Technion - Israel Institute of Technology
Faculty of Aerospace Engineering
Numerical Methods in Transonic Flows
Exercise no. 0

Problem Definition

Consider the following differential equation:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \mu\frac{\partial^2 u}{\partial y^2} \\ \mu &= 1.0\end{aligned}$$

The equation is obtained by neglecting the convective term in Burgers equation.

Solution Method

Solve the equation using the following implicit/explicit method:

$$\left(I - \frac{\alpha\Delta t\mu}{\Delta y^2}\delta_{yy}\right)\Delta u_i^n \quad = \quad \frac{\mu\Delta t}{\Delta y^2}\delta_{yy}u_i^n$$

where $\alpha = 0$ corresponds to an explicit method, $\alpha = 0.5$ corresponds to the Crank-Nicolson scheme and $\alpha = 1$ corresponds to a fully implicit (Euler Implicit) scheme. You may use 11 grid points.

Boundary and Initial Conditions

Use the following boundary conditions:

$$\begin{aligned} u\,(y=0) &= 0.0 \\ u\,(y=1) &= 1.0 \end{aligned}$$

and the following initial conditions:

$$u\,(t=0) = 1.0$$

Case Studies

Use the program to solve for the following cases:

1. $\alpha = 0$, choose Δt appropriately and then verify the stability limit. Examine the stability and convergence behavior as you vary the time step.

2. $\alpha = 0.5$, choose a small to moderate Δt and a very large Δt. Explain the results.

3. $\alpha = 1$, Δt should be very large.

Short Report Preparation Instructions

The report should include the following:

1. A short description of the mathematical problem.

2. A short description of the numerical scheme and how it is developed.

3. Stability analysis of the scheme.

4. Results (plots, NOT pages with meaningless numbers).

5. Discussion and conclusions.

6. The computer program.

In any event it has to be a SHORT report.