Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(6 marks)**

(a) Determine, giving answers rounded to one decimal place,

(i) the vector projection of $12\mathbf{i}+37\mathbf{j}$ onto $75\mathbf{i}-94\mathbf{j}$. (2 marks)

$$\frac{-2578}{14461}(75\mathbf{i}-94\mathbf{j})$$

$$=-13.4\mathbf{i}+16.8\mathbf{j}$$

(ii) the vector projection of a force of 60N on bearing 333º onto a force of 30N on a bearing 115º. (2 marks)

$$60\cos(27+115)=-47.3$$

$$115+180=295$$

Projection is force of 47.3N on bearing 295º.

(b) Determine the values of a and b given that the vectors (2, -3) and (a , 6) are parallel and the vectors (2, -3) and (6, b) are perpendicular. (2 marks)

$$a=\frac{6}{-3}\times 2=-4$$

$$2\times 6+(-3)b=0$$

$$b=4$$

Question 9 **(8 marks)**

(a) A teacher has to choose 3 girls and 4 boys to sit in a row for a photograph from a group of 7 girls and 6 boys who volunteered. How many possible ways can she do this, if the boys chosen have to sit next to each other? (4 marks)

Choose first: ${}^{7}C_3 \times {}^{6}C_4 = 525$

Now arrange: $4! \times (1+3)! = 576$

$525 \times 576 = 302400$

(b) A calculator is programmed to generate random numbers between 0 and 1, such as 0.9155629523 and then round them to one decimal place. How many such numbers must be generated to be certain of obtaining two identical numbers? (2 marks)

11 possible numbers can be generated (0.0, 0.1, 0.2, ..., 0.9, 1.0). Using pigeonhole principle, will need 12 to be certain of a duplicate.

(c) A student has a large selection of music tracks by four different bands (INXS, Spooky Tooth, KISS and The Clash) on their phone. Determine the smallest number of tracks on a playlist so that they will be certain to have at least six tracks by the same band. (2 marks)

Suppose have 20 tracks – five by each of the four bands. Adding one more track will ensure one of the bands must have six tracks, so require 21 tracks.

Question 10 **(7 marks)**

The work done, in joules, by a force $\mathbf{F}$ Newtons in changing the displacement of an object $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$.

(a) Calculate the work done by a force (15, 22) N in moving an object (3, 2) m. (1 mark)

$\boxed{15\times 3+22\times 2=89 \text{ J}}$

(b) Calculate the work done by a force of 25 N that moves an object 6 m if

(i) the force acts parallel to the direction of movement. (1 mark)

$\boxed{25\times 6\times \cos(0)=150 \text{ J}}$

(ii) the force acts perpendicular to the direction of movement. (1 mark)

$\boxed{25\times 6\times \cos(90)=0 \text{ J}}$

(iii) the force acts at an angle of 25º to the direction of movement. (1 mark)

$\boxed{25\times 6\times \cos(25)=135.9 \text{ J}}$

(c) The work done by a force in moving an object (50, -80) cm is 590 joules. If the force acts on a bearing of 115º, determine the magnitude of the force. (3 marks)

Angle between force and displacement:

$$\tan^{-1}\tfrac{-80}{50}=-58$$
$$58-25=33^\circ$$

Force:

$$F\times\sqrt{(0.5)^2+(-0.8)^2}\times\cos(33)=590$$
$$F=745.7 \text{ J}$$

Question 11 **(8 marks)**

A sub-committee of four, consisting of a chairperson, a secretary and two ordinary members is to be chosen from a larger committee of 20 people (consisting of a chairperson, a secretary and 18 ordinary members).

(a) Determine, for the sub-committee, the number of possible choices for

(i) the posts of chairperson and secretary, (1 mark)

$$\boxed{{}^{20}P_2 = 20 \times 19 = 380}$$

(ii) the two ordinary members, (1 mark)

$$\boxed{{}^{20}C_2 = 190}$$

(iii) the chairperson, secretary and two ordinary members. (2 marks)

$$\boxed{{}^{20}C_1 \times {}^{19}C_1 \times {}^{18}C_2 = 58140}$$

(b) How many sub-committees are possible in which

(i) the chairman of the larger committee is not included? (2 marks)

