Professor Avi Sadeh a pioneer in the field of pediatric sleep and actigraphy died on September Dr Sadeh earned his BA in psychology and MA in clinical psychology at Haifa University Israel and his DSc from the School of Medicine The Technion Israel Institute of Technology with Dr Peretz Lavie He completed a postdoctoral fellowship with Dr Mary Carskadon at Brown University Providence, Rhode Island before joining the faculty at Tel Aviv University Israel in where he was a professor of clinical psychology Dr Sadeh was one of the world’s leaders in the field of pediatric sleep, publishing over scientific papers and book chapters

Dr. Sadeh was among the first researchers to establish actigraphy as a valid way to estimate sleep wake patterns. His pioneering work included the development of the Sadeh algorithm, which continues to be widely used today He also incorporated actigraphy into many of his own research studies, from newborns to adolescents

In Dr Sadeh published a model of infant sleep disturbances from a systems perspec tive This guiding framework shaped his research and clinical work and resulted in many of his well-known measures This includes the Brief Infant Sleep Questionnaire BISQ, a well validated measure of infant and toddler sleep The BISQ has been translated into over languages and utilized in studies around the world Dr Sadeh also had the vision of how important the Internet would be to research and from the earliest validation study he demon strated that the BISQ could be completed via the Internet

Dr Sadehs early theoretical model also suggested the role of parent cognitions and parent infant mediating factors This model contributed to the development of the Infant Sleep Vignettes Interpretation Scale which has been used in a number of research studies highlighting how parental both mother and father cognitions about infant sleep directly and indirectly predict an infants actual sleep behaviors This model also contributed to the examination of the role of fathers involvement in caregiving on infant sleep and to the development of the Parental Involvement Questionnaire

Discrete mathematics is the study of [mathematical structures](https://en.wikipedia.org/wiki/Mathematical_structures) that are fundamentally [discrete](https://en.wikipedia.org/wiki/Discrete_space) rather than [continuous](https://en.wiktionary.org/wiki/continuous). In contrast to [real numbers](https://en.wikipedia.org/wiki/Real_number) that have the property of varying "smoothly", the objects studied in discrete mathematics – such as [integers](https://en.wikipedia.org/wiki/Integer), [graphs](https://en.wikipedia.org/wiki/Graph_(discrete_mathematics)), and [statements](https://en.wikipedia.org/wiki/Statement_(logic)) in [logic](https://en.wikipedia.org/wiki/Mathematical_logic)[[1]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-1) – do not vary smoothly in this way, but have distinct, separated values.[[2]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-2)Discrete mathematics therefore excludes topics in "continuous mathematics" such as [calculus](https://en.wikipedia.org/wiki/Calculus) and [analysis](https://en.wikipedia.org/wiki/Mathematical_analysis). Discrete objects can often be [enumerated](https://en.wikipedia.org/wiki/Enumeration) by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with [countable sets](https://en.wikipedia.org/wiki/Countable_set)[[3]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-3) (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). However, there is no exact definition of the term "discrete mathematics."[[4]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-4) Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions.

The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of [digital computers](https://en.wikipedia.org/wiki/Digital_computers) which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of [computer science](https://en.wikipedia.org/wiki/Computer_science), such as [computer algorithms](https://en.wikipedia.org/wiki/Computer_algorithm), [programming languages](https://en.wikipedia.org/wiki/Programming_language), [cryptography](https://en.wikipedia.org/wiki/Cryptography), [automated theorem proving](https://en.wikipedia.org/wiki/Automated_theorem_proving), and [software development](https://en.wikipedia.org/wiki/Software_development). Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in [operations research](https://en.wikipedia.org/wiki/Operations_research).

Although the main objects of study in discrete mathematics are discrete objects, analytic methods from continuous mathematics are often employed as well.

