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1 4 digits - MPxx

- $m = M/100$
- $p = P/10 = x_{mc}$
- $t = xx$ - The maximum thickness as chord percentage, so $t = 12$ means maximum thickness is 12% of the chord.
- The leading edge approximates a cylinder with a radius of $r = 1.1019t^2$.

$$y_c = \begin{cases} \dfrac{m}{p^2}\left(2px - x^2\right) & 0 \leq x < p \\ \\ \dfrac{m}{(1-p)^2}\left((1-2p) + 2px - x^2\right) & p \leq x \leq 1 \\ \end{cases} \tag{1}$$

$$\frac{dy_c}{dx} = \begin{cases} \dfrac{2m}{p^2}\left(p - x\right) & 0 \leq x < p \\ \\ \dfrac{2m}{(1-p)^2}\left(p - x\right) & p \leq x \leq 1 \\ \end{cases} \tag{2}$$

1.1 Surfaces Generation

Open airfoil:

$$y_t = 5t\left(0.969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right) \tag{3}$$

To close the airfoil, the sum of the coefficient needs to be equal to 1. Changing the last coefficient results in the smallest change to the overall shape of the airfoil.

$$y_t = 5t\left(0.969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\right) \tag{4}$$

The respectively upper and lower airfoil surface are given by:

$$\begin{array}{rclcrcl} x_U & = & x - y_t\sin(\theta) & & y_U & = & y_c + y_t\cos(\theta) \\ x_L & = & x + y_t\sin(\theta) & & y_L & = & y_c - y_t\cos(\theta) \end{array} \tag{5}$$

Where:

- $\theta = \arctan\left(\dfrac{dy_c}{dx}\right)$

2 5 digits - LPSxx

- $x_{mc} = 0.05P$.
- $CL_i = 0.15L$.
- $t = xx$ - The maximum thickness as chord percentage, so $t = 12$ means maximum thickness is 12% of the chord.
- The S digit is only between 1 and 5 (because the polynomial approximations).
- The parameters are calculated at $L = 2$, and linearly scaled for a different desired design lift coefficient CL_i as explained at Sec.2.3.

2.1 $S = 0$

$$y_c = \begin{cases} \frac{k_1}{6}\left(x^3 - 3rx^2 + r^2(3-r)x\right) & 0 \le x < r \\ \\ \frac{k_1 r^3}{6}(1-x) & r \le x \le 1 \end{cases} \tag{6}$$

$$\frac{dy_c}{dx} = \begin{cases} \frac{k_1}{6}\left(3x^2 - 6rx + r^2(3-r)\right) & 0 \le x < r \\ \\ -\frac{k_1 r^3}{6} & r \le x \le 1 \end{cases} \tag{7}$$

2.1.1 r

$$\begin{gathered} x_{mc} = r\left(1 - \sqrt{\frac{r}{3}}\right) \\ x_{mc} = r - \sqrt{\frac{r^3}{3}} \\ \sqrt{\frac{r^3}{3}} = r - x_{mc} \\ \frac{r^3}{3} = (r - x_{mc})^2 \\ \frac{r^3}{3} = r^2 - 2rx_{mc} + x_{mc}^2 \\ \frac{1}{3}r^3 - r^2 + 2rx_{mc} - x_{mc}^2 = 0 \\ \vdots \\ \text{solving numerically} \end{gathered} \tag{8}$$

2.1.2 k_1

$$k_1 = \frac{6}{N}CL_i \tag{9}$$

$$N = \frac{3r - 7r^2 + 8r^3 - 4r^4}{\sqrt{r - r^2}} - \frac{3}{2}(1 - 2r)\left(\frac{\pi}{2} - \arcsin(1 - 2r)\right) \tag{10}$$

2.1.3 fittings - L=2

P	$x_m c$	r	k_1
1	0.05	0.0580	361.40
2	0.10	0.1260	51.649
3	0.15	0.2025	15.957
4	0.20	0.290	6.643
5	0.25	0.391	3.23

$$\begin{aligned} r &= 3.333x_{mc}^3 + 0.7x_{cm}^2 + 1.197x_{cm} - 0.004 \\ k1 &= 1.5149e6x_{mc}^4 - 1.0877e6x_{mc}^3 + 2.8646e5x_{mc}^2 - 3.2968e4x_{mc} + 1.4202e3 \end{aligned} \tag{11}$$

