

Satellite Orbit Control

HW2

Almog Dobrescu

ID 214254252

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1 Given

$$\begin{aligned}
 T_1 &= 100 [min] = 6 \cdot 10^3 [sec] & T_2 &= T_1 = 6 \cdot 10^3 [sec] \\
 e_1 &= 0 & e_2 &= 0 \\
 a_1 &= \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3 [km] & a_2 &= a_1 = 7.1366 \cdot 10^3 [km] \\
 \alpha &= \Delta i = 0.01^\circ
 \end{aligned}$$

In CW fram with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [km] \quad \begin{pmatrix} \dot{x}_2(0) = ?? \\ \dot{y}_2(0) = ?? \\ \dot{z}_2(0) < 0 \end{pmatrix}$$

Desired:

$$\begin{pmatrix} x_2(t_1) = 0 \\ y_2(t_1) = 0 \\ z_2(t_1) = 0 \end{pmatrix} \quad \begin{pmatrix} \dot{x}_2(t_1) = 0 \\ \dot{y}_2(t_1) = 0 \\ \dot{z}_2(t_1) = 0 \end{pmatrix}$$

Limitations:

$$\Delta v = t \begin{pmatrix} 0 \\ \Delta v_y \\ \Delta v_z \end{pmatrix}$$

2 A

General equations of motion in CW fram:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2z = f_z \end{cases} \quad (1)$$

The velocities:

$$\begin{cases} u_x = \dot{x} - ny(t) \\ u_y = \dot{y} + nx(t) \\ u_z = \dot{z} \end{cases} \quad (2)$$

The solution without external forces (i.e. $\vec{f} = \vec{0}$):

$$\begin{cases} x(t) = (4 - 3\cos(nt)) \cdot x_0 + \frac{\dot{x}_0}{n} \sin(nt) + \frac{2}{n} (1 - \cos(nt)) \cdot \dot{y}_0 \\ y(t) = 6(\sin(nt) - nt) \cdot x_0 + y_0 + \frac{2}{n} (\cos(nt) - 1) \cdot \dot{x}_0 + (4\sin(nt) - 3nt) \frac{\dot{y}}{n} \\ z(t) = z_0 \cos(nt) + \frac{\dot{z}_0}{n} \sin(nt) \end{cases} \quad (3)$$

Because the two satellite have the same period then the motion is periodical. The condition for periodical motion is:

$$\dot{y}_0 = -2nx_0 \quad (4)$$

So the equations of motion becomes:

$$\begin{cases} x(t) = x_0 \cos(nt) + \frac{\dot{x}_0}{n} \sin(nt) \\ y(t) = y_0 + \frac{2}{n} (\cos(nt) - 1) \cdot \dot{x}_0 - 2x_0 \sin(nt) \\ z(t) = z_0 \cos(nt) + \frac{\dot{z}_0}{n} \sin(nt) \end{cases} \quad (5)$$