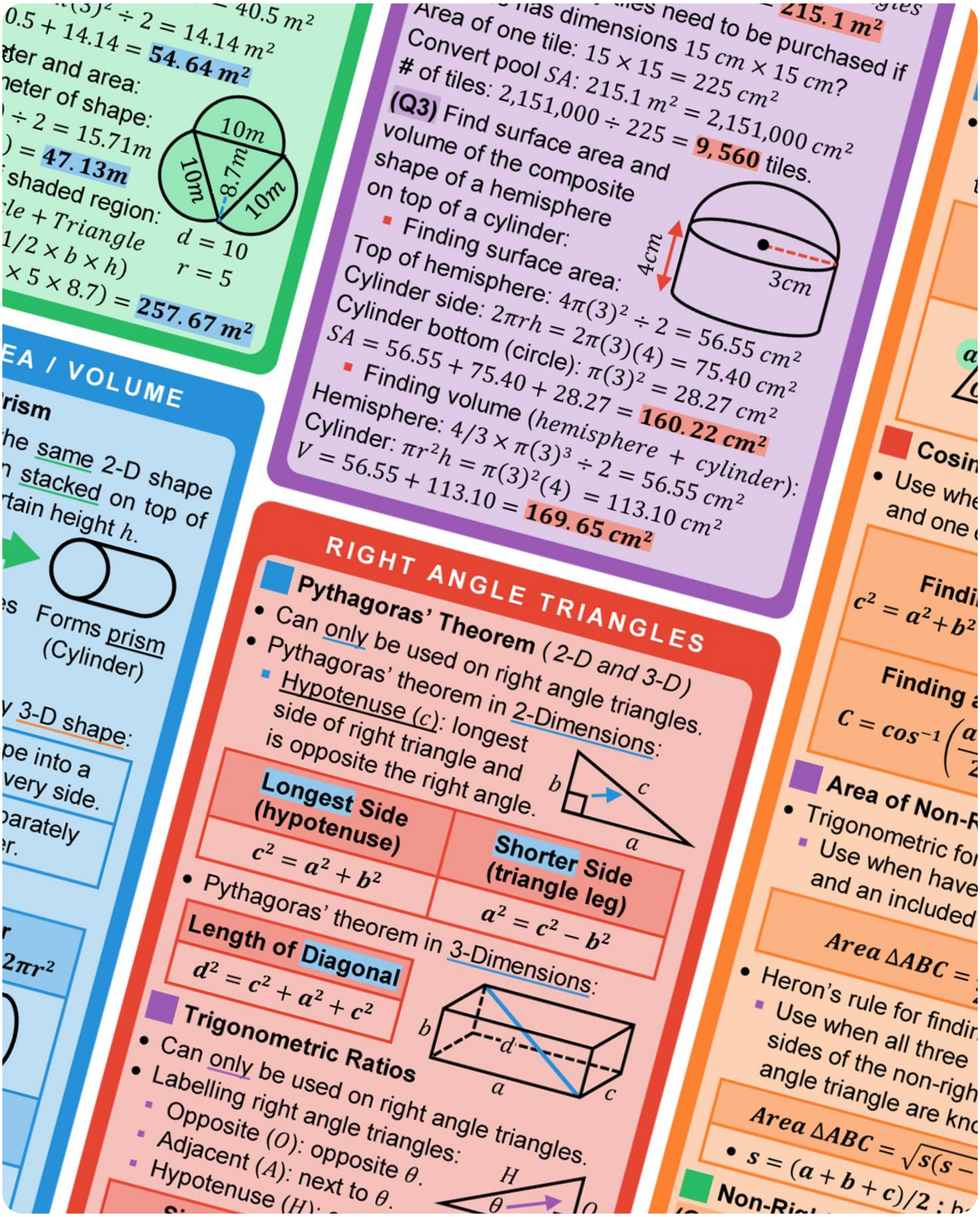

ATAR Mathematics Applications Units 1 & 2

Exam Notes for Western Australian Year 11 Students

Created by Anthony Bochrinis
Version 3.0 (Updated 05/01/20)

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▶ About the Author - Anthony Bochrinis

Hello! My name is Anthony and I graduated from high school in 2012, completed a Bachelor of Actuarial Science in 2015, completed my Graduate Diploma in Secondary Education in 2017 and am now a secondary mathematics teacher!

My original exam notes (created in 2013) were inspired by Severus Snape's copy of Advanced Potion Making in Harry Potter and the Half-Blood Prince; a textbook filled with annotations containing all of the pro tips and secrets to help gain a clearer understanding.

Thank you for being a part of my journey in realising that teaching is my lifelong vocation!

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FINANCE

TIME AND DATE

Time Conversions (*24 Hour Time*)

0000 0400 0800 1200 1600 2000 0000

AM (12 HRS) **PM** (12 HRS)

12:00 4:00 8:00 12:00 4:00 8:00 12:00

Date Conversions

- Frequency of time periods per year:

Yearly/Annual	1	**Monthly**	12
Six-Monthly	2	**Fortnightly**	26
Quarterly	4	**Weekly**	52

- Days in each month of the year:

Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**
31	28/29	31	30	31	30
Jul	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
31	31	30	31	30	31

PERCENTAGES

Percentage and Decimal Conversions

$\div$ **100** (*i.e.* move decimal point 2 places left)

Percentage → **Decimal**

Decimal → **Percentage**

$\times$ **100** (*i.e.* move decimal point 2 places right)

Percentage of Quantities

- Finding the percentage of a quantity:
  - *e.g.* what is A% of B?

%	Method of Finding Percentage
50%	Find half of the number (*i.e.* $\div$ by 2).
10%	Move decimal point 1 place left.
5%	Find 10% and then halve it.
1%	Move decimal point 2 places left.
0.5%	Find 1% and then halve it.

- Finding percentage between two quantities:
  - *e.g.* what percentage is A out of B?

$$(\boldsymbol{Quantity\ A} \div \boldsymbol{Quantity\ B}) \times \mathbf{100}$$

- $\boldsymbol{Quantity\ A}$ **:** typically smaller amount.
- $\boldsymbol{Quantity\ B}$ **:** typically larger amount.

Percentage Change

- Percentage increase (*i.e.* markups):
  - Markup: amount added to cost price.

$$(\mathbf{1} + (\% \div \mathbf{100})) \times \boldsymbol{Quantity}$$

- Percentage decrease (*i.e.* discounts):
  - Discount: amount taken from cost price.

$$(\mathbf{1} - (\% \div \mathbf{100})) \times \boldsymbol{Quantity}$$

- Percentage change (*i.e.* profit and loss):
  - Profit: a positive difference between the total amount earned and amount spent.
  - Loss: a negative difference between the total amount earned and amount spent.

$$\%\ \boldsymbol{Change} = (\boldsymbol{New} - \boldsymbol{Old}) \div \boldsymbol{Old} \times \mathbf{100}$$

$\%\ \boldsymbol{Change\ is\ Negative} = \boldsymbol{Loss}$	$\%\ \boldsymbol{Change\ is\ Positive} = \boldsymbol{Profit}$

- $\boldsymbol{New}$ **:** most recent price of good/service.
- $\boldsymbol{Old}$ **:** price before the change occurred.

- Reverse a percentage increase/decrease:
  - Finds the original price before a change.
  - Mistake to "undo" a % increase with the same % as a decrease. e.g. increasing 100 by 10% $=$ 110 and decreasing 110 by 10% $=$ 99 (*i.e.* not original number).

Reverse a x% Increase *i.e.* divide new amount by:	$\div\left(1 + \frac{x}{100}\right)$
Reverse a x% Decrease *i.e.* divide new amount by:	$\div\left(1 - \frac{x}{100}\right)$

Percentage Application Examples

(Q1) Determine the value of 96% of 500km:

$10\%\ of\ 500 = 50$ and $1\%\ of\ 500 = 5$

$96\% = 90\% + 6\% = (9 \times 10\%) + (6 \times 1\%)$

$= (9 \times 50) + (6 \times 5) = 450 + 30 = \mathbf{480}\boldsymbol{km}$

(Q2) Find the percentage of 42 out of 50:

$\frac{42}{50} \times 100 = \frac{42}{50} \times \frac{100}{1} = \frac{42}{1} \times \frac{2}{1} = \mathbf{84\%}$

(Q3) I bought a calculator for $150 and sold it for $120. What is the percentage profit/loss?

$New = \$120$ $\quad \frac{(120 - 150)}{150} \times 100 = -20\%$

$Old = \$150$

As the % is negative, result is a **20% loss**

(Q4) A $75 item is discounted by 40% and then discounted a further 60%. What single discount is equivalent to this discount?

- 1st discount: $75 \times \big(1 - (40 \div 100)\big) = 45$
- 2nd discount: $45 \times \big(1 - (60 \div 100)\big) = 18$

Cost is $18 which is $75 - 18 = \$57$ discount, $(1 - 0.4) \times (1 - 0.6) = 0.24$ which means an overall discount of $100\% - 24\% = \mathbf{76\%}$

(Q5) What was the original price on a $12,500 car before a 6% discount was applied?

$12500 \div \left(1 - \frac{6}{100}\right) = \$\mathbf{13297.87}$ original price.

GOODS AND SERVICES TAX

Goods and Services Tax (*GST*)

- Sales tax of 10% that's added onto the price of most goods and services in Australia.

Find GST Inclusive Price (*i.e.* $Total\ Price = Price + GST$)	$\times\ 1.1$
Find GST Amount (*i.e.* $GST\ = Total\ Price - Price$)	$\div\ 11$
Find GST Exclusive Price (*i.e.* $Price\ = Total\ Price - GST$)	$\div\ 1.1$

Goods and Services Tax Example

(Q1) A TV was bought for $550 GST inclusive.

(Q1a) Find how much GST included in price.

$550 \div 11 = \$\mathbf{50}$ was total amount of GST.

(Q1b) Find original price before adding GST.

$550 \div 1.1 = \$\mathbf{500}$ was amount before GST.

