Includes Sixteen Lab Experiments

Basic Electrical Engineering

D C Kulshreshtha

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ABOUT THE AUTHOR

D C Kulshreshtha is working since July 2003 as Professor in Electronics and Communication Engineering, **Jaypee University of Information Technology**, Waknaghat, Distt. SOLAN (HP). He served for more than 25 years in **Delhi College of Engineering, Delhi**. From time to time, he has been a visiting faculty at Netaji Subhash Institute of Technolgy, New Delhi; Institute of Technolgy and Management, Gurgaon; Guru Tegh Bahadur College of Engineering, New Delhi; Maharaja Agrasen College of Engineering, Delhi; Galgotia College of Engineering and Technology, Greater Noida; and IEC College of Engineering and Technology, Greater Noida.

He earned his B.Tech. (Honours) in Electronics and Electrical Communication Engineering from IIT Kharagpur in 1967, and Masters Degree in Advanced Electronics from Delhi College of Engineering, Delhi in 1980. For 3 years, he was at IIT Delhi, pursuing research in Digital Signal Processing.

He secured 2nd rank in All India Merit in Engineering Services Examination conducted by UPSC, and served for five years as Engineer, Ministry of Communications, Government of India. For two years, he was In-charge of Frequency Section, responsible for the management of electromagnetic spectrum on national and international basis. For 3 years, he worked as Officer-in-Charge, Wireless Monitoring Station, Srinagar (J & K), and was responsible for installation of equipments and antenna system for the Monitoring Station.

He has conducted or attended short-term refresher courses in Microelectronics at IIT Kharagpur; Measurement Techniques at University of Roorkee; Modern Process Control Techniques at DCE, Delhi; Digital Signal Processing at REC, Warangal; and Microwave Techniques and Systems at REC, Surathkal.

He has authored/co-authored many excellent textbooks, namely, • **Electronic Devices, Applications and Integrated Circuits** (1980) • **Basic Electronics and Linear Circuits** (1984) • **Engineering Network Analysis** (1989) • **Elements of Electronics and Instrumentation** (1992) • **Electronic Devices and Circuits** (2005) and • **Electronics Engineering** (for UPTU) (2006) by various publishers.

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Dedicated

to

My Dear Wife

Aruna

FOREWORD

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The present textbook on "Basic Electrical Engineering" is yet another contribution by Prof. DC Kulshreshtha. This textbook is based on teacher-taught-interaction pattern. The effort is praiseworthy. Having a long experience of teaching and authoring books, the author could put the text matter in the present form, making it primarily learning-specific.

Writing a textbook is indeed a very challenging task. Besides possessing the expertise in the subject, one must have the talent of presenting the text in such a way that the student can understand easily. The author has fully succeeded in achieving this goal.

The most important aspect of this book is the perfect manuscript that has evolved after testing it in the class-room teaching for two years in Jaypee University and its constituent institutions. The appreciation received from the students as well as the teachers confirms the quality of the text. The remarks and suggestions of the experts during the review of this book at the pre-publishing stage helped in further enhancing the usefulness of the book. I highly admire the author for this contribution of an excellent textbook.

I am sure that the teachers and students of various engineering colleges would approve this book as a basic textbook for teaching and learning of the subject.

DSChauhan

Prof. D.S. Chauhan
Vice-Chancellor

PREFACE

The Need for this Book

Since the beginning of my career as a teacher, I have been teaching Basic Electrical Engineering—a core course usually taught in 1st or 2nd semester of B. Tech. common to all disciplines. I observed that the students had difficulty in understanding the available textbooks, and therefore resorted to memorising the statements and formulae of the basic principles. It forced my teacher's conscience to prepare a textbook that presented the important ideas in depth and left many of the details for future learning in further study. So, I authored this book emphasising important principles as the foundation of electrical engineering. The main goal of this book is to:

- explain the basic ideas of electrical engineering in simple language,
- bring easy understanding of circuits, electrical power systems, generators and motors, and
- provide enough number of 'Practice Problems' to enhance the grasp of the basic principles in different ways.

The book is a culmination of various stages such as drafting, testing and feedback on prepared material by students' editorial board.

Prerequisites

This book is meant to be used by students who have just passed 10+2, who lack the knowledge of higher mathematics. It uses derivatives at some places and integrals scantly. Simple algebra, vector algebra, complex-number algebra and matrices are extensively used with detailed explanations. Thus, it will serve the purpose of a textbook to the undergraduate students of all engineering disciplines and diploma students of Electrical Engineering and Electronics Engineering streams.

Pedagogy of the Book

When an engineering subject is considered by a teacher for textbook or class-teaching, he moves from general to specific—first ideas, and then laws, followed by equations, and examples. However, most students seem to learn in the opposite order—first examples, then equations, followed by laws and finally ideas. Some students never go beyond studying the examples, and many believe that only the equations are important. When the teacher probes their understanding of the general principles through a quiz, many students give memorised 'solutions', or protest that this thing was never taught in the class.

The goals and needs of both the teacher and the taught are met in the design and the pedagogy of this book. The principles and applications of the subject are clearly and concisely presented using a step-by-step approach.

Structure of the Book

The book, spanning over 19 chapters, has been structured to cover all important topics required by the syllabus in a single volume. It begins with an introductory chapter, and ends with a chapter on electrical wiring and illumination needed by a practicing engineer. Chapters 2 to 4 explain the basic ideas, principles, circuit analysis techniques and theorems by considering simple dc circuits. Chapters 5 to 7 deal with magnetic circuits. Chapter 8 explains first-order transients in *RC* and *RL* dc circuits. Chapters 9 to 12 are on single-phase and three-phase ac circuits. Chapters 13 to 17 cover electrical machines. Lastly, chapter 18 gives basic information of electrical measuring instruments.

Each chapter ends with 'Summary', Check Your Understanding', 'Review Questions', Multiple Choice Questions', 'Problems', and 'Experimental Exercises'.

Aids to Learning

This textbook addresses the viewpoints of both students and teachers in a number of ways:

- **Objectives and Summary** Each chapter begins with the Learning Objectives and ends with Summary giving key Terms and Concepts, and Important Formulae.
- **Causality** In complicated situations, the cause-effect relationships are given. Understanding consists largely in knowing the causal connections between various factors in a problem so that the equations are written with a purpose. Students often have trouble solving problems because they memorise equations without clearly understanding what the variables mean.
- **Major Equations** Important major equations are boxed with a thick border, with a view to highlight them. This format, however, is sparingly used lest the students think that electrical engineering consists of a set of equations to be memorised.
- **Key and Important Terms** Key terms are made bold-italic when they are first introduced and defined. Important terms are italicised wherever they appear.
- **Solved Examples/Problems** The book has 528 examples/problems, solved step-by-step clearly bringing out the causal-effect relationships.
- **Additional Solved Examples** More solved problems have been included for difficult concepts and further integrations of subject areas.
- **Check Your Understanding** Students can check their understanding of the principles studied in a chapter by answering 10 True/False (objective) questions given at the end of each chapter.
- **Review Questions** These questions allow students to review the key concepts and assess their understanding.
- **MCQs** There are 646 Multiple Choice Questions, which prepare the students for competitive tests conducted by GRE, UPSC, NTPC, ONGC, Infosys, Accenture, etc.
- **Practice Problems** The book provides 840 numerical problems for the practice of the students in the category of simple, tricky and challenging. The answers to the problems are also provided to enable students to gain confidence in their attempts. Most of the problems have been taken from examination papers of different universities.
- **Experimental Exercises** Experimental Exercises aid students to perform experiments in the laboratory in a systematic way.

