# Satellite Orbit Control
# HW2

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# 1 Given

$$T_1 = 100\,[min] = 6\cdot 10^3\,[sec] \qquad T_2 = T_1 = 6\cdot 10^3\,[sec]$$
$$e_1 = 0 \qquad e_2 = 0$$
$$a_1 = \sqrt[3]{\frac{\mu T_1^2}{4\pi^2}} = 7.1366\cdot 10^3\,[km] \qquad a_2 = a_1 = 7.1366\cdot 10^3\,[km]$$
$$\alpha = \Delta i = 0.01^\circ$$

In CW frame with origin at Satellite #1 and at $t = 0$:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [km] \qquad \begin{pmatrix} \dot{x}_2(0) = ?? \\ \dot{y}_2(0) = ?? \\ \dot{z}_2(0) < 0 \end{pmatrix}$$

Desired:

$$\begin{pmatrix} x_2(t_1) = 0 \\ y_2(t_1) = 0 \\ z_2(t_1) = 0 \end{pmatrix} \qquad \begin{pmatrix} \dot{x}_2(t_1) = 0 \\ \dot{y}_2(t_1) = 0 \\ \dot{z}_2(t_1) = 0 \end{pmatrix}$$

Limitations:

$$\Delta v = t \begin{pmatrix} 0 \\ \Delta v_y \\ \Delta v_z \end{pmatrix}$$

# 2 A

General equations of motion in CW fram:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{1}$$

The velocities:

$$\begin{cases} u_x = \dot{x} - ny(t) \\ u_y = \dot{y} + nx(t) \\ u_z = \dot{z} \end{cases} \tag{2}$$

The solution without external forces (i.e. $\vec{f} = \vec{0}$):

$$\begin{cases} x(t) = (4 - 3\cos(nt))\cdot x_0 + \dfrac{\dot{x}_0}{n}\sin(nt) + \dfrac{2}{n}(1-\cos(nt))\cdot \dot{y}_0 \\ y(t) = 6\left(\sin(nt) - nt\right)\cdot x_0 + y_0 + \dfrac{2}{n}(\cos(nt) - 1)\cdot \dot{x}_0 + (4\sin(nt) - 3nt)\,\dfrac{\dot{y}}{n} \\ z(t) = z_0\cos(nt) + \dfrac{\dot{z}_0}{n}\sin(nt) \end{cases} \tag{3}$$

Because the two satellite have the same period, then in CW fram, it is a no-drift orbit:

$$\dot{y}_0 = -2nx_0 \tag{4}$$

From the angle between the plains we can claculate $\dot{z}_2(0)$:

$$\tan\alpha = \frac{z_{max}}{a_1} = \frac{\sqrt{z_2(0)^2 + \left(\frac{\dot{z}_2(0)}{n}\right)^2}}{a_1} \tag{5}$$

$$\frac{\dot{z}_2(0)}{n} = \underbrace{-}_{\dot{z}_2(0)<0}\sqrt{(a_1\cdot\tan(\alpha))^2 - z_2(0)^2} \tag{6}$$

$$\frac{\dot{z}_2(0)}{n} = -0.7426; \tag{7}$$

Because we can't apply a velocity pulse in the x direction, $\dot{x}_2(0) = 0$. The CW frame is therefor:

$$\begin{pmatrix} x_2(0) = 0 \\ y_2(0) = -1 \\ z_2(0) = 1 \end{pmatrix} [km] \qquad \begin{pmatrix} \dot{x}_2(0) = 0 \\ \dot{y}_2(0) = 0 \\ \dot{z}_2(0) = -0.7426 \end{pmatrix} \left[\frac{km}{sec}\right] \tag{8}$$

The CW equations in state-space form:

$$\begin{array}{cc} \text{x-y plane:} & \text{z-direction:} \\ x1 = x & \\ x2 = \dot{x} & z1 = z \\ y1 = y & \\ y2 = \dot{y} & z2 = \dot{z} \end{array} \tag{9}$$

The homogeneous solution:

$$\vec{x}(t) = \Phi_{(t,t_0)}\vec{x}_0 \tag{10}$$

x-y plane:

$$\Phi_{(t,t_0)} = \left(\begin{array}{cc|cc} 4 - 3\cos(n\tau) & 0 & \frac{1}{n}\sin(n\tau) & \frac{2}{n}(1 - \cos(n\tau)) \\ 6(\sin(n\tau) - n\tau) & 1 & \frac{2}{n}(\cos(n\tau) - 1) & \frac{1}{n}(4\sin(n\tau - 3n\tau)) \\ \hline 3n\sin(n\tau) & 0 & \cos(n\tau) & 2\sin(n\tau) \\ 6n(\cos(n\tau) - 1) & 0 & -2\sin(n\tau) & 4\cos(n\tau) - 3 \end{array}\right) = \begin{pmatrix} \Phi_{11(t,t_0)} & \Phi_{12(t,t_0)} \\ \Phi_{21(t,t_0)} & \Phi_{22(t,t_0)} \end{pmatrix}$$

z-direction:

$$\Phi_{(t,t_0)} = \begin{pmatrix} \cos(n\tau) & \frac{1}{n}\sin(n\tau) \\ -n\sin(n\tau) & \cos(n\tau) \end{pmatrix} \tag{11}$$

Where:

$$\tau = t - t_0$$

Desired:

$$\begin{pmatrix} x1 \\ y1 \\ x2 \\ y2 \end{pmatrix}(t_1) = \vec{0} \qquad \begin{pmatrix} z1 \\ z2 \end{pmatrix}(t_1) = \vec{0} \tag{12}$$

The required velocity components are found from the first 2 lines of $\vec{x}(t_1) = \Phi_(t_1, 0)x\vec{(0)}$:

$$\begin{pmatrix} x_{(t_1)} \\ y_{(t_1)} \end{pmatrix} = \vec{0} = \Phi_{11(t_1,0)}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} + \Phi_{12(t_1,0)}\begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} \tag{13}$$

$$\Downarrow$$

$$\begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} = -\Phi^{-1}_{12(t_1,0)}\Phi_{11(t_1,0)}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} \equiv \mathbf{C}^*_{(t_1,0)}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} \tag{14}$$

The terminal velocity is determined from the last two rows of $\vec{x}(t_1) = \Phi_(t_1, 0)x\vec{(0)}$:

$$\begin{pmatrix} \dot{x}_{t_1} \\ \dot{y}_{t_1} \end{pmatrix} = \Phi_{21(t_1,0)}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} + \Phi_{22(t_1,0)}\begin{pmatrix} \dot{x}_{req} \\ \dot{y}_{req} \end{pmatrix} \tag{15}$$

Because we can't apply a velocity pulse in the x direction, we set $\dot{x}_{t_1}$ to be zero. Using *MatLab*, the first equation after substituting the initial conditions is:

$$\frac{2\pi\left(\cos\left(\frac{\pi\tau}{3000}\right)-1\right)}{24000\cos\left(\frac{\pi\tau}{3000}\right)+3\tau\pi\sin\left(\frac{\pi\tau}{3000}\right)-24000}=0 \tag{16}$$

$$\cos\left(\frac{\pi\tau}{3000}\right)=1 \tag{17}$$

$$\frac{\pi\tau}{3000}=2\pi \tag{18}$$

$$\pi\tau=6000\pi \tag{19}$$

$$\tau=6000 \tag{20}$$

$$t_1=\tau=6000\,[sec] \qquad \text{(exactly on period)}$$

# 3 B