

Mandatory Assignment1 – Dynamic programming

Problem description

The task is to find the minimum number of coins to make 1 040 528 NOK using Norwegian coins with denominations of 1, 5, 10, and 20 NOK.

Approaches

Two methods were used to solve this problem:

- Greedy algorithm
- Dynamic programming

Greedy approach

Steps

1. Sort coins from highest to lowest value
2. Use the highest value coin possible at each step
3. Keep a count of coins used

Result

Total coins is 52 030 and total coins used 52 026 x 20 NOK, 1 x 5 NOK, 3 x 1 NOK

Dynamic programming approach

Steps

1. Create a dp list filled with infinity, except $dp[0] = 0$ for base case
2. For each amount from 1 to target: For each coin: If $coin \leq amount$: Update $dp[amount]$ if using this coin is better
3. The answer is in $dp[target]$
4. Backtrack to find coin usage: Start at target, subtract largest helpful coin, repeat until 0
5. Count coins used

Result

This method breaks the problem into smaller subproblems. For each amount from 1 to 1 040 528 NOK, calculate the minimum number of coins needed. Result is that minimum number of coins for 1 040 528 NOK is 52 030 coins

Discussions and comparison of approaches

Both methods produced the same result 52 030 coins. It seems that it happens because of the specific properties of the Norwegian coin system. The greedy method works best for these coins because of how they are set up. Each coin is either a multiple of smaller coins or it's just one more than what one can make with smaller coins. This means that always picking the biggest coin one can use will give the best answer.

The same results between greedy and dynamic programming approaches would not be the same for all coin systems. In many cases, the greedy approach may yield suboptimal solutions.

Example where the approaches differ:

- Coins {1, 15, 25} with target sum: 30
- Greedy solution gives $25 + 5 \times 1 = 6$ coins when the optimal solution is $2 \times 15 = 2$ coins

Conclusion

Both greedy and dynamic programming approaches gave the optimal solution (52 030 coins) for the Norwegian coin system. The dynamic programming solution is more valuable as it gives the optimal solution for any coin system making it more agile.