Satellite Orbit Control
HW3

Almog Dobrescu

ID 214254252

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1   Given

$$T_1 = 100\,[min] = 6 \cdot 10^3\,[sec] \qquad T_2 = T_1 = 6 \cdot 10^3\,[sec]$$
$$e_1 = 0 \qquad\qquad e_2 = 0$$
$$a_1 = \sqrt[3]{\tfrac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3\,[km] \qquad a_2 = a_1 = 7.1366 \cdot 10^3\,[km]$$
$$\alpha = \Delta i = 0.01°$$

In CW frame with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix}\,[\text{km}] \qquad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.74267 \cdot n \end{pmatrix}\left[\frac{\text{km}}{\text{sec}}\right]$$

1.1   Desired

$$\begin{pmatrix} x_2(t_f) = 0 \\ y_2(t_f) = 0 \\ z_2(t_f) = 0 \end{pmatrix} \qquad \begin{pmatrix} \dot{x}_2(t_f) = 0 \\ \dot{y}_2(t_f) = 0 \\ \dot{z}_2(t_f) = 0 \end{pmatrix}$$

1.2   Limitations

The engine can't create thrust in the x direation and:

$$a_{\max} = 8 \cdot 10^{-6}\left[\frac{\text{km}}{\text{sec}^2}\right]$$

2   The CW equations

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{1}$$

2.1   x-y

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{2}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{3}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & 0 & 1 \\ 0 & -2n & 0 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{4}$$

2.2   z

$$\vec{x} = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \tag{5}$$

$$\dot{\vec{x}} = F\vec{x} + Gf \tag{6}$$

Where:

$$F = \begin{pmatrix} 0 & 1 \\ -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad f = (f_z) \tag{7}$$

2.3 x-y-z

The equations of motion in state space form are therefor:

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{pmatrix} \tag{8}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{9}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \tag{10}$$

3   case A

The desired maneuver time is 2000 [sec]. The desired approach trajectory is a straight line with constant velocity from the initial point to the origin.
From geometric considerations, the required velocity on the straight line is:

$$\dot{\vec{x}}_{req} = \begin{pmatrix} \dfrac{x_f - x_0}{t_f} \\[2mm] \dfrac{y_f - y_0}{t_f} \\[2mm] \dfrac{z_f - z_0}{t_f} \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix} \cdot 10^{-4} \left[\tfrac{km}{\sec^2} \right] \tag{11}$$

Which mean that the initial pulse:

$$\Delta v_1 = \dot{\vec{x}}_{req} - \dot{\vec{x}}_0 = \begin{pmatrix} 0 \\ 0.5 \\ 0.2777 \end{pmatrix} \cdot 10^{-3} \left[\tfrac{km}{\sec^2} \right] \tag{12}$$

and the final pulse:

$$\Delta v_f = \dot{\vec{x}}_f - \dot{\vec{x}}_{req} = \begin{pmatrix} 0 \\ -5 \\ 5 \end{pmatrix} \cdot 10^{-4} \left[\tfrac{km}{\sec^2} \right] \tag{13}$$

Since the velocity is constant:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \vec{0}$$

By substituting the constraints in the CW equations we get:

$$\begin{cases} 0 - 2n\dot{y} = f_x \\ 0 = f_y \\ 0 + n^2 z = f_z \end{cases} \tag{14}$$

$$\vec{a} = \vec{f} = \begin{pmatrix} -2n \cdot 0.5 \cdot 10^{-3} \\ 0 \\ n^2 z \end{pmatrix} \tag{15}$$

Check that the maximum acceleration hasn't been reached:

$$|\vec{a}| = \sqrt{(2n \cdot 0.5 \cdot 10^{-3}) + (n^2 z)} < \sqrt{(2n \cdot 0.5 \cdot 10^{-3}) + (n^2 \cdot 1)}$$

$$|\vec{a}| < 1.5163 \cdot 10^{-6} < 8 \cdot 10^{-6} \qquad \checkmark \tag{16}$$

