

$r = \text{altitude} + r_{earth} = 6.37 \times 10^6 + 1.91 \times 10^7 = 2.547 \times 10^7$ m ✓

$v^2 / r = GM / r^2$ (substituting $v = 2\pi r / T$)

$r^3 = (G.M.T^2) / (4.\pi^2)$

$T^2 = (r^3.4.\pi^2)/ (G.M)$

$T^2 = ((2.547 \times 10^7)^3.4.\pi^2)/ (6.67 \times 10^{-11} \times 5.98 \times 10^{24})$ ✓

$T = 4.044 \times 10^4$ s ✓

$T = 4.044 \times 10^4 / (60 \times 60) = 11.2$ hours ✓

b) Explain whether or not a satellite can be geostationary at this altitude.

(2)

No

The equation $T^2 = (r^3.4.\pi^2)/(G.M)$ shows that the period of a satellite is fixed at a given radius of separation. The radius must increase to give a period of 24 hours. ✓ ✓

c) Place **labelled** arrow(s) on the diagram to show the direction of the **net acceleration** of the satellite.

(1)

Towards centre of circle. ✓

d) Give two examples of the uses of artificial satellites in everyday life.

(2)

Communications satellites to transmit telephone signals around the globe. ✓

GPS system for navigation.✓

Any 2 reasonable answers

The Earth is a natural satellite that orbits the Sun. (Assume a circular orbit for this question)

e) Calculate the orbital speed of the Earth as it goes around the Sun.

$M = 1.99 \times 10^{30}$ kg $r = 1.50 \times 10^{11}$ m ✓

$v^2 / r = GM / r^2$

$v^2 = GM / r$

$v^2 = 6.67 \times 10^{-11} \times 1.99 \times 10^{30} / 1.50 \times 10^{11}$ ✓

$v = 2.97 \times 10^4$ m s^{-1} ✓

Alternatively $v = 2\pi r/T$ $(T = 365.25 \times 24 \times 60 \times 60)$

(3)

f) If the Sun was 90% of its current mass, describe how the orbital speed of the Earth would be affected if it remained at the same distance from Sun. *(A calculation is not required)*

Referring to previous equation $v^2 = GM / r$

If mass of the sun M decreases then v also decreases. ✓

Question 5 (13 marks)

The orbit of Venus lies between the Earth's orbit and the Sun. The radius of Venus is 6.05×10^6 m. The Magellan spacecraft was launched by NASA in 1995 for the purpose of radar mapping Venus. At one stage Magellan was put into a circular orbit of Venus at an altitude of 346 km. It took Magellan 94 minutes to complete this orbit. Magellan had a mass of 1035 kg.

a) Calculate the centripetal acceleration of the Magellan satellite in this orbit. (3)

Orbital radius = $6.05 \times 10^6 + 346 \times 10^3 = 6.396 \times 10^6$ m

Orbital period = $94 \times 60 = 5640$ s

$v = 2\pi r / T = (2\pi \times 6.396 \times 10^6) / 5640 = 7125.4$ m/s ✓

$a_{centripetal} = v^2 / r = 7125.4^2 / 6.396 \times 10^6$ ✓

$a_{centripetal} = 7.93798 = 7.94$ m s^{-2} towards Venus ✓

(Alternatively $a = 4\pi^2 r / T^2$)

b) Calculate the mass of the planet Venus using the satellite data provided. (3)

$a_{centripetal} = 7.93798$ m s^{-2}

Orbital radius = $6.05 \times 10^6 + 346 \times 10^3 = 6.396 \times 10^6$ m

Orbital period = $94 \times 60 = 5640$ s

$a_{centripetal} = v^2 / r$ = gravitational field strength = GM / r^2

$7.93798 = G M / (6.396 \times 10^6)^2$ ✓

$M = (7.93798 \times (6.396 \times 10^6)^2)/(6.67 \times 10^{-11})$ ✓

$M = 4.87 \times 10^{24}$ kg ✓

Alternatively

derive $r^3 = (G.M.T^2)/ 4\pi^2$

$M = (r^3 \times 4\pi^2) / (6.67 \times 10^{-11} \times T^2)$

c) If the Magellan spacecraft was double the mass in this orbit explain how its orbital period would be affected.

(2)

$a_{centripetal}$ = v^2 / r = gravitational field strength = GM / r^2

It can be shown that $v_{satellite}$ = √(G M / r) where M is the mass of the host planet. ✓

Therefore, the mass of the satellite has no effect on the period. ✓

d) There is a location between the Earth and the Sun where the net gravitational field strength due to the Earth and the Sun is zero. Calculate the distance from Earth to this location.

(5)

Earth Sun distance = 1.50 × 10^{11} m

Let distance from Earth to this location = x

Then distance from Sun to this location = (1.50 × 10^{11} – x) ✓

Magnitude of gravitational field strength is equal at this location

$GM_{sun} / r_{sun}{}^2 = GM_{Earth} / r_{Earth}{}^2$ ✓ (concept)

$$\frac{M_{sun}}{M_{Earth}} = \frac{(1.50 \times 10^{11} - x)^2}{x^2}$$

$$\sqrt{\frac{1.99 \times 10^{30}}{5.97 \times 10^{24}}} = \frac{1.50 \times 10^{11} - x}{x}$$

✓

$$577.350 = \frac{1.50 \times 10^{11} - x}{x}$$

$$578.350x = 1.50 \times 10^{11}$$

✓

x = 2.60 × 10^8 m from Earth ✓

Question 6

A satellite provides information about the receding glaciers on the Earth's surface. It has a mass of 395 kg and is in a circular orbit of radius 1.45 x 10^4 km. By orbiting for 12 days it can map most of the Earth's glaciers.

(a) Calculate the orbital speed of the satellite. (3 marks)

$r = 1.45 \times 10^7$ m

$m_s = 395$ kg

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$M_e = 5.98 \times 10^{24}$

$v^2 = GM_e/r$

$v^2 = (6.67 \times 10^{-11}) \times (5.98 \times 10^{24})/(1.45 \times 10^7)$

$v^2 = 27.508 \times 10^6$

$v = 5.20 \times 10^3$ m s^{-1}

b) At what **altitude** above the Earth is the satellite orbiting? (3 marks)

$r_e = 6.37 \times 10^6$ m

$r_s = 1.45 \times 10^7$ m

altitude = $r_s - r_e$

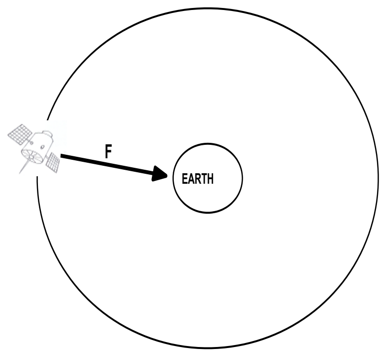
altitude = $(1.45 \times 10^7) - (6.37 \times 10^6)$

the altitude of the satellite = 8.13×10^6 m

c) List the force(s) that keep the satellite in its stable circular orbit. (2 marks)

The force that keeps the satellite in orbit is the centripetal force that acts towards the centre of the orbit. This force is provided by the gravitational attraction between the Earth and satellite.

d) On the diagram below draw one or more **labelled** arrows to show the direction of the force(s) on the satellite as it orbits the Earth. (2 marks)



e) Would you expect this satellite to be in a geostationary orbit about the Earth? Explain your answer. (2 marks)

The satellite would not be in a geostationary orbit. If it was it would remain in orbit above the same location on the Earth and would not be able to map most of the glaciers on Earth. For the mapping to be complete the satellite would be in an orbit other than geostationary.

(T required = 24 hrs but calculated to be otherwise

Question 7. (10 marks)
In your lifetime it is expected that there will be a manned mission to the planet Mars. One strategy is to launch the rocket from a base on the moon and that rocket will have a capsule that will be ejected and land on the surface of Mars. This is similar to the lunar missions 40 years ago. The capsule launches itself from the surface of Mars and reunites with the rocket. That is the plan.

