Satellite Orbit Control
HW9

Almog Dobrescu

ID 214254252

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1 Given

A communication satellite is placed in a geostationary orbit above longitude 30°E.

$$T = 8.6164 \cdot 10^4 \,[\text{sec}] \rightarrow a = 4.2164 \cdot 10^4 \,[\text{km}] \qquad \lambda_n = 30^\circ \qquad e = 0 \qquad i = 0 \tag{1}$$

The longitude tolerance is $\Delta\lambda = \pm 0.05^\circ$

1.1 Desired

1.2 Limitations

The thrust is only at the $\pm y$ direction and:

$$a_{\max} = 1 \cdot 10^{-7} \left[\frac{\text{km}}{\text{sec}^2}\right] \tag{2}$$

2 Control

2.1 East-West Control

2.1.1 Drift Duo to Tesseral Harmonics

The gravity component normal to the radius changes the orbit period so the satellite moves toward the closest equilibrium point. The force component creates a tangential acceleration, which is equivalent to the angular acceleration of the longitude line underneath the satellite:

$$a\ddot{\lambda} = \vec{f} \cdot \vec{v} \tag{3}$$

The change in the semi-major axis and the longitude line over one cycle:

$$\begin{aligned} \frac{da}{dt} &= -12anJ_{22}\left(\frac{Re}{a}\right)^2 \sin\left(2\left(\lambda - \lambda_{22}\right)\right) \\ \frac{d^2\lambda}{dt^2} &= -\frac{3}{2}\frac{n}{a}\frac{da}{dt} \end{aligned} \tag{4}$$

Where:

$$\begin{aligned} J_{22} &= \left(C_{22}^2 + S_{22}^2\right)^{\frac{1}{2}} \\ \lambda_{22} &= \tan^{-1}\left(\frac{S_{22}}{C_{22}}\right) = -14.9^\circ \end{aligned} \tag{5}$$

After substitution:

$$\begin{aligned} \dot{a} &= 0.132 \sin\left(2\left(\lambda - \lambda_s\right)\right) \left[\frac{\text{km}}{\text{day}}\right] \\ \ddot{\lambda} &= -0.00168 \sin\left(2\left(\lambda - \lambda_s\right)\right) \left[\frac{\text{deg}}{\text{day}^2}\right] \end{aligned} \qquad \begin{array}{c}\lambda_s \text{ is the closest stable} \\ \text{equilibrium point } 75^\circ \text{ or } 255^\circ\end{array} \tag{6}$$

Since the tolerance is small:

$$\begin{aligned} \ddot{\lambda} &= -K \sin\left(2\left(\lambda_n - \lambda_s\right)\right) = \ddot{\lambda}_n \\ \lambda &= \frac{1}{2\ddot{\lambda}_n}\left(\dot{\lambda} - \dot{\lambda}_0\right)^2 + \frac{\dot{\lambda}_0}{\ddot{\lambda}_n} + \lambda_0 \end{aligned} \tag{7}$$

The initial conditions of the satellite:

$$\lambda_0 = \lambda_n - \Delta\lambda \qquad \dot{\lambda}_0 = -2\ \mathrm{sgn}\left(\ddot{\lambda}_n\right)\sqrt{-\ddot{\lambda}_n \Delta\lambda} \tag{8}$$

The velocit pulse needes to change $-\dot{\lambda}_0$ into $\dot{\lambda}_0$:

$$\Delta n = 2\dot{\lambda}_0 \tag{9}$$

The relation between the change in angular velocit and the semi-major axis:

$$\begin{gathered} n^2 = \frac{\mu}{a^3} \to \frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta n}{n} \\ \Downarrow \\ \Delta v = \left|\frac{\mu}{2a^2 v}\Delta a\right| = \frac{a}{3}\left|\Delta n\right| = \frac{2a}{3}\left|\dot{\lambda}_0\right| \end{gathered} \tag{10}$$

and the friquency of pulses:

$$t_m = 2\left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right| \tag{11}$$

The change in altitude between to corections:

$$\Delta a = 0.132 \sin\left(2\left(\lambda - \lambda_s\right)\right) t_m\ [\mathrm{km}] \tag{12}$$

3 The Results

3.1 Part A

$$\begin{aligned}
\ddot{\lambda}_n &= -0.00168 \sin\left(2\left(30^\circ - 75^\circ\right)\right) = 0.0017 \left[\frac{\text{deg}}{\text{day}^2}\right] \\
\dot{\lambda}_0 &= -2 \ \text{sgn}\left(\ddot{\lambda}_n\right) \sqrt{-\ddot{\lambda}_n \Delta\lambda} = -0.0183 \left[\frac{\text{deg}}{\text{day}}\right] \\
&\Downarrow \\
&\boxed{t_m = 2\left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right| = 2\left|\frac{-0.0183}{0.0017}\right| = 21.8218\,[\text{day}]} \\
&\Downarrow \\
&\boxed{\Delta a = 0.132 \sin\left(2\left(30^\circ - 75^\circ\right)\right) t_m = -2.8805\,[\text{km}]}
\end{aligned} \tag{13}$$

