Machine Learning

Ariel University

Semester II, 2022

Problem 1.

Consider a die with six sides, and values {1,2,3,4,5,6}. You will roll the die **m** times. How large must **m** be in order to simultaneously estimate the bias of every side within ±**.01**, with probability at least **99%**?

Problem 2.

Let our set of rules **H** include all programs of size exactly **50** bits. Suppose we sampled **n** points from a distribution **D** and found that one of the programs is consistent with **85**% of the sample points.

a) What is the smallest **δ** for which we can say that: With probability at least 1− **δ**, the program misclassifies at most a **.15+ϵ** fraction of all points of **D**? (**δ** is a function of **n,ϵ**.)
b) Now suppose **n=1000**. Give a minimum value ϵ such that with probability at least **95**%, the program misclassifies at most a **.15+ϵ** fraction of all points of **D**.
c) In problem (b) above, what can you say about the true error of the program on points of a different distribution **D'**?

Problem 3.

Consider the following card game: We have a (infinite) deck of cards with values {1, 2, 3, 4}, and a distribution **D** assigning each value the probability **¼**. In each round, players **A** and **B** both draw two cards according to **D**. The player with the higher sum wins the round. If there is a tie, neither player wins the round.

a) What's the probability that **A** wins a given round? What's the probability that **B** wins a given round? What is the probability of a tie?
b) If there are **100** rounds, what is the expected number of rounds that **A** wins? What is the expected number of ties?
c) If there are **100** rounds, give an upper bound on the probability that **A** will win 50 or more rounds. Derive bounds using Markov, Chernoff and Hoeffding.
d) At least how many rounds does **A** need to play so that with at least 99% probability, **A** will win **35** or more rounds? Derive bounds using Chernoff and Hoeffding.