Short Paper

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Abstract

This is the abstract.

It consists of two paragraphs.

Methods

Spatial Autocorrelation and Map Pattern

Spatial autocorrelation is a condition whereby the value of a variable at one location is correlated with the value(s) of the same variable at one or more proximal locations. A tool widely used to measure spatial autocorrelation is Moran's coefficient of autocorrelation, or MC for short. In matrix form, MC can be formulated as follows:

$$MC = \frac{n}{\sum_i \sum_j w_{ij}} \frac{x'Wx}{x'x} \tag{1}$$

where x is a vector $(n \times 1)$ of mean-centered values of a georeferenced variable, and W is a spatial weights matrix of dimensions $(n \times n)$ with elements w_{ij}. The elements of the spatial weights matrix take non-zero values if locations i and j are deemed to be spatially proximate in some sense, and 0 otherwise. It can be appreciated that the coefficient is composed to two elements: the variance of the random variable (i.e., $(x'x)/n$) and its spatial autocovariance $\frac{(x'Wx)}{\sum_i \sum_j w_{ij}}$. As an alternative, the numerator of the right-hand term of Equation 1 can be expressed as follows:

$$x'\Big(I - \frac{11'}{n}\Big)W\Big(I - \frac{11'}{n}\Big)x \tag{2}$$

with I as the identity matrix of size $n \times n$ and 1 a conformable vector of ones.

One possible interpretation of spatial autocorrelation is as map pattern. More concretely, the eigenvalues of the following matrix represent the range of possible values of MC given a spatial weights matrix W, and the extreme eigenvalues

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are in fact associated with the minimum and maximum values of MC for the system of relationships represented by W:

$$\left(I - \frac{11'}{n}\right) W \left(I - \frac{11'}{n}\right) \tag{3}$$

A remarkable discovery is that the eigenvectors associated with the eigenvalues of the matrix in Expression 3 represent a catalogue of latent map patterns, each with a level of autocorrelation (as measured by MC) given by its corresponding eigenvalue. Furthermore, the patterns represented by the eigenvectors are orthogonal by design, and so they furnish n maps that are independent from each other. Since these map patterns depend only on the spatial weights matrix – and not the spatial random variable – they constitute an extensive set of latent map patterns that can be used in regression analysis as filters. This is explained next.

References