

Calibration Problem

In this assignment you will implement and demonstrate optimal calibration for a linear estimation. The underlying process imitates a simple signal measurement experiment and is heavily based on Homework 6. Here are suggested phases of the project.

1. Measurement Simulation:

- (a) Randomly generate some “unknown” profile $x \sim (0, F)$.
- (b) Create a matrix A .
- (c) Simulate a measurement

$$y = Ax + \nu, \quad \nu \sim (0, \sigma^2 I).$$

2. Calibration (assuming that A is unknown):

- (a) Randomly generate calibration signals φ_k , $k = 1, \dots, K$ in the same way as you generated x .
- (b) Simulate calibration measurements

$$\psi_k = A\varphi_k + \nu_k.$$

- (c) Collect canonical calibration information (G, H) .
- (d) Compute A_0 (an estimate of A) and J .

3. Using your simulated observation y construct an optimal linear estimate \hat{x} and the variance matrix $\text{Var}(\hat{x} - x)$. Show on the same graph:

- (a) The original signal x (a curve with components x_i),
- (b) Its estimate \hat{x} (a curve with components \hat{x}_i),
- (c) Standard deviations for the estimates \hat{x}_i

$$\sqrt{\mathbb{E}(\hat{x}_i - x_i)^2} = \sqrt{\text{Var}(\hat{x} - x)_{ii}} = \sqrt{Q_{ii}}$$

can be illustrated by showing the corresponding “corridor” around \hat{x}_i .

4. Illustrate estimation (Phase 3) when you not only increase the number of calibration measurements K but also measure the original signal x N times:

- (a) Simulate N measurements $y_n = Ax + \nu_n$ for $n = 1, \dots, N$ and collect the appropriate information.
- (b) Simulate K calibration measurements, collect canonical calibration information, and compute A_0 and J .
- (c) Using these two types of information construct and show (as in Phase 3) an optimal linear estimate \hat{x} and its precision.
- (d) Using these two types of information construct and show (as in Phase 3) an optimal linear estimate \hat{x} and its precision.

- (e) Do the above for several cases with numbers N and K “small” “medium”, and “large”. Reminder: N and K are “balanced” when $K \approx N \cdot M$, where M is the dimension of the unknown x .
- (f) (Optional) Show how estimation precision depends on N and K . To do that you could show total estimation error

$$E\|\hat{x} - x\|^2 = \text{tr}Q$$

as a function of N and K . To illustrate a function of two variables you can show it, e.g., as a surface or as a pseudocolor image. It might be interesting to indicate contour lines (curves along which the function has constant values).