Computational Fluid Dynamics
HW1

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1   Inviscid Burgers Equation

The Inviscid Burgers equation, in conservation law form, is given by:

$$\frac{\partial u}{\partial t} + \frac{\partial F}{\partial x} = 0 \quad F = F_{(u)} = \frac{u^2}{2} \tag{1}$$

In non-conservation law form, is given by:

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0 \quad A = \frac{\partial F}{\partial u} = u \tag{2}$$

The equation is obtained by neglecting the viscous term from the viscous Burger equation.

1.1   Boundary and Initial Conditions

$$\begin{aligned} u_{(x=0,t)} &= 1.0 \\ u_{(x=1,t)} &= u_1 \\ u_{(x,t=0)} &= 1 - (1 - u_1)\cdot x \end{aligned} \tag{3}$$

1.2   First Order Roe Method ($u_1 = 0.0$)

Roe scheme is based on the solution of the linear problem:

$$\frac{\partial u}{\partial t} + \bar{A}\frac{\partial u}{\partial x} = 0 \tag{4}$$

Where $\bar{A}$ is a constant matrix that is dependent on local conditions. The matrix is constructed in a way to guarantee unifrom validity across discontinuities:

1. For any u_i, u_{i+1}:
$$F_{i+1} - F_i = \bar{A}\cdot(u_{i+1} - u_i)$$

2. When $u = u_i = u_{i+1}$ then:

$$\bar{A}_{(u_i,u_{i+1})} = \bar{A}_{(u,u)} = \frac{\partial F}{\partial u} = u$$

In case of the Burgers equation, the matrix $\bar{A}$ is a scalar, namely, $\bar{A} = \bar{u}$. The equation becomes:

$$\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} = 0 \tag{5}$$

The value of $\bar{u}$ for the cell face between i and $i+1$ is determined from the first conditions:

$$\bar{u} = \bar{u}_{i+\frac{1}{2}} = \frac{F_{i+1} - F_i}{u_{i+1} - u_i} = \frac{\frac{1}{2}u_{i+1}^2 - \frac{1}{2}u_i^2}{u_{i+1} - u_i} = \tag{6}$$

1.3   Second Order Roe ($u_1 = 0.5$)

1.3.1   With Limiters

1.3.2   Without Limiters

2   Generalized Burgers Equation