UNIT COST METHOD

Unit Cost Method (*Best Buy*)

- Finds cost of an item per lowest single unit.
- Used to compare multiple sizes of the same item to find best value for money.

$$\boldsymbol{Unit\ Price} = \boldsymbol{Total\ Price} \div \boldsymbol{Amount}$$

Unit Cost Method Examples

(Q1) Rank each of the following:

- Option One: $5.20 for 2000 mL of water
- Option Two: $1.75 for 600 ml of water
- Option Three: $0.95 for 0.45 litres of water

Option One: $5.2/2 = \$2.60/L$

Option Two: $1.75/0.6 = \$2.92/L$

Option Three: $0.95/0.45 = \$2.11/L$

*Convert to cost per one litre

Rank: **3, 1, 2** is order from best buy to worst.

WAGES / SALARY / COMMISSION

Wages, Salary and Commission

- Wages: being paid for work by the hour
  - Multipliers: employees can be paid more than base rate depending on day/hours:

Standard Rate	$\times\ 1$	**Double Time**	$\times\ 2$
Time and a Half	$\times\ 1.5$	**Triple Time**	$\times\ 3$

- Annual Salary: set amount earned per year.
- Commission: paid depending on how much revenue an employee earns for a business.

Wage, Salary & Commission Examples

(Q1) Mark earns $8 for every shirt and $6 for every pair of pants he sells at a store.

(Q1a) How much is Mark paid for selling 20 shirts and 12 pairs of pants in a day?

$(20 \times 8) + (12 \times 6) = \$\mathbf{232}$

(Q1b) Mark earned $1166 this week and sold 112 shirts, how many pairs of pants did he sell?

$1166 - (112 \times 8) = 270 \rightarrow 270 \div 6 = \mathbf{45}$

(Q2) Find Lisa's weekly pay if her salary is $90,000 p.a.? $90000 \div 52 = \$\mathbf{1,730.77}$

(Q3) Ben earns $20 p/h working these times:

Mon	0600 – 1300 with 30 min unpaid break
Tue	0800 – 1700 with 30 min unpaid break
Wed	0900 – 1600 with no break
Thu	0600 – 1900 with 90 min unpaid break
Fri	0530 – 1715 with 15 min unpaid break

- If Ben works more than 8 hours a day, then the next 2 hours are paid "time and a half" and next 2 hours after are paid "double time".
- Any Friday hours paid at "double time".

(Q3a) Breakdown the hours worked per week:

Day	Normal	Time & Half	Double	Total
Mon	6.5	0	0	6.5
Tue	8	0.5	0	8.5
Wed	7	0	0	7
Thu	8	2	1.5	11.5
Fri	0	0	11.5	11.5
Total	**29.5**	**2.5**	**13**	**45**

(Q3b) What is Ben's weekly pay?

$(29.5 \times 20) + (2.5 \times 20 \times 1.5) + (13 \times 20 \times 2)$

$= 590 + 75 + 520 = \$\mathbf{1185}$

GOVERNMENT ALLOWANCES

Government Allowance Example

(Q1) A person qualifies for youth allowance of $420 per fortnight if doesn't earn more than $427 in that time. Allowance is reduced by 50c in the dollar for each dollar over $427.

(Q1a) Find monthly payment for youth allowance of Joshua who earns $0 per week.

No deductions $\therefore (420 \times 26) \div 12 = \$\mathbf{910}$

(Q1b) Find weekly payment of Sarah who earns $18 per hour working 15 hours a week.

$18 \times 19 = \$270\ p/week = \$540\ \ per\ fortnight$

$540 - 427 = \$113 \rightarrow 50\% \times 113 = \$56.50$

$\$420 - \$56.50 = \$363.50\ \ per\ fortnight$

$= \$363.50 \div 2 = \$\mathbf{181.75}\ \ per\ week$

BUDGETING

Fixed and Discretionary Spending

- Fixed: spending on necessities for daily life (*e.g.* rent, electricity bills or insurance).
- Discretionary: spending on non-essential items that can be removed (*e.g.* dining out).

$$\boldsymbol{Savings} = \ \boldsymbol{Income} - \ \boldsymbol{Expenditure}$$

Budgeting Example

(Q1) Make a weekly budget for the following:

- Income from job: $114,400 per year.
- Investment income: $950 per week.
- Rent for apartment: $380 per week.
- Utilities (*i.e.* gas and electricity): $5 per day.
- Entertainment/dining: $200 per week.
- Car and health insurance: $45 per fortnight.

Income		**Expenditure**	
Investments	$950	Rent	$380
Income	$2200	Utilities	$35
Total	**$3150**	Entertainment	$200
Total Savings		Insurance	$22.50
$2491.50		**Total**	**$658.50**

SIMPLE INTEREST

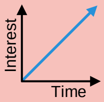
Simple Interest Formula

$\boldsymbol{I = P \times R \times T}$	$\boldsymbol{A = I + P}$

- $\boldsymbol{A}$ **:** total amount (principal plus interest).
- $\boldsymbol{P}$ **:** principal (starting amount).
- $\boldsymbol{I}$ **:** total interest earned/ owed.
- $\boldsymbol{R}$ **:** interest rate (as a decimal).
- $\boldsymbol{T}$ **:** time (must be converted to years).

Graphing Simple Interest

- Amount of interest earned in simple interest is linear (*i.e.* amount of interest earned is constant over time).



Simple Interest Examples

(Q1) Find simple interest on investment of $1500 for 3 years 6 months at a rate of 0.8%?

$I = P \times R \times T = 1500 \times 0.008 \times 3.5 = \$\mathbf{42.00}$

(Q2) Ellie purchased a mobile phone worth $600 using her credit card that charges 19.8% p.a. simple interest on the 30th of March. She paid the account on the 11th of April.

(Q2a) What was the total interest charged?

$I = 600 \times 0.198 \times (13/365) = \$\mathbf{4.23}$

(Q2b) Find the total amount that Ellie paid:

$A = I + P = 4.23 + 600 = \$\mathbf{604.23}$

COMPOUND INTEREST

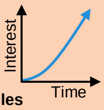
Compound Interest Formula

$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$	$\boldsymbol{I = A - P}$

- $\boldsymbol{A}$ **:** total amount (principal plus interest).
- $\boldsymbol{P}$ **:** principal (initial/starting amount).
- $\boldsymbol{I}$ **:** total amount of interest earned/owed.
- $\boldsymbol{r}$ **:** annual interest rate (as a decimal).
- $\boldsymbol{n}$ **:** number of times in which interest is compounded per year.
- $\boldsymbol{t}$ **:** time (must be converted to years).

Graphing Compound Interest

- Amount of interest earned in simple interest is exponential (*i.e.* interest amount earned increases over time).



Compound Interest Examples

(Q1) $50,000 is invested into a bank with a rate of 7.67% p.a., compounding half-yearly over 3 years. How much interest does it accrue?

$\boldsymbol{A} = 50000\left(1 + \frac{0.0767}{2}\right)^{2\times3} = \$62666.09$

$\boldsymbol{I} = A - P = 62666.09 - 50000 = \$\mathbf{12666.09}$

(Q2) Invest $25,000 using choice of schemes:

- **X**: 6.22% p.a. compounding monthly
- **Y**: 6.25% p.a. compounding quarterly

Which would pay more after 3 years?

$\boldsymbol{X} = 25000\left(1 + \frac{0.0622}{12}\right)^{12\times3} - 1 = \$30,114.11$

$\boldsymbol{Y} = 25000\left(1 + \frac{0.0625}{4}\right)^{4\times3} - 1 = \$30,112.07$

$\therefore$ **Scheme X** pays more interest than Y.

INFLATION

Inflation Definition and Formula

- Consistent rise in level of wages and prices.

$$\boldsymbol{New\ Price} = \boldsymbol{P}(\mathbf{1} + \boldsymbol{r})^{\boldsymbol{n}}$$

- $\boldsymbol{P}$ **:** cost or wage of current item/person.
- $\boldsymbol{r}$ **:** annual inflation rate (as a decimal).
- $\boldsymbol{n}$ **:** time (must be converted to years).

Inflation Examples

(Q1) Inflation is 2.1% p.a. What would be the price of an $2010 TV 10 years from now and how much has inflation added to the price?

$2010(1 + 0.021)^{10} = \$\mathbf{2474.31}$ is new price.

$\$2474.31 - \$2010 = \$\mathbf{464.31}$ due to inflation.

EXCHANGE RATES

Exchange Rate Definition

- Price of country's currency in terms of another currency for the purpose of conversion.

Exchange Rate Examples

(Q1a) 1 Aus. Dollar (AUD) buys 3.25 Malaysian Ringgit (RM). How much AUD is 450 RM?

$450\ RM = 450 \div 3.25 = \$\mathbf{138.46\ \boldsymbol{AUD}}$

(Q1b) Convert a price of $55 AUD to RM:

$\$55\ AUD = 55 \times 3.25 = \$\mathbf{178.75\ \boldsymbol{RM}}$

(Q1c) Conversion rate increases to 1:3.5. How much more RM can be made from $10 AUD?

$\$10AUD = 35\ RM \rightarrow 35 - 32.50 = \mathbf{2.50}$ more.

SHARE MARKET

Share Market Terminology

- Shares: a small part of a company, entitling the holder to a proportion of company profits.
- Share Portfolio: collection of different shares.
- Purchase Price: how much an investor spent to initially purchase shares in a company.
- Market Value: most recent price for a share.
- Earnings Per Share: profit after tax that is available to be distributed to shareholders.
- Dividends Per Share: profit after tax that is actually paid to shareholders for each share held (normally on a bi-annual or annual basis).
- Price-to-Earnings Ratio (PE): how much an investor expects to invest in order to receive one dollar of company earnings.
  - The lower the PE ratio, the more attractive the company is to investors to buy shares.
- Brokerage: fee paid to a stockbroking firm to buy and sell shares of your choice for you.