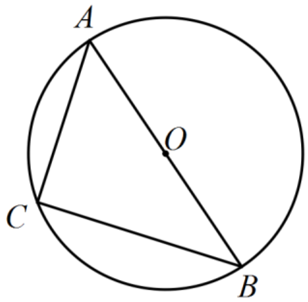
$$\boxed{{}^{19}C_1 \times {}^{18}C_1 \times {}^{17}C_2 = 46512}$$

(ii) the chairman of the larger committee is chosen as the secretary and the secretary of the larger committee is chosen as an ordinary member? (2 marks)

$$\boxed{{}^{1}C_1 \times {}^{1}C_1 \times {}^{18}C_1 \times {}^{17}C_1 = 306}$$

Question 12 **(8 marks)**

(a) Determine, with justification, the radius of the circle shown below, given that $AC = 8$ cm and $BC = 15$ cm. (2 marks)

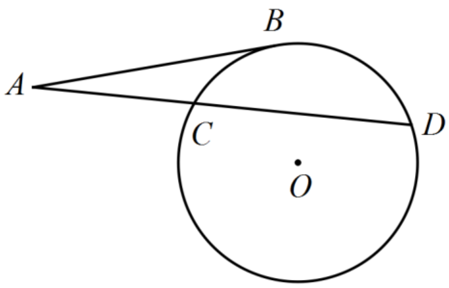


$\triangle ACB$ is right angled at C.

$$d = \sqrt{8^2 + 15^2}$$
$$= 17$$

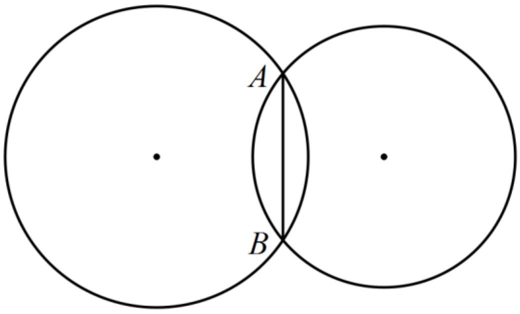
$r = 8.5$ cm

(b) Determine the length of the chord CD given that the length of the tangent AB is 15 cm and the length of the secant AD is 26 cm. (3 marks)



$$AB^2 = AC \times AD$$
$$15^2 = (26 - x) \times 26$$
$x = 17.35$ cm

(c) Two circles of radii 18 cm and 24 cm intersect at points A and B. The length of the chord AB is 28 cm. Determine how far apart the centre of the circles lie, giving your answer to three significant figures. (3 marks)



$$AB \div 2 = 14$$

$$\sqrt{24^2 - 14^2} = 19.4936$$

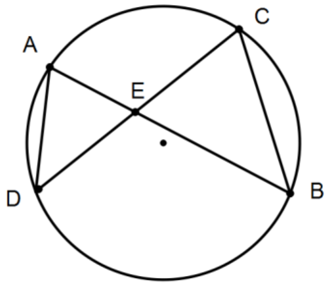
$$\sqrt{18^2 - 14^2} = 11.3137$$

$$19.4936 + 11.3137 = 30.8073$$
≈ 30.8 cm (3 sf)

Question 13 **(12 marks)**

(a) In the diagram below the chords AB and CD intersect at the point E.

The area of $\triangle EAD$ is 15cm^2.



(i) Explain why $\angle EAD = \angle ECB$ (1 mark)

Both angles stand on the arc BD.

(ii) Prove that $\triangle EAD$ is similar to $\triangle ECB$. (3 marks)

$\angle EAD = \angle ECB$	Stand on arc BD
$\angle EDA = \angle EBC$	Stand on arc AC
$\angle AED = \angle CEB$	Vertically opposite
$\triangle EAD \approx \triangle ECB$	AAA

(iii) Use your result from (ii) to show that $AE \times BE = DE \times CE$. (1 mark)

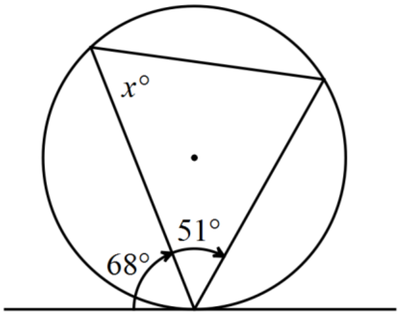
$\frac{AE}{DE} = \frac{CE}{BE}$ Ratio of sides

$\therefore AE \times BE = DE \times CE$

(iv) Find the area of $\triangle ECB$ if $CE = 2 \times AE$. (2 marks)

If $CE = 2 \times AE$ then $DE = 2 \times BE$.
Hence area of $\triangle ECB = 2 \times 2 \times Area \triangle EAD$.
Area $= 4 \times 15 = 60$ cm^2.