In university curricula, "Discrete Mathematics" appeared in the 1980s, initially as a computer science support course; its contents were somewhat haphazard at the time. The curriculum has thereafter developed in conjunction with efforts by [ACM](https://en.wikipedia.org/wiki/Association_for_Computing_Machinery) and [MAA](https://en.wikipedia.org/wiki/Mathematical_Association_of_America) into a course that is basically intended to develop [mathematical maturity](https://en.wikipedia.org/wiki/Mathematical_maturity) in freshmen; therefore it is nowadays a prerequisite for mathematics majors in some universities as well.[[5]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-LevasseurDoerr-5)[[6]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-Howson1988-6) Some high-school-level discrete mathematics textbooks have appeared as well.[[7]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-Rosenstein-7) At this level, discrete mathematics is sometimes seen as a preparatory course, not unlike [precalculus](https://en.wikipedia.org/wiki/Precalculus) in this respect.[[8]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-8)

The [Fulkerson Prize](https://en.wikipedia.org/wiki/Fulkerson_Prize) is awarded for outstanding papers in discrete mathematics.

The history of discrete mathematics has involved a number of challenging problems which have focused attention within areas of the field. In graph theory, much research was motivated by attempts to prove the [four color theorem](https://en.wikipedia.org/wiki/Four_color_theorem), first stated in 1852, but not proved until 1976 (by Kenneth Appel and Wolfgang Haken, using substantial computer assistance).[[9]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-4colors-9)

In [logic](https://en.wikipedia.org/wiki/Mathematical_logic), the [second problem](https://en.wikipedia.org/wiki/Hilbert%27s_second_problem) on [David Hilbert](https://en.wikipedia.org/wiki/David_Hilbert)'s list of open [problems](https://en.wikipedia.org/wiki/Hilbert%27s_problems) presented in 1900 was to prove that the [axioms](https://en.wikipedia.org/wiki/Axioms) of [arithmetic](https://en.wikipedia.org/wiki/Arithmetic) are [consistent](https://en.wikipedia.org/wiki/Consistent). [Gödel's second incompleteness theorem](https://en.wikipedia.org/wiki/G%C3%B6del%27s_second_incompleteness_theorem), proved in 1931, showed that this was not possible – at least not within arithmetic itself. [Hilbert's tenth problem](https://en.wikipedia.org/wiki/Hilbert%27s_tenth_problem) was to determine whether a given polynomial [Diophantine equation](https://en.wikipedia.org/wiki/Diophantine_equation) with integer coefficients has an integer solution. In 1970, [Yuri Matiyasevich](https://en.wikipedia.org/wiki/Yuri_Matiyasevich) proved that this [could not be done](https://en.wikipedia.org/wiki/Matiyasevich%27s_theorem).

The need to [break](https://en.wikipedia.org/wiki/Cryptanalysis) German codes in [World War II](https://en.wikipedia.org/wiki/World_War_II) led to advances in [cryptography](https://en.wikipedia.org/wiki/Cryptography) and [theoretical computer science](https://en.wikipedia.org/wiki/Theoretical_computer_science), with the [first programmable digital electronic computer](https://en.wikipedia.org/wiki/Colossus_computer) being developed at England's [Bletchley Park](https://en.wikipedia.org/wiki/Bletchley_Park) with the guidance of [Alan Turing](https://en.wikipedia.org/wiki/Alan_Turing) and his seminal work, On Computable Numbers.[[10]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-10) At the same time, military requirements motivated advances in [operations research](https://en.wikipedia.org/wiki/Operations_research). The [Cold War](https://en.wikipedia.org/wiki/Cold_War) meant that cryptography remained important, with fundamental advances such as [public-key cryptography](https://en.wikipedia.org/wiki/Public-key_cryptography) being developed in the following decades. Operations research remained important as a tool in business and project management, with the [critical path method](https://en.wikipedia.org/wiki/Critical_path_method) being developed in the 1950s. The [telecommunication](https://en.wikipedia.org/wiki/Telecommunication) industry has also motivated advances in discrete mathematics, particularly in graph theory and [information theory](https://en.wikipedia.org/wiki/Information_theory). [Formal verification](https://en.wikipedia.org/wiki/Formal_verification) of statements in logic has been necessary for [software development](https://en.wikipedia.org/wiki/Software_development) of [safety-critical systems](https://en.wikipedia.org/wiki/Safety-critical_system), and advances in [automated theorem proving](https://en.wikipedia.org/wiki/Automated_theorem_proving) have been driven by this need.

[Computational geometry](https://en.wikipedia.org/wiki/Computational_geometry) has been an important part of the [computer graphics](https://en.wikipedia.org/wiki/Computer_graphics_(computer_science)) incorporated into modern [video games](https://en.wikipedia.org/wiki/Video_game) and [computer-aided design](https://en.wikipedia.org/wiki/Computer-aided_design) tools.

