Introduction to Data Processing and Representation

(236201)

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**Homework 1**

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**Theory**

**Question 1 - Solving the problem using the solution**

1. Under the constraints and when

Similarly:

And the optimal value for each will be .

Under the same constraints and when the optimal value will be, for each

1. For general w(x)

Let’s differentiate, with respect to :

Find the minimum:

Thus, for each , will be the optimal.

1. For general w(x), and given :

Let’s differentiate, with respect to :

Like in the lecture, for :

Find the minimum, while notate:

– points in where

– points in where

* The minimum is given when: , meaning at the weighted median of f(x), as the weighted values of f(x) above the optimal equal to the weighted values of f(x) below it.

Then Thus, for each , the optimal will be the weighted median of f(x) in .

1. Let’s define the functions for and look at the optimization problem - :

*for* by the definition of , , therefore:

So:

1. We wish to find , s.t:

Let’s define (given that ), therefore:

1. Given w(x) from the optimizing problem, let’s define:

= and we’ll get:

1. If was indepented of it’d be much easier problem as we won’t take it into account when differentiating, and we’ll get the solution we got in b:
2. Without the assumption of =

may not be defined or the error may diverge to .

But when replacing with fixed (limited) we won’t be able to diverge as we’ll have an upper limit.

1. Algo:
2. Init
3. While
4. *Return*
5. Algo:

1.Init   
2. For

3. return

1. Hmm idk, remind me some kind of gradient decent algo

**Implementation**

**Question 1 – Quantization**

*A person sitting on a couch with a can of beer

Description automatically generated*The image we chose: and it’s 256 gray levels version:

A person sitting on a couch with a can of soda

Description automatically generated

1. Our image histogram:  
    A graph of a graph

   Description automatically generated

The histogram doesn’t seems too uniform.

1. a. MSE with different number of quantization bits:

A graph with a line

Description automatically generated

As excepted, the more bits we have, the smaller the MSE we’ll get.  
We can also see, that where the number of bits is 8, which is the actual amounts of bit the original gray image use – we’ll get 0 MSE, as we’ll get the original gray image again.

b. We chose 1,2,3,4 bits for the representative b values (as the rest are too noisy):

A graph with a line and a line

Description automatically generated with medium confidenceA graph with a line and a line

Description automatically generated with medium confidence

A graph with a blue line

Description automatically generatedA graph with a blue line

Description automatically generated

1. Implemented the Max-Lloyd Algorithm – max\_lloyd\_algo, and ran it with  
   epsilon=0.001.
2. After applying the Max-Lloyd quantizer starting with uniform quantization:
   1. The MSE graph against the values of b = 1,…,8:  
      A graph with a line

      Description automatically generated

As expected, as the representation bits budgets is higher, the MSE is lower.

* 1. A graph with a line and a line

     Description automatically generated with medium confidenceWe chose 1,2,3,4 bits for the representative b values (as the rest are too noisy):

A graph with a line and a line

Description automatically generated

A graph of a line graph

Description automatically generated with medium confidence

A graph with a line and a line

Description automatically generated with medium confidence

* 1. First let’s compare between the MSE graphs:  
     A graph of a graph with blue and orange lines

     Description automatically generated  
     Even though both the MSE converged to 0 at when the number of quantization bits is 8, it seems like overall, the Max-Lloyd quantization performed a bit better than the Uniform one.  
     This was due to the fact that the decision boundaries are determined using also the the PDF, and take into account the density of the most common levels (in the image/histogram) – as we can see at the histogram, the PDF is denser at lower levels of gray, which lad to denser decision boundaries at this area (as can be seen in 4.b).

**Question 2 – Subsampling and Reconstruction**

1. a. The subsampled images for - with MSE sensing:

|  |  |  |  |
| --- | --- | --- | --- |
| A graph of a number of squares  Description automatically generated with medium confidence | A graph of a number of squares  Description automatically generated with medium confidence | A graph of a number of squares  Description automatically generated with medium confidence | A graph of a number of images  Description automatically generated with medium confidence |
| A person sitting in a chair  Description automatically generated | A person sitting in a chair  Description automatically generated | A person sitting on a couch with a can  Description automatically generated | A person sitting on a couch with a can of food  Description automatically generated |

A graph with a line

Description automatically generated

And the MSE graph:

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1. The subsampled images for - with MAD sensing:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |

A graph with a line

Description automatically generated  
And the MAD graph:

1. The reconstructed images for - with MSE sensing:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |

The reconstructed images for - with MSE sensing:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |

1. For both of the MAD and MSE results, the integer sub-sampling factor D affects the “resolution” of the re-constructed images: smaller values of D, means fewer sampling but bigger domains (on the reconstructed image), which led to 1 value (mean or median) that supposed to represent bigger domains (smaller D -> bigger domains).  
   Therefore, not only the images will look better when D is bigger, but also, as expected, the error (MAD or MSE) will be lower.