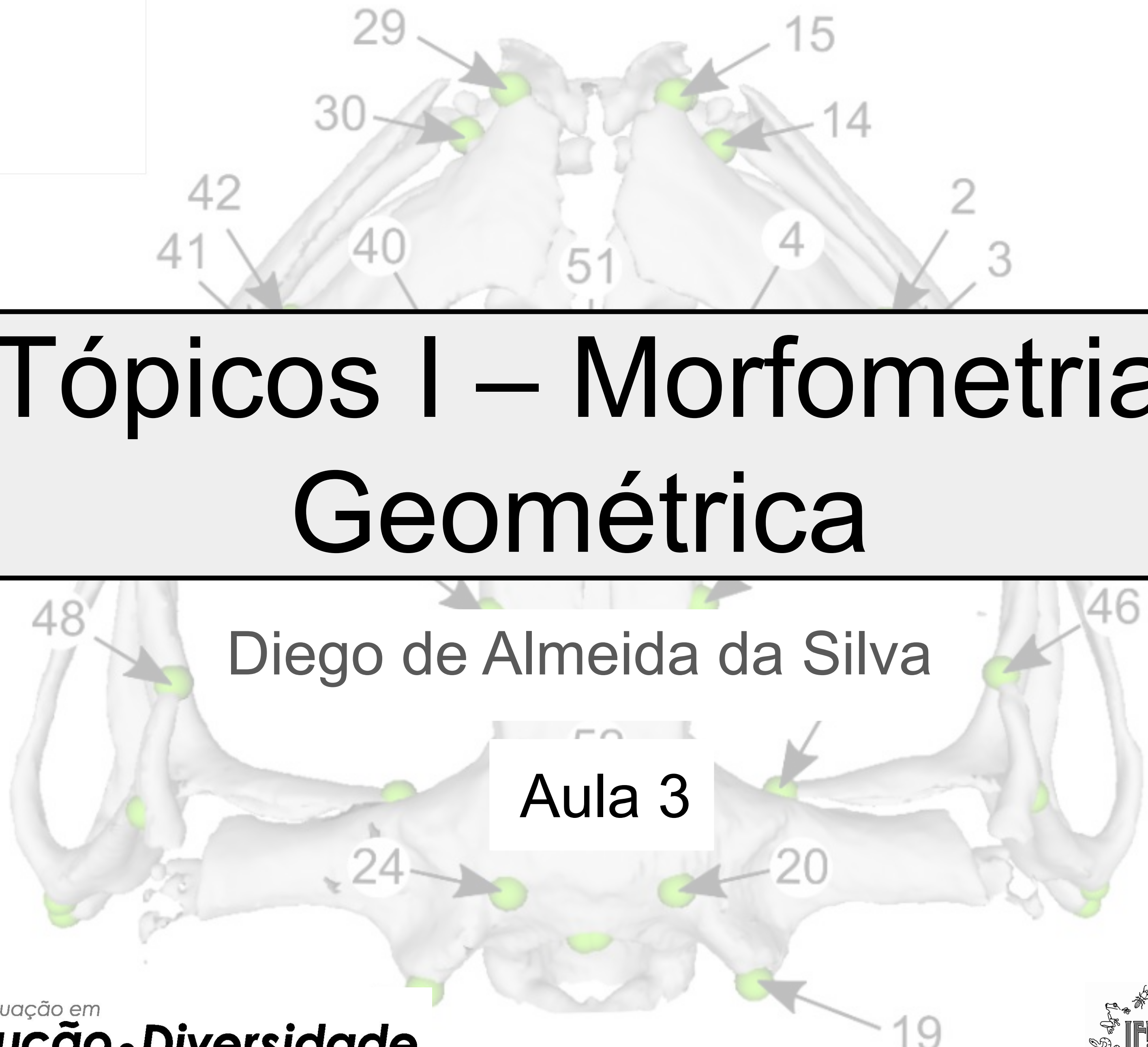


Tópicos I – Morfometria Geométrica

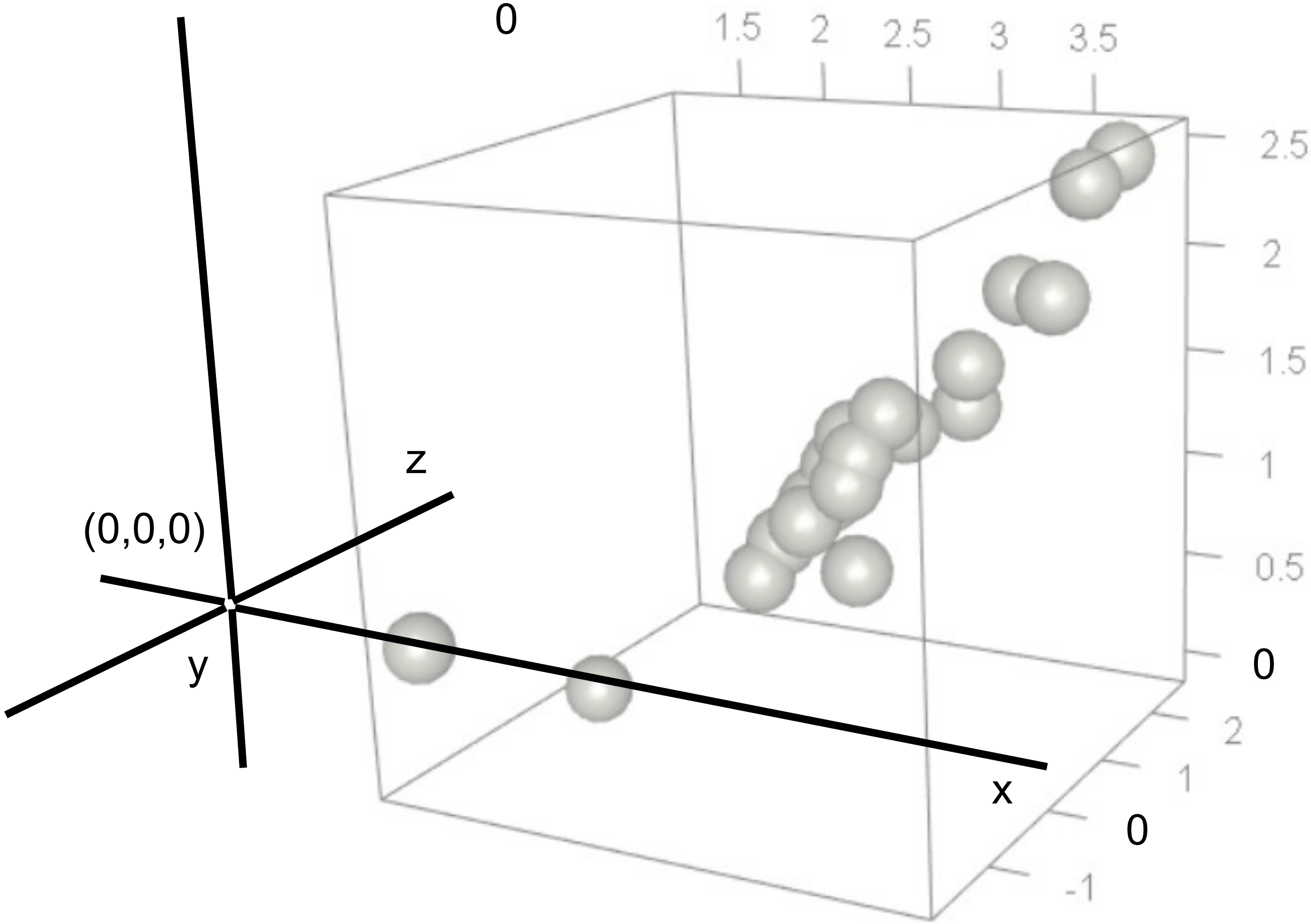
Diego de Almeida da Silva

Aula 3

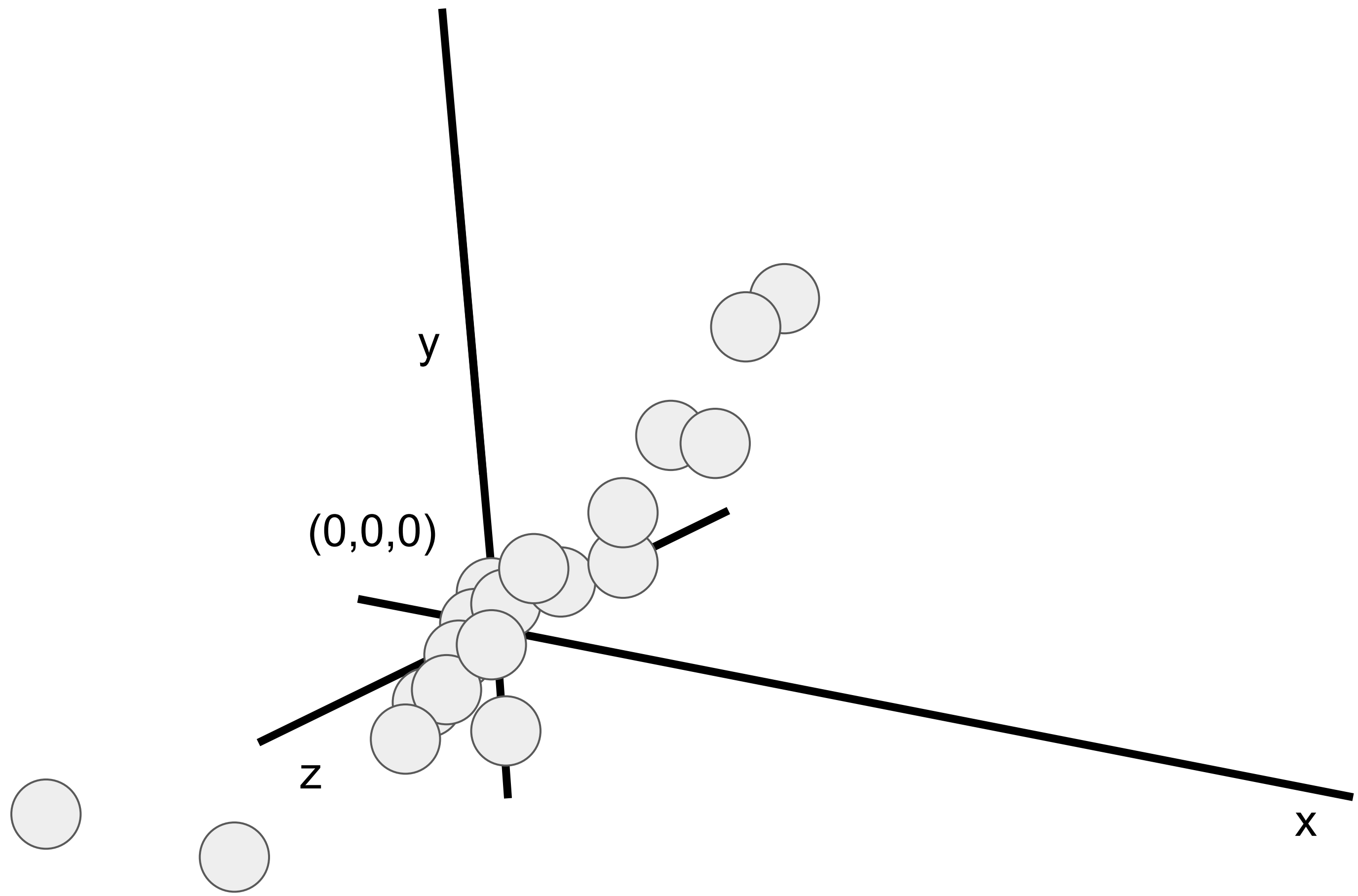
Pós-Graduação em Evolução e Diversidade
Universidade Federal do ABC