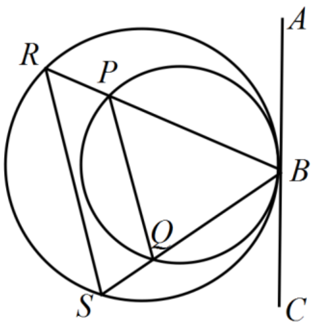
(b) Determine the size of x in the diagram below. (2 marks)



$$180-68-51=61°$$

$$x=61°$$

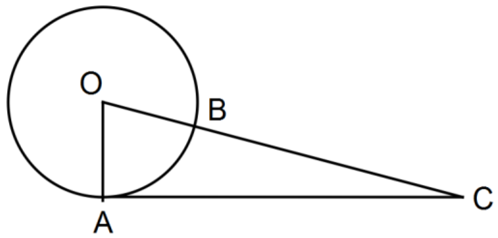
(c) The line segment ABC is a common tangent to both circles shown below. Prove that PQ is parallel to RS. (3 marks)



$\angle BPQ=\angle CBQ$ (alt seg theorem)
$\angle BRS=\angle CBQ$ (alt seg theorem)
$\angle BPQ=\angle BRS \Rightarrow PQ \parallel RS$ (corresponding angles)

Question 14 **(8 marks)**

(a) In the diagram, AC is a tangent to the circle at A, OC cuts the circle at B and $BC = 2OB$.

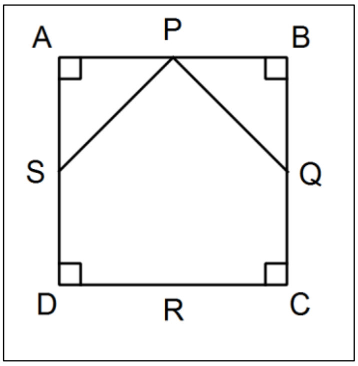


If $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, prove that $\mathbf{a} \bullet \mathbf{b} = \dfrac{|\mathbf{a}|^2}{3}$. (4 marks)

$$\begin{aligned} AC &= AO + OB + BC \\ &= -\mathbf{a} + \mathbf{b} + 2\mathbf{b} \\ &= 3\mathbf{b} - \mathbf{a} \end{aligned}$$

$$\overrightarrow{OA} \bullet \overrightarrow{AC} = 0$$
$$a \bullet (3\mathbf{b} - \mathbf{a}) = 0$$
$$3\mathbf{b} \bullet \mathbf{a} - \mathbf{a} \bullet \mathbf{a} = 0$$
$$\mathbf{a} \bullet \mathbf{b} = \frac{|\mathbf{a}|^2}{3}$$

(b) The midpoints of square $ABCD$ are $PQRS$ respectively. Use a vector method to prove that PS is perpendicular to PQ. (4 marks)



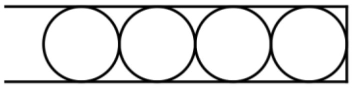
$$\overrightarrow{AP} = \mathbf{a}, \overrightarrow{AS} = \mathbf{b}$$
$$\overrightarrow{PS} = \mathbf{b} - \mathbf{a}$$
$$\overrightarrow{PQ} = \mathbf{a} + \mathbf{b}$$

$$\begin{aligned} \overrightarrow{PS} \bullet \overrightarrow{PQ} &= (\mathbf{b} - \mathbf{a}) \bullet (\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} \bullet \mathbf{b} + \mathbf{b} \bullet \mathbf{b} - \mathbf{a} \bullet \mathbf{a} - \mathbf{a} \bullet \mathbf{b} \\ &= |\mathbf{b}|^2 - |\mathbf{a}|^2 \\ &= 0 \end{aligned}$$

Hence PS is perpendicular to PQ.

Question 15 **(7 marks)**

Four different coloured balls (yellow, green, blue, red, purple or orange) are to be placed, one after another, into a tube as shown.



