Numerical Methods in Aeronautical Engineering
HW1 - Theoretical Questions

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May 10, 2025

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1 Q2

Given:

$$\frac{dy}{dt} = f_{(t,y)} \qquad y_{(0)} = 1 \tag{1}$$

1.1 A

Since the Euler method is based on Taylor series expansion, let's derive the expansion for y_{i+1}:

$$y_{i+1} = y_i + \frac{h^1}{1!}\frac{dy}{dt}\bigg|_i + \frac{h^2}{2!}\frac{d^2y}{dt^2}\bigg|_i + \frac{h^3}{3!}\frac{d^3y}{dt^3}\bigg|_i + \cdots \tag{2}$$

According the Euler method, the local error can be written as:

$$y_{i+1} = y_i + \frac{h^1}{1!}f_i + R_i \qquad R_i = \frac{h^2}{2!}\frac{df}{dt}\bigg|_i \approx O\left(h^2\right) \tag{3}$$

1.2 B

Given the function f, we can derive f and get the supremum of the local error over the hole field:

$$\max_i R = \frac{\max_i h^2}{2!} \max_i \left|\frac{df}{dt}\right| \tag{4}$$

1.3 C

Assuming h is constant, the supremum of the local error is:

$$\max_i R = \frac{h^2}{2!} \max_i \left|\frac{df}{dt}\right|$$

Defining the final time as T, the number of integration steps is:

$$N = \frac{T}{h} \tag{5}$$

Hence the supremum of the global error is:

$$R_{\text{global}} = \sum_{i=1}^{N} \max_i R = \sum_{i=1}^{N} \frac{h^2}{2!} \max_i \left|\frac{df}{dt}\right| = \frac{T}{h}\frac{h^2}{2!} \max_i \left|\frac{df}{dt}\right| = \frac{h}{2!} \max_i \left|\frac{df}{dt}\right| \approx O\left(h\right) \tag{6}$$

1.4 D

If we have the equation of f, we can derive and find the maximum value of $\frac{df}{dt}$, in the filed.

2 Q3

Given the ODE:

$$\frac{dy}{dx} = -y \qquad y_{(0)} = 1 \tag{7}$$

The analytical solution for the problem is:

$$\begin{aligned} \frac{dy}{y} &= -dx \\ \int \frac{dy}{y} &= \int -dx \\ \ln(y) &= -x \end{aligned} \tag{8}$$

$$\boxed{y = e^{-x}} \tag{9}$$

2.1 A - Forward differencing