Test One

Semester One 2016

Year 12 Mathematics Methods

Calculator Free

PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Teacher:

_____ **Mr Staffe**

_____ **Mrs. Carter**

_____ **Mr Bertram**

_____ **Mr Roohi**

_____ **Ms Cheng**

Name:

- ***Complete all questions***
- ***Show all necessary working***
- ***Total Marks = 25***
- ***25 minutes***

1. [12 marks]

Find $\frac{dy}{dx}$ in each of the following, by using the appropriate rule.

(a) $y = (3x^2 - x)(x^3 - 4x^2 - 5x + 3)$ (Do not simplify) [2]

$\frac{dy}{dx} = (x^3 - 4x^2 - 5x + 3)(6x - 1) + (3x^2 - x)(3x^2 - 8x - 5)$ ✓ ✓

(b) $y = 2x - \sqrt{x} + 3\pi^3 + \frac{4}{x^2}$ (Leave with positive indices.) [2]

$\frac{dy}{dx} = 2 - \frac{1}{2}x^{-1/2} - 8x^{-3}$

$= 2 - \frac{1}{2\sqrt{x}} - \frac{8}{x^3}$ ✓✓

(c) $y = \frac{2x^3}{(5 - 3x^4)^2}$ (Do not simplify) [3]

$\frac{dy}{dx} = \frac{(5 - 3x^4)(6x^2) - 2x^3 . 2(5 - 3x^4)(-12x^3)}{(5 - 3x^4)^4}$ ✓ ✓ ✓

(d) $y = \sqrt{x^4 - 3x^3 + 2}$ [3]

$\frac{dy}{dx} = \frac{1}{2}(x^4 - 3x^3 + 2)^{-1/2} . (4x^3 - 9x^2)$ ✓ ✓

$= \frac{4x^3 - 9x^2}{2\sqrt{x^4 - 3x^3 + 2}}$ ✓

(e) $y = \sqrt{u^2 - 3}$ using the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, where $u = 2x^3 + 3$ [2]

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{2}(u^2 - 3)^{-1/2} . 2u \times 6x^2$ ✓

$= \frac{\cancel{2}(2x^3 + 3) . 6x^2}{\cancel{2}\sqrt{(2x^3 + 3)^2 - 3}}$ ✓

$= 6x^2$

12

2. **[3 marks]**

Consider the function $f(x) = x^3 - 5x^2 - 8x + p$ where p is a constant.

(a) Determine where the local (relative) extrema points occur. [2]

$f'(x) = 3x^2 - 10x - 8$

$3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$x = -\frac{2}{3}, 4$

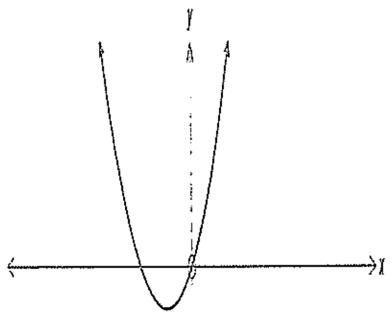
(b) What can we say about value of p given that two of the three roots are negative [1]

p is negative.

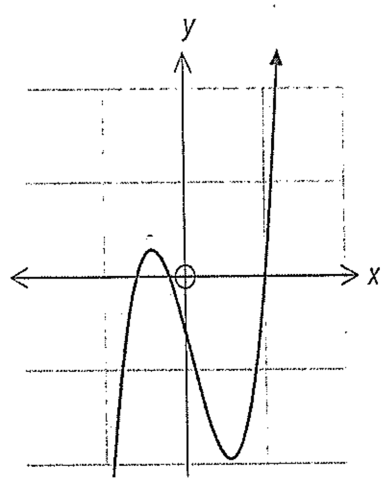
3. **[4 marks]**

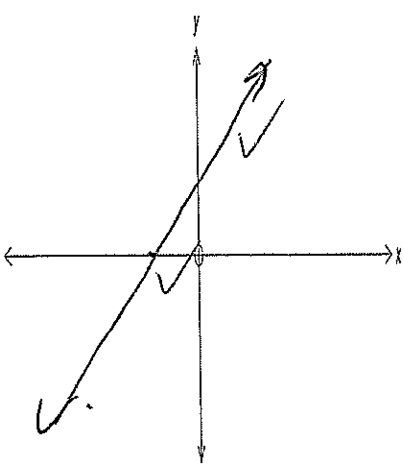
Draw a sketch below of each of the gradient functions formed by each of the following functions

