

# Check yourself problems

## Lecture slides

Lecture slides are available at the LMS. The slides are not exhaustive – some topics are explained on the blackboard. The dates in the titles below correspond to the seminar(s) when the respective topic was discussed.

### 1 Percentage calculus (seminar 06.09)

1. Look through the reference problems<sup>1</sup> titled “Percents”.
2. Pay attention to the effective annual rates (EAR). Find the EAR if you invest now \$100 and get \$135 in 2.3 years.
3. Read [1, pp.91-93] and Section 4.3

### 2 Discounted cash flow valuation (seminar 06.09)

1. Look through the reference problems titled “Cash flows”
2. Read [1, pp.95-101] and the entire Section 4.4
3. Problems 15,16,47,52,58 from [1, Ch.4]. Solutions are in the folder “Solutions”, subfolder “Ross”, see the LMS.
4. Consider a cash flow  $CF = \{(0, -\$100); (1, \$40); (2, \$90)\}$  where the first number in the parentheses denotes a point of time (0 is the current moment, 1 is a year from now etc) and the second number is the respective cash inflow/outflow. Let the discount rate ( $\approx$  market rate of return) be  $r\%$ . By no-arbitrage argument show that the fair price of such cash flow is its present value. Describe the arbitrage opportunities if the current price  $\neq$  PV.

### 3 Making capital budgeting decisions (seminar 13.09)

1. Read the entire Chapter 5 of [1]
2. Problems 11-14,16,20,28 from [1, Ch.5]. Solutions are in the folder “Solutions”, subfolder “Ross”, see the LMS.
3. IRR examples for margin trading and short sales:
  - (a) See the solutions in the folder “Solutions”, subfolder “Ross”, at the LMS
  - (b) Read more about trading on margin and short sales in [2, pp.76,80]

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<sup>1</sup>By “looking through” I mean that you don’t need to solve all of the problems. Think of those that look difficult. Ask the instructor if you do not know how to approach them.

## **4 Structure of CFs, impact of inflation etc (seminar 13.09)**

1. Read Chapter 6 of [1]
2. Problems 2,10,12,13,15 from [1, Ch.6], see the solutions at the LMS.

## **5 Bond valuation (seminars 20.09 and 27.09)**

1. Look through the reference problems titled “Bond prices and yields”
2. Look through the reference problems titled “Duration of bonds” (special attention to #6,7)
3. Problems 1-6,13,16,18,19,24 from [1, Ch.8]. See the solutions at the LMS.
4. Term structure of interest rates: read [2, Ch.15], do end-of-chapter exercises 11,13 (the solutions are in subfolder “Bodie”).
5. **Problems for test II to be held on October, 15.** Problems 1-6 from the list at the end of this document.

## **6 CAPM, factor models and DDM (seminar 11.10)**

All the problems below are likely to appear in test II scheduled on October, 15.

1. Pure CAPM. Problems 14-15 from the list at the end of this document.
  - (a) For analogs, see Problems 1,4,5,9a,19; 8,9,12 from the CFA section, see [2, Ch.9]. Solutions are available at the LMS.
2. Factor models, problems 8-13 from the list at the end of this document.
  - (a) For analogs, see Problems 1,4,5,6; Example 10.4 from [2, Ch.10]. Solutions are available at the LMS.
3. Discount dividend model, problems 16-20 from the list at the end of this document.
  - (a) For analogs, see Problems 2,5,9 from [1, Ch.9]. Solutions are available at the LMS.

