Satellite Orbit Control
HW5

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1 Given

$$T_1 = 100\,[min] = 6 \cdot 10^3\,[sec] \qquad T_2 = T_1 = 6 \cdot 10^3\,[sec]$$
$$e_1 = 0 \qquad e_2 = 0$$
$$a_1 = \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3\,[km] \qquad a_2 = a_1 = 7.1366 \cdot 10^3\,[km]$$
$$\alpha = \Delta i = 0.01^\circ$$

In CW frame with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [\text{km}] \qquad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.74267 \cdot n \end{pmatrix} \left[\frac{\text{km}}{\text{sec}}\right]$$

1.1 Desired

$$\begin{pmatrix} x_2(t_f) = 0 \\ y_2(t_f) = 0 \\ z_2(t_f) = 0 \end{pmatrix} \qquad \begin{pmatrix} \dot{x}_2(t_f) = 0 \\ \dot{y}_2(t_f) = 0 \\ \dot{z}_2(t_f) = 0 \end{pmatrix}$$

1.2 Limitations

$$a_{\max} = 4 \cdot 10^{-5} \left[\frac{\text{km}}{\text{sec}^2}\right]$$

2 The CW equations

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{1}$$

2.1 x-y

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{2}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{3}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & 0 & 1 \\ 0 & -2n & 0 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{4}$$

2.2 z

$$\vec{x} = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \tag{5}$$

$$\dot{\vec{x}} = F\vec{x} + Gf \tag{6}$$

Where:

$$F = \begin{pmatrix} 0 & 1 \\ -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_z \end{pmatrix} \tag{7}$$

2.3 x-y-z

The equations of motion in state space form are therefor:

$$\vec{x} = \begin{pmatrix} x & \dot{x} & y & \dot{y} & z & \dot{z} \end{pmatrix}^T \tag{8}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{9}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \tag{10}$$

3 Target Trajectory

The target trajectory:

$$\dot{\vec{x}}_r = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{F_r} \vec{x}_r \tag{11}$$

The initial state for the approach trajectory:

$$\vec{x}_{r(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ |v_{ref}| \cdot t_f \\ -v_{ref} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.06 \\ -3 \cdot 10^{-5} \end{pmatrix} \tag{12}$$

The system will be solved by subtracting the equation of motion of the traget trajectory from the equation of motion of the satellite

$$\vec{\delta x} = \vec{x} - \vec{x}_r \tag{13}$$

$$\dot{\vec{\delta x}} = F\vec{\delta x} + (F - F_r)\,\vec{x}_r + G\vec{f} \qquad \text{Where : } \; G\vec{f} = -\,(F - F_r)\,\vec{x}_r - GK\vec{\delta x} \tag{14}$$

$\vec{x}_r$ is determaned by solving Eq.11

4 Poles and Gains

The desired poles are given by the following equation:

$$P = 10 \cdot \begin{pmatrix} -n + i \cdot n \\ -n - i \cdot n \\ -4n + i \cdot 3n \\ -4n - i \cdot 3n \\ -3n + i \cdot n \\ -3n - i \cdot n \end{pmatrix} = \begin{pmatrix} -0.0105 + i \cdot 0.0105 \\ -0.0105 - i \cdot 0.0105 \\ -0.0419 + i \cdot 0.0314 \\ -0.0419 - i \cdot 0.0314 \\ -0.0314 + i \cdot 0.0105 \\ -0.0314 - i \cdot 0.0105 \end{pmatrix} \tag{15}$$

By using the function *place* in Matlab, we get:

$$K = \begin{pmatrix} 0.0014 & 0.0649 & -0.0009 & -0.0240 & 0.0002 & 0.0201 \\ -0.0002 & -0.0012 & 0.0007 & 0.0516 & -0.0004 & -0.0062 \\ 0.0001 & -0.0138 & 0 & -0.0068 & 0.0007 & 0.0511 \end{pmatrix} \tag{16}$$

