הטכניון – מכון טכנולוגי לישראל

NUMERICAL METHODS IN AEROSPACE ENGINEERING

HOMEWORK ASSIGNMENT x

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Abstract

hello

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Nomenclature

α	mass ratio between the liquid fuel and environment oxigen
β	ratio between the latent heat of the liquid fuel and between the chemical reaction heat of liquid vapor with oxigen
Λ	empirical constant of droplet vaporation
h	size of each cell in the domain
i	cell index
m_d	mass fraction of liquid fuel in droplets
N	number of elements
s	temporery variable
T	temperature
T_u	environment temperature of the liquid fuel upstream
T_v	vaporization temperature of the liquid fuel
x	spatial coordinate
$\boxed{\cdot}^{(n)}$	value at time step n

1 The Physical Problem

The physical problem at hand is the location of the flame front which depends on the point at which the fuel spray evaporates.

2 The Mathematical Model

The evaporation front of fuel spray can be described by solving the following equations:

$$\begin{gathered}\frac{d^2T}{d\zeta^2} = \Lambda e^T \left(T_v - T_u + \alpha\beta - \frac{dT}{d\zeta} \right) \\ m_d = \left(\alpha\beta\Lambda e^T \right)^{-1} \frac{d^2T}{d\zeta^2}\end{gathered} \tag{1}$$

The boundary condition of the problem:

$$\begin{array}{lccc} \zeta \to -\infty : & m_d \to 1 & T \;\; \to & \zeta \cdot (T_v - T_u) \\ \zeta \to +\infty : & T \to \zeta \cdot (T_v - T_u + \alpha\beta) & m_d \;\; \to & 0 \end{array} \tag{2}$$

- According to the defenition of T: $T|_{\zeta=0} = 0$

3 The Numerical Methods

Eq.1 can be rewrite as:

$$\frac{d^2T}{d\zeta^2} + \Lambda e^T \frac{dT}{d\zeta} - \Lambda e^T (T_v - T_u + \alpha\beta) = 0$$

In our case:

- ∞ is at around 30
- $\Lambda = 0.1$
- $T_v = 0.203$
- $T_u = 0.152$
- $\alpha\beta = 0.0234$

3.1 Finite Difference Method

Using central difference we can write the difference equations:

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} + \Lambda e^{T_i} \frac{T_{i+1} - T_{i-1}}{2h} - \Lambda e^{T_i} (T_v - T_u + \alpha\beta) = 0 \qquad \begin{gathered} i = 1, 2, \cdots, N \\ h = \frac{\zeta|_{i=N+1} - \zeta|_{i=0}}{N + 1 - 0} \end{gathered} \tag{3}$$

We will use the 'point Jacobi' method:

1. Set the filed with initial condition (linear interpolation).

2. Calculate the temperature at index i and time step $n+1$ from the previous time step:

$$\Lambda e^{T_i^{(n+1)}} = \frac{-\dfrac{T_{i+1}^{(n)} - 2T_i^{(n)} + T_{i-1}^{(n)}}{h^2}}{\dfrac{T_{i+1}^{(n)} - T_{i-1}^{(n)}}{2h} - (T_v - T_u + \alpha\beta)}$$

$$\Downarrow \qquad i = 1, 2, \cdots, N \tag{4}$$

$$T_i^{(n+1)} = \ln\left(\frac{1}{\Lambda}\frac{-\dfrac{T_{i+1}^{(n)} - 2T_i^{(n)} + T_{i-1}^{(n)}}{h^2}}{\dfrac{T_{i+1}^{(n)} - T_{i-1}^{(n)}}{2h} - (T_v - T_u + \alpha\beta)}\right)$$

3. The solution is considered converged when:

$$\left|y_i^{n+1} - y_i^n\right| < \varepsilon \qquad \forall i \in [1, n] \tag{5}$$

3.2 Shooting Method

Let's rewrite Eq.3 as a system of 2 ODE:

$$\begin{cases} \dfrac{dT}{d\zeta} &= s & T|_{\zeta\to-\infty} \to \zeta \cdot (T_v - T_u) \\ \dfrac{ds}{d\zeta} &= -\Lambda e^T s - \Lambda e^T (T_v - T_u + \alpha\beta) & T|_{\zeta\to+\infty} \to \zeta \cdot (T_v - T_u + \alpha\beta) \end{cases} \tag{6}$$

To solve the system of equations using the shooting method, we will guess $s_{(i=0)}^{(n)}$ and solve the system of equations using forward difference. Namely:

$$\begin{cases} T_{i+1}^{(n)} &= s_i^{(n)} \cdot h + T_i^{(n)} \\ s_{i+1}^{(n)} &= \left(-\Lambda e^{T_i^{(n)}} s_i^{(n)} - \Lambda e^{T_i^{(n)}} (T_v - T_u + \alpha\beta)\right) \cdot h + s_i^{(n)} \end{cases} \qquad \begin{array}{l} s_{i=0}^n = s_0^n \\ T_{i=0}^n = \zeta \cdot (T_v - T_u + \alpha\beta) \\ i = 0, 1, \cdots, N \end{array} \tag{7}$$

To correct the guess of $s_{(i=0)}^{(n)}$, let's define:

$$F_{\left(s_{(i=0)}\right)} = T_{(i=N+1)}^{(n)} - T|_{\zeta\to+\infty} \tag{8}$$

- When $F = 0$, the guess of $s_{(i=0)}^{(n)}$ is correct

The next guess of s $s_{(i=0)}^{(n+1)}$ we will use a numerical method to find the root of an equation. Namely:

$$s_{(i=0)}^{(n+1)} = s_{(i=0)}^{(n)} - F_{\left(s_{(i=0)}^{(n)}\right)} \cdot \frac{s_{(i=0)}^{(n)} - s_{(i=0)}^{(n-1)}}{F_{\left(s_{(i=0)}^{(n)}\right)} - F_{\left(s_{(i=0)}^{(n-1)}\right)}} \tag{9}$$

4 Influence of The Numerical Methods

5 Results and Discussion

6 Summary and Conclusion

A Listing of The Computer Program