Homework 4: Jet Dynamics Analysis (Numerical HW)

The behavior of a jet exiting a nozzle is a common phenomenon observed in everyday life, from simple applications such as faucets to advanced engineering systems like propulsion devices. Understanding and predicting the jet's shape and dynamics is essential for various practical and theoretical applications.

1. A liquid jet is illustrated in figure 1. Given the parameters $\sigma, \rho, P_0, U_0, g$ derive the general expression for the velocity profile $U(z)$, as a function of z, without neglecting the term $\frac{1}{R_2}$. You may assume that the jet is incompressible and inviscid.
2. Formulate an ordinary differential equation governing the jet radius $r(z)$. Write all the assumptions you make and clearly define and justify the boundary conditions you assume, provide a physical explanation for each.
3. Write a numerical solver for the derived ODE using the specified boundary conditions and the physical properties of water. Explain the discretization and numerical method you developed. (Do not use available functions such as ode45)
4. Assume the following jet parameters: $U_0 = 1\frac{m}{s}$, $a = 5\ cm$ and solve the problem for a *water jet in air* (in standard atmosphere at sea level) using the numerical method developed.
5. Compare your numerical results with those obtained using the approximated method developed in the lecture, where the simplification $R_2 \to \infty$ was applied. Discuss the differences and highlight any significant deviations.
6. Examine whether your numerical solution exhibits oscillatory behavior.

- If oscillations are present, discuss the potential causes. Analyze whether these oscillations are significant or negligible in your specific case (and numerical scheme).
- Add the code to your report.

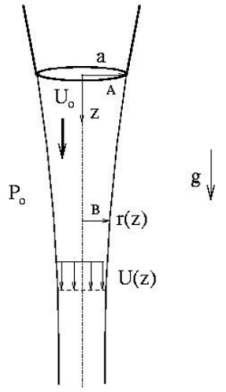

Figure 1: Schematic of a fluid jet extruded from an orifice of radius a, accelerates under the influence of gravity.