Several fields of discrete mathematics, particularly theoretical computer science, graph theory, and [combinatorics](https://en.wikipedia.org/wiki/Combinatorics), are important in addressing the challenging [bioinformatics](https://en.wikipedia.org/wiki/Bioinformatics) problems associated with understanding the [tree of life](https://en.wikipedia.org/wiki/Phylogenetic_tree).[[11]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-11)

Currently, one of the most famous open problems in theoretical computer science is the [P = NP problem](https://en.wikipedia.org/wiki/P_%3D_NP_problem), which involves the relationship between the [complexity classes](https://en.wikipedia.org/wiki/Complexity_class) [P](https://en.wikipedia.org/wiki/P_(complexity)) and [NP](https://en.wikipedia.org/wiki/NP_(complexity)). The [Clay Mathematics Institute](https://en.wikipedia.org/wiki/Clay_Mathematics_Institute) has offered a $1 million [USD](https://en.wikipedia.org/wiki/USD) prize for the first correct proof, along with prizes for [six other mathematical problems](https://en.wikipedia.org/wiki/Millennium_Prize_Problems).[[12]](https://en.wikipedia.org/wiki/Discrete_mathematics#cite_note-CMI_Millennium_Prize_Problems-12)

A random walk is a [mathematical](https://en.wikipedia.org/wiki/Mathematical) object, known as a stochastic or [random process](https://en.wikipedia.org/wiki/Random_process), that describes a path that consists of a succession of [random](https://en.wikipedia.org/wiki/Random) steps on some mathematical space such as the integers. An elementary example of a random walk is the random walk on the [integer](https://en.wikipedia.org/wiki/Integer) number line,

Z

, which starts at 0 and at each step moves +1 or −1 with equal probability. Other examples include the path traced by a [molecule](https://en.wikipedia.org/wiki/Molecule) as it travels in a liquid or a gas, the search path of a [foraging](https://en.wikipedia.org/wiki/Foraging) animal, the price of a fluctuating [stock](https://en.wikipedia.org/wiki/Random_walk_hypothesis) and the financial status of a [gambler](https://en.wikipedia.org/wiki/Gambler) can all be approximated by random walk models, even though they may not be truly random in reality. As illustrated by those examples, random walks have applications to many scientific fields including [ecology](https://en.wikipedia.org/wiki/Ecology), [psychology](https://en.wikipedia.org/wiki/Psychology), [computer science](https://en.wikipedia.org/wiki/Computer_science), [physics](https://en.wikipedia.org/wiki/Physics), [chemistry](https://en.wikipedia.org/wiki/Chemistry), [biology](https://en.wikipedia.org/wiki/Biology) as well as [economics](https://en.wikipedia.org/wiki/Economics). Random walks explain the observed behaviors of many processes in these fields, and thus serve as a fundamental [model](https://en.wikipedia.org/wiki/Statistical_model) for the recorded [stochastic activity](https://en.wikipedia.org/wiki/Stochastic_process). As a more mathematical application, the value of pi can be approximated by the usage of random walk in agent-based modelling environment.[1][2] The term random walk was first introduced by [Karl Pearson](https://en.wikipedia.org/wiki/Karl_Pearson) in 1905.[3]

Various types of random walks are of interest, which can differ in several ways. The term itself most often refers to a special category of [Markov chains](https://en.wikipedia.org/wiki/Markov_chain) or [Markov processes](https://en.wikipedia.org/wiki/Markov_process), but many time-dependent processes are referred to as random walks, with a modifier indicating their specific properties. Random walks (Markov or not) can also take place on a variety of spaces: commonly studied ones include [graphs](https://en.wikipedia.org/wiki/Graph_theory), others on the integers or the real line, in the plane or in higher-dimensional vector spaces, on [curved surfaces](https://en.wikipedia.org/wiki/Surface_(differential_geometry)) or higher-dimensional [Riemannian manifolds](https://en.wikipedia.org/wiki/Riemannian_manifold), and also on [groups](https://en.wikipedia.org/wiki/Group_theory) finite, [finitely generated](https://en.wikipedia.org/wiki/Finitely_generated_group) or [Lie](https://en.wikipedia.org/wiki/Lie_group). The time parameter can also be manipulated. In the simplest context the walk is in discrete time, that is a sequence of [random variables](https://en.wikipedia.org/wiki/Random_variable) (X