Share Market Formulae

$$\boldsymbol{P/E\ Ratio} = \frac{\boldsymbol{Market\ Price\ Per\ Share}}{\boldsymbol{Earnings\ Per\ Share}}$$

$$\boldsymbol{Dividend\ Per\ Share} = \frac{\boldsymbol{Net\ Profit}}{\boldsymbol{Total\ Shares}}$$

Share Market Application Examples

(Q1) A share portfolio is shown below:

Company	CBA	RIO	NMC
Number of Shares	85	120	220
Market Value per Share	$8.00	$5.60	$9.65
Earnings per Share	$2.50	$0.80	$3.40
Annual Dividend	$0.12	6%	Nil

(Q1a) Find the total market value of portfolio:

$(85 \times 8) + (120 \times 5.6) + (220 \times 9.65) = \$\mathbf{3475}$

(Q1b) Find the total dividend of the portfolio:

$(85 \times 0.12) + (120 \times 0.06) = \$\mathbf{17.40}$

(Q1c) Find the price to earnings ratio of all 3 companies and recommend an investment:

$CBA's\ P/E = 8 \div 2.5 = 3.2$

$RIO's\ P/E = 5.6 \div 0.8 = 7$

$NMC's\ P/E = 9.65 \div 3.4 = 2.8$

NMC is the best choice as it has the lowest P/E.

SPREADSHEETS

Spreadsheeting Cell References

Equation	Description
=A1+A2	Adds value of two cells.
=A1–A2	Subtracts value of two cells.
=A1*A2	Multiplies value of two cells.
=A1/A2	Divides value of two cells.
=SUM(A1:A5)	Adds the values of all cells together between A1 and A5.
=$A1	Absolute cell reference (cell reference does not change when copied across cells).

Spreadsheeting Example

	A	B	C	D	E
1	**Food**	**Cost**	**Price**	**# Sold**	**Profit**
2	Burger	$3.40	$7.00	88	$316.80
3	Panini	$3.10	$8.50	72	$388.80
4	Fries	$1.10	$3.50	125	$300.00
5	Salad	$4.20	$9.00	45	$216.00
6				**Total**	$1211.60

(Q1a) What is the formula that calculates the total cost of selling fries today?

(*i.e.* $Cost = price \times quantity$): $= \boldsymbol{C4} \times \boldsymbol{D4}$

(Q1b) What is formula that calculates cell E3?

Cell E3 finds profit for paninis (*i.e.* $Profit = revenue - cost$): $= (\boldsymbol{C3} - \boldsymbol{B3}) \times \boldsymbol{D3}$

(Q1c) What is formula that calculates cell E6?

Cell E6 finds total profit for all foods items (*i.e.* add individual profits): $= \boldsymbol{SUM}(\boldsymbol{E2}{:}\,\boldsymbol{E5})$

ATAR Math Applications

Units 1 & 2 Exam Notes

Created by **Anthony Bochrinis**
More resources at **reviseonline.com**

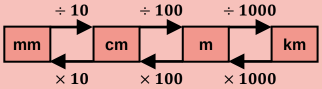
Version: **3.0**

MEASUREMENT

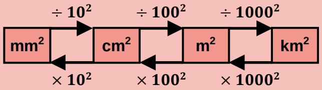
CONVERSIONS

1-D / 2-D / 3-D Distance Conversions

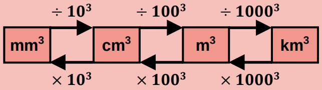
- 1-D Conversions (*i.e.* distance):



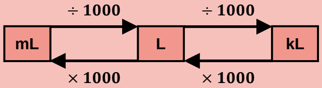
- 2-D Conversions (*i.e.* area/surface area):



- 3-D Conversions (*i.e.* volume):



Capacity Conversions

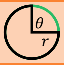
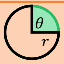


$1mL = 1cm^3$	$1L = 1000cm^3$	$1kL = 1m^3$

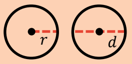
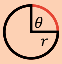
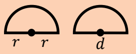
PERIMETER / AREA

Circle Terminology

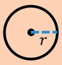
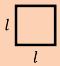
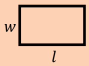
- Circumference (C): the perimeter of a circle.
- Radius (r): distance from circle centre to the edge (*i.e.* radius is half of the diameter).
- Diameter (d): distance across a circle that goes through the centre of the circle.

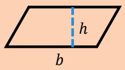
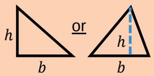
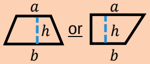
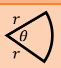
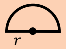
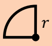
Arc		Part of the circumference of a circle (arc length depends on angle θ).
Sector		Area between arc and centre of circle (sector depends on angle θ).

Perimeter Formulae

Circumference $C = 2\pi r$ or $C = \pi d$	**Arc** $P = \frac{\theta}{360} \times 2\pi r$
	 *For any angle θ
Sector $P = \frac{\theta}{360} \times 2\pi r + 2r$	**Semi-Circle** $P = \pi r + 2r$
 *For any angle θ	

Area Formulae

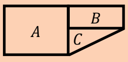
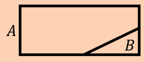
Circle $A = \pi r^2$	**Square** $A = l^2$	**Rectangle** $A = l \times w$
		

Parallelogram $A = b \times h$	**Triangle** $A = \frac{1}{2} \times b \times h$
	
Trapezium $A = \frac{1}{2} \times (a + b) \times h$	**Sector** $A = \frac{\theta}{360} \times \pi r^2$
	 *For any angle θ
Semi-Circle $A = \pi r^2 \div 2$	**Quarter Circle** $A = \pi r^2 \div 4$
	

Area of Composite Shapes

- Composite shape: complex shape made up of two or more simpler and smaller shapes.

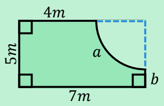
Method 1: Adding Shapes	
Step 1	Break down composite shape into small shapes that can be added.
Step 2	Find area of each smaller shape separately and then add them.
Method 2: Subtracting Shapes	
Step 1	Break down composite shape into shapes that can be subtracted.
Step 2	Find area of smaller shapes and subtract them from larger shape.

Method 1: Add	**Method 2: Subtract**
	
$Area = \mathbf{A + B + C}$ *3 different shapes	$Area = \mathbf{A - B}$ *A is whole rectangle

PERIMETER / AREA EXAMPLES

Perimeter and Area Examples

(Q1) Find the perimeter and area of the shaded region of the following composite shape:



- Finding perimeter:

Radius of the quarter circle: $7 - 4 = 3m$

Length of curve a: $a = 2\pi(3) \div 4 = 4.71m$

Length of line b: $a = 5 - 3 = 2m$

Shape perimeter: $5 + 4 + 7 + a + b = \mathbf{22.71m}$

- Finding area of the shaded region:

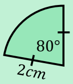
$Area = Whole\ Rectangle - Quarter\ Circle$

Area of whole rectangle: $5 \times 7 = 35\ m^2$

Area of quarter circle: $\pi(3)^2 \div 4 = 7.07\ m^2$

Shaded Area: $35 - 7.07 = \mathbf{27.93\ m^2}$

(Q2) Find perimeter and area of the sector:



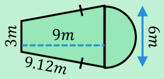
- Finding perimeter of sector:

$P = \frac{80}{360} \times 2\pi(2) + 2(2) = \mathbf{6.79\ cm}$

- Finding area of sector:

$A = \frac{80}{360} \times \pi(2)^2 = \mathbf{2.79\ cm^2}$ $\quad \theta = 80°, r = 2\ cm$

(Q3) Find the perimeter and area of the shaded region of the following composite shape:



- Finding perimeter of shape: $d = 6, r = 3$

Perimeter of semi-circle: $2\pi(3) \div 2 = 9.42m$

Perimeter: $3 + 2(9.12) + 9.42 = \mathbf{30.66m}$

- Finding area of the shaded region:

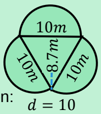
$Area = Trapezium + Semicircle$

Area of trapezium: $\frac{1}{2} \times (3 + 6) \times 9 = 40.5\ m^2$

Area of semi-circle: $\pi(3)^2 \div 2 = 14.14\ m^2$

Shaded Area: $40.5 + 14.14 = \mathbf{54.64\ m^2}$

(Q4) Find perimeter and area:



- Finding perimeter of shape:

Semi-circle: $2\pi(5) \div 2 = 15.71m$

Perimeter: $3(15.71) = \mathbf{47.13m}$

- Finding area of shaded region: $d = 10$, $r = 5$

$Area = 3 \times Semicircle + Triangle$

$Area = 3 \times (\pi r^2) + (1/2 \times b \times h)$

$= 3 \times (\pi \times 5^2) + (1/2 \times 5 \times 8.7) = \mathbf{257.67\ m^2}$

SURFACE AREA / VOLUME

Definition of a 3-D Prism

- A prism is a series of the same 2-D shape (*i.e.* base) that has been stacked on top of each other to obtain a certain height h.

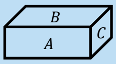
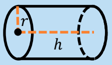
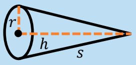
Base (Circle) — Stacked multiple times on top of each other — Forms prism (Cylinder)

Surface Area Formulae

- Finding the surface area of any 3-D shape:

Step 1	Break down the 3-D shape into a separate 2-D shape for every side.
Step 2	Find area of each side separately and then add them together.

- Surface area formulae:

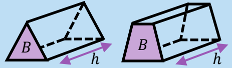
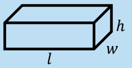
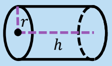
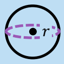
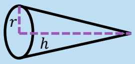
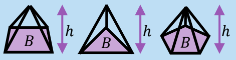
Rectangular Prism $SA = 2A + 2B + 2C$	**Cylinder** $SA = 2\pi rh + 2\pi r^2$
	
Sphere $SA = 4\pi r^2$	**Cone** $SA = \pi rs + \pi r^2$
	

Volume Formulae

- Finding the volume of any prism:

Step 1	Identify which side of the prism is the base (*i.e.* stackable 2-D shape).
Step 2	Find area of the 2-D base and multiply it by height of the prism.

