Computational Fluid Dynamics
HW2

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Nomenclature

γ	ratio of specific heats
κ	coefficient of thermal conductivity
μ	coefficient of viscodity
ρ	fluid density
E	inviscid convective vector
e	total energy
E_v	viscous convective vector
p	pressure
Q	conservation state space
R	gas constant
T	temperature
t	time
u	fluid velocity
x	spatial coordinate

1 Problem Definition

1.1 Governing Equations

Consider the one-dimensional Navier-Stokes Equations:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = \frac{\partial E_v}{\partial x} \tag{1}$$

Where:

$$Q = \begin{Bmatrix} \rho \\ \rho u \\ e \end{Bmatrix}, \quad E = \begin{Bmatrix} \rho u \\ p + \rho u^2 \\ (e+p)\,u \end{Bmatrix}, \quad E_v = \begin{Bmatrix} 0 \\ \frac{4}{3}\mu \frac{\partial u}{\partial x} \\ \frac{4}{3}\mu u \frac{\partial u}{\partial x} - \kappa \frac{\partial T}{\partial x} \end{Bmatrix} \tag{2}$$

$$p = (\gamma - 1)\left(e - \frac{1}{2}\rho u^2\right), \qquad T = \frac{p}{\rho R},$$

$$\mu = 1.458 \cdot 10^{-6} \frac{T^{\frac{3}{2}}}{T + 110.4}, \quad \mu = 2.495 \cdot 10^{-3} \frac{T^{\frac{3}{2}}}{T + 194}$$

The constants are:

- $\gamma = 1.4$ for air under standard atmospheric conditions
- $R = 287.0$ for air

1.2 Physical Domain

The physical domain is a tube extended between $x = 0.2$ and $x = 1.0$. At both ends there are impermeable walls.

1.3 Initial Conditions

The initial conditions are showen in Fig.1:

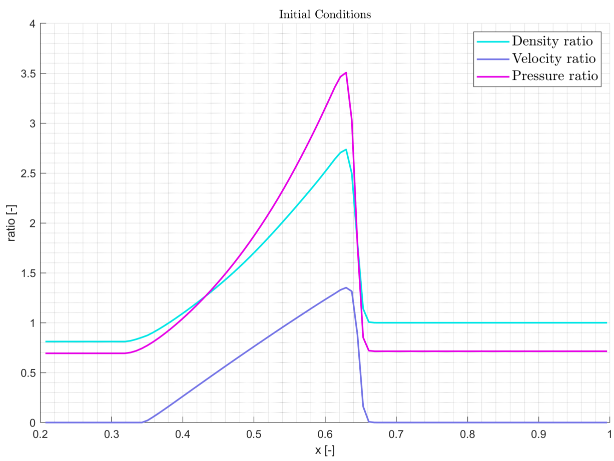

Figure 1: Initial conditions

1.4 Boundary Conditions

On each side of the tube there is an adiabatic, soild wall boundary conditions.

2 The Numerical Shemes

2.1 Boundary Conditions

In order to set the boundary conditions on the edge faces we will define ghost cells that will be calculated like so:

$$\begin{aligned} u_{(i=0)} &= -u_{(i=1)} \\ u_{(i=N+1)} &= -u_{(i=N)} \end{aligned} \tag{3}$$

in order to mentain velocity zero on the boundary and like so:

$$\begin{aligned} T_{(i=0)} &= T_{(i=1)} \\ T_{(i=N+1)} &= T_{(i=N)} \end{aligned} \tag{4}$$

in order to mentain adiabatic boundary conditions.