Percents. Questions and problems

1. The capital of a company has become K times bigger that it was three years ago. Find the percentage increase of the capital.

2. The percentage increase of the capital is equal to r. In how many times has the capital increased?

3. If the capital has decreased in K times ($K > 1$), what is the percentage decrease?

4. The percentage decrease of the capital is equal to r ($r < 100\%$). In how many times has the capital decreased?

5. Prove that the simple interest is more profitable for investors than the compound interest rate if the time of the deposit is less than a year and the interest rates are the same.

6. Manhattan Island was bought for \$ 24 in 1624. If this amount of money was invested at that time and went up 6.5% a year, what the sum would be got in 2008?

7. The annual compound interest rate is equal to 12%. Initial capital is equal to \$ 1000. Find the value of the capital three years later if the interest is compounded (a) 1, (b) 2, (c) 3, (d) 4, (e) 6, (f) 12 times? compounded continiously?

8. The annual compound interest rate and the initial capital were equal to 12% and \$ 1000 respectively. What is the percentage increase of the capital got three years later if the interest is compounded (a) 1, (b) 2, (c) 3, (d) 4, (e) 6, (f) 12 times? compounded continiously?

9. Find the effective interest rate for each scheme of compounding given in problem 7.

10. For each scheme of compounding given in problem 7 find the principal that leads to \$ 10 000 three years later.

11. For each scheme of compounding given in problem 7 find the interest rate such that the principal would have been doubled for 6 years.

12. Prove that the number of periods required to get FV given the PV and the interest rate r satisfies the formulat

$$n = \frac{\log(FV) - \log(PV)}{\log(1+r)}.$$

13. The interest rate is compounded m times a year. Prove that the initial investment would be N times bigger n years later if

$$n = \frac{\log N}{m \log\left(1 + \frac{r}{m}\right)}.$$

14. The interest rate r is compound 2 times a year. Find the equivalent interest rate, which is compounded each month.

15. Suppose that α_1, ..., α_n are the inflation rates during periods t_1, ..., t_n. Find the infliation rate during the period $T = \sum_{t=1}^{n} t_i$.

16. The monthly inflation rate is equal to 2%. Find the annual inflation rate.

17. The annual inflation rate is expected to be 24%. What is expected monthly inflation rate?

18. The monthly inflation rate is expected to be 2%. The nominal interest of 12% is compounded quarterly. Find the effective interest rate.

19. One thousand dollars are deposited. The nominal interest rate is 12% compounded two times a year. The inflation rate is 1% a month. Find the real interest rate.

Cash flows

1. Bank pays 10% interest rate compounded yearly at the moment of the investment. Find the effective interest rate for half a year deposit made under these conditions.

2. Show that the present value of the annuity (all the payments are the same) decreases if the interest rate increases.

3. Show that the present value of the annuity (all the payments are the same) increases if the maturity goes up.

4. Find the present value of a \$1000 annuity (all the payments equal \$1000) that lasts 10 periods. Each periods is half a year. The interest rate is equal to 10% compounded once a year.

5. Find the present and future values of a \$1000 annuity (all the payments equal \$1000) that lasts 10 years. The interest rate is equal to 10% compounded continuously.

6. Find the present value of the cash flow $CF = \{(1, -1120); (2, 6272); (3, 21952)\}$. The interest rate is equal to 12% compounded once a year.

7. Find the present and future values of the cash flow $CF = \{(1, -1120); (2, 6272); (93, 21952)\}$. The interest rate is equal to 12% a year compounded monthly.

8. Find the internal rate of returns of the cash flow $CF = \{(0, -2700); (1, 1500); (2, 5000)\}$.

9. Each year \$12 000 are deposited under compound interest rate 12%. What would be the amount seven year later if (a) each payment is deposited at the end of the year and the interest rate is compounded yearly; (b) each payment is deposited at the end of the year and the interest rate is compounded each half a year; (c) payments are deposited at the end of each half a year and the interest rate is compounded yearly; (d) payments are deposited at the end of each month a year and the interest rate is compounded monthly?

10. You have a \$18 000 car loan at 14.25% (a year compounded yearly) for 36 months. You have just made your 24th payment of 617.39 and would like to know the pay off amount (current balance remaining after the payments made).

11. You are buying \$250 000 house on credit at 0.65% a month for 20 years. What is the monthly payment? What is your loan balance after the first year?