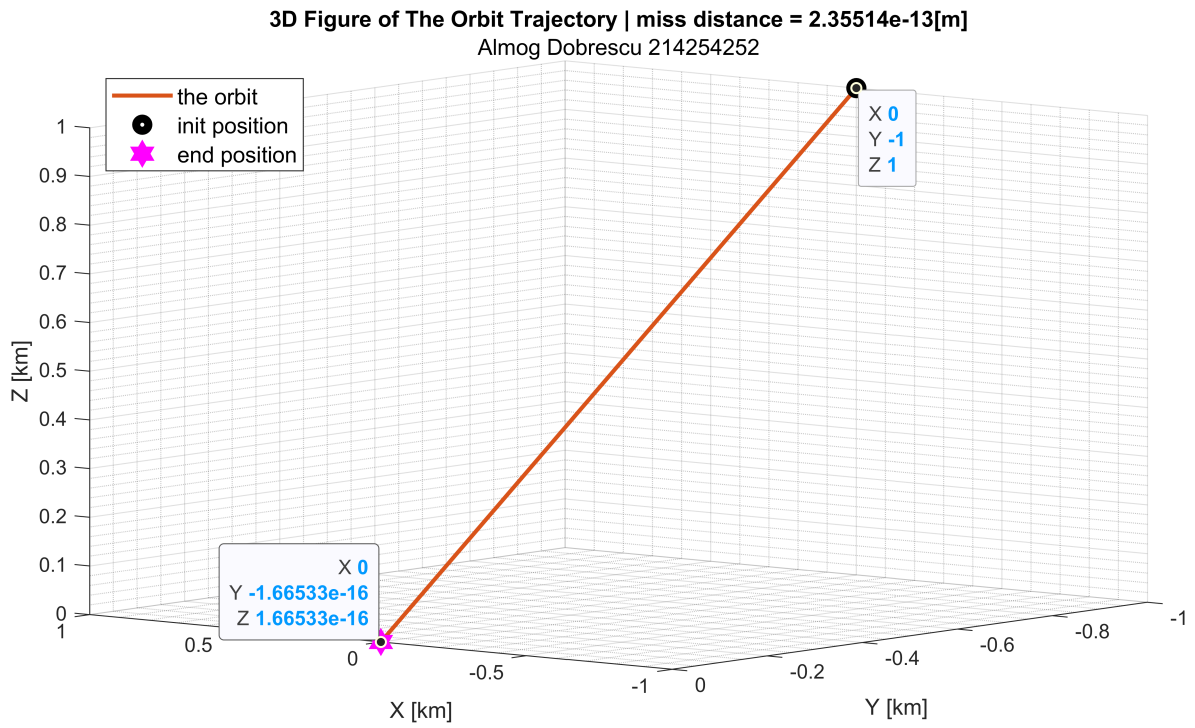By solving this system numerically, we obtain the approach trajectory.

Figure 1: 3D figure of the orbit trajectory
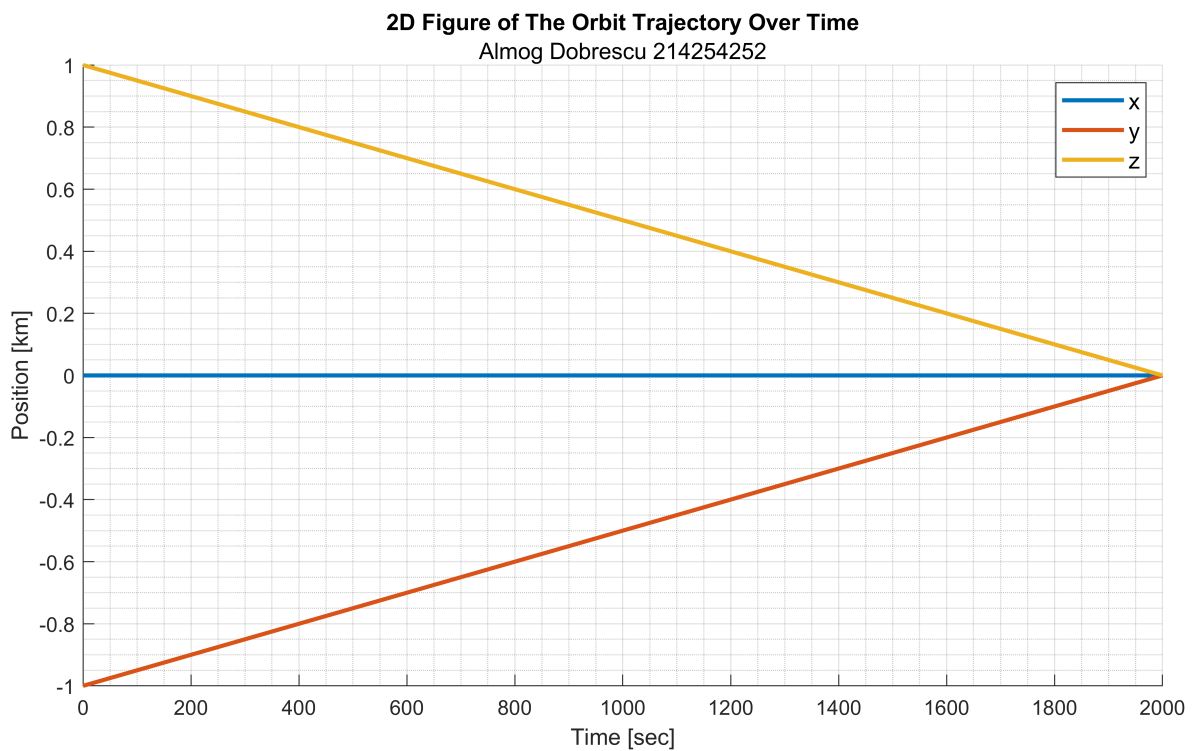