(a) Why would they even consider a launch from the moon rather than Cape Canaveral in the USA? (1)

(b) Calculate the radius of Mars given the gravitational field strength on Mars is 3.73 m s-2 and its mass is 6.42 x 1023 kg. (3)

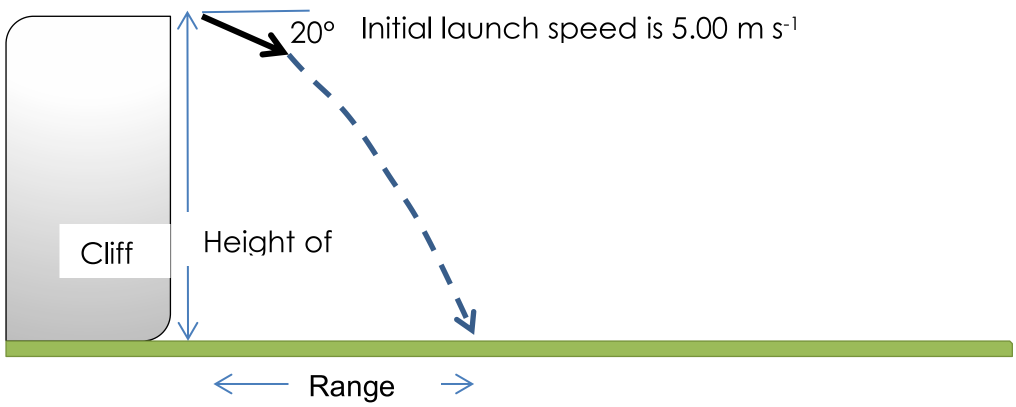
c) If the orbiting space craft will be at an altitude of 150km, what will be the time for one orbit around Mars? If you were unable to do part (b) assume a value of 4×10^6 as the radius of mars (4)

d) How many orbits will occur in one “earth day” of 24 hours (1)

PROJECTILE MOTION

Question 1 (13 marks)

A physics student observes a stone of mass 380 g being catapulted from the top of a cliff. The stone takes a time of 4.00 s to reach the ground. The stone is launched at 5.00 ms^{-1} at an angle of 20.0° below the horizontal. You may ignore air resistance for the calculations.



a. Calculate the height of the cliff.

(3)

Let up be positive (or alternative defined reference frame)

u (vertical) = u.sin θ (down) = -(5 x sin 20) = -1.71 m s^{-1}

t (flight) = 4.00 s   a = -9.80 m s^{-1} ✓

$s = ut + \frac{1}{2} at^2$

s = (-1.71 x 4) – (4.9 x 4^2) ✓

s = -85.24 m

Height of cliff = 85.2 m ✓

b. Calculate the horizontal range of the stone.

(3)

u (horizontal) = u.cosθ (right) = -(5 x cos 20) = 4.6984631 m s^{-1} right

t (flight) = 4.00 s   a = 0 ✓

s = u (horizontal) x t

s = (4.6984631 x 4) ✓

s = 18.79385 m

s (horizontal) = 18.8 m right ✓

c. Calculate the kinetic energy of the stone after 3 seconds.

(4)

u (horizontal) = 4.6984631 m s^{-1} right   t = 3.00 s

v (vertical) = u + at

v = -1.71 + (-9.80 x 3) = -31.11 m s^{-1} down ✓

speed after 3 seconds = $\sqrt{-31.11^2 + 4.6984631^2} = 31.4629\ m\ s^{-1}$ ✓

KE = ½ m v^2 = ½ x 0.38 x 31.4629^2 ✓= 188 J ✓

d. The final velocity of the stone is achieved as the stone reaches ground level. If the stone had been catapulted at the same launch speed but at an angle of 20° above the horizontal how would the magnitude of final velocity compare to a launch 20° below the horizontal. Circle a response and explain briefly.

(3)

final velocity greater   final velocity the same   final velocity less

u (horizontal) = is constant and is the same value whether launched at an angle above or below the horizontal.

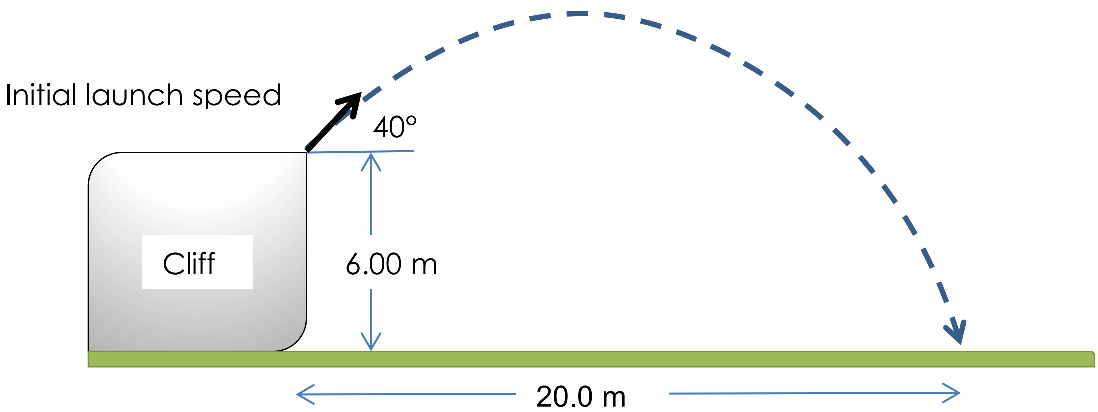
If launching above horizontal then u (vertical) is positive, below the horizontal u (vertical) is the same magnitude but negative ✓

When calculating the vertical component of velocity at a given displacement we use the equation $v^2 = u^2 + 2as$

u^2 is the same value whether u is positive or negative, this leads to the same result for the final velocity (vertical) and the final velocity in 2 dimensions. ✓   (or similar)

Question 2 (13 marks)

A physics student observes a stone of mass 450 g being catapulted from the top of a cliff. The launch position at the top of the cliff is 6.00 m above ground level. The stone lands 20.0 m in front of the launch position. The initial launch speed u is at an angle of 40.0° to the horizontal. You may ignore air resistance for the calculations.



a. Calculate the initial launch speed **u** of the stone. You must show clear algebraic steps in your solution.

Hint: consider the flight time for both the horizontal and vertical components of motion. (5)

Consider the total flight time t_f

$s_y = -6.00$ $u_y = u.\sin 40$ $a_y = -9.80\ m/s^2$ ✓

$u_x = s_x / t_f$ $u.\cos 40 = 20 / t_f$ $\therefore t_f = 20/u.\cos 40$ ✓

$s_y = u_y t_f + \frac{1}{2} a_y t_f^2$

$-6 = 20(u.\sin 40/u.\cos 40)-((4.9\times 20^2)/(u^2\cos^2 40)$ ✓

$-6 = 20(\tan 40)-(3340.02/u^2)$

$-6 = 16.78199 - 3340.02/u^2$

$-22.7819 = -3340.02/u^2$

$u^2 = 3340.02 / 22.7819$ ✓

$u^2 = 146.6$

$u = 12.1\ m/s$ ✓

b. Calculate the flight time of the stone. (If you were not able to solve part a), use a numerical value of 12.1 m s^{-1} for the initial launch speed **u**).

(3)

$u = 12.1$ m/s $s_x = 20.0$ m $\theta = 40°$ $u_x = u.\cos 40$ ✓

$u_x = s_x / t_f$

$t_f = 20/(12.1 \times \cos 40)$ ✓

$t_f = 2.157698$ $t_f = 2.16$ s ✓

c. Calculate the minimum value of kinetic energy of the stone whilst in flight. (If you were not able to solve part a), use a numerical value of 12.1 m s^{-1} for the initial launch speed **u**).

