Technion - Israel Institute of Technology
Faculty of Aerospace Engineering
Numerical Methods in Transonic Flows
Exercise no. 2

1 Problem Definition

1.1 Governing Equations

Consider the one-dimensional Navier-Stokes Equations:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = \frac{\partial E_v}{\partial x} \tag{1}$$

where

$$Q = \left\{ \begin{array}{c} \rho \\ \rho u \\ e \end{array} \right\} \tag{2}$$

The term ρ signifies the fluid density, u is the fluid velocity, and e denotes the total energy. The inviscid convective vector is given by:

$$E = \left\{ \begin{array}{c} \rho u \\ p + \rho u^2 \\ (e + p)\, u \end{array} \right\} \tag{3}$$

where is p is the pressure, given by:

$$p = (\gamma - 1)\left(e - \frac{1}{2}\rho u^2\right) \tag{4}$$

and γ is the (constant) ratio of specific heats ($\gamma = 1.4$ for air under standard atmospheric conditions). The viscous convective vector is:

$$E_v = \left\{ \begin{array}{c} 0 \\ \frac{4}{3}\mu\frac{\partial u}{\partial x} \\ \frac{4}{3}\mu u\frac{\partial u}{\partial x} - \kappa\frac{\partial T}{\partial x} \end{array} \right\} \tag{5}$$

where T is the temperature given by:

$$T = \frac{p}{\rho R} \tag{6}$$

and R is the gas constant ($R = 287.0$ for air). The coefficients of viscosity, μ, and thermal conductivity, κ are given by the Sutherland formulae as follows:

$$\begin{aligned} \mu &= 1.458 \times 10^{-6}\frac{T^{\frac{3}{2}}}{T + 110.4} \\ \kappa &= 2.495 \times 10^{-3}\frac{T^{\frac{3}{2}}}{T + 194} \end{aligned} \tag{7}$$

1.2 Physical Domain

The domain is a tube extended between $x = 0.2$ and $x = 1.0$. At both ends there are impermeable walls.

2 Boundary and Initial Conditions

2.1 Initial Conditions

The initial conditions are given by an attached text file. Figure 1 shows the initial conditions.

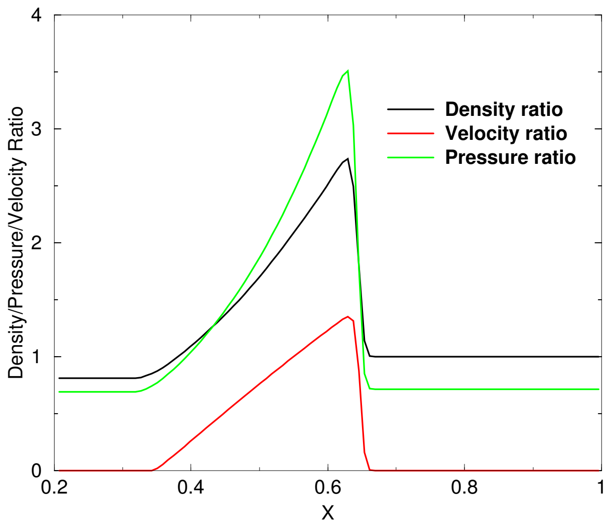

Figure 1: Initial conditions

2.2 Boundary Conditions

Use the appropriate, adiabatic, solid wall boundary conditions.

3 Solution Method

Solve the equations using the following methods:

1. First order approximate Riemann Roe/HLLC/AUSM method (explicit, choose one)

2. First order Steger-Warming (explicit)

3. First order Steger-Warming (Implicit)

4 Computational Mesh

Choose 101 points and compute until the shock wave hits both walls.

5 Comparisons

Compare between flux difference splitting (Roe/HLLC/AUSM) and flux vector splitting (Steger-Warming), explicit and Implicit (Steger-Warming), and viscous and inviscid for at least one case.

6 Helper Program

A 3×3 block tri-diagonal solver and a test program are also attached.