Satellite Orbit Control
HW4

Almog Dobrescu

ID 214254252

December 29, 2024

Contents

1 Given **2**
1.1 Desired 2
1.2 Limitations 2

2 The CW equations **2**
2.1 x-y 2
2.2 z 2
2.3 x-y-z 3
2.4 The homogeneous solution 3

3 A **4**

List of Figures

1 Given

$$T_1 = 100\,[min] = 6 \cdot 10^3\,[sec] \qquad T_2 = T_1 = 6 \cdot 10^3\,[sec]$$
$$e_1 = 0 \qquad e_2 = 0$$
$$a_1 = \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3\,[km] \qquad a_2 = a_1 = 7.1366 \cdot 10^3\,[km]$$
$$\alpha = \Delta i = 0.01^\circ$$

In CW frame with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [\text{km}] \qquad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.74267 \cdot n \end{pmatrix} \left[\frac{\text{km}}{\text{sec}}\right]$$

1.1 Desired

$$\begin{pmatrix} x_2(t_f) = 0 \\ y_2(t_f) = 0 \\ z_2(t_f) = 0 \end{pmatrix} \qquad \begin{pmatrix} \dot{x}_2(t_f) = 0 \\ \dot{y}_2(t_f) = 0 \\ \dot{z}_2(t_f) = 0 \end{pmatrix}$$

1.2 Limitations

$$a_{\max} = 4 \cdot 10^{-5} \left[\frac{\text{km}}{\text{sec}^2}\right]$$

2 The CW equations

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{1}$$

2.1 x-y

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{2}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{3}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & 0 & 1 \\ 0 & -2n & 0 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{4}$$

2.2 z

$$\vec{x} = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \tag{5}$$

$$\dot{\vec{x}} = F\vec{x} + Gf \tag{6}$$

Where:

$$F = \begin{pmatrix} 0 & 1 \\ -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_z \end{pmatrix} \tag{7}$$

2.3 x-y-z

The equations of motion in state space form are therefor:

$$\vec{x} = \begin{pmatrix} x & \dot{x} & y & \dot{y} & z & \dot{z} \end{pmatrix}^T \tag{8}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{9}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \tag{10}$$

2.4 The homogeneous solution

$$\vec{x} = \begin{pmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{pmatrix}^T \tag{11}$$

x-y plane:

$$\Phi_{\{t,t_0\}} = \left(\begin{array}{cc|cc} 4-3\cos(n\tau) & 0 & \frac{1}{n}\sin(n\tau) & \frac{2}{n}\left(1-\cos(n\tau)\right) \\ 6\left(\sin(n\tau)-n\tau\right) & 1 & \frac{2}{n}\left(\cos(n\tau)-1\right) & \frac{1}{n}\left(4\sin(n\tau)-3n\tau\right) \\ \hline 3n\sin(n\tau) & 0 & \cos(n\tau) & 2\sin(n\tau) \\ 6n\left(\cos(n\tau)-1\right) & 0 & -2\sin(n\tau) & 4\cos(n\tau)-3 \end{array}\right) = \begin{pmatrix} \Phi_{11(t,t_0)} & \Phi_{12(t,t_0)} \\ \Phi_{21(t,t_0)} & \Phi_{22(t,t_0)} \end{pmatrix}$$

z-direction:

$$\Phi_{(t,t_0)} = \begin{pmatrix} \cos(n\tau) & \frac{1}{n}\sin(n\tau) \\ -n\sin(n\tau) & \cos(n\tau) \end{pmatrix} \tag{12}$$

Where:

$$\tau = t - t_0$$

So the full homogeneous solution in state space from:

$$\Phi_{(t,t_0)} = \left(\begin{array}{ccc|ccc} 4-3\cos(n\tau) & 0 & 0 & \frac{1}{n}\sin(n\tau) & \frac{2}{n}\left(1-\cos(n\tau)\right) & 0 \\ 6\left(\sin(n\tau)-n\tau\right) & 1 & 0 & \frac{2}{n}\left(\cos(n\tau)-1\right) & \frac{1}{n}\left(4\sin(n\tau)-3n\tau\right) & 0 \\ 0 & 0 & \cos(n\tau) & 0 & 0 & \frac{1}{n}\sin(n\tau) \\ \hline 3n\sin(n\tau) & 0 & 0 & \cos(n\tau) & 2\sin(n\tau) & 0 \\ 6n\left(\cos(n\tau)-1\right) & 0 & 0 & -2\sin(n\tau) & 4\cos(n\tau)-3 & 0 \\ 0 & 0 & -n\sin(n\tau) & 0 & 0 & \cos(n\tau) \end{array}\right) =$$

$$\begin{pmatrix} \Phi_{11(t,t_0)} & \Phi_{12(t,t_0)} \\ \Phi_{21(t,t_0)} & \Phi_{22(t,t_0)} \end{pmatrix} \tag{13}$$

3 A

The desired poles are given by the following equation:

$$P = 10 \cdot \begin{pmatrix} -n + i \cdot n \\ -n - i \cdot n \\ -4n + i \cdot 3n \\ -4n - i \cdot 3n \\ -3n + i \cdot n \\ -3n - i \cdot n \end{pmatrix} = \begin{pmatrix} -0.0105 + i \cdot 0.0105 \\ -0.0105 - i \cdot 0.0105 \\ -0.0419 + i \cdot 0.0314 \\ -0.0419 - i \cdot 0.0314 \\ -0.0314 + i \cdot 0.0105 \\ -0.0314 - i \cdot 0.0105 \end{pmatrix} \tag{14}$$

By using the function *place* in Matlab, we get:

$$K = \begin{pmatrix} 0.0013 & 0.0623 & -3.0786 \cdot 10^{-5} & 0.0122 & -9.0833 \cdot 10^{-4} & -0.0308 \\ 1.8104 \cdot 10^{-4} & -0.0121 & 6.1079 \cdot 10^{-4} & 0.0464 & -3.0853 \cdot 10^{-5} & -0.0077 \\ -2.4680 \cdot 10^{-4} & 0.0059 & -3.4367 \cdot 10^{-4} & -0.0033 & 9.6612 \cdot 10^{-4} & 0.0588 \end{pmatrix} \tag{15}$$

We can see that

$$\tau = \frac{1}{\mathbb{R}e\left\{P_i\right\}} < \frac{1}{10n} = \frac{1}{0.0105} = 95.4930 < 2000$$