(2)

This occurs at the top of the flight when the there is no vertical component of velocity.

m = 0.450 kg

speed = u.cos 40 = 9.269 m/s

$KE = \frac{1}{2} m v^2 = \frac{1}{2} 0.45 \times 9.269^2$ ✓ = 19.3 J ✓

d. Calculate the maximum height above ground level that the stone reaches on its flight path. (If you were not able to solve part a), use a numerical value of 12.1 m s^{-1} for the initial launch speed **u**).

(3)

when v = 0 $u = 12.1 \times \sin 40° = 7.77773$ m/s up

$a = -9.80$ m/s^2 s = ?

$v^2 = u^2 + 2as$

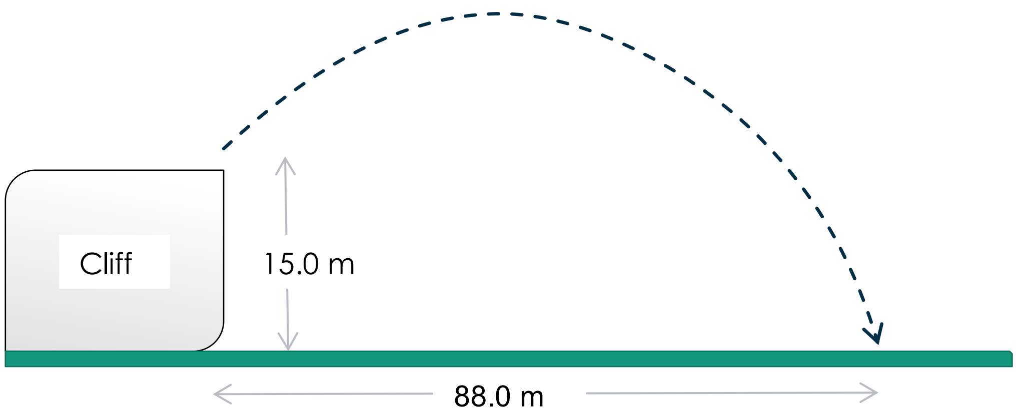
$0 = 7.77773^2 - (19.6 \times s)$ ✓

$s = 7.77773^2/19.6 = 3.086$ m above launch ✓

above ground = 3.086 + 6.00 = 9.09 m ✓

Question 3 (12 marks)

A physics student observes a stone of mass 350 g being catapulted from the top of a cliff. The launch position at the top of the cliff is 15.0 m above ground level and it takes the stone a time of 5.00 seconds to reach the ground. The stone lands 88.0 m in front of the launch position. You may ignore air resistance for the calculations.



a) Calculate the vertical component of the velocity when the stone is launched. (3)

In the vertical let up = positive

$s = -15.0$ m $\quad t = 5.00$ s $\quad u = ?$ ✓

$s = ut + \frac{1}{2}at^2$

$-15 = (u \times 5) + (-4.9).25$ ✓

$u = +21.5 \text{ m s}^{-1}$ (positive = up) ✓

b) Considering the kinetic energy of the stone along its flight path. Circle the best response for the following statement. The kinetic energy of the stone at maximum height is:

Maximum 50% of maximum Zero Minimum Equal to all other positions

(1)

c) Calculate the initial velocity of the stone.

(4)

In the horizontal

s_{range} = 88.0 m t_{flight} = 5.00 s ✓

$v_h = s_{range} / t_{flight}$

v_h = 88.0 / 5 = 17.6 m s^{-1} right ✓

Considering a vector diagram

$u = \sqrt{(21.5^2 + 17.6^2)} = \sqrt{(772.01)} = 27.8$ m s^{-1} ✓

Elevation = $\tan^{-1}$ (21.5 / 17.6) = 50.7° ✓

d) Calculate the kinetic energy of the 350 g stone just before it hits the ground.

(4)

Considering the conservation of mechanical energy

Total Mechanical Energy = constant at any height.

At this position TME = KE + GPE = $\frac{1}{2}mv^2 + mgh$

identifies variables✓

TME = ($\frac{1}{2}$ × 0.35 × 772.01) + (0.35 × 9.8 × 15) ✓

TME = 186.55 J ✓

At end of flight GPE = zero ∴ TME is all kinetic

Kinetic energy just before stone hits ground = 187 J ✓

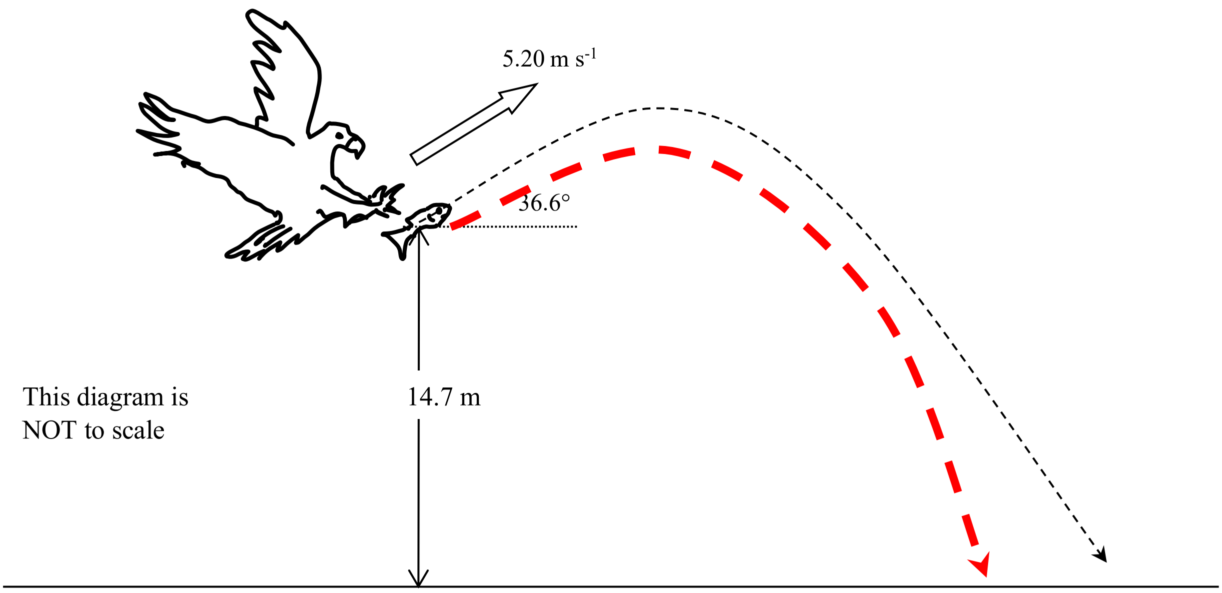
Alternatively in vertical v = u + at = 21.5 + (-9.8*5) = -27.5 ✓ ✓

in horizontal v = 17.6 final $v^2 = 27.5^2 + 17.6^2 = 1066.01$ ✓

KE = $\frac{1}{2}$ m v^2 = 0.5*0.35*1066.01 = 187 J ✓

Question 4 (13 marks)

An eagle has captured a fish and is 14.7 m directly above the water when it releases the fish. The eagle is moving with a velocity of 5.20 m s^{-1} at an angle of 36.6° above the horizontal when the fish is released. Ignore air resistance for calculations.



a) Calculate the time taken for the fish to reach the water.

(4)

Positive is taken as upwards for vertical motion in this solution

$u_v = 5.20 \times \sin 36.6° = +3.10$ m/s

$s_v = -14.7$ m ✓

$a = -9.8$ m/s^2

Solve for final vertical velocity at displacement of -14.7 m

$v^2 = u^2 + 2.a.s$

$v^2 = +3.10^{\,2} + (2 \times -9.8 \times -14.7)$ ✓

v = -17.2549 m/s (take negative root as fish is travelling down) ✓

v = u + at

t = (v – u) / a

t = (-17.2549 – 3.10) / -9.8

t = 2.077 = <u>2.08 seconds</u> (3 sig figs) ✓

Or general solution of a quadratic

b) Calculate the horizontal distance that the fish travels during its flight back to the water.