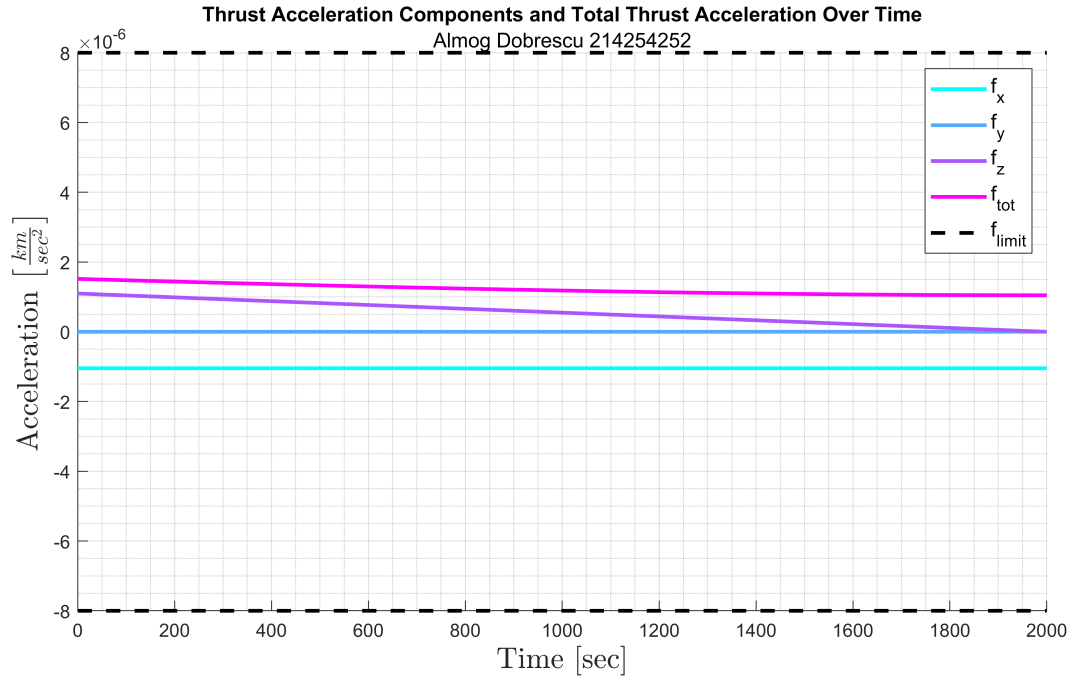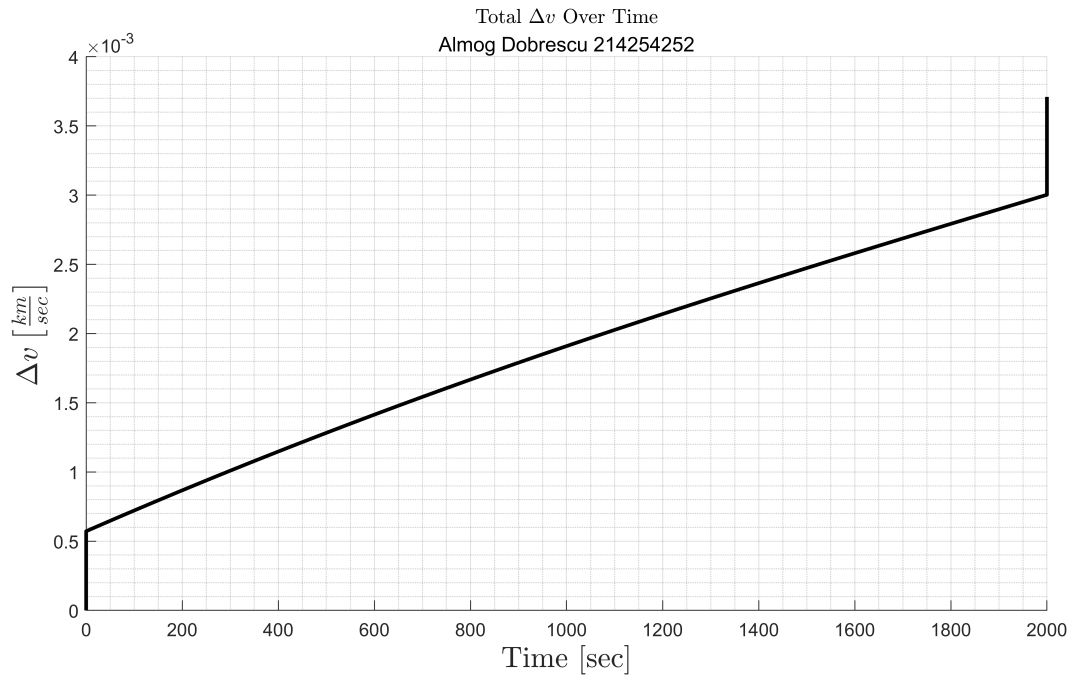Figure 2: 2D figure of the position over time

