

# Satellite Orbit Control

## HW5

Almog Dobrescu

ID 214254252

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## 1 Given

$$\begin{aligned}
 T_1 &= 100 [\text{min}] = 6 \cdot 10^3 [\text{sec}] & T_2 &= T_1 = 6 \cdot 10^3 [\text{sec}] \\
 e_1 &= 0 & e_2 &= 0 \\
 a_1 &= \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366 \cdot 10^3 [\text{km}] & a_2 &= a_1 = 7.1366 \cdot 10^3 [\text{km}] \\
 \alpha &= \Delta i = 0.01^\circ
 \end{aligned}$$

In CW frame with origin at Satellite #1 and at  $t = 0$ :

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [\text{km}] \quad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.74267 \cdot n \end{pmatrix} \left[ \frac{\text{km}}{\text{sec}} \right]$$

### 1.1 Desired

$$\begin{pmatrix} x_2(t_f) = 0 \\ y_2(t_f) = 0 \\ z_2(t_f) = 0 \end{pmatrix} \quad \begin{pmatrix} \dot{x}_2(t_f) = 0 \\ \dot{y}_2(t_f) = 0 \\ \dot{z}_2(t_f) = 0 \end{pmatrix}$$

### 1.2 Limitations

$$a_{\max} = 4 \cdot 10^{-5} \left[ \frac{\text{km}}{\text{sec}^2} \right]$$

## 2 The CW equations

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2z = f_z \end{cases} \quad (1)$$

### 2.1 x-y

$$\vec{x} = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \quad (2)$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \quad (3)$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & 0 & 1 \\ 0 & -2n & 0 & 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \vec{f} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} \quad (4)$$

### 2.2 z

$$\vec{x} = \begin{pmatrix} z \\ \dot{z} \end{pmatrix} \quad (5)$$

$$\dot{\vec{x}} = F\vec{x} + Gf \quad (6)$$

Where:

$$F = \begin{pmatrix} 0 & 1 \\ -n^2 & 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad f = (f_z) \quad (7)$$

### 2.3 x-y-z

The equations of motion in state space form are therefor:

$$\vec{x} = (x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z})^T \quad (8)$$

$$\dot{\vec{x}} = F\vec{x} + G\vec{f} \quad (9)$$

Where:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \quad (10)$$

## 3 Target Trajectory

The target trajectory:

$$\dot{\vec{x}}_r = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{F_r} \vec{x}_r \quad (11)$$

The initial state for the approach trajectory:

$$\vec{x}_{r(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ |v_{ref}| \cdot t_f \\ -v_{ref} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.06 \\ -3 \cdot 10^{-5} \end{pmatrix} \quad (12)$$

The system will be solved by subtracting the equation of motion of the target trajectory from the equation of motion of the satellite

$$\vec{\delta x} = \vec{x} - \vec{x}_r \quad (13)$$

$$\dot{\vec{\delta x}} = F\vec{\delta x} + (F - F_r)\vec{x}_r + G\vec{f} \quad \text{Where : } G\vec{f} = -(F - F_r)\vec{x}_r - GK\vec{\delta x} \quad (14)$$

$\vec{x}_r$  is determined by solving Eq.11

## 4 Poles and Gains

The desired poles are given by the following equation:

$$P = 10 \cdot \begin{pmatrix} -n + i \cdot n \\ -n - i \cdot n \\ -4n + i \cdot 3n \\ -4n - i \cdot 3n \\ -3n + i \cdot n \\ -3n - i \cdot n \end{pmatrix} = \begin{pmatrix} -0.0105 + i \cdot 0.0105 \\ -0.0105 - i \cdot 0.0105 \\ -0.0419 + i \cdot 0.0314 \\ -0.0419 - i \cdot 0.0314 \\ -0.0314 + i \cdot 0.0105 \\ -0.0314 - i \cdot 0.0105 \end{pmatrix} \quad (15)$$

