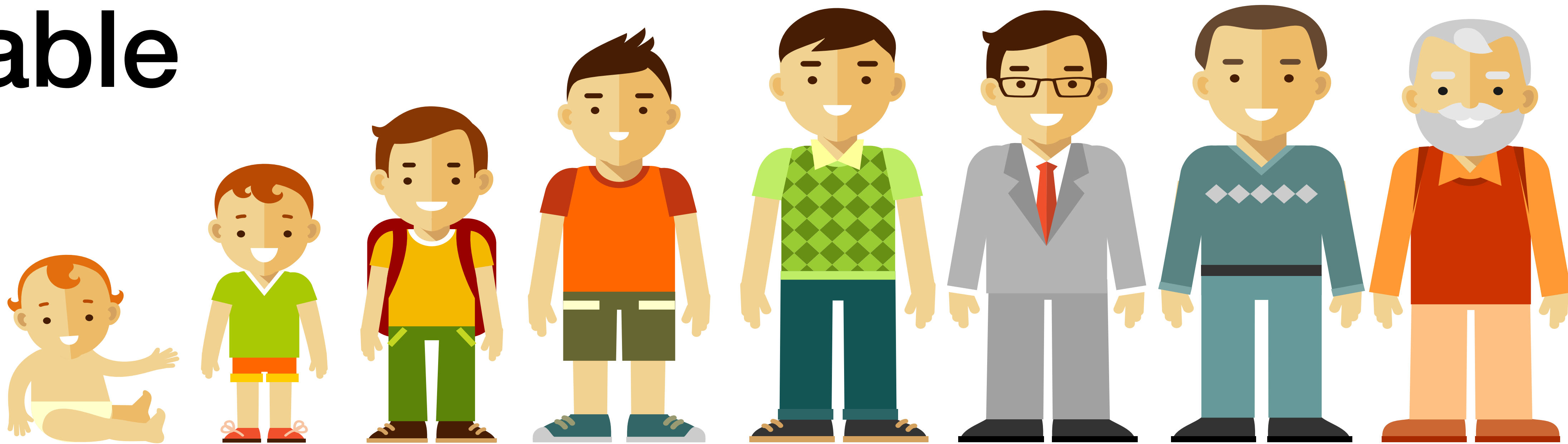# Variable

Modifications
(aka functions)

Functions of a Random Variable

Random variables X take values in $\mathbb{R}$

Often interested in related variable $Y = g(X)$ $g:\mathbb{R}\rightarrow\mathbb{R}$ is a fixed function

X   Random salary in $

$10 raise $Y = X + 10$

10% raise $Y = 1.1X$

→ CEO $Y = X^2$

Deterministic Functions

Y = g(X)   g is a deterministic function over $\mathbb{R}$ (or Ω)