5 The Results

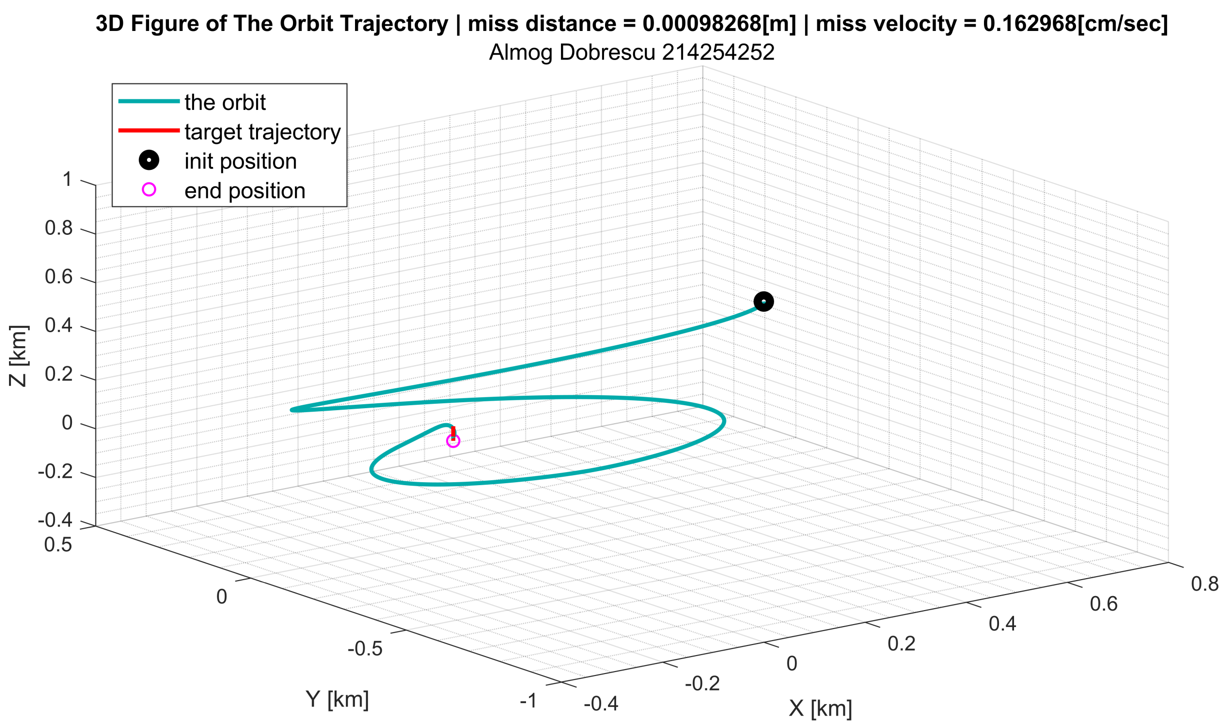

Figure 1: 3D figure of the orbit trajectory

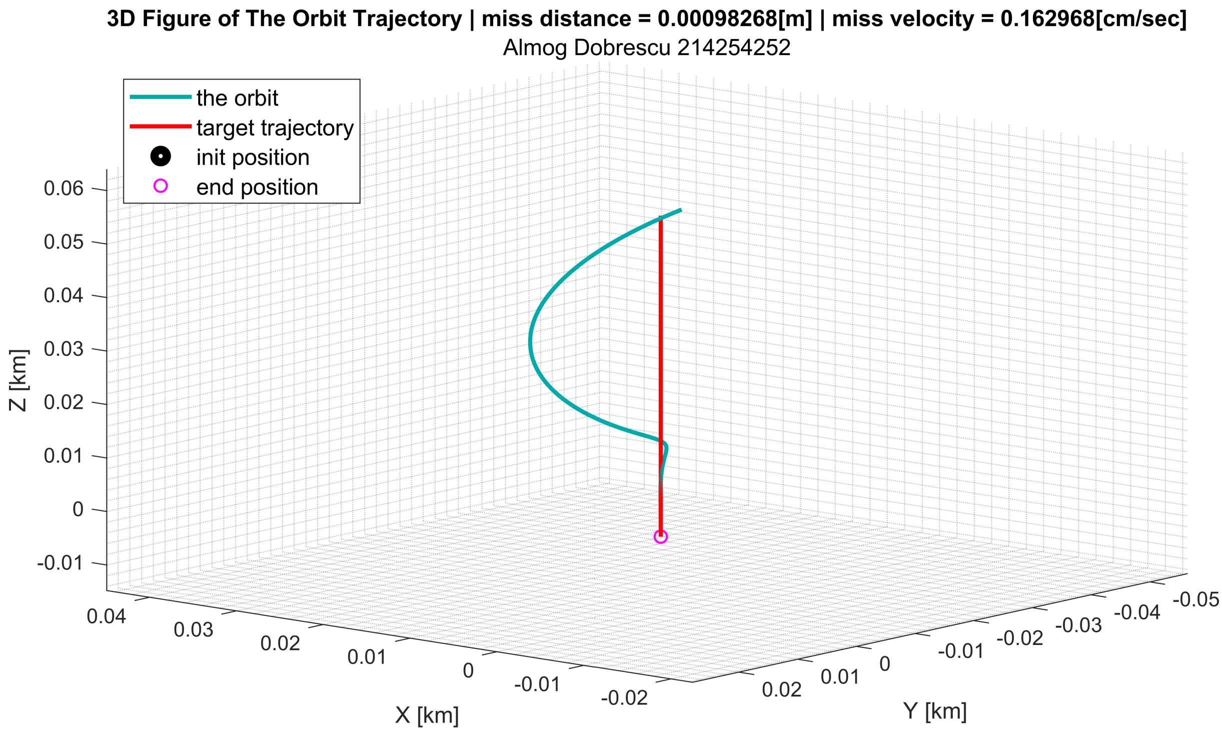

Figure 2: 3D figure of the orbit trajectory - zoomed

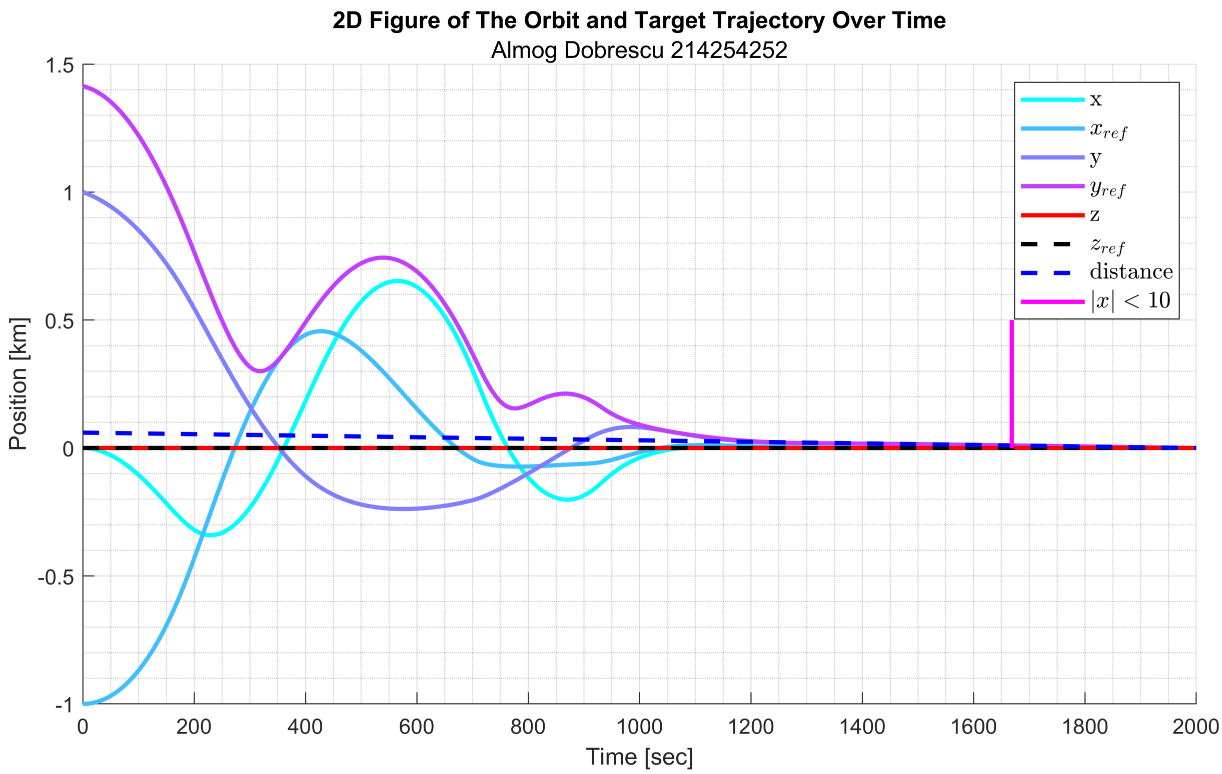