By using the function *place* in Matlab, we get:

$$K = \begin{pmatrix} 0.0014 & 0.0649 & -0.0009 & -0.0240 & 0.0002 & 0.0201 \\ -0.0002 & -0.0012 & 0.0007 & 0.0516 & -0.0004 & -0.0062 \\ 0.0001 & -0.0138 & 0 & -0.0068 & 0.0007 & 0.0511 \end{pmatrix} \quad (16)$$

## 5 The Results

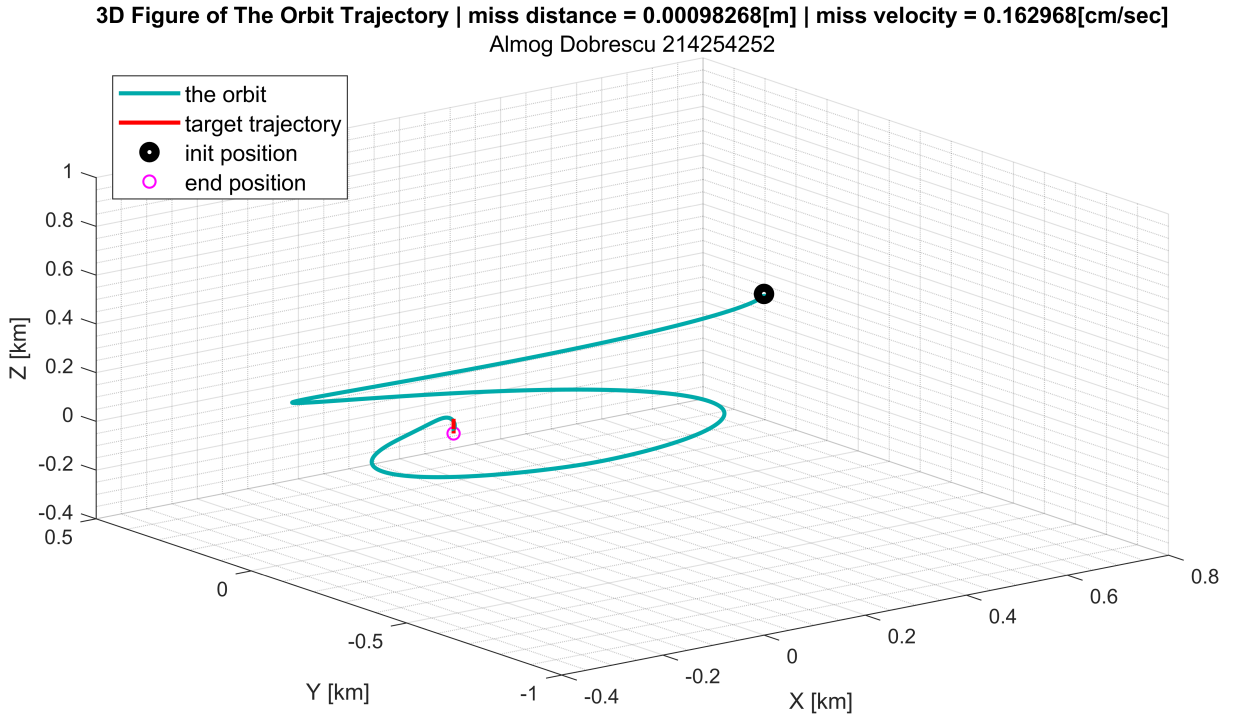