(3)

$u_h = 5.20 \times \cos 36.6° = 4.17465$ m/s right ✓

$s_h = u_h \times t_f$

$s_h = 4.17465 \times 2.077$ ✓

<u>$s_h = 8.67$ m right</u> ✓

c) Calculate the maximum height above the water that the fish reaches during its flight.

(3)

v_v at max height = 0 $a = -9.8$ m/s^2 $u_v = +3.10$ m/s

$v^2 = u^2 + 2.a.s$

$s_v = (v^2 - u^2) / 2a$

$s_v = (0 - 3.1^2) / -19.6$ ✓

$s_v = 0.490$ m above launch ✓

s above water = 0.490 + 14.7 = 15.19 m = 15.2 m (3 sig figs) ✓

If air resistance is taken into account then the flight path is altered.

d) Sketch the altered flight path onto the diagram.

(1)

Shows reduced max height, range and steeper descent ✓

e) Will the time of flight be longer or shorter when air resistance is taken into account? Discuss the factors that affect this.

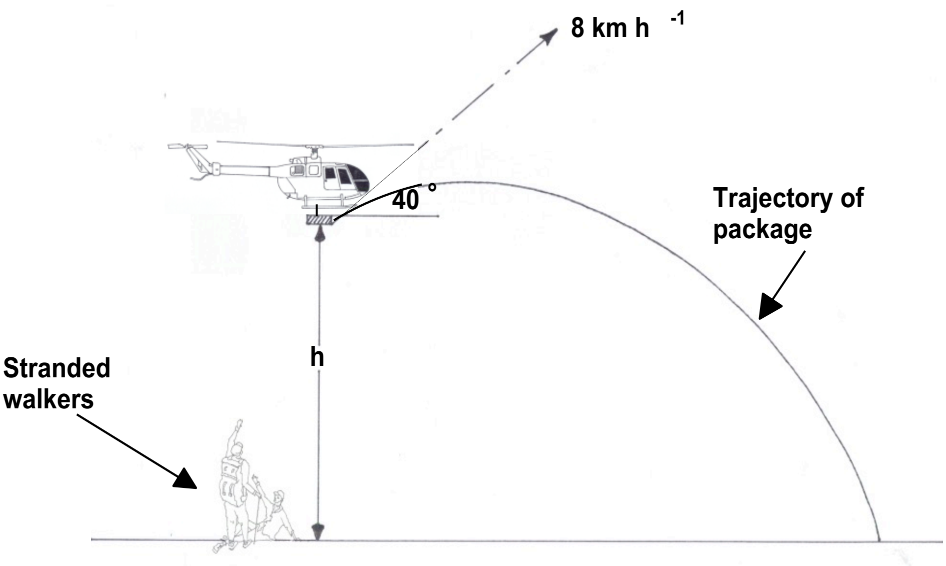
(2)

Air resistance acting against the projectile will slow its vertical ascent and descent (e.g. like a parachute).

The maximum height (0.49 m above launch position) is reduced but the time to fall the smaller total distance is increased by air resistance. ✓ So flight time will probably increase. ✓ Any well structured discussion

Question 5 (12 marks)

A helicopter is required to drop emergency equipment to a group of walkers stranded in rugged bushland. A package is released from the helicopter at altitude (h) directly above the group. The helicopter is moving with a velocity of 8 km h^{-1} at an angle of 40° above the horizontal when the package is released. The package lands on the ground 2.5 s after being released.



(a) Calculate the value of h. (4 marks)

To calculate where the package is with respect to the helicopter:

Because the horizontal speed of the helicopter and the package is the same, and the time of flight is the same, the package will land directly below the helicopter. That is 27.1 m plus the vertical distance the helicopter has travelled during the 2.5 s. That distance is 3.535 m

So the package lands (27.1 + 3.535) = 30.6 m directly below the helicopter.

(b) If the helicopter continues to fly with its initial velocity, calculate the distance between the helicopter and the package at the instant the package hits the ground. (4 marks)

To calculate vertical speeds of the package so the graph can be constructed.

Time for the package to reach its highest point ($v = 0\ m\ s^{-1}$)

$u = -1.414\ m\ s^{-1}$

$a = 9.8\ m\ s^{-2}$

$v = 0\ m\ s^{-1}$

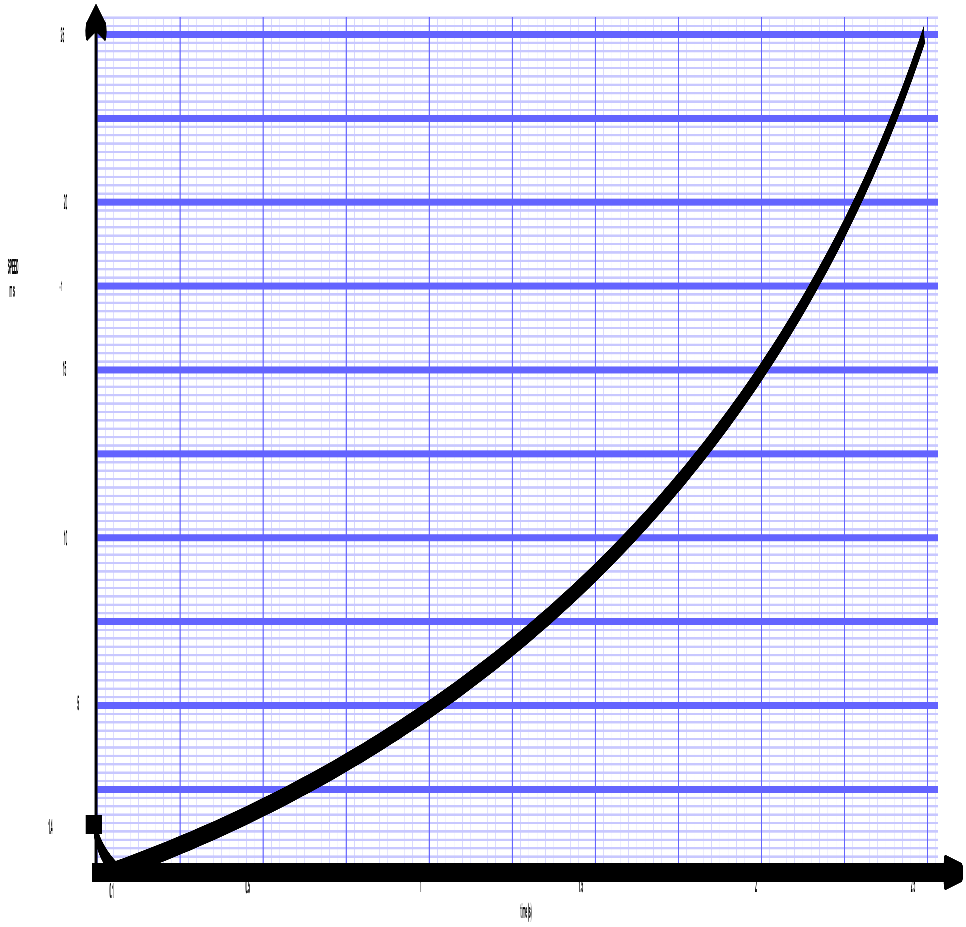
$t = ?$

$v = u + at$

$0 = -1.414 + 9.8 \times t$

$t = 1.414 / 9.8$

(c) On the axes below draw a graph that best represents the vertical speed of the package as a function of time. Include actual values on the axes. Show calculations that determine significant points on the graph. (2 marks)



Time (s)	Speed (m s^{-1})
0	1.4 upwards
0.1	0
2.5	23 down

(d)If the helicopter was travelling **horizontally** at the same speed (8 km h^{-1}) and height (h) when it released the package, would you expect the package to land closer or further away from the group? Explain your answer. (2 marks)

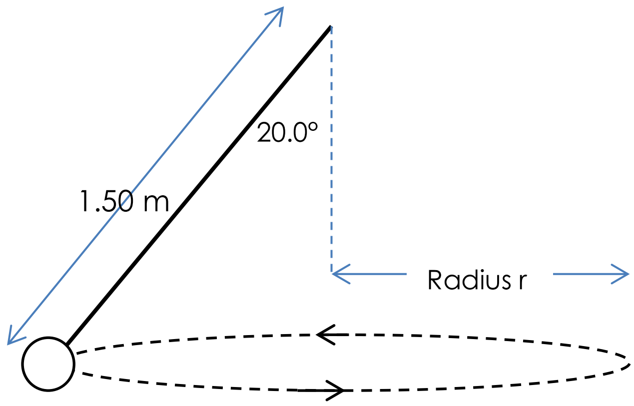
The package would land further away from the group. The horizontal component of the helicopter velocity is now greater so the package will travel further horizontally than before in the 2.5 seconds it is in flight

CIRCULAR MOTION

Question 1

A ball of mass **m**, suspended from a ceiling moves along a horizontal circle of radius **r** at a constant speed **v**. The string connecting the mass to the ceiling makes an angle of 20.0° to the vertical. The string has a length of 1.50 metres.