Figure 3: Thrust acceleration components and total thrust acceleration over time

Figure 4: Total Δv over time

We can see that we indeed acomplished the design criteria:

- The thrust doesn't exceed the maximum available thrust.

- The miss distance at the final desired time is less than 1[m] $\left(2.35517 \cdot 10^{-13}\,[\text{m}]\right)$.

4   case B

The velocity to be gained:

$$\vec{v}_g = \vec{v}_r - \vec{v} \tag{17}$$

The equation of motion:

$$\frac{d\vec{v}}{dt} = \vec{g}_{(\vec{r})} + \vec{a}_T \tag{18}$$

The equation of motion for the velocity to be gained:

$$\frac{d\vec{v}_g}{dt} = \frac{d\vec{v}_r}{dt} - \frac{d\vec{v}}{dt} = \frac{d\vec{v}_r}{dt} - \vec{g}_{(\vec{r})} - \vec{a}_T \tag{19}$$

$\vec{v}_r$ is a function of time and space, so:

$$\frac{d\vec{v}_{r(t,\vec{r})}}{dt} = \frac{\partial \vec{v}_r}{\partial t} + \frac{\partial \vec{v}_r}{\partial \vec{r}} \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} = \frac{\partial \vec{v}_r}{\partial t} + \frac{\partial \vec{v}_r}{\partial \vec{r}} (\vec{v}_r - \vec{v}_g)$$

The required velocity $\vec{v}_r$ also fulfill the equation of motion with no thrust:

$$\frac{d\vec{v}_r}{dt} = \frac{\partial \vec{v}_r}{\partial t} + \frac{\partial \vec{v}_r}{\partial \vec{r}} \vec{v}_r = \vec{g}_r$$

so:

$$\frac{d\vec{v}_r}{dt} = \vec{g}_r - \frac{\partial \vec{v}_r}{\partial \vec{r}} \vec{v}_g \tag{20}$$

The equation of motion becomes:

$$\frac{d\vec{v}_g}{dt} = \vec{g}_r - \frac{\partial \vec{v}_r}{\partial \vec{r}} \vec{v}_g - \vec{g}_{(\vec{r})} - \vec{a}_T = - \underbrace{\frac{\partial \vec{v}_r}{\partial \vec{r}}}_{C^*} \vec{v}_g - \vec{a}_T \tag{21}$$

where C^* is:

The Cross-Product fixed thrust control law:

$$\vec{v}_g \parallel \frac{d\vec{v}_g}{dt} \tag{22}$$

$$\Downarrow$$

$$\frac{d\vec{v}_g}{dt} \times \vec{v}_g = 0$$