Determine the number of different arrangements of balls that can be made using

(a) six balls, all of different colours. (1 mark)

$$^6P_4 = 360$$

(b) two yellow and two green balls. (1 mark)

$$\frac{4!}{2! \times 2!} = 6$$

(c) one red, one purple and two blue balls. (1 mark)

$$\frac{4!}{2!} = 12$$

(d) twelve balls, two of each colour. (4 marks)

Split into possibilities:

Four, all different:
$^6P_4 = 360$

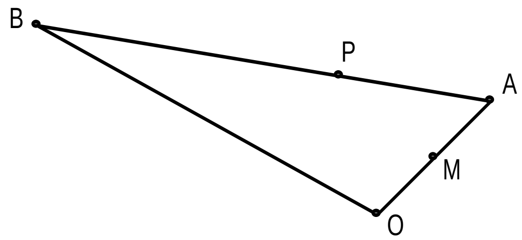
Choose two same, other two different then arrange:
$$^6C_1 \times 1 \times {}^5C_2 \times \frac{4!}{2!} = 720$$

Choose two same, another two same then arrange:
$$\frac{6 \times 5}{2!} \times \frac{4!}{2! \times 2!} = 90$$

Total of 1170 arrangements.

Question 16 **(5 marks)**

In the triangle below, $\mathbf{a}=\overrightarrow{OA}$, $\mathbf{b}=\overrightarrow{OB}$, M is the midpoint of OA and P is a point on AB such that $\overrightarrow{AP}:\overrightarrow{PB}=1:3$.



(a) Express each of the following in terms of **a** and /or **b**.

(i) $\overrightarrow{BA}$ (1 mark)

$$\boxed{\mathbf{a}-\mathbf{b}}$$

(ii) $\overrightarrow{OP}$ (1 mark)

$$\boxed{\begin{aligned}\overrightarrow{OP}&=\mathbf{b}+\frac{3}{4}(\mathbf{a}-\mathbf{b})\\&=\frac{3}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}\end{aligned}}$$

(iii) $\overrightarrow{MP}$ (1 mark)

$$\boxed{\begin{aligned}\overrightarrow{MP}&=-\frac{1}{2}\mathbf{a}+\frac{3}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}\\&=\frac{1}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}\end{aligned}}$$

(b) If $\mathbf{a}=\mathbf{i}+2\mathbf{j}$ and $\mathbf{b}=-9\mathbf{i}+4\mathbf{j}$, determine $\left|\overrightarrow{MP}\right|$. (2 marks)

$$\boxed{\begin{aligned}&\frac{1}{4}\left|\begin{pmatrix}1-9\\2+4\end{pmatrix}\right|\\&=\frac{1}{4}\left|\begin{pmatrix}-8\\6\end{pmatrix}\right|\\&=2.5\end{aligned}}$$

Question 17 **(7 marks)**

A small ball leaves point M and travels with a constant velocity of $2\mathbf{i}+3\mathbf{j}$ ms^{-1}.

(a) Determine

(i) the distance travelled by the ball in 3 seconds, rounding your answer to two decimal places. (2 marks)

$$3\times\left\|\begin{bmatrix}2\\3\end{bmatrix}\right\|=3\times\sqrt{13}\approx 10.82 \text{ m}$$

(ii) the time taken for the ball to travel 40 m, to one decimal place. (1 mark)

$$\frac{40}{\sqrt{13}}=11.1 \text{ s}$$

(b) Determine the least distance between the ball and a point located at $6\mathbf{i}+5\mathbf{j}$ relative to M. (4 marks)

Distance MP $\sqrt{6^2+5^2}=\sqrt{61}$

Angle of MP $\tan^{-1}\dfrac{5}{6}=39.8°$

Angle of path $\tan^{-1}\dfrac{3}{2}=56.3°$

Min dist: $\sqrt{61}\times\sin(56.3-39.8)\approx 2.22$ m

Question 18 **(8 marks)**

(a) How many integers between 1000 and 9999 inclusive are multiples of 2, 3 or 7? (4 marks)

4500 multiples of 2
3000 of 3
1285 of 7
8785

1500 of 2 and 3
642 of 2 and 7
428 of 3 and 7
2570

214 of 2, 3 and 7
214

8785-2570+214 = 6429 total multiples

(b) Determine the number of different arrangements of three letters selected from those in the word LEVELLED. (4 marks)