12. Your friend offers to lend you \$2500 at 6% a year (compounded yearly) for a new notebook. You are able to pay \$100. How long will it take to pay off the loan? Suppose that you are able to pay \$20 a month instead of \$100. How long will it take to pay off the loan then?

13. You have \$15 000 in 5% saving account which is compounded monthly. How long will it take to run down the account if you withdraw \$100 a month?

14. You are able to pay \$100 monthly. The interest rate is 15% a year. How big an apartment you can afford to buy if you have to pay off the loan within ten years?

15. You are looking to buy the furniture. You can afford to pay \$60 a month over the next three years and your credit card charges 16.9% interest. How much furniture can you buy?

16. You want to purchase a 20-year annuity that will pay \$500 a month. If the interest rate is 4%, how much will the annuity cost?

17. You have a 600 000 education loan for 3 years at 5% a year. The loan will be paid off at the end of each year after the education will have been finished. The payment is 10% of the salary which is expected to be 1 200 000 a year. How long will it take to pay off the loan?

18. Find the present value of perpetuity (annuity that continues forever) if the interest rate is 12.5% and each payment is 100 000.

19. Four year loan of 1 million dollars is paid off according to the following scheme: \$500 000, \$300 000, and \$250 000 are returned at the end of the first, second, and third year respectively. Find the amount that has to be returned at the end of the fourth year to pay off the loan. The interest rate is equal to 10%.

20. Mr. Smith has been underpaid 1000 roubles a month for 10 years. A court makes the company to pay its debts now taking into account 12% interest rate. Find the payment.

21. You win a lottery. The prise is \$4 million in annual payments of \$200 000. The organizers offer you a lump sum of \$3 million now instead of the cash flow. Should you take it, if the interest rate is 5% a year? Figure interest rate when you take the proposition.

Bond prices and yields

1. Which security has a higher *effective* annual interest rate?

(a) A three-month treasure bill[1] selling at \$97 645 with par value \$100 000.

(b) A 10% coupon bond selling at par and paying coupon semiannually.

2. Treasury bonds paying an 8% coupon rate with semiannual payments sell at par value. What coupon rate would they have to pay in order to sell at par if they paid their coupons annually?

3. Consider a bond with a 10% coupon and with yield to maturity 8%. If the bond's yield to maturity remains constant, then in one year, will the bond price be higher, lower, or unchanged? Why?

4. Two bonds have indentical times to maturity, par values, and yield to maturity. The price of the first bond is higher. Which bond should have higher coupon rate? Why?

5. Consider an 8% coupon bond selling for \$924.81 with three years until maturity making annual coupon payments. The par value is \$1000. The interest rate in the next three years will be 10% a year. Calculate the yield to maturity.

6. Consider an 8% coupon bond selling for \$970.16 with three years until maturity making annual coupon payments. The par value is \$1000. The interest rate in the next three years will be 11% a year. Calculate the yield to maturity.

7. Consider an 8% coupon bond selling for \$900.10 with three years until maturity making annual coupon payments. The par value is \$1000. The interest rate in the next three years will be, with certainty, $r_1 = 8\%$ $r_1 = 10\%$ $r_1 = 12\%$ Calculate the yield to maturity.

8. A 10% coupon bond with anual payments and the par value of 2 000 sell at par value, mature in two years. Find the yield to maturity.

9. A 10% coupon bond with semianual payments and the par value of 2 000 sell at par value, mature in two years. Find the yield to maturity.

10. Bonds of Zello Corporation with a par value of 1 000 sell for 960, mature in five years, and have a 6% annual coupon rate paid semiannually. Calculate realized compound yield for an investor with a three-year holding period and a reinvestment rate of 3% over each half a year. At the end of three years the 7% coupon bonds with two years remaining to maturite will sell to yield 7%.

11. There are three bonds that mature in 10 years. The first is a zero-coupon bond that pays 1 000 at maturity. The second has an 8% coupon rate and pays the 80 coupon once per year. The third has a 10% coupon rate and pays the 100 coupon once per year. If all three bonds are now priced to yield 8% to maturity, what are their prices? If you expect their yields to maturity to be 8% at the beginning of next year, what will their prices be then?

12. A 20-year maturity bond with par value of \$1 000 makes semiannual coupon payments at a coupon rate of 8%. Find the yeild to maturity of the bond if the bond price is \$950? \$1000? \$1050?