UFABC

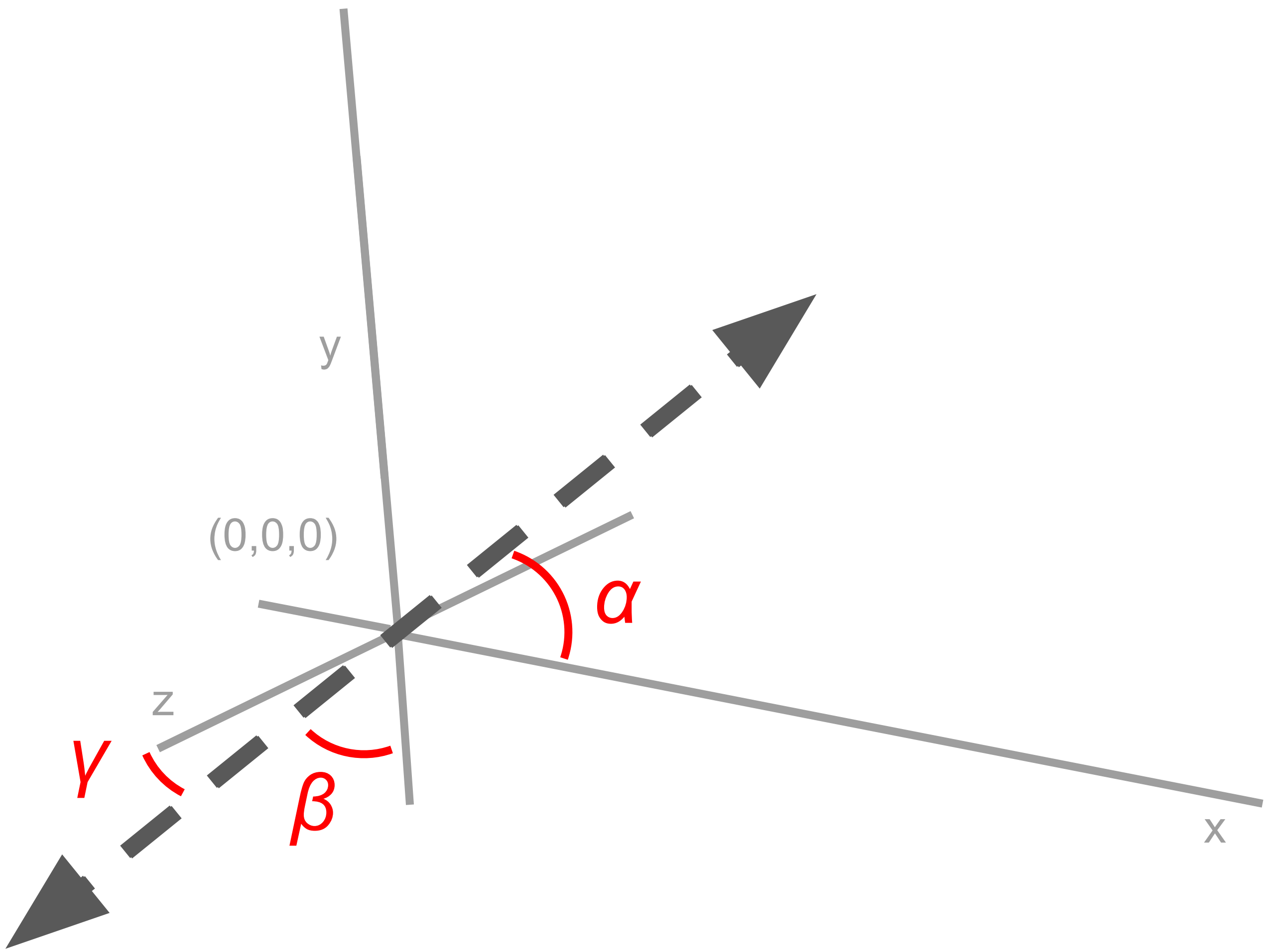
Recapitulando



Recapitulando



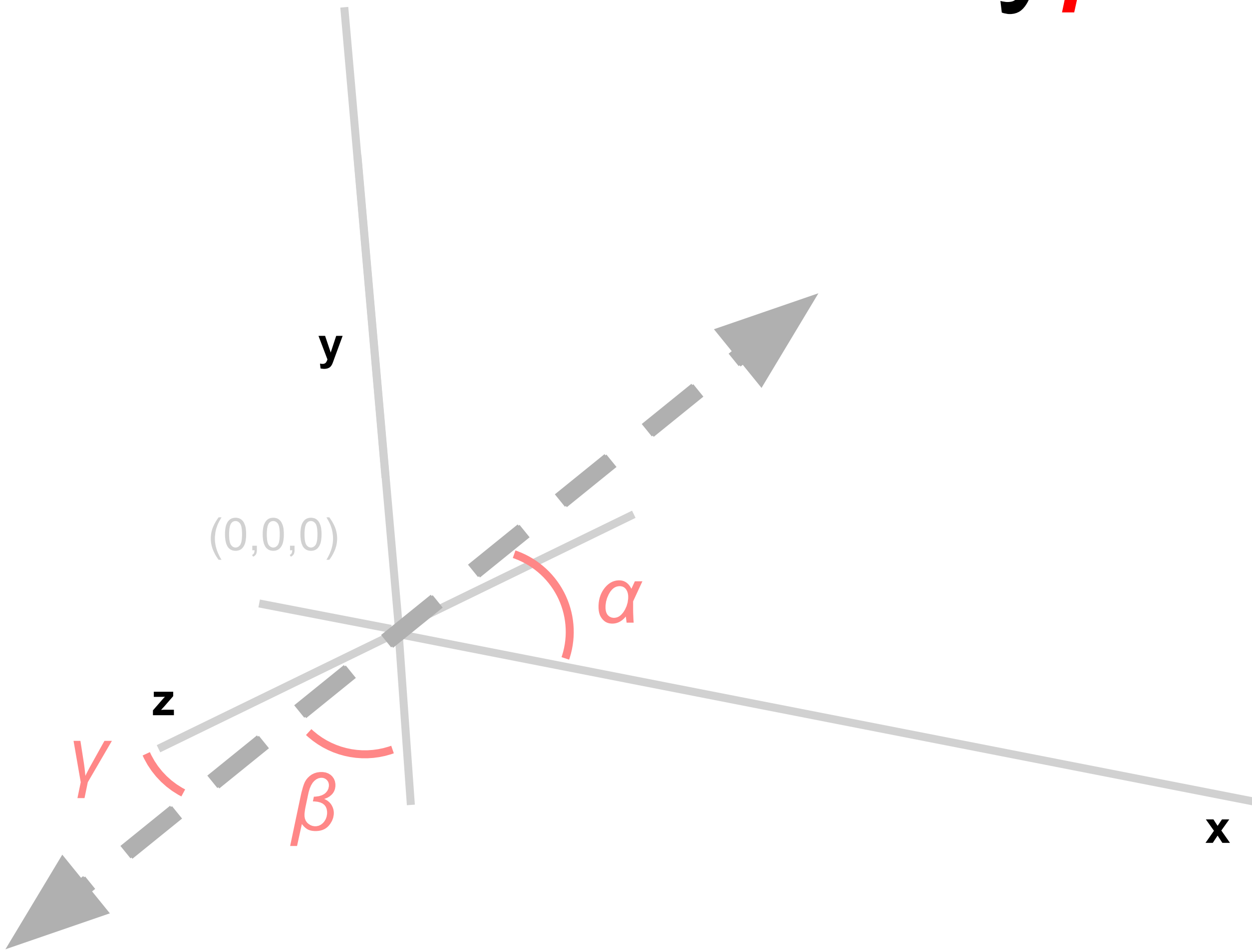
Recapitulando



youtu.be/FgakZw6K1QQ

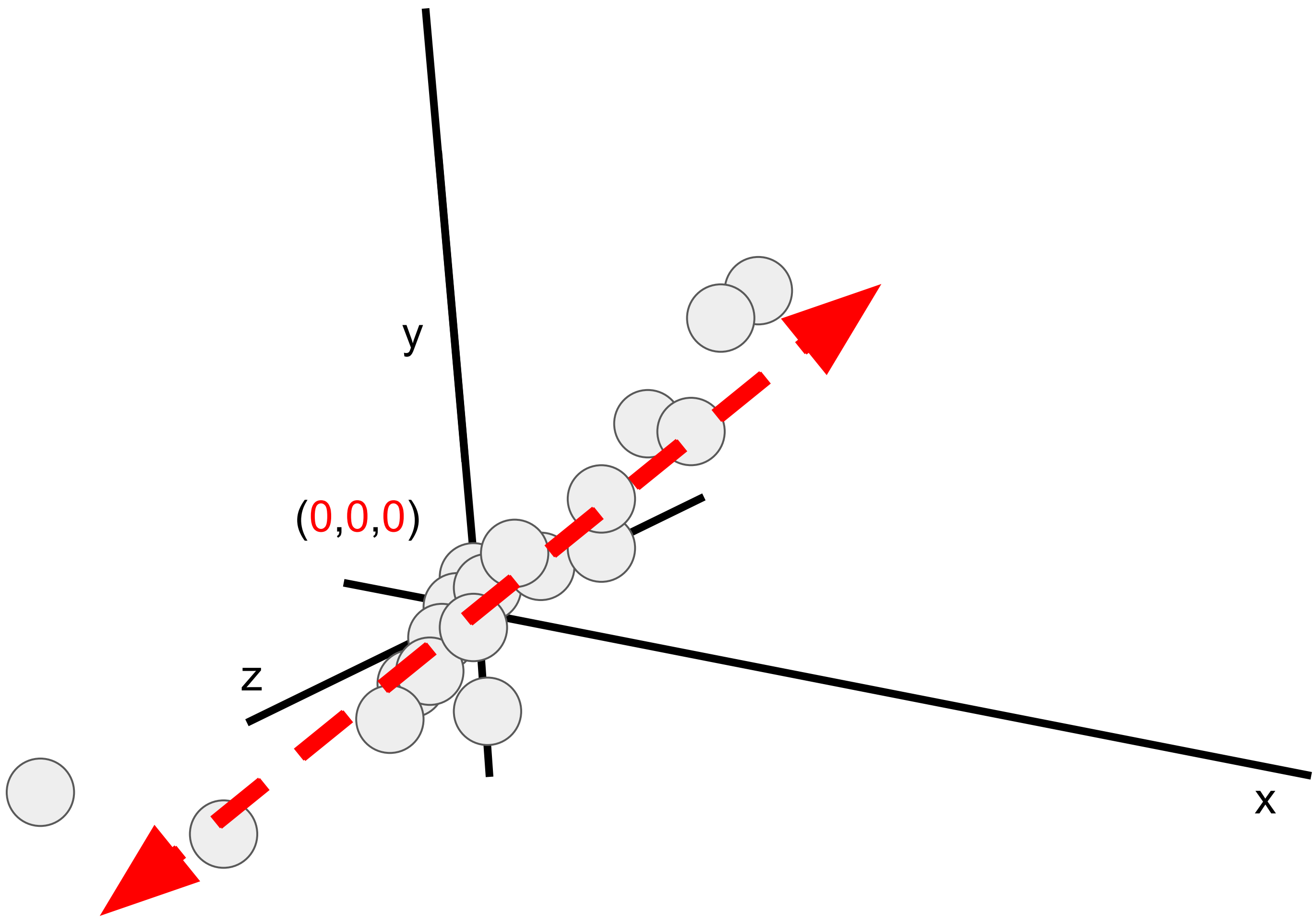
Recapitulando

$$x\alpha + y\beta + z\gamma$$



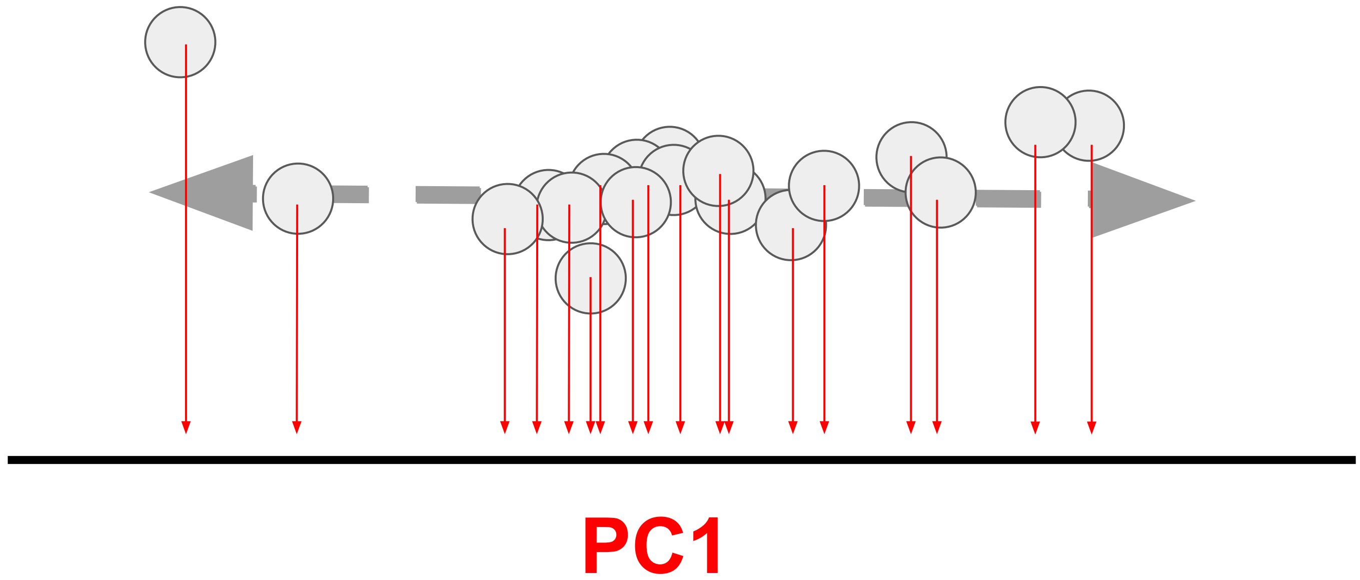
youtu.be/FgakZw6K1QQ

Recapitulando



youtu.be/FgakZw6K1QQ

Recapitulando



youtu.be/FgakZw6K1QQ

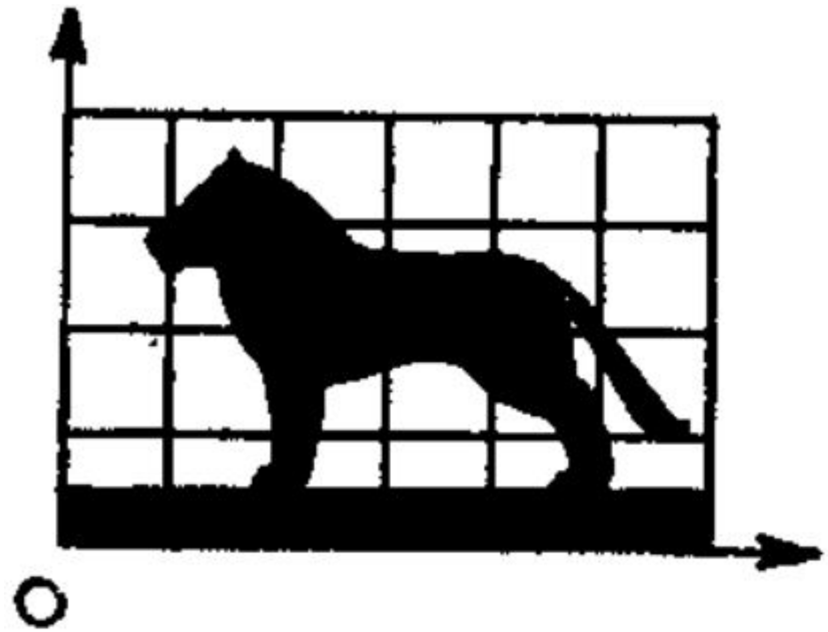
∴ *a PCA é um exemplo de*

Projeção de matrizes

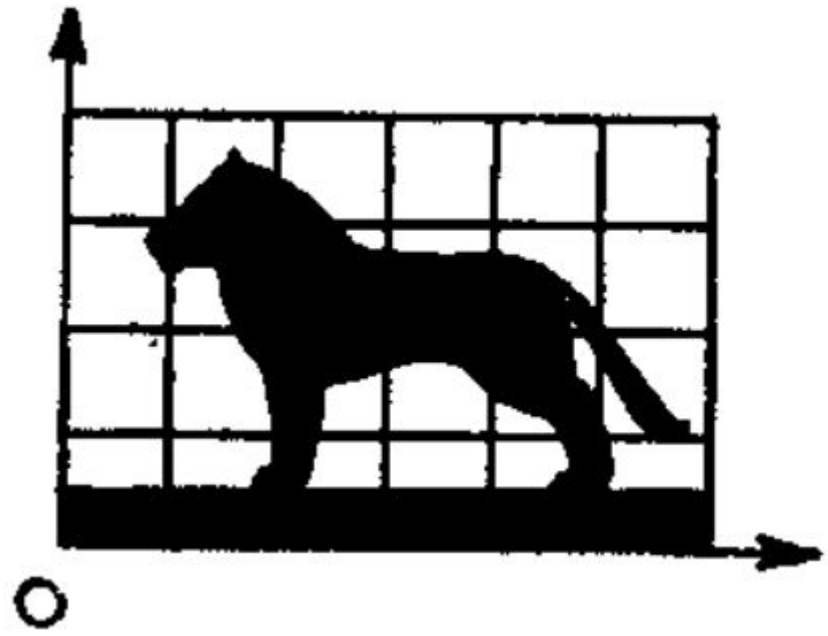
O mesmo conceito é aplicado na Morfometria Geométrica:

$$Z = \frac{1}{CS}\left(Y - \bar{Y}\right) H$$

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



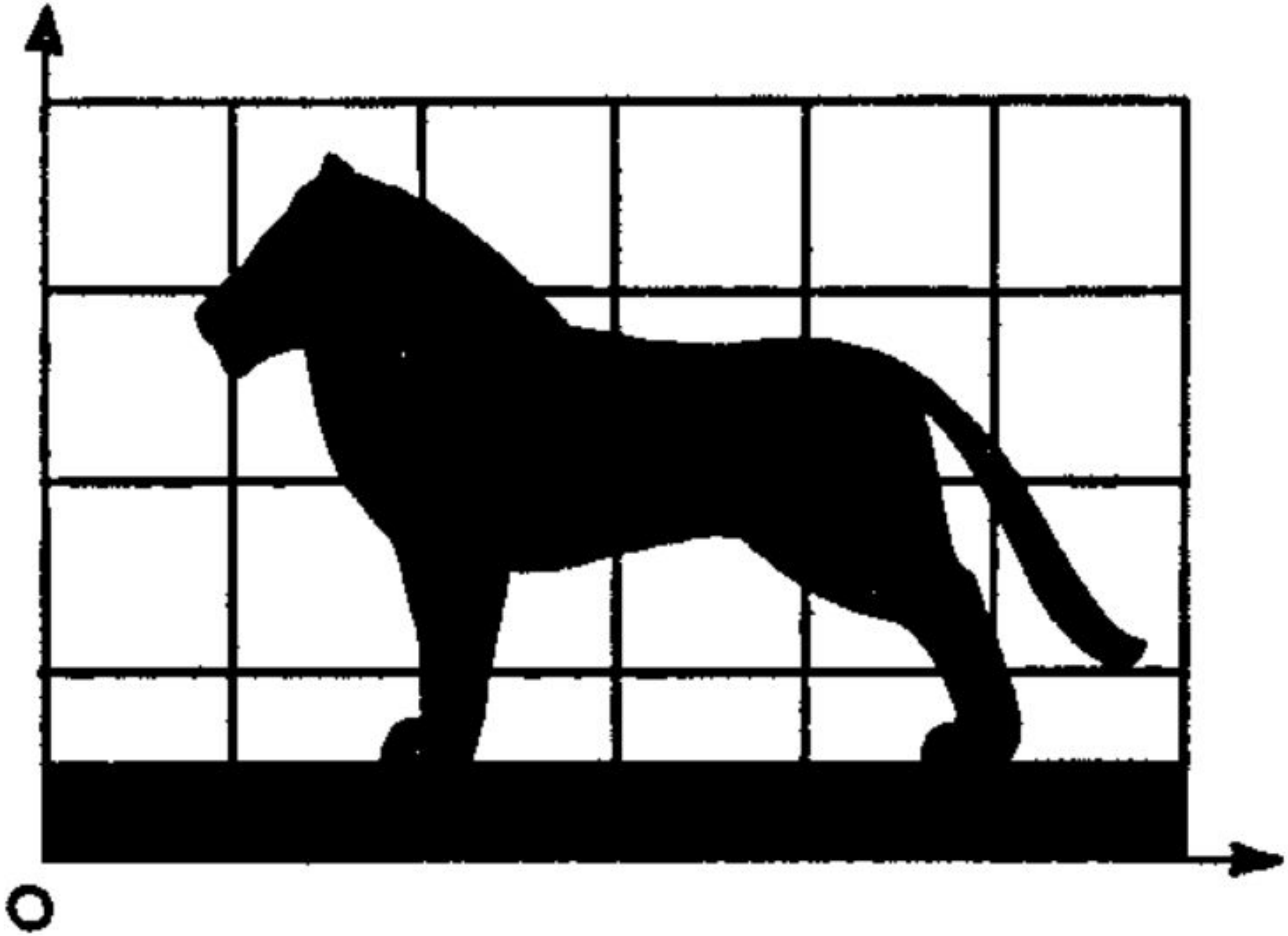
https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Original

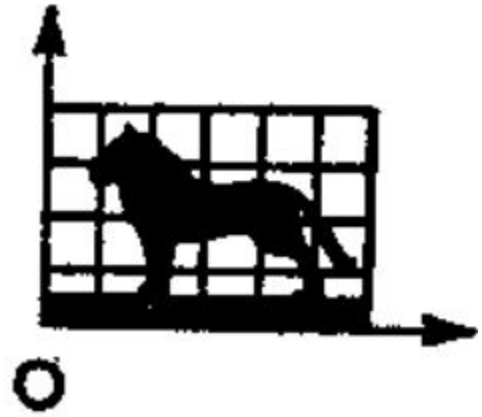
https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\,I$$

Escalar

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \frac{1}{2}I$$

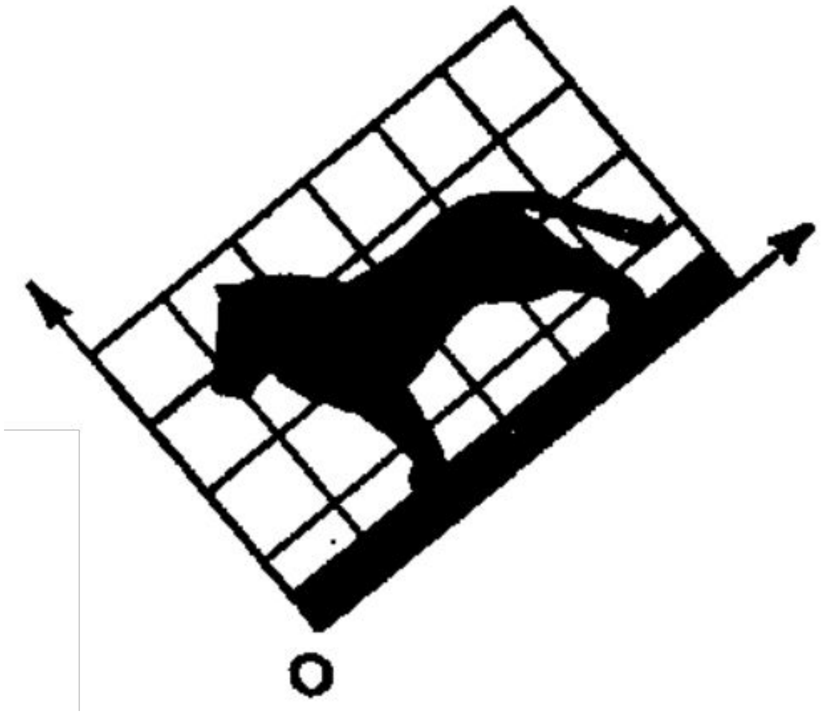
Escalar

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

Rotacionar

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



$$\begin{pmatrix} 0{,}8 & -0{,}6 \\ 0{,}6 & 0{,}8 \end{pmatrix}$$

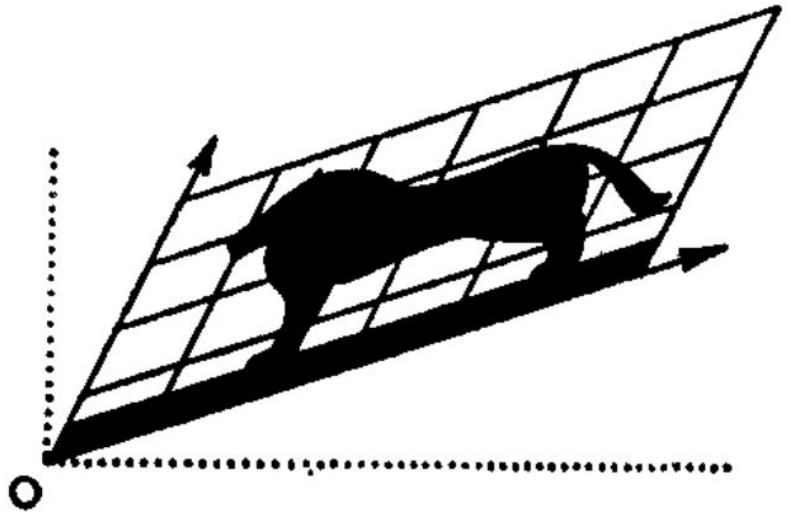
Rotacionar

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html

$$\begin{pmatrix} 2/3 & 0 \\ 0 & 3/2 \end{pmatrix}$$

Distorções / Projeções

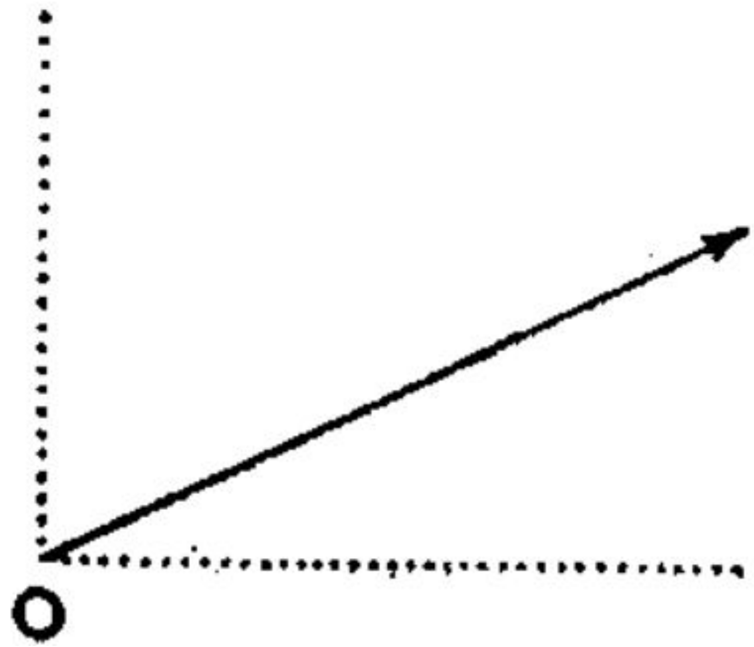
https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html



