Satellite Orbit Control

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1 Given

- In ECI

$$\vec{r}_{an} = \begin{pmatrix} 5000 \\ 8500 \\ ??? \end{pmatrix} [km] \qquad \vec{v}_{an} = \begin{pmatrix} 2.15 \\ -2.05 \\ ??? \end{pmatrix} \left[\frac{km}{sec}\right]$$

- $$T = 7800[sec]$$

- $$m = 450[kg]$$

- $$A_{drag} = 2[m^2]$$

- $$C_D = 2.2[-]$$

- $$A_{solar} = 5[m^2]$$

2 Q1

Given: Because the ascending node position is given in ECI, $r_{an,3} = 0$ so:

$$\vec{r}_{an} = \begin{pmatrix} 5000 \\ 8500 \\ 0 \end{pmatrix} [km] \quad r = 9.8615 \cdot 10^3 [km] \tag{1}$$

From the period:

$$T = \frac{2\pi}{\sqrt{\frac{\mu}{a^3}}} \Rightarrow a = \sqrt[3]{\frac{\mu T^2}{4\pi^2}} \Rightarrow a = 8.5007 \cdot 10^3 \tag{2}$$

The velocity at the ascending node is:

$$v = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} \Rightarrow v = 5.8266 \left[\frac{km}{sec}\right] \tag{3}$$

$$v = \sqrt{2.15^2 + 2.05^2 + v_{an,3}^2} \Rightarrow v_{an,3} = \pm 5.0124 \left[\frac{km}{sec}\right] \tag{4}$$

Find the angular momentum in order to find the ascending node line:

$$\vec{h} = \vec{r} \times \vec{v} = \begin{vmatrix} i & j & k \\ 5000 & 8500 & 0 \\ 2.15 & -2.05 & \pm 5.0124 \end{vmatrix} = \begin{pmatrix} \pm 42605.4 \\ \mp 25062 \\ -28525 \end{pmatrix} \tag{5}$$

$$\vec{n} = \vec{z} \times \vec{h} = -h_y \vec{x} + h_x \vec{y} = \begin{pmatrix} \pm 25062 \\ \pm 42605.4 \\ 0 \end{pmatrix} \quad n = 4.9430 \cdot 10^4 \tag{6}$$

The radius at the ascending node is parallel to the ascending node line which means that they have the same sign in every direction:

$$v_{an,3} = 5.0124 \left[\frac{km}{sec}\right] \Rightarrow \vec{v}_{an} = \begin{pmatrix} 2.15 \\ -2.05 \\ 5.0124 \end{pmatrix} \left[\frac{km}{sec}\right] \tag{7}$$

$$\vec{h} = \begin{pmatrix} 42605.4 \\ -25062 \\ -28525 \end{pmatrix} \left[\frac{km^2}{sec}\right] \quad h = 5.7070 \cdot 10^4 \left[\frac{km^2}{sec}\right] \tag{8}$$

The eccentricity is given by:

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r} = \begin{pmatrix} -0.0452 \\ -0.1723 \\ 0.0839 \end{pmatrix} \quad e = 0.1969 \tag{9}$$

The angle of inclination ($0° \leq i \leq 180°$):

$$\cos(i) = \frac{h_z}{h} = \frac{-28525}{57070} \Rightarrow i = 2.0942[rad] = 119.9888° \tag{10}$$

Argument of perigee:

$$\cos(\omega) = \frac{\vec{n} \cdot \vec{e}}{n \cdot e}, \quad \sin(\omega) = \sqrt{1 - \cos^2(\omega)} \tag{11}$$

$$\Downarrow$$

$$\omega = \text{atan2}\left[\text{sign}(e_z) \cdot \sqrt{1 - \left(\frac{\vec{n} \cdot \vec{e}}{n \cdot e}\right)^2}, \frac{\vec{n} \cdot \vec{e}}{n \cdot e}\right] \tag{12}$$

$$\Downarrow$$

$$\omega = 2.6270[rad] = 150.5160° \tag{13}$$

Longitude of ascending node:

$$\cos(\Omega) = \frac{n_x}{n}, \quad \sin(\Omega) = \frac{n_y}{n} \Rightarrow \Omega = \text{atan2}\left(\frac{n_y}{n}, \frac{n_x}{n}\right) \tag{14}$$

$$\Downarrow$$

$$\Omega = 1.0391[rad] = 59.5303° \tag{15}$$

3 Q2

In order to perform the simulation of the orbit based on the equations of motion we need to list all the forces that act on the satellite:

