Short Paper

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Abstract

This is the abstract.

It consists of two paragraphs.

Introduction

The economy of a nation is tied to its consumption of energy, since every process of production requires energy as an input. However, the strength of the relationship between the economy and the consumption of energy varies. Some countries (e.g., Japan) were more successful than others in terms of decoupling their productive processes from energy after the oil shocks of the 1970s. This was achieved by increasing the efficiency of production, so that the same output could be produced using less energy, or in somewhat different terms, by improving their energy intensity.

The relationship between economic output and energy consumption is of interest at a time when the effects of a carbon-intense economy is creating a heavy environmental burden. A relevant question is, what countries are more energy-efficient, and can we learn from them. To explore this question we will consider data on national energy use (in barrels of oil per day), economic output (GDP), and CO_2 emissions.

Figure **??** is a scatterplot of energy consumption to GPD. It can be seen that in general, greater economic output is associated with greater consumption of energy. However, there are some important differences. If we fit a regression line to this relationship, the line would indicate the *expected* economic output for a given level of energy consumption. Points below the line would use more energy for a lower level of economic output than expected, whereas points above the line would represent greater economic output than expected, given their energy consumption.

The regression line is estimated as follows:

$$GDP_i = \beta_0 + \beta_1 \text{bblpd}_i + \epsilon_i$$

This is a linear model and can be estimated using ordinary least squares. The scatterplot of energy to GDP with this line is shown in Figure **??**. Clearly,

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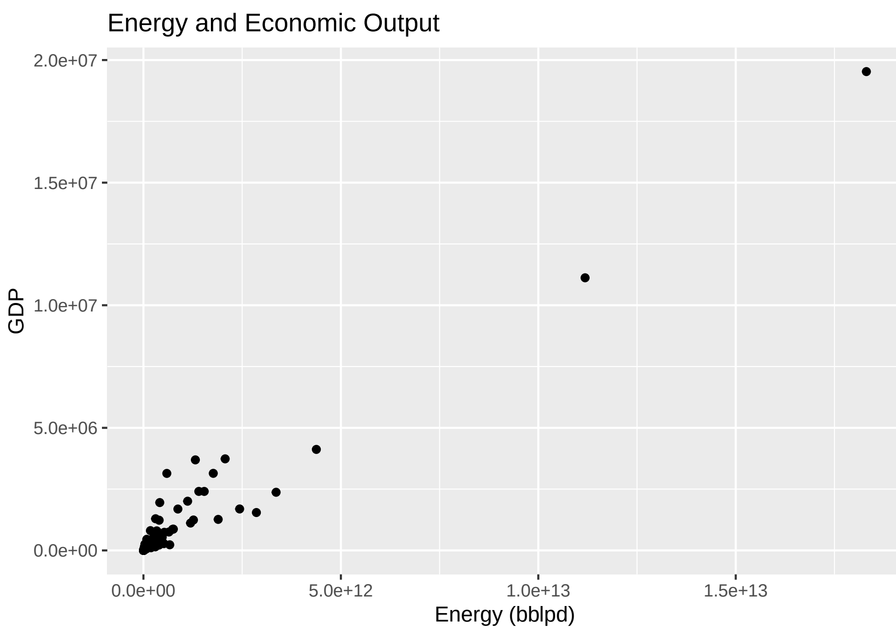

Figure 1: The relationship between energy consumption and economic output by world countries

some countries are more efficient than others in that they can produce more with less energy. The more a point deviates from the regression line, the more efficient that economy is. It would be interesting to

Figure **??** shows Figures **??** and **??** side by side.

Figure **??** shows Figures **??** and **??** as a two-panel figure with one column and two rows.

Methods

Spatial Autocorrelation and Map Pattern

Spatial autocorrelation is a condition whereby the value of a variable at one location is correlated with the value(s) of the same variable at one or more proximal locations. A tool widely used to measure spatial autocorrelation is Moran's coefficient of autocorrelation, or MC for short. In matrix form, MC can be formulated as follows:

$$MC = \frac{n}{\sum_i \sum_j w_{ij}} \frac{x'Wx}{x'x} \tag{1}$$

where x is a vector $(n \times 1)$ of mean-centered values of a georeferenced variable, and W is a spatial weights matrix of dimensions $(n \times n)$ with elements w_{ij}. The elements of the spatial weights matrix take non-zero values if locations i and