$$\begin{pmatrix} 1{,}5 & 0{,}5 \\ 0{,}5 & 1{,}0 \end{pmatrix}$$

Distorções / Projeções

https://geomorphr.github.io/GMcourse/Lectures/02-LinearAlgebra.html

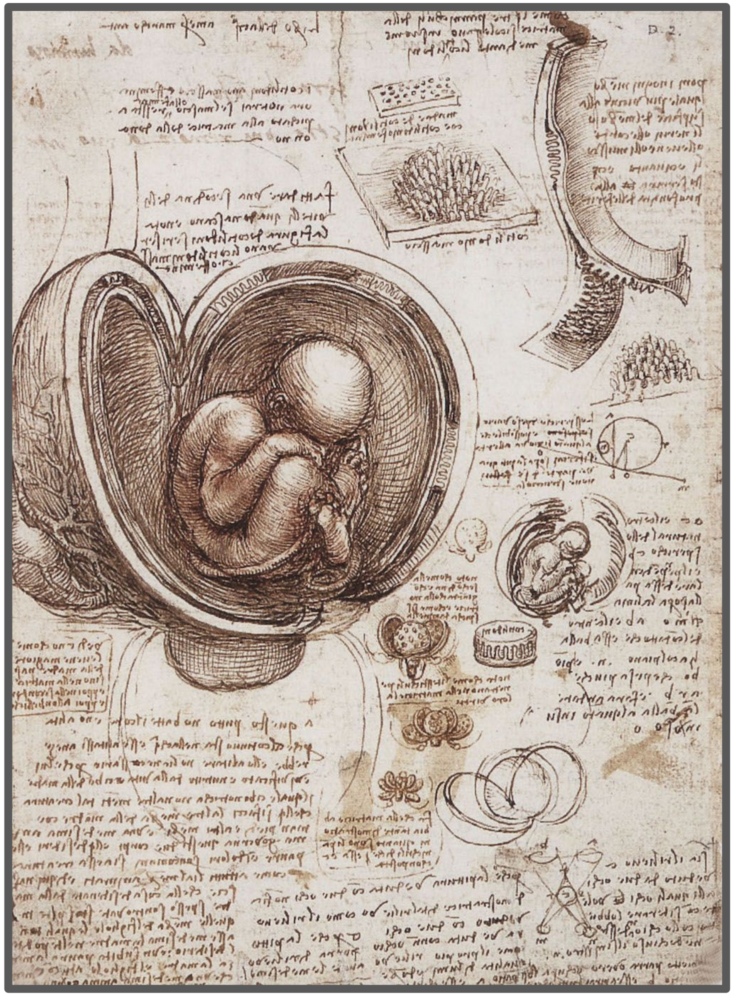


$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} \end{pmatrix}$$

Distorções / Projeções

Forma

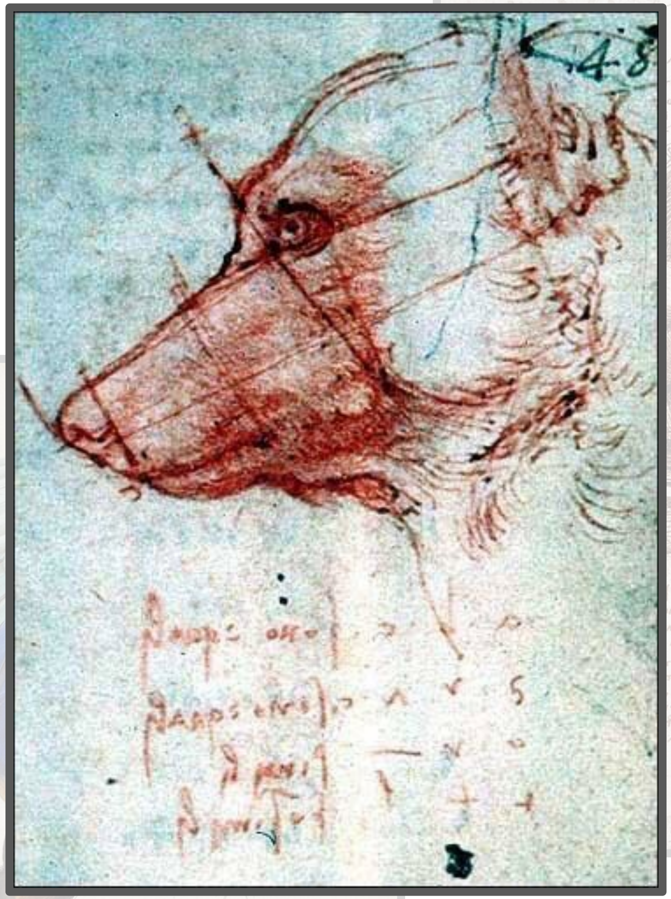
Albrecht Dürer



Leonardo da Vinci

Forma

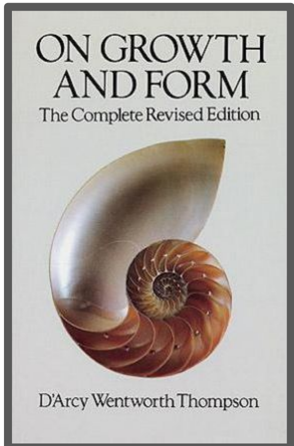
Albrecht Dürer

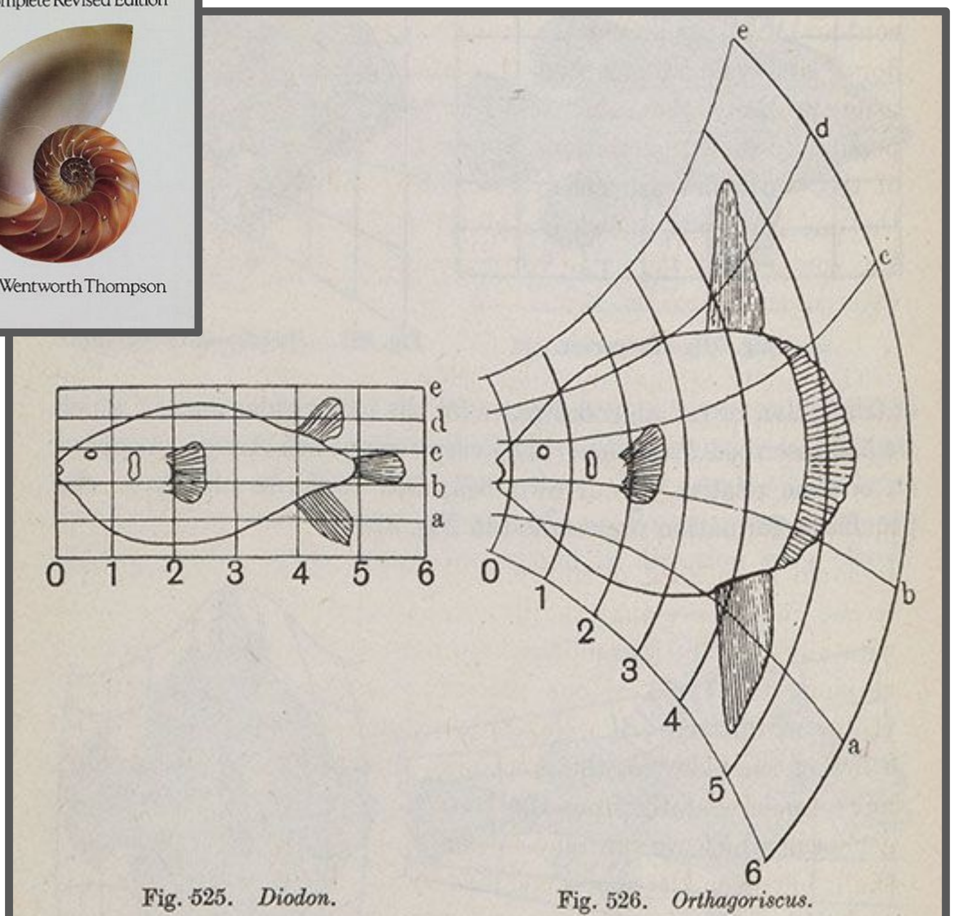


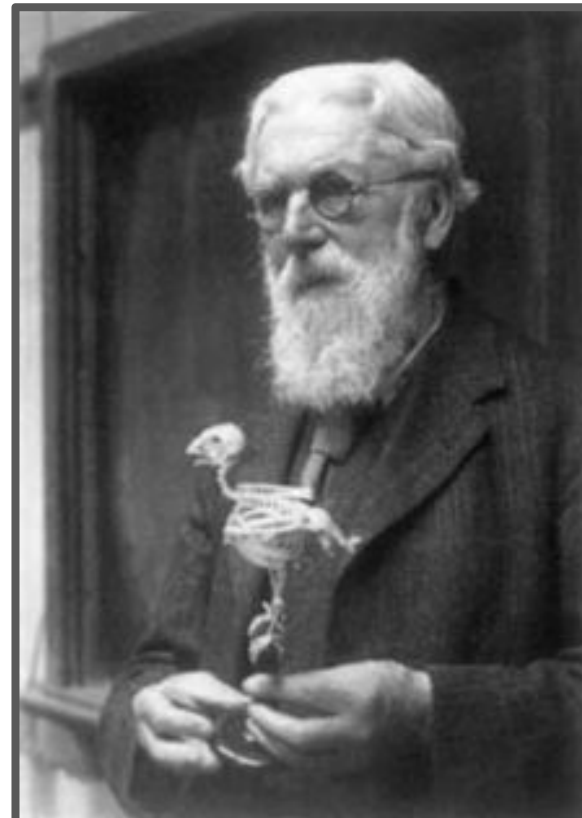
Leonardo da Vinci

Forma

Grades de deformação



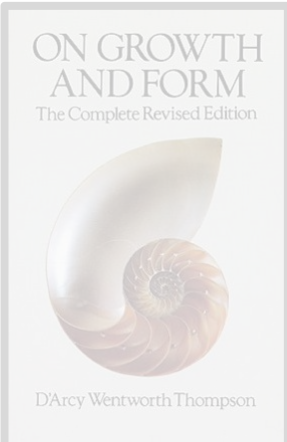




D'Arcy Thompson

Forma

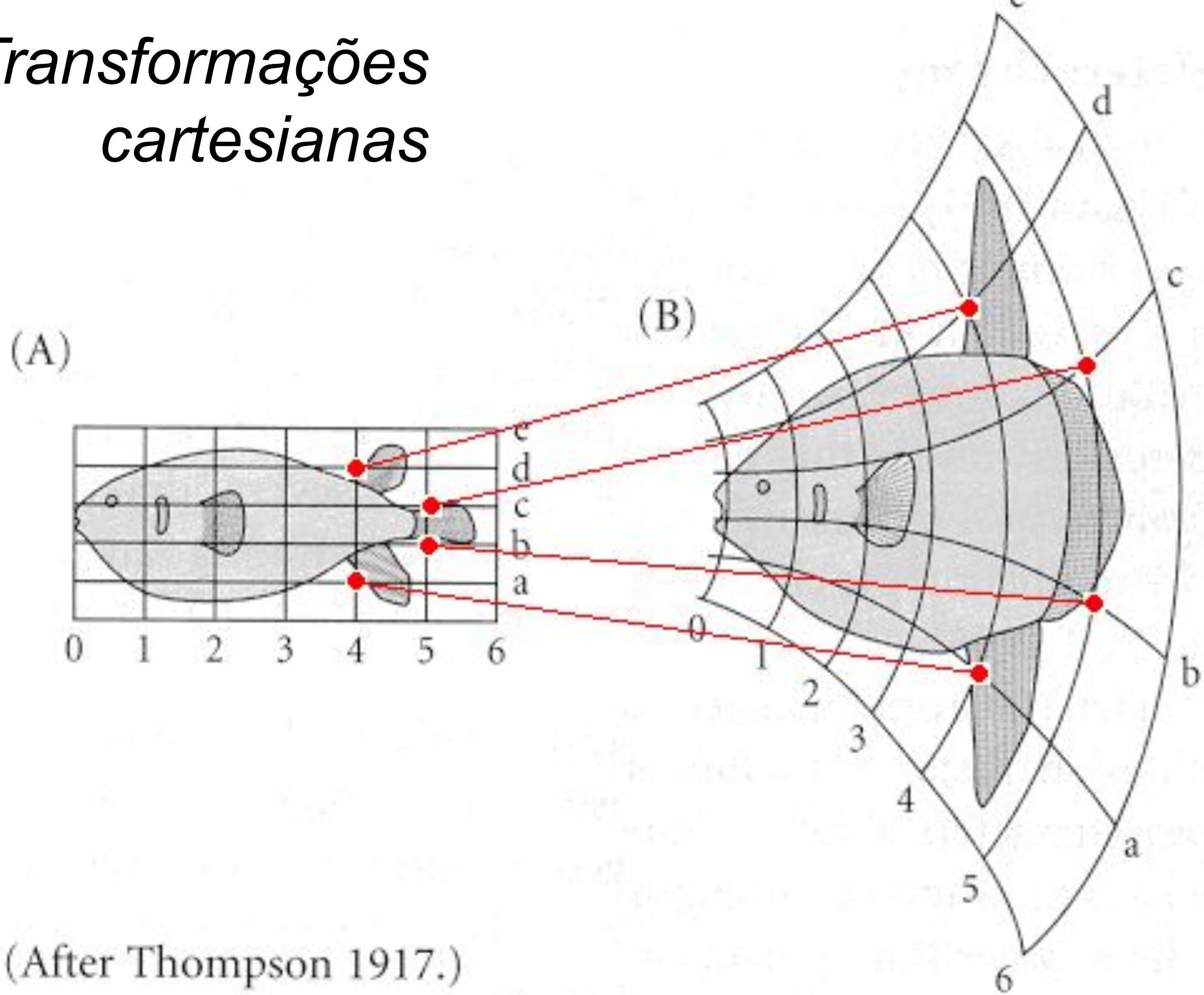
Grades de deformação



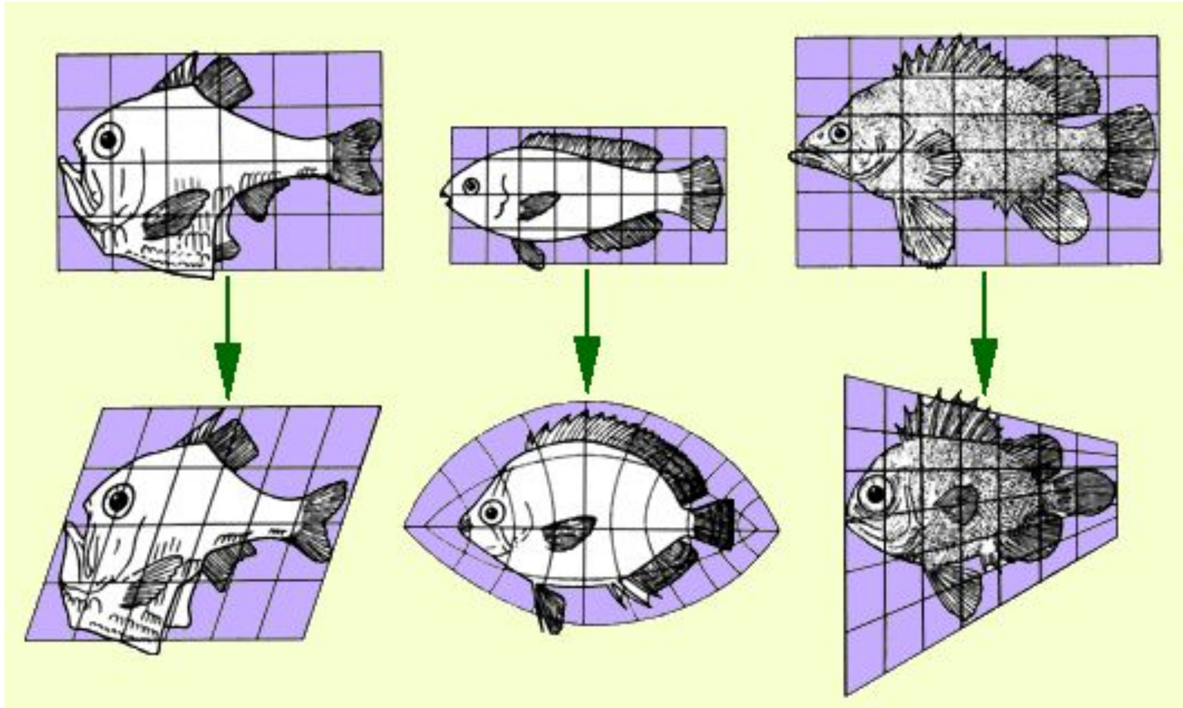
Como comparar?

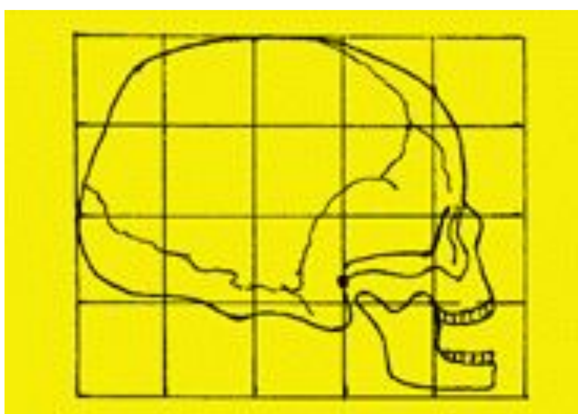
D'Arcy Thompson

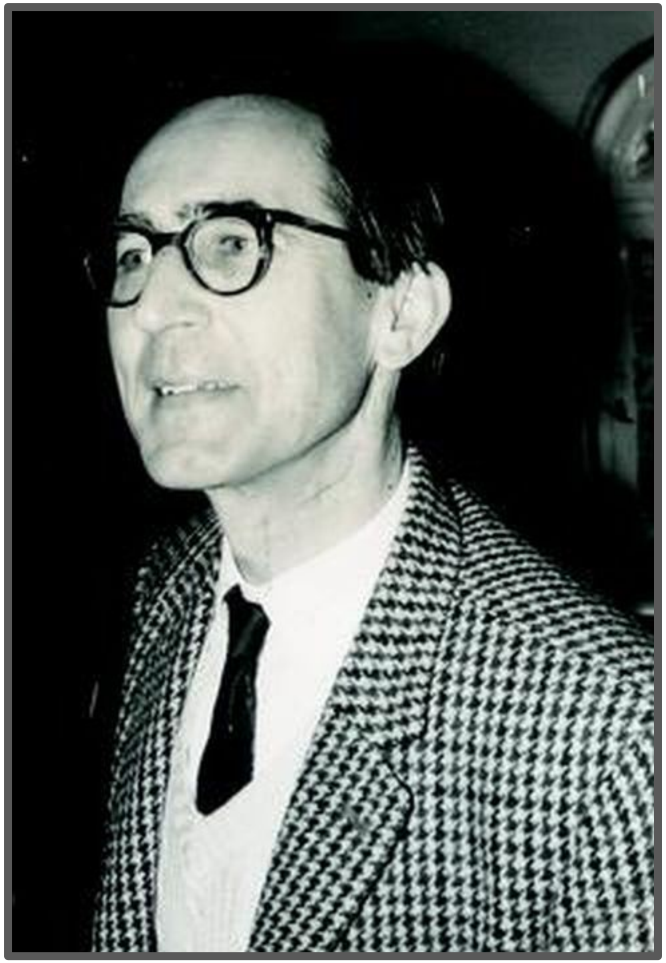
Transformações cartesianas



(After Thompson 1917.)

