

Satellite Orbit Control

HW2

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1 Given

$$\begin{aligned}
 T_1 &= 100 [\text{min}] = 6 \cdot 10^3 [\text{sec}] & T_2 &= T_1 = 6 \cdot 10^3 [\text{sec}] \\
 e_1 &= 0 & e_2 &= 0 \\
 a_1 &= \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3 [\text{km}] & a_2 &= a_1 = 7.1366 \cdot 10^3 [\text{km}] \\
 \alpha &= \Delta i = 0.01^\circ
 \end{aligned}$$

In CW fram with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [\text{km}] \quad \begin{pmatrix} \dot{x}_2(0) = ?? \\ \dot{y}_2(0) = ?? \\ \dot{z}_2(0) < 0 \end{pmatrix}$$

Desired:

$$\begin{pmatrix} x_2(t_1) = 0 \\ y_2(t_1) = 0 \\ z_2(t_1) = 0 \end{pmatrix} \quad \begin{pmatrix} \dot{x}_2(t_1) = 0 \\ \dot{y}_2(t_1) = 0 \\ \dot{z}_2(t_1) = 0 \end{pmatrix}$$

Limitations:

$$\Delta v = t \begin{pmatrix} 0 \\ \Delta v_y \\ \Delta v_z \end{pmatrix}$$

2 A

General equations of motion in CW fram:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2z = f_z \end{cases} \quad (1)$$

The velocities:

$$\begin{cases} u_x = \dot{x} - ny(t) \\ u_y = \dot{y} + nx(t) \\ u_z = \dot{z} \end{cases} \quad (2)$$

The solution without external forces (i.e. $\vec{f} = \vec{0}$):

$$\begin{cases} x(t) = (4 - 3\cos(nt)) \cdot x_0 + \frac{\dot{x}_0}{n} \sin(nt) + \frac{2}{n} (1 - \cos(nt)) \cdot \dot{y}_0 \\ y(t) = 6(\sin(nt) - nt) \cdot x_0 + y_0 + \frac{2}{n} (\cos(nt) - 1) \cdot \dot{x}_0 + (4\sin(nt) - 3nt) \frac{\dot{y}}{n} \\ z(t) = z_0 \cos(nt) + \frac{\dot{z}_0}{n} \sin(nt) \end{cases} \quad (3)$$

Because the two satellite have the same period then the motion is periodical. The condition for periodical motion is:

$$\dot{y}_0 = -2nx_0 \quad (4)$$

So the equations of motion becomes:

$$\begin{cases} x(t) = x_0 \cos(nt) + \frac{\dot{x}_0}{n} \sin(nt) \\ y(t) = y_0 + \frac{2}{n} (\cos(nt) - 1) \cdot \dot{x}_0 - 2x_0 \sin(nt) \\ z(t) = z_0 \cos(nt) + \frac{\dot{z}_0}{n} \sin(nt) \end{cases} \quad (5)$$

$$\begin{cases} \dot{x}(t) = -nx_0 \sin(nt) + \dot{x}_0 \cos(nt) \\ \dot{y}(t) = -\frac{2\dot{x}_0}{n} (n \sin(nt) + 1) - 2nx_0 \cos(nt) \\ \dot{z}(t) = -nz_0 \sin(nt) + \dot{z}_0 \cos(nt) \end{cases} \quad (6)$$

From the angle between the plains we can claculate $\dot{z}_2(0)$:

$$\tan \alpha = \frac{z_{max}}{a_1} = \frac{\sqrt{z_2(0)^2 + \left(\frac{\dot{z}_2(0)}{n}\right)^2}}{a_1} \quad (7)$$

$$\frac{\dot{z}_2(0)}{n} = \underbrace{-}_{\dot{z}_2(0) < 0} \sqrt{(a_1 \cdot \tan(\alpha))^2 - z_2(0)^2} \quad (8)$$

$$\frac{\dot{z}_2(0)}{n} = -0.7426; \quad (9)$$

The CW equations in state-space form:

$$\begin{array}{ll} \text{x-y plane:} & \text{z-direction:} \\ x1 = x & z1 = z \\ x2 = \dot{x} & \\ y1 = y & z2 = \dot{z} \\ y2 = \dot{y} & \end{array} \quad (10)$$

The homogeneous solution with no drift condition:

$$\vec{x}(t) = \Phi_{(t,t_0)} \vec{x}_0 \quad (11)$$

x-y plane:

$$\Phi_{(t,t_0)} = \left(\begin{array}{cc|cc} \cos(n\tau) & 0 & \frac{1}{n} \sin(n\tau) & 0 \\ -2n \cos(n\tau) & 1 & \frac{2}{n} (\cos(n\tau) - 1) & 0 \\ \hline -n \sin(n\tau) & 0 & \cos(n\tau) & 0 \\ -2n \cos(n\tau) & 0 & -\frac{2}{n} (n \sin(n\tau) + 1) & 0 \end{array} \right) = \begin{pmatrix} \Phi_{11(t,t_0)} & \Phi_{12(t,t_0)} \\ \Phi_{21(t,t_0)} & \Phi_{22(t,t_0)} \end{pmatrix} \quad (12)$$

z-direction:

$$\Phi_{(t,t_0)} = \begin{pmatrix} \cos(n\tau) & \frac{1}{n} \sin(n\tau) \\ -n \sin(n\tau) & \cos(n\tau) \end{pmatrix}$$

Where:

$$\tau = t - t_0$$

Desired:

$$\begin{pmatrix} x1 \\ y1 \\ x2 \\ y2 \end{pmatrix} (t_1) = \vec{0} \quad (13)$$

$$\begin{pmatrix} z1 \\ z2 \end{pmatrix} (t_1) = \vec{0}$$

The required velocity components are found from the first 2 lines of $\vec{x}(t_1) = \Phi_{(t_1,0)} \vec{x}(0)$:

$$\begin{pmatrix} x(t_1) \\ y(t_1) \end{pmatrix} = \vec{0} = \Phi_{11(t_1,0)} \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} + \Phi_{12(t_1,0)} \begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} \quad (14)$$

\Downarrow

$$\begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} = -\Phi_{12(t_1,0)}^{-1} \Phi_{11(t_1,0)} \begin{pmatrix} x \\ y \end{pmatrix} \equiv \mathbf{C}_{(t_1,0)}^* \begin{pmatrix} x \\ y \end{pmatrix} \quad (15)$$

The terminal velocity is determined from the last two rows of $\vec{x}(t_1) = \Phi(t_1, 0)\vec{x}(0)$:

$$\begin{pmatrix} \dot{x}_{t_1} \\ \dot{y}_{t_1} \end{pmatrix} = \Phi_{21(t_1, 0)} \begin{pmatrix} x \\ y \end{pmatrix} + \Phi_{22(t_1, 0)} \begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} \quad (16)$$

Because we can't apply a velocity pulse in the x direction, we set \dot{x}_{t_1} to be zero. Using *MatLab*, the first equation is:

$$abc \quad (17)$$