Calculate the time taken for the ball to make one revolution.



(5)

From vector diagram of forces: ✓ $\Sigma F = mv^2 / r$ $W = m.g$

By substitution of $v = 2\pi r / T$ $\Sigma F = m.4.\pi^2.r / T^2$

$\tan 20° = O / A = (m.4.\pi^2.r / T^2) \times (1 / m.g) = (4.\pi^2.r) / (T^2.g)$ ✓

From triangle of displacements:

$\sin 20° = O/H = r / 1.5$ so $r = 1.50 \times \sin 20°$ ✓

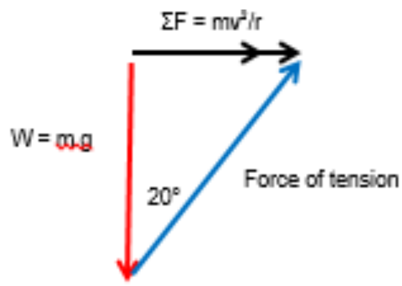
$\tan 20 = (4.\pi^2.r) / (T^2.g) = (4.\pi^2 \times 1.50 \times \sin 20°) / (T^2 \times 9.80)$

$T^2 = (4.\pi^2 \times 1.50 \times \sin 20°) / (\tan 20° \times 9.80)$ ✓

$T^2 = 5.6782$

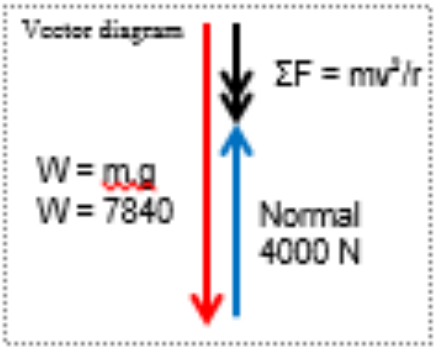
$T = 2.38$ seconds ✓

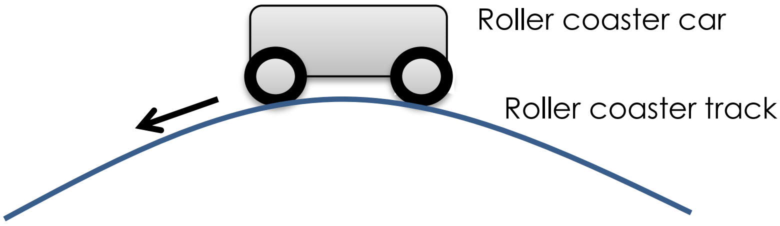
(or similar)



Question 2

A roller coaster car of mass 800 kg is going over the apex of a circular section of track. The car has a speed of 6.00 m s^{-1}. Calculate the radius of the curve for the car to experience a normal reaction force of 4000 N from the track. You must refer to a vector diagram in your answer.





(4)

Vector diagram ✓

W = 800 x 9.8 = 7840 N down

From vector diagram ΣF = 7840 – 4000 = 3840 N to centre ✓

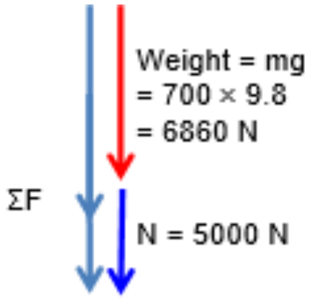
$\Sigma F = mv^2 / r = 3840$ N

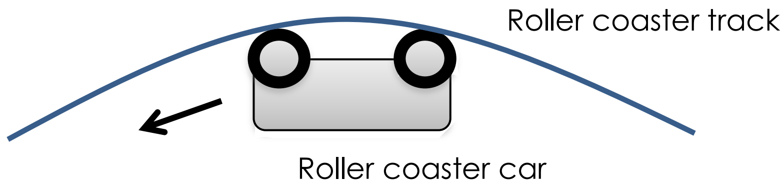
$r = mv^2 / 3840$ $r = 800 \times 6^2 / 3840$ ✓ = 7.50 m ✓

(Or similar alternative derivations)

Question 3

A roller coaster car of mass 700 kg is upside down whilst doing a 'loop the loop'. The radius of the loop is 12.0 m. Calculate the speed required at the top of the loop for the car to experience a normal reaction force of 5000 N from the track. You must refer to a vector diagram in your answer.





(4)

Vector diagram ✓

$r = 12.0$ m $\quad m = 700 \quad W = mg = 6860$ N ✓

$\Sigma F = 5000 + 6860 = 11860$ N

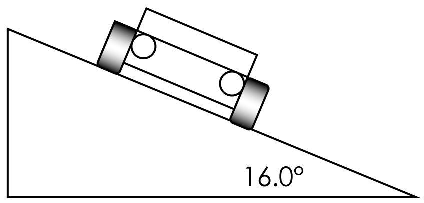
$mv^2/r = 11860$ N

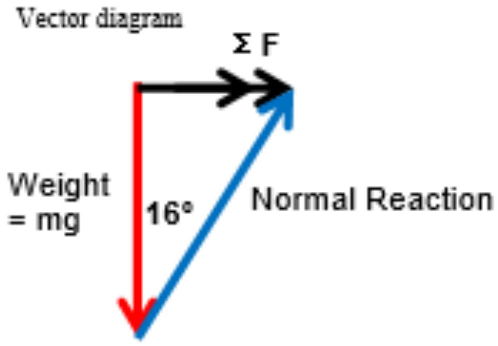
$v = ((11860 \times 12 / 700))^{0.5}$ ✓

$v = 14.3$ m/s ✓

Question 4

By banking the curves of racetracks it is possible for vehicles to turn in a horizontal circle without relying on friction. For a car of mass 2100 kg the angle of banking is set at 16.0° above the horizontal. The car drives at a speed 24.0 m s^{-1} to maintain its height on the bank.





a) Draw a vector diagram in the space above showing the forces acting on the car and the sum of those forces.

(1)

b) Calculate the horizontal radius of the car's path.

$\Sigma F = mv^2/r$

$\tan 16° = (mv^2/ r) / (mg)$ ✓

$\tan 16° = (v^2/ gr)$

$r = v^2 / g.\tan 16°$ ✓

$r = 24^2 / 9.8.\tan 16°$

$r = 205$ m

(3)

c) The speed of the car increases to greater than 24.0 m s^{-1}. Explain what other change must occur if the magnitude of forces on your vector diagram remain the same on this frictionless track.

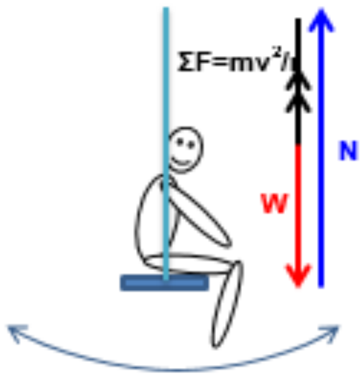
(2)

If $\Sigma F = mv^2/r$ is fixed ✓then as v increases the radius must increase as well. ✓

(The car will move to a position higher up the track)

Question 5

A person is sitting on a swing that is moving through the arc of a circle. It has reached the lowest point and is moving at maximum speed. Explain with reference to a vector diagram how the person's apparent weight is different compared to being at rest on the swing.



