Satellite Orbit Control
HW9

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Contents

List of Figures

1   Given

A communication satellite is placed in a geostationary orbit above longitude 30°E.

$$T = 8.6164 \cdot 10^4 \,[\text{sec}] \rightarrow a = 4.2164 \cdot 10^4 \,[\text{km}] \qquad \lambda_n = 30° \qquad e = 0 \qquad i = 0 \qquad (1)$$

The longitude tolerance is $\Delta\lambda = \pm 0.05°$

1.1   Desired

1.2   Limitations

The thrust is only at the $\pm y$ direction and:

$$a_{\max} = 1 \cdot 10^{-7} \left[\frac{\text{km}}{\text{sec}^2}\right] \qquad (2)$$

2   Control

2.1   East-West Control

2.1.1   Drift Duo to axial asymmetry

The gravity component normal to the radius changes the orbit period so the satellite moves toward the closest equilibrium point. The force component creates a tangential acceleration, which is equivalent to the angular acceleration of the longitude line underneath the satellite:

$$a\ddot{\lambda} = \vec{f} \cdot \vec{v} \qquad (3)$$

The change in the semi-major axis and the longitude line over one cycle:

$$\frac{da}{dt} = -12an J_{22} \left(\frac{Re}{a}\right)^2 \sin\left(2\left(\lambda - \lambda_{22}\right)\right)$$

$$\frac{d^2\lambda}{dt^2} = -\frac{3}{2}\frac{n}{a}\frac{da}{dt} \qquad (4)$$

Where:

$$J_{22} = \left(C_{22}^2 + S_{22}^2\right)^{\frac{1}{2}}$$

$$\lambda_{22} = \tan^{-1}\left(\frac{S_{22}}{C_{22}}\right) = -14.9° \qquad (5)$$

After substitution:

$$\dot{a} = 0.132 \sin\left(2\left(\lambda - \lambda_s\right)\right) \left[\frac{\text{km}}{\text{day}}\right]$$

λ_s is the closest stable equilibrium point 75° or 255°

$$\ddot{\lambda} = -0.00168 \sin\left(2\left(\lambda - \lambda_s\right)\right) \left[\frac{\text{deg}}{\text{day}^2}\right] \qquad (6)$$

Since the tolerance is small:

$$\ddot{\lambda} = -K \sin\left(2\left(\lambda_n - \lambda_s\right)\right) = \ddot{\lambda}_n$$

$$\lambda = \frac{1}{2\ddot{\lambda}_n}\left(\dot{\lambda} - \dot{\lambda}_0\right)^2 + \frac{\dot{\lambda}_0}{\ddot{\lambda}_n} + \lambda_0$$

(7)

The initial conditions of the satellite:

$$\lambda_0 = \lambda_n - \Delta\lambda \qquad \dot{\lambda}_0 = -2\,\mathrm{sgn}\left(\ddot{\lambda}_n\right)\sqrt{-\ddot{\lambda}_n\Delta\lambda}$$

(8)

The velocit pulse needes to change $-\dot{\lambda}_0$ into $\dot{\lambda}_0$:

$$\Delta n = 2\dot{\lambda}_0$$

(9)

The relation between the change in angular velocit and the semi-major axis:

$$n^2 = \frac{\mu}{a^3} \rightarrow \frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta n}{n}$$
$$\Downarrow$$
$$\Delta v = \left|\frac{\mu}{2a^2 v}\Delta a\right| = \frac{a}{3}\left|\Delta n\right| = \frac{2a}{3}\left|\dot{\lambda}_0\right|$$

(10)

and the friquency of pulses:

$$t_m = 2\left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right|$$

(11)

The change in altitude between to corections:

$$\Delta a = 0.132\sin\left(2\left(\lambda - \lambda_s\right)\right)t_m\,[\mathrm{km}]$$

(12)