3.2 Part C

In order to test the worst scenario, the initial condition for the rate of change of the longitude line will be its maximal value:

$$\dot{\lambda}_0 = \pm 0.0183 \left[\frac{\text{deg}}{\text{day}}\right] \tag{14}$$

and the initial longitude line will be the nominal one:

$$\lambda_0 = \lambda_n = 30^\circ \tag{15}$$

By assuming that the amplitude of the change of the semi-major axis is constant we get the range of longitudes:

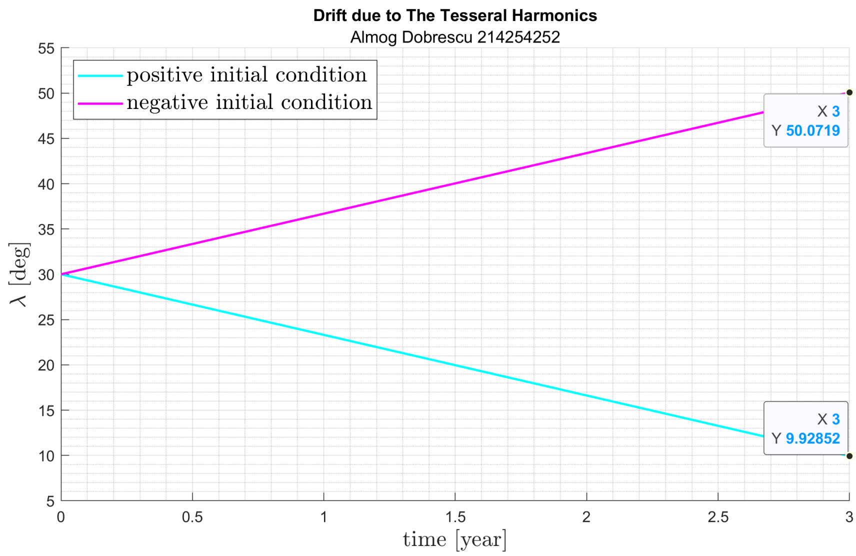

Figure 1: Longitude vs. Time

We can see that the maximal longitude range is $\pm 20.0719^\circ$.

A The Code

```matlab
clc; clear; close all;

%% Part A

mu = 3.986e5; % [km^3/s^2]
R_earth = 6378; % [km]
T = 23*60*60+56*60+4.09; % [sec]
a = (mu * T^2 / 4 / pi^2)^(1/3); % [km]
n = 2*pi/T; % [1/sec]
f_max = 0.0001e-3; % [km/sec]

lambda_n_deg = 30;
lambda_s_deg = 75;

lambda_n_ddot = -0.00168*sind(2*(lambda_n_deg - lambda_s_deg))

delta_lambda_0 = -sign(lambda_n_ddot)*0.05; % [deg]
delta_lambda = abs(delta_lambda_0);
Delta_L = delta_lambda * pi / 180 * a; % [km]

lambda_0_dot = -2*sign(lambda_n_ddot)*sqrt(-lambda_n_ddot*delta_lambda_0)

tm = 2*abs(lambda_0_dot / lambda_n_ddot)

delta_a = 0.132*sind(2*(lambda_n_deg - lambda_s_deg))*tm

%% Part C
time_interval = 0:1:365*3;

state = [lambda_n_deg; lambda_0_dot; a]; % [lambda, labmbda_dot, a]
 % This is where we integrate the equations of motion.
[t_out, state_out] = ode45(@ODE, time_interval, state, odeset('RelTol',5e-14,'AbsTol',5e-14));
state_out_positive_lambda_0_dot = state_out;

state = [lambda_n_deg; -lambda_0_dot; a]; % [lambda, labmbda_dot, a]
 % This is where we integrate the equations of motion.
[t_out, state_out] = ode45(@ODE, time_interval, state, odeset('RelTol',5e-14,'AbsTol',5e-14));

state_out_negative_lambda_0_dot = state_out;

fig1 = figure ("Name","1",'Position',[100 300 900 500]);
colors = cool(2);
hold all

plot(t_out/365, state_out_positive_lambda_0_dot(:,1), "LineWidth",1.5, "Color", colors(1,:))
```

```
plot(t_out/365, state_out_negative_lambda_0_dot(:,1), "LineWidth",1.5, "Color", colors(2,:))

xlabel('time [year]','FontSize',15,Interpreter='latex')
ylabel('$\lambda$ [deg]','FontSize',15,Interpreter='latex')
grid on
grid minor
title("Drift due to The Tesseral Harmonics")
subtitle("Almog Dobrescu 214254252")
legend({'positive initial condition', 'negative initial condition'},'FontSize',15,'Location','northwest',Interpreter='latex')
% exportgraphics(fig1, 'graph1.png','Resolution',300);



%% Functions
function d_state_dt = ODE(t, state)
    mu = 3.986e5; % [km^3/s^2]

    lambda = state(1);
    lambda_dot = state(2);
    a = state(3);

    n = sqrt(mu/a^3);

    temp1 = abs(lambda-75);
    temp2 = abs(lambda-255);
    if (temp1<temp2)
        lambda_s_deg = 75;
    else
        lambda_s_deg = 255;
    end

    a_dot = 0.132*sind(2*(lambda-lambda_s_deg));

    lambda_ddot = -3/2*n/a*a_dot;

    d_state_dt = [lambda_dot; lambda_ddot; a_dot];
end
```

Listing 1: The code