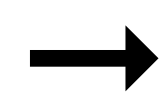Bayes' Rule

Asymmetry

Won Mega Millions → Wealthy ←?

Alive today → Born after 1800 ←?

Forward - Backward

At times   P(F | E) - easy   P(E | F) - hard

2 coins H_i - coin i is h $\exists H$ - at least one h

   $P(\exists H \mid H_1) = 1$ $P(H_1 \mid \exists H)$?

2 dice D_i - face of die i $S = D_1+D_2$ sum of 2 faces

   $P(S=5 \mid D_1=2) = P(D_2=3) = \frac{1}{6}$ $P(D_1=2 \mid S=5)$?

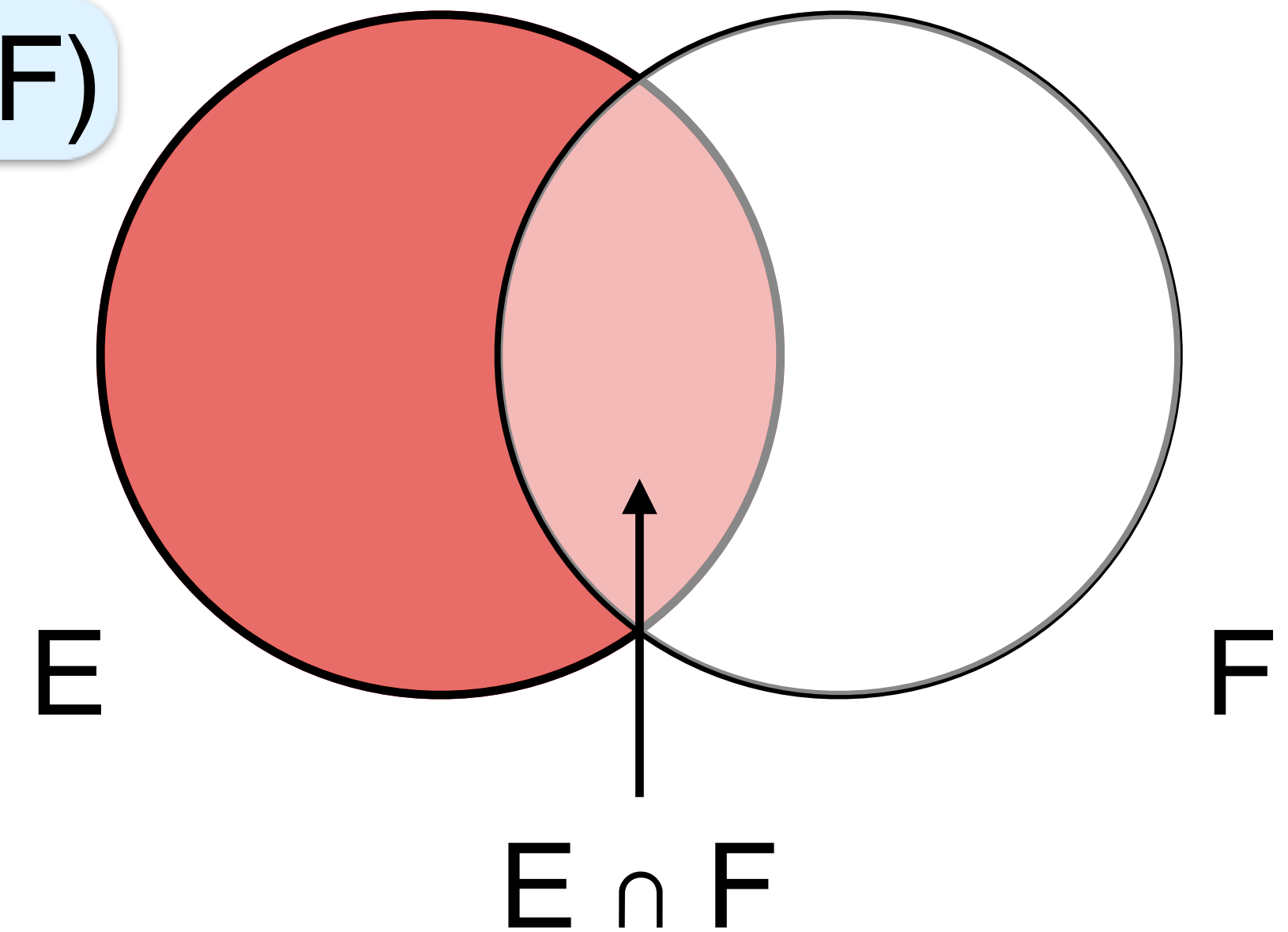**Bayes' Rule**   Method for converting P(F | E) to P(E | F)

Bayes' Rule

P(E), P(F)