13. A 20-year maturity bond with par value of \$1 000 makes annual coupon payments at a coupon rate of 8%. Find the yeild to maturity of the bond if the bond price is \$950? \$1000? \$1050?

14. A 20-year maturity bond makes annual payments at a coupon rate of 5%. Its yield to maturity is 8%. The bond is sold with 19 years remaining to maturity at a yield to maturity of 7%. Find the holding period return.

15. The interest rate has fallen dramatically. What can be said about bold price?

16. A firm issued a 9% coupon bond with par value of \$1 000 20 years ago. The bond now has 10 years left until its maturity date but the firm is having financial difficulties. Investors believe that the firm will be able to make good on the remaining interest payments, but that at the maturity date, the firm will be forced into bankruptce, and bondholders will receive only 70% of par value. The bond is selling at \$700. Find stated and expected yield to maturity.

[1] T-bill is sold at a discount of the par value to create a positive yield to maturity

Duration of bonds

1. Bonds with a par value of \$1 000 sell for \$960, mature in five years, and have a 7% annual coupon rate paid semiannually. Calculate (a) the yield-to-maturity; (b) Total return for an investor with a three-year holding period and a reinvestment rate of 6% over the period. At the end of three years the 7% coupon bonds with two years remaining will be sold such that the yield-to-maturity will be 7%.

2. What can you say about duration if the yield-to-maturity increases?

3. Bonds XYZ and YZX have the same characteristics except the coupon rate. If the coupon rate of XYZ is higher, what can you say about price? duration?

4. A 3-year coupon bond has payments as follow

Bond Cash Flows by Year		
Year 1	Year 2	Year 3
8	8	108

This 8% coupon bond is currently trading at par (\$100). (a) What is the yield to maturity? (b) Compute duration of the bond. (c) The yield to maturity declines by 2%. How do you expect this bond's price to change?

5. Put in order (from smallest to largest) the following bonds with respect to their duration

Bond	Time to maturity	Coupon rate	Yield-to-maturity
A	11 years	8%	8%
B	11 years	0%	8%
C	11 years	8%	5%
D	4 years	8%	8%

6. A financial institution has raised \$1 million by selling a number of 2-year zero-coupon bonds to individuals. These bonds have a yield-to-maturity of 6%. The institution has used the proceeds to buy a number of long-term coupon bonds. These bonds have the duration of 12 years and a yield-to-maturity of 7%.Use the concept of duration to explain how this institution is exposed to changes in interest rates. In particular, what happens to the value of the zero-coupon bonds and the value of the firm as a whole, if the yields on these bonds change by 1%.

7. You are managing a portfolio of \$1 million. Your target duration is 10 years, and you choose from two bonds: a zero-coupon bond with maturity of 5 years, and a perpetuity, each yielding 5%. (a) How much of each bond will you hold in your portfolio? (b) How these fractions change next year if target duration is now nine years?

8. You have an obligation to pay \$1 488 in four years and 2 months. In which bond (shown below) would you invest your \$1 000 to accumulate this amount if you do not expect their yield-to-maturity increasing bigger than by 1%.

9. Consider a bond selling at par with duration of 10.6 years and convexity of 210. A 2% decrease in yield would cause the price to increase by 21.2% , according to the duration rule (verify!). What would be the percentage price change, if you take the convexity into consideration?

10. A 30-year maturity bond making annual coupon payments with a coupon rate of 12% has duration of 11.54 years and convexity of 192.4. The bond currently sells at a yield-to-maturity of 8%. Find the price of the bond if its yield-to-maturity falls to 7% or rises to 9% (use, first, the duration rule and, second, the duration-with-convexity rule). What is the percentage error for each rule? What do you conclude about the accurace of the two rules?

11. You have to pay \$10 000 in four years. How many zero-coupon two year bonds and zero-coupon ten year bonds with 10% yield-to-maturity can you buy to hedge your risk against changes of the yield-to-maturity.

12. A zero-coupon one year bond with nominal 909090 has the yield-to-maturity 10%. A 5%-coupon three year bond with nominal 943800 has the yield-to-maturity 10% too. What combination of these two bonds will give you 1 000 000 two years later?