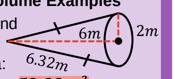
- Volume formulae:

Prism $V = Area\ of\ base \times h$	
 *For any 2-D base	
Rectangular Prism $V = l \times w \times h$	**Cylinder** $V = \pi r^2 \times h$
	
Sphere $V = \frac{4}{3} \times \pi r^3$	**Cone** $V = \frac{1}{3} \times \pi r^2 h$
	
Pyramid $V = \frac{1}{3} \times Area\ of\ Base \times h$	
 *For any 2-D base	

SURFACE / VOLUME EXAMPLES

Surface Area and volume Examples

(Q1) Find surface area and volume of the cone:



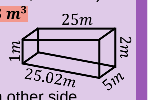
- Finding surface area:

$SA = \pi(2)(6.32) + \pi(2)^2 = \mathbf{52.28\ m^2}$

- Finding volume ($r = 2, s = 6, h = 6.32$):

$V = 1/3 \times \pi(2)^2(6) = \mathbf{25.13\ m^3}$

(Q2) A lap pool is 25m long and 5m wide. At the shallow end it is 1m deep and then evenly gets deeper to 2m on other side.



(Q2a) Find the volume of the pool:

Pool is a prism with trapezium as the base:

Trapezium area: $1/2 \times (1 + 2) \times 25 = 37.5\ m^2$

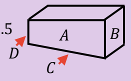
Volume: $area\ base \times h = 37.5 \times 5 = \mathbf{187.5\ m^3}$

(Q2b) Find the capacity of the pool in litres:

$1\ m^3 = 1\ kL \rightarrow 187.5\ m^3 = 187.5\ kL$ $\quad$ *$\times 1000$

$1\ kL = 1000\ L \rightarrow 187.5\ kL = \mathbf{187,500\ L}$

(Q2c) Find the surface area of the pool to find how many tiles to buy (*i.e.* don't include top):



$SA = 2A + B + C + D$

$A = 0.5 \times (1 + 2) \times 25 = 37.5$

$B = 5 \times 2 = 10\ m^2$

$C = 25.02 \times 5 = 125.1\ m^2$

$D = 5 \times 1 = 5\ m^2$ $\quad$ *$B, C, D\ are\ rectangles$

$SA = 2(37.5) + 10 + 125.1 + 5 = \mathbf{215.1\ m^2}$

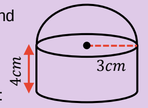
(Q2d) How many tiles need to be purchased if each tile has dimensions $15\ cm \times 15\ cm$?

Area of one tile: $15 \times 15 = 225\ cm^2$

Convert pool SA: $215.1\ m^2 = 2{,}151{,}000\ cm^2$

\# of tiles: $2{,}151{,}000 \div 225 = \mathbf{9,560}$ tiles.

(Q3) Find surface area and volume of the composite shape of a hemisphere on top of a cylinder:



- Finding surface area:

Top of hemisphere: $4\pi(3)^2 \div 2 = 56.55\ cm^2$

Cylinder side: $2\pi rh = 2\pi(3)(4) = 75.40\ cm^2$

Cylinder bottom (circle): $\pi(3)^2 = 28.27\ cm^2$

$SA = 56.55 + 75.40 + 28.27 = \mathbf{160.22\ cm^2}$

- Finding volume ($hemisphere\ +\ cylinder$):

Hemisphere: $4/3 \times \pi(3)^3 \div 2 = 56.55\ cm^2$

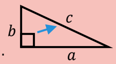
Cylinder: $\pi r^2 h = \pi(3)^2(4)\ = 113.10\ cm^2$

$V = 56.55 + 113.10 = \mathbf{169.65\ cm^2}$

RIGHT ANGLE TRIANGLES

Pythagoras' Theorem (*2-D and 3-D*)

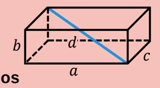
- Can only be used on right angle triangles.
- Pythagoras' theorem in 2-Dimensions:
  - Hypotenuse (c): longest side of right triangle and is opposite the right angle.



Longest Side (hypotenuse)	**Shorter Side (triangle leg)**
$c^2 = a^2 + b^2$	$a^2 = c^2 - b^2$

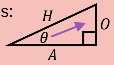
- Pythagoras' theorem in 3-Dimensions:

Length of Diagonal
$d^2 = c^2 + a^2 + c^2$



Trigonometric Ratios

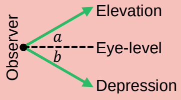
- Can only be used on right angle triangles.
- Labelling right angle triangles:
  - Opposite (O): opposite θ.
  - Adjacent (A): next to θ.
  - Hypotenuse (H): opposite right angle.



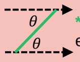
Sin	**Cos**	**Tan**
$sin\theta = \frac{O}{H}$	$cos\theta = \frac{A}{H}$	$tan\theta = \frac{O}{A}$
$\theta = \sin^{-1}\left(\frac{O}{H}\right)$	$\theta = \cos^{-1}\left(\frac{A}{H}\right)$	$\theta = \tan^{-1}\left(\frac{O}{A}\right)$

Angles of Elevation and Depression

- Elevation (a): angle of looking up at an object.
- Depression (b): angle of looking down at an object.



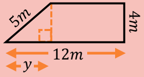
- Alternate angles (z-rule): if there are 2 parallel lines, alternating angles are equal.



*Angles are equal (z-rule)

Right Angle Triangle Examples

(Q1) Find the length of x:

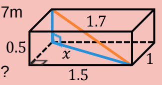


Pythagoras: $y^2 = 5^2 - 4^2$

$y = \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$

$y = 3, x = 12 - 3 = \mathbf{9m}$

(Q2) Will a rod that is 1.7m long fit in a rectangular prism that's 1.5m wide, 1m deep and 0.5m high?



- Method 1: Using 2-D Pythagoras formula:

Find x on the bottom: Use x to find rod length:

$x = \sqrt{1^2 + 1.5^2}$ $\qquad rod = \sqrt{1.8^2 + 0.5^2}$

$x = \sqrt{3.25} = 1.8$ $\qquad x = \sqrt{3.5} = 1.87\ m$, **Yes**.

- Method 2: Using 3-D Pythagoras formula:

$rod^2 = 0.5^2 + 1^2 + 1.5^2$ $\quad rod = 1.87\ m > 1.7\ m$

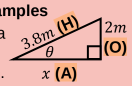
$rod = \sqrt{0.25 + 1 + 2.25}$ **Yes**, the rod will fit.

► Topic Is Continued In Next Column ◄

RIGHT ANGLE TRIANGLES

Right Angle Triangle Examples

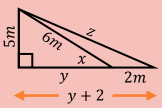
(Q3a) Draw diagram to show a 3.8m ladder leaning against a wall that reaches height of 2m.



(Q3b) Find the angle of elevation that the ladder has between the ground and the wall:

$sin(\theta) = \frac{O}{H} = \frac{2}{3.8} \rightarrow \theta = \sin^{-1}\left(\frac{2}{3.8}\right) = \mathbf{31.76°}$

(Q4) Find length of x, y and z in the diagram:



- Find angle size of x:

$x = sin^{-1}(5/6) = \mathbf{56.44°}$

- Find length of y:

$y = \sqrt{6^2 - 5^2} = \mathbf{3.32m}$

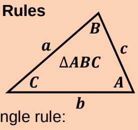
- Find length of z:

$z = \sqrt{5^2 + (y+2)^2} = \sqrt{5^2 + 5.32^2} = \mathbf{7.3m}$

NON-RIGHT ANGLE TRIANGLES

Triangle Notation and Rules

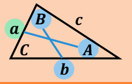
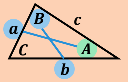
- Angles are capitalized.
- Sides are in lower case.
- Opposing angles and sides have same letter.
- Sum of angles in any triangle rule:



$$A + B + C = 180°$$

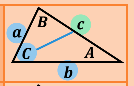
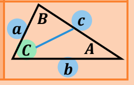
Sine Rule (*Finding Angles or Sides*)

- Use when two pairs of opposite angles and sides are given and one element (*i.e.* one of the angles or sides in either pair) is missing.

Finding Sides: $\frac{a}{sinA} = \frac{b}{sinB}$	**Finding Angles:** $\frac{sinA}{a} = \frac{sinB}{b}$
	

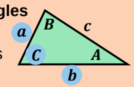
Cosine Rule (*Finding Angles or Sides*)

- Use when three sides and one angle is given and one element (*i.e.* angle or side) is missing.

Finding a Side: $c^2 = a^2 + b^2 - 2ab \times cos(C)$	
Finding an angle: $C = cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2 \times a \times b}\right)$	

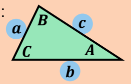
Area of Non-Right Triangles

- Trigonometric formula:
  - Use when have two sides and an included angle.



$$Area\ \Delta ABC = \frac{1}{2} \times a \times b \times sin(C)$$

- Heron's rule for finding area:
  - Use when all three sides of the non-right angle triangle are known.

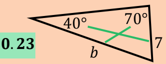


$Area\ \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$
• $s = (a + b + c)/2$: half of the perimeter.

Non-Right Angle Triangle Examples

(Q1) Find side length b in the following triangle:

- Two pairs of angle and side: sine rule

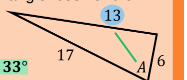


Side rule: $\frac{7}{sin(40)} = \frac{b}{sin(70)}$

$b = si\,n(70) \times \frac{7}{sin(40)} = \mathbf{10.23}$

(Q2) Find angle A in the following triangle:

- Three sides and one angle: cosine rule



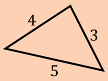
$A = cos^{-1}\left(\frac{17^2 + 6^2 - 13^2}{2 \times 17 \times 13}\right)$

$A = cos^{-1}(156/442)$

$A = \cos^{-1}(0.3529) = \mathbf{69.33°}$

(Q3) Find area of the following triangle:

- Three sides: Heron's rule



$s = (3 + 4 + 5)/2 = 6$

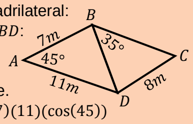
$Area = \sqrt{6(6-3)(6-4)(6-5)}$

$Area = \sqrt{36} = 6\ units^2$

(Q4) Find area of quadrilateral:

- Find side length BD:

Two sides with the included angle are given; use cosine rule.



