Optimal Linear Estimation - Many Observations & Prior Information

In this homework you are asked to write a program which would implement and demonstrate various aspects of optimal linear estimation. The underlying process imitates a simple signal measurement experiment.

Items 1 and 2 below describe in more details certain suggestions for your program. The problem itself (in fact, four closely related ones) is formulated in item 3.

1. Measurement Simulation.

(a) Choose some profile x or generate it randomly:

i. Generate random "white noise" $\mu \sim (0, I)$, i.e., μ_i are i.i.d. with $\mathsf{E}\mu_i = 0$ and $\mathrm{Var}(\mu)_i = 1$.

ii. "Smooth" it with some matrix B.

iii. Then $x = B\mu \sim (0, F)$, where $F = BB^T$. Use this x (one and the same) in all your numeric simulations.

(b) Create a matrix A.

(c) Simulate a measurement $y = Ax + \nu$.

2. Estimation: Construct an optimal linear estimate $\widehat{x}$ and the variance matrix $Q = \mathrm{Var}(\widehat{x} - x)$. Show on the same graph:

(a) The original signal x (a curve with components x_i),

(b) Its estimate $\widehat{x}$ (a curve with components $\widehat{x}_i$),

(c) Standard deviations for the estimates $\widehat{x}_i$ ($= \sqrt{\mathrm{Var}(\widehat{x}_i - x_i)} = \sqrt{Q_{ii}}$). It can be illustrated by showing the corresponding "corridor" around $\widehat{x}_i$).

3. Illustrate estimation (see item 2) in different settings:

(a) Single measurement (y, A, S).

i. Transform (y, A, S) to canonical form (T, v).

ii. Construct the estimate, based on the canonical information.

(b) Single measurement (y, A, S) with the prior information $x \sim (0, F)$:

i. Transform the prior information to canonical form.

ii. Transform the measurement to canonical form.

iii. Combine the pieces of canonical information.

iv. Construct the estimate, based on the combined canonical information.

(c) Many measurements (y_j, A_j, S_j), no the prior information.

i. Simulate a sequence of measurements of the same signal x, but with differing matrices A_j (and, possibly, S_j).

ii. Extract canonical information from each measurement.

iii. Combine pieces of canonical information.

iv. Construct the estimate, based on the combined canonical information.

(d) Many measurements (y_j, A_j, S_j), now with the prior information $x \sim (0, F)$. Same steps as in item (c).

i. Extract canonical information from the prior information.
ii. Simulate a sequence of measurements of the same signal x, but with differing matrices A_j (and, possibly, S_j).
iii. Extract canonical information from each measurement.
iv. Combine the pieces of canonical information.
v. Construct the estimate, based on the combined canonical information.