Sensation of weight from the normal reaction force ✓

Vector diagram ✓

The person is in circular motion so the net force acting on the person is the centripetal force directed to the centre of the circle. ✓

Considering all forces acting on the person

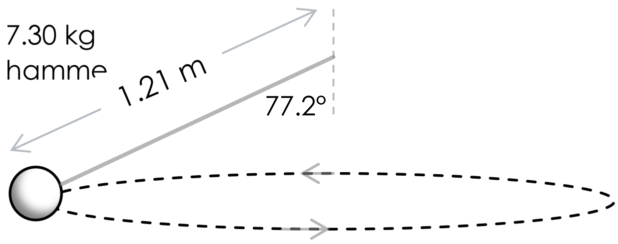
In a vector diagram $\Sigma F = mv^2/r = N + W$ and acts up (to centre)

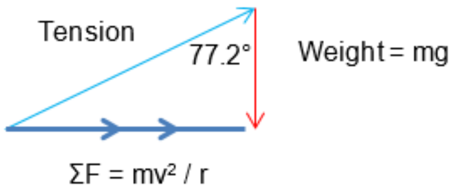
So the magnitude of N (normal reaction) acting up must be greater than magnitude of weight (mg) acting down ✓

(Or similar.)

Question 6

A student is investigating the physics of the hammer throw event at the London Olympics. A hammer of mass 7.30 kg is describing uniform circular motion at a constant height. The length of the hammer is 1.21 m and the wire makes an angle of 77.2° with the vertical. Calculate the time taken for the hammer to make one revolution.





$m = 7.30$ kg $l = 1.21$ m $\theta = 77.2°$

$\sin\theta = r / 1.21$ so $r = 1.21 \times \sin 77.2 = 1.18$ m ✓

Vector diagram or other analysis of vector components ✓ ✓

$\tan 77.2 = mv^2 / r \div mg = v^2 / gr$

$v^2 = \tan 77.2 \times 9.8 \times 1.18 = 50.896$

$v = 7.13415$ ✓

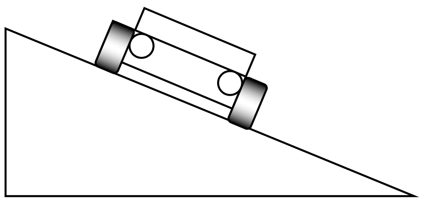
$T = 2\pi r / v = 2\pi \times 1.18 / 7.13415 = 1.04$ s ✓

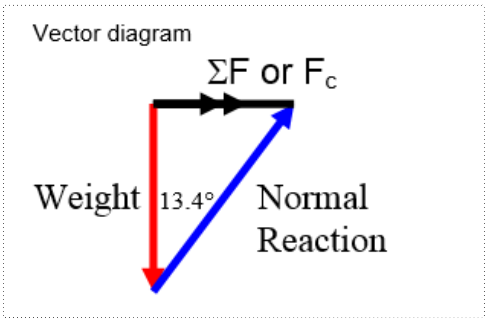
(Alternative methods that lead to correct answer acceptable)

(5)

Question 7

By banking the curves of racetracks it is possible for vehicles to turn in a horizontal circle without relying on friction. For a car of mass 1 700 kg the angle of banking is set at 13.4° above the horizontal. The curve has a radius of 171 m and the car drives at a speed to maintain its height.





13.4°

d) Draw a vector diagram showing the forces acting on the car and the sum of those forces in the space indicated above.

(1)

e) Calculate the centripetal force acting on the car.

(3)

$W = m.g = 1700 \times 9.8 = 16\ 660$ N

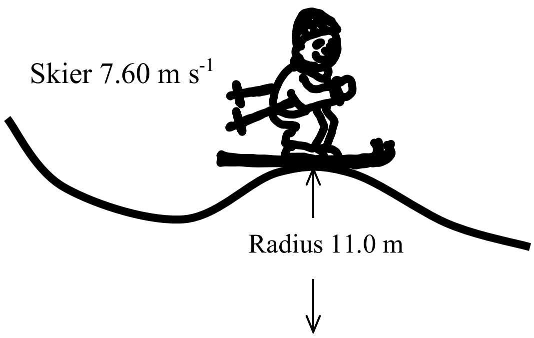
$\tan 13.4 = opp / adj = \Sigma F / W$

$\Sigma F = mg \times \tan 13.4° = 16\ 660 \times \tan 13.4°$ ✓

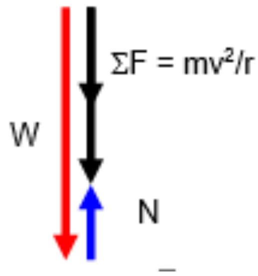
$\underline{\Sigma F = 3.97 \times 10^3 \text{ N}}$ ✓ <u>towards centre of circle</u> ✓

Question 8

A 70 kg skier is on a frictionless slope. He follows a circular path of radius 11.0 m as he goes over a mound and has a speed of 7.60 m s^{-1} at the top of the circle.



Calculate the normal reaction force he experiences from the mound at the top of the circle.



Consider forces going towards the centre of the circle

Correct vectorial analysis (with or without diagram✓)

$\Sigma F = mv^2 / r = W - N$

$N = W - mv^2 / r$

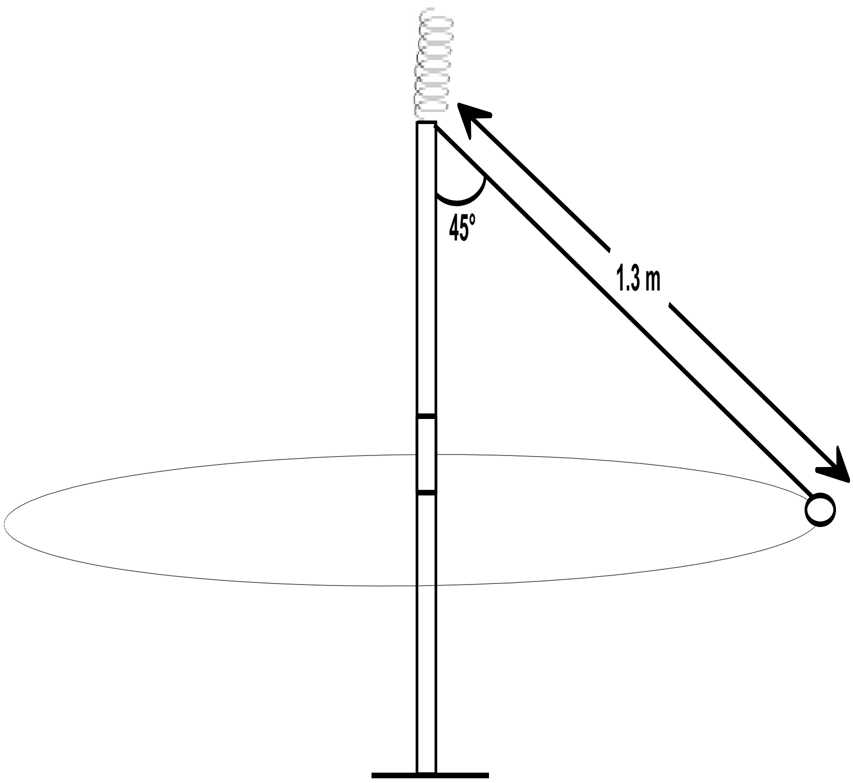
$N = (70 \times 9.8) - (70 \times 7.6^2 / 11)$ ✓

$N = 318.436 = 318$ N ✓ up / away from centre of circle ✓

(4)

Question 9

During a game of totem tennis a ball of mass 60.0 g swings freely in a horizontal circular path. The string is 1.30 m long and is at an angle of 45° to the vertical as shown in the diagram.



