הטכניון – מכון טכנולוגי לישראל

NUMERICAL METHODS IN AEROSPACE ENGINEERING

HOMEWORK ASSIGNMENT x

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Abstract

hello

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Nomenclature

α	mass ratio between the liquid fuel and environment oxigen
β	ratio between the latent heat of the liquid fuel and between the chemical reaction heat of liquid vapor with oxigen
$.^{(n)}$	value at time step n
Λ	empirical constant of droplet vaporation
h	size of each cell in the domain
i	cell index
m_d	mass fraction of liquid fuel in droplets
N	number of elements
s	temporery variable
T	temperature
T_u	environment temperature of the liquid fuel upstream
T_v	vaporization temperature of the liquid fuel
x	spatial coordinate

1 The Physical Problem

The physical problem at hand is the location of the flame front which depends on the point at which the fuel spray evaporates.

2 The Mathematical Model

The evaporation front of fuel spray can be described by solving the following equations:

$$\begin{aligned} \frac{d^2T}{d\zeta^2} &= \Lambda e^T \left(T_v - T_u + \alpha\beta - \frac{dT}{d\zeta} \right) \\ m_d &= \left(\alpha\beta\Lambda e^T \right)^{-1} \frac{d^2T}{d\zeta^2} \end{aligned} \tag{1}$$

The boundary condition of the problem:

$$\begin{array}{lccc} \zeta \to -\infty : & m_d \to 1 & T \;\; \to & \zeta \cdot (T_v - T_u) \\ \zeta \to +\infty : & T \to \zeta \cdot (T_v - T_u + \alpha\beta) & m_d \;\; \to & 0 \end{array} \tag{2}$$

- According to the defenition of T: $T|_{\zeta=0} = 0$

3 The Numerical Methods

Eq.1 can be rewrite as:

$$\frac{d^2T}{d\zeta^2} + \Lambda e^T \frac{dT}{d\zeta} - \Lambda e^T (T_v - T_u + \alpha\beta) = 0$$

In our case:

- ∞ is at around 30
- $\Lambda = 0.1$
- $T_v = 0.203$
- $T_u = 0.152$
- $\alpha\beta = 0.0234$

3.1 Finite Difference Method

Using central difference we can write the difference equations:

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} + \Lambda e^{T_i} \frac{T_{i+1} - T_{i-1}}{2h} - \Lambda e^{T_i} (T_v - T_u + \alpha\beta) = 0 \qquad \begin{array}{c} i = 1, 2, \cdots, N \\ h = \dfrac{\zeta|_{i=N+1} - \zeta|_{i=0}}{N + 1 - 0} \end{array} \tag{3}$$

$$\ddots \; \ddots$$

3.2 Shooting Method

Let's rewrite Eq.3 as a system of 2 ODE:

$$\begin{cases} \dfrac{dT}{d\zeta} = s & T|_{\zeta\to-\infty} \to \zeta \cdot (T_v - T_u) \\ \dfrac{ds}{d\zeta} = -\Lambda e^T s - \Lambda e^T (T_v - T_u + \alpha\beta) & T|_{\zeta\to+\infty} \to \zeta \cdot (T_v - T_u + \alpha\beta) \end{cases} \tag{4}$$

To solve the system of equations using the shooting method, we will guess $s^{(n)}_{(i=0)}$ and solve the system of equations using forward difference. Namely:

$$\begin{cases} T^{(n)}_{i+1} = s^{(n)}_i \cdot h + T^{(n)}_i \\ s^{(n)}_{i+1} = \left(-\Lambda e^{T^{(n)}_i} s^{(n)}_i - \Lambda e^{T^{(n)}_i} (T_v - T_u + \alpha\beta)\right) \cdot h + s^{(n)}_i \end{cases} \tag{5}$$

To correct the guess of $s^{(n)}_{(i=0)}$, let's define:

$$F_{\left(s_{(i=0)}\right)} = T^{(n)}_{(i=N+1)} - T|_{\zeta\to+\infty} \tag{6}$$

- When $F = 0$, the guess of $s^{(n)}_{(i=0)}$ is correct

The next guess of s $s^{(n+1)}_{(i=0)}$ we will use a numerical method to find the root of an equation. Namely:

$$s^{(n+1)}_{(i=0)} = s^{(n)}_{(i=0)} - F_{\left(s^{(n)}_{(i=0)}\right)} \cdot \frac{s^{(n)}_{(i=0)} - s^{(n-1)}_{(i=0)}}{F_{\left(s^{(n)}_{(i=0)}\right)} - F_{\left(s^{(n-1)}_{(i=0)}\right)}} \tag{7}$$

4 Influence of The Numerical Methods

5 Results and Discussion

6 Summary and Conclusion

A Listing of The Computer Program