Random Variables

Motivation

Examples

Definitions

Visualization

What Matters

So far coins, dice, cards, dominoes, marbles, ….

Often subscribers clicks viewers yield weight sales

time congestion delay age temperature heart rate

GPA tuition assignment income cost

Numbers!

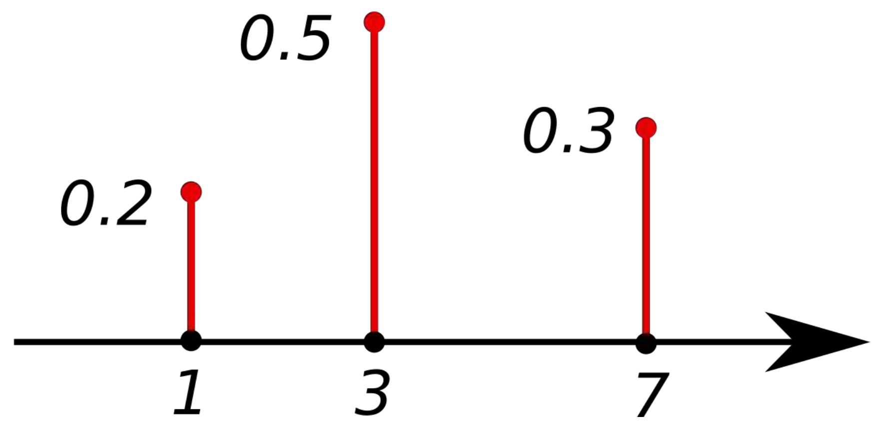
Random variable Number-valued random outcome

Xtra with Numbers

Distri-
bution

p(x)

View on a line

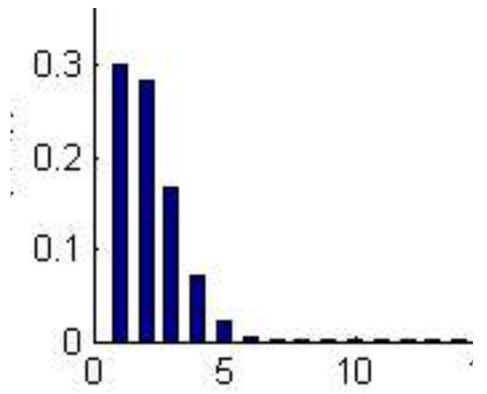


Express as function

$p(x)=1/x^2$

Consider properties

Decreasing



concentrated

Random
Variable

X

Perform operations

$X+1$

X^2

Combine variables

$X+Y$

Consider properties

average value of X

Two Types

Size of sample space Ω

Finite $\{1, 2, 3\}$ $\{e, \pi\}$ or countably infinite $\mathbb{N}$ $\mathbb{Z}$ Discrete

