1.a) Find H of $J(\theta)$ and show that $z^T . H . z \geq 0$ for any z.

Average Impirical loss for logistic Regression

$$J(\theta) = \frac{-1}{n} \sum_{i=1}^{n} y^{(i)} . \log(h_\theta(x^{(i)})) + (1 + y^{(i)}) . \log(1 - h_\theta(x^{(i)}))$$

where:

$$h_\theta = g(\theta^T . x) \quad \text{and} \quad g(z) = \frac{1}{1 + e^{-z}}$$

each element of H:

$$H_{jK} = (\nabla_\theta^2 J(\theta))_{jK} = \frac{\partial^2 J(\theta)}{\partial \theta_j . \partial \theta_K}$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{-1}{n} \sum_{i=1}^{n} (y^{(i)} - g(\theta^T . x^{(i)})) . x_j^{(i)}$$

$$\frac{\partial^2 J(\theta)}{\partial \theta_j \partial \theta_K} = \frac{+1}{n} \sum_{i=1}^{n} g(\theta^T . x^{(i)}) . (1 - (\theta^T . x^{(i)})) . x_j^{(i)} . x_K^{(i)}$$

for any $z \in \mathbb{R}^n$:

$$z^T . H . z = \frac{1}{n} \sum_{i=1}^{n} \sum_{j,K=1}^{d} g(\theta^T . x^{(i)})(1 - g(\theta^T . x^{(i)})) . x_j^{(i)} . x_K^{(i)} . z_j . z_K$$

$$= \frac{1}{n} \sum_{i=1}^{n} g(\theta^T . x^{(i)})(1 - g(\theta^T . x^{(i)})) . ((x^{(i)})^T . z)^2 \geq 0$$

$\therefore H \succeq 0$