- **Equations of Motion of a Point Mass in a Central Force Field:**
  Vector equation of motion:
  $$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{f} \tag{16}$$
  Where:
  - $\mu = \mathrm{G} \cdot \mathrm{M}$
  - $\vec{r}(t_0) = \vec{r}_0$
  - $\dot{\vec{r}}(t_0) = \frac{d\vec{r}}{dt}(t_0) = \vec{v}_0$

  The equation is based on:
  - Gravitational attraction between two point masses
  - Newton's second law
  - Assumptions: m¡¡M, M is homogeneous sphere
  - $\vec{f}$ is all the external forces

- **$\vec{f}$:**
  1. **Atmospheric Drag:**
     $$\vec{f}_d = -\frac{1}{2}\rho v^2 K_D \vec{v} \tag{17}$$
     Where:
     - ρ is the atmospheric density
     - $K_D = \dfrac{A_{drag} \cdot C_D}{m}$

     Density modle:
     $$\rho(h) = \rho_b \cdot e^{\frac{h-h_b}{H}} \tag{18}$$
     Where:
     - H is the Scale Height
     - ρ_b is the density at the bottom of h_b height layer
     - h_b is the base height
  2. **Earth Oblateness (J_2 term):**
     First order perturbation acceleration:
     $$\begin{cases} f_r &= -\dfrac{3}{2}\dfrac{\mu}{r^2}J_2\left(\dfrac{R_e}{r}\right)^2\left(1 - 3\sin^2(i)\sin^2(\theta^*)\right) \\ f_\theta &= -3\dfrac{\mu}{r^2}J_2\left(\dfrac{R_e}{r}\right)^2 \sin^2(i)\sin(\theta^*)\cos(\theta^*) \\ f_h &= -3\dfrac{\mu}{r^2}J_2\left(\dfrac{R_e}{r}\right)^2 \sin(i)\cos(i)\sin(\theta^*) \end{cases} \tag{19}$$

Where:

- $J_2 = 0.0010826$
- R_e is the earth radius
- $\theta^* = \theta + \omega$
- θ is the true anomaly

And in ECI coordinate system:

$$\begin{cases} f_x &= -\frac{3}{2}\frac{\mu \cdot x}{r^3} J_2 \left(\frac{R_e}{r}\right)^2 \left(5\frac{z^2}{r^2} - 1\right) \\ f_y &= -\frac{3}{2}\frac{\mu \cdot y}{r^3} J_2 \left(\frac{R_e}{r}\right)^2 \left(5\frac{z^2}{r^2} - 1\right) \\ f_z &= -\frac{3}{2}\frac{\mu \cdot z}{r^3} J_2 \left(\frac{R_e}{r}\right)^2 \left(5\frac{z^2}{r^2} - 3\right) \end{cases} \tag{20}$$

3. **Solar Radiation Pressure:**

The solar radiation acceleration is:

$$\vec{f_s} = -(1+\varepsilon)\frac{G_s}{c}\frac{A_{solar}}{m}\hat{r}_{sun} \tag{21}$$

Where:

- ε is the reflectivity coefficient $\in [0,1]$ (in our case $\varepsilon = 1$)
- $G_s = 1358 \left[\frac{W}{m^2}\right]$
- c is the speed of light $\left(2.9979 \cdot 10^8 \left[\frac{m}{s}\right]\right)$

The unit vector from Earh to the sun:

$$\hat{r}_s = \begin{pmatrix} \cos(\delta)\cos(RA) \\ \cos(\delta)\sin(RA) \\ \sin(\delta) \end{pmatrix} \tag{22}$$

Where:

- $RA = 0.98563(N-80)^\circ$
- $\sin(\delta) = 0.39795 \cdot \cos\left(0.98563(N-173)^\circ\right)$
- N = datenum(['our date']) - datenum(['1-Jan']) + 1 (in MatLab)

The angle between the sun and the orbit plane (β):