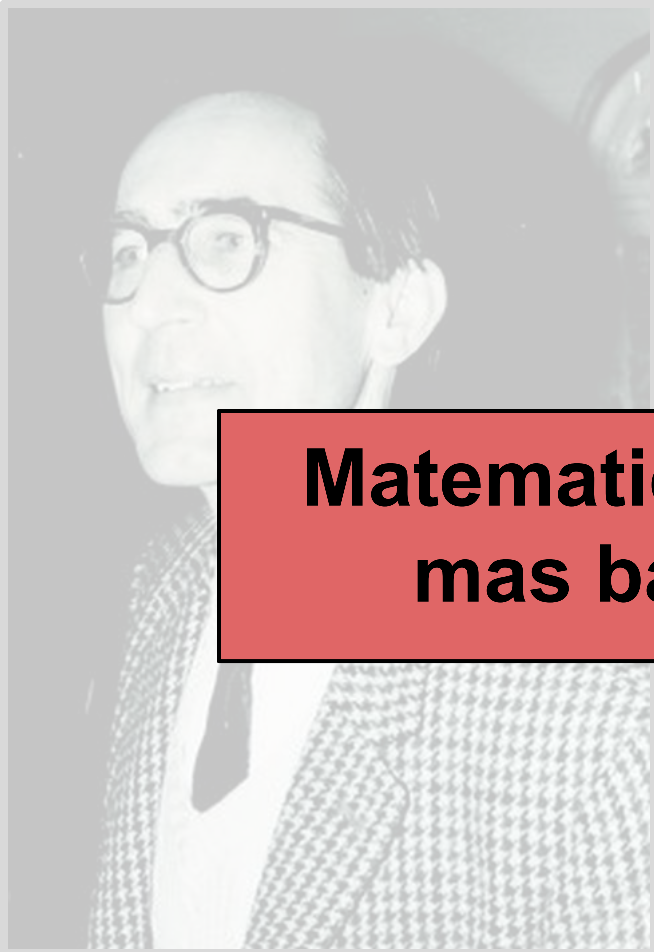






David Kendall

Desenvolvimento matemático formal implementando as análises multivariadas como mecanismo para visualizar a forma, a partir dos anos 1970.



David Kendall

Desenvolvimento matemático formal implementando as análises multivariadas

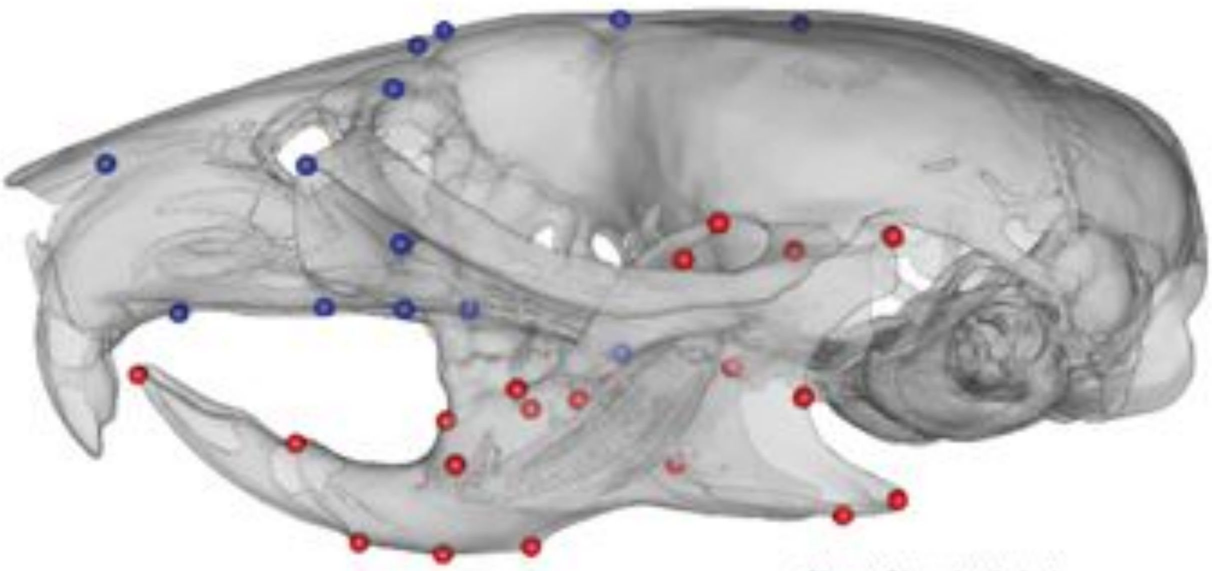
ra

partir

dos anos 1970.

Matematicamente importante, mas bastante complexo

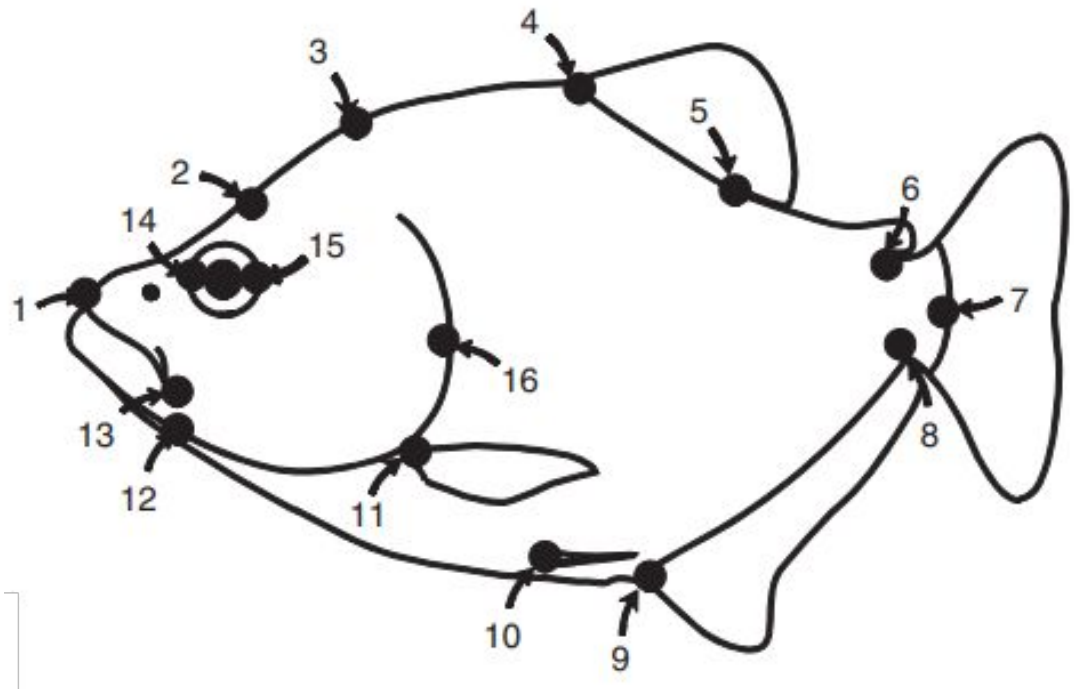
Morfometria geométrica



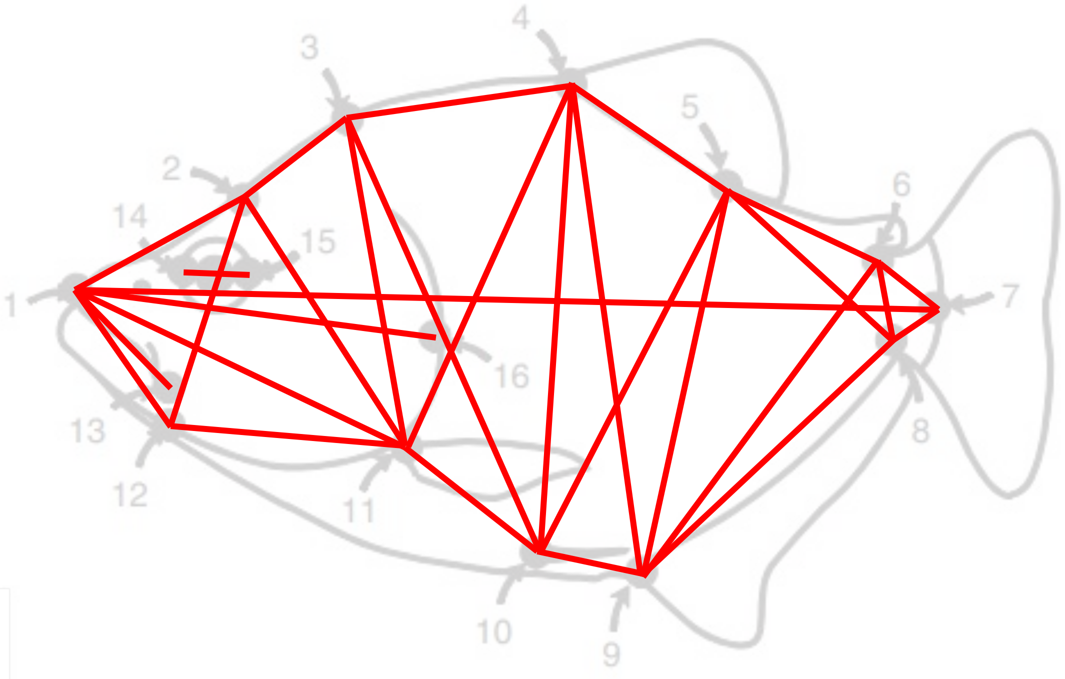
Descrição da forma em função de marcos anatômicos

	x	y
Espécie 1	5.1	7.0
Espécie 2	4.8	6.5
Espécie 3	3.2	4.1
(...)	(...)	(...)

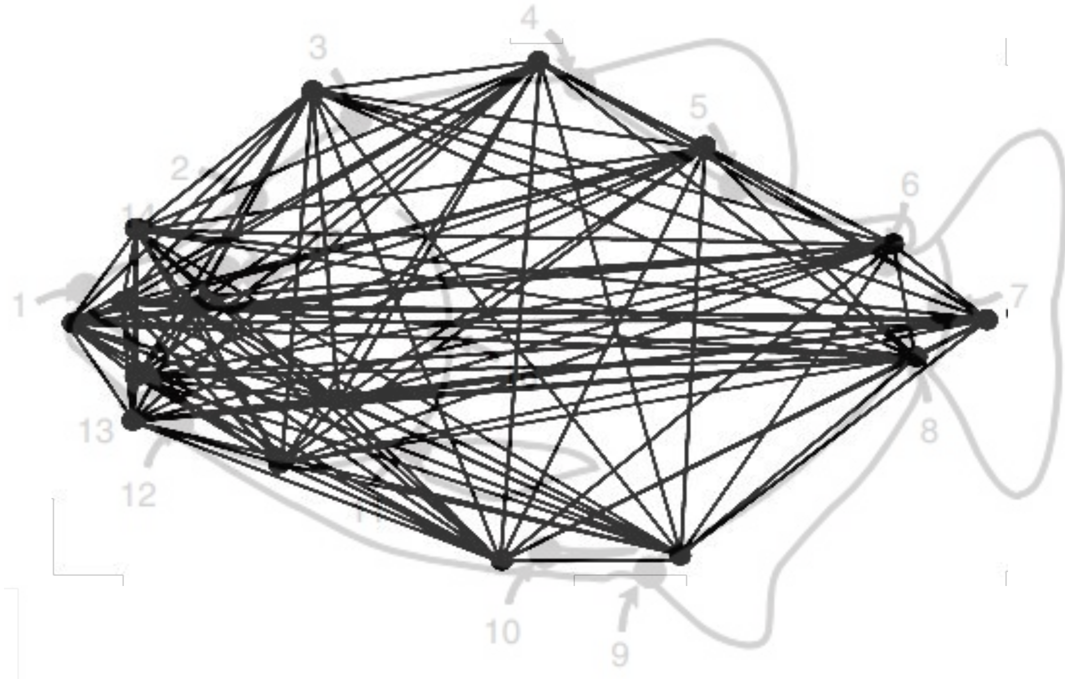
*Método **eficiente** em descrever a forma*



Ex: morfometria geométrica



Ex: medidas lineares



Ex: medidas lineares (todas)

Agora sim, analisemos a fórmula:

$$Z = \frac{1}{CS}(\mathrm{Y} - \bar{\mathrm{Y}})\,\mathrm{H}$$

Agora sim, analisemos a fórmula:

$$Z = \frac{1}{\boxed{CS}}(Y - \bar{Y})H$$

Tamanho do centróide

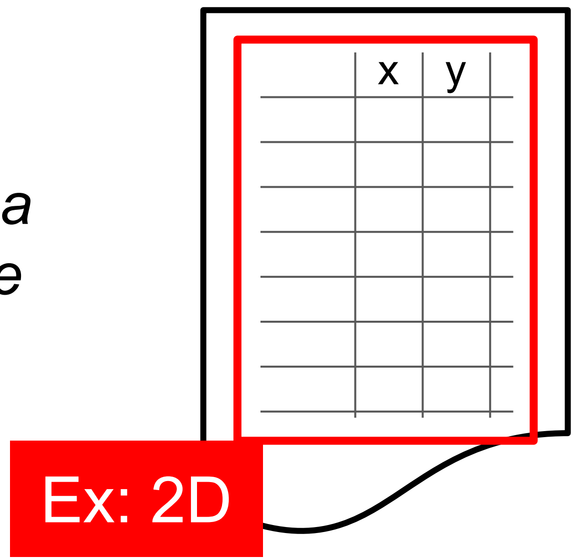
Tamanho do centróide

$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij} - C_j)^2}$$

Tamanho do centróide

$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij}-C_j)^2}$$

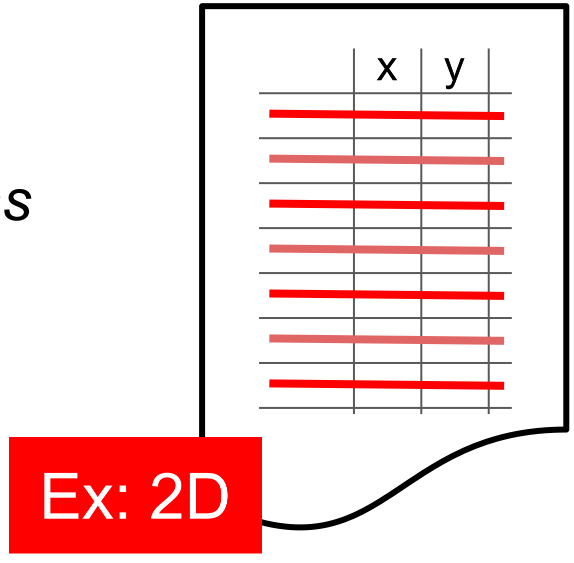
tamanho do centróide de uma determinada configuração de landmarks



Tamanho do centróide

$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij} - C_j)^2}$$

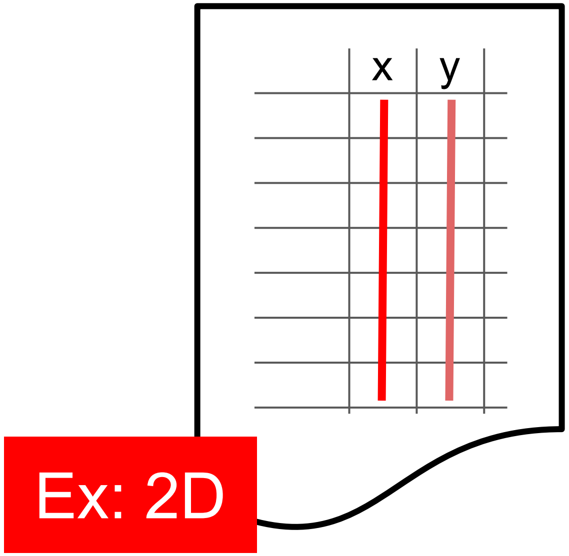
somatório ao longo das linhas



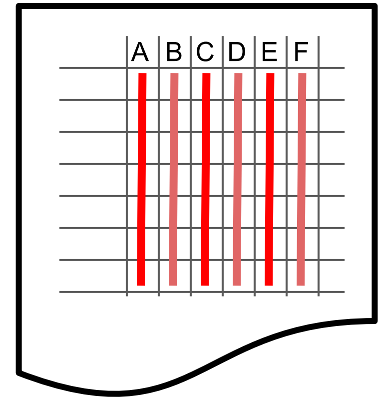
Tamanho do centróide

$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij}-C_j)^2}$$

e ao longo das colunas

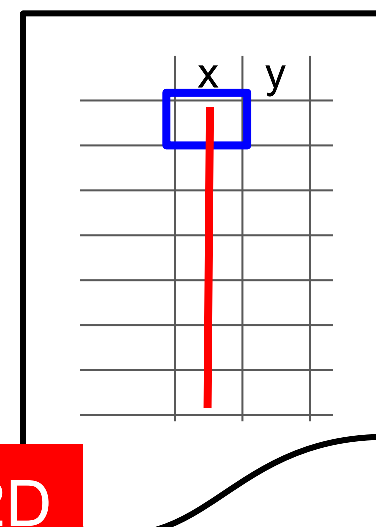


Tamanho do centróide



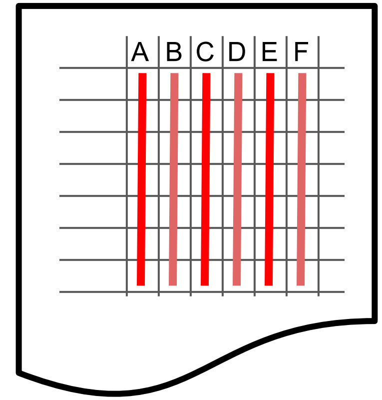
$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij} - C_j)^2}$$

das distâncias entre cada ponto os demais em sua respectiva dimensão



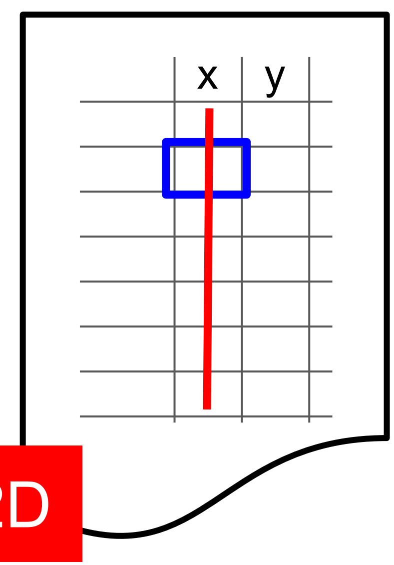
Ex: 2D

Tamanho do centróide



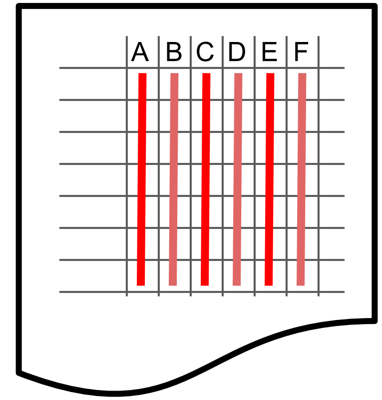
$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}\left(\mathbf{X}_{ij} - C_{j}\right)^{2}}$$