Portfolio theory

1. A portfolio consists of assets A, B, and C with equal weights. Calculate the expected returns of the portfolio, if the expected returns $r_A = 20\,\%$, $r_B = 10\,\%$, $r_C = 30\,\%$.

2. The returns of assets A and B will be equal 0.2 and 0 respectively with probability of 1/2 and 0 and 0.4 with the same probability. (a) Find the expected returns of assets A and B.
(b) Find the probability mass function for the joint distribution of returns of assets A and B.
(c) Calculate the expected returns and risk of the portfolio $\Pi = 0.5(A + B)$.

3. The expected returns and risk of assets A and B are written in the table

Assets	Expected returns	Risks
A	0.12	0.15
B	0.20	0.45

Portfolio $\Pi = tA + (1 - t)B$.
(a) Calculate the expected returns and risk of the portfolios corresponding to $t = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0, 7, 0.8, 0.9, 1. Consider coefficients of correlation ρ_{AB} being equal to -1, -0.5, 0, 0.5, 1.
(b) If A and B are anti-correlate ($\rho_{AB} = -1$), what is the minimal possible risk of a portfolio consisting of these two assets?
(c) If A and B are correlate ($\rho_{AB} = 1$), what is the minimal possible risk of a portfolio consisting of these two assets?
(d) The question of part (c), if the weight of B in the portfolio can be negative (the short sale of B is possible).
(f) The question of part (c), if the weight of A in the portfolio can be negative (the short sale of A is possible).

4. The expected returns r_A and r_B of assets A and B are equal to 0.1 and 0.4 respectively. Their risk $\sigma_A = 0.2$, $\sigma_B = 0.3$. Draw the set of the admissible portfolios. The two assets do not correlate ($\rho_{AB} = 0$).

5. The expected returns and risk of assets A and B are defined as follows: $r_A = 0.2$, $r_B = 0.3$, $\sigma_A = 0.1$, $\sigma_B = 0.2$. Their coefficient of correlation $\rho_{AB} = 0.5$. The returns of a risk-free asset $r_F = 0.05$. If your desired level of returns is equal to 0.20, choose the portfolio with the minimal risk.

6. A fair game is a risky prospect that has a zero-risk premium. It will not be undertaken be a risk-averse investor. Discuss.

7. Is $U(\sigma, r) = r - 2\sigma^2$ a utitlity function?

8. Draw the indifference curves for the utility function $U(\sigma, r) = 2r - \sigma^2$.

9. Can indifference curves of some unitility function intersect one another?

10. The utility function $U(r, \sigma) = r - \sigma^2$, where $r \in [0, 0.35]$ and σ are the expected returns and risk respectively. Find the optimal portfolio that consists of uncorrelated assets A and B with $r_A = 0.1$, $r_B = 0.4$, $\sigma_A = 0.2$, $\sigma_B = 0.3$.

11. The utility function $U(r, \sigma) = r - r^2 - \sigma^2$, where $r \in [0, 0.35]$ and σ are the expected returns and risk respectively. Find the optimal portfolio that consists of two assets A and B with $r_A = 0.1$, $r_B = 0.4$, $\sigma_A = 0.2$, $\sigma_B = 0.3$, $\rho_{AB} = 0.5$.

12. The utility function $U(r, \sigma) = r - r^2 - \sigma^2$, where $r \in [0, 0.35]$ and σ are the expected returns and risk respectively. Find the optimal portfolio that consists of two assets A and B with $r_A = 0.1$, $r_B = 0.4$, $\sigma_A = 0.2$, $\sigma_B = 0.3$, $\rho_{AB} = -0.5$.

Optimal portfolio

1. Can be the portfolio standard deviation greater than the weighted average of the individual security standard deviations (the weights are the same as in the construction of the portfolio)?

2. Two assets have the same characteristics of their risk and expected returns: $r = 0.1$, $\sigma = 0.4$. Find the expected returns and risk of the portfolio consisting of these assets, if the weights of the assets in the portfolio are equal. The returns of the assets are not correlated ($\rho = 0$). Generalize the answer to the case of n identical non-correlated assets.

3. Admissible portfolios are given by $r^2 - 4\sigma^2 = 1$ $(\sigma, r > 0)$. Find the minimum variance portfolio.

4. Admissible portfolios are the combinations of assets A and B with the expected returns 0.1 and 0.3 and risk 0.2 and 0.4 respectively. The coefficient of correlation of the returns of A and B is -0.5. Find the minimum variance portfolio.