Given P(F | E) (and a bit more) determine P(E | F)

$$P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

μ-proof

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$
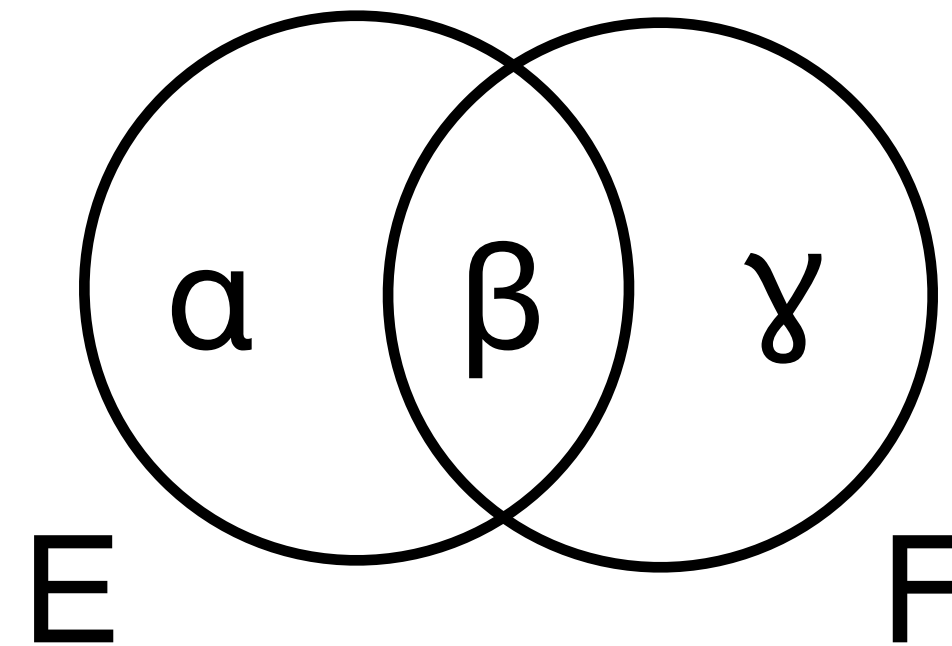
E

F

E ∩ F

Another View

$$P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

$$P(F) \cdot P(E \mid F) = P(E \cap F) = P(E) \cdot P(F \mid E)$$

$$P(F \mid E) = \frac{\beta}{\alpha + \beta}$$

$$P(E \mid F) = \frac{\beta}{\beta + \gamma}$$

$$P(E \mid F) = \frac{\beta}{\beta + \gamma} = \frac{\beta}{\alpha + \beta} \cdot \frac{\alpha + \beta}{\beta + \gamma} = \frac{P(F \mid E) \cdot P(E)}{P(F)}$$

Two Fair Coins

H_i - coin i is h $\exists H$ - at least one h

$$P(H_1 \mid \exists H) = \frac{|H_1 \cap \exists H|}{|\exists H|}$$

$P(H_1 \mid \exists H) = ?$

$$\exists H \begin{cases} h\ h \\ h\ t \\ t\ h \\ t\ t \end{cases} \} H_1$$

$$P(H_1 \mid \exists H) = P(\exists H \mid H_1) \cdot \frac{P(H_1)}{P(\exists H)} \quad = 1 \cdot \frac{½}{¾} \quad = ⅔$$

$P(\exists H \mid H_1) = 1$

$P(H_1) = ½$

$P(\exists H) = ¾$ Last lecture

Two Fair Dice

D_i - outcome of die i $S = D_1 + D_2$ Sum of 2 dice

$P(D_1 = 2 \mid S = 5)$?

$$P(D_1 = 2 \mid S = 5) = \frac{P(S = 5 \mid D_1 = 2) \cdot P(D_1 = 2)}{P(S = 5)} = \frac{\frac{1}{6} \cdot \frac{1}{6}}{\frac{1}{9}} = \frac{1}{4}$$

$P(S = 5 \mid D_1 = 2) = P(D_2 = 3 \mid D_1 = 2) = P(D_2 = 3) = \frac{1}{6}$

$P(D_1 = 2) = \frac{1}{6}$

$P(S = 5) = \frac{1}{9}$ Last lecture

$$S = 5 \begin{cases} 1\ 4 \\ 2\ 3 \} \ D_1 = 2 \\ 3\ 2 \\ 4\ 1 \end{cases}$$

$$P(D_1 = 2 \mid S = 5) = \frac{|D_1 = 2 \cap S = 5|}{|S = 5|}$$

Foxconn

Foxconn has 3 factories producing 50%, 30%, and 20% of its iPhones

Factory defective fractions 4%, 10%, and 5% respectively

Overall fraction of defective phones?