Figure 3: 2D figure of the orbit target trajectory over time

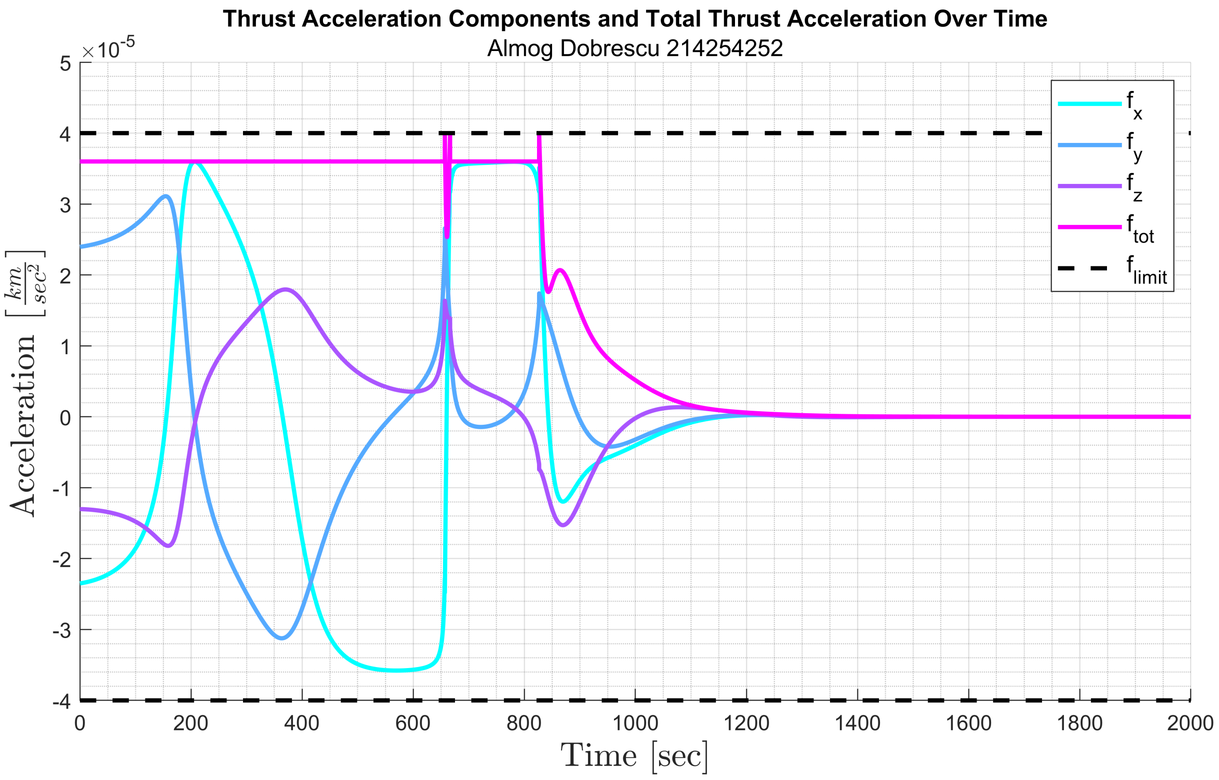

Figure 4: Thrust acceleration components and total thrust acceleration over time

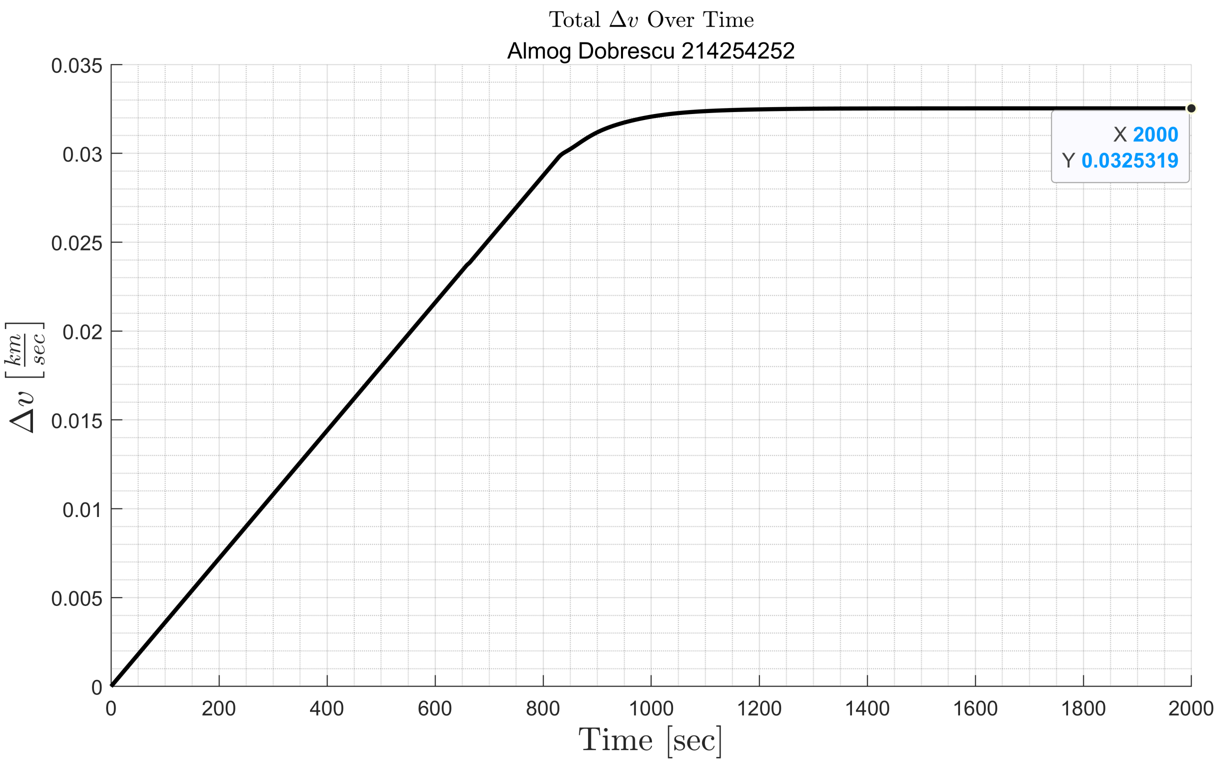