t) = (X

1, X

2,...) indexed by the natural numbers. However, it is also possible to define random walks which take their steps at random times, and in that case the position X

t has to be defined for all times t ∈ [0,+∞). Specific cases or limits of random walks include the [Lévy flight](https://en.wikipedia.org/wiki/L%C3%A9vy_flight) and [diffusion](https://en.wikipedia.org/wiki/Diffusion) models such as [Brownian motion](https://en.wikipedia.org/wiki/Brownian_motion).

Random walks are a fundamental topic in discussions of Markov processes. Their mathematical study has been extensive. Several properties, including dispersal distributions, first-passage or hitting times, encounter rates, recurrence or transience, have been introduced to quantify their behaviour.

A popular random walk model is that of a random walk on a regular lattice, where at each step the location jumps to another site according to some probability distribution. In a simple random walk, the location can only jump to neighboring sites of the lattice, forming a [lattice path](https://en.wikipedia.org/wiki/Lattice_path). In simple symmetric random walk on a locally finite lattice, the probabilities of the location jumping to each one of its immediate neighbours are the same. The best studied example is of random walk on the d-dimensional integer lattice (sometimes called the hypercubic lattice)

If the state space is limited to finite dimensions, the random walk model is called simple bordered symmetric random walk and the transition probabilities depend on the location of the state, because on margin and corner states the movement is limited.[5]

An elementary example of a random walk is the random walk on the [integer](https://en.wikipedia.org/wiki/Integer) number line,

starts at 0 and at each step moves +1 or −1 with equal probability.

This walk can be illustrated as follows. A marker is placed at zero on the number line and a fair coin is flipped. If it lands on heads, the marker is moved one unit to the right. If it lands on tails, the marker is moved one unit to the left. After five flips, the marker could now be on 1, −1, 3, −3, 5, or −5. With five flips, three heads and two tails, in any order, will land on 1. There are 10 ways of landing on 1 (by flipping three heads and two tails), 10 ways of landing on −1 (by flipping three tails and two heads), 5 ways of landing on 3 (by flipping four heads and one tail), 5 ways of landing on −3 (by flipping four tails and one head), 1 way of landing on 5 (by flipping five heads), and 1 way of landing on −5 (by flipping five tails). See the figure below for an illustration of the possible outcomes of 5 flips.

In higher dimensions, the set of randomly walked points has interesting geometric properties. In fact, one gets a discrete [fractal](https://en.wikipedia.org/wiki/Fractal), that is, a set which exhibits stochastic [self-similarity](https://en.wikipedia.org/wiki/Self-similarity) on large scales. On small scales, one can observe "jaggedness" resulting from the grid on which the walk is performed. Two books of Lawler referenced below are a good source on this topic. The trajectory of a random walk is the collection of points visited, considered as a set with disregard to when the walk arrived at the point. In one dimension, the trajectory is simply all points between the minimum height and the maximum height the walk achieved (both are, on average, on the order of √n).

To visualize the two dimensional case, one can imagine a person walking randomly around a city. The city is effectively infinite and arranged in a square grid of sidewalks. At every intersection, the person randomly chooses one of the four possible routes (including the one originally traveled from). Formally, this is a random walk on the set of all points in the [plane](https://en.wikipedia.org/wiki/Plane_(mathematics)) with [integer](https://en.wikipedia.org/wiki/Integer) [coordinates](https://en.wikipedia.org/wiki/Coordinate_system).

Will the person ever get back to the original starting point of the walk? This is the 2-dimensional equivalent of the level crossing problem discussed above. It turns out that the person [almost surely](https://en.wikipedia.org/wiki/Almost_surely) will in a 2-dimensional random walk, but for 3 dimensions or higher, the probability of returning to the origin decreases as the number of dimensions increases. In 3 dimensions, the probability decreases to roughly 34%.[[10]](https://en.wikipedia.org/wiki/Random_walk#cite_note-10)

The asymptotic function for a two dimensional random walk as the number of steps increases is given by a [Rayleigh distribution](https://en.wikipedia.org/wiki/Rayleigh_distribution). The probability distribution is a function of the radius from the origin and the step length is constant for

A [Wiener process](https://en.wikipedia.org/wiki/Wiener_process) is a stochastic process with similar behaviour to [Brownian motion](https://en.wikipedia.org/wiki/Brownian_motion), the physical phenomenon of a minute particle diffusing in a fluid. (Sometimes the [Wiener process](https://en.wikipedia.org/wiki/Wiener_process) is called "Brownian motion", although this is strictly speaking a [confusion of a model with the phenomenon being modeled](https://en.wikipedia.org/wiki/Map-territory_relation).)