Y = X + 3

All randomness in Y derives from X

Deterministically modified by g

X = 5   Y = 8

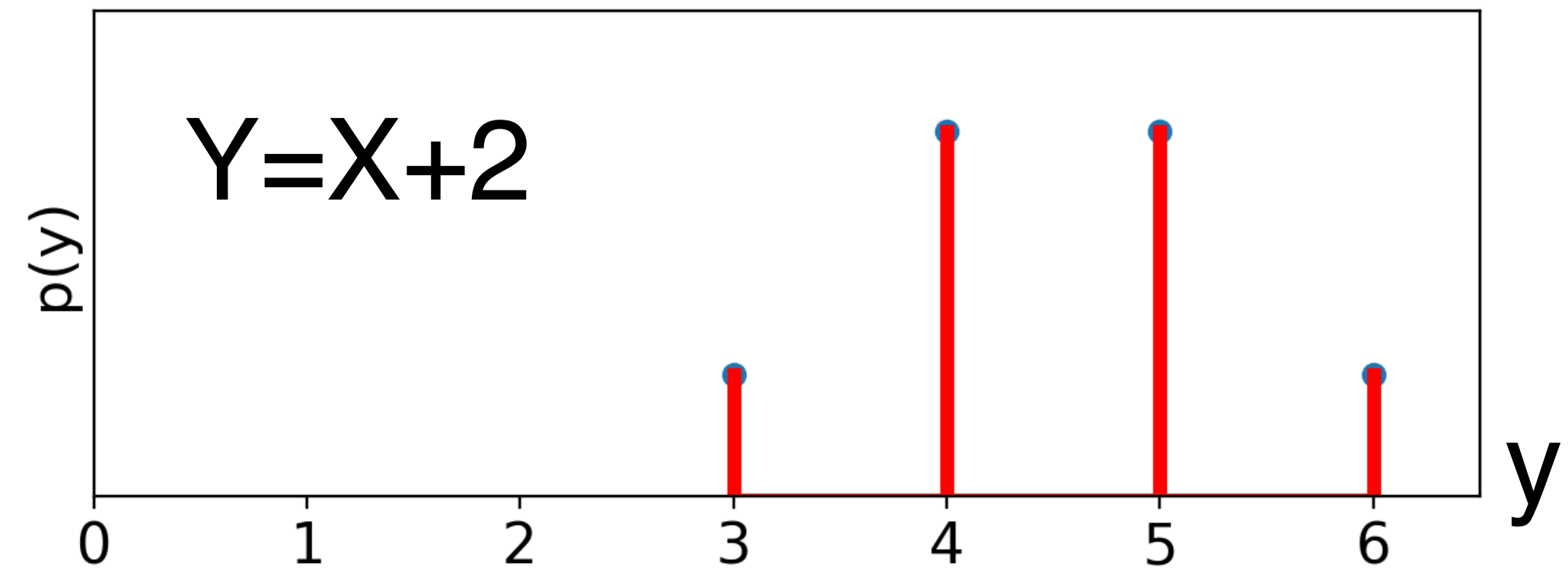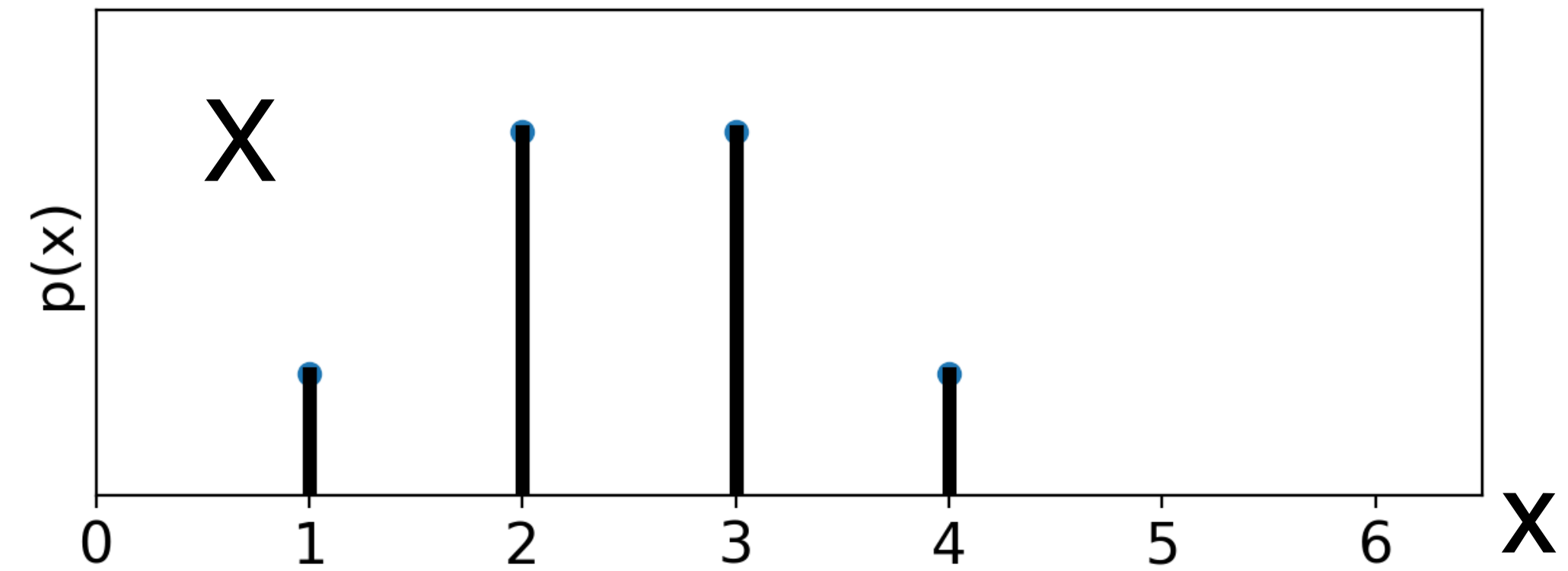# Translation

