Computational Fluid Dynamics
HW2

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Nomenclature

Δx	size of each cell in the domain
γ	ratio of specific heats
κ	coefficient of thermal conductivity
μ	coefficient of viscodity
ρ	fluid density
E	inviscid convective vector
e	total energy
E_v	viscous convective vector
L	characteristic length
p	pressure
Q	conservation state space
R	gas constant
T	temperature
t	time
u	fluid velocity
x	spatial coordinate
x_F	x coordinate of the end of the domain
x_i	x coordinate of the i-th cell

1 Problem Definition

1.1 Governing Equations

Consider the one-dimensional Navier-Stokes Equations:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = \frac{\partial E_v}{\partial x} \tag{1}$$

Where:

$$Q = \begin{Bmatrix} \rho \\ \rho u \\ e \end{Bmatrix}, \quad E = \begin{Bmatrix} \rho u \\ p + \rho u^2 \\ (e+p)\,u \end{Bmatrix}, \quad E_v = \begin{Bmatrix} 0 \\ \frac{4}{3}\mu \frac{\partial u}{\partial x} \\ \frac{4}{3}\mu u \frac{\partial u}{\partial x} - \kappa \frac{\partial T}{\partial x} \end{Bmatrix} \tag{2}$$

$$p = (\gamma - 1)\left(e - \frac{1}{2}\rho u^2\right), \qquad T = \frac{p}{\rho R},$$

$$\mu = 1.458 \cdot 10^{-6} \frac{T^{\frac{3}{2}}}{T + 110.4}, \quad \mu = 2.495 \cdot 10^{-3} \frac{T^{\frac{3}{2}}}{T + 194}$$

The constants are:

- $\gamma = 1.4$ for air under standard atmospheric conditions
- $R = 287.0$ for air

1.2 Physical Domain

The physical domain is a tube extended between $x = 0.2$ and $x = 1.0$. At both ends there are impermeable walls.

1.3 Initial Conditions

The initial conditions are showen in Fig.1:

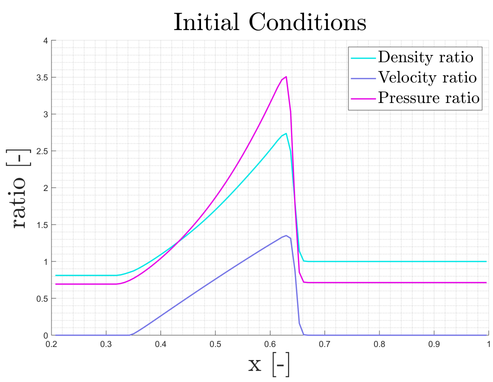

Figure 1: Initial conditions

1.4 Boundary Conditions

On each side of the tube there is an adiabatic, soild wall boundary conditions.

$$u_{(x=0.2)} = u_{(x=1.0)} = 0 \quad \Big\| \quad \left.\frac{\partial p}{\partial x}\right|_{x=0.2} = \left.\frac{\partial p}{\partial x}\right|_{x=1.0} = 0 \quad \Big\| \quad \left.\frac{\partial T}{\partial x}\right|_{x=0.2} = \left.\frac{\partial T}{\partial x}\right|_{x=1.0} = 0$$

2 Normalizing The Navier-Stokes Equations

Since the initial conditions are normalized, there is a need to normalize the N-S equations. We will use the following normalizations:

$$\rho = \rho_\infty \tilde{\rho}, \quad u = a_\infty \tilde{u}, \quad p = \gamma p_\infty \tilde{p}, \quad T = \gamma T_\infty \tilde{T}, \quad x = L\tilde{x}, \quad t = \tfrac{L}{a_\infty}\tilde{t}, \quad \mu = \mu_\infty \tilde{\mu}, \quad \kappa = \kappa_\infty \tilde{\kappa} \tag{3}$$

The normalization of the temperature was choosen to cancel out the γ in the normalization of the pressure:

$$\begin{aligned} p &= \rho R T \\ \gamma p_\infty \tilde{p} &= \rho_\infty \tilde{\rho} R \gamma T_\infty \tilde{T} \\ \tilde{p} &= \tilde{\rho}\tilde{T} \end{aligned} \tag{4}$$

3 The Computational Domain

3.1 Discretization

The physical domain $[x_I, x_F]$ is discretizes into N equispaced cells. The size of each cell is there for:

$$\Delta x = \frac{x_F - x_I}{N} = \frac{L}{N} \tag{5}$$

so the x coordinate of the i-th cell x_i is:

$$x_i = x_I + \frac{1}{2}\Delta x + \Delta x \cdot (i-1) \qquad \text{when starting from } i = 1 \tag{6}$$

3.2 Boundary Conditions

In order to set the boundary conditions on the edge faces we will define ghost cells that will be calculated like so:

$$\begin{aligned} u_{(i=0)} &= -u_{(i=1)} \\ u_{(i=N+1)} &= -u_{(i=N)} \end{aligned} \tag{7}$$

in order to mentain velocity zero on the boundary and like so:

$$\begin{aligned} T_{(i=0)} &= T_{(i=1)} \\ T_{(i=N+1)} &= T_{(i=N)} \end{aligned} \tag{8}$$

in order to mentain adiabatic boundary conditions. Since the gradient of the pressure on the wall is zero, we get:

$$\begin{aligned} p_{(i=0)} &= p_{(i=1)} \\ p_{(i=N+1)} &= p_{(i=N)} \end{aligned} \tag{9}$$

From equations 2, 8, and 9 we can conclude:

$$\begin{aligned} \rho_{(i=0)} &= \rho_{(i=1)} \\ \rho_{(i=N+1)} &= \rho_{(i=N)} \end{aligned} \tag{10}$$

and from equations 2, 7, 9, and 10 we can conclude:

$$\begin{aligned} e_{(i=0)} &= e_{(i=1)} \\ e_{(i=N+1)} &= e_{(i=N)} \end{aligned} \tag{11}$$

4 The Numerical Schemes

4.1 First Order Approximate Riemann Roe Method

4.2 First Order Steger-Warming – Explicit

4.3 First Order Steger-Warming – Implicit