Satellite Orbit Control
HW8

Almog Dobrescu

ID 214254252

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1 Given

A satellite is placed in an Earth Repeat circular orbit ($e = 0$) with $T = 100\,[\text{min}] = 6 \cdot 10^3\,[\text{sec}]$. The satellite performs a D maneuver cycle (point A) and the ground track deviation was $20\,[\text{km}]$. At $t = 5\,[\text{day}]$ the gournd track deviation was $-10\,[\text{km}]$ and the rate of change of the ground track was negative.

1.1 Desired

1. Altitude decay rate
2. Altitude difference between A and C (the end of the maneuver cycle)
3. Velocity change that is required to raise the altitude from C to A

2 D Maneuver

The allowed tolerance is ΔL, which means the allowed deviation is $\pm\Delta L$. Point A is Δh above the nominal height and ΔL before the nominal position. The satellite is in a higher orbit than the nominal so the velocity decreases. The height of the satellite decreases because of the drag and with the lower velocity, the satellite goes backward towards the nominal point.
At point B the height is equal to the nominal height but ΔL after the nominal position. The height continues to decrease and the velocity increases and passes the nominal velocity.
At point C the satellite is the height $-\Delta h$ and $DeltaL$ before the nominal point. At this point, using 2 pulses, the satellite transfers to point A using Hohmann transfer.

2.1 Semi-major Axis Decay Because of Drag

$$\dot{a} = \frac{2a}{r}\left(2a - r\right)\frac{f_t}{V} = -\frac{2a}{r}\left(2a - r\right)\frac{\rho V K_D}{2} \tag{1}$$

For semi-circular orbit:

$$\dot{a} = -\rho K_D\sqrt{\mu a} = -\rho K_D n a^2 \tag{2}$$

Assuming the change of height is small (the density is constant), we will mark the constant rate of decent $k \equiv -\dot{a}$. The change of the semi-major axis when ($t_A = 0$):

$$\Delta a = -kt \tag{3}$$

2.2 Change in Longitudinal Position

To calculate the change in the longitudinal position, we will find the change in the mean anomaly:

$$\Delta\dot{M} = \Delta n \qquad \Rightarrow \qquad \Delta M = \textstyle\int_0^t \Delta n dt \tag{4}$$

Δn is the change in n because the change in a:

$$\begin{aligned} n^2 = \frac{\mu}{a^3} \qquad &\rightarrow \qquad \Delta n = -\frac{3}{2}\frac{n}{a}\Delta a \\ &\Downarrow \\ \Delta M \qquad &= \qquad \frac{3n}{2a}\int_0^t ktdt = \frac{3nk}{4a}t^2 \end{aligned} \tag{5}$$

2.3 Required Δv

$$\Delta v = \frac{n}{2}\Delta a = \frac{n}{2}k \cdot 4\sqrt{\frac{2\Delta L}{3nk}} = 2\sqrt{\frac{2nk\Delta L}{3}} \tag{6}$$

3 The Solution

3.1 Altitude Decay Rate

$$\Delta L = 20\,[\mathrm{km}]$$

$$\Delta t = 5 - 0\,[day]$$

$$T = 6 \cdot 10^3 \qquad \Rightarrow \qquad a = 7.1366 \cdot 10^3\,[\mathrm{km}] \quad n = 1.0472 \cdot 10^{-3}$$

$$\Delta L_{\Delta t} = \Delta a M_{\Delta t} = \frac{3nk}{4}\,(\Delta t)^2 \tag{7}$$

$$\Downarrow$$

$$k = \frac{4\Delta L_{\Delta t}}{3n\,(\Delta t)^2}$$

$$\boxed{k = 2.0467 \cdot 10^{-7}\left[\frac{\mathrm{km}}{\mathrm{sec}}\right]} \tag{8}$$

3.2 Altitude Difference Between A and C

$$t_B = \sqrt{\frac{8\Delta L}{3nk}} \tag{9}$$

$$\Delta h = \Delta a = -2kt_B$$

$$\boxed{\Delta h = -0.2042\,[\mathrm{km}]} \tag{10}$$

3.3 Velocity Change to Raise the Altitude From C to A

$$\Delta v = 2\sqrt{\frac{2nk\Delta L}{3}} \tag{11}$$

$$\boxed{\Delta v = 1.0692 \cdot 10^{-4}\left[\frac{\mathrm{km}}{\mathrm{sec}}\right]} \tag{12}$$