2.2 $S = 1$

$$y_c = \begin{cases} \frac{k_1}{6}\left((x-r)^3 - \frac{k_2}{k_1}(1-r)^3 x - r^3 x + r^3\right) & 0 \le x < r \\ \frac{k_1}{6}\left(3(x-r)^2 - \frac{k_2}{k_1}(1-r)^3 - r^3\right) & r \le x \le 1 \end{cases} \tag{12}$$

$$\frac{dy_c}{dx} = \begin{cases} \frac{k_1}{6}\left(\frac{k_2}{k_1}(x-r)^3 - \frac{k_2}{k_1}(1-r)^3 x - r^3 x + r^3\right) & 0 \le x < r \\ \frac{k_1}{6}\left(3\frac{k_2}{k_1}(x-r)^2 - \frac{k_2}{k_1}(1-r)^3 - r^3\right) & r \le x \le 1 \end{cases} \tag{13}$$

2.2.1 fittings - L=2

P	$x_m c$	r	k_1	$\frac{k_2}{k_1}$
2	0.10	0.130	51.999	0.000764
3	0.15	0.217	15.793	0.00677
4	0.20	0.318	6.520	0.0303
5	0.25	0.441	3.191	0.1355

$$\begin{aligned} r &= 10.6667x_{mc}^3 - 2x_{cm}^2 + 1.7333cm - 0.034 \\ k_1 &= -2.7973e4x_{mc}^3 + 1.7973e4x_{cm}^2 - 3.8884e3x_{cm} + 289.076 \\ \frac{k_2}{k_1} &= 85.528x_{mc}^3 - 34.9828x_{cm}^2 + 4.8032x_{cm} - 0.2153 \end{aligned} \tag{14}$$

2.3 Scaling for Different L digit

In order to scale for L values different from 2, just multiply the y_c values:

$$\begin{aligned} y_{c\text{scaled}} &= \frac{L}{2} y_c \\ \frac{dy_c}{dx}_{\text{scaled}} &= \frac{L}{2}\frac{dy_c}{dx} \end{aligned} \tag{15}$$

2.4 Surfaces Generation

Open airfoil:

$$y_t = 5t\left(0.969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right) \tag{16}$$

To close the airfoil, the sum of the coefficient needs to be equal to 1. Changing the last coefficient results in the smallest change to the overall shape of the airfoil.

$$y_t = 5t\left(0.969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\right) \tag{17}$$

The respectively upper and lower airfoil surface are given by:

$$\begin{array}{rclrcl} x_U &=& x - y_t \sin(\theta) & y_U &=& y_c + y_t \cos(\theta) \\ x_L &=& x + y_t \sin(\theta) & y_L &=& y_c - y_t \cos(\theta) \end{array} \tag{18}$$

Where:

- $\theta = \arctan\left(\frac{dy_c}{dx}\right)$

References

[1] K. Ata, "Naca 5 digit airfoils." https://web.itu.edu.tr/~atares/courses/CA/3.1.2_NACA5.html, 2025.

[2] Wikipedia, "Naca airfoil." https://en.wikipedia.org/wiki/NACA_airfoil, 2025.

[3] C. L. Ladson, C. W. Brooks Jr, A. S. Hill, and D. W. Sproles, "Computer program to obtain ordinates for naca airfoils," tech. rep., 1996.

[4] E. N. Jacobs and R. M. Pinkerton, "Tests in the variable-density wind tunnel of related airfoils having the maximum camber unusually far forward," tech. rep., National Advisory Committee for Aeronautics, 1935.

[5] N. Eastman, E. Kenneth, and R. Pinkerton, "The characteristics of 78 related airfoil sections from tests in the variable-density wind tunnel," *NACA-report-460*, 1933.