Rich pool of pedagogy of more than 2000 solved problems and students' practice problems needed by practicing engineers.

Solved Examples/Problems:	**528**	*Review Questions*:	**268**
Multiple Choice Questions:	**244**	*Students' Practice Problems*:	**840**
Experimental Exercises:	**16**	*True/False Questions*:	**180**

Online Learning Centre

This book is accompanied by a comprehensive website—http://www.mhhe.com/kulshreshtha/bee—designed to provide valuable resources for students, instructors and professionals. Students can access a sample chapter, 402 additional Multiple Choice Questions and link to reference material on the website. Teachers using this book as the main text can request the Publisher for the Solution Manual of numerical problems. Supplementary teaching materials include chapter-wise PowerPoint slides with animations for effective lecture presentations, and an exhaustive test-bank.

Supplementary Exercises

For a thorough understanding of electrical principles, one needs to work out an exhaustive number of numerical problems. Supplementary Exercises—in the category of Solved Problems and Students' Practice Problems—is an additional feature of the book. These sets of exercises are meant to provide a standard methodology of sincerely learning the basic principles of Electrical Engineering. The questions and problems in this set of exercises are categorised into *five* parts :

I. Part A—*DC Circuits* : Assemblage of
Ch. 2 : Ohm's Law; ***Ch. 3*** : Network Analysis; and ***Ch. 4*** : Network Theorems.

II. Part B—*Electromagnetic Circuits* : Assemblage of
Ch. 5 : Electromagnetism; ***Ch. 6*** : Magnetic Circuits; and ***Ch.7*** : Self and Mutual Inductances.

III. Part C—*AC Circuits* : Assemblage of
Ch. 9 : Alternating Voltage and Current; ***Ch. 10*** : AC Circuits; ***Ch. 11*** : Resonance in AC Circuits; and ***Ch. 12*** : Three-Phase Circuits and Systems.

IV. Part D—*Electrical Machines* : Assemblage of
Ch. 13 : Transformers; ***Ch. 14*** : Alternators and Synchronous Motors; ***Ch. 15*** : Induction Motors; ***Ch. 16*** : DC Machines; and ***Ch. 17*** : Fractional Horse Power Motors.

V. Part E—*Miscellaneous* : Assemblage of
Ch. 8 : DC Transients; ***Ch. 18*** : Electrical Measuring Instruments; and ***Ch. 19*** : Electrical Installation and Illumination.

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Despite the best efforts put in by me and my team, it is possible that some unintentional error might have eluded us. I shall acknowledge with gratitude if any of these is pointed out. Any suggestions from the readers for the improvement in future edition of this book are most welcome.

D C Kulshreshtha

Publisher's Note :

Tata McGraw Hill Education looks forward to receiving from teachers and students their valuable views, comments and suggestions for improvements, all of which may be sent to tmh.corefeedback@gmail.com, mentioning the title and author's name on the subject Line.

CONTENTS

WALKTHROUGH

OBJECTIVES

After completing this Chapter, you will be able to:

- State the meaning of electro-mechanical energy-conversion (EMEC) machines.
- Explain how the power flows from a prime-mover to an electrical load by means of a generator.
- Explain how the power flows from electric supply to a mechanical load by means of a motor.
- State how a generator differs from a motor.
- Explain the two ways (alignment and interaction) in which a mechanical force is generated in an electromechanical system.
- State the general characteristics of a synchronous machine.
- Derive the expression for synchronous speed in terms of number of poles and frequency of induced emf.
- Describe the construction of stator and rotor of a synchronous machine.
- Explain how a rotating magnetic flux is produced by three-phase currents in three windings on the stator in a synchronous machine.
- Explain why a synchronous machine has armature on its stator and field on its rotor.
- Derive the expressions for pitch factor and distribution factor.
- Derive the expression for induced emf in a generator.
- Explain the meaning of 'armature reaction' and its effect on generated induced emf.
- Draw the equivalent circuit for the stator of an alternator.
- Explain the meaning of synchronous reactance (X_s) and synchronous impedance (Z_s).
- Derive expressions of real power and reactive power generated by a generator.
- Describe how to determine the synchronous impedance of a synchronous machine by conducting open-circuit and short-circuit tests on it.
- Determine voltage regulation for lagging and leading power factors.
- Explain in what way the operation of a synchronous motor differs from that of a generator, with the help of suitable phasor diagrams.
- Justify that a synchronous motor is a constant-speed motor in true sense.
- State different methods of starting a synchronous motor.

OBJECTIVES

Chapter objectives provide a concise statement of expected learning outcomes.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 5.9

A horizontal overhead line carries a current of 90 A in east-to-west direction. What is the magnitude and direction of the magnetic field due to this current at a point 1.5 m below the line?

Solution The magnitude of the magnetic field is given by Eq. 5.2, as

$$B = \frac{\mu_0 I}{2\pi x} = \frac{4\pi \times 10^{-7} \times 90}{2\pi \times 1.5} = 12 \times 10^{-6}\ \text{T} = \mathbf{12\ \mu T}$$

By applying the right-hand thumb rule, we find that the direction of the magnetic field is from **north-to-south**.

EXAMPLE 5.10

What is the magnitude of the magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of uniform magnetic field of 0.15 T?

Solution Using Eq. 5.8, the force per unit length of the wire is given as

$$F_u = \frac{F}{l} = IB \sin\theta = 8 \times 0.15 \times \sin 30° = \mathbf{0.6\ Nm^{-1}}$$

EXAMPLE 5.11

A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of uniform horizontal magnetic field of magnitude 0.8 T. What is the magnitude of the torque experienced by the coil?

Solution As shown in Fig. 5.17, the coil MNOP is kept such that the normal to its plane makes an angle of 30° with the uniform magnetic field. The magnetic force F experienced by each of the sides MN and OP is given by Eq. 5.7, as

$$F = IBlN = 12 \times 0.8 \times 0.1 \times 20 = 19.2\ \text{N}$$

The normal distance between these two forces is

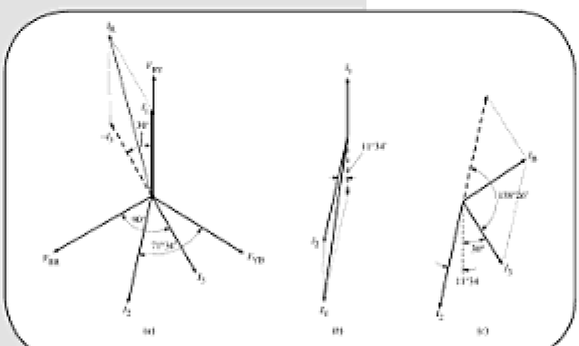
$$x = l \sin 30° = 0.1 \times 0.5 = 0.05\ \text{m}$$

Therefore, the torque experienced by the coil is

$$\tau = Fx = 19.2 \times 0.05 = \mathbf{0.96\ Nm}$$

SOLVED EXAMPLES

Provided at appropriate locations, solved examples aid in learning the technique of applying concepts to practical problems.