das distâncias entre cada ponto os demais em sua respectiva dimensão



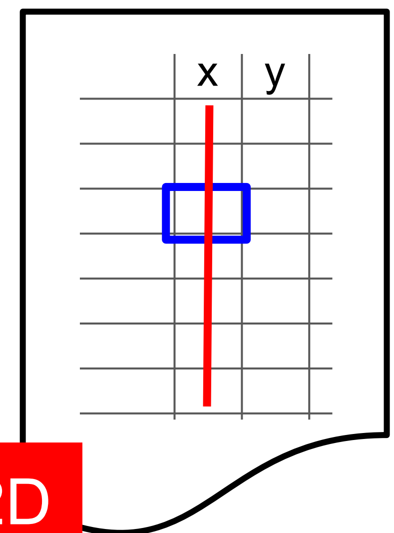
Ex: 2D

Tamanho do centróide



$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij}-C_j)^2}$$

das distâncias entre cada ponto os demais em sua respectiva dimensão

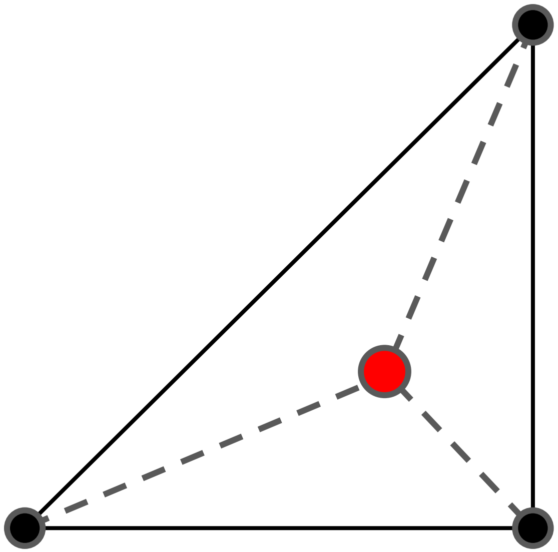


Ex: 2D

Tamanho do centróide

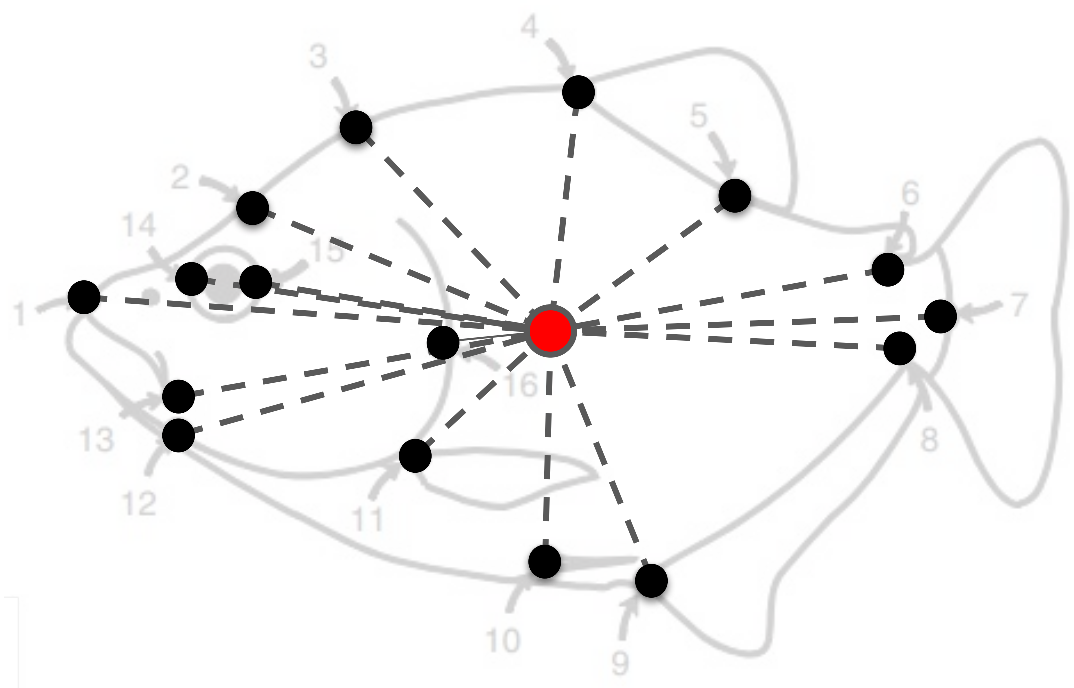
$$CS(\mathbf{X}) = \sqrt{\sum_{i=1}^{K}\sum_{j=1}^{M}(\mathbf{X}_{ij}-C_j)^2}$$

Tamanho do centróide



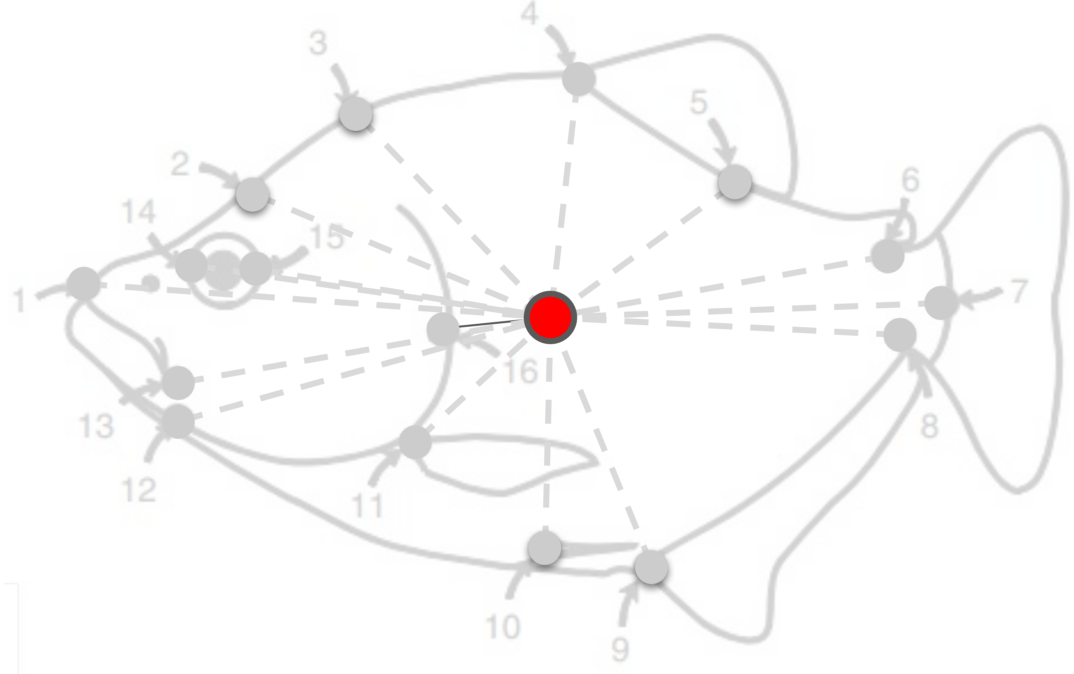
medida escalar que descreve o tamanho geral de uma configuração de landmarks

Tamanho do centróide



medida escalar que descreve o tamanho geral de uma configuração de landmarks

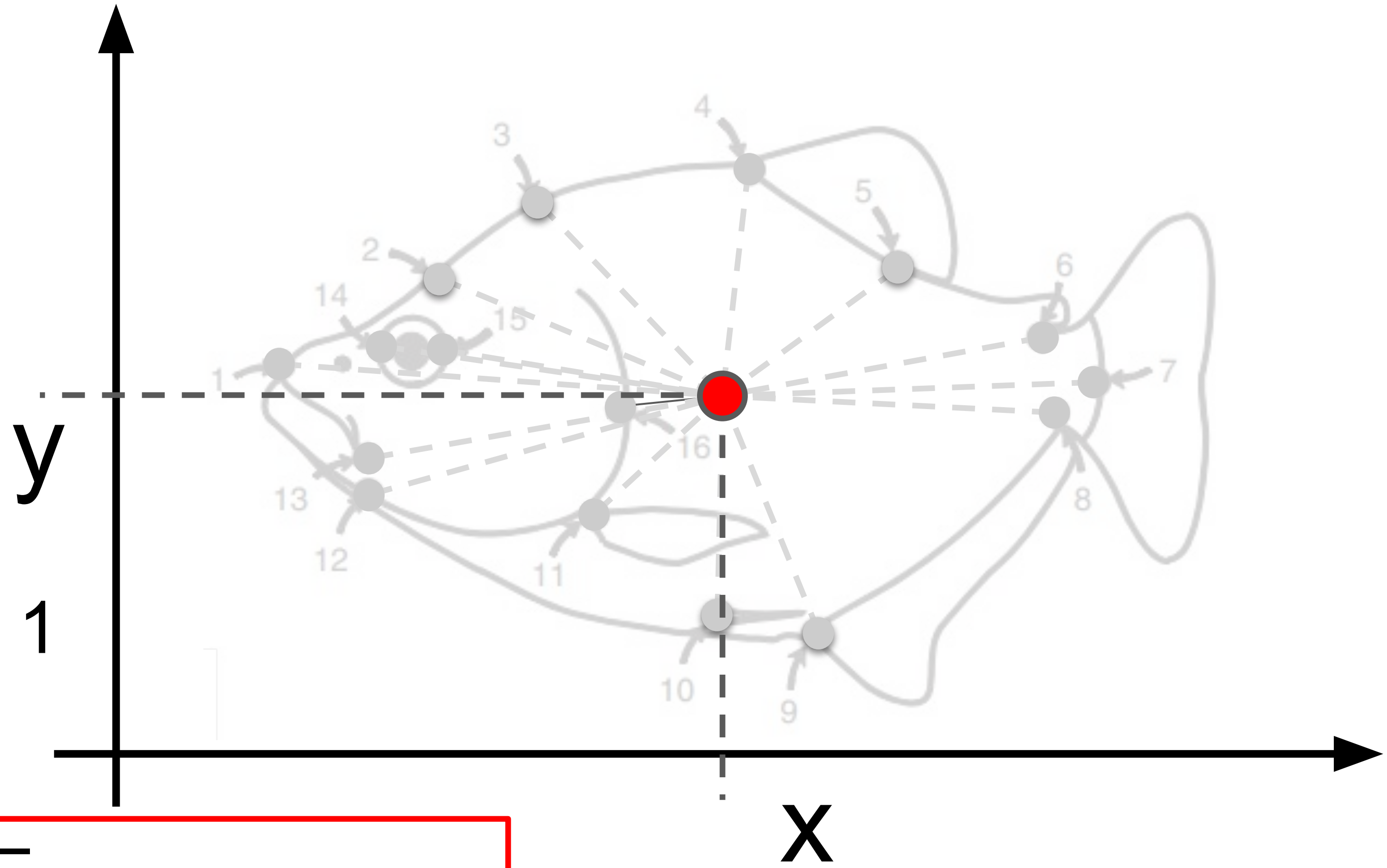
Centróide



Então, existe um ponto que representa a otimização do CS

Centróide

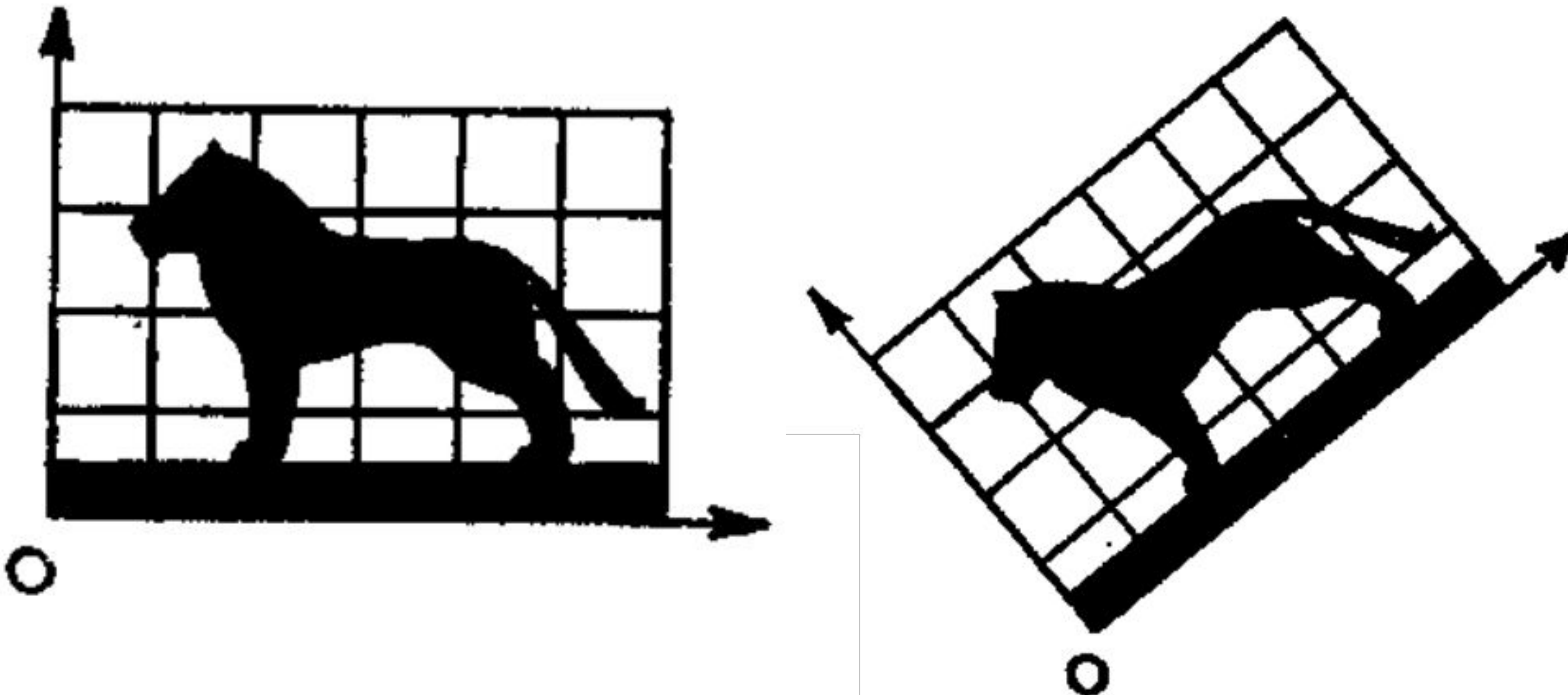
Que pode ser, portanto, descrito por um par ordenado



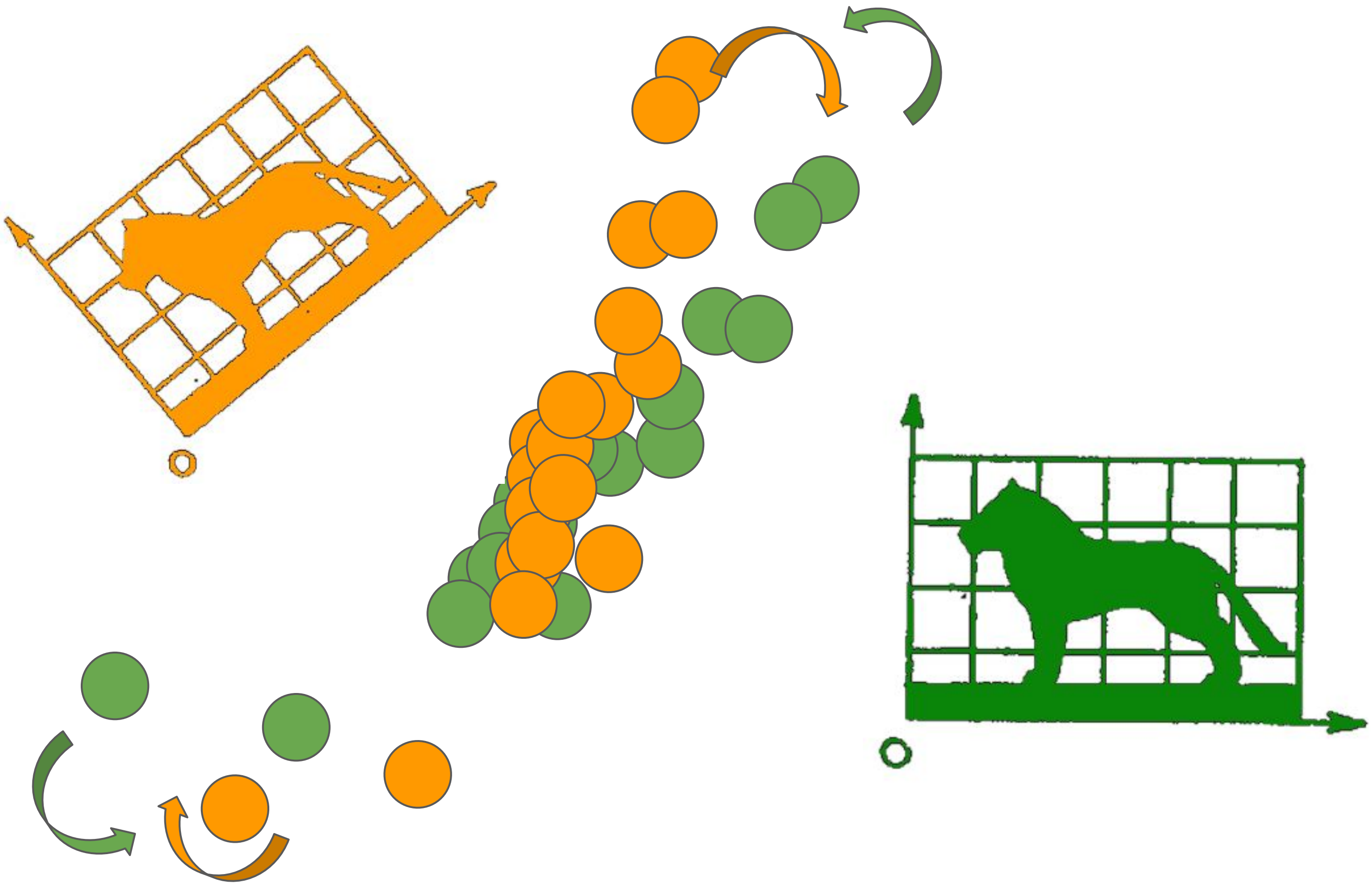
$$\bar{Y} = x_1 - y_1$$

1

Essa distância poderia variar de acordo com o alinhamento de cada indivíduo



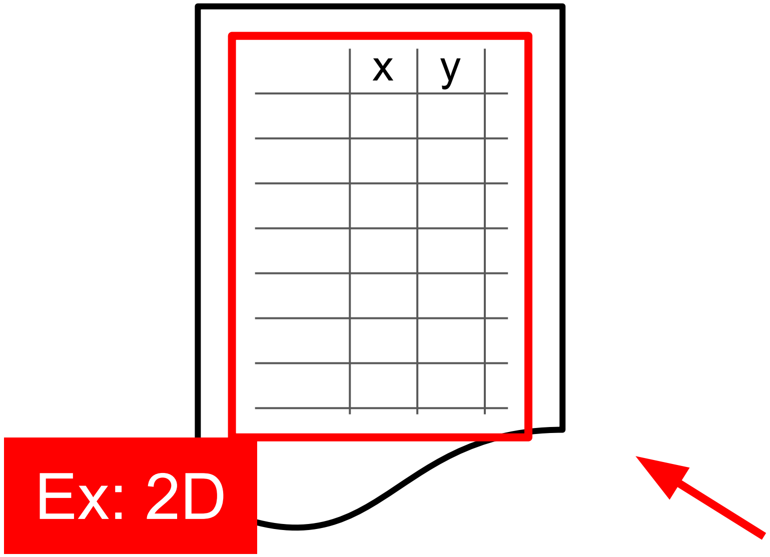
Por isso, é necessária a projeção de matrizes