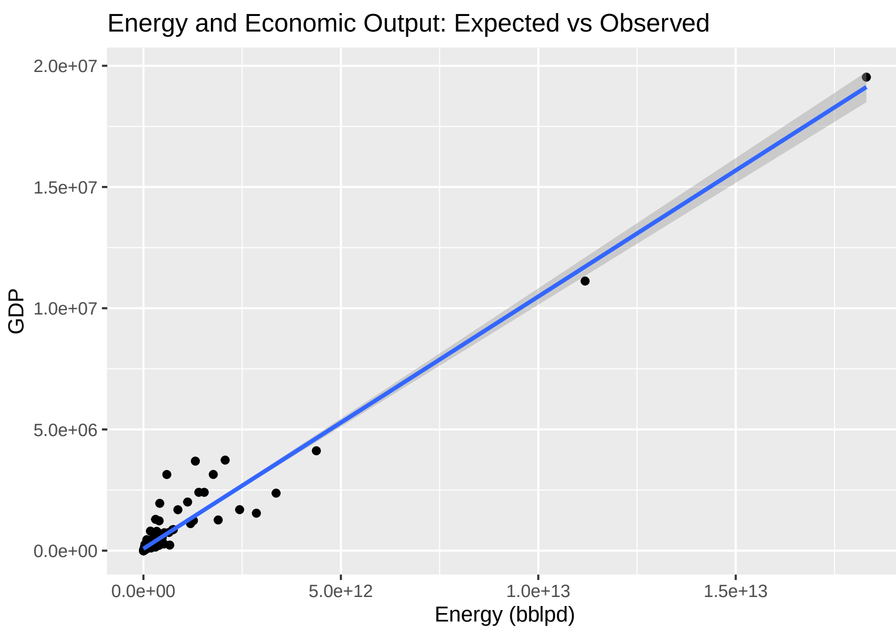

Figure 2: A regression line gives the expected values of GDP given energy consumption

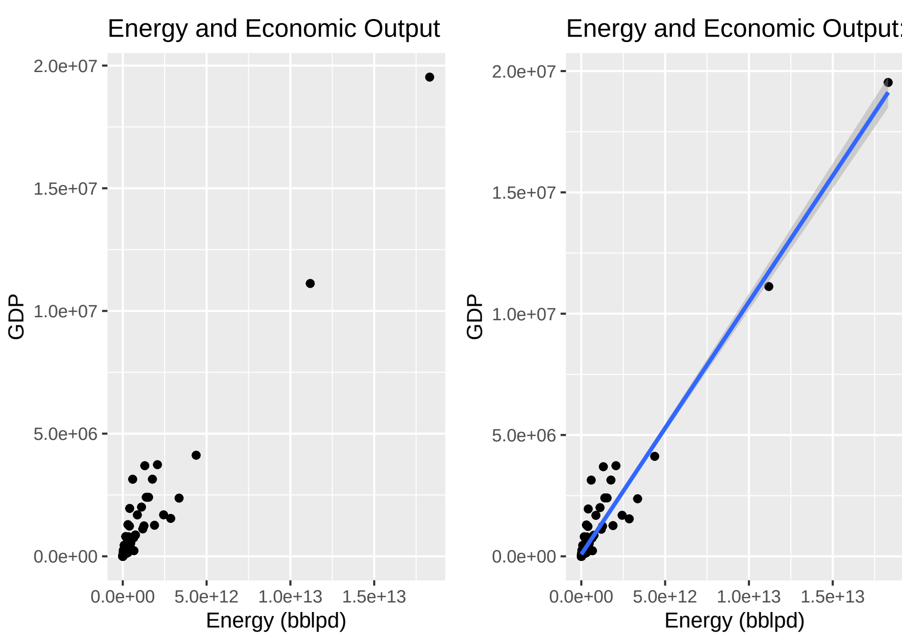

Figure 3: Two plots in a single figure; left panel is Figure 1 and right panel is Figure 2