5. The efficient portfolios are defined by the curve $r = \sqrt{1 + 4\sigma^2}$. The utility function is $U(\sigma, r) = r - \sigma^2$. Find the optimal portfolio.

6. Draw schematically effective portfolios, if a risk-free asset is available.

7. The efficient portfolios are defined by the curve $r = 0.2\sigma + 0.05$, $\sigma > 0$. The utility function is $U(\sigma, r) = r^2/25 - (\sigma + 0.02)^2/100$, $\sigma > 0$. Find the optimal portfolio.

8. A toy stock market consists of two risk assets A_1, A_2 and risk-free asset F. The returns and expected returns of F, A_1, and A_2 are equal to 0.05, 0.1? 0.4 respectively. Let $\begin{pmatrix} 0.1 & -0.1 \\ -0.1 & 0.4 \end{pmatrix}$ be the covariance matrix of the returns of A_1 and A_2. Find the tangent portfolio.

9. The return of each \$0.9 invested in risk assets A_1, A_2, and risk-free asset F are given in the following table.

Probability	$q = 0.6$	$q = 0.3$	$q = 0.1$
F	1.00	1.00	1.00
A_1	1.05	1.20	1.10
A_2	1.30	1.00	1.05

In other words, there are three states of the world. The probability of each state is given in the table. For example, in the first state of the world (which comes with probability of 0.6) \$0.9 invested into A_1 gives you \$1.05 and \$0.9 invested into A_2 gives you \$1.30. The yield of F is known with certainty. Find the returns r_F of risk-free asset F, the expected returns r_1 and r_2 of assets A_1? A_2, their risks σ_1 and σ_2, and the coefficient of correlation $\rho(R_1, R_2)$ corresponding to the returns R_1 and R_2 of A_1 and A_2.

10. Under the conditions of the previous problem find the tangent portfolio.

11. Under the conditions of the previous problem find the efficient portfolios.

12. Exhibit efficient portfolios in the Markovitz model, if short sales of the risk-free portfolio are forbidden.

13. Exhibit efficient portfolios in the Markovitz model, if you see different interest rates for borrowing and lending. Suppose that the riskless borrowing rate is higher than the riskless lending rate.

CAPM and Market Model

1. What are the assumptions of the CAPM?

2. If the return of the risk-free asset is equal to 9%, draw the capital market line. The expected return and risk of the tangent portfolio are equal 0.2 and 0.18 respectively.

3. The return of a risk-free T-bill is 9%, the expected return and risk of the market portfolio are 12% and 3% respectively. Use the Capital Asset Pricing Model to answer the following questions. (a) If a security has a covariance with the market of 0.00045, what is the beta of the security?
(b) What is the expected return on the above security?
(c) If a security has an expected return of 10%, what is its beta? What is its covariance with the market?
(d) What is the beta of a security with an expected return of 15%? What is the covariance of this security with the market?

4. Under the assumptions of the CAPM can the market portfolio contains an asset whose expected return is less than the return of the risk-free asset?

5. Under the assumptions of the CAPM can the market portfolio contains such two assets $A(\sigma_A, r_A)$, $B(\sigma_B, r_B)$ that $\sigma_A < \sigma_B$, $r_A > r_B$?

6. The market portfolio T consists of three assets A_1, A_2, and A_3 taken in equal proportions. Find the risk of T, if

$$\mathbf{Cov}(R_1, R_T) = 230, \quad \mathbf{Cov}(R_2, R_T) = 280, \quad \mathbf{Cov}(R_3, R_T) = 250,$$

where the random variable R_i is the return of A_i, R_T is the return of T.

7. Does the risk-free asset belong to security market line?

8. The expected return and risk of the market portfolio are equal to 0.18 and 0.09 respectively. Draw CML and SML. Specify assets A and B with expected return 0.1 and 0.4 respectively on the SML.

9. Suppose an investor uses the quadratic utility $U(\sigma, r) = r - \sigma^2/2$. Consider a return-risk diagram with σ on the horizontal axis.
(a) What is the slope of the indifference curves for $\sigma = 0$?
(b) Assume the market rate is larger than the risk-free rate. "No investor with this utility function would hold 100% of the risk-free asset". Discuss.