## **7 Pairs trading, Glosten-Milgrom and Kyle models (seminar 12.10)**

1. Pairs trading (problem 21 from the list at the end of this document)
  - (a) For the solution, see the LMS.
2. Glosten-Milgrom and Kyle – problems 22,23. See [3, Ch.5, p.22]. For Kyle,

## References

- [1] S. Ross, R. Westerfield, J. Jaffe, [Corporate Finance with S&P card](#), McGraw-Hill Companies, Incorporated, 2009.  
URL <https://books.google.ru/books?id=bMiCPwAACAAJ>
- [2] A. Marcus, A. Kane, Z. Bodie, R. Jain, [Investments](#), The McGraw-Hill/Irwin series in finance, insurance and real estate, McGraw-Hill Education, 2014.  
URL <https://books.google.ru/books?id=4VMVnwEACAAJ>
- [3] J. Hasbrouck, [Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading](#), Oxford University Press, 2007.  
URL <https://books.google.ru/books?id=aaReNv846eMC>

## Problems for test II, October 15

**Problem 1.** The discount rate is  $r = 6\%$ . A corporation issues 70 6-year bonds with the face value  $N_1 = \$300$ . To immunize this obligation against a possible change of interest rates, the firm is considering 3-year bonds with face value of  $N_2 = \$300$  and 7-year bonds with face value  $N_3 = \$250$ .

*Bonpoc 1.* Find the present value of the liabilities:

*Bonpoc 2.* What should be done with bonds #2 and #3 for immunizing against the interest rate change?:

*Bonpoc 3.* Find the formula describing the change in the current price of liabilities when the interest rate changes by  $\Delta r$ :

*Bonpoc 4.* Work out the equation for the amount  $x$  that needs to be invested in bonds #2 to provide immunization:

*Bonpoc 5.* Find the numerical value of  $x$  from the preceding question:

*Bonpoc 6.* Find the number of bonds #3 in the immunizing portfolio:

*Bonpoc 7.* Rebalance the immunizing portfolio in 2 year(s) (if needed): find the number of bonds #2 and #3 in the new portfolio.

**Problem 2.** The discount rate is currently equal to  $r = 10\%$ . A corporation just sold an obligation to make the following payments in the future:  $CF = (3, 500); (6, 1200); (7, 3500)$ . To hedge against a possible interest rate change the firm is considering 2-year bonds with face value  $N_2 = \$150$  and 8-year bonds with face value  $N_3 = \$300$ .

*Bonpoc 1.* Find the present value of the liabilities:

*Bonpoc 2.* What should be done with bonds #2 and #3 for immunizing against the interest rate change?:

*Bonpoc 3.* Find the expression for the weight  $w_6$  in the formula for calculating the duration of the liabilities:

*Bonpoc 4.* Find the numerical value of the duration of the liabilities:

*Bonpoc 5.* Work out the equation for the amount  $x$  that needs to be invested in bonds #2 to provide immunization:

*Bonpoc 6.* Find the numerical value of  $x$  from the preceding question:

*Bonpoc 7.* Find the number of bonds #3 in the immunizing portfolio:

*Bonpoc 8.* Rebalance the immunizing portfolio in 1 year(s) (if needed): find the number of bonds #2 and #3 in the new portfolio.

**Problem 3.** The discount rate is currently  $r = 9\%$ . A corporation just sold an obligation to pay \$25000 every year in the foreseeable future (in perpetuity). To hedge against a possible change in interest rates the firm is considering 2-year bonds with face value  $N_2 = \$200$  and 25-year bonds with face value  $N_3 = \$100$ .

*Bonpoc 1.* Find the present value of the liabilities:

*Bonpoc 2.* What should be done with bonds #2 and #3 for immunizing against the interest rate change?:

*Bonpoc 3.* Find the duration of the liabilities:

*Bonpoc 4.* Work out the equation for the amount  $x$  that needs to be invested in bonds #2 to provide immunization:

*Bonpoc 5.* Find the numerical value of  $x$  from the preceding question:

*Bonpoc 6.* Find the number of bonds #3 in the immunizing portfolio:

*Bonpoc 7.* Rebalance the immunizing portfolio in 1 year(s) (if needed): find the number of bonds #2 and #3 in the new portfolio.

**Problem 4.** 1-year and 2-year zero coupon bonds with face value \$955 и \$1065 are currently selling for \$925 and \$995, respectively. Consider a 2-year bond with coupon rate 8% and face value of \$1000.

