iii) Assume each $y^{(i)}$ is drawn from a Gaussian distribution with mean $\theta^T . x^{(i)}$ and variance $(\sigma^{(i)})^2$; σ is constant.

$$p(y^{(i)} \mid x^{(i)} ; \theta) = \frac{1}{\sqrt{2\pi\sigma^{(i)}}} \exp\left(- \frac{(y^{(i)} - \theta^T . x^{(i)})^2}{2.(\sigma^{(i)})^2} \right)$$

Show that finding the maximum likelihood estimate of θ reduces to solving a weighted linear regression.

$$\ell(\theta) = \sum_{i=1}^{n} \log p(y^{(i)} \mid x^{(i)} ; \theta)$$

$$= \sum_{i=1}^{n} \Big(- \log \sqrt{2\pi} - \log \sigma^{(i)} \underbrace{- \frac{1}{2(\sigma^{(i)})^2} . (y^{(i)} - \theta^T . x^{(i)})^2}_{J(\theta)} \Big)$$

this is equivalent to minimize $J(\theta) = \sum_{i=1}^{m} \frac{1}{2} . \frac{1}{\sigma^2} (y^{(i)} - \theta^T . x^{(i)})^2$

so, here $w^{(i)} = \frac{1}{\sigma^2}$