10. You believe that company UTMore will be worth \$100 per share one year from now. How much are willing to pay for one share of UTMore today if the risk-free rate is 8%, the expected rate of return on the market is 18%, and the company's beta is 2.0? (Assume there are no dividends.)

11. The risk of the market portfolio is $\sqrt{490}$, the covariance of assets A and B is 470. Under the assumptions of the market model find the beta of B, if the beta of A is 1.20.

12. Estimation of stocks A_1 and A_2 gives the follwoing results:

$$R_1 = 0.01 + 0.8R_M + \varepsilon_1, \quad R_2 = 0.02 + 1.2R_M + \varepsilon_2,$$

where R_1, R_2, R_M are the expected returns of A_1, A_2, and the market portfolio M. Suppose that $\sigma_M = 0.20$, $\sigma_{\varepsilon_1} = 0.20$, $\sigma_{\varepsilon_2} = 0.10$. What does the market model predict for the values of σ_{A_1}, σ_{A_2}, $\mathbf{Cov}(R_1, R_2)$? Suppose we construct a portfolio that has weights $w_{A_1} = 0.25$, $w_{A_2} = 0.25$, $w_0 = 0.5$, where w_0 corresponds to a risk-free T-bill. What is the risk of this portfolio?

13. Under the assumptions of the market model the coefficients α_i, $i = 1, \ldots, n$, of n assets are the same. If utility function is $r^2 - \beta^2$, find the optimal portfolio consisting of these assets.

Case of a risk-free asset (additional problems)

1. The efficient portfolios which consist of risk securities is given by equation

$$r = 2 + 0.5\sigma - \frac{1}{1.5\sigma - 0.09}.$$

The return of a risk-free asset is 0.04. Find the tangent portfolio.

2. Under the conditions of problem 1 find the equation of the CML.

3. The utility function of an investor is $U(\sigma, r) = \sqrt{r - \sigma^2}$. Suppose that borrowing is admissible and find the optimal portfolio for this investor if the conditions of problem 1 are valid.

4. Solve the previous problem if borrowing is forbidden.

5. Find the equation of the security market line for problem 1

1. Let σ_T and r_T be the risk and expected return of the tangent portfolio. Then they solves the following system of equations: $r_T = r(\sigma_T)$, $r'(\sigma_T) = (r - r_F)/\sigma_T$, where $r = 2 + 0.5\sigma - \frac{1}{1.5\sigma - 0.09}$ is the efficient porfolios which consist of risk secrutities only. Elementary calculations yield that

$$0.5 + \frac{1.5}{1.5\sigma_T - 0.09)^2} = \frac{0.5\sigma_T + 2.0 - \frac{1}{1.5\sigma_T - 0.09} - r_F}{\sigma_T}$$

This equation has two roots. One of them is very close to 0.03. The corresponding r is negative. Therefore this root is irrelevant. The second root $\sigma_T \approx 0.77$ is the solution in question. Then $r_T \approx 1.44$.

2. $r = 1.82\sigma + 0.04$.

3. The indifference curves are $r - \sigma^2 = const$. Equalizing the derivatives of the indifference curves and the CML we find that

$$2\sigma = 1.82 \quad \text{or} \quad \sigma = 0.91.$$

Finally, $r = 1.67$. Then the optimal portfolio are characterized by weights $(-0.16, 1.16)$, where the first weight corresponds to the risk-free asset.

4. Since the weight of the risk-free asset in the optimal portfolio found in the previous problem is negative this portfolio is not admissible if borrowing is forbidden. Therefore the optimal portfolio (in the case) does not belong on the CML. It lies on the tangent point of an indifferent curve (of the utility function) and the curve of the efficient risky portfolios. The derivative 2σ of the indifference curves is equal to $0.5 + \frac{1.5}{(1.5\sigma - 0.09)^2}$. This equation can be transformed to a cubic equation. Any mathematical package gives evidence that two of its roots are complex. Thus its real root $\sigma \approx 0.82$ is the risk of the optimal portfolio; $r = 1.53$.

Factor Model

1. Explain the similarities between the CAPM and the signle-factor models?

2. In what significant does the single-factor model differ from CAPM?

3. Explain, why the diversification reduces the non-factor risk.

4. The sensitivity β and α of a security are 0.5 and 0.04 respectively. Let the value of the factor be 10%. If the return of the security is 11%, what part of the return is not connected with the factor?

