Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS SPECIALIST ATAR COURSE

FORMULA SHEET

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Differentiation and integration

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\|x\| + c$
$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln\|f(x)\| + c$
$\frac{d}{dx}(\sin f(x)) = f'(x) \cos(f(x))$	$\int \sin(ax)\, dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos f(x)) = -f'(x) \sin(f(x))$	$\int \cos(ax)\, dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan f(x)) = f'(x) \sec^2(f(x)) = \frac{f'(x)}{\cos^2 f(x)}$	$\int \sec^2(ax)\, dx = \frac{1}{a} \tan(ax) + c$

Product rule	If $y = uv$ then $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$	or	If $y = f(x)\, g(x)$ then $y' = f'(x)\, g(x) + f(x)\, g'(x)$
Quotient rule	If $y = \frac{u}{v}$ then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	or	If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$
Chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	or	If $y = f(g(x))$ then $y' = f'(g(x))\, g'(x)$
Fundamental theorem	$\frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x)$	and	$\int_a^b f'(x)\, dx = f(b) - f(a)$

Applications of calculus

Growth and decay	
Exponential equation	$\frac{dP}{dt} = kP \Leftrightarrow P = P_0 e^{kt}$
Logistic equation	$\frac{dP}{dt} = rP(k-P) \Leftrightarrow P = \frac{kP_0}{P_0 + (k-P_0)e^{-rkt}}$
Volumes of solids of revolution	
About the x-axis	$V = \pi \int_a^b [f(x)]^2 dx$
About the y-axis	$V = \pi \int_c^d [f(y)]^2 dy$
Simple harmonic motion	
If $\frac{d^2x}{dt^2} = -k^2x$ then $x = A\sin(kt+\alpha)$ or $x = A\cos(kt+\beta)$ where A is the amplitude, α and β are phase angles, v is the velocity and x is the displacement	
$v^2 = k^2(A^2 - x^2)$ Period: $T = \frac{2\pi}{k}$ Frequency: $f = \frac{1}{T}$	
Incremental formula	$\delta y \approx \frac{dy}{dx} \times \delta x$
Acceleration	$\frac{dv}{dt}$ or $v\frac{dv}{dx}$ or $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Functions

Quadratic function	If $f(x) = ax^2 + bx + c$ and $f(x) = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Absolute value function	$\lvert x\rvert = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Statistical inference

Confidence interval for the mean of the population:	$\overline{X} - z\frac{s}{\sqrt{n}} \leq \mu \leq \overline{X} + z\frac{s}{\sqrt{n}}$
Sample size:	$n = \left(\frac{z \times s}{d}\right)^2$

Mensuration

Parallelogram	$A = bh$	
Triangle	$A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab\sin C$	
Trapezium	$A = \frac{1}{2}(a+b)h$	
Circle	$A = \pi r^2$ and $C = 2\pi r = \pi d$	
Prism	$V = Ah$, where A is the area of the cross section	
Pyramid	$V = \frac{1}{3}Ah$, where A is the area of the cross section	
Cylinder	$V = \pi r^2 h$	$S = 2\pi rh + 2\pi r^2$
Cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi rs + \pi r^2$, where s is the slant height
Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

Vectors in 3D

Magnitude	$\lvert(a_1, a_2, a_3)\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
Dot product	$\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$	
Cross product	$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	
Equation of a line	One point and direction	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}$
	Two points A and B	$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$
Equation of a plane	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}_1 + \mu\mathbf{u}_2$ or $\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$	
Equation of a sphere	$\lvert\mathbf{r} - \mathbf{d}\rvert = r$ or $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$	
Cartesian equation of a line	$\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3}$	
Cartesian equation of a plane	$ax + by + cz = d$	
Parametric equation of a line	$x = a_1 + \lambda u_1 \ldots\ldots (1)$ $y = a_2 + \lambda u_2 \ldots\ldots (2)$ $z = a_3 + \lambda u_3 \ldots\ldots (3)$	

Complex numbers

Cartesian form	
$z = a + bi$	$\bar{z} = a - bi$
$\text{Mod}(z) = \lvert z\rvert = \sqrt{a^2 + b^2} = r$	$\text{Arg}(z) = \theta, \quad \tan\theta = \dfrac{b}{a}, \quad -\pi < \theta \le \pi$
$\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$	$\left\lvert\dfrac{z_1}{z_2}\right\rvert = \dfrac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$	$\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
$z\bar{z} = \lvert z\rvert^2$	$z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{\lvert z\rvert^2}$
$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$	$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
Polar form	
$z = a + bi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$	$\bar{z} = r\,\text{cis}(-\theta)$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$\text{cis}(\theta_1 + \theta_2) = \text{cis}\,\theta_1\,\text{cis}\,\theta_2$	$\text{cis}(-\theta) = \dfrac{1}{\text{cis}\,\theta}$
De Moivres theorem	
$z^n = \lvert z\rvert^n\,\text{cis}(n\theta)$	$(\text{cis}\,\theta)^n = \cos n\theta + i\sin n\theta$
$z^{\frac{1}{q}} = r^{\frac{1}{q}}\left(\cos\dfrac{\theta + 2\pi k}{q} + i\sin\dfrac{\theta + 2\pi k}{q}\right)$, for k an integer	

Trigonometry

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	Length of arc $= r\theta$
$a^2 = b^2 + c^2 - 2bc\cos A$	Area of segment $= \dfrac{1}{2} r^2 (\theta - \sin\theta)$
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	Area of sector $= \dfrac{1}{2} r^2 \theta$
Identities	
$\cos^2 x + \sin^2 x = 1$	$1 + \tan^2 x = \sec^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sin 2x = 2\sin x \cos x$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
$\cos A \cos B = \dfrac{1}{2}(\cos(A - B) + \cos(A + B))$	$\sin A \cos B = \dfrac{1}{2}(\sin(A + B) + \sin(A - B))$
$\sin A \sin B = \dfrac{1}{2}(\cos(A - B) - \cos(A + B))$	$\cos A \sin B = \dfrac{1}{2}(\sin(A + B) - \sin(A - B))$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.