E voltemos à fórmula:

$$Z = \frac{1}{CS}\boxed{(\mathrm{Y} - \bar{\mathrm{Y}})}\,\mathrm{H}$$

Translação



$$(Y - \bar{Y})$$

$$\begin{pmatrix} 0{,}11; & 0{,}75 \\ 2{,}04; & -0{,}92 \\ 0{,}04; & 1{,}26 \end{pmatrix}$$

Ex: 2D

$$(\color{red}{Y} - \bar{Y})$$

$$\begin{pmatrix} 0{,}11;\ 0{,}75 \\ 2{,}04; -0{,}92 \\ 0{,}04;\ 1{,}26 \end{pmatrix}$$

Ex: 2D

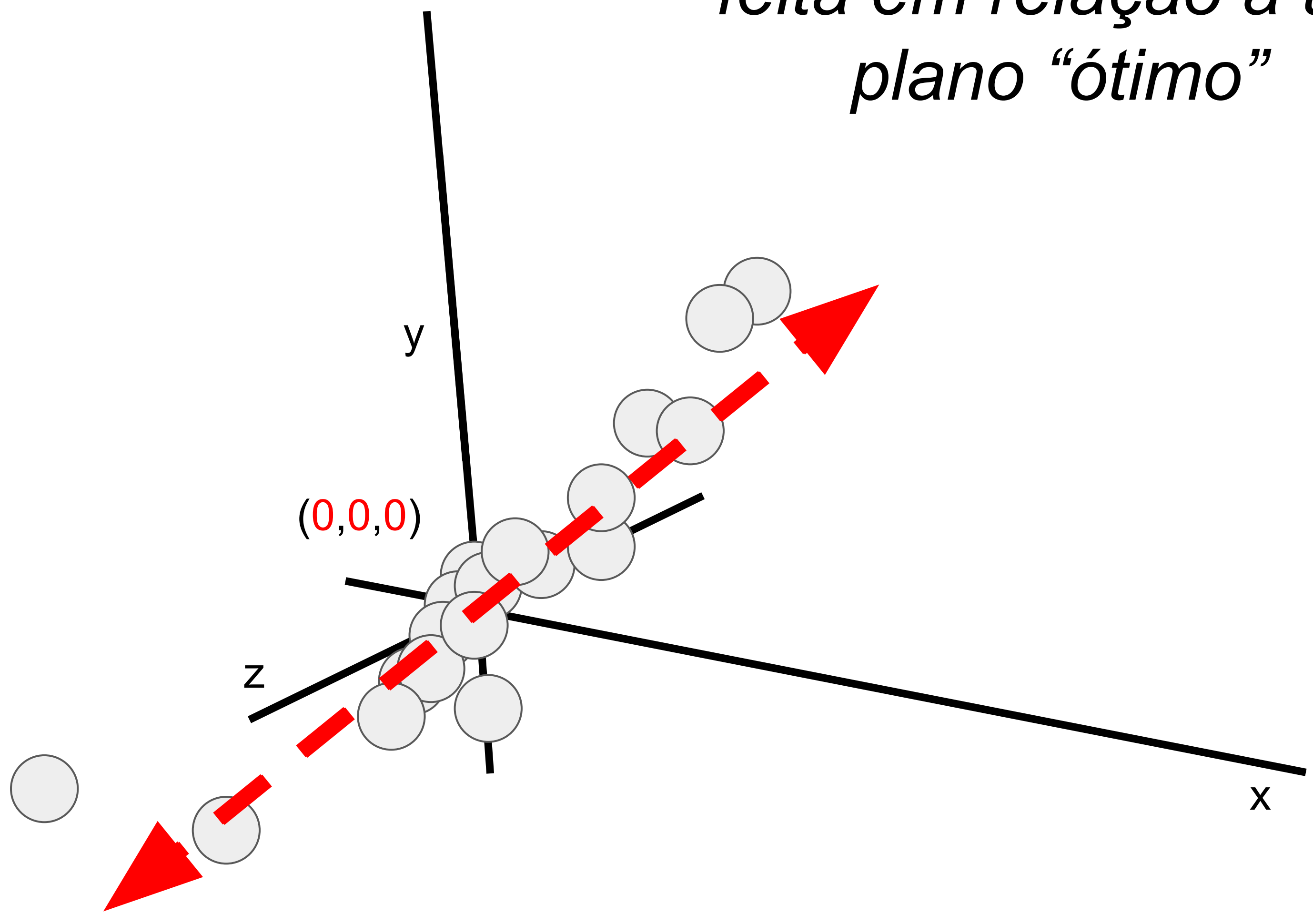
$$(Y - \bar{Y})$$

Posição do centróide

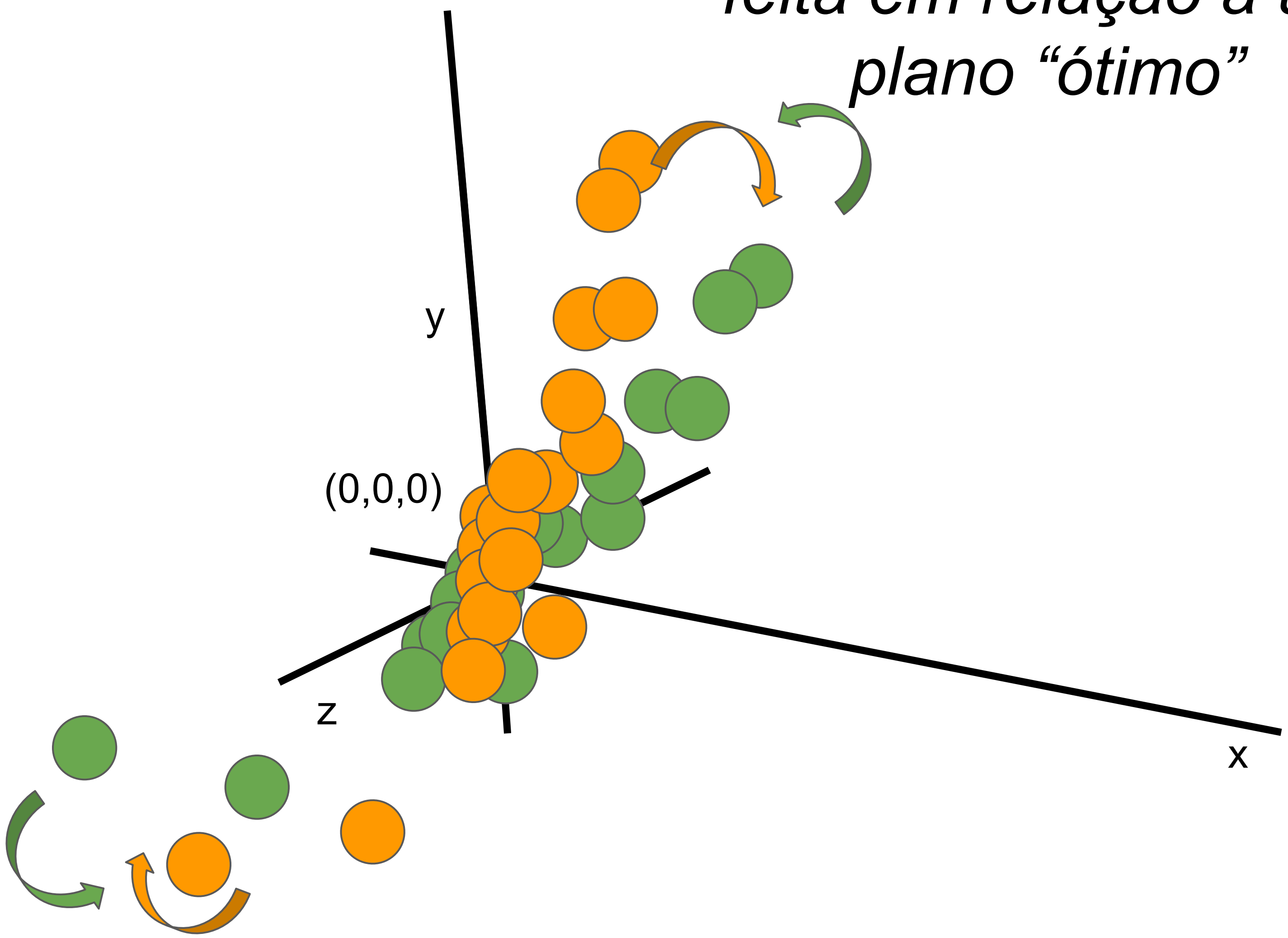
$$\begin{pmatrix} 0{,}11 - x_1 ; 0{,}75 - y_1 \\ 2{,}04 - x_1 ; -0{,}92 - y_1 \\ 0{,}04 - x_1 ; 1{,}26 - y_1 \end{pmatrix}$$

Ex: 2D

Essa projeção deve ser feita em relação a um plano “ótimo”



Essa projeção deve ser feita em relação a um plano "ótimo"



E voltemos à fórmula:

$$Z = \frac{1}{CS}(\mathrm{Y} - \bar{\mathrm{Y}})\,\boxed{\mathrm{H}}$$

Rotação

E voltemos à fórmula:

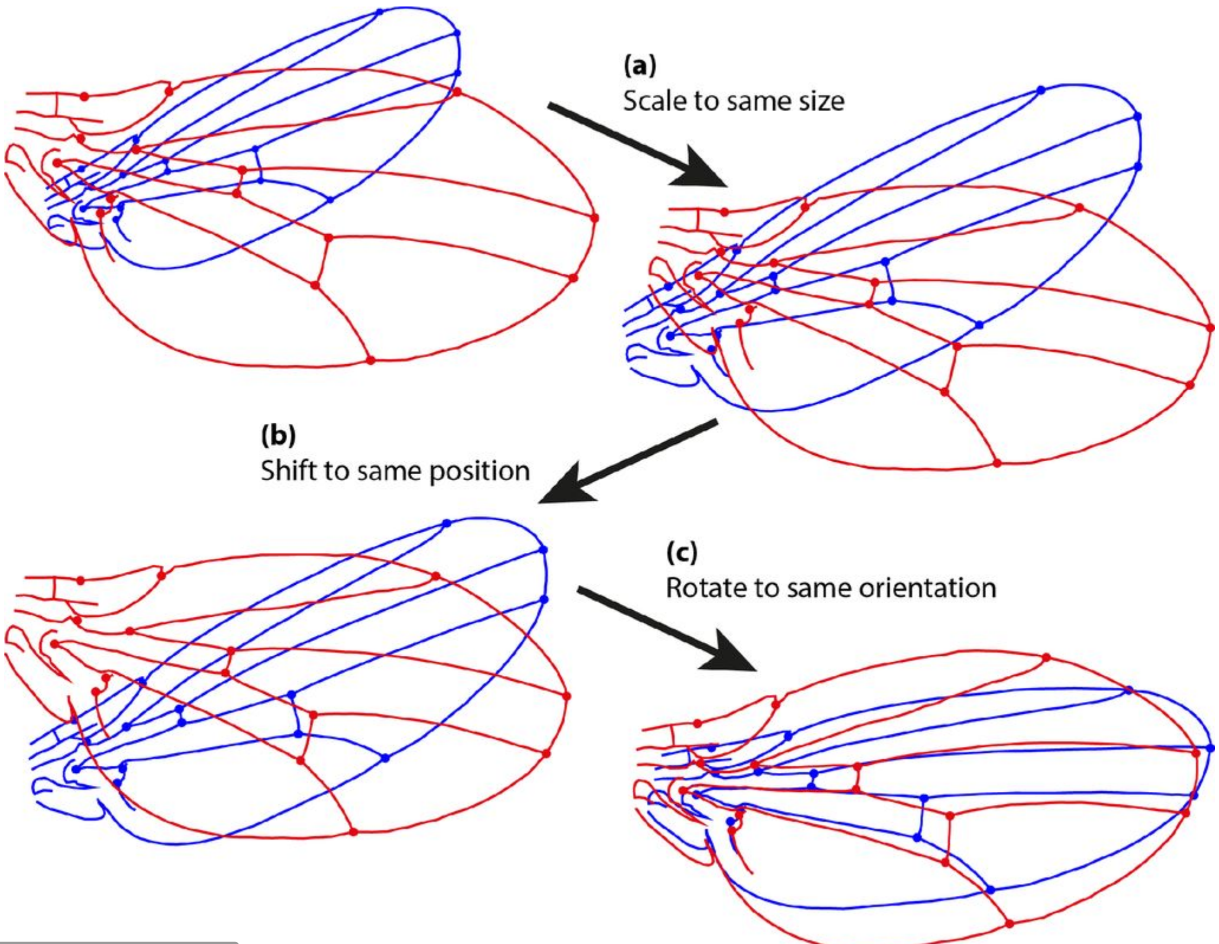
$$Z = \frac{1}{CS}(Y - \bar{Y})\,H$$

$$\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}$$

Ex: 2D

Ufa…

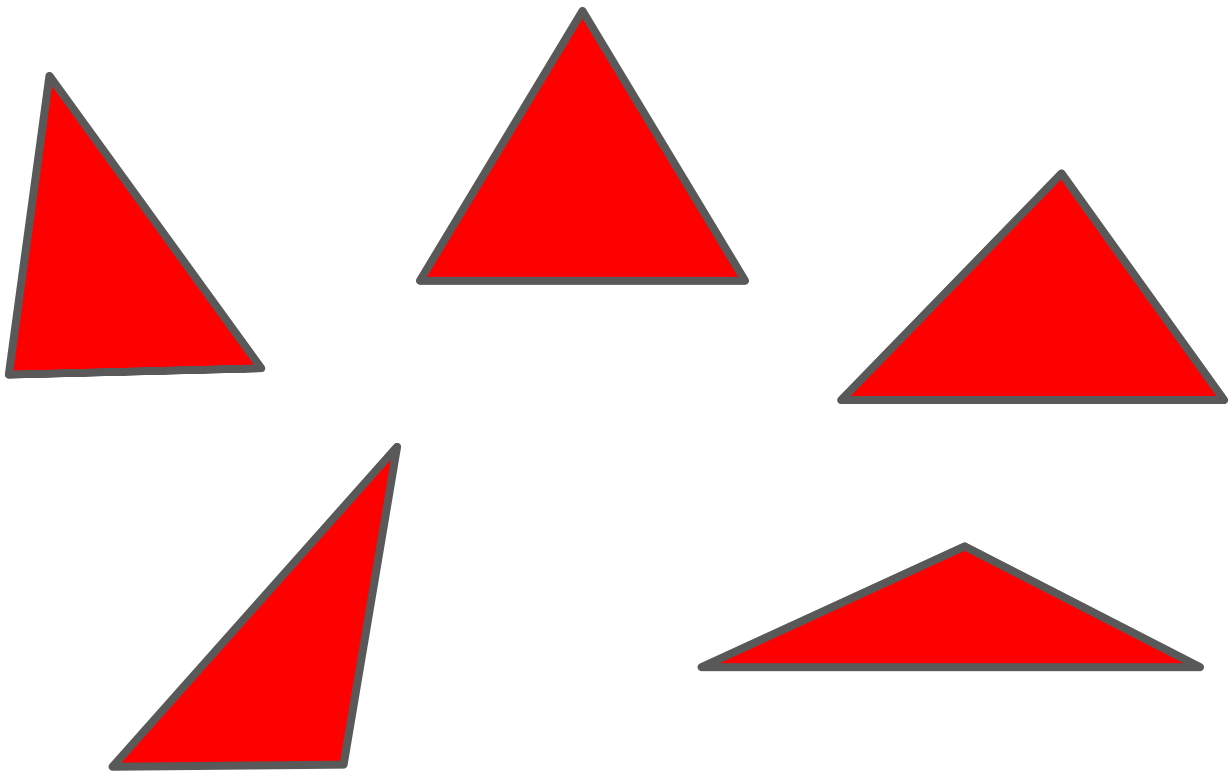
Análise Generalizada de Procrustes (GPA)

