Technion - Israel Institute of Technology
Faculty of Aerospace Engineering
Numerical Methods in Transonic Flows
Exercise no. 1

1 Inviscid Burgers Equation

Consider the inviscid Burgers equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

The equation is obtained by neglecting the viscous term from the viscous Burgers equation.

1.1 Boundary and Initial Conditions

Use the following boundary conditions:

$$\begin{aligned} u(x=0,t) &= 1.0 \\ u(x=1,t) &= u_1 \end{aligned}$$

and the following initial conditions:

$$u(x,t=0) = 1-(1-u_1)*x$$

1.2 Solution Methods

Solve the equation using the following methods:

1. First order Roe method ($u_1 = 0.0$).
2. Second order Roe with and without limiters ($u_1 = 0.5$).

1.3 Case Studies

Use the program to study the following effects:

1. Effect of CFL number on the solution.
2. Effect of limiter (where applicable).

2 Generalized Burgers Equation

Consider the generalized Burgers equation:

$$\begin{aligned} \frac{\partial u}{\partial t} + (c + bu)\frac{\partial u}{\partial x} &= \mu \frac{\partial^2 u}{\partial x^2} \\ c &= \frac{1}{2} \\ b &= -1 \\ \mu &= 0.25 \\ \mu &= 0.001 \end{aligned}$$

2.1 Initial Conditions

The initial conditions are given by:

$$u = \frac{1}{2}\left\{1 + \tanh\left[250\left(x - 20\right)\right]\right\}$$

Boundary Conditions

Use Dirichlet boundary conditions (based on the initial conditions).

2.2 Solution Method

Solve the equation using the following methods:

1. First order Roe method (explicit)
2. MacCormack method
3. First and second order Beam and Warming, with and without smoothing.

2.3 Domain and Computational Mesh

Choose 41 points with $\Delta x = 1$ and compute until t = 18.0. Solve for $\Delta t = 1.0$ and $\Delta t = 0.5$, and compare with the exact stationary solution for $\mu = 0.25$ and $\mu = 0.001$

2.4 Case Studies

Use the program to study the following effects:

1. Effect of smoothing.
2. Effect of time step.