Uncountably infinite $[0,2]$ $(-1,3) \cup [4,5)$ $\mathbb{R}$ Continuous

Combination $[0,2] \cup \{e, \pi\}$ Mixed

Discrete first

Number Outcomes

Several past examples had number outcomes

Outcome of a die roll $\{1,\ldots,6\}$

Values of a domino tile $\{0,\ldots,6\}$

Number of heads in 3 coin tosses $\{0,\ldots,3\}$

Did not use numerical features → Use extensively

Familiar and new examples

General formulation

Heads

3 fair coins

$\Omega = \{$ ttt, tth, tht, thh, htt, hth, hht, hhh $\}$

$|\Omega| = 8$

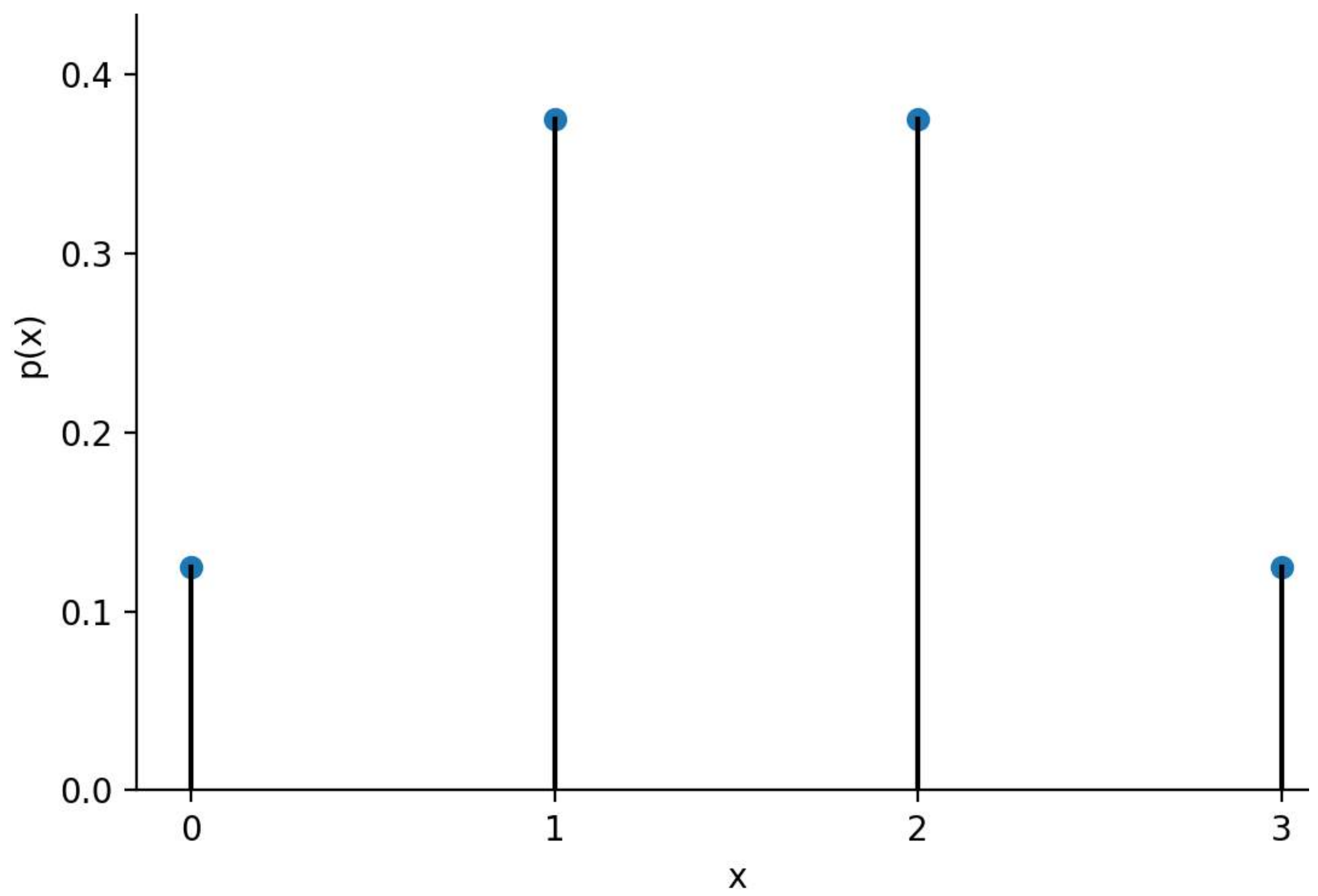
Equiprobable

$p = 1/8$

X

heads

x	Outcomes	p(x)
0	ttt	⅛
1	tth, tht, htt	⅜
2	thh, hth, hht	⅜
3	hhh	⅛



Specification

As before

Explicit

$p(1)=.1 \quad p(2)=.2 \quad p(3)=.3 \quad p(4)=.4$

Table

x	1	2	3	4
p(x)	.1	.2	.3	.4

With numbers

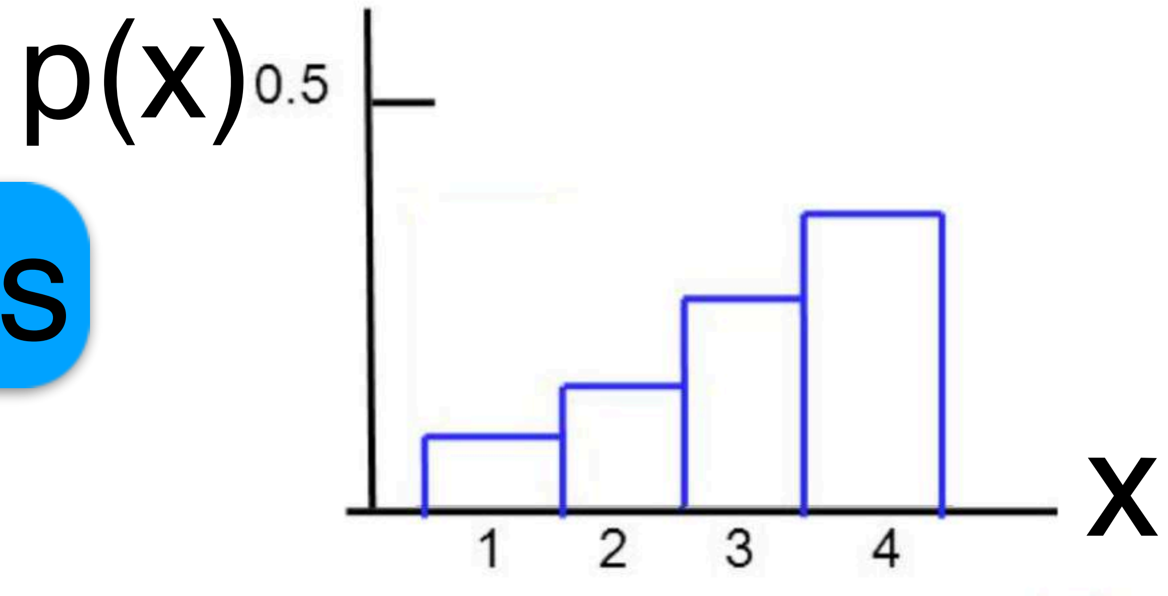
Function

$p(x) = x / 10$

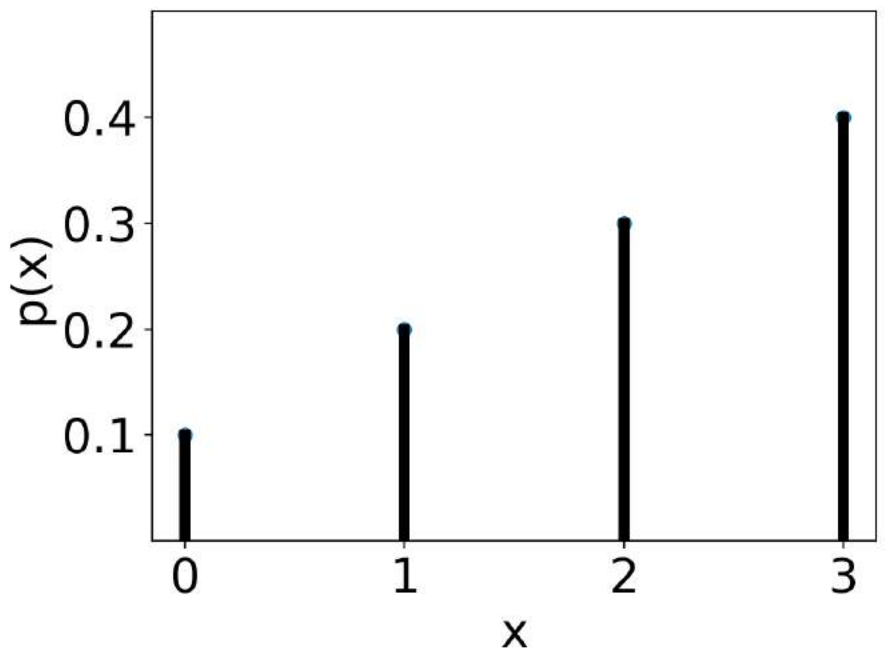
$x \in \{1,2,3,4\}$

Graphs

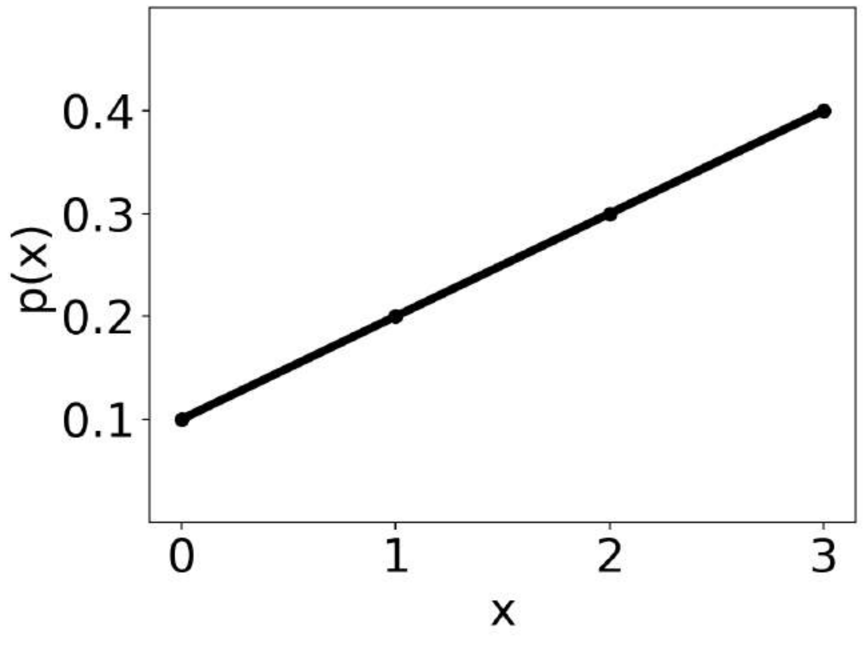
Histogram



Stem plot



Plot



Probability Mass Function

As before

pmf

$p : \Omega \rightarrow \mathbb{R}$

Specify Ω and p

Ω

Random variable $\rightarrow$ $\subseteq \mathbb{R}$

Discrete $\rightarrow$ finite or countably infinite

p

$p(x) \geq 0 \quad \forall x \in \Omega$

$$\sum_{x \in \Omega} p(x) = 1$$

If X is distributed according to p, we write $X \sim p$

Alternative Notation

Discrete

$\Omega \subseteq \mathbb{R}$ Often $\mathbb{Z}$ $\mathbb{N}$ $\mathbb{P}$ $\{1,\ldots n\}$