(a) Calculate the radius of the ball's circular path. (2 marks)

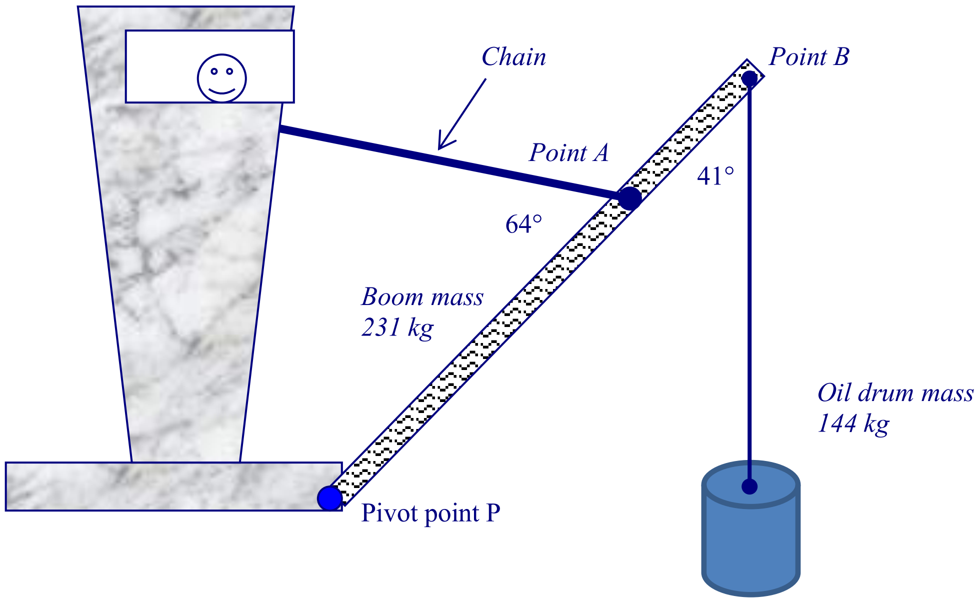
(b) Calculate the net force acting on the ball (3 marks)

TORQUE AND EQUILIBRIUM

Question 1 (9 marks)

A crane at Fremantle port is unloading an oil drum from a ship.

- The boom of the crane has a mass of 231 kg and is pivoted at point P.
- The oil drum of mass 144 kg is suspended from point B. Its rope makes an angle of 41° with the boom.
- A chain attached at point A is holding the boom in position. The distance from P to A is 3.80 m.
- The chain makes an angle of 64° with the boom.
- The boom has a length of 4.50 m from P to B with uniform mass distribution.



a. Demonstrate by calculation that the tension in the chain = 2.20 x 10^3 N.

(4)

Consider boom in static equilibrium, ΣM = 0

Select pivot at P and take moments

Σacwm = Σcwm   Concept ✓

$3.80 \times F_T \times \sin 64$ ✓ = $(4.50 \times 144 \times 9.8 \times \sin 41) + (2.25 \times 231 \times 9.8 \times \sin 41)$ ✓

$F_T = 7507.9 / (3.80 \times \sin 64)$

$F_T = 2198.23 = 2.20 \times 10^3$ N ✓

b. Calculate the magnitude of the **reaction force** acting on the boom from the pivot.

(3)

Consider boom in static equilibrium, ΣF = 0

Construct vector diagram / solve by components (Concept)

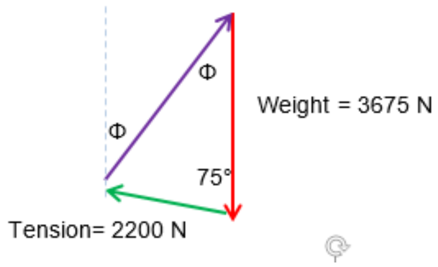
θ = 180 −(41+64) = 75°

Combined weight = (231 + 144) x 9.8 = 3675 N down ✓

By Cosine Rule $R^2 = W^2 + T^2 - 2.W.T.\cos 75°$

$R^2 = 3675^2 + 2200^2 - 2 \times 3675 \times 2200 \times \cos 75°$
✓

R = 3763 = 3.76 × 10^3 N ✓



c. Calculate the direction of the **reaction force** acting on the boom from the pivot.

(2)

By Sine rule $\frac{T}{\sin\Phi} = \frac{R}{\sin 75}$

$\sin\Phi = \frac{T \times \sin 75}{R} = \frac{2200 \times \sin 75}{3763}$ ✓

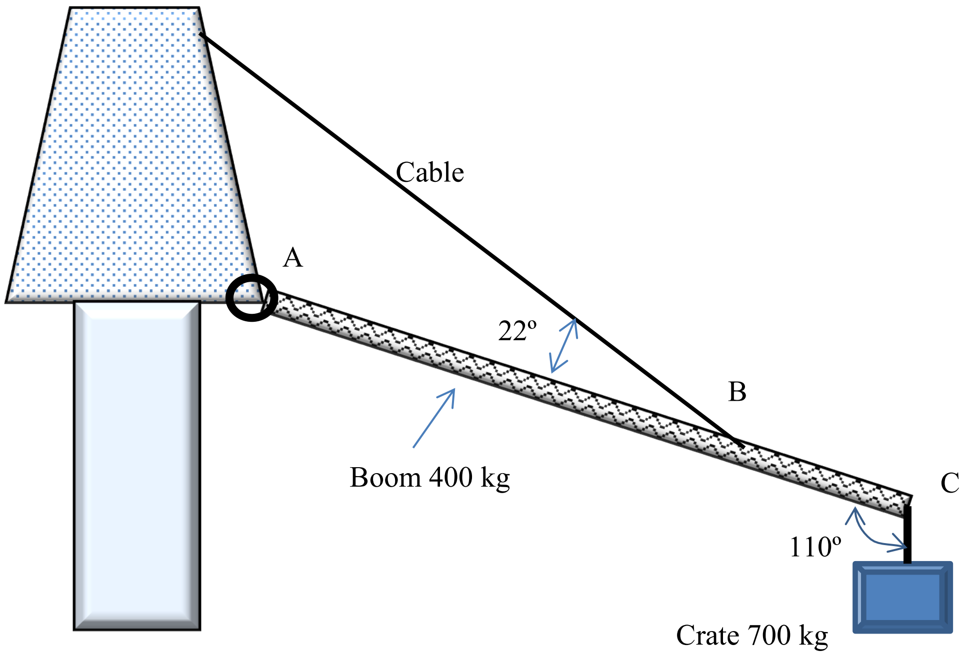
$\sin\Phi = 0.56471879$

Φ = 34.4° from vertical (or 55.6° above horizontal)

Must correspond to angle shown on diagram. ✓

Question 2 (10 marks)

The 400 kg boom of a crane is pivoted at point A. The length of the uniform boom AC is 8.00 m. A crate of mass 700 kg is lifted by a rope attached at C. A flexible cable is attached at point B where the length AB is 6.00 m. The cable makes an angle of 22° with the boom. The rope lifting the crate makes an angle of 110° with the boom.



a. Demonstrate by calculation that the tension in the cable is 2.95×10^4 N

(4)

$\Sigma M = 0$ $\Sigma acwm = \Sigma cwm$ $M = r.F.\sin\theta$

$AC = 8.00$ m $AB = 6.00$ m $\theta_B = 22°$ $\theta_C = 110°$ ✓

Take moments from A

$6 \times F_{tension} \times \sin 22°$ ✓

$= 4 \times 400 \times 9.8 \times \sin 110° + 8 \times 700 \times 9.8 \times \sin 110°$ ✓