*Bonpoc 1.* Find one-year  $y_1$  and two-year  $y_2$  spot rate:

*Bonpoc 2.* Find the expression for the current value of a 2-year bond with coupon rate 8%:

*Bonpoc 3.* Find the forward rate for the second year assuming (i) the expectation theory holds (ii) liquidity preference theory with liquidity premium  $\Delta = 1\%$ :

*Bonpoc 4.* Find the current price of the 2-year coupon bond in a year under the liquidity preference assumption with  $\Delta = 1\%$ :

**Problem 5.** A 9-year bond with face value \$1000 currently shows a yield-to-maturity of 7% and makes annual coupon payments at a rate of 9%.

*Bonpoc 1.* Find the expression for the holding period return (HPR) for the 1st year:

*Bonpoc 2.* Find the numerical value of the HPR for the second year if the yield-to-maturity by the beginning of the 2nd year is going to be 9%:

**Problem 6.** The discount rate is  $r = 7\%$ . Consider a 4-year bond with face value  $N = \$350$  and coupon rate  $q = 6\%$ .

*Bonpoc* 1. Find the formula for the current price of the bond:

*Bonpoc* 2. Find the current price numerically:

*Bonpoc* 3. Write down the equation for the yield-to-maturity ( $y$ ) of the bond if it is currently selling for \$328:

*Bonpoc* 4. Let the market price of the bond be equal to the face value. What is the yield-to-maturity?:

*Bonpoc* 5. Let the yield-to-maturity be 4.5%. What is the market price of the bond with respect to the face value?:

*Bonpoc* 6. As the market price increases, the yield-to-maturity...

*Bonpoc* 7. Find the formula for the cuurent value of the bond if it were zero coupon:

**Problem 7.** Consider two assets with  $r_A = 0.11$ ,  $\sigma_A = 0.08$  and  $r_B = 0.36$ ,  $\sigma_B = 0.4$ , the correlation coefficient  $\rho_{AB} = 0.3$ . The risk-free return is  $r_F = 0.03$ . Your utility function is  $U(r) = r - A/2\sigma^2$ , where  $A = 4$ .

*Bonpoc* 1. Find the expression for the Sharpe ratio  $S_p$  on the feasible set ( $w_A$  and  $w_B$  are the weights):

*Bonpoc* 2. Choose the condition for finding the tangent portfolio:

*Bonpoc* 3. Find the tangent portfolio (i.e., the weights) numerically:

*Bonpoc* 4. Find the equation for the CML:

*Bonpoc* 5. Find the return on the portfolio providing maximal utility:

**Problem 8.** There are two diversified portfolios, A and B. Portfolio A has  $\beta_A = 1.2$  and the expected return 15%. Portfolio B has  $\beta_B = 1.3$  and the expected return of 19%. The risk-free rate is 3%.

*Bonpoc* 1. Is there an arbitrage opportunity in such a market? What should be done with assets A and B?

*Bonpoc* 2. Find the weights of assets A, B and risk-free asset providing the arbitrage opportunity:

*Bonpoc* 3. Find the risk of the resulting portfolio. What is the measure of this risk? Write down the formula, otherwise the

*Bonpoc* 4. Does one need to make the initial investment to fulfil this transaction?

*Bonpoc* 5. Find the profit of such a transaction for \$4000:

*Bonpoc* 6. Consider the following tactics (the risk-free asset is not used): sell short asset A, buy asset B for \$1000. The initial investment is zero. The profit is poditive:  $1000 \times |0.15 - 0.19|$ . What is wrong here? It it an arbitrage opportunity? If not, why?

**Problem 9.** Consider a single-factor model with factor  $F$ . There are two diversified portfolios, A and B. Portfolio A has  $\beta_A = 1.2$  and the observed expected return 19%. Portfolio B has  $\beta_B = 1.5$  and the observed expected return of 25%.

*Bonpoc* 1. Work out the formula for the expected return on asset B according to the single-factor model:

*Bonpoc* 2. Under no-arbtrage condition find the risk-free rate:

**Problem 10.** Consider a single-factor model with factor  $F$ . There are two diversified portfolios, A and B. Portfolio A the observed expected return 15%. Portfolio B has  $\beta_B = 1.1$  and the observed expected return of 17%. The risk-free rate is 7%.