$p(x) \rightarrow p_x \quad p_i$

$p_i \geq 0 \qquad \sum_i p_i = 1$

Types of Discrete Distributions

Finite $|\Omega| = n \in \mathbb{P}$

Infinite $|\Omega| = \infty = \aleph_0$

Finite Distributions

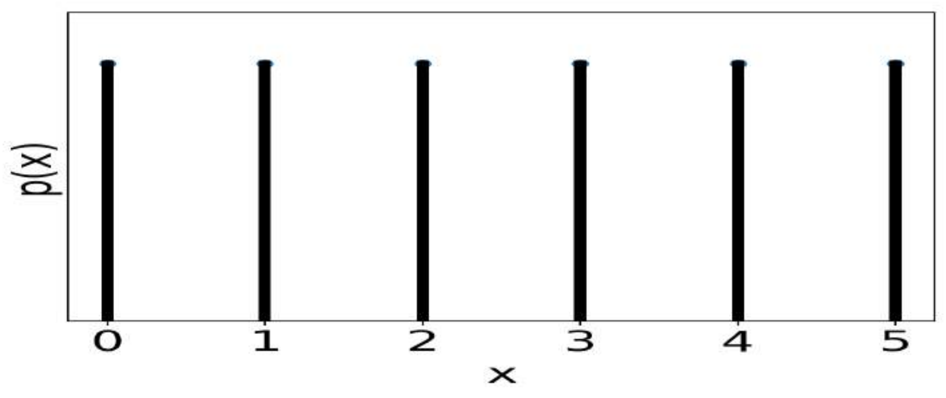
|Ω| = n

Specify pmf

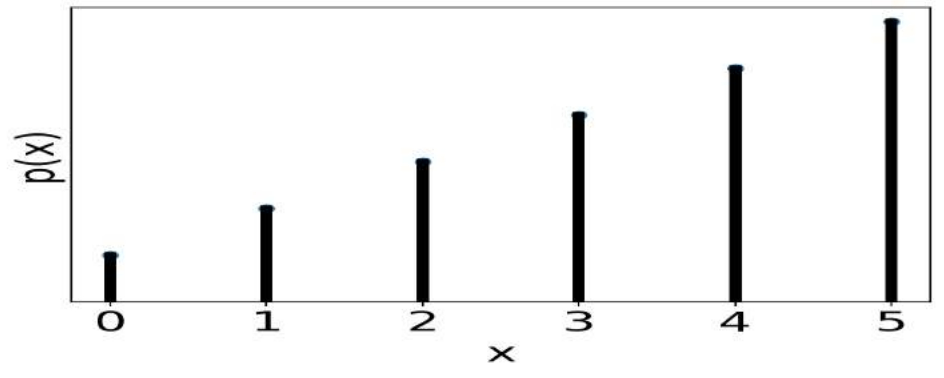
$p_1, p_2, \ldots, p_n$

$$\forall_{1 \le i \le n} \quad p_i \ge 0 \qquad \sum_{i=1}^{n} p_i = 1$$

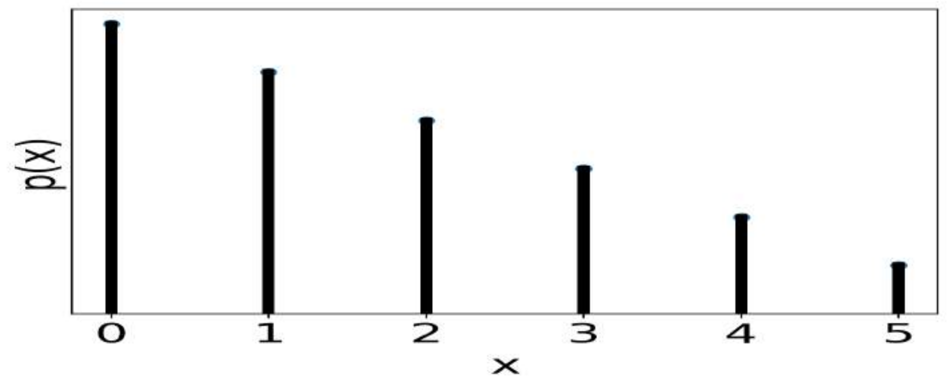
Uniform $p_1 = p_2 = \ldots = p_n$ $= 1/n$