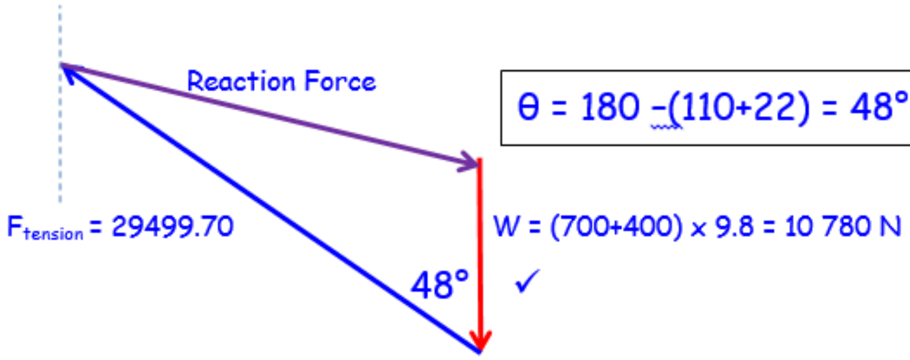
$F_{tension} =$

$(4 \times 400 \times 9.8 \times \sin 110° + 8 \times 700 \times 9.8 \times \sin 110°)/(6 \times \sin 22)$

$F_{tension} = 66304.711 / 2.24763956$

$F_{tension} = 29499.70 = 2.95 \times 10^4$ N ✓

b. Construct a vector diagram (approximately to scale) to show that $\Sigma F = 0$ when considering the weight of the boom, the weight of the crate, tension in the cable and reaction force from the pivot.



(1)

θ = 180 -(110+22) = 48°

(1)

c. Calculate the magnitude of the reaction force from the pivot.

(3)

θ = 180 -(110+22) = 48° ✓

By Cosine Rule $R^2 = W^2 + T^2 - 2.W.T.\cos 48°$

$R^2 = 10780^2 + 29499.7^2 - 2 \times 10780 \times 29499.7 \times \cos 48°$ ✓

R = 23682.58 = 2.37×10^4 N ✓

d. Calculate the direction of the reaction force relative to the vertical and show this angle on your vector diagram. (note that the reaction force acts below the horizontal)

(2)

By Sine rule $\frac{T}{\sin\Phi} = \frac{R}{\sin 48}$

$\sin\Phi = \frac{T \times \sin 48}{R} = \frac{29499.7 \times \sin 48}{23682.58}$ ✓

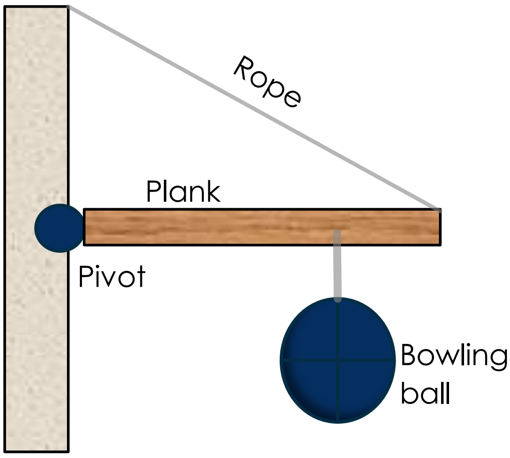
$\sin\Phi = 0.92568248$

Φ = 67.77 or 112.23°

Must correspond to angle shown on diagram. ✓

Question 3

A rigid wooden plank of mass 2.5 kg is attached to a wall by a pivot and is supported by a rope in tension. A 3.5 kg bowling ball is suspended from the plank. The diagram is to scale. **Estimate** the tension in the rope. Express your answer to an appropriate number of significant figures.



Let length of plank = 1.00 m = lever arm to rope

lever arm to ball = 0.750 m   lever arm to plank CofM = 0.500 m

θ_{rope} = 30.0°   ✓ (or other reasonable estimates)

$\theta_{plank} = \theta_{ball}$ = 90.0°   moment = r.F.sin θ

Σacwm = Σcwm about pivot

$r_{rope}.F_{tension}.\sin 30° = r_{ball}.F_{ball}.\sin 90° + r_{plank}.F_{plank}.\sin 90°$

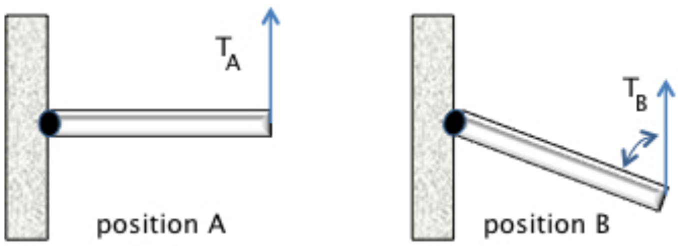
$1 * F_{tension} * 0.5 = 0.75*3.5*9.8 + 0.50*2.5*9.8$ ✓

$F_{tension}$ = 75.95 ✓ = 76 N (appropriate sig figs ✓)

(4)

Question 4

A rigid boom of mass m is free to rotate about a frictionless pivot P. The boom is held in static equilibrium by a rope that is in tension. The boom is held in two different positions where the tension in position A is T_A and the tension in position B is T_B. The positions are shown in the diagram below.



a) When comparing the magnitude of tension in each position, circle the best response: (1)

$T_A = T_B$ $T_A > T_B$ $T_A < T_B$ Insufficient information for a response

b) Clearly explain your choice.

Let boom length = l Σacwm = Σcwm about pivot

Position A $0.5 \times l \times mg \times \sin 90 = l \times T_A \times \sin 90$ ✓

$T_A = 0.5 \times mg$ ✓

Position B $0.5 \times l \times mg \times \sin 60 = l \times T_B \times \sin 120$ (same method)

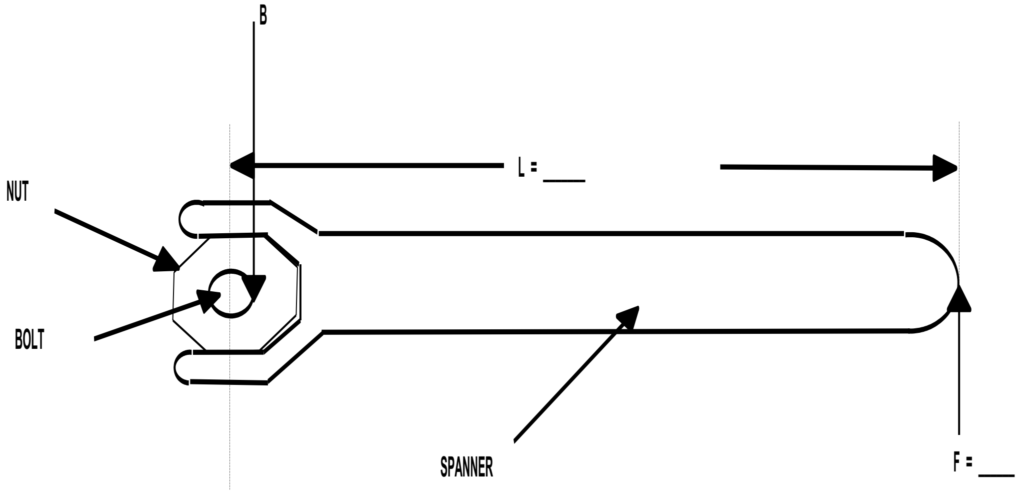
$T_B = 0.5 \times mg$ ✓   (Or acceptable alternative proof)

(3)

Question 5

A nut on a bolt on a bicycle requires a torque of 6 N m to just loosen it.

(a) Label the diagram below and estimate realistic values for the length (L) and force (F) that would just supply enough torque to loosen the nut. (4 marks)



Any reasonable combination of length and force whose product is 6 N m. However the length of the spanner should not exceed say 0.25 m

(b) ESTIMATE the binding force (B), between the nut and the bolt, which is just sufficient to stop the nut from coming loose. (2 marks)

B = _______________

A reasonable estimate of the radius of the bolt would be between 0.002 m and 0.004 m. Using these values a force of between 3000 N and 1500 N would represent the frictional force between the bolt and nut.