

RIGHT ANGLE TRIANGLES

1 Right Angle Triangle Examples


(Q3a) Draw diagram to show a 3.8m ladder leaning against a wall that reaches height of 2m. **x (A)**


(Q3b) Find the angle of elevation that the ladder has between the ground and the wall:
 $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2}{4} = 0.5$
 $\theta = \sin^{-1}(0.5) = 30^\circ$

2 Find angle size of a:
 $x = \sin^{-1}(5/6) = 56.44^\circ$

Sides are in lower case.
Opposing angles and sides are labeled opposite.
Sum of angles in any triangle rule:
 $A + B + C = 180^\circ$

Sine Rule (Finding Angles or Sides)
 Use when two sides and one angle or sides are given and one element (i.e. one of the angles or sides in either pair) is missing.

Finding Sides: $\frac{\sin A}{a} = \frac{\sin B}{b}$


Finding Angles: $\frac{\sin A}{a} = \frac{\sin B}{b}$


Cosine Rule (Finding Angles or Sides)
 Use when three sides and one angle are given and one element (i.e. one of the angles or sides in either pair) is missing.

Finding a side:
 $c^2 = a^2 + b^2 - 2ab \times \cos(C)$

Finding an angle:
 $C = \cos^{-1} \left(\frac{a^2 + b^2 - c^2}{2 \times a \times b} \right)$

Area of Non-Right Triangles
 Trigonometric formula:
 Use when have two sides and an included angle
 $\text{Area} = \Delta ABC = \frac{1}{2} \times a \times b \times \sin(C)$

Area of Non-Right Triangles
 Heron's rule for finding area:
 Use when all three sides of the non-right triangle are known.
 $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
 $s = \frac{1}{2} \times (a + b + c) = \frac{1}{2} \times \text{length of the perimeter.}$

Non-Right Angle Triangle Examples
 (Q1) Find side length b in the following triangle:
 Two pairs of angle and side, sine rule
 $b = \frac{a \sin B}{\sin A} = \frac{10 \sin 70^\circ}{\sin 50^\circ} = 10.23$

(Q2) Find angle A in the following triangle:
 Three sides and one angle: cosine rule
 $A = \cos^{-1} \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$
 $A = \cos^{-1} \left(\frac{15^2 + 4^2 - 10^2}{2 \times 15 \times 4} \right) = 69.33^\circ$

(Q3) Find area of the following triangle:
 Three sides, Heron's rule
 $s = \frac{1}{2} (4 + 5 + 7) = 8$
 $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$
 $\text{Area} = \sqrt{8(8-4)(8-5)(8-7)} = 9.92$
 Area = $\sqrt{36} = 6$ cm

(Q4) Find area of the quadrilateral:
 Two sides with the included angle and one diagonal
 Find side length BD:
 $BD = \sqrt{7^2 + 11^2 - 2(7)(11)\cos(45^\circ)}$
 $BD = 9.6$ cm

Find angle C using the sine angle rule:
 $\frac{\sin C}{c} = \frac{\sin A}{a} \Rightarrow \sin C = \frac{a \sin A}{c} = \frac{0.561}{1} \Rightarrow C = 34.1^\circ$
 Angle $\Delta BDC = 180 - 35 + 34.1 = 110.9^\circ$
 Find area of quadrilateral using trig rule:
 $\Delta ABC = \frac{1}{2} \times 7 \times 11 \times \sin(45^\circ) = 27.22 \text{ m}^2$
 $\Delta BDC = \frac{1}{2} \times 5 \times 7.82 \times \sin(110.9^\circ) = 19.22 \text{ m}^2$
 $\Delta ABCD = 27.22 + 19.22 = 46.44 \text{ m}^2$

[illegible]

1-Step \rightarrow $\begin{bmatrix} 1 & 3 & 5 & 2 \\ 2 & 1 & 6 & 25 \\ 4 & 14 & 1 & 4 \\ 5 & 1 & 1 & 1 \end{bmatrix}$ working out:
 $\begin{bmatrix} 1 & 3 & 5 & 2 \\ 2 & 1 & 6 & 25 \\ 4 & 14 & 1 & 4 \\ 5 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 5 & 2 \\ 2 & 1 & 6 & 25 \\ 4 & 14 & 1 & 4 \\ 5 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 34 & 12.5 & 3.5 & 16.845 \end{bmatrix}$
(G1e) Find matrix X given that $C + X = I$
 $C = \begin{bmatrix} 1 & 3 & 5 & 2 \\ 2 & 1 & 6 & 25 \\ 4 & 14 & 1 & 4 \\ 5 & 1 & 1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} 0 & -2 & -4 & -1 \\ -1 & -4 & -5 & -23 \\ -3 & -13 & -2 & -3 \\ -4 & -2 & -2 & -20 \end{bmatrix}$
(G2) Matrix A shows burgers sold at a canteen for each row (row 1) shows row (2) over a Monday and Tuesday this is an adjacency matrix
(G3) Find matrix Z if $X + Z = I$ and explain it's meaning
 $Z = \begin{bmatrix} 14 & 20 & 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 14 & 20 & 2 & 1 \end{bmatrix}$ shows the total amount of burgers sold at canteen and school
(G4) Find profit if profit matrix $P = \begin{bmatrix} 2.5 & 3.5 \\ 1 & 1 \end{bmatrix}$
 $P = \begin{bmatrix} 34 & 12.5 & 3.5 & 16.845 \end{bmatrix}$
 means that profit is equal to **\$668.50**

ROUTE MATRICES

- Properties of an Adjacency Matrix**
 - Matrix that shows how many times each vertex is connected to each other vertex by a single edge.
 - From vertices on left, To vertices are above.
 - Matrix may only contain 0 or 1 adjacency matrices
- Adjacency matrix for an undirected graph:

A	To
C	1
B	1
C	1
B	2
C	2
- Adjacency matrix for a directed graph:

A	To
C	0
B	1
C	1
B	0
C	1

Properties of a Route Matrix

- Any entries in an adjacency matrix from the raised to the n^{th} power indicates how many ways it is possible to move to and from the points contained in n steps.

1-Step \rightarrow M^1 **2-Step** \rightarrow M^2

- M^1 : adjacency matrix
- M^2 : adjacency matrix squared
- 1-Step**: a matrix showing number of ways to travel between vertices in 1 step.
- 2-Step**: a matrix showing number of ways to travel between vertices in 2 steps.
- 1 or 2**: a matrix showing number of ways to travel between vertices in 1 or 2 steps.

Transition Matrix Example

(G1) A connected graph and related transition matrices M^1 and M^2 are shown below:

	A	B	C	D
A	0	2	1	0
B	2	0	1	0
C	1	1	0	1
D	0	0	1	1

	A	B	C	D
A	5	1	2	1
B	5	3	3	1
C	2	2	3	1
D	1	1	2	2

(G1a) Explain how the highlighted entry "2" is calculated in transition matrix M^1 .
 Look at matrix M^1 - 1-Step Transitions.
 The highlighted entry in matrix shows that there are **2 ways** of going from **A to B in 1 step**.
Matrix M^2 shows 2-Step Transitions.
 Look at matrix M^2 - 2-Step Transitions.
 The highlighted entry in matrix shows that there are **3 ways** of going from **C to A in 2 steps**.
impossible to go from **E to D in 2 steps**.

(G2) Explain how the highlighted entry "3" is calculated in transition matrix $M^1 - M^2$.
 Look at matrix M^1 - 1-Step Transitions.
 The highlighted entry in matrix shows that there are **3 ways** of going from **C to A in 1 or 2 steps**.