*Bonpoc* 1. Work out the expression for the expected return on asset B according to the factor model

*Bonpoc* 2. Under no-arbitrage condition find  $\beta_A$ :

**Problem 11.** In a single-factor model, find the unsystematic risk of an equally weighted portfolio consisting of 16 stocks. The unsystematic risk of each stock is 0.1.

*Bonpoc* 1. Provide the answer:

**Problem 12.** Consider a two-factor model:  $r = 0.13 + 1.1 \times F_1 + 0.6 \times F_2 + \epsilon$ . Variabce of factor 1 is 0.04, variance of factor 2 is 0.09, variance of the unsystematic component is 0.09. The risk free rate is 5%.

*Bonpoc* 1. What is the expected return on the stock?:

*Bonpoc* 2. What is the expected value of the factor  $F_1$ ?

*Bonpoc* 3. What is the expected value of the unsystematic risk?:

*Bonpoc* 4. Find the risk of this asset

**Problem 13.** Consider a two-factor model with two independent factors  $F_1$  and  $F_2$ . The risk-free rate is 4%, the unsystematic risk for all stocks is 0.2. Consider two well-diversified portfolios. Portfolio A: expected return 18%,  $\beta_{F_1} = 1.2$ ,  $\beta_{F_2} = 1.1$ . Portfolio B: expected return 23%,  $\beta_{F_1} = 1.4$ ,  $\beta_{F_2} = 1.5$ .

*Bonpoc* 1. Find the equation for the expected return in this market:

**Problem 14.** The company's stock has  $\beta = 0.7$ . The risk-free rate is  $r_F = 4\%$ , the return on the market portfolio is  $r_M = 15\%$ .

*Bonpoc* 1. Find the risk-premium of the market portfolio:

*Bonpoc* 2. Find the formula for the expected return on the stock:

*Bonpoc 3.* Suppose the observed return on the stock is 0.15%:

*Bonpoc 4.* How can the stock be characterized provided its observed return is equal to the value given in Question 3?:

*Bonpoc 5.* What should be done with the stock provided its observed return is equal to the value given in Question 3?

**Problem 15.** Karen Kay, the asset manager at Collins Asset Management, frequently uses the CAPM when advising her clients. The research department provides Karen with the following data on two stocks, X and Y. Stocks X and Y have betas  $\beta_X = 0.8$  and  $\beta_Y = 1.6$ . The risk-free rate is  $r_F = 6\%$ , the market portfolio return is  $r_M = 13\%$ . Let the *observed* expected returns on these stocks are equal to  $r_X = 13\%$  and  $r_Y = 17\%$ , respectively. Also, risk associated with the stocks is known,  $\sigma_X = 37\%$ ,  $\sigma_Y = 24\%$ , as well as the risk of the market portfolio  $\sigma_M = 15\%$ . Tasks: 1) Compute the expected return and the alpha for each stock. Depict the assets on the graph containing the SML 2) If these stocks are going to be a part of a well-diversified portfolio, which one should be selected? 3) The same question, if the stocks are bought as an isolated investment. Hint: compute the Sharpe and Treynor coefficients.

*Bonpoc 1.* Найти риск-премию рыночного портфеля:

**Problem 16.** The dividend yield for Martin's Mills, Inc. stocks is 2%. The current dividend per share is \$1.9. Next year the firm plans to pay \$2.014 dividend per share. In the future the dividend growth rate will remain at the current level.

*Bonpoc 1.* Find the current stock price:

*Bonpoc 2.* Compute the dividend growth rate:

*Bonpoc 3.* Choose the formula for the company's return  $r$ :

**Problem 17.** The firm's discount rate is  $r = 11\%$  and ROE is 13%. Retention ratio is 0.5. The projected earnings per share will be \$6 next year.

*Bonpoc 1.* Find dividend growth rate  $g$ :

*Bonpoc 2.* Obtain the current stock price:

*Bonpoc 3.* Choose the formula for the 'cash cow' firm, i.e., assume that all earnings are spent on dividends:

*Bonpoc 4.* Find NPVGO:

**Problem 18.** The discount rate for Flexsteel Company is  $r = 12\%$ , ROE is 13%. The retention ratio is 0.5. Earnings per share is 4.5 ден.ед.