50%   30%   20%

4%   10%   5%

2%   3%   1%
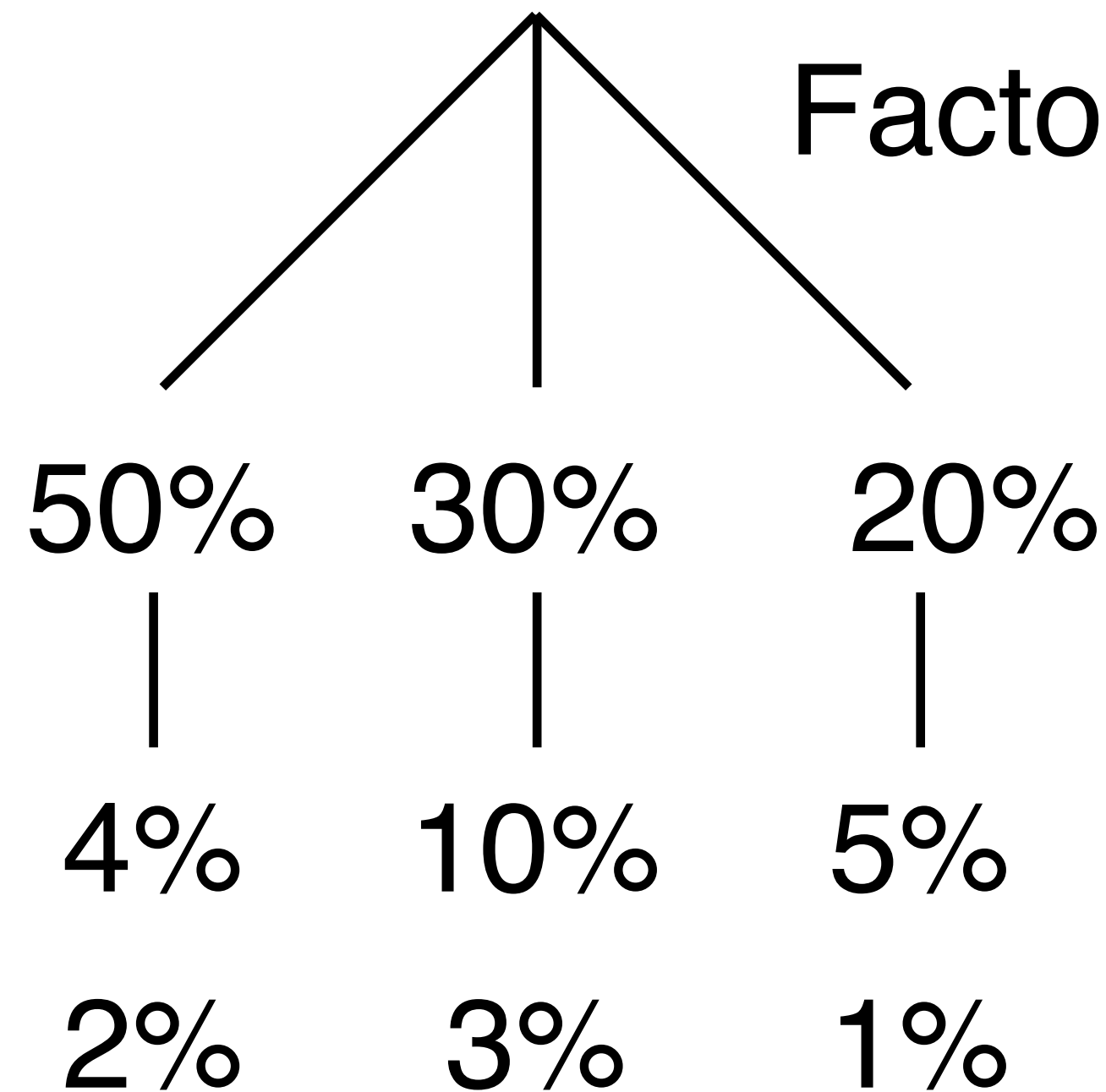
$P(D) = P(D \cap F_1) + P(D \cap F_2) + P(D \cap F_3)$

$= P(F_1)P(D \mid F_1) + P(F_2)P(D \mid F_2) + P(F_3)P(D \mid F_3)$

$= .5 \times .04 + .3 \times .1 + .2 \times .05$

$= .02 + .03 + .01$

$= .06$

Culprit?

$$P(F_1 \mid D) = \frac{P(D \mid F_1) \cdot P(F_1)}{P(D)} = \frac{.04 \cdot .5}{.06} = \frac{.02}{.06} = \frac{1}{3}$$

$$P(D \mid F_1) = .04 \qquad P(F_1) = .5 \qquad P(D) = .06$$

$$P(F_2 \mid D) = \frac{.1 \cdot .3}{.06} = \frac{.03}{.06} = \frac{1}{2}$$

$$P(F_3 \mid D) = \frac{.05 \cdot .2}{.06} = \frac{.01}{.06} = \frac{1}{6}$$

Conditional probabilities add to 1

Conditional order determined by both $P(F_i)$ and $P(D \mid F_i)$

Bayes' Rule

From P(F|E), find P(E|F)

$$P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

Next   Conditional conundrums

Taxi

Hong Kong Taxicab Problem
Witness says taxi is green