Satellite Orbit Control
HW3

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1 Given

$$T_1 = 100\,[min] = 6 \cdot 10^3\,[sec] \qquad T_2 = T_1 = 6 \cdot 10^3\,[sec]$$
$$e_1 = 0 \qquad e_2 = 0$$
$$a_1 = \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3\,[km] \qquad a_2 = a_1 = 7.1366 \cdot 10^3\,[km]$$
$$\alpha = \Delta i = 0.01^\circ$$

In CW frame with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [\mathrm{km}] \qquad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.74267 \cdot n \end{pmatrix} \left[\frac{\mathrm{km}}{\mathrm{sec}}\right]$$

1.1 Desired

$$\begin{pmatrix} x_2(t_f) = 0 \\ y_2(t_f) = 0 \\ z_2(t_f) = 0 \end{pmatrix} \qquad \begin{pmatrix} \dot{x}_2(t_f) = 0 \\ \dot{y}_2(t_f) = 0 \\ \dot{z}_2(t_f) = 0 \end{pmatrix}$$

1.2 Limitations

The engine can't create trust in the x direation and:

$$a_{\max} = 8 \cdot 10^{-6} \left[\frac{\mathrm{km}}{\mathrm{sec}^2}\right]$$

2 The CW equations

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{1}$$

2.1 x-y

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{2}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{3}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & 0 & 1 \\ 0 & -2n & 0 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{4}$$

2.2 z

$$\vec{x} = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \tag{5}$$

$$\dot{\vec{x}} = F\vec{x} + Gf \tag{6}$$

Where:

$$F = \begin{pmatrix} 0 & 1 \\ -n^2 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad f = \begin{pmatrix} f_z \end{pmatrix} \tag{7}$$

3 case A

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{pmatrix} \tag{8}$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \tag{9}$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \tag{10}$$

The desired maneuver time is 2000 [sec]. The desired approach trajectory is a straight line with constant velocity from the initial point to the origin.
From geometric considerations, the required velocity on the straight line is:

$$\dot{\vec{x}}_{req} = \begin{pmatrix} \dfrac{x_f - x_0}{t_f} \\ \\ \dfrac{y_f - y_0}{t_f} \\ \\ \dfrac{z_f - z_0}{t_f} \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix} \cdot 10^{-4} \left[\frac{\text{km}}{\text{sec}^2}\right] \tag{11}$$

Which mean that the initial pulse:

$$\Delta v_1 = \dot{\vec{x}}_{req} - \dot{\vec{x}}_0 = \begin{pmatrix} 0 \\ 0.5 \\ 0.2777 \end{pmatrix} \cdot 10^{-3} \left[\tfrac{\text{km}}{\text{sec}^2}\right] \tag{12}$$

and the final pulse:

$$\Delta v_f = \dot{\vec{x}}_f - \dot{\vec{x}}_{req} = \begin{pmatrix} 0 \\ -5 \\ 5 \end{pmatrix} \cdot 10^{-4} \left[\tfrac{\text{km}}{\text{sec}^2}\right] \tag{13}$$

Since the velocity is constant:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \vec{0}$$

By substituting the constraints in the CW equations we get:

$$\begin{cases} 0 - 2n\dot{y} = f_x \\ 0 = f_y \\ 0 + n^2 z = f_z \end{cases} \tag{14}$$

$$\vec{a} = \vec{f} = \begin{pmatrix} -2n \cdot 0.5 \cdot 10^{-3} \\ 0 \\ n^2 z \end{pmatrix} \tag{15}$$

Check that the maximum acceleration hasn't been reached:

$$\begin{gathered} |\vec{a}| = \sqrt{(2n \cdot 0.5 \cdot 10^{-3}) + (n^2 z)} < \sqrt{(2n \cdot 0.5 \cdot 10^{-3}) + (n^2 \cdot 1)} \\ |\vec{a}| < 1.5163 \cdot 10^{-6} < 8 \cdot 10^{-6} \qquad \checkmark \end{gathered} \tag{16}$$