FIGURES

Figures are used exhaustively to illustrate the concepts and methods described in the text.

MULTIPLE CHOICE QUESTIONS

Chapter-end MCQs help the students in clarifying concepts. Also, a useful tool to prepare for various competitive examinations.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. The specific resistance ρ depends upon
   (*a*) the material, the area of cross-section and the length of the conductor
   (*b*) the area of cross-section and the length of the conductor only
   (*c*) the area of cross-section of the conductor only
   (*d*) the nature of the material of the conductor only
2. The resistance of a conductor increases when
   (*a*) its length increases
   (*b*) its area increases
   (*c*) both its length and area increase
   (*d*) its resistivity is kept constant
3. On increasing its temperature, the resistance of a conductor made of a metal
   (*a*) decreases (*b*) increases
   (*c*) remains constant (*d*) varies either way
5. The 'ampere second' could be the unit of
   (*a*) conductance (*b*) power
   (*c*) energy (*d*) charge
6. The polarity of voltage drop across a resistor is determined by
   (*a*) the value of the resistor
   (*b*) the value of current through the resistor
   (*c*) the direction of current through the resistor
   (*d*) the polarity of the source
7. If 110 V is applied across a 220-V, 100-W bulb, the power consumed by it will be
   (*a*) 100 W (*b*) 50 W
   (*c*) 25 W (*d*) 12.5 W
8. A resistance of 10 Ω is connected across a supply of 200 V. When another resistance of R ohms is connected in parallel with the above 10-Ω resistor, the current drawn from the supply doubles. The value of R is
   (*a*) 5 Ω (*b*) 10 Ω
   (*c*) 20 Ω (*d*) 40 Ω
9. Three resistances each of R ohms are connected in star. Its equivalent delta will comprise three

REVIEW QUESTIONS

1. What is the magnetic field pattern due to a long current-carrying straight conductor?
2. When a current-carrying conductor is placed in a magnetic field, it experiences a force. How do you find the magnitude and direction of this force?
3. State and explain (*a*) Fleming's left-hand rule, and (*b*) Fleming's right-hand rule.
4. State how you will determine the nature of force between two parallel current-carrying conductors.
5. Sketch the magnetic field around two adjacent parallel current-carrying conductors, when the currents flowing through them are (*a*) in opposite directions, and (*b*) in the same direction.
6. Explain how the unit of current is defined.
7. What is a solenoid? Write the expression for the magnetic field at a point (*a*) inside the solenoid, and (*b*) just at one of its ends.
8. As you move away from the middle of a solenoid, why does the magnetic field decrease?
9. What is the difference between a solenoid and a toroid?
10. Suppose that you are sitting in a room with your back to the wall. Imagine that an electron beam travelling horizontally from the back wall to the direction of the magnetic field that exists in the room?
11. What is meant by electromagnetic induction? State and explain Faraday's laws of electromagnetic induction.
12. State Lenz's law. Show, by means of an example, that the Lenz's law and Fleming's right-hand rule give the same direction of induced emf in a circuit.
13. Show that Lenz's law is a consequence of the principle of conservation of energy.
14. One end of a bar magnet is thrust into a coil. It is noted that the induced current in the coil is in clockwise direction as viewed from the front end. Is the end of the bar magnet its N-pole or S-pole?
15. A metallic loop is placed in a nonuniform magnetic field. Will an emf be induced in the loop?
16. Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop? Do the loops attract or repel each other?
17. The battery in the above question is suddenly disconnected. Is a current induced in the other

REVIEW QUESTIONS

Review questions at the end of each chapter are meant to give good practice for answering theoretical questions in examinations.

PROBLEMS WITH ANSWERS

*Problems given at the end of each chapter are divided, based on the difficulty level, into three categories—**(A) Simple, (B) Tricky, and (C) Challenging with answers**.*

PROBLEMS

(A) SIMPLE PROBLEMS

1. If the moving coil of an electric meter consists of 150 turns wound on a square former which has a length of 4 cm and the flux density in the air gap is 0.09 T, calculate the turning moment acting on the coil when it is carrying a current of 12 mA.
   [Ans. 172.8×10^{-6} N m**]**
2. A moving coil instrument gives a full-scale deflection of 10 mA when the potential difference across its terminals is 100 mV. Calculate the series resistance to measure 1000 V on full scale.
   [Ans. 99 990 Ω**]**
3. A moving coil instrument gives a full-scale deflection of 10 mA when a potential difference of 10 mV is applied across its terminals. Show how will you use the instrument to measure (*i*) currents up to 100 A, and (*ii*) voltage up to 500 V.
   [Ans. (*i*) 0.000 100 01 Ω; (*ii*) 49 999 Ω**]**
4. A moving coil ammeter has a resistance of 0.01 Ω and full scale deflection current of 0.25 A. How this meter can be made to read (*i*) voltage up to 250 V, and (*ii*) current up to 20 A?
   [Ans. (*i*) 999.99 Ω; (*ii*) 1.2658×10^{-4} Ω**]**

(B) TRICKY PROBLEMS

5. The coil of a moving-iron instrument has a resistance of 500 Ω and an inductance of 1 H. An additional resistance of 2000 Ω is connected in series with the instrument to make it a voltmeter. It reads 250 V when a dc voltage of 250 V is applied. What will it read when 250-V, 50-Hz is applied?
   [Ans. 248 V**]**

EXPERIMENTAL EXERCISE 10.2

PARALLEL AC CIRCUIT

Objectives

1. To observe the variation of current I when the resistance in the parallel ac circuit is varied.
2. To draw the phasor diagram for the parallel ac circuit for *four* sets of impedances obtained by varying the resistance.
3. To calculate the circuit parameters (R, L and C) for the four sets of observations, assuming the ac supply frequency to be 50 Hz.
4. To compare the two values of power-factors—one obtained from the readings and the other obtained from the phasor diagram—for the four sets of observations.

Apparatus Single-phase ac supply; One Variac 0-250 V, 5 A; One choke coil with negligible resistance; One wattmeter; Three ammeters (MI type) 0-5 A; Three voltmeters (MI Type) 0-300 V; One rheostat 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 10.21.

EXPERIMENTAL EXERCISES

Experimental exercies given at the end of relevant chapters help students to perform laboratory experiments in a systematic way.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** *marks for each correct answer and* ***minus one*** *for each wrong answer. If your score is* ***12*** *or more, go to the next Chapter; otherwise study this Chapter again.*

No.	*Statement*	*True*	*False*	*Marks*
1.	The ratio of bandwidth to the resonance frequency of a resonant circuit is called its quality factor.	☐	☐	
2.	In a *series resonant circuit*, the lower the resistance in the circuit, the steeper is its current response.	☐	☐	
3.	When a capacitor is connected in parallel to an inductive circuit, the phase angle increases and the power factor decreases.	☐	☐	
4.	In a practical resonant circuit, the value of the resistance affects the resonant frequency.	☐	☐	
5.	The impedance of both the series and parallel resonant circuit increase with increase in frequency.	☐	☐	
6.	In a series resonant circuit, the impedance for the frequencies above resonant frequency is inductive.	☐	☐	
7.	When the frequency is much greater than the resonant frequency of a series resonant circuit, the angle of impedance Z approaches 0°.	☐	☐	
8.	For a series resonant circuit, the resonance curve is a plot of frequency versus voltage.	☐	☐	

CHECK YOUR UNDERSTANDING

A set of 10 objective questions enable students gauge their mastery of chapter content.

