Observations for Sample Covariance matrix (SCM)

$$V = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - X)(x_i - X)^T,$$

What if we expand the parentheses for the sum of $(x_i - X)(x_i - X)^T$, so

$$V = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i x_i^T - x_i X^T - X x_i^T + XX^T\right) =$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i x_i^T - \sum_{i=1}^{n} x_i X^T - \sum_{i=1}^{n} X x_i^T + \sum_{i=1}^{n} XX^T\right) =$$

$$= \frac{1}{n-1}\left(\underbrace{\sum_{i=1}^{n} x_i x_i^T}_{R} - X^T \underbrace{\sum_{i=1}^{n} x_i}_{S} - X \underbrace{\sum_{i=1}^{n} x_i^T}_{S^T} + nXX^T\right)$$

S - from Observations for SMV

The final equation for SCM is

$$V = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i x_i^T - X^T \sum_{i=1}^{n} x_i - X \sum_{i=1}^{n} x_i^T + nXX^T\right)$$

From observation for SMV we have to store the n - count of vectors, S - sum of received vectors; so, to SMC we need one extra canonical variable that represents the sum of multiply x_i and x_i^T (called as R for simplicity).

$$(n, S, R), \Rightarrow \quad S = \sum_{i=1}^{n} x_i$$

$$R = \sum_{i=1}^{n} x_i x_i^T$$

n - count of observ.