$BD^2 = 7^2 + 11^2 - 2(7)(11)(\cos(45))$

$BD = \sqrt{61.11} = 7.82\ m$

- Find angle C using the sine angle rule:

$\frac{sinC}{7.82} = \frac{sin(35)}{8} \rightarrow sinC = 0.561 \rightarrow C = 34.1°$

$Angle\ \angle BDC = 180 - 35 - 34.1 = 110.9°$

- Finding area of quadrilateral using trig rule:

$\Delta ABD = 0.5 \times 7 \times 11 \times sin(45) = 27.22\ m^2$

$\Delta BCD = 0.5 \times 8 \times 7.82 \times sin(110.9) = 29.22\ m^2$

$Quadrilateral = \Delta ABD + \Delta BCD = \mathbf{56.44\ m^2}$

ATAR Math Applications
Units 1 & 2 Exam Notes

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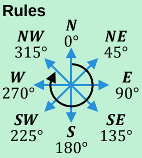
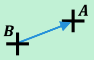
BEARINGS

Bearing Notation and Rules

- True bearing (°T): angle between 0 and 360 that is measured clockwise around the compass starting from true north.
- Finding the bearing from one point to another:

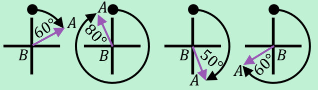


$$\begin{gathered}\textit{Bearing of A from B}\\ = \textit{Start at B and go to A}\end{gathered}$$



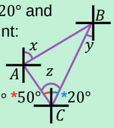
Using Bearings Examples

(Q1) Find true bearing of point A from B of:



$0 + 60$	$360 - 10$	$90 + 50$	$180 + 60$
$= \mathbf{60°T}$	$= \mathbf{350°T}$	$= \mathbf{140°T}$	$= \mathbf{240°T}$

(Q2) The diagram below has three locations: a, b and c. Angle x is 40°, y is 20° and z is 70°. Find the bearing of point:

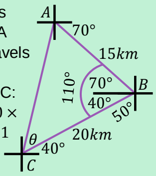


(Q2a) B from A? $\mathbf{40°T}$

(Q2b) B from C? If $y = 20°$ then blue angle at point $C = 20°$ (using alternate angles). $\mathbf{20°T}$

(Q2c) A from C? If blue angle at point $C = 20°$ and $z = 70°$ then red angle is $70 - 20 = 50°$. Then the bearing is equal to $360 - 50 =$ $\mathbf{310°T}$

(Q3) A small plane travels 20km at 160° from point A to point B. Then plane travels 15km at 230° to point C.



(Q3a) Find distance A to C:
$AC^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \times \cos(110)\,, AC = 28.81$

(Q3b) Bearing A from C:
$\theta = cos^{-1}\left(\frac{20^2+28.81^2-15^2}{2\times20\times28.81}\right)$ *Subtract from 90°
$\theta = 29.30°$, bearing$= 90 - 40 - 29.3 = \mathbf{20.7°T}$

SIMILAR FIGURES

Similar Figures and Scale Factors

- Similar figures: two shapes with identical internal angles but are different in size.
- Scale factor: a ratio that compares matching side lengths of two similar figures.

$$\boldsymbol{y = kx}$$

- $\boldsymbol{k}$ **:** scale factor between shapes.
- $\boldsymbol{y}$ **:** length of a side of a shape y.
- $\boldsymbol{x}$ **:** length of a side of another shape x.

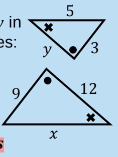
Scale Factors and Length/Area/Volume

- Find scale factor "k" between side lengths of the small shape to the larger shape:

1-Dimension: Shape Side Lengths
First Shape → $\times k$ → Second Shape
2-Dimension: Area and Surface Area
First Shape → $\times k^2$ → Second Shape
3-Dimension: Volume and Capacity
First Shape → $\times k^3$ → Second Shape

Similar Figure Examples

(Q1) Find the length of x and y in following pair of similar triangles:



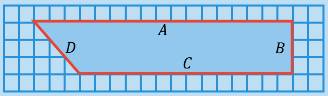
- Finding scale factor k:

$k = 9 \div 3 = 12 \div 4 = 3$

- Finding side x and y:

$x = k \times 5 = 3 \times 5 = \mathbf{15\ units}$
$y = 12 \div k = 12 \div 3 = \mathbf{4\ units}$

(Q2) A drawing of a plan for a pool is shown below. Each square on grid is $2mm \times 2mm$ and represents $1m \times 1m$ in real life.



(Q2a) How much longer is A than C in real life?
Scale factor is $2mm : 1000mm = 1{:}\,500$
$A - C = (17 \times 500) - (14 \times 500) = \mathbf{1500mm}$

(Q2b) Find area of the top of the real pool:
Plan drawing has 46.5 squares (removing half squares for triangle) $= 46.5 \times 2 \times 2 = 186mm^2$
$186 \times 500^2 = 46{,}500{,}000mm^2 = \mathbf{46.5m^2}$

(Q3) A cylindrical water tank has a height of 12m. A model water tank is built that is 3m tall.

(Q3a) What is the scale factor of the model water tank compared to the real water tank?
$k = model \div real = 12 \div 3 = \mathbf{4}$

(Q3b) If the radius of the top of the model 1m, what is the radius of the real water tank?
$= 1 \times k = 1 \times 4 = \mathbf{4m}$

(Q3b) If the surface area of the model is 14m^2, what is surface area of the real water tank?
$= 14 \times k^2 = 14 \times 4^2 = 14 \times 16 = \mathbf{224m^2}$

(Q3b) If the volume of the real tank is 1800m^3, what is the volume of the model water tank?
$= 1800 \div k^3 = 1800 \div 4^3 = \mathbf{28.125m^3}$

UNIVARIATE DATA

STATISTICAL INVESTIGATIONS

Statistical Investigation Process

- A cyclical (*i.e.* repeated) process that reflects how statisticians solve real-world problems:

Step 1	Analyse the problem and create relevant questions to be answered.
Step 2	Design and implement a plan to collect or obtain the data.
Step 3	Calculate statistics and plot graphs to analyse the collected data.
Step 4	Interpret the results by relating to the initial question and then communicate the findings.
Step 5	Return to step 1 if additional problems are identified in findings.

TYPES OF VARIABLES

Types of Statistical Variables

- Two primary types of data:

Numerical or Quantitative Data	**Categorical or Qualitative Data**
Discrete or Continuous	Ordinal or Nominal

- Numerical or Quantitative: have values that describe a measurable quantity as a number, like 'how many' or 'how much'.
  - Discrete: can take whole values (e.g. number of children or number of cars).
  - Continuous: can take any value including decimals (e.g. height, weight or time).
- Categorical or Qualitative: have values that describe a 'quality' or 'characteristic' of data.
  - Ordinal: observations that can logically ordered or ranked (e.g. academic grades such as A, B, C, D, E or clothing sizes such as small, medium, large, extra large).
  - Nominal: observations that cannot be ordered logically (e.g. eye colour, brands, gender, religion, car models).

DESCRIBING DATA SETS

Measures of Location and Spread

- Mean (*a.k.a.* average):

Step 1	Add together all the data values in the entire set.
Step 2	Divide the sum from step 1 by the total amount of numbers in the set.

- Median (*a.k.a.* middle number and Q$_2$):

Step 1	Order all numbers in ascending order (i.e. from smallest to largest).
Step 2	If there's an odd number of values, median is the middle number. If there's an even number of values, median average of two numbers.

- Mode: the most common data value.
  - There can be more than one mode.
  - If all data values appear once, no mode.
- Range: largest value subtract the smallest.
- Upper and Lower Quartile (*a.k.a.* Q$_1$ / Q$_3$):
  - If there's no number halfway, find the average of two central numbers instead.

Q$_1$ *Lower*	Data value halfway between the numbers before the value of Q$_2$.
Q$_3$ *Upper*	Data value halfway between the numbers after the value of Q$_2$.

- Interquartile Range (*a.k.a.* IQR):

$$\boldsymbol{IQR = Q_3 - Q_1}$$

- Standard Deviation: measure of how far the data set is away from the mean (average).

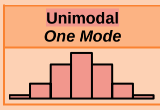
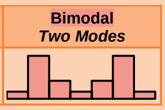
Outliers in a Data Set

- Value too large/small compared to data set:

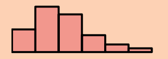
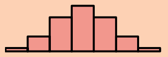
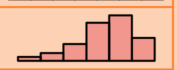
Lower Outlier	Lower outlier if a data value is less than $Q_1 - (1.5 \times IQR)$.
Upper Outlier	Upper outlier if a data value is greater than $Q_3 + (1.5 \times IQR)$.

Modality and Shape of Distributions

- Modality of a distribution:

Unimodal *One Mode*	**Bimodal** *Two Modes*
	

- Shape (*a.k.a.* skewness) of a distribution:

Skewed Right (Positive) $Mean > Median$	
Symmetrical $Mean = Median$	
Skewed Left (Negative) $Mean < Median$	

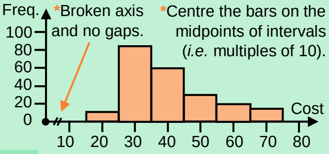
HISTOGRAMS

Histogram Example

(Q1) Costs of customers buying petrol are:

Cost ($)	**Freq.**	**Cost ($)**	**Freq.**
$15 \le x < 25$	9	$45 \le x < 55$	30
$25 \le x < 35$	85	$55 \le x < 65$	20
$35 \le x < 45$	62	$65 \le x < 75$	15

(Q1a) Create a histogram of this data:



*Broken axis and no gaps.

