Problem Set 1

Intro to Turb. Flow
Released: *April 20, 2025*
Recommended Submission: *May 5, 2025*
Please note how much time each problem takes to complete.

1. Computing a Turbulent Flow

Consider a commercial airliner with a chord $L = 5\text{m}$ cruising at $U = 250\text{m/s}$.

(a) Estimate the thickness of the boundary layer, δ, at the trailing edge of the wing using Prandtl's one-seventh power law (from White's Viscous Fluid Flow, eq. 6-70):

$$\frac{\delta}{x} \approx \frac{0.16}{\text{Re}_x^{1/7}}$$

(b) What are the smallest dynamically important scales in the flow over the wing?

(c) I want to calculate this flow on my computer. How does the number of spatial grid points in a computational calculation, N, scale with Re_δ? Roughly how many grid points are needed to resolve the flow over the whole wing?

(d) How many time steps are needed to resolve the instantaneous flow over one eddy turnover time, δ/U?

2. Heating a Room

Consider trying to heat a room of characteristic dimension, L. The air has thermal diffusivity, α. There is a small heater in the corner of size h.

(a) What is the characteristic time scale for heating the room, assuming no air flow? Evaluate it for a typical bedroom.

(b) Imagine that the space heater induces motion via buoyancy effects as the air heats up, where the local temperature is T and the temperature difference between it and the newly heated air is ΔT. Under the Boussinesq approximation, we can write the momentum equation as:

$$\frac{D}{Dt}\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + g\frac{\Delta T}{T}$$

Assuming the buoyancy-driven flow is turbulent, use dimensional analysis to estimate the magnitude of the induced velocity, u, in the neighborhood of the heater.

(c) What is the characteristic time scale for heating the room? Evaluate it for reasonable values of the relevant parameters.

3. Help Karl Pearson

In 1905, the following request appeared in the journal Nature (vol 72, no 1865).

The Problem of the Random Walk.

Can any of your readers refer me to a work wherein I should find a solution of the following problem, or failing the knowledge of any existing solution provide me with an original one? I should be extremely grateful for aid in the matter.

A man starts from a point O and walks l yards in a straight line; he then turns through any angle whatever and walks another l yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r+\delta r$ from his starting point, O.

The problem is one of considerable interest, but I have only succeeded in obtaining an integrated solution for *two* stretches. I think, however, that a solution ought to be found, if only in the form of a series in powers of $1/n$, when n is large.

Karl Pearson.

The Gables, East Ilsley, Berks.

Lord Rayleigh eventually came to Mr. Pearson's rescue with a theoretical result for large values of n after which Mr. Pearson responded

The lesson of Lord Rayleigh's solution is that in open country the most probable place to find a drunken man who is at all capable of keeping on his feet is somewhere near his starting point!

Karl Pearson.

Can you help Mr. Pearson find a probability density function (PDF) for his random walk **numerically**? (You can decide how many steps n a drunk walker can take before he collapses, but assume $n \gg 1$).

(a) How many realizations do you need for your ensemble PDF to converge? Is it close to Lord Rayleigh's result of $\frac{2}{n}re^{-r^2/n}$ for $\ell = 1$?

(b) What is the average displacement, $\langle r \rangle$ of your walker?

(c) Take a single random walk realization: what is its autocovariance as a function of steps, $Q(\Delta\ell)$? its autocorrelation? How many walks do you need in the ensemble for the autocorrelation to converge?