

Verify that all the "desirable" properties of canonical information are satisfied:

1. Existence and Uniqueness: NO,  $(n, S, R)$  - represents vectors without respect of order of vectors

2. Completeness: YES:

$$X = \frac{1}{n} \sum_{i=1}^n x_i, \text{ where can. info } (n, S, R), \text{ so}$$

$$\boxed{X = \frac{1}{n} S'}$$

$$V = \frac{1}{n-1} \sum (x_i - X)(x_i - X)^T = \boxed{\frac{1}{n-1} (R - S - S^T + nXX^T)}$$

3. Elementary: YES

Canonical information for single observation will be like

$$(n, S, R) = (1, x_i, x_i x_i^T)$$

4. Empty: YES

Canonical information for empty dataset is

$$(n, S, R) = (0, [ \dots ], [ \begin{smallmatrix} \vdots \\ \vdots \\ \vdots \end{smallmatrix} ])$$

↓ Nil vector, ↓ Nil matrix

5. Combination

$$\begin{matrix} (n_1, S_1, R_1) \\ (n_2, S_2, R_2) \end{matrix} \xrightarrow{+} (n_1 + n_2, S_1 + S_2, R_1 + R_2)$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

$$\begin{matrix} (n_1, S_1, R_1) \\ (0, [ \dots ], [ \begin{smallmatrix} \vdots \\ \vdots \\ \vdots \end{smallmatrix} ] \end{matrix} \xrightarrow{+} (n_1, S_1, R_1)$$