5. In the framework of the single-factor model the portfolio is defined by the following table.

Security	Sencitivity, β_i	Non-factor risk, $\sigma_{\varepsilon i}$	Fraction
A	0.20	0.07	0.4
B	3.50	0.10	0.6

(a) If the risk σ_F of the factor is 15%, what is the factor risk of the portfolio?
(b) Find the non-factor risk of the portfolio.
(c) What is the risk of the portfolio?

6. Resolve the previous problem, if the part of the portfolio is invested in a risk-free security. The corresponding weights x_A, x_B, and x_0 are 0.36, 0.54, and 0.10 respectively.

7. Under the assumptions of the single-factor model the sensitivity of securities A and B are -0.5 and 1.25 respectively. The covariance between their returns is -312.50. What is the risk of the factor?

8. In the framework of the single-factor model the returns of securities A and B satisfy the following equations

$$R_A = 0.05 + 0.8F + \varepsilon_A,$$
$$R_B = 0.07 + 1.2F + \varepsilon_B.$$

It is given that $\sigma_F = 0.18$, $\sigma_{\varepsilon A} = 0.25$, $\sigma_{\varepsilon B} = 0.15$. Find the risk of each security.

9. Let $\langle\sigma_A\rangle = \sqrt{\frac{\sum \beta_i^2 \sigma_{\varepsilon i}^2}{N}}$ be the average non-factor risk of a portfolio A. Suppose that portfolio A consists of n securities. Let $\langle\sigma_A\rangle = 15$. Find the non-factor risk of the portfolio, if (a) $N = 10$, (b) $N = 100$, (c) $N = 1000$.

10. Under the assumptions of problem 8 find the expected returns of securities A and B, if the expected return of the factor is equal to 0.1.

11. Your preferences are determined by utitlity function $U(\sigma, r) = \sqrt{r - \sigma^2}$. Under the assumptions of problem 8 find your optimal portfolio which consists of securities A and B only.

Multi-factor Model

The multifactor model is defined by equation

$$R_i = \alpha_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{in}F_n + \varepsilon_i,$$

where R_i is the return (which is a random variable) of the ith security; F_j is the jth factor, α_i and β_{ij} are some coefficients, ε_i is a random error. The random errors do not correlate with one another and with the factors.

1. Write the equation for two-factor model.

2. What is the expected return and risk of a security under the framework of the two-factor model?

3. What parameters of the two-factor model should you know to find the expected return and risk of a security? Answer the same question for m-factor model. How many parameters should you know?

4. Why a multi-factor model can be more precise than a single-factor model?

5. The returns R_1 and R_2 of securities S_1 and S_2 are given by the equations: $R_1 = 0.05 + 2.2F_1 - 0.7F_2 + \varepsilon_1$, $R_2 = 0.10 + 1.5F_1 - 0.2F_2 + \varepsilon_2$. Let the expected values of the factors F_1 and F_2 be 0.03 and 0.04 respectively; their risks be 0.1 and 0.15; the covariance be 0.60. The variance of the random errors are 0.04 and 0.09 respectively. Find the expected returns and risk of the securities S_1 and S_2. What is the covariance between R_1 and R_2?

6. Under the assumptions of the previous problem find the expected return and risk of the portfolio which consists of securities S_1 and S_2 with equal weights.

7. Under the assumptions of problem 5 find the set of feasible portfolios which consist of securities S_1 and S_2 only.

8. Under the assumptions of problem 5 the return of a risk-free security S_0 is equal to 0.05. Find the expected return and risk of the portfolio $\Pi = 0.2S_0 + 0.5S_1 + 0.3S_2$.

9. Can two securities with the same betas have different expected returns under the assumptions of the multi-factor model? Answer the same question, if you suppose that the CAPM is valid (instead of the multi-factor model).

10. In the framework of the multi-factor model the portfolio is defined by the following table.

Security	Sencitivity, β_1	β_2	β_3	Non-factor risk, $\sigma_{\varepsilon i}$	Weight
A	0.20	-0.10	1.00	7	0.4
B	3.50	0.80	-2.00	10	0.6

(a) What is the sencitivities of the portfolio to the factors? (b) If the risks σ_{F_1}, σ_{F_2}, σ_{F_3} of the factors are 10%, 15%, and 20% and the factors are uncorrelated, what is the risk of the portfolio?