Add constant b to X   Translate X by b

Y = X + b

P(Y=y) = P(X+b=y) = P(X=y-b)
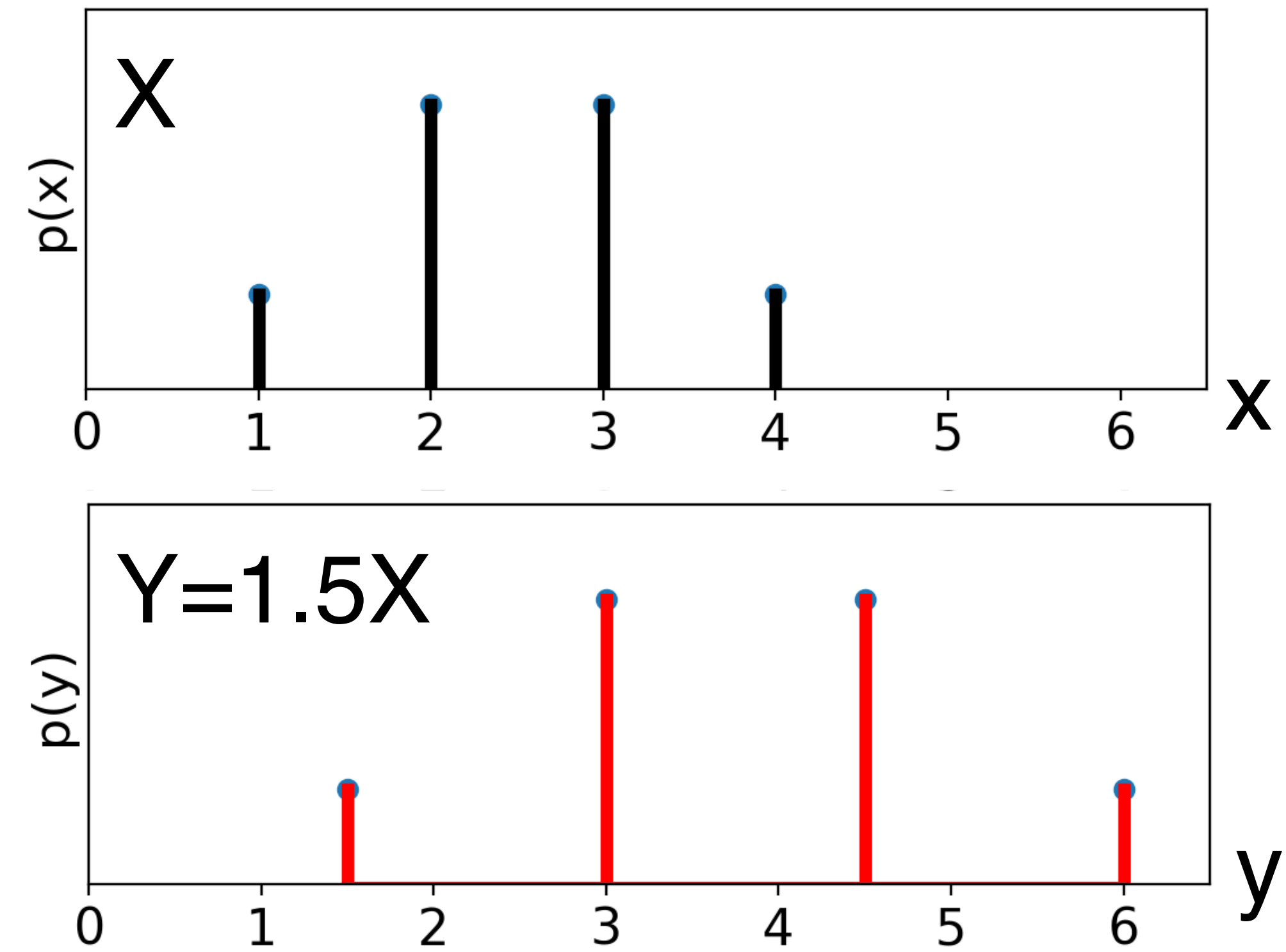
Scaling

Multiply X by a constant b

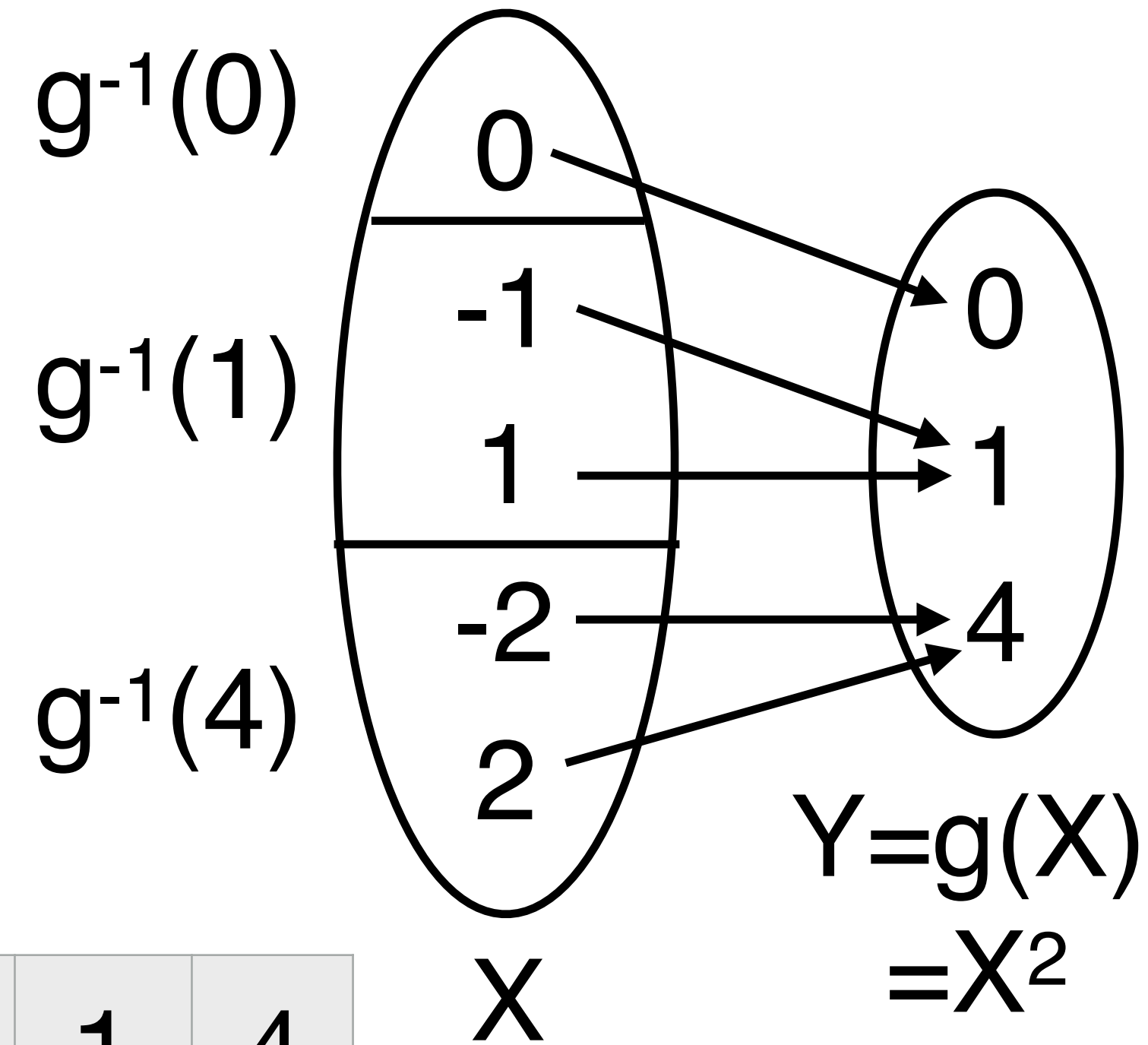Scale X by a factor b

$Y = b \cdot X$

$P(Y=y) = P(bX=y) = P(X=y/b)$

Two Square Examples

$g^{-1}(0)$

x	0	1	2
p(X = x)	½	⅓	⅙

X

$Y = X^2$

y	0	1	4
p(Y = y)	½	⅓	⅙

$g^{-1}(1)$

$g^{-1}(4)$

$Y = g(X)$
$= X^2$

Square is many to 1

x	-2	-1	0	1	2
p(X = x)	⅕	⅕	⅕	⅕	⅕

X

$Y = X^2$

y	0	1	4
p(Y = y)	⅕	⅖	⅖

$$P(Y=y) = P(g(X)=y) = P(X \in g^{-1}(y)) = \sum_{x \in g^{-1}(y)} P(X=x)$$

g(x) = constant   g(x) = 1

$P(Y=1) = P(X \in \Omega) = 1$

Variable

Modifications

(aka functions)

Expectation of modification