# Satellite Orbit Control
# HW9


Almog Dobrescu

ID 214254252


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# 1 Given

A communication satellite is placed in a geostationary orbit above longitude 30°E.

$$T = 8.6164 \cdot 10^4\,[\text{sec}] \rightarrow a = 4.2164 \cdot 10^4\,[\text{km}] \qquad \lambda_n = 30^\circ \qquad e = 0 \qquad i = 0 \tag{1}$$

The longitude tolerance is $\Delta\lambda = \pm 0.05^\circ$

## 1.1 Desired

## 1.2 Limitations

The thrust is only at the $\pm y$ direction and:

$$a_{\max} = 1 \cdot 10^{-7} \left[\frac{\text{km}}{\text{sec}^2}\right] \tag{2}$$

# 2 Control

## 2.1 East-West Control

### 2.1.1 Drift Duo to axial asymmetry

The gravity component normal to the radius changes the orbit period so the satellite moves toward the closest equilibrium point. The force component creates a tangential acceleration, which is equivalent to the angular acceleration of the longitude line underneath the satellite:

$$a\ddot{\lambda} = \vec{f} \cdot \vec{v} \tag{3}$$

The change in the semi-major axis and the longitude line over one cycle:

$$\begin{aligned} \frac{da}{dt} &= -12anJ_{22}\left(\frac{Re}{a}\right)^2 \sin\left(2\left(\lambda - \lambda_{22}\right)\right) \\ \frac{d^2\lambda}{dt^2} &= -\frac{3}{2}\frac{n}{a}\frac{da}{dt} \end{aligned} \tag{4}$$

Where:

$$\begin{aligned} J_{22} &= \left(C_{22}^2 + S_{22}^2\right)^{\frac{1}{2}} \\ \lambda_{22} &= \tan^{-1}\left(\frac{S_{22}}{C_{22}}\right) = -14.9^\circ \end{aligned} \tag{5}$$

After substitution:

$$\begin{aligned} \dot{a} &= 0.132\sin\left(2\left(\lambda - \lambda_s\right)\right)\left[\frac{\text{km}}{\text{day}}\right] \\ \ddot{\lambda} &= -0.00168\sin\left(2\left(\lambda - \lambda_s\right)\right)\left[\frac{\text{deg}}{\text{day}^2}\right] \end{aligned} \qquad \begin{array}{c}\lambda_s \text{ is the closest stable} \\ \text{equilibrium point } 75^\circ \text{ or } 255^\circ\end{array} \tag{6}$$

Since the tolerance is small:

$$\begin{aligned}\ddot{\lambda} &= -K\sin\left(2\left(\lambda_n-\lambda_s\right)\right)=\ddot{\lambda}_n \\ \lambda &= \frac{1}{2\ddot{\lambda}_n}\left(\dot{\lambda}-\dot{\lambda}_0\right)^2+\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}+\lambda_0\end{aligned} \tag{7}$$

The initial conditions of the satellite:

$$\lambda_0=\lambda_n-\Delta\lambda \qquad \dot{\lambda}_0=-2\ \mathrm{sgn}\left(\ddot{\lambda}_n\right)\sqrt{-\ddot{\lambda}_n\Delta\lambda} \tag{8}$$

The velocit pulse needes to change $-\dot{\lambda}_0$ into $\dot{\lambda}_0$:

$$\Delta n=2\dot{\lambda}_0 \tag{9}$$

The relation between the change in angular velocit and the semi-major axis:

$$\begin{gathered} n^2=\frac{\mu}{a^3}\rightarrow\frac{\Delta a}{a}=-\frac{2}{3}\frac{\Delta n}{n} \\ \Downarrow \\ \Delta v=\left|\frac{\mu}{2a^2v}\Delta a\right|=\frac{a}{3}\left|\Delta n\right|=\frac{2a}{3}\left|\dot{\lambda}_0\right| \end{gathered} \tag{10}$$

and the friquency of pulses:

$$t_m=2\left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right| \tag{11}$$

The change in altitude between to corections:

$$\Delta a=0.132\sin\left(2\left(\lambda-\lambda_s\right)\right)t_m\,[\mathrm{km}] \tag{12}$$

## 2.2 Drift due to The Tesseral Harmonics

# 3 The Results

## 3.1 Part A

$$
\begin{aligned}
\ddot{\lambda}_n &= -0.00168 \sin\left(2\left(30^\circ - 75^\circ\right)\right) = 0.0017 \left[\frac{\text{deg}}{\text{day}^2}\right] \\
\dot{\lambda}_0 &= -2 \operatorname{sgn}\left(\ddot{\lambda}_n\right) \sqrt{-\ddot{\lambda}_n \Delta\lambda} = -0.0183 \left[\frac{\text{deg}}{\text{day}}\right] \\
& \Downarrow \\
& \boxed{t_m = 2\left|\frac{\dot{\lambda}_0}{\ddot{\lambda}_n}\right| = 2\left|\frac{-0.0183}{0.0017}\right| = 21.8218\,[\text{day}]} \\
& \Downarrow \\
& \boxed{\Delta a = 0.132 \sin\left(2\left(30^\circ - 75^\circ\right)\right) t_m = -2.8805\,[\text{km}]}
\end{aligned}
\tag{13}
$$

## 3.2 Part C