SUMMARY

TERMS AND CONCEPTS

- ***Self inductance*** of a coil arises when an emf is induced in itself by changing the current flowing through it.
- Isolated inductances in series and parallel are added the same way as resistances.
- An inductor may be fixed or variable, iron-cored or air-cored.
- When a magnetic field is set up by an inductor, it stores energy.
- ***Mutual inductance*** is the property of two magnetically coupled coils because of which there is an induced emf in one coil due to change in current in the other coil.
- ***Coefficient of coupling*** (k) is the ratio of flux linkage between primary and secondary coils to the flux produced by primary current.
- The mutual voltage is present independently of and in addition to any voltage due to self-induction.
- Both L and M are measured in henrys (H).

IMPORTANT FORMULAE

- Self-induced emf, $e = L\dfrac{di}{dt}$.
- Energy stored, $W = \dfrac{1}{2}LI^2$.
- EMF induced by mutual inductance, $e_2 = M\dfrac{di_1}{dt}$.
- $L = \dfrac{N^2 \mu A}{l}$.
- $M_{21} = N_2\dfrac{d\Phi_{21}}{di_1}$.
- Coupling coefficient, $k = \dfrac{M}{\sqrt{L_1 L_2}}$; $0 \le k \le 1$.

SUMMARY

The chapter-end summary divided into: (1) Terms and Concepts and (2) Important Formulae can be used by students for a quick review during examinations.

IMPORTANT NOTE

In both the rules—Fleming's right-hand rule and Fleming's left-hand rule—the first finger, the central finger and the thumb represent the same quantities. It may therefore be helpful to associate

First finger	with	Field Flux;
Central finger	with	Current; and
thuMb	with	Motion of the conductor

Often, students get confused which rule to apply where. In electrical engineering, we come across two types of situations. One may be called *generator action*, and the other *motor action*. In generator action, the induced emf is the result when a conductor is moved in a magnetic field (this is what happens in a dynamo). Whereas, in motor action, the motion of a conductor is the result when a current is passed through the conductor placed in a magnetic field (this is what happens in an electric fan).

You can easily remove the confusion by noting that it is your right hand that (usually) *generates* most of the things (like writing, painting, tightening of a screw, etc). Thus, for the sake of remembering, the right-hand rule can be associated with the *generator* action of the right hand. The other rule (*i.e.*, Fleming's left-hand rule) then applies to the motor action.

COMMON MISTAKES

Wrong concepts or wrong solutions to problems are deliberately given at many places to emphasise the mistakes commonly committed by students.

MNEMONICS

As an aid to remember the sequence of colour codes given above, the student can memorise one of the following (all the capital letters stand for colours):

(*a*) Bill Brown Realised Only Yesterday Good Boys Value Good Work

(*b*) Bye Bye Rosie Off You Go Bristrol Via Great Western

(*c*) B B Roy of Great Britain had a Very Good Wife.

AID TO MEMORY/ MNEMONICS

By giving examples drawn from day-to-day experience, aids to memory remove any confusion from the minds of the students.

SCIENTIFIC CALCULATOR

The author strongly advises using an *advanced scientific calculator* (such as CASIO *fx*-991ES), which requires no programming. Such a calculator is capable of :

- Directly solving quadratic and simultaneous equations (up to three variables), just by keying in the constants of the equation.
- Making calculations with complex numbers (even in mixed mode—polar and Cartesian), as is often required for the analysis of ac circuits

Using a good scientific calculator not only saves a lot of time, but also reduces the possibility of making errors.

A **word of caution** about the use of a calculator is called for. While using a calculator, the student is tempted to show a false accuracy in the results obtained. Remember that the result of a calculation cannot produce a value of greater accuracy than the accuracy of the information used to formulate the calculation. For example, consider a current of 11 A passing through a 23-Ω resistor, each figure having an accuracy of two significant places. Expressing the voltage across the resistor as 253 V is wrong as it suggests an accuracy of three significant places. It should be correctly expressed as 250 V.

> **Note:** In this book, we have used the calculator CASIO ***fx*-991ES** for solving equations and making other calculations. Therefore, often the intermediate steps of calculations have been omitted.

STANDARD SYMBOLS USED IN ELECTRICAL ENGINEERING

Description	*Symbol*	*Description*	*Symbol*
Cell	or	Capacitor (general)	
Battery of cells	or	Capacitor (polarized)	+
DC voltage source	+ − or + −	Inductor or winding	
DC current source	or	Inductor with core	
AC voltage source	+ − or + −	Transformer	
AC current source	or	Ammeter	A
		Voltmeter	V
Controlled voltage source	+ − or + −	Wattmeter	W or W
Controlled current source	or	Galvanometer	
Fixed resistor		Closed switch	
Variable resistor		Open switch	
Resistor with moving contact		Crossing of conductor (no electrical connections)	or
Filament lamp		Junction of conductors	
		Double junction of conductors	
		Earth (ground)	

INTRODUCTION

1

OBJECTIVES

After completing this Chapter, you will be able to:

- *Understand the basic concepts of 'charge', 'charge carriers', 'drift velocity', 'electric current', 'current density', 'potential difference' and 'voltage rise or voltage drop'.*
- *Calculate 'power', if voltage and current are known.*
- *Express electrical energy in correct units.*
- *Differentiate between emf and terminal voltage.*
- *Calculate efficiency of a machine, if input and output powers are known.*
- *Express different electrical quantities with correct notations of the units, as per SI units.*
- *To state the need of doing experiments in laboratory.*
- *To state the general instructions and precautions that must be followed while doing experiments.*

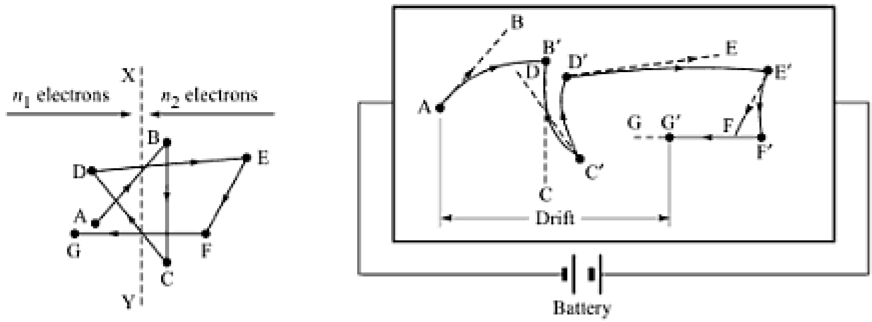
1.1 CHARGE

All materials are made up of one or more elements. An element is a substance composed entirely of atoms of the same kind. An atom consists of a massive core (*nucleus*) carrying *protons* and *neutrons*. *Electrons* move around nucleus in orbits at distances that are large as compared to the size of the nucleus. Each electron has a mass of 9.11×10^{-31} kg and a *negative* charge, $-e$, equal to -1.602×10^{-19} coulomb. Each proton carries a *positive* charge, e, equal in magnitude to that of an electron. A neutron carries no charge and its mass is almost same as that of a proton.