*Centre the bars on the midpoints of intervals (*i.e.* multiples of 10).

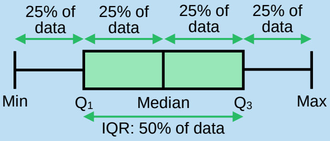
(Q1b) Describe the distribution for the costs:

- Location: Estimated mean cost of petrol is \$40.54. Modal class cost is \$25 to \$35 and the median class cost is \$35 to \$45.
- Spread: range of the costs is \$60 (\$75 - \$15) and the estimated standard deviation of the petrol costs is \$12.86.
- Shape: distribution is skewed to the right, is unimodal and contains no gaps/outliers.

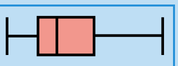
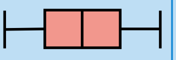
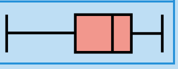
BOXPLOTS

Five Number Summary

- To draw a boxplot, collect and calculate the five statistics: min, Q$_1$, median, Q$_3$ and max.
  - Outliers are separated from a boxplot and represented by an asterisk symbol (*i.e.* *).

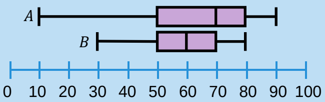


Shape and Spread of Boxplots

Skewed Right (Positive) $Mean > Median$	
Symmetrical $Mean = Median$	
Skewed Left (Negative) $Mean < Median$	

Analysing Boxplot Examples

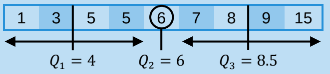
(Q1) Exam scores from classes A and B are:



(Q1a) Which class performed better?
Class A had higher scores as it had a higher Q$_2$, Q$_3$ and max. However, **Class B** was more consistent as it had a lower IQR than Class A.

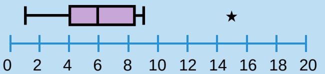
(Q1b) Which class may have an outlier? Verify.
Class A has a score well below score for Q$_1$.
$10 < 50 - \big(1.5 \times (80 - 50)\big) \rightarrow 10 < 5$
Therefore, **not** an outlier, however, quite close.

(Q2) Create a boxplot for the following data:



- Using formula to test if 15 is upper outlier:

$15 > 8 + \big(1.5 \times (8.5 - 4)\big) \rightarrow 15 > 14.75 \rightarrow Yes$



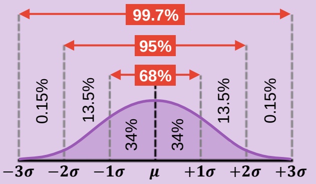
NORMAL DISTRIBUTION

Normal Distribution *(Bell Curve)*

- Has greater probability closer to the mean.
- As standard deviation (*i.e.* σ) increases, the bell curve becomes more spread out and flat.

$\boldsymbol{X \sim N(\mu, \sigma^2)}$	• $\boldsymbol{\mu}$ **:** mean • $\boldsymbol{\sigma}$ **:** standard deviation

Bell Curve Shape and 68/95/99.7 Rule



- Symmetrical: 50% of scores are above the mean and 50% of scores are below mean.
- 68% of scores lie within 1 S.D. of the mean.
- 95% of scores lie within 2 S.D. of the mean.
- 99.7% of scores lie within 3 S.D. of the mean.

► Topic Is Continued In Next Column ◄

NORMAL DISTRIBUTION

Z-Scores *(Standardised Scores)*

- Simplifies all normal distributions to a mean of 0 and a standard deviation of 1.
- Shows how many standard deviations above or below the mean that each score (*i.e.* x) is.

$\boldsymbol{Z \sim N(0, 1^2)}$	$\boldsymbol{z = \frac{x - \mu}{\sigma}}$

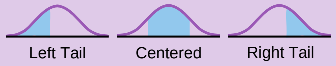
Distribution Quantiles/Percentiles

- a% of data lies below the a^{th} quantile.

$\boldsymbol{P(X < k) = a}$	• $\boldsymbol{a}$ **:** quantile $0 < a < 1$

ClassPad Main App Normal Distribution

$\boldsymbol{P(A \le X \le B)}$	$normCDF(A, B, \sigma, \mu)$
Find $\boldsymbol{k}$ given $\boldsymbol{P(X \le k)}$	$invNormCDF$ $(Tail\ Setting, P(X \le k), \sigma, \mu)$

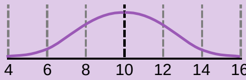


Finding Probabilities

$P(X = a)$ cannot be calculated	
$P(X < a) =$ $P(X \le a)$	$P(X > a) =$ $P(X \ge a)$

Normal Distribution Examples

(Q1) X is normal distributed with a mean of 10 and a standard deviation of 2. Determine:



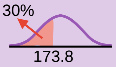
*Create bell curve with μ and σ.

(Q1a) $P(x = 10) = \mathbf{0}$ Cannot determine.	**(Q1b)** $P(x < 10)$ $P(x < \mu) = \mathbf{0.5}$
(Q1c) $P(8 < x < 12)$ $P(-1\sigma < x < +1\sigma)$ $= \mathbf{0.64}$	**(Q1d)** $P(4 < x < 16)$ $P(-3\sigma < x < +3\sigma)$ $= \mathbf{0.997}$
(Q1e) $P(6 < x < 10)$ $P(-2\sigma < x < \mu)$ $0.135 + 0.34 = \mathbf{0.475}$	**(Q1f)** $P(8 < x < \infty)$ $P(x > 8) = P(x > -1\sigma)$ $0.34 + 0.5 = \mathbf{0.84}$

(Q2) Cost of weekly food shopping at a grocery store is normal distributed with $X \sim N(200, 50^2)$

(Q2a) Find the probability that a customer will spend between \$175 and \$225 at the store:
$X \sim N(200, 50^2)$ and finding $P(175 < x < 225)$
$= normCDF(175{,}225{,}50{,}200) = \mathbf{0.3829}$

(Q2b) Find cost that sits in the 30% quantile:



$P(x < k) = 0.30$ and finding the value of k gives
$invNormCDF(Left, 0.3{,}50{,}200)$
$k = 30\%\ quantile = \mathbf{173.8}$

(Q3) Maria scored 65% & 70% in her english & maths exam respectively. English exam $\mu = 60$ & $\sigma = 3$ and maths exam $\mu = 65$ & $\sigma = 2.5$. Use z-scores to find which was her best result.

English: $z = \frac{65-60}{3} = 1.67$ Maths: $z = \frac{70-65}{2.5} = 2$

Therefore, **maths** was her best result.

ALGEBRA

SUBSTITUTION

Order of Operations *(BIMDAS)*

B Brackets
I Indices
M Multiply
D Divide
A Addition
S Subtract

- If there is × and ÷ or + and – in the same question, work through it from left to right.
- Indices multiply a number by itself: $(-2)^2 = -2 \times -2 = 4$
- Find which number multiplied by itself to give the number under a square root: $\sqrt{16} = 4$

Substitution Examples

(Q1) $u = -3$, $a = 0.5$, $s = 16$ find $v = \sqrt{u^2 + 2as}$
$v = \sqrt{(-3 \times -3) + (2 \times 0.5 \times 16)} = \sqrt{25} = \mathbf{5}$

(Q2) $A = 5.8$, $a = 4.2$, $s = 5$ find $x = \frac{1}{2}(A - B)T$
$x = \frac{1}{2}(5.8 - 4.2) \times 5 = \frac{1}{2} \times 1.6 \times 5 = 0.8 \times 5 = \mathbf{4}$

SOLVING LINEAR EQUATIONS

Solving Linear Equations Examples

(Q1) Solve the following linear equations for x:

(Q1a) $15 = 6x - 3$	**(Q1b)** $-3(x - 5) = 6$
$15 + 3 = 6x$	$x - 5 = 6 \div -3$
$18 = 6x$	$x - 5 = -2$
$x = 18 \div 6 = \mathbf{3}$	$x = -2 + 5 = \mathbf{3}$

(Q1c) $3(2x - 7) - 5(4 - 2x) = 7(x + 1) + 6$
$6x - 21 - 20 + 10x = 7x + 7 + 6$
$16x - 41 = 7x + 13 \rightarrow 9x = 54 \rightarrow x = \mathbf{6}$

(Q1d) $-4(-3 + x) = 7x - 10$
$12 - 4x = 7x - 10 \rightarrow 22 = 11x \rightarrow x = \mathbf{2}$

► Topic Is Continued On Next Page ◄

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SOLVING LINEAR EQUATIONS

Solving Linear Equations Examples

(Q1) Solve the following linear equations for x:

(Q1e) $\frac{3x}{2}-\frac{1}{5}=\frac{5}{6}$

$\frac{5(3x)-2(1)}{2\times 6}=\frac{5}{6}$

$\frac{15x-2}{12}=\frac{5}{6}$

$6(15x-2)=12(5)$

$90x-12=60$

$90x=60+12$

$90x=72$

$x=72\div 90=\mathbf{4/5}$

(Q1f) $\frac{2x-3}{5}=\frac{3-4x}{2}$

$2(2x-3)=5(3-4x)$

$4x-6=15-20x$

$24x=21$

$x=21\div 24=\mathbf{7/8}$

LINEAR EQUATIONS

Forms of Linear Equations

- Standard form of a linear equation:

$\boldsymbol{y=mx+c}$	$\boldsymbol{m=\frac{rise}{run}=\frac{y_2-y_1}{x_2-x_1}}$

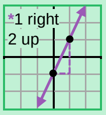
- $\boldsymbol{m}$**:** gradient (*i.e.* slope of the line).
  - $\boldsymbol{m>0}$**:** positive gradient (*i.e.* line travels from bottom left to top right).
  - $\boldsymbol{m<0}$**:** negative gradient (*i.e.* line travels from top left to bottom right).
- $\boldsymbol{c}$**:** $y-$intercept (*i.e.* where the equation crosses the $y-$*axis at the point* $(0,c)$).
  - Standard form link with table of values:

$\boldsymbol{y=2x-1}$									
$\boldsymbol{x}$	-4	-3	-2	-1	0	1	2	3	4
$\boldsymbol{y}$	-9	-7	-5	-3	-1	1	3	5	7

$\boldsymbol{m}$: +2 +2 +2 +2 +2 +2 +2 +2

$\boldsymbol{c}$: value of y when x is 0 is y-intercept

  - Standard form link with the graph:



$\boldsymbol{y=2x-1}$

$\boldsymbol{m}$ **:** For every 1 to the right, move m up (if m is positive) or down (if m is negative).