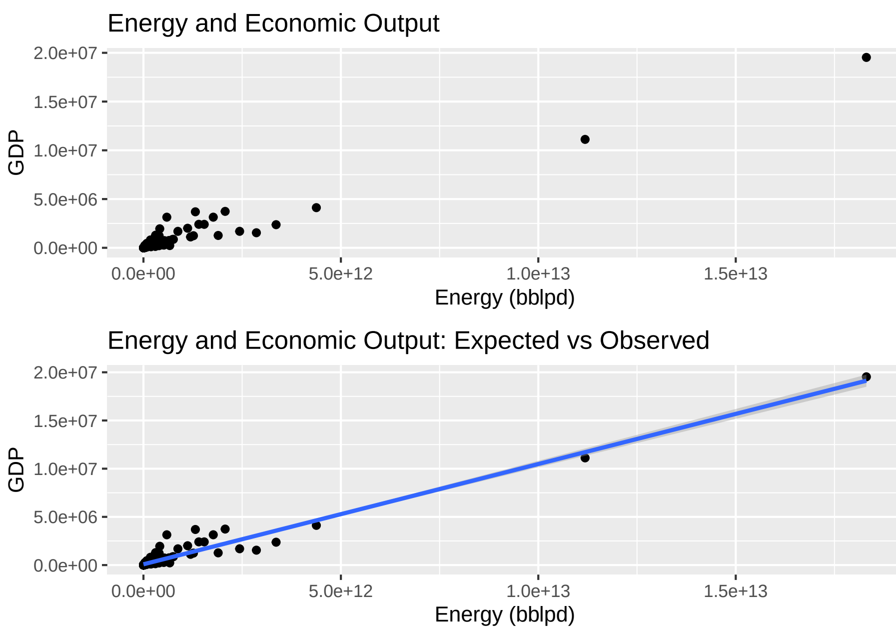

Figure 4: Two plots in a single figure; top panel is Figure 1 and bottom panel is Figure 2

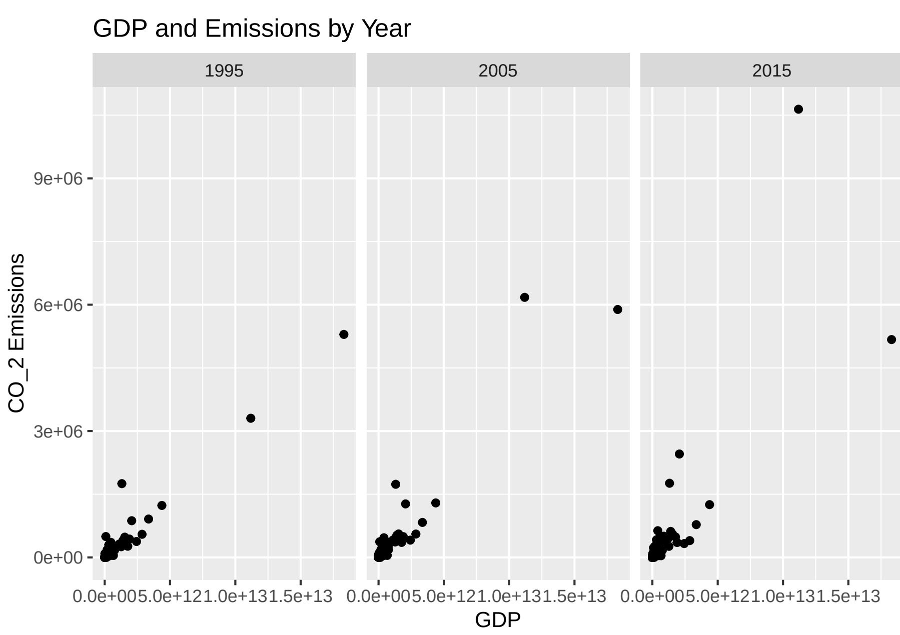

Figure 5: CO_2 emissions versus GDP by year

j are deemed to be spatially proximate in some sense, and 0 otherwise. It can be appreciated that the coefficient is composed to two elements: the variance of the random variable (i.e., $(x'x)/n$) and its spatial autocovariance $\frac{(x'Wx)}{\sum_i \sum_j w_{ij}}$. As an alternative, the numerator of the right-hand term of Equation **??** can be expressed as follows:

$$x'\Big(I - \frac{11'}{n}\Big)W\Big(I - \frac{11'}{n}\Big)x \tag{2}$$

with I as the identity matrix of size $n \times n$ and 1 a conformable vector of ones.

One possible interpretation of spatial autocorrelation is as map pattern. More concretely, the eigenvalues of the following matrix represent the range of possible values of MC given a spatial weights matrix W, and the extreme eigenvalues are in fact associated with the minimum and maximum values of MC for the system of relationships represented by W:

$$\Big(I - \frac{11'}{n}\Big)W\Big(I - \frac{11'}{n}\Big) \tag{3}$$

A remarkable discovery is that the eigenvectors associated with the eigenvalues of the matrix in Expression **??** represent a catalogue of latent map patterns, each with a level of autocorrelation (as measured by MC) given by its corresponding eigenvalue. Furthermore, the patterns represented by the eigenvectors are orthogonal by design, and so they furnish n maps that are independent from each other. Since these map patterns depend only on the spatial weights matrix – and not the spatial random variable – they constitute an extensive set of latent map patterns that can be used in regression analysis as filters. This is explained next.

References