Problem Set 2

Intro to Turb. Flow
Released: *April 30, 2025*
Recommended Submission: *May 14, 2025*
Please note how much time each problem takes to complete.

1. Stationarity

Consider the following random processes

$$x_k = a_k \sin(2\pi f t + \theta) \tag{1}$$
$$x_k = a \sin(2\pi f t + \theta_k) \tag{2}$$
$$x_k = a_k \sin(2\pi f t + \theta_k) \tag{3}$$

where a_k is a uniformly distributed random variable on $[0, 1]$ and θ_k is uniformly distributed on $[0, 2\pi]$. Are these processes stationary? Are they ergodic? Why or why not?

2. Getting Closure

Consider a 'toy' version of the Navier Stokes equation, where instead of a differential equation, we use a difference equation:

$$u_{n+1} = r u_n (1 - u_n)$$

(a) What is the ensemble average, $\langle u \rangle$? (Hint: consider the limit of infinite 'realizations', $n \to \infty$ and assume this limit exists)

(b) What does the 'Reynolds stress' $\langle uu \rangle$ depend on?

(c) Why can't you solve for $\langle u \rangle$ directly? Make reasonable assumptions in order to solve for $\langle u \rangle$? (Hint: assume u has a 'convenient' PDF; also make sure to note the difference between central and raw moments.)

(d) Verify your solution for $\langle u \rangle$ numerically as a function of r. How close is it?

3. Scalar 'TKE' Equation

Consider the instantaneous transport equation for a passive scalar $\tilde{T}$:

$$\frac{\partial \tilde{T}}{\partial t} + \frac{\partial}{\partial x_j}(\tilde{T}\tilde{u}_j) = \alpha \frac{\partial^2 \tilde{T}}{\partial x_j^2}$$

We can derive an equivalent to the TKE for the temperature fluctuations, $\frac{1}{2}\overline{T^2}$, where $\tilde{T} = \overline{T} + T$, in order to describe the magnitude of the fluctuations. What is the rate at which these temperature fluctuations dissipate?