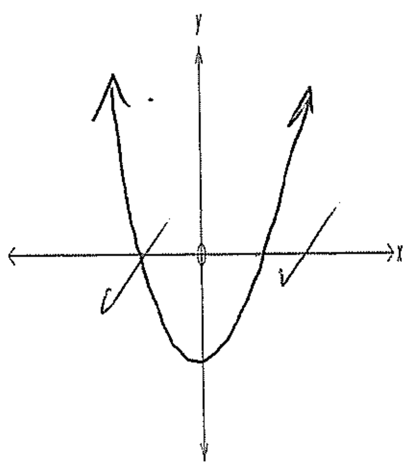
(a)



(b)

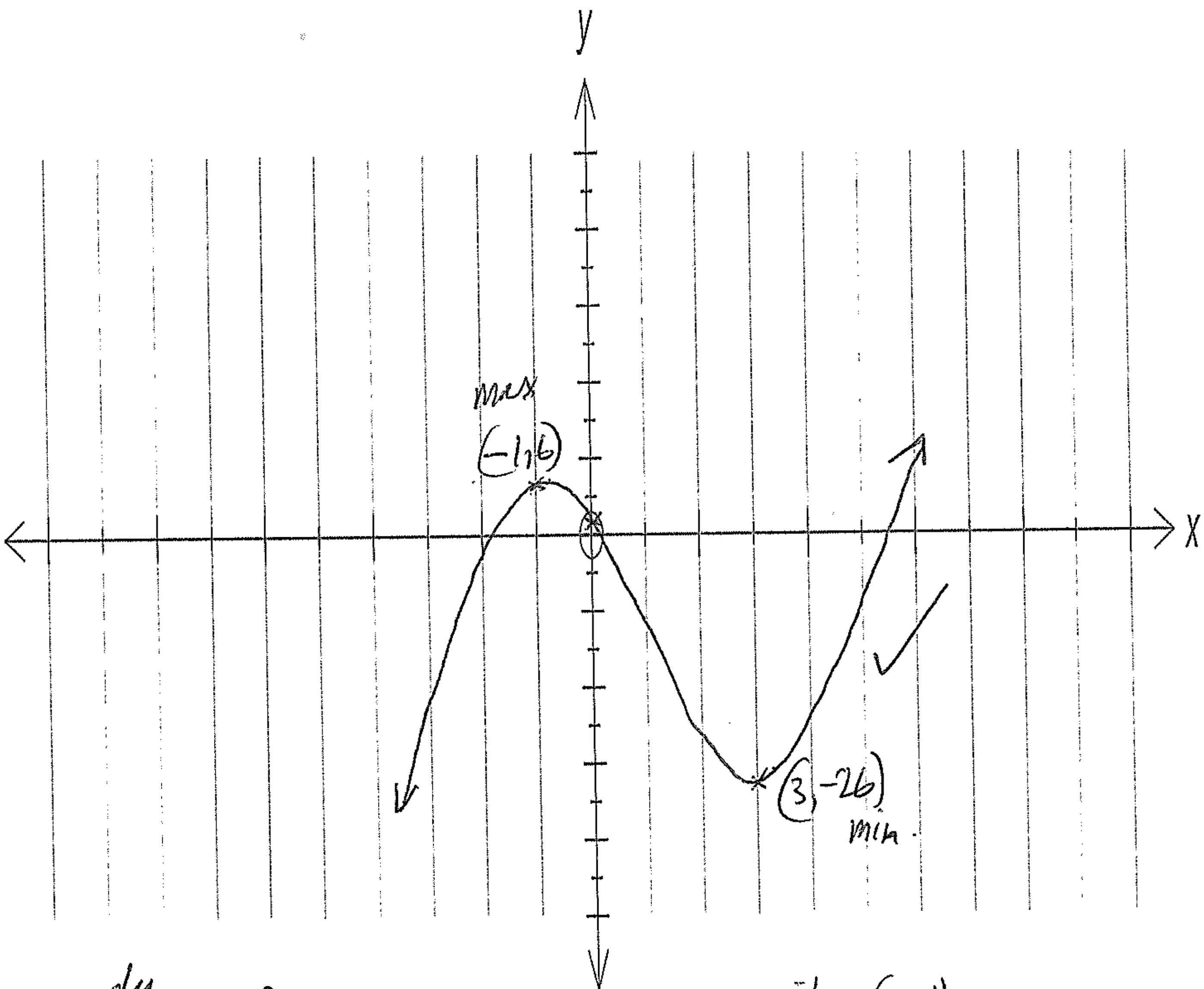






4. [6 marks]

Find the turning points, points of inflection and intercepts for the function $y = x^3 - 3x^2 - 9x + 1$. Then graph a sketch of the function on the axes provided below, clearly showing these key points.