Under normal conditions, an atom is supposed to be electrically neutral. The total positive charge on protons is equal to the total negative charge on electrons. If an atom loses electron(s), it becomes positively charged. On the other hand, if an atom acquires excess electrons, it becomes negatively charged. The *charge is considered as the quantity of electricity.* The unit of charge is ***coulomb*** (C).

Charge Carriers

The atoms in a metal (such as Cu, Al, etc.) are packed together to form a solid. In such a case, the loosely bound electron in the outermost orbit is free to move from one atom to the other throughout the solid. Such electrons are called *free electrons*. The path of a free electron deviates whenever it collides with a nucleus or with other free electrons. After the collision, the path may deviate in any direction. Thus, the free electrons move in a *haphazard, zig-zag,* or *random* way (from A to B, B to C, …) in the solid, as shown in Fig. 1.1*a*.



(*a*) Random motion of electrons. (*b*) Drift of electrons.

Fig. 1.1 *Motion of free electrons in a metal.*

When a free electron moves, it carries a negative charge with it. Hence, the free electrons in a metal work as *charge carriers*.

In semiconductors (such as germanium and silicon), there are two kinds of charge carriers. First is the negatively charged ***electrons***, and the second is the positively charged ***holes***. In liquids, the molecules split into positive and negative ions. Both type of these ions work as charge carriers.

Drift Velocity

The random motion of an electron (see Fig. 1.1*a*) in a metal does not produce any net displacement. If you observe any imaginary plane XY, at any instant the number n_1 of electrons crossing from left to right will be the same as the number n_2 of electrons crossing from right to left. There is no net flow of electrons across the plane XY. Hence, *there is no flow of electric current due to random motion of electrons.*

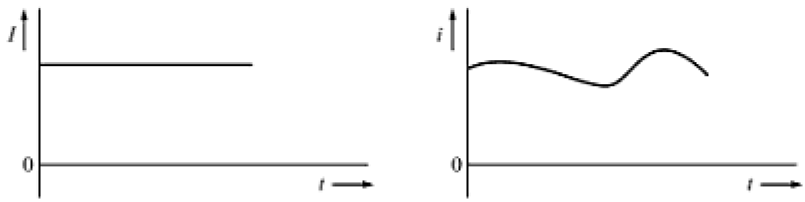
When a voltage is applied across a piece of metal, the random movement of free electrons gets modified, as shown in Fig. 1.1*b*. Negatively charged electrons experience a constant force towards the positive terminal. Instead of going straight from A to B, the electron takes a curved path and reaches B′. Again, instead of straight path B′C, it takes a curved path B′C′, and so on. This way, in addition to the random movement, the electrons ***drift*** towards positive terminal. This drift of charge carriers (here, electrons) results in an electric current, called ***drift current***.

The *average velocity of the charge carriers moving under the influence of an applied electric field is called* ***drift velocity*** (v_d). The magnitude of drift velocity is quite small, of the order of mm/s. The drift velocity of electrons must not be confused with the speed with which electrical signals travel along a wire. Remember, when you switch on an electric light, the current reaches the bulb in almost no time. Electrical signals travel along a wire almost at the speed of light (3×10^8 m/s). This situation may be compared to the action of water supply system. When a tap is turned to fill a glass, the response of the water flow is immediate. However, it takes quite a long time for a particular molecule of water to travel from the main water pipe in the street to the tap in the kitchen.

Electric Current

If one metre length of the conductor has n electrons, each having a charge $-e$, the total charge passing the plane XY in one second will be nev_d. Since, ***electric current*** *is defined as the rate of flow of positive charge*, the current I due to drift of free electrons is given as

$$I = \frac{dq}{dt} = -nev_d \tag{1.1}$$



(*a*) DC current. (*b*) Pulsating current.

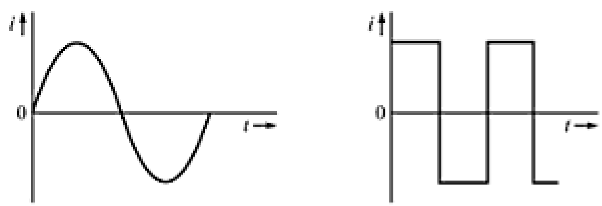
Fig. 1.2 *Unidirectional current.*

The negative sign in the above equation indicates that the conventional direction of electric current is opposite to the motion of free electrons.

The unit of current is ***ampere*** (A), which means the flow of 1 C of charge past any point in a conductor in one second.

Although a current in a conductor has a magnitude in a certain direction, it is ***not*** a vector quantity. The direction of current merely indicates the sense of flow from one side to the other.

Types of Electric Current If the current flow always remains in one direction only, it is called *unidirectional current* (Fig. 1.2). If the magnitude of this current remains constant with time, it is called ***direct current*** or ***dc current*** (Fig. 1.2*a*). On the other hand, if the magnitude of the unidirectional current varies with time, it is called *pulsating current* (Fig. 1.2*b*). However, a current which keeps on reversing its direction continually is called ***alternating current*** or ***ac current*** (Fig. 1.3). Figure 1.3*a* shows a *sinusoidal* ac current, and Fig. 1.3*b* shows *square-wave* ac current.



(*a*) Sinusoidal ac current. (*b*) Square-wave ac current.

Fig. 1.3 *Alternating current.*

NOTE

The abbreviations 'dc' for 'direct current' and 'ac' for 'alternating current' have been taken as adjectives. Hence, it is perfectly alright to say, 'ac current' or 'dc voltage', etc.

Defining Current We establish a graphical symbol for current by placing an arrow next to the conductor, as shown in Fig. 1.4. Thus, in Fig. 1.4*a* the direction of the arrow and the value "3 A" indicate *either* that a net current of 3 A is flowing from left to right *or* that a net current of −3 A is flowing from right to left. In Fig. 1.4*b*, there are again two possibilities: *either* a net current of −3 A is flowing from right to left *or* a net current of 3 A is flowing from left to right. In fact, all of these four statements and both figures represent the same current.

Fig. 1.4 *Two methods of representation for the same current.*

It is essential to realize that the current arrow does not indicate the "actual" direction of current flow, but is simply a part of defining a specific current. Remember that *the arrow is a fundamental part of the definition of a current.* Thus, to talk about the value of a current i without specifying the arrow is to discuss an undefined entity.

Current Density

The *current density* in a conductor carrying current is the current per unit area of the cross-section of the conductor. The area is in the direction normal to the current. The current density J is given as

$$J = \frac{I}{A} \tag{1.2}$$

It is a vector quantity and its unit is ***ampere/metre²*** (A/m²).

1.2 VOLTAGE OR POTENTIAL

The ***absolute potential*** of a point is defined as the amount of work done to bring a unit positive charge from infinity to that point. Most of the time, we are not interested in absolute potential of a point, but in ***potential difference*** (pd) (also called ***voltage***) between two points.