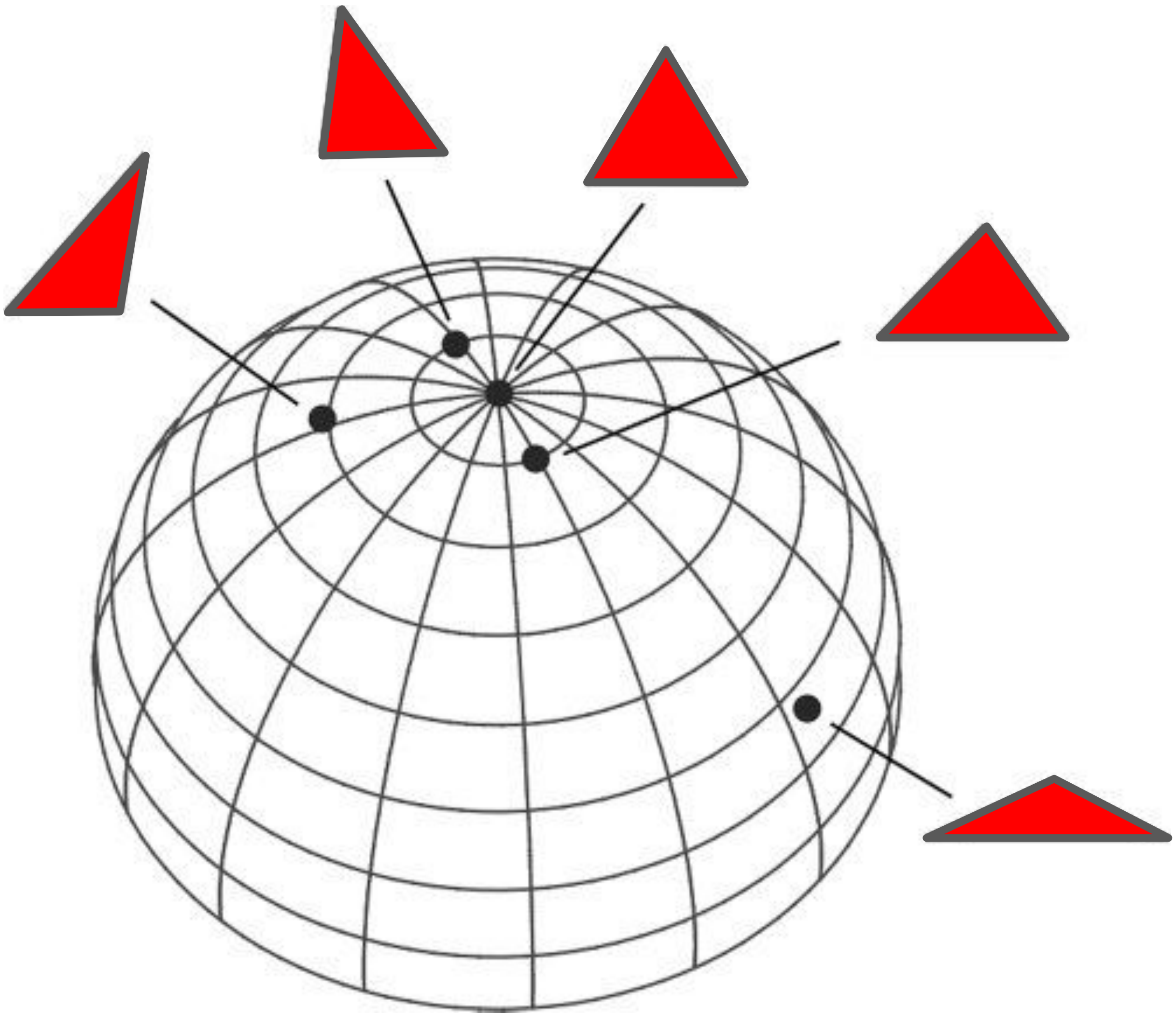


Em resumo

Procrustes

Após a GPA, cada forma representa apenas um ponto em um novo espaço amostral





pena que o espaço é curvo kkkk

pena que o espaço é curvo kkkk

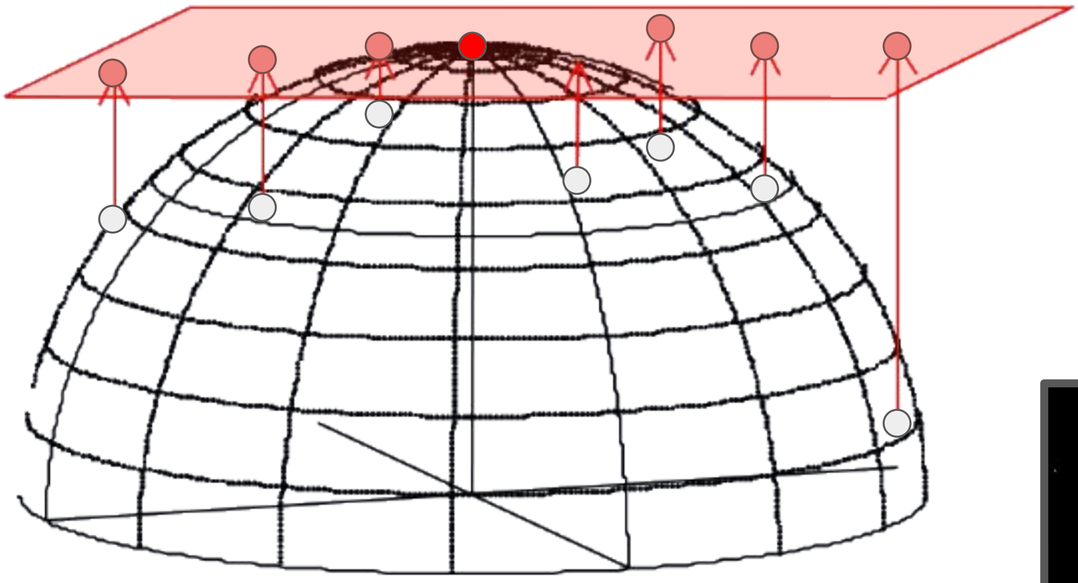
Após a GPA, cada forma representa apenas um ponto em um novo espaço amostral

$$D_{Proc} = \sqrt{\sum_{i=1}^{K} \sum_{j=1}^{M} (\mathbf{Z}_{1.i} - \mathbf{Z}_{2.ij})^2}$$

j

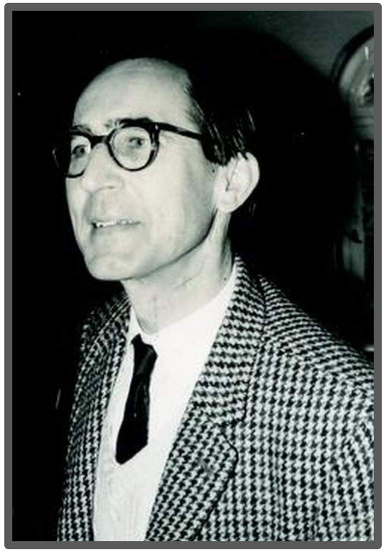
Espaço de Kendell

Espaço tangente



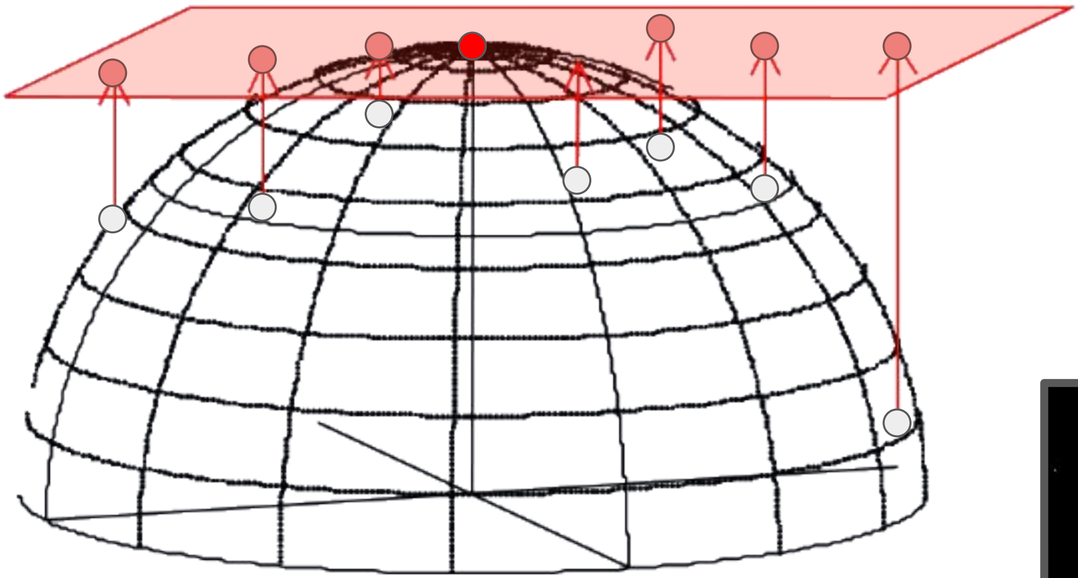
Espaço da forma

Representação linear do espaço da forma…

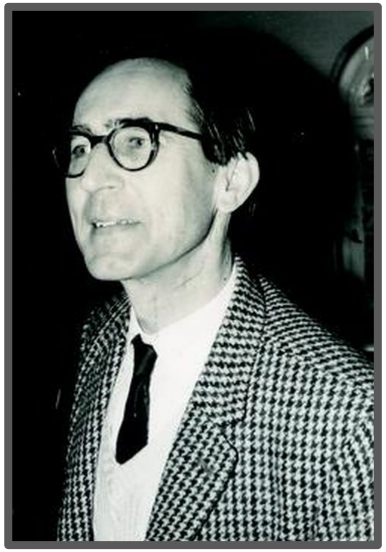


Espaço de Kendell

Espaço tangente



Espaço da forma



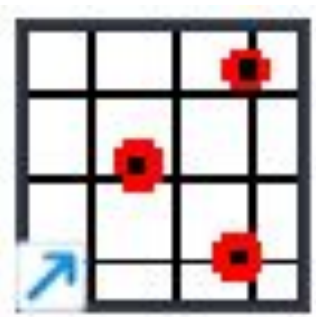
*… o que é obtido a partir de uma **reprojeção***

Ufa…

E como fazer tudo isso?