Increasing $p_1 \le p_2 \le \ldots \le p_n$



Decreasing $p_1 \ge p_2 \ge \ldots \ge p_n$



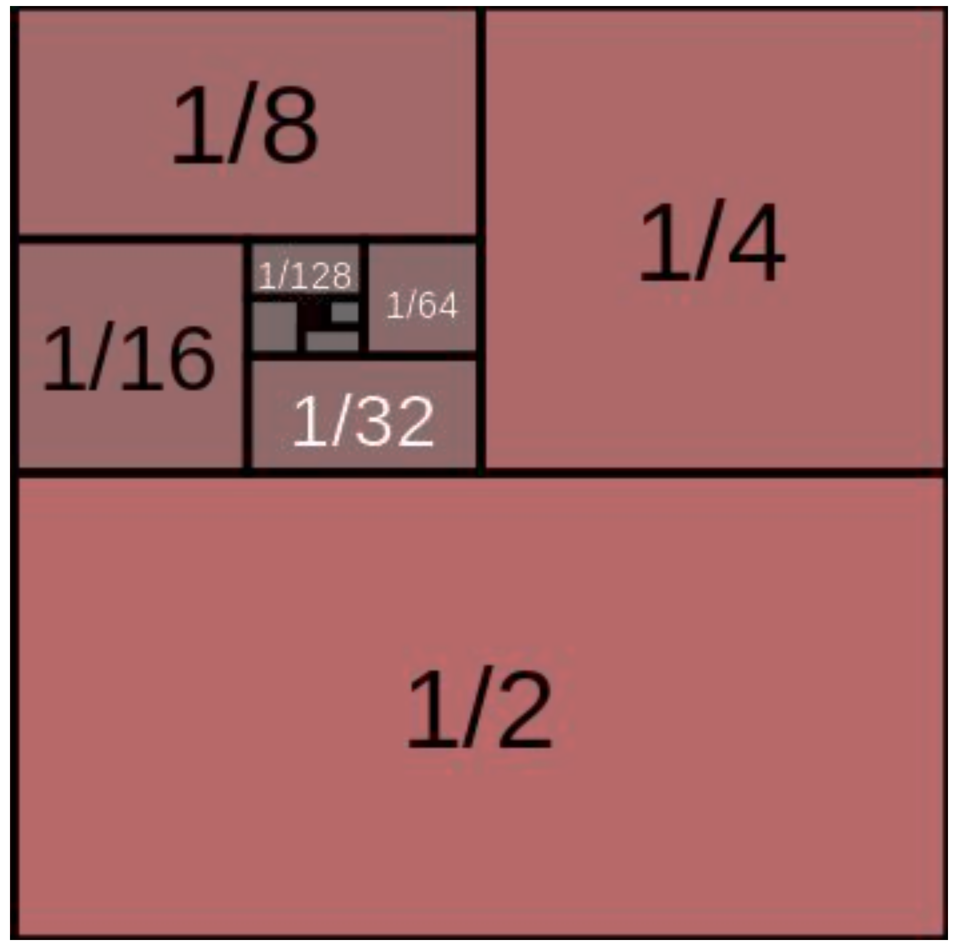
Infinite Distributions

$|\Omega| = \infty$

One-sided infinite $p_1, p_2, p_3, \ldots$

Cannot be uniform $p = 0 \rightarrow \sum = 0$ $p > 0 \rightarrow \sum = \infty$

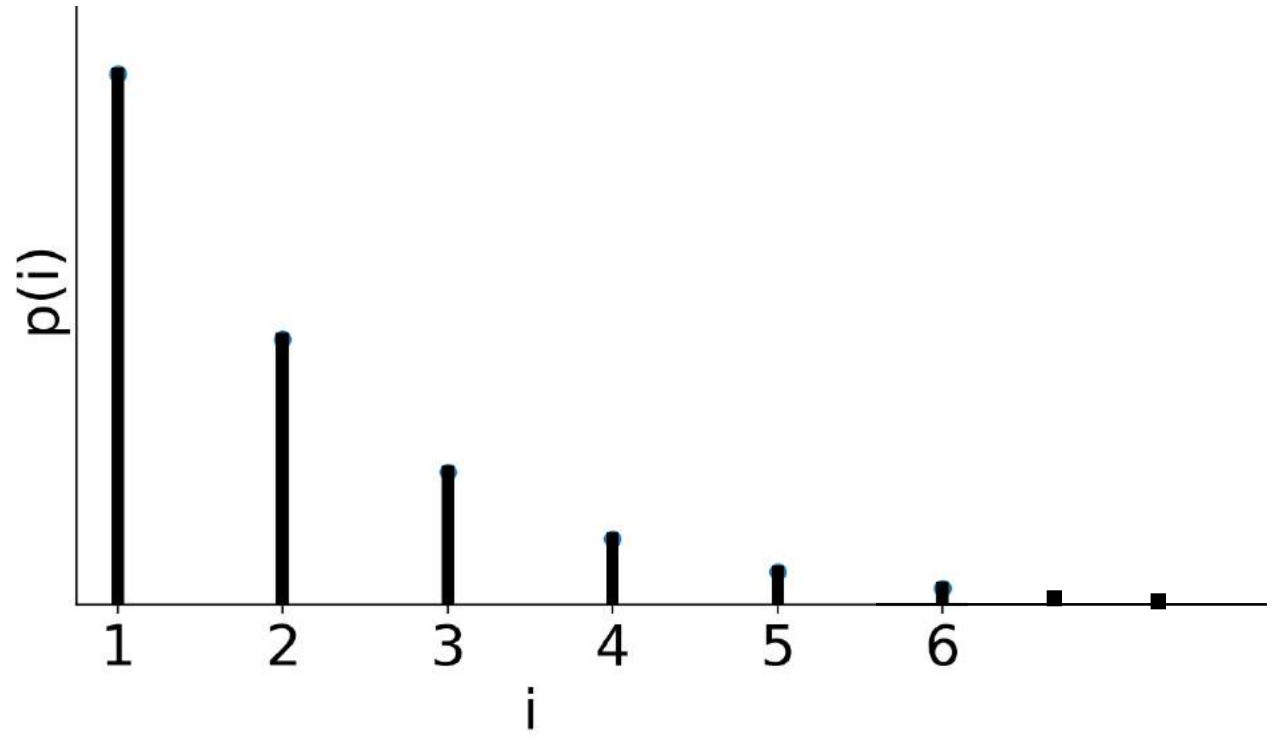
Cannot increase $p_i > 0 \rightarrow p_{i+1}, p_{i+2}, \ldots > p_i \rightarrow \sum = \infty$



Can decrease $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = 1$$



Doubly infinite $\ldots, p_{-2}, p_{-1}, p_0, p_1, p_2, \ldots$

$\ldots, \frac{1}{8}, \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{8}, \ldots$

Formal Definition

Random variable is a mapping $f : \Omega \to \mathbb{R}$

Simplify terminology, focus on math

Number-valued random experiment

Random Variables

Motivation Numbers Operations

Examples

Definitions pmf

Visualization Histogram Plot Stem