Cumulative Distribution Functions

Interesting Probabilities?

Coin

Poker

Random variables

Intervals

Salary > 80K

GPA < 3.0

Temperature between 60 and 80 °F

One function determines all interval probabilities!

Cumulative Distribution Function

Probability mass function (pmf)

p: $\Omega \rightarrow \mathbb{R}$

Cumulative distribution function (cdf)

F: $\mathbb{R} \rightarrow \mathbb{R}$

$$F(x) \stackrel{\text{def}}{=} P(X \in (-\infty, x])$$

$$\stackrel{\text{def}}{=} P(X \leq x)$$

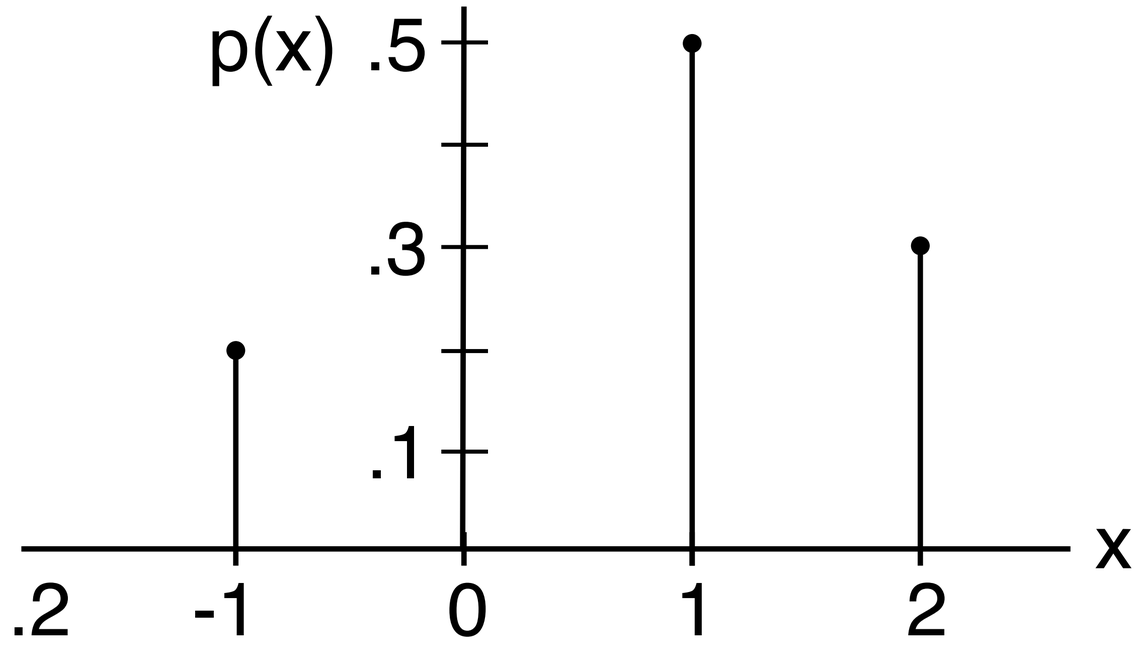
$$= \sum_{u \leq x} p(u)$$

X discrete, still F defined over $\mathbb{R}$

Example

PMF

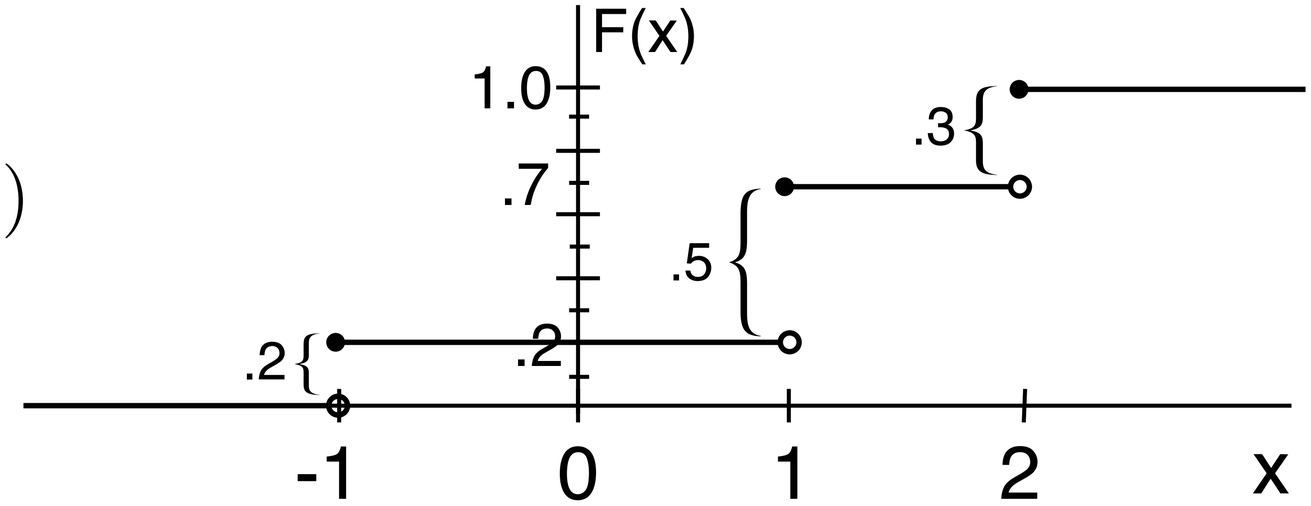
$$p(x) = \begin{cases} .2 & -1 \\ .5 & 1 \\ .3 & 2 \end{cases}$$