$\frac{dy}{dx} = 3x^2 - 6x - 9$ ✓

$y \cdot \text{int} = (0, 1)$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$ ✓

$(x-3)(x+1) = 0$

$x = -1, 3. \rightarrow y = (-26), 6$ ✓

$\frac{d^2y}{dx^2} = 6x - 6 \therefore$ pt of inflection $x = (1, -10)$ ✓

When $x = -1$ $\frac{d^2y}{dx} < 0$. max.

When $x = 3$. $\frac{d^2y}{dx} > 0$ min ✓

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Test One

Semester One 2016
Year 12 Mathematics Methods
Calculator Assumed

Teacher:

______Mr Staff[illegible]

______**Mrs. Carter**

______**Mr Bertram**

______**Mr Roohi**

______**Ms Cheng**

Name:

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- ***Total Marks = 25***
- ***25 minutes***

1. [5 marks]

A particle's position along the x-axis, in meters, is given by the function $s = 3t^3 - 5t + 9$.

(a) Find the Velocity and Acceleration of this particle when t = 2 seconds [3]

$v = 9t^2 - 5$ ✓

$a = 18t$ ✓

At $t = 2$ $v = 31$ m/s , $a = 36$ m/s^2 ✓

(b) When does the particle stop moving, and how far from the origin is it at this time? [2]

$9t^2 - 5 = 0$

$t = \sqrt{5}/3$. Ignore -ve value.

✓ 0.74535 secs.

$s(\sqrt{5}/3) = 6.51$m.

Stops after $\sqrt{5}/3$ s at 6.51 m. ✓

(6.52 if using exact value

2. [8 marks]

The volume of a certain rectangular box is given by the equation $f(x) = x^3 - 5x^2 - 8x + 48$.

(a) If the height of the box is $(4 - x)$ units, determine an algebraic expression for the area of the base of the box. [3]

$$\text{Area of base} = \frac{x^3 - 5x^2 - 8x + 48}{4 - x}$$

$$= -x^2 + x + 12$$

(b) Calculate the value of x for which the volume is a maximum. [5]

$$f'(x) = 3x^2 - 10x - 8$$

$$= (3x + 2)(x - 4) = 0$$

$$x = -2/3,\ 4$$

$$f''(x) = 6x - 10$$

$f''(-2/3) < 0$ max

$f''(4) > 0$ min

$\therefore$ max when $x = -2/3$

3. **[7 marks]**

(a) If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage change in V when r changes by $\frac{1}{2}$% [3]

$V = 2\pi r^3$

$\frac{dV}{dr} = 6\pi r^2 \quad \frac{\delta r}{r} = 0.005$

$\delta V \approx \frac{dV}{dr} \times \delta r$

$\frac{\delta V}{V} = \frac{dV}{dr} \times \frac{\delta r}{2\pi r^2}$

$= 3 \times 0.005$

$= 0.015 = 1.5\%$ change of 1.5%.

$\frac{3}{6\pi r^2} \cdot 2 \cdot 0.005$ / $2\pi r^3$

.5%

= 0.005

(b) If the volume of the solid generated by rotating a shaded region is given by $V = \pi[0.05h^5 + \frac{2}{3}h^3 + 4h]$, use the incremental formula, $\delta V \approx \frac{dV}{dh}\delta h$, to estimate the change in volume when h increases from 3 to 3.01. [4]

$\frac{dV}{dh} = \frac{\pi(h^4 + 8h^2 + 16)}{4}$ off classpad.

For small change on h $\frac{\delta V}{\delta h} \approx \frac{\pi(h^4 + 8h^2 + 16)}{4}$

$\delta V = \frac{\pi(3^4 + 8.3^2 + 16)}{4} \times (0.01)$

$0.25h^4 + 2h^2 + 4$

$= \frac{169\pi}{400}$

≈ 1.33 units.

0.4225π

The increase would be 1.33 units as h increase 3 to 3.01.

4. **[5 marks]**

Sketch the graph of y = f(x) given the data below:

(i) $f(2) = -9$ $f(-4) = 27$ $f(-1) = 9$

(ii) $f'(2) = 0$ and $f''(2) > 0$ min t.p at $x=2$

(iii) $f'(-4) = 0$ and $f''(-4) < 0$ max at $x=-4$.

(iv) $f''(-1) = 0$ inflection when $x=-1$.

(v) $f'(x) > 0$ for $x > 2$, $x < -4$

(vi) $f'(x) < 0$ for $-4 < x < 2$

(vii) $f(0) = 3$