A Wiener process is the [scaling limit](https://en.wikipedia.org/wiki/Scaling_limit) of random walk in dimension 1. This means that if you take a random walk with very small steps you get an approximation to a Wiener process (and, less accurately, to Brownian motion). To be more precise, if the step size is ε, one needs to take a walk of length L/ε2 to approximate a Wiener length of L. As the step size tends to 0 (and the number of steps increases proportionally) random walk converges to a Wiener process in an appropriate sense. Formally, if B is the space of all paths of length L with the maximum topology, and if M is the space of measure over B with the norm topology, then the convergence is in the space M. Similarly, a Wiener process in several dimensions is the scaling limit of random walk in the same number of dimensions.

A random walk is a discrete [fractal](https://en.wikipedia.org/wiki/Fractal) (a function with integer dimensions; 1, 2, ...), but a Wiener process trajectory is a true fractal, and there is a connection between the two. For example, take a random walk until it hits a circle of radius r times the step length. The average number of steps it performs is r2.[[citation needed](https://en.wikipedia.org/wiki/Wikipedia:Citation_needed" \o "Wikipedia:Citation needed)] This fact is the discrete version of the fact that a Wiener process walk is a fractal of [Hausdorff dimension](https://en.wikipedia.org/wiki/Hausdorff_dimension) 2.[[citation needed](https://en.wikipedia.org/wiki/Wikipedia:Citation_needed)]

In two dimensions, the average number of points the same random walk has on the boundary of its trajectory is r4/3. This corresponds to the fact that the boundary of the trajectory of a Wiener process is a fractal of dimension 4/3, a fact predicted by [Mandelbrot](https://en.wikipedia.org/wiki/Beno%C3%AEt_Mandelbrot) using simulations but proved only in 2000 by [Lawler](https://en.wikipedia.org/wiki/Greg_Lawler), [Schramm](https://en.wikipedia.org/wiki/Oded_Schramm) and [Werner](https://en.wikipedia.org/wiki/Wendelin_Werner).[[11]](https://en.wikipedia.org/wiki/Random_walk#cite_note-11)

A Wiener process enjoys many [symmetries](https://en.wikipedia.org/wiki/Symmetry) random walk does not. For example, a Wiener process walk is invariant to rotations, but random walk is not, since the underlying grid is not (random walk is invariant to rotations by 90 degrees, but Wiener processes are invariant to rotations by, for example, 17 degrees too). This means that in many cases, problems on random walk are easier to solve by translating them to a Wiener process, solving the problem there, and then translating back. On the other hand, some problems are easier to solve with random walks due to its discrete nature.

Random walk and [Wiener process](https://en.wikipedia.org/wiki/Wiener_process) can be [coupled](https://en.wikipedia.org/wiki/Coupling_(probability)), namely manifested on the same probability space in a dependent way that forces them to be quite close. The simplest such coupling is the Skorokhod embedding, but there exist more precise couplings, such as [Komlós–Major–Tusnády approximation](https://en.wikipedia.org/wiki/Koml%C3%B3s%E2%80%93Major%E2%80%93Tusn%C3%A1dy_approximation) theorem.

The convergence of a random walk toward the Wiener process is controlled by the [central limit theorem](https://en.wikipedia.org/wiki/Central_limit_theorem), and by [Donsker's theorem](https://en.wikipedia.org/wiki/Donsker%27s_theorem). For a particle in a known fixed position at t = 0, the central limit theorem tells us that after a large number of [independent](https://en.wikipedia.org/wiki/Statistical_independence) steps in the random walk, the walker's position is distributed according to a [normal distribution](https://en.wikipedia.org/wiki/Normal_distribution) of total [variance](https://en.wikipedia.org/wiki/Variance):