Figure 1: 3D figure of the orbit trajectory

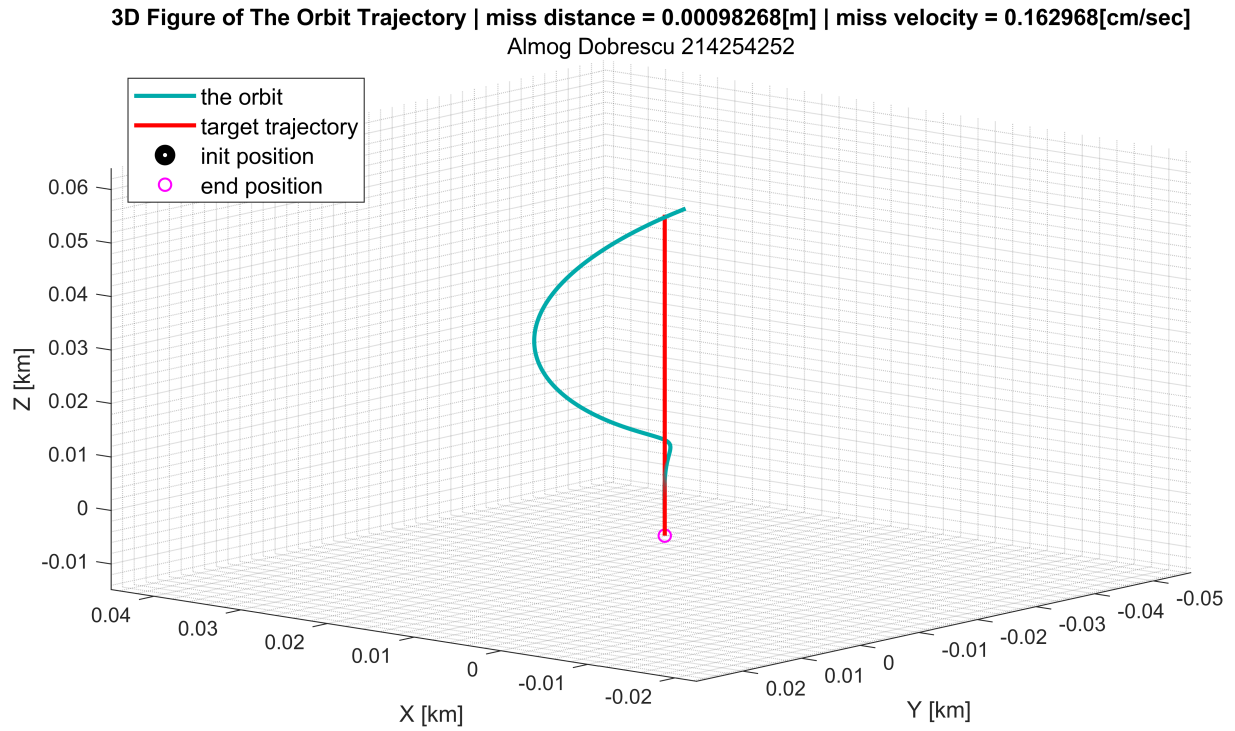


Figure 2: 3D figure of the orbit trajectory - zoomed