CDF

$$F(x) = P(X \leq x)$$

$$= \sum_{u \leq x} p(u)$$



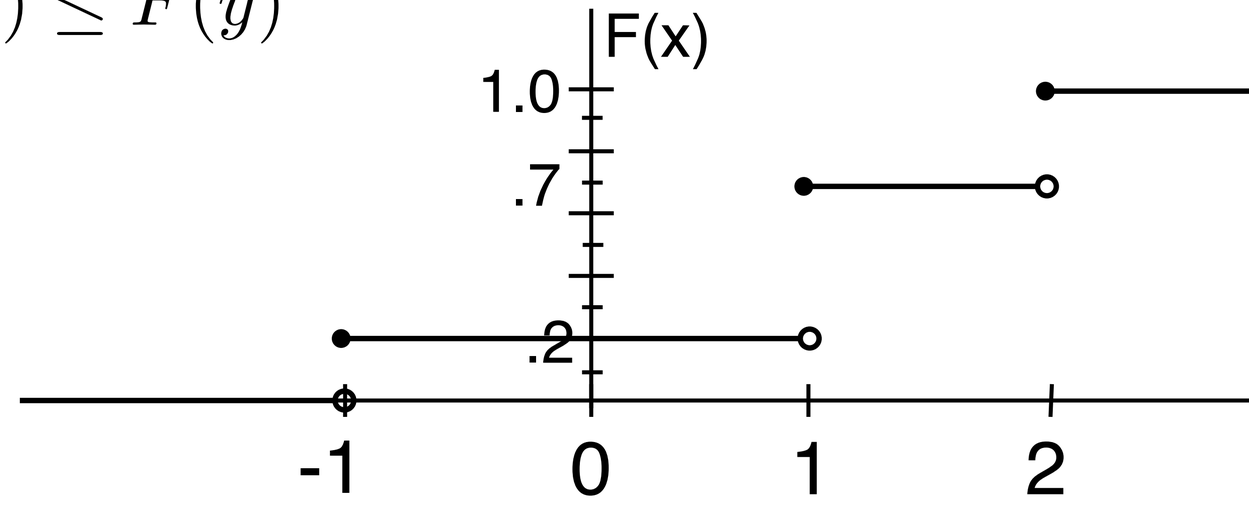
Properties

Nondecreasing $x \leq y \quad \rightarrow \quad F(x) \leq F(y)$

Limits $\lim_{x \to -\infty} F(x) = 0$

$\lim_{x \to \infty} F(x) = 1$

Right-continuous $\lim_{x \searrow a} F(x) = F(a)$



Interval Probabilities

$$P(X \leq a) = F(a) \quad - \text{by definition}$$

$$P(X > a) = 1 - P(X \leq a) = 1 - F(a)$$

$$P(a < X \leq b) = P((X \leq b) - (X \leq a))$$

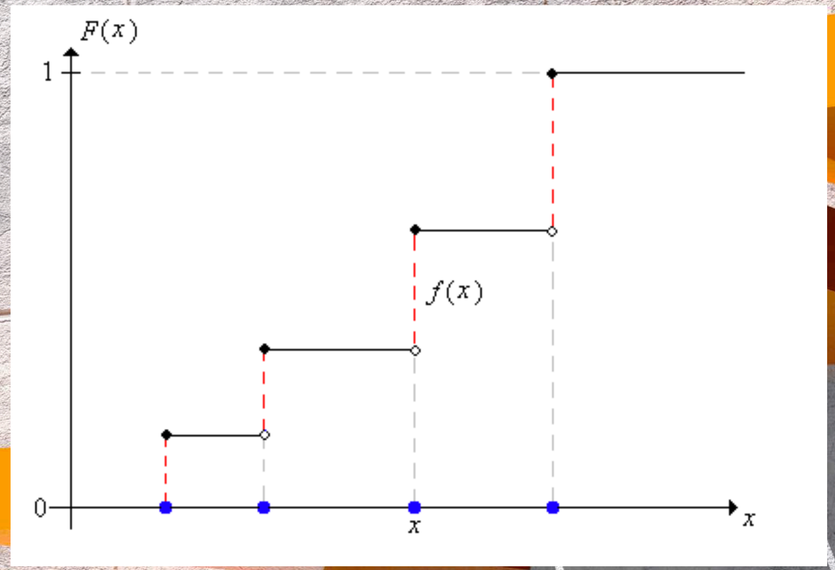
$$= P(X \leq b) - P(X \leq a)$$

$$= F(b) - F(a)$$

Cumulative Distribution Functions

$$F(x) \stackrel{\text{def}}{=} P(X \leq x) = \sum_{u \leq x} p(u)$$

Yields interval probability



Expectations