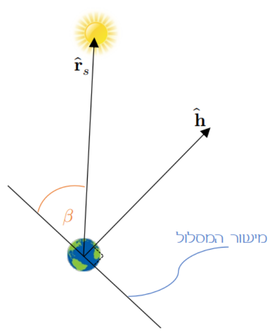

Figure 1: defention of β [1]

$$\hat{h} \cdot \hat{r}_s = |\hat{h}| \cdot |\hat{r}_s| \cdot \cos(90^\circ - \beta) = \sin\beta \tag{23}$$

Where:

- $-90^\circ \leq \beta \leq 90^\circ$

The angle between the projection of $\hat{r}_s$ on the orbit plane and $\hat{e}$ (α):

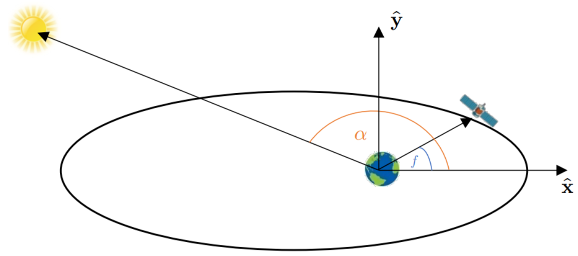

Figure 2: defention of α [1]

$$\begin{cases} sin(\alpha) = -\dfrac{\left(\hat{h} \times \hat{r}_s\right) \cdot \hat{e}}{\cos(\beta)} \\ \cos(\alpha) = \dfrac{\left(\hat{h} \times \left(\hat{r}_s \times \hat{h}\right)\right) \cdot \hat{e}}{\cos(\beta)} \end{cases} \tag{24}$$

Decomposing $\vec{f}_s$ into two forces: $\vec{f}_{sh}$ in the orbit plane, and $\vec{f}_{sp}$ normal to the orbit plane. splitting the horizontal component to a radial and tangent parts:

$$\begin{cases} f_r = \vec{f}_{sh} \cdot \overbrace{(\cos f \hat{x} + \sin f \hat{y})}^{\hat{r}} = -f_{sh}(\cos\alpha \cos f + \sin\alpha \sin f) \\ f_f = \vec{f}_{sh} \cdot \underbrace{(-\sin f \hat{x} + \cos f \hat{y})}_{\hat{f}} = -f_{sh}(\cos\alpha \cos f - \sin\alpha \sin f) \end{cases} \tag{25}$$

<u>Shadow Model</u>:

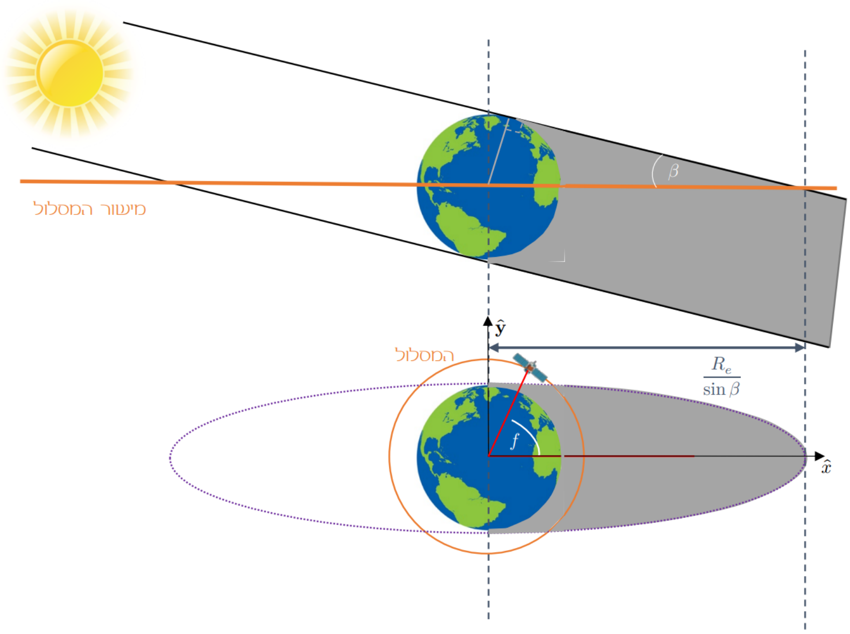

Figure 3: shadow model [1]

References

[1] S. Levi, “Space mechanics - tutorial 10,” 2024.