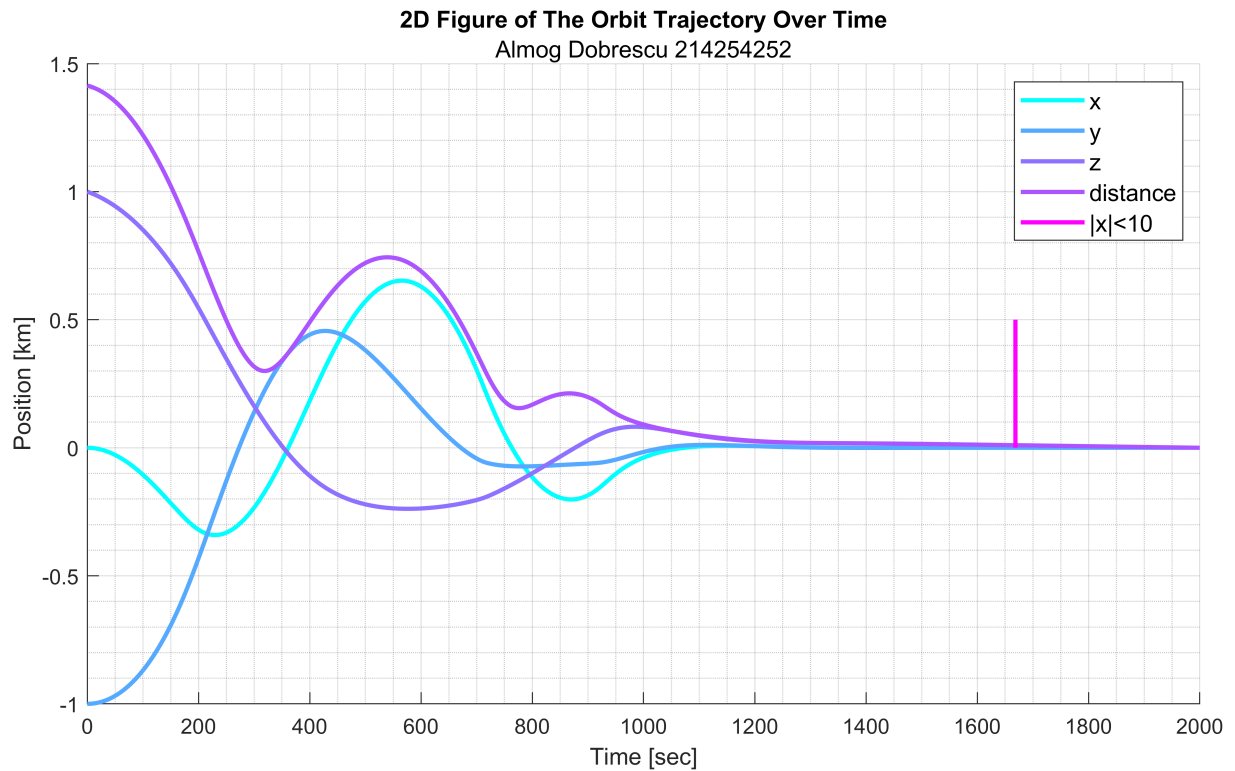


Figure 3: 2D figure of the position over time

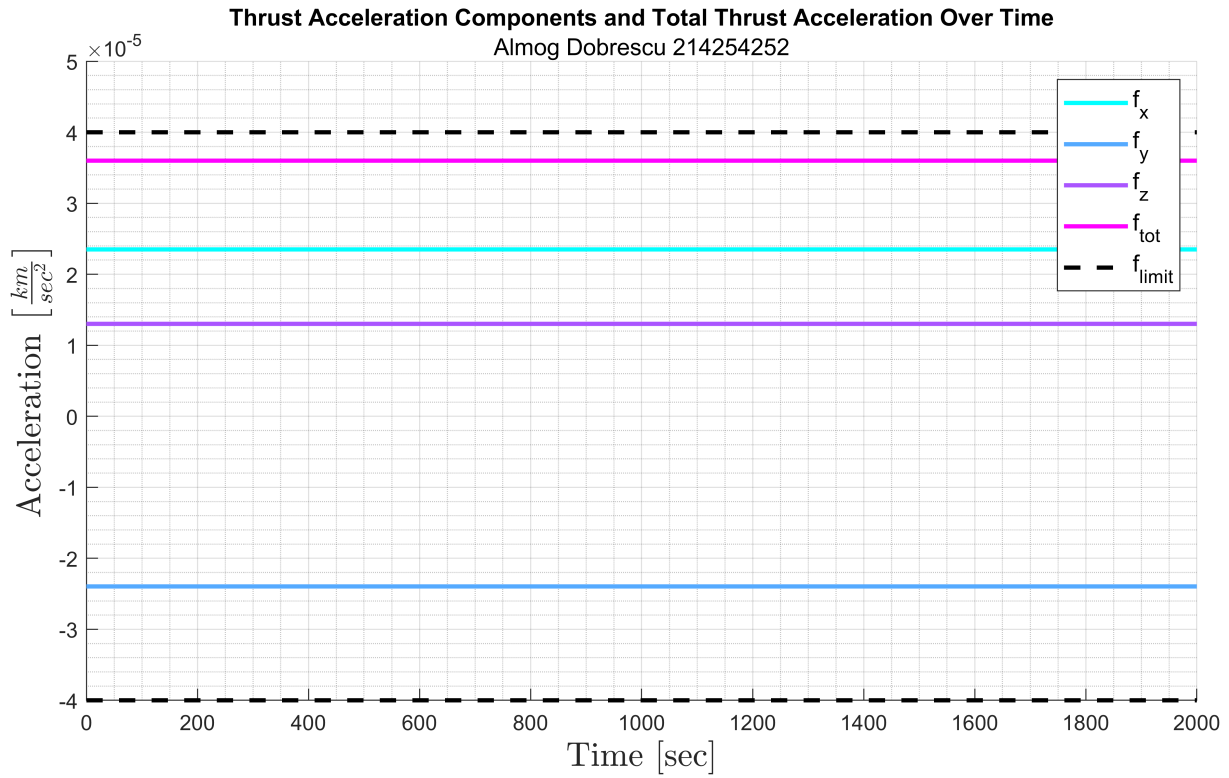
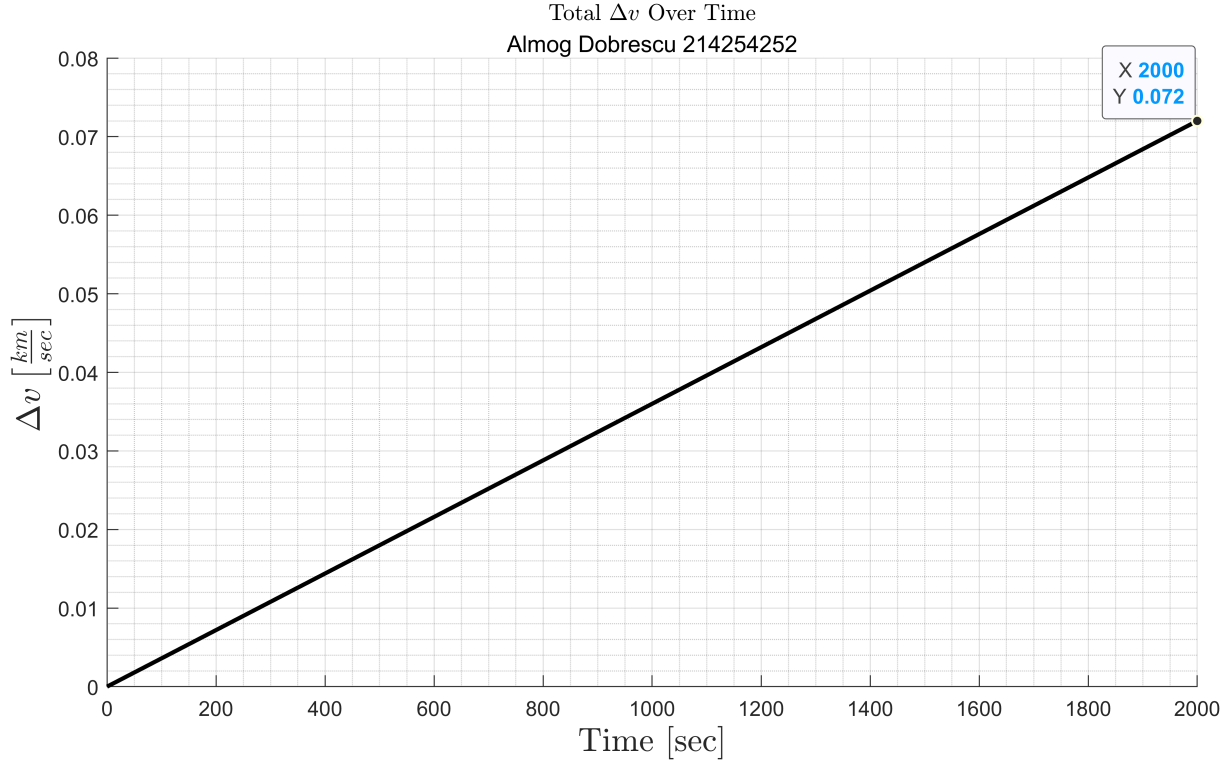