A Code Examples

A.1 MatLab

```
clc; clear; close all

NACA = '23121';
num_of_points = 300;

% ========================

if length(NACA) == 4
    m = str2num(NACA(1))/100;
    p = str2num(NACA(2))/10;
    t = str2num(NACA(3:4))/100;
elseif length(NACA) == 5
    L = str2num(NACA(1));
    P = str2num(NACA(2));
    if P > 5 || P < 1
        fprintf('unsupported NACA: %s (LPSTT)!\n1<= S <=5\n', NACA)
    end
    S = str2num(NACA(3));
    t = str2num(NACA(4:5))/100;
end


delta_x = 1 / (num_of_points-1);
x         = zeros(num_of_points,1);
dy_c__dx = zeros(num_of_points,1);
y_t       = zeros(num_of_points,1);
theta     = zeros(num_of_points,1);
y_c       = zeros(num_of_points,1);
x_L       = zeros(num_of_points,1);
x_U       = zeros(num_of_points,1);
y_L       = zeros(num_of_points,1);
y_U       = zeros(num_of_points,1);
for i = 0:num_of_points-1
    x(i+1)= delta_x * i;
    y_t(i+1) = 5 * t * (0.2969 * sqrt(x(i+1)) - 0.1260 * x(i+1) - 0.3516 * x(i+1)^2 + 0.2843 * x(i+1)^3 - 0.1036 * x(i+1)^4);

    if length(NACA) == 4
        if p == 0 || m == 0
            x_U(i+1) = x(i+1);
            x_L(i+1) = x(i+1);
            y_U(i+1) = y_t(i+1);
            y_L(i+1) = -y_t(i+1);
        else
            if x(i+1) <= p
                y_c(i+1)      = m / p^2 * (2 * p * x(i+1) - x(i+1)^2);
                dy_c__dx(i+1) = m / p^2 * (p - x(i+1));
            else
                y_c(i+1)      = m / (1 - p)^2 * ((1 - 2 * p) + 2 * p * x(i+1) - x(i+1)^2);
                dy_c__dx(i+1) = 2 * m / (1 - p)^2 * (p - x(i+1));
            end
        end
    elseif length(NACA) == 5
```

```
        x_mc = 0.05*P;
        CL_i = 0.15*L;
        if S == 0
            r  = 3.3333*x_mc^3 + 0.7*x_mc^2 + 1.1967*x_mc - 0.0040;
            k1 = 1.5149e6*x_mc^4 - 1.0877e6*x_mc^3 + 2.8646e5*x_mc^2 - 3.2968e4*x_mc + 1.4202e3;

            if x(i+1) <= r
                y_c(i+1) = L/2 * (k1 / 6 * (x(i+1)^3 - 3 * r * x(i+1)^2 + r^2 * (3 - r) * x(i+1)));
                dy_c__dx(i+1) = L/2 * (k1 / 6 * (3 * x(i+1)^2 - 6 * r * x(i+1) + r^2 * (3 - r)));
            else
                y_c(i+1) = L/2 * (k1 * r^3 / 6 * (1 - x(i+1)));
                dy_c__dx(i+1) = - L/2 * k1 * r^3 / 6;
            end
        elseif S == 1
            r   =   10.6667*x_mc^3 -        2*x_mc^2 +   1.7333*x_mc - 0.0340;
            k1  = -2.7973e4*x_mc^3 + 1.7973e4*x_mc^2 - 3.8884e3*x_mc + 289.0760;
            k21 =   85.5280*x_mc^3 -  34.9828*x_mc^2 +   4.8032*x_mc - 0.2153;

            if x(i+1) <= r
                y_c(i+1) = L/2 * (k1 / 6 * ((x(i+1) - r)^3 - k21 * (1 - r)^3 * x(i+1) - r^3 * x(i+1) + r^3));
                dy_c__dx(i+1) = L/2 * (k1 / 6 * (3 * (x(i+1) - r)^2 - k21 * (1 - r)^3 - r^3));
            else
                y_c(i+1) = L/2 * (k1 / 6 * (k21 * (x(i+1) - r)^3 - k21 * (1 - r)^3 * x(i+1) - r^3 * x(i+1) + r^3));
                dy_c__dx(i+1) = L/2 * (k1 / 6 * (3 * k21 * (x(i+1) - r)^2 - k21 * (1 - r)^3 - r^3));
            end
        else
            fprintf('unable to create this NACA: %s, S is only 1 of 0\n', NACA);
        end
    elseif length(NACA) == 6
        fprintf('still not supporting NACA 6 digit\n');
    end
    theta(i+1) = atan(dy_c__dx(i+1));

    x_U(i+1) = x(i+1) - y_t(i+1) * sin(theta(i+1));
    x_L(i+1) = x(i+1) + y_t(i+1) * sin(theta(i+1));
    y_U(i+1) = y_c(i+1) + y_t(i+1) * cos(theta(i+1));
    y_L(i+1) = y_c(i+1) - y_t(i+1) * cos(theta(i+1));
end

fig1 = figure('Name','1', 'Position',[900,200,700,500]);
hold all
plot(x_U, y_U)
plot(x, y_c, '--k')
plot(x_L, y_L)
axis equal
grid on
grid minor
box on
```

Listing 1: Example code listing