If the energy required to move a charge of Q coulombs from point A to point B is W joules, the voltage V between A and B is given as

$$V = \frac{W}{Q} \tag{1.3}$$

The unit of voltage is ***volt*** (V), and is given as

$$1 \text{ volt} = 1 \text{ joule}/1 \text{ coulomb}$$

Voltage Rise or Voltage Drop

While talking about potential difference or voltage, it is more appropriate to designate it as either voltage rise or voltage drop. In Fig. 1.5, the potential difference in all cases is 4 V. But in Fig. 1.5*a* there is a *voltage rise* of 4 V from B to A, and in Fig. 1.5*b* there is a *voltage drop* of 4 V from B to A. Double subscript notation is used to designate voltage rise or voltage drop. Thus, **the notation V_{AB} denotes the voltage of point A with respect to point B.** It is simply *the voltage rise from B to A*, which is same thing as *the voltage drop from A to B.*

For Fig. 1.5*a*, we can write, $V_{AB} = 4$ V, as point A is 4 V above point B. The same thing can also be written as $V_{BA} = -4$ V, as point B is 4 V below point A. In general, we have

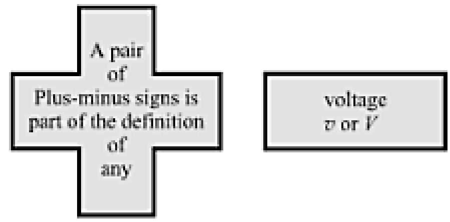
$$V_{AB} = -V_{BA} \tag{1.4}$$

For Fig. 1.5*c*, the voltage of A with respect to B is $V_{AB} = -(-4)$ V $= 4$ V. Therefore, this voltage representation is equivalent to that of Fig. 1.5*a*.

(*a*) Voltage $V_{AB} = 4$ V. (*b*) Voltage $V_{AB} = -4$ V. (*c*) Voltage $V_{AB} = -(-4)$ V.

Fig. 1.5 *Potential difference between points A and B.*

Defining Voltage Voltage or potential difference across a pair of terminals in a circuit is defined by putting a pair of plus-minus signs and then writing the value of the voltage. Thus, in Fig. 1.6*a*, placing the plus sign at terminal A indicates that the terminal A is v volts positive with respect to the terminal B. If we later find that v has a numerical value of −5 V, we may say *either* that A is −5 V positive with respect to B *or* that B is 5 V positive with respect to A.



Just as we noted in our definition of current, it is essential to realize that the plus-minus pair of signs does not indicate the "actual" polarity of the voltage. It is simply part of a convention that enables us to talk unambiguously about a specific voltage. *The definition of a voltage must include a pair of plus-minus signs.* Using a quantity v without specifying the location of the plus-minus-sign pair is like using an undefined term.

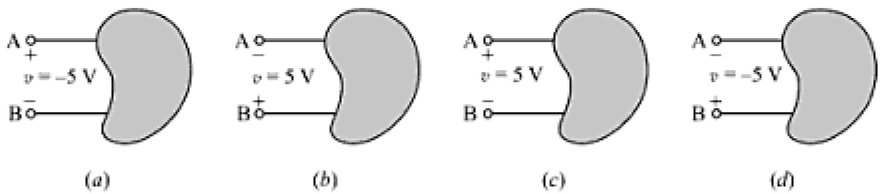


Fig. 1.6 *In (a) and (b), terminal B is 5 V positive with respect to terminal A; in (c) and (d), terminal A is 5 V positive with respect to terminal B.*

Electric Field in a Conductor

When a battery is connected across the two ends of a conductor (as in Fig. 1.1*b*), an electric field is set up at every point within the conductor. It is because of this electric field that the electrons keep flowing in the conductor. The flow of these electrons constitutes an electric current.

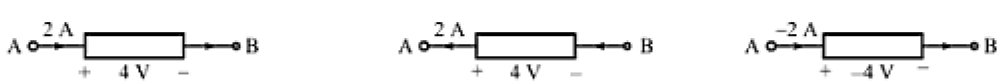
Electric field (**E**) is a vector quantity. It is directed from the positive terminal to the negative terminal. Its magnitude is given as

$$E = \frac{V}{L} \tag{1.5}$$

where, V is the voltage across the terminals of the conductor and L is its length. The unit of electric field is ***volt/metre*** (V/m).

Energy

Work has to be done in transferring a charge through an element. If the current is entering the positive terminal of an element (as in Fig. 1.7*a*), there must be some external force that is driving the current through the element. This external force is delivering energy, and the element is absorbing energy. On the other hand, if the current is leaving the positive terminal (as in Fig. 1.7*b*), the element is delivering energy to the external circuit. The unit of energy is ***joule*** (J).



(*a*) The element is absorbing energy. (*b*) The element is delivering energy. (*c*) Equivalent of Fig. (*b*).

Fig. 1.7 *Different voltage-current relationships.*

Power

Consider an element having a voltage v across it. A small charge Δq is moved through the element from the positive terminal to the negative terminal in time Δt. The energy ΔW absorbed by the element in this process is given as $\Delta W = v\Delta q$. Therefore, by dividing the two sides by Δt, we get

$$\frac{\Delta W}{\Delta t} = v\frac{\Delta q}{\Delta t} \quad \text{or} \quad \underset{\Delta t \to 0}{Lt}\frac{\Delta W}{\Delta t} = v\underset{\Delta t \to 0}{Lt}\frac{\Delta q}{\Delta t} \quad \text{or} \quad \frac{dW}{dt} = v\frac{dq}{dt}$$

The left hand side of the above equation is *the rate of doing work*, and as per definition, is called ***power***. Thus, the ***power absorbed*** is given as

$$p = vi \tag{1.6}$$

The unit of power is ***watt*** (W).

Caution! While using above equation to calculate the power consumed by an element, the direction of current through the element and the reference polarity of the voltage across it must be in conformity. The ***passive sign convention*** *says that the current must enter the plus-marked terminal of the element* (as in Fig. 1.7*a*). Thus, for the element in Fig. 1.7*a*, the power absorbed is

$$p = vi = (4 \text{ V})\,(2 \text{ A}) = 8 \text{ W}$$

The direction of current and voltage polarity for the element in Fig. 1.7*b* are not in conformity as per the passive sign convention. Here, the current of 2 A is leaving the positive terminal. This is equivalent to saying that a current of –2 A is entering the positive terminal, as shown in Fig. 1.7*c*. These two representations are equivalent. We can now calculate the *power absorbed* by the element in Fig. 1.7*b*, by using its equivalent in Fig. 1.7*c*, as follows :

$$p = vi = (4 \text{ V})\,(-2 \text{ A}) = -8 \text{ W}$$

The power absorbed is –8 W. This is equivalent to saying that the *power delivered* by the element of Fig. 1.7*b* is 8 W.

Horse power (hp) is also used as a unit of power [1 hp = 746 W].

Unit of Energy in Electrical Engineering

If a charge Q flows through an element for time t, and if the potential difference across the element is V, the energy expended or the work done is given as

$$\text{Energy} = \text{Work} = \text{Voltage} \times \text{Charge} = V \times Q = V\,(It) = VIt = pt \tag{1.7}$$

When 1 watt of power is delivered to an electrical load for 1 second, the energy consumed is 1 watt-second (1 Ws) or 1 joule (J). Watt-second is a very small unit. Practical unit of electrical energy is ***kilowatt-hour*** (kWh). This is also called the *commercial unit of electrical energy*, as electric supply companies send the bills to the customers for kilowatt-hours of energy consumed.