Figure 4: Thrust acceleration components and total thrust acceleration over time

Figure 5: Total  $\Delta v$  over time

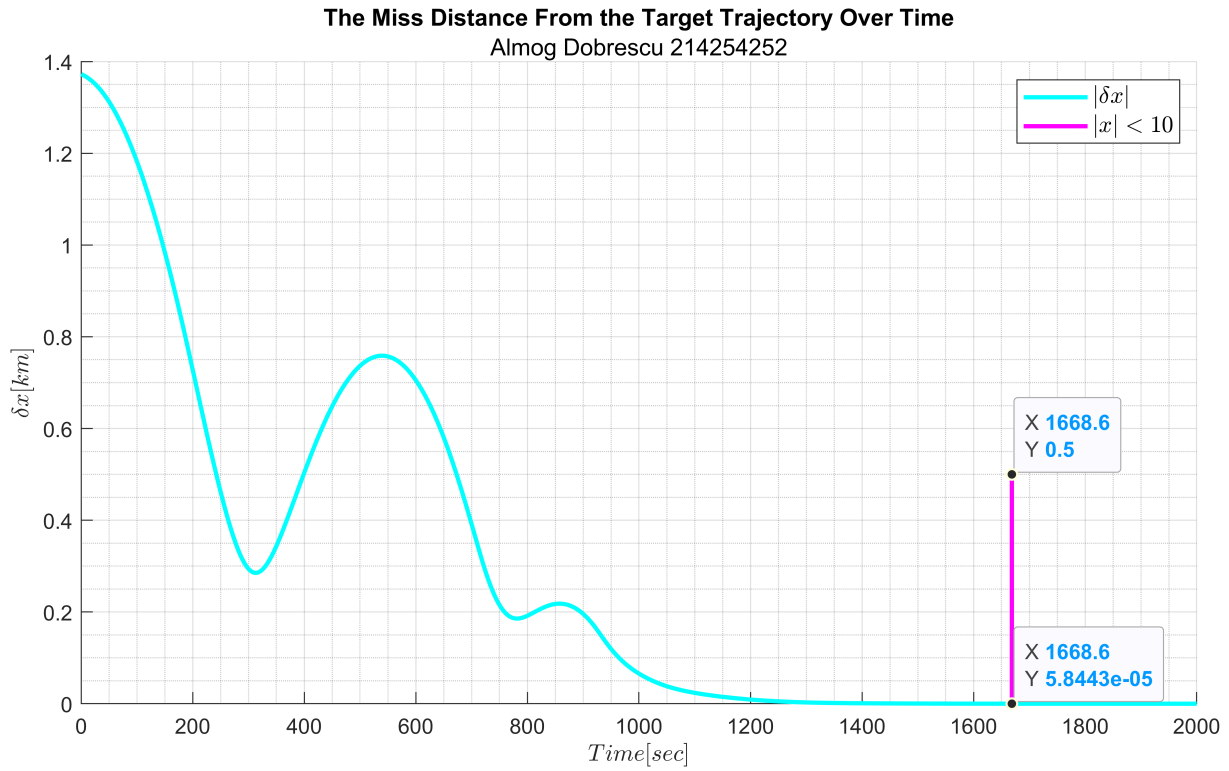


Figure 6: Distance from target trajectory over time

We can see that we indeed accomplished the design criteria:

- The thrust doesn't exceed the maximum available thrust.
- The time at which we are at the last  $10[m]$  is  $t = 1668.6[sec]$ . At this time, the distance from the target trajectory is  $5.8443 \cdot 10^{-5}[km] = 5.8443 \cdot 10^{-2}[m] < 1[m]$

The total  $\Delta v$  is:  $0.072 \left[ \frac{km}{sec} \right]$