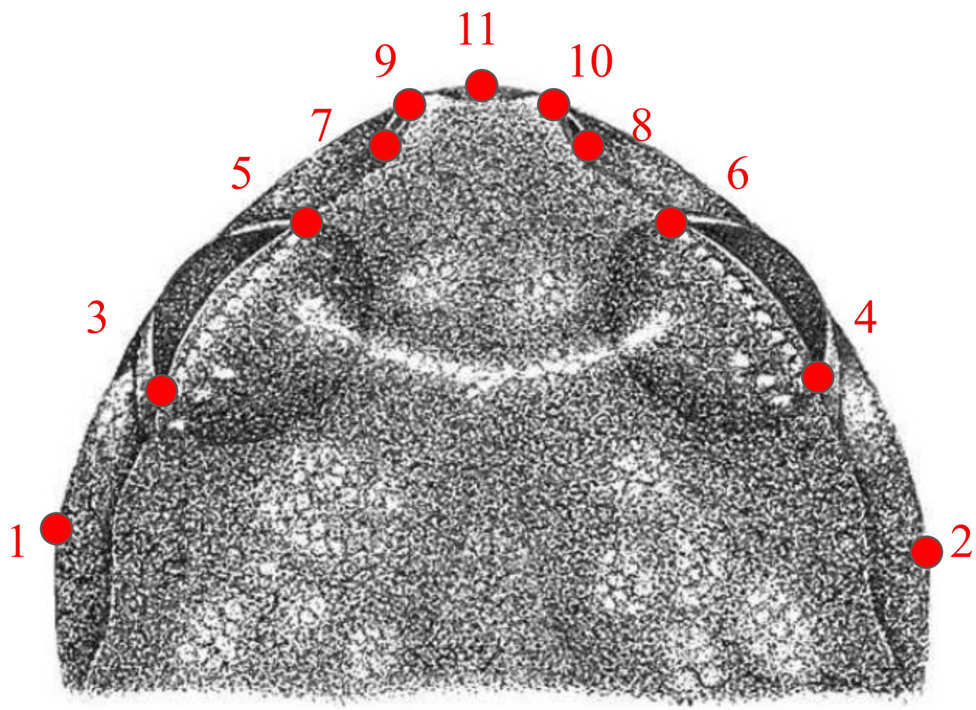
F. James Rohlf

tpsUtil

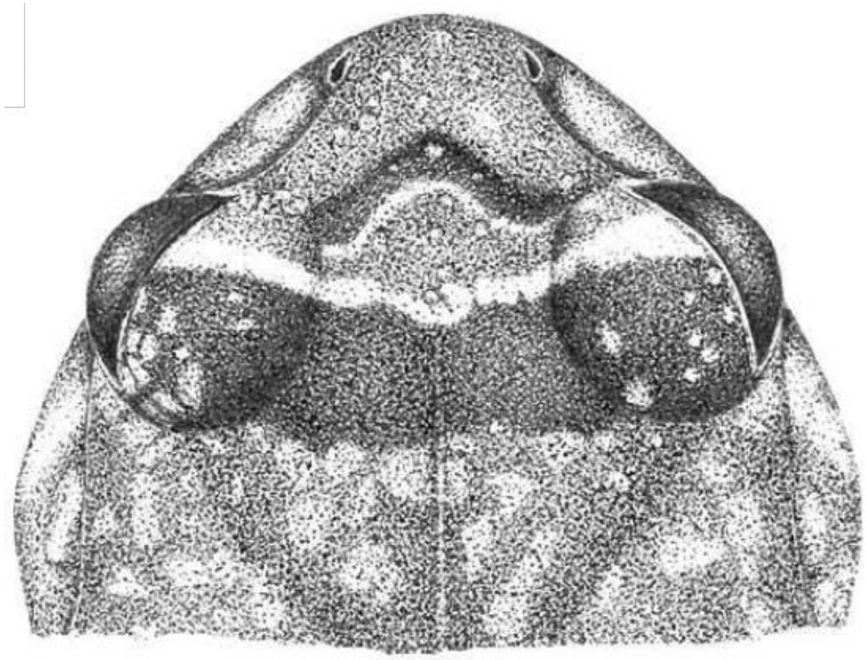


tpsDig

Escolhendo landmarks

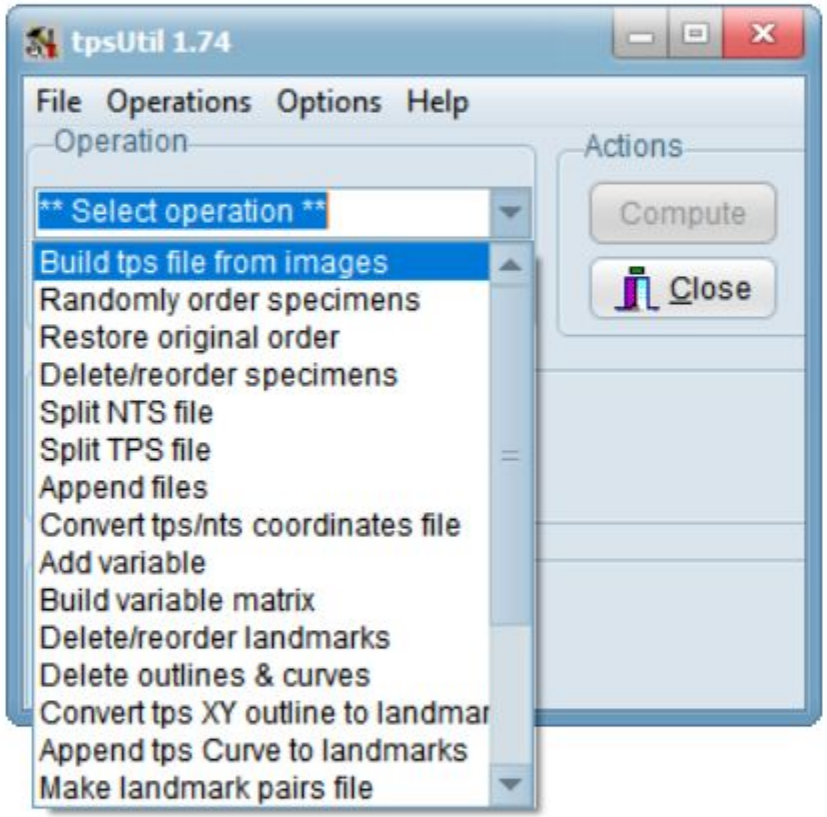


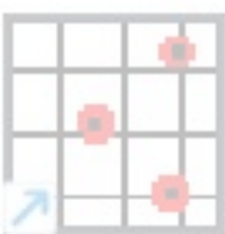
Cycloramphus faustoi



Cycloramphus eleutherodactylus

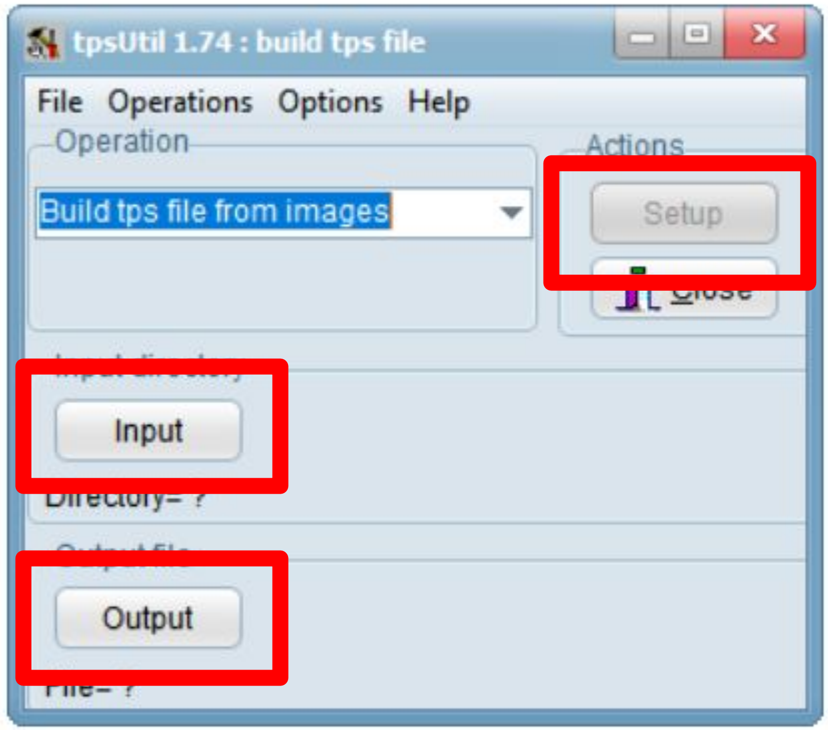
tpsUtil





tpsDig

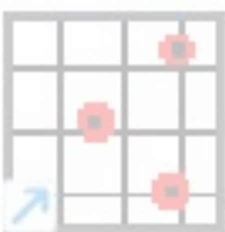
tpsUtil



3

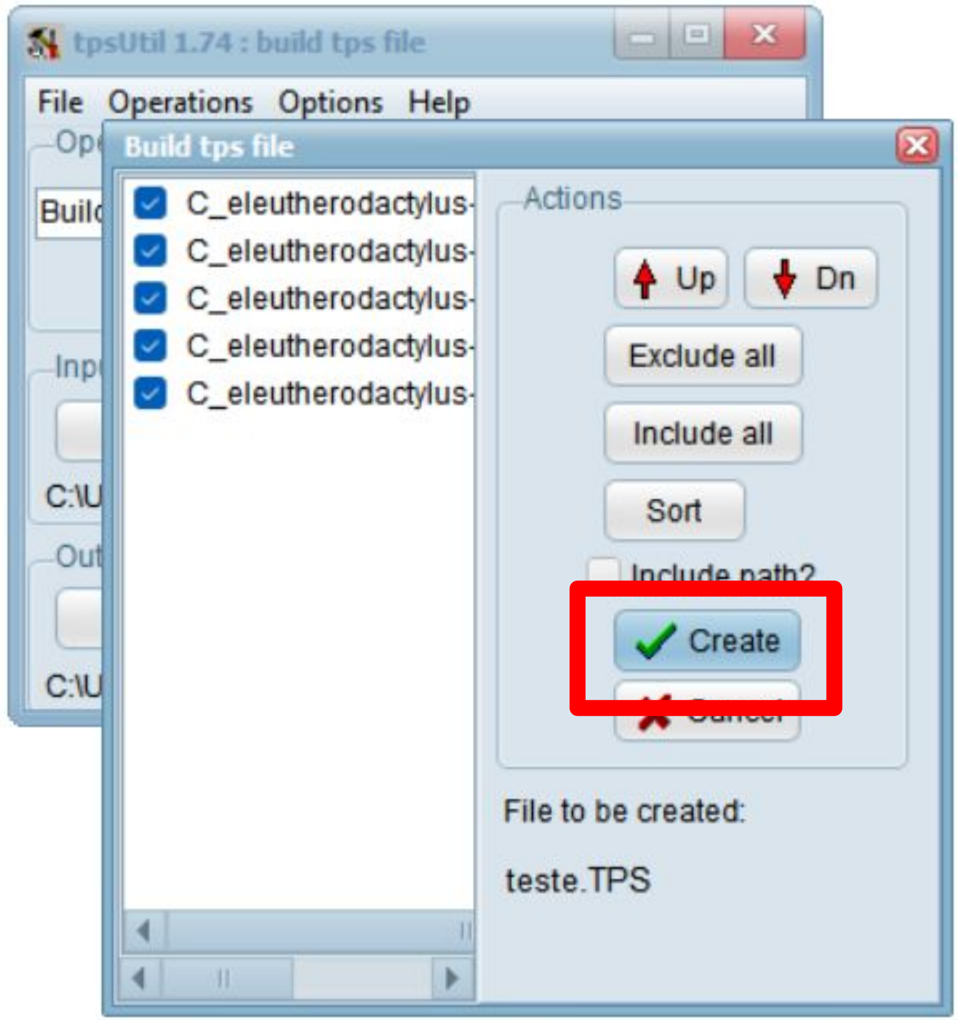
1

2



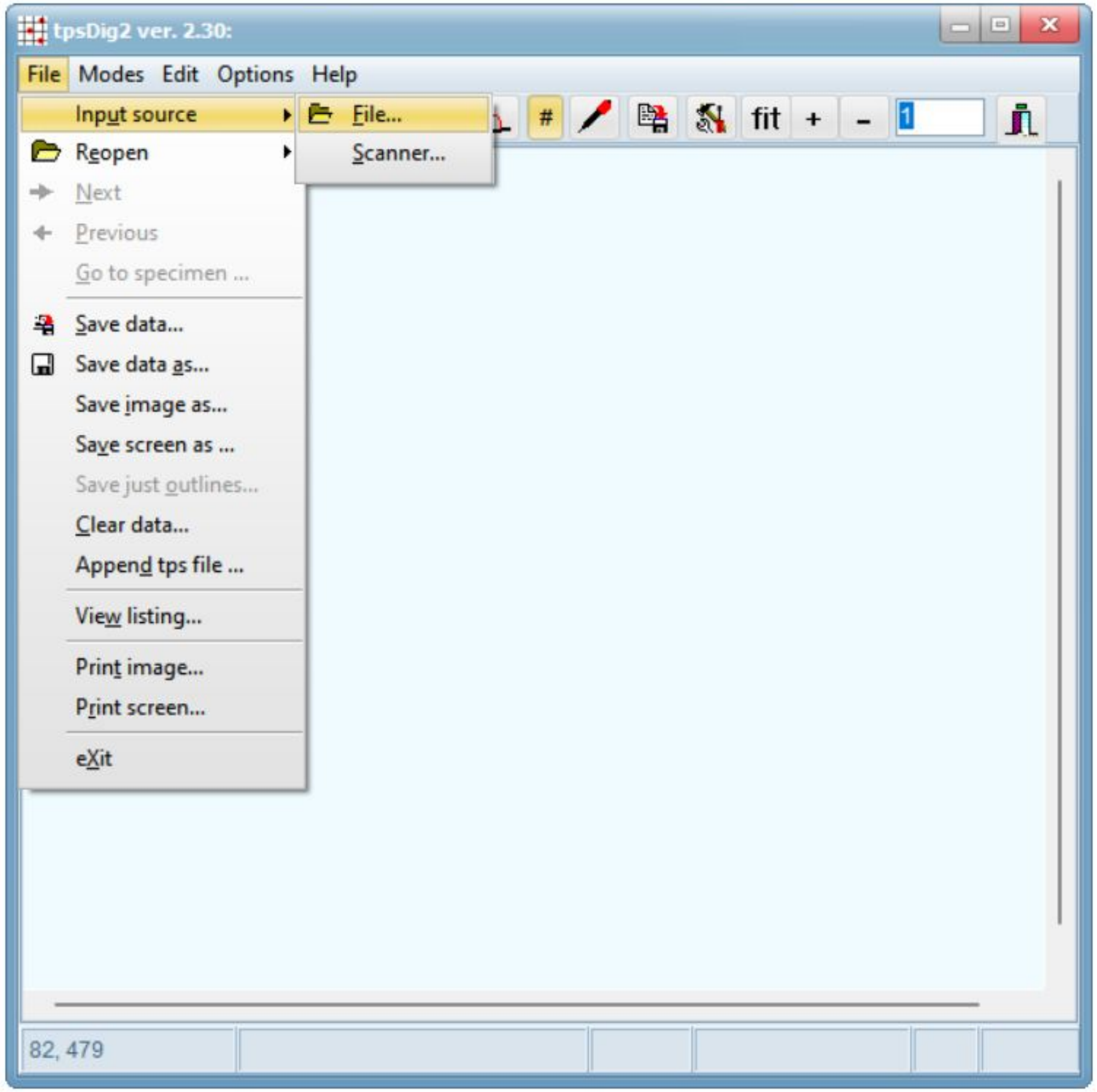
tpsDig

tpsUtil



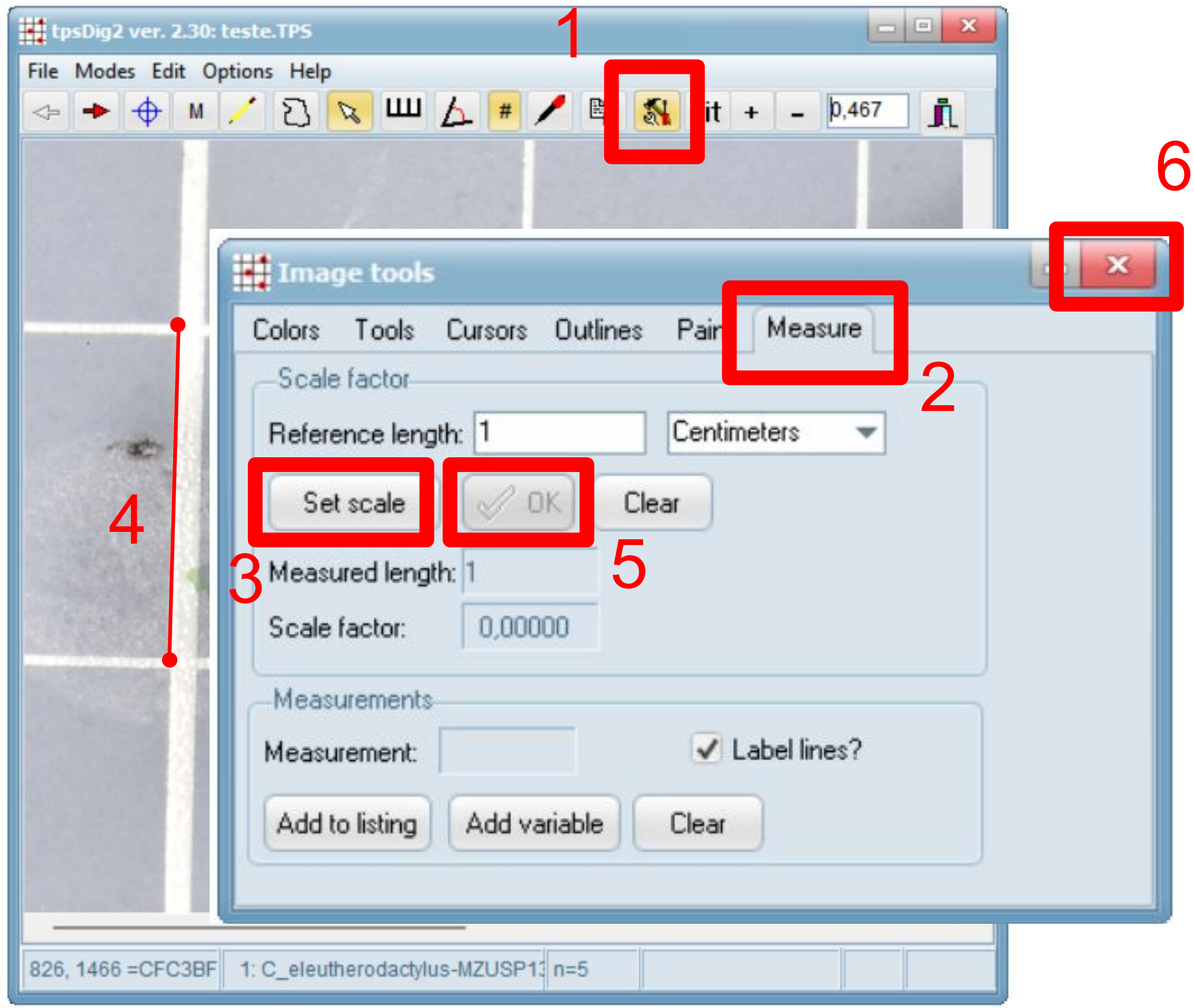
tpsDig

tpsUtil

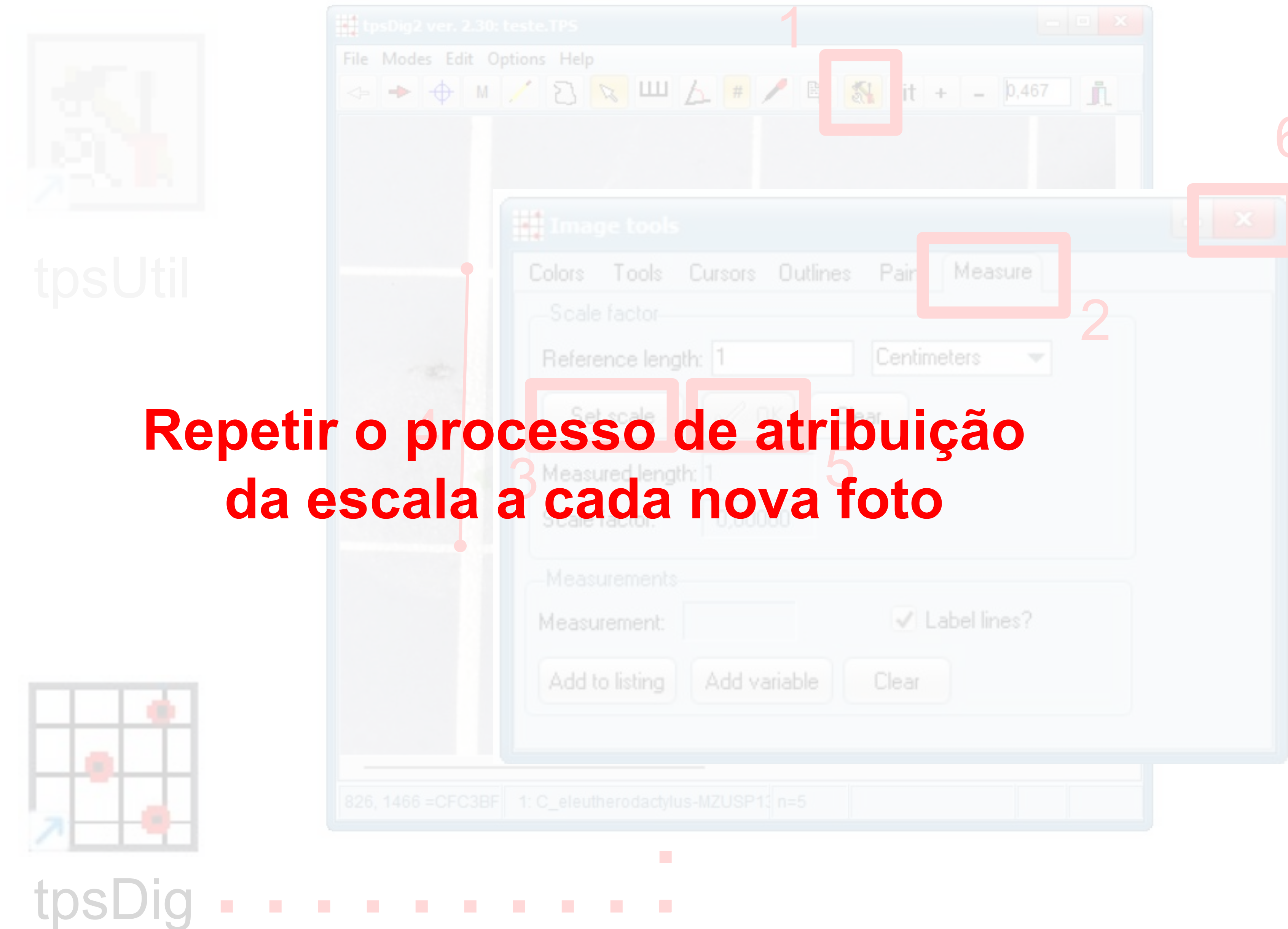


tpsDig

tpsUtil

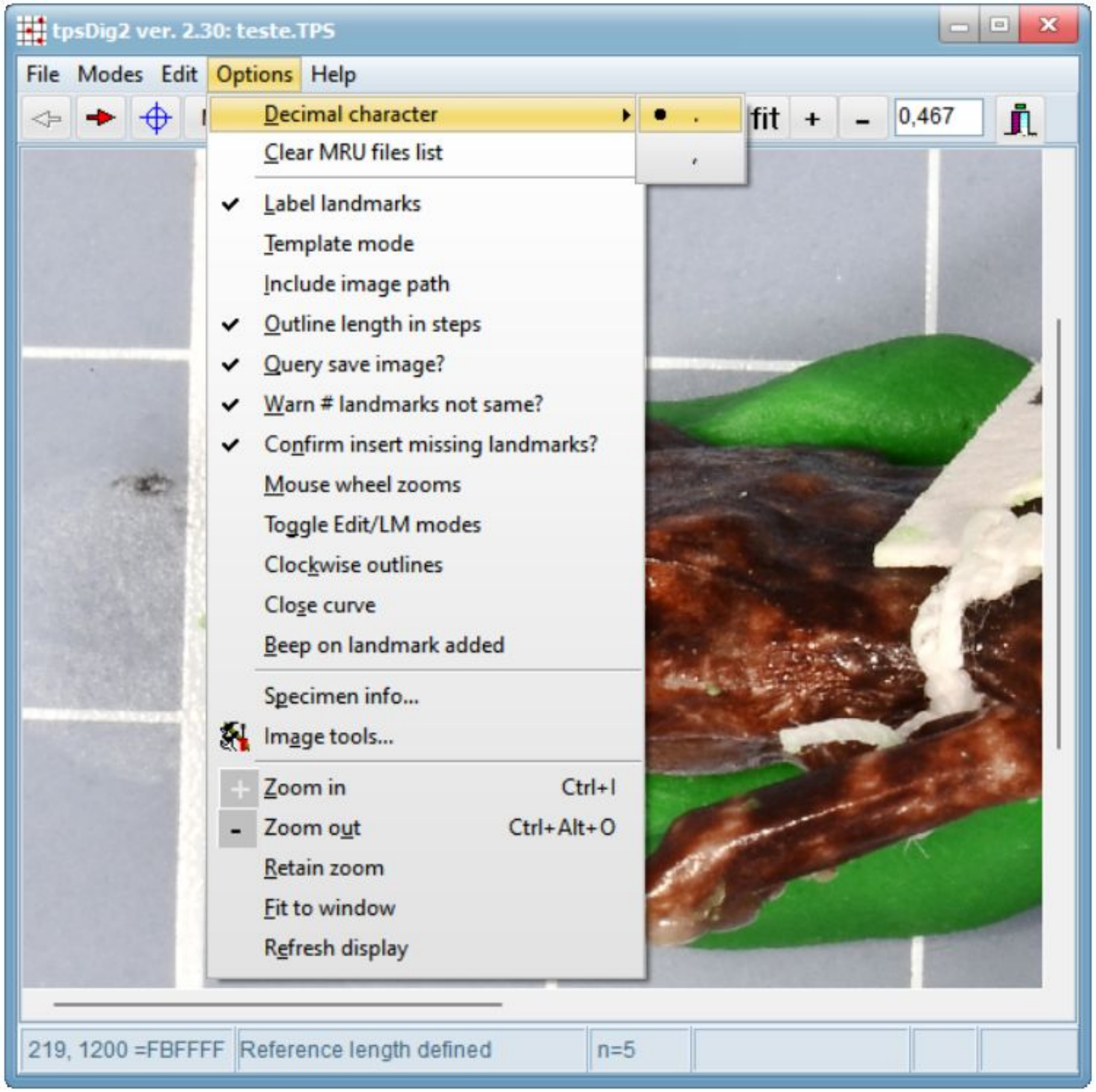


tpsDig



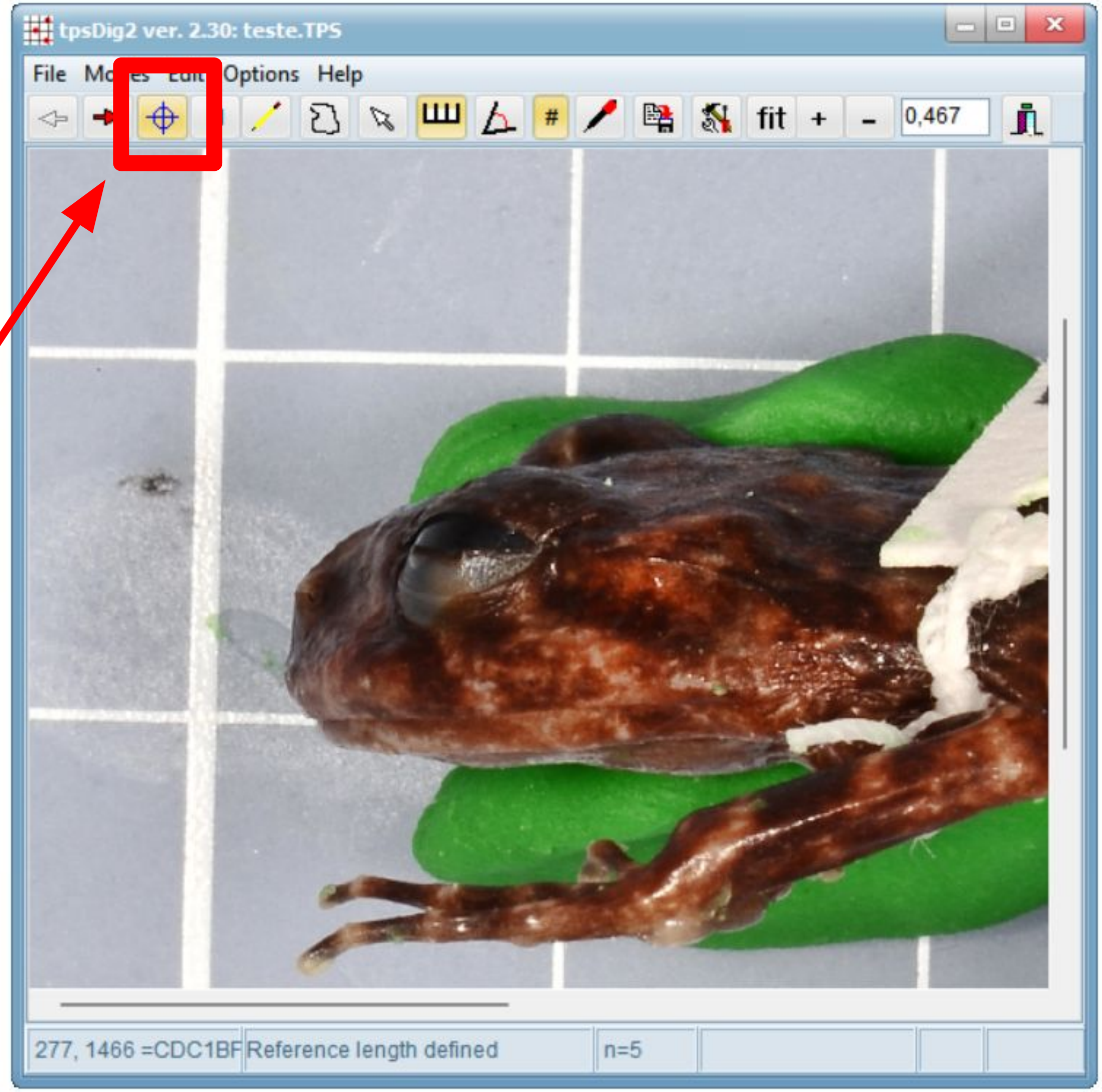
Repetir o processo de atribuição da escala a cada nova foto

tpsUtil



tpsDig

tpsUtil



landmarks
(pontos)

tpsDig

tpsUtil

landmarks (pontos)

semilandmarks (curvas)

Quando pronto:
File > Save Data

tpsDig

Exemplo

Conhecendo o geomorph, um dos pacotes do R para morfometria geométrica

Agora, vamos pro