Heat is a form of energy. It is normally measured in ***kilocalories*** (kcal) [1 kcal = 4.2 kJ].

1.3 BASIC CIRCUIT

A basic electric circuit consists of a *source*, a pair of *connecting wires* and a *load*, as shown in Fig. 1.8. Here, a battery is shown as the source and a resistance R_L is shown as the load. The source forces a current I to flow through the load. To complete the path for the current flow, an outgoing conductor and an incoming conductor are needed.

Electromotive Force (EMF)

It is the voltage of the source (such as a battery) when nothing is connected to it. When connected in an electric circuit, it delivers energy (or power) to the other elements of the circuit. **Note** that the electromotive force (abbreviated as ***emf***) is not a force. It is a voltage and is measured in **volts** (V). It is called a force because it forces current to flow in a circuit.

Terminal Voltage

The voltage across the terminals of a source is called its ***terminal voltage***. In Fig. 1.8, V_1 is the terminal voltage of the source. When current flows through the source, there is a small voltage drop in its internal resistance, r. Thus, the terminal voltage is slightly less than the emf of the source.

From the source to the load, there will be some voltage drop due to the resistance of the conducting wires. The terminal voltage V_2 across the load will be slightly less than the terminal voltage V_1 across the source. Normally, the length of the connecting wires is small, and they offer very small resistance. The voltage drop across the wires is negligibly small. Hence, $V_1 = V_2$ for all purposes.

Efficiency

Electrical equipments (such as heater, motor, generator, etc.) cannot convert the entire input power into the required output power. The ratio of the output power to the input power is known as ***efficiency*** (η). It is usually expressed in percentage. For example, consider the battery in the circuit of Fig. 1.8. The power generated due to the chemical reactions is the product of its emf E and the current I. This can be considered as input power to the battery. The power delivered by the battery (i.e., its output power) is the product of its terminal voltage V_1 and the current I. Hence, its efficiency is

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\% = \frac{V_1 I}{EI} \times 100\%$$

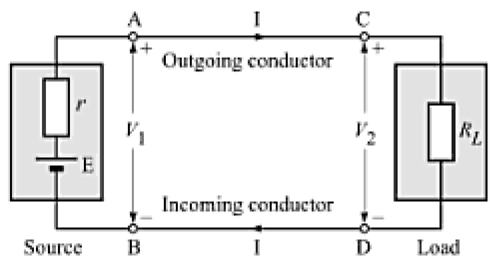


Fig. 1.8 *A basic electric circuit.*

1.4 UNITS AND SYMBOLS

The International System of Units (abbreviated as **SI Units**) is coherent, rational and comprehensive. It is now followed everywhere in the world—at least in engineering. It has seven base units as given in Table 1.1 and many derived units.

Table 1.1 *Base units*

Physical quantity	*Name of SI unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin*	K
Amount of substance	mole	mol
Luminous	candela	cd

*It should be written as kelvin only, and not degree kelvin or °K.

Table 1.2 *Prefixes for multiples and sub-multiples*

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	µ
10^9	giga*	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

*Pronounced as jeega, as it is derived from the word 'gigantic'.

Note :

(i) The prefixes for factors greater than unity have Greek origin; those for factors less than unity have Latin origin (except femto and atto, recently added, which have Danish origin).

(ii) Almost all abbreviations of prefixes for magnitudes < 1, are English lowercase letters. An exception is *micro* (Greek letter μ).

(iii) Abbreviations of prefixes for magnitudes > 1 are English upper-case letters. Exceptions are kilo, hecto, and deca.

(iv) The prefixes hecto, deca, deci and centi should not be used unless there is a strongly-felt need.

⇨ Rules to Use SI Prefixes

1. Multiples of the fundamental unit should be chosen in powers of $\pm 3n$ where n is an integer. However, the unit 'centimetre (cm)' owing to its convenient size and well-established usage, cannot be given up readily.
2. Double or compound prefixes should be avoided, e.g., instead of micromicrofarad (µµF) or millinanofarad (mnF), use picofarad (pF).
3. To simplify calculation, attach the prefix to the numerator and not to the denominator. For example, use MN/m^2 instead of N/mm^2, even though mathematically both forms are equivalent.
4. The rules for *binding-in indices* are not those of ordinary algebra. For example, cm^2 means $(cm)^2 = (0.01)^2\ m^2 = 0.0001\ m^2$, and not $c \times (m)^2 = 0.01\ m^2$.

⇨ Guidelines for Using SI Units

Following are the rules and conventions regarding the use of SI units :

1. Full name of units, even when they are named after a person, are not written with a capital (or upper-case) initial letter, e.g., kelvin, newton, joule, watt, volt, ampere, etc.
2. The symbols for a unit, named after a person, has a capital initial letter, e.g., W for watt (after James Watt), J for joule (after James Prescott Joule) and Wb for weber (after Wilhem Eduard Weber).
3. The symbols for other units are not written with a capital letter, e.g., m for metre.
4. Units may be written out in full or using the agreed symbols, but no other abbreviation may be used. They are printed in full or abbreviated, in roman (upright) type, e.g., amp. is not a valid abbreviation for ampere.

5. Symbols for units do not take a plural form with added 's' ; the symbol merely names the unit in which the preceding magnitude is measured, e.g., 50 kg, and not 50 kgs.
6. No full stops or hyphens or other punctuation marks should be used within or at the end of the symbol for units. However, when a unit symbol prefix is identical to a unit symbol, a raised dot may be used between the two symbols to avoid confusion. For example, while writing, say, metre second it should be abbreviated as m·s to avoid confusion with ms, the symbol for millisecond.
7. There is a mixture of capital and lower-case letters in the symbols for the prefixes as shown in Table 1.2, but the full names of the prefixes commence with lower-case letters only, e.g., 5 MW (5 megawatt), 2 ns (2 nanosecond).
8. A space is left between a numeral and the symbol except in case of the permitted non-SI units for angular measurements, e.g., 57° 16′ 44″.
9. A space is left between the symbols for compound units, e.g., N m for newtons × metres and kW h for kilowatt hour. This reduces the risk of confusion when an index notation instead of the solidus (/) is used. In the former notation, a velocity in metres per second is written as $\mathrm{m\,s^{-1}}$ instead of m/s, but $\mathrm{ms^{-1}}$ may mean 'per millisecond' This type of confusion will not occur if we follow the rule that the denominators of compound units are always expressed in the base units and not in their multiples or submultiples. Thus a heat flow rate will not be given as J/ms but only as kJ/s = kW.
10. When a compound unit is formed by dividing one unit by another, this may be indicated in one of the two forms as m/s or $\mathrm{m\,s^{-1}}$. In no case, should more than one solidus sign (/) on the same line be included in such a combination unless a parenthesis be inserted to avoid all ambiguity. In complicated cases, negative powers or parenthesis should be used.
11. Algebraic symbol representing "quantities" are written in *italics*, while symbols for "units" are written as upright characters, e.g.,

current	$I = 3$ A
energy	$E = 2.75$ J
terminal voltage	$V = 1.5$ V

12. When expressing a quantity by a numerical value and certain unit, it has been found suitable in most applications to use units resulting in numerical value between 1.0 and 1000. To facilitate the reading of numerals, the digits may be separated into groups of three—counting from the decimal sign towards the left and the right. The groups should be separated by small space, but not by a comma or a point. In numerals of four digits, the space is usually not necessary. (It is recognized, however, that to drop the comma from commercial accounting will involve difficulties, particularly with the adding machines in use at present). A few examples are give below:

Incorrect	*Correct*
(i) 40,000 or 40000	(i) 40 000
(ii) 81234.765	(ii) 81 234.765
(iii) 764213.876	(iii) 764 213.876
(iv) 6 543.21	(iv) 6543.21

Note

(a) The recommended decimal sign is a full stop (.). The sign of multiplication of numbers is a cross (×).