LLL EEE V D

Choose then arrange method

3 different: ${}^4C_3 \times 3! = 24$

2 E's, 1 other: ${}^3C_1 \times \frac{3!}{2!} = 9$

2 L's, 1 other: 9

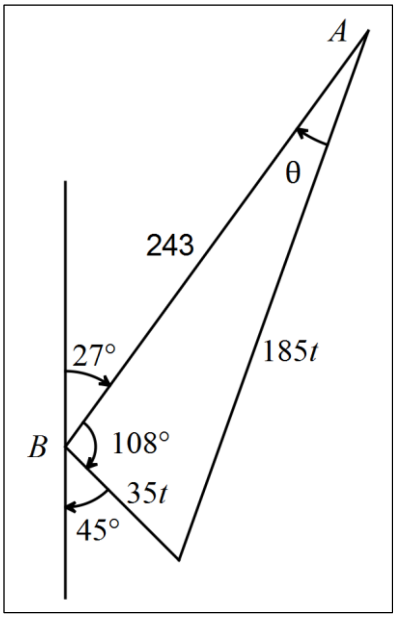
3 E's: 1

3 L's: 1

Total permutations: 44

Question 19 **(7 marks)**

Location B is 243km away from location A on a bearing of 207º. A helicopter leaves A to fly to B on a day when a steady wind of 35km/h is blowing from the SE. If the helicopter has a cruising airspeed of 185km/h, determine the bearing, to the nearest tenth of a degree, the pilot should steer to fly directly to B and find how long the flight will take, to the nearest minute.



$$(185t)^2 = (35t)^2 + 243^2 - 2(35t)(243)\cos 108°$$

$t = -1.26$ or 1.42 hours

$t = 1$ hour 25 minutes (Time)

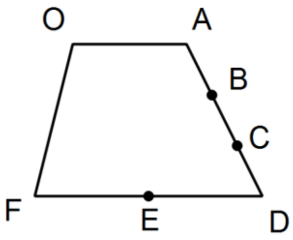
$$\frac{\sin\theta}{35} = \frac{\sin(108)}{185}$$

$\theta = 10.4°$

$207 - 10.4 = 196.6°$ (Bearing)

Question 20 **(7 marks)**

The diagram shows a trapezium in which $\overrightarrow{FD} = 2\overrightarrow{OA}$, E is the midpoint of FD and AD is divided into thirds by points B and C.



Let $\overrightarrow{OA} = \mathbf{m}$ and $\overrightarrow{OF} = \mathbf{n}$.

Use a vector method to prove that $\overrightarrow{FB} = k\overrightarrow{EC}$ and determine the value of k.

$$OE = \mathbf{n} + \mathbf{m}$$

$$OC = OA + \tfrac{2}{3}AD = OA + \tfrac{2}{3}(AO + OF + FD)$$
$$= \mathbf{m} + \tfrac{2}{3}(-\mathbf{m} + \mathbf{n} + 2\mathbf{m}) = \tfrac{5}{3}\mathbf{m} + \tfrac{2}{3}\mathbf{n}$$

$$EC = \tfrac{5}{3}\mathbf{m} + \tfrac{2}{3}\mathbf{n} - (\mathbf{n} + \mathbf{m})$$
$$= \tfrac{2}{3}\mathbf{m} - \tfrac{1}{3}\mathbf{n}$$

$$OF = \mathbf{n}$$

$$OB = OA + \tfrac{1}{3}AD = OA + \tfrac{1}{3}(AO + OF + FD)$$
$$= \mathbf{m} + \tfrac{1}{3}(-\mathbf{m} + \mathbf{n} + 2\mathbf{m}) = \tfrac{4}{3}\mathbf{m} + \tfrac{1}{3}\mathbf{n}$$

$$FB = \tfrac{4}{3}\mathbf{m} + \tfrac{1}{3}\mathbf{n} - \mathbf{n}$$
$$= \tfrac{4}{3}\mathbf{m} - \tfrac{2}{3}\mathbf{n}$$
$$= 2\left(\tfrac{2}{3}\mathbf{m} - \tfrac{1}{3}\mathbf{n}\right)$$
$$= 2EC$$

$$k = 2$$

End of questions