$\boldsymbol{c}$ **:** y co-ord of y-intercept.

- Intercept form of a linear equation:

$\boldsymbol{ax+by=c}$	$\boldsymbol{x-int=\frac{c}{a}}$	$\boldsymbol{y-int=\frac{c}{b}}$

- $\boldsymbol{x-int}$**:** x-intercept of the line $(x,0)$.
- $\boldsymbol{x-int}$**:** y-intercept of the line $(0,y)$.

Finding Linear Equations

- Determine formula given two random co-ordinates (x_1,y_1) and (x_2,y_2) on the line.

Step 1	Calculate gradient $m=\frac{rise}{run}=\frac{y_2-y_1}{x_2-x_1}$
Step 2	Using $y=mx+c$, substitute either (x_1,y_1) or (x_2,y_2) into x and y, sub in m and rearrange to solve for c.

- Find rule given (x_1,y_1) and line gradient m.

Step 1	Using $y=mx+c$, sub (x_1,y_1) into x and y, sub in m and then rearrange to solve for c.

Linear Equations Examples

(Q1) Determine the equations of the following:

$\boldsymbol{x}$	5	4	3	2
$\boldsymbol{y}$	8	6	4	2

$+2$ $+2$ $+2$

Jump (m) is $+2$ as x goes backwards. Continuing pattern gives $y=-2$ when $x=0$ giving $c=-2$.

$\boldsymbol{y=2x-2}$

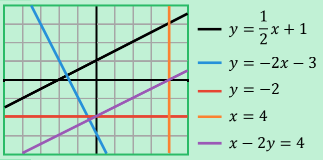
$\boldsymbol{x}$	0	2	4	6
$\boldsymbol{y}$	9	6	3	0

-3 -3 -3

Jump (m) is -1.5 as x skips 2 values each time ($\div$ by 2). y-intercept is $(0,9)$ which gives $c=9$.

$\boldsymbol{y=-1.5x+9}$

(Q2) Graph the following linear equations:



(Q3) Does $(2,-14)$ lie on $y=-x-10$?

$y=-2-10\rightarrow y=-12\neq -14$, **No** it doesn't.

(Q4) What is the equation of the line that passes through the points $(1,2)$ and $(-3,10)$?

$m=\frac{rise}{run}=\frac{y_2-y_1}{x_2-x_1}=\frac{10-2}{-3-1}=-\frac{8}{2}=-4$

$y=mx+c\rightarrow 2=4(1)+c\rightarrow c=-2$

Therefore, the equation is $\boldsymbol{y=-4x-2}$

(Q5) Mark's current age is multiplied by three and two is subtracted from answer, result is equal to his Dad's age. In 10 years the ages show that Mark's Dad is twice as old as Mark. How old is Mark and his Dad currently?

- Create expression for Mark and his Dad:

Let $x=$ Mark currently, $3x-2=$ Dad currently.

- Find expressions after 10 years:

$x+10=$ Mark in 10 yrs, $3x+8=$ Dad in 10 yrs

- Equate expression and solve for Mark (x):

Double Mark's age equals Dad's age in 10 yrs:

$2(x+10)=3x+8$

$2x+20=3x+8$

$x=\mathbf{12}$ years

	Now	**10 yrs**
Mark	12	22
Dad	34	44

SIMULTANEOUS EQUATIONS

Solving Equations by Substitution

Step 1	Substitute one equation into the other. Place it inside brackets.
Step 2	Expand the brackets and simplify by collecting like terms.
Step 3	Solve the equation found in step 2 for the first variable.
Step 4	Substitute answer from step 3 back into one of the original equations and solve for second variable.
Step 5	Present both answers (*i.e.* display the values of both x and y).

Solving Equations by Elimination

Step 1	Stack the two equations on top of each other (*i.e.* in a single column).
Step 2	$+$, $-$, $\times$ and/or $\div$ the two equations so that one of the two variables (either x or y) are eliminated to solve for the first variable.
Step 3	Substitute this answer back into either of the original equations and solve for the second variable.
Step 4	Present both answers (*i.e.* display the values of both x and y).

Solving Equations Graphically

Step 1	Graph both lines on the same set of axes using plotting techniques.
Step 2	Find the co-ordinates of where the two lines intercept, this is the solution for both variables x and y.

- Where the two equations represent cost and revenue functions, the break-even point (*i.e.* where profit $=$ \$0) is the intercept co-ords).

Simultaneous Equations Examples

(Q1) Solve these equations by substitution: $x+2y=10$ and $y=x+2$.

- Substitute one equation into the other:

$x+2y=10$

$x+2(x+2)=10$

$x+2x+4=10$

$3x+4=10$

$3x=6$

$x=2$

- Substitute x back into original equation:

$y=x+2$

$y=2+2=4$

Solution for equations: $\boldsymbol{x=2}$ and $\boldsymbol{y=4}$

(Q2) Solve these equations by elimination: $5x+3y=11$ and $3x+2y=6$.

- Stack equations and eliminate a row:

R1: $5x+3y=11$

R2: $3x+2y=6$

$2R1\ -\ 3R2$ eliminates y

2R1: $10x+6y=22$

3R2: $9x+6y=18$

3R1 - 2R2: $x=4$

- Substitute x back into either equation:

R1: $5x+3y=11$

$5(4)+3y=11$

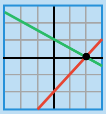
$20+3y=11$

$3y=-9\rightarrow y=-3$

Solution for equations: $\boldsymbol{x=4}$ and $\boldsymbol{y=-3}$

(Q3) Solve these equations graphically: $y=x-2$ and $2x+4y=4$.

- Graph both equations and find intercept:



— $y=x-2$ — $2x+4y=4$

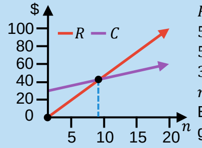
The two lines intersect at the point $(2,0)$, therefore the solution for both equations is $\boldsymbol{x=2}$ and $\boldsymbol{y=0}$.

(Q4) A lemonade stand sells a glass (n) for \$5 to each customer. To run the stand, it costs \$30 plus \$1.50 per glass of lemonade sold.

(Q4a) Find the equation for revenue and cost:

Revenue: $\boldsymbol{R=5n}$ Cost: $\boldsymbol{C=1.5n+30}$

(Q4b) Plot both lines and determine how many glasses need to be sold to break-even:



$R=C$

$5n=1.5n+30$

$5n-1.5n=30$

$3.5n=30$

$n=30\div 3.5=8.6$

Break-even at **9** glasses sold.

(Q4c) Find the profit/loss at 15 glasses sold.

At $n=15$, it is clear that $R>C\rightarrow$ Profit made.

When $n=15$, $R=5n=5\times 15=\$75$

When $n=15$, $C=1.5\times 15+30=\$52.50$

$Profit=R-C=75-52.5=\boldsymbol{\$22.50}$

PIECEWISE AND STEP GRAPHS

Graph Inequality Notation

○ **Empty Dot**	Less than (<) or Greater than (>)
● **Filled Dot**	Less than or equal to (≤) or Greater than or equal to (≥)

Step Graphs

- Series of non-overlapping horizontal lines plotted on same axes in form $y=number$.

Piecewise Graphs

- Series of different linear equations with only small parts are plotted together on axes.

$$\begin{cases} \boldsymbol{equation\ 1} & \boldsymbol{inequality\ condition} \\ \boldsymbol{equation\ 2} & \boldsymbol{inequality\ condition} \end{cases}$$

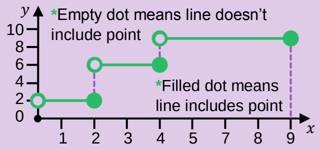
► Topic Is Continued In Next Column ◄

PIECEWISE AND STEP GRAPHS

Piecewise and Step Graphs Examples

(Q1) Graph the step graph based on the table:

$\boldsymbol{x}$	$0<x\le 2$	$2<x\le 4$	$4<x\le 9$
$\boldsymbol{y}$	2	6	9



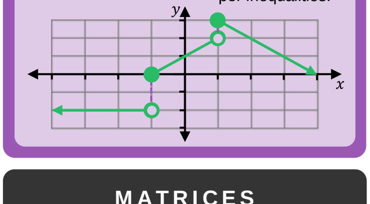
*Empty dot means line doesn't include point

*Filled dot means line includes point

(Q2) Graph the piecewise function below:

$$y=\begin{cases} -2 & x<-1 \\ x+1 & -1\le x<1 \\ -x+4 & x\ge 4 \end{cases}$$

*Graph all three lines and trim as per inequalities.



MATRICES

MATRIX ARITHMETIC

Matrix Terminology

- A matrix (the plural is matrices) is an array (*a.k.a.* a grid) of numbers of a certain size.
- Matrix order/size: the number of rows and columns that are in a matrix.
  - When writing a matrix, the number of rows always goes first then number of columns.

$\boldsymbol{n\times m\ Matrix}$
$\boldsymbol{m}$ **:** number of rows in a matrix. $\boldsymbol{n}$ **:** number of columns in a matrix.