*Bonpoc 1.* Find dividend growth rate  $g$ :

*Bonpoc 2.* Calculate the price-earnings ratio:

**Problem 19.** The discount rate is  $r = 12\%$ , ROE is 12%. The retention ratio is 0.4. The dividends for the previous year were \$2 per share.

*Bonpoc 1.* Find the next year dividends:

*Bonpoc 2.* Investors prefer high  $g$  (dividend growth rate) when:

*Bonpoc 3.* If the discount rate = ROE, then...:

**Problem 20.** Stocks F&G Manufacturing Company have  $\beta = 1.2$ . The risk free rate is  $r_F = 2\%$ , the market portfolio return is  $r_M = 16\%$ . ROE is 14%. The retention ratio is 0.5. The year-end dividends will be \$2 per share.

*Bonpoc 1.* Find the discount rate  $r$ :

*Bonpoc 2.* Choose the formula for the company's stock :

**Problem 21.** Pairs trading. Suppose the price for two assets  $A$  and  $B$ ,  $P_A$  and  $P_B$  respectively, tend to move together. Currently,  $P_A = \$95$   $P_B = \$70$ . Demonstrate a trading strategy that delivers a trading profit if the prices converge in the future, regardless of the convergence level.

*Bonpoc 1.* Describe your current transactions to create the profitable strategy:

*Bonpoc 2.* Consider a strategy when you make a profit at time of convergence rather than now. Compute your total cash inflow(+) or outflow(-) as of now:

*Bonpoc 3.* Describe your actions at the moment when the prices  $P_A$  and  $P_B$  converge:

*Bonpoc 4.* Compute your overall profit when the prices converge. The common price level is  $P$ :

*Bonpoc 5.* Let  $S_t$ ,  $S_{t-1}$  be the price spreads distributed independently at two successive moments  $t$  and  $t - 1$ . Provide the graph suggesting that the percentage of time you may make a profit on pairs trading is... (a brief explanation is required)

**Problem 22.** Discrete Kyle model. Consider an asset whose value is either  $+1$  or  $-1$  with equal probabilities. Also consider the asset being traded by uninformed noise traders who either buy  $q$  units or sell  $q$  units with equal probabilities. An informed insider (who knows whether the asset is worth  $+1$  or  $-1$ ) is considering trading the same amount of asset to make a profit. A market maker who cannot tell which trader submits which order, clears the market by observing the order flow and then setting a price at which the market maker makes zero expected profits.

*Bonpoc 1. Draw the probability tree.* Compute the price set by MM given the price is  $+1$  and a noise trader

is buying:

*Bonpoc 2.* Compute the profit of the insider given the price is  $-1$  and a noise trade is selling:

*Bonpoc 3.* Compute the average profit of the insider:

*Bonpoc 4.* To maximize his profit, the insider should :

*Bonpoc 5.* The Kyle model holds for

*Bonpoc 6.* Consider the full Kyle model. Why do the expected profits of the insider depend on the quantity traded by the noise traders?

**Problem 23.** Glosten-Milgrom model. Suppose noise traders trade 85% of the time, when they buy and sell equally often. The rest of the time insiders trade, and assume that insiders have perfect knowledge of the payoff of the asset. The asset has value \$36 or \$10, each equally likely as seen by all individuals other than the insiders.

*Bonpoc 1. Draw the probability tree.* Compute the probability of asset price \$36 given the market maker

receives a buy order

*Bonpoc 2.* Explain why the probability computed in Task 1 is not equal to 1/2. Explain why it is higher or lower.

:

*Bonpoc 3.* Compute the bid and ask price. **Explain the principle** the MM uses in setting those prices :

*Bonpoc 4.* Explain the difference between the Glosten-Milgrom and Kyle models :

*Bonpoc 5.* What is the autocovariance of the ask (bid) price in the Glosten-Milgrom model? **Explain.**