Figure 5: Total Δv over time

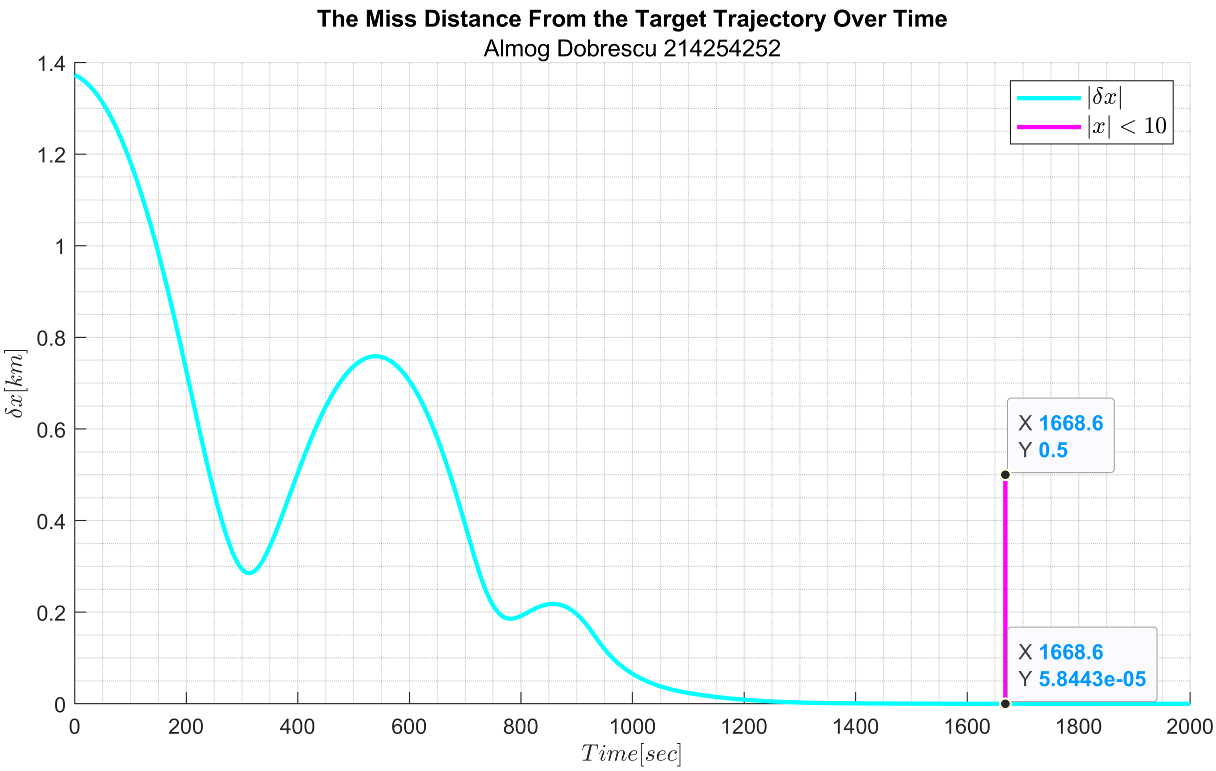

Figure 6: Distance from target trajectory over time

We can see that we indeed acomplished the design criteria:

- The thrust doesn't exceed the maximum available thrust.
- The time at which we are at the last $10[m]$ is $t = 1668.6[sec]$. At this time, the distance from the target trajectory is $5.8443 \cdot 10^{-5}[km] = 5.8443 \cdot 10^{-2}[m] < 1[m]$

The total Δv is: $0.0325 \left[\frac{\text{km}}{\text{sec}}\right]$