- Matrix elements/entries (a_{ij}): represents the element i^{th} row and j^{th} column in matrix A.

$$\boldsymbol{2\times 3\ Matrix\ A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{22} \end{bmatrix}}$$

Common Types of Matrices

$\begin{bmatrix} 1 & 4 & 1 \end{bmatrix}$	**Row Matrix:** consists of only one row.
$\begin{bmatrix} 5 \\ 9 \end{bmatrix}$	**Column Matrix:** consists of only one column.
$\begin{bmatrix} 2 & 6 \\ 5 & 3 \end{bmatrix}$	**Square Matrix:** a matrix of any size with a condition that $\#\ of\ rows\ =\ \#\ of\ columns$
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	**Zero Matrix (0):** a matrix of any size with 0 as all entries.
$I_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**Identity Matrix ($\boldsymbol{I_n}$):** square matrix with all elements in the leading diagonal (goes from top left to bottom right) as 1 and all other entries as 0.

Matrix Arithmetic

- Adding and subtracting matrices: can only be possible if both matrices have same size.

$\boldsymbol{A+B=a_{ij}+b_{ij}}$	$\boldsymbol{A-B=a_{ij}-b_{ij}}$
$\boldsymbol{a_{ij}\pm b_{ij}}$ **:** add/subtract matching entries.	

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\pm\begin{bmatrix} e & f \\ g & h \end{bmatrix}=\begin{bmatrix} a\pm e & b\pm f \\ c\pm g & d\pm h \end{bmatrix}$$

- Scalar multiplication: can use on any size.

$\boldsymbol{kA=ka_{ij}}$
$\boldsymbol{k}$**:** scalar multiplier (*i.e.* a number). $\boldsymbol{ka_{ij}}$**:** multiply all entries in matrix by k.

$$k\times\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}=\begin{bmatrix} k\times a & k\times b & k\times c \\ k\times d & k\times e & k\times f \end{bmatrix}$$

- Multiplying matrices: multiply each element in row of 1st matrix with matching element from each column of 2nd matrix and add.

$\boldsymbol{Matrix\ A=m\times n}$ **&** $\boldsymbol{Matrix\ B=p\times q}$ $\boldsymbol{A\times B}$ **only possible if** $\boldsymbol{n=p}$ **Matrix of size** $\boldsymbol{m\times q}$ **is created**

  - E.g. 1: $(1\times 3)(3\times 1)=1\times 1$ Matrix

$$\begin{bmatrix} a & b & c \end{bmatrix}\times\begin{bmatrix} d \\ e \\ f \end{bmatrix}=\begin{bmatrix} ad+be+cf \end{bmatrix}$$

  - E.g. 2: $(1\times 3)(3\times 2)=1\times 2$ Matrix

$$\begin{bmatrix} a\ b\ c \end{bmatrix}\times\begin{bmatrix} d & e \\ f & g \\ h & j \end{bmatrix}=\begin{bmatrix} ad+bf+ch & ae+bg+cj \end{bmatrix}$$

  - E.g. 3: $(2\times 2)(2\times 2)=2\times 2$ Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\times\begin{bmatrix} e & f \\ g & h \end{bmatrix}=\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Common Rules of Matrix Arithmetic

$\boldsymbol{AI=A}$	Multiplying matrix by identity matrix returns original matrix.
$\boldsymbol{0A=0}$	Multiplying matrix by zero matrix returns zero matrix.
$\boldsymbol{AB\neq BA}$	Matrix multiplication is not commutative; order matters.
$\boldsymbol{A(B\pm C)=AB\pm AC}$	Matrix addition/subtraction is associative; can expand.

APPLYING MATRICES

Applying Matrices Examples

(Q1) Given the following matrices, determine:

$A=\begin{bmatrix} 5 & 1 \\ -1 & 7 \end{bmatrix}$ $B=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $C=\begin{bmatrix} 0 & 6 \\ 2 & 2 \end{bmatrix}$ $D=\begin{bmatrix} 3 & 5 & 2 \\ 1 & 0 & 4 \end{bmatrix}$

(Q1a) $-2B$

$-2B=-2\begin{bmatrix} 2 \\ 5 \end{bmatrix}=\begin{bmatrix} \mathbf{-4} \\ \mathbf{-10} \end{bmatrix}$

(Q1b) $5a_{21}-c_{22}\times d_{12}$

$(5\times -1)-2\times 5$

$=-5-2\times 5=\mathbf{-15}$

(Q1c) $A+C$

$\begin{bmatrix} 5+0 & 1+6 \\ -1+2 & 7+2 \end{bmatrix}=\begin{bmatrix} \mathbf{5} & \mathbf{7} \\ \mathbf{1} & \mathbf{9} \end{bmatrix}$

(Q1d) BD **impossible**

$(2\times 1)(2\times 3)$ is not compatible (as $1\neq 2$)

(Q1d) AD $(2\times 2)(2\times 3)$ is compatible and will produce a 2×3 matrix as the answer:

$\begin{bmatrix} 5 & 1 \\ -1 & 7 \end{bmatrix}\times\begin{bmatrix} 3 & 5 & 2 \\ 1 & 0 & 4 \end{bmatrix}=\begin{bmatrix} \mathbf{16} & \mathbf{25} & \mathbf{14} \\ \mathbf{4} & \mathbf{-5} & \mathbf{26} \end{bmatrix}$ working out:

$$\begin{bmatrix} 5\times 3+1\times 1 & 5\times 5+1\times 0 & 5\times 2+1\times 4 \\ -1\times 3+7\times 1 & -1\times 5+7\times 0 & -1\times 2+7\times 4 \end{bmatrix}$$

(Q1e) Find matrix X given that $C+X=I_2$

$\begin{bmatrix} 0 & 6 \\ 2 & 2 \end{bmatrix}+X=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\rightarrow X=\begin{bmatrix} 0 & 6 \\ 2 & 2 \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \mathbf{-1} & \mathbf{6} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$

(Q2) Matrix Y shows burgers sold at a canteen for recess (row 1) & lunch (row 2) over a Monday and Tuesday this week: $Y=\begin{bmatrix} 14 & 20 \\ 63 & 98 \end{bmatrix}$

(Q2a) Find matrix $Z=Y\times\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and explain it.

$Z=\begin{bmatrix} 14 & 20 \\ 63 & 98 \end{bmatrix}\times\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\begin{bmatrix} \mathbf{34} \\ \mathbf{161} \end{bmatrix}$ and shows the total amount of burgers sold at recess and lunch.

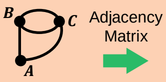
(Q2b) Find profit if profit matrix $P=\begin{bmatrix} 2.5 & 3.5 \end{bmatrix}$

$Profit=\begin{bmatrix} 34 \\ 161 \end{bmatrix}\times\begin{bmatrix} 2.5 & 3.5 \end{bmatrix}=\begin{bmatrix} 648.5 \end{bmatrix}$ which means that profit is equal to **\$648.50**.

ROUTE MATRICES

Properties of an Adjacency Matrix

- Matrix that shows how many times each vertex is connected (adjacent) to another vertex by a single edge.
- From vertices on left, To vertices are above.
- Loops only count once in adjacency matrices.
- Adjacency matrix for an undirected graph:



$$\begin{array}{cc|ccc} & & & \boldsymbol{To} & \\ & & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} \\ & \boldsymbol{A} & 0 & 1 & 1 \\ \boldsymbol{From} & \boldsymbol{B} & 1 & 0 & 2 \\ & \boldsymbol{C} & 1 & 2 & 0 \end{array}$$

- Adjacency matrix for a directed graph:

$$\begin{array}{cc|ccc} & & & \boldsymbol{To} & \\ & & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} \\ & \boldsymbol{A} & 0 & 0 & 0 \\ \boldsymbol{From} & \boldsymbol{B} & 1 & 0 & 1 \\ & \boldsymbol{C} & 1 & 1 & 0 \end{array}$$

Properties of a Route Matrix

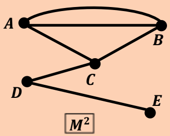
- Any entries in an adjacency matrix raised to the n^{th} power indicates how many ways it is possible to move to and from the points corresponding to that entry in n steps.

1-Step $=$ $\boldsymbol{M}$	**2-Step** $=$ $\boldsymbol{M^2}$
1-Step or 2-Step $=$ $\boldsymbol{M}$ $+$ $\boldsymbol{M^2}$	

- $\boldsymbol{M}$ **:** adjacency matrix.
- $\boldsymbol{M^2}$ **:** adjacency matrix squared.
- **1-Step :** a matrix showing number of ways to travel between vertices in 1 step.
- **2-Step :** a matrix showing number of ways to travel between vertices in 2 steps.
- **1 or 2-Step :** a matrix showing number of ways to travel between vertices in 1 or 2 steps (*i.e.* combines 1-Step and 2-Step).

Transition Matrix Examples

(Q1) A connected graph and related transition matrices M, M^2 and $M+M^2$ are shown below:



$$\boxed{\boldsymbol{M}}\quad\begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{\boldsymbol{M^2}}\quad\begin{bmatrix} 5 & 1 & 2 & 1 & 0 \\ 1 & 5 & 2 & 1 & 0 \\ 2 & 2 & 3 & 0 & 1 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\boxed{\boldsymbol{M+M^2}}\quad\begin{bmatrix} 5 & 3 & 3 & 1 & 0 \\ 3 & 5 & 3 & 1 & 0 \\ 3 & 3 & 3 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(Q1a) Explain how the highlighted entry "2" is calculated in transition matrix M.

- Matrix M shows 1-Step transitions.

The highlighted entry in matrix shows that there are **2 ways** of going from **A to B in 1 step**.

(Q1b) Explain how the highlighted entry "0" is calculated in transition matrix M^2.

- Matrix M^2 shows 2-Step transitions.

The highlighted entry shows that it is **impossible** to go from **E to D in 2 steps**.

(Q1c) Explain how the highlighted entry "3" is calculated in transition matrix $M+M^2$.

- $M+M^2$ shows 1- or 2-Step transitions.

The highlighted entry in matrix shows that there are **3 ways** of going from **C to A in 1 or 2 steps**.

ATAR Math Applications
Units 1 & 2 Exam Notes

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