2.2   Drift due to The Tesseral Harmonics

$$\frac{da}{dt} = 6.06\sin\left(2\left(\lambda - 27.8°\mathrm{E}\right)\right) - 1.48\sin\left(4\left(\lambda - 30.28°\mathrm{E}\right)\right) + 0.19\sin\left(2\left(\lambda - 31.92°\mathrm{E}\right)\right)\left[\frac{\mathrm{m}}{\mathrm{day}}\right]$$

$$\frac{d^2\lambda}{dt^2} = \frac{3n}{4a}\frac{da}{dt}$$

(13)

3   The Results

3.1   Part A

$$\ddot{\lambda}_n = -0.00168 \sin\left(2\left(30° - 75°\right)\right) = 0.0017 \left[\frac{\text{deg}}{\text{day}^2}\right]$$

$$\dot{\lambda}_0 = -2 \operatorname{sgn}\left(\ddot{\lambda}_n\right) \sqrt{-\ddot{\lambda}_n \Delta\lambda} = -0.0183 \left[\frac{\text{deg}}{\text{day}}\right] \tag{14}$$

$$\Downarrow$$

$$t_m = 2 \left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right| = 2 \left|\frac{-0.0183}{0.0017}\right| = 21.8218 \,[\text{day}]$$

$$\Downarrow$$

$$\Delta a = 0.132 \sin\left(2\left(30° - 75°\right)\right) t_m = -2.8805 \,[\text{km}]$$

3.2   Part C

A The Code

```matlab
1  clc; clear; close all;
2
3  %% Part A
4
5  mu = 3.986e5; % [km^3/s^2]
6  R_earth = 6378; % [km]
7  T = 23*60*60+56*60+4.09; % [sec]
8  a = (mu * T^2 / 4 / pi^2)^(1/3); % [km]
9  n = 2*pi/T; % [1/sec]
10 f_max = 0.0001e-3; % [km/sec]
11
12 lambda_n_deg = 30;
13 lambda_s_deg = 75;
14
15 lambda_n_ddot = -0.00168*sind(2*(lambda_n_deg - lambda_s_deg))
16
17 delta_lambda_0 = -sign(lambda_n_ddot)*0.05; % [deg]
18 delta_lambda = abs(delta_lambda_0);
19 Delta_L = delta_lambda * pi / 180 * a; % [km]
20
21 lambda_0_dot = -2*sign(lambda_n_ddot)*sqrt(-lambda_n_ddot*delta_lambda_0)
22
23 tm = 2*abs(lambda_0_dot / lambda_n_ddot)
24
25 delta_a = 0.132*sind(2*(lambda_n_deg - lambda_s_deg))*tm
26
27 %% Part C
28 time_interval = 0:1:365*3;
29
30 state = [lambda_n_deg; 0; a]; % [lambda, labmbda_dot, a]
31
32  % This is where we integrate the equations of motion.
33 [t_out, state_out] = ode45(@ODE, time_interval, state, odeset('RelTol',5e-14,'
      AbsTol',5e-14));
34
35 fig1 = figure ("Name","1",'Position',[100 300 900 500]);
36
37 plot(t_out/365, state_out(:,1), "LineWidth",1.5)
38
39 xlabel('time [year]','FontSize',15,Interpreter='latex')
40 ylabel('$\lambda$ [deg]','FontSize',15,Interpreter='latex')
41 grid on
42 grid minor
43 title("Drift due to The Tesseral Harmonics")
44 subtitle("Almog Dobrescu 214254252")
45 % legend({''},'FontSize',11 ,'Location','northwest')
46 % exportgraphics(fig1, 'graph1.png','Resolution',300);
47
48
```

```matlab
49
50  %% Functions
51  function d_state_dt = ODE(t, state)
52      mu = 3.986e5; % [km^3/s^2]
53
54      lambda = state(1);
55      lambda_dot = state(2);
56      a = state(3);
57
58      n = sqrt(mu/a^3);
59
60      a_dot = 6.06*sind(2*(lambda-27.8))-1.48*sind(4*(lambda-30.28))+0.19*sind(2*(
            lambda-31.92));
61
62      d_state_dt = [lambda_dot; 3*n/4/a*a_dot; a_dot];
63  end
```

Listing 1: The code