(b) If the magnitude of a number is less than unity, the decimal sign should be preceded by a zero

13. Full stop should be omitted after a unit symbol. For example, write 'a current of 6 mA flows' and **not** 'a current of 6 mA. flows'.
14. Full stop should not be used in a multi-word abbreviation. For example, write emf, pd, ac, dc, etc., and **not** e.m.f., p.d., a.c., d.c., etc.
15. The abbreviated forms ac and dc should be used only as adjectives. For example, dc motor, ac current.
16. A unit symbol should be used only after a numerical value of the quantity and **not** after the symbol of the quantity. For example, write 5 kg, 7.5 V, but m kilograms, I amperes.
17. A hyphen is inserted between the numerical value and the unit when the combination is used as an adjective. For example, a 240-V motor, a 2-ohm resistor, etc.

1.5 BASICS OF EXPERIMENTATION

Doing Experimental Exercises in a Laboratory helps in deeper understanding of physical concepts. At the end of each Chapter, this book provides guidelines to perform suitable experiments. Conducting experiments serves the following purposes :

(i) To be familiar with the basic components, measuring instruments and other equipments.
(ii) To learn the techniques of measuring basic electrical and non-electrical quantities.
(iii) To realise the limitations of accuracies of measuring instruments.
(iv) To physically verify theorems/laws pertaining to a topic.
(v) To get training of technical report writing.

Before coming to the laboratory, a student should become familiar with the theoretical background and the necessary circuit diagram for conducting the experiment. Merely making the connections and taking the readings *mechanically* does not help in developing an understanding of the matter.

⇨ General Instructions and Precautions

Since you are going to work with equipments and machines operating at high voltages, such as 220-V dc supply, 230-V, 50-Hz ac supply, or 440-V, 50-Hz, 3-phase ac supply, you should take utmost care to avoid electric shock hazards. Furthermore, enough care should be taken so as to get meaningful results, without damaging any equipment or instrument. Therefore, it is important to adhere to the following general instructions and precautions.

(i) Never work *alone* in the laboratory.
(ii) Always put on shoes with rubber soles so as to provide insulation from ground.
(iii) Don't wear loose dress while working in the laboratory.
(iv) Power should be switched off before changing any connection.
(v) Familiarise yourself with the first aid instructions for electrical shocks.
(vi) Keep away from moving parts.
(vii) Use fuse wire or MCB (Mini Circuit Breaker) of proper rating.
(viii) Use suitable wires for different parts of the circuit. For example, flexible wires may be used for connecting voltmeters or pressure coils of wattmeters as the current carried is small. Thicker wires are to be used where current carried is large.

(ix) Take down complete specifications from the name plate of the machines and equipments. This will enable you to decide the range of all instruments to be used.

(x) Select an instrument that gives the reading in its upper range of the scale. This provides better accuracy of the measurement.

(xi) Switch on the supply *only* after the connections have been checked (preferably by another person).

(xii) Don't touch any live terminal while the supply is ON.

(xiii) While applying electrical load, gradually increase it. Similarly, while removing the electrical load, gradually decrease it.

(xiv) Never apply full voltage to a motor. While switching ON, first apply a low voltage and then gradually increase it.

Power Supply Systems

Following power supply systems are normally available in the electrical engineering laboratories.

(i) 230-V, 50-Hz, *Single-Phase Supply* It has two wires—phase/line wire and neutral wire (Fig. 1.9).

(ii) 400-V, 50-Hz, *Three-Phase Supply* Normally, it has four wires—three for phase/line and one for neutral (Fig. 1.10).

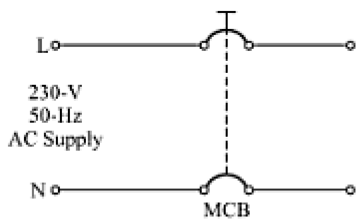


Fig. 1.9 *Single phase ac supply.*

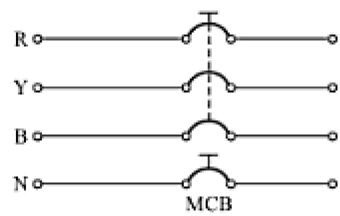


Fig. 1.10 *Three-phase ac supply.*

(iii) 230-V *DC Supply* This may be obtained either from a dc generator or a rectifier. It has two wires—positive and negative.

Single phase ac supply is also available for lighting purpose. For low current requirements, the ac supply can be taken from the sockets.

Variable DC Supply can be obtained by using a suitable ***rheostat*** (a variable resistance), as shown in Fig. 1.11. This arrangement has the disadvantage of incurring considerable power loss in the resistance.

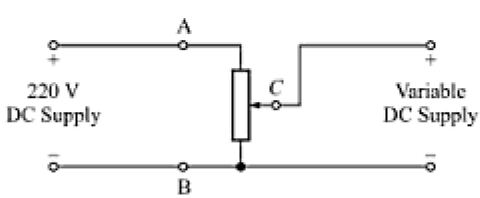


Fig. 1.11 *Variable dc supply.*

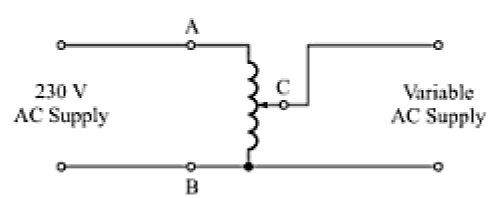


Fig. 1.12 *Variable ac supply.*

Variable AC Supply can be obtained by using an auto-transformer with a variable tap (also known as '*variac*' or '*dimmerstat*'), as shown in Fig. 1.12. For 3-phase supply, we use a 3-phase variac (Fig. 1.13).

Common Instruments

For measuring an electrical quantity, it is important to select a proper instrument with *proper range*. The following are the important instruments and equipments used in the laboratories.

(1) *Ammeters and Voltmeters* The basic principle of operation of these two instruments is the same. An ammeter is used for measuring amperes (or current). It has low resistance and must be connected in series with the circuit. A voltmeter is used for measuring voltage across two points of a circuit. It has high resistance and must be connected across (in parallel with) the two points.

For measurements in dc circuits, we should use a permanent magnet moving coil (PMMC) type of ammeters and voltmeters. The deflection of the pointer in such instruments is directly proportional to the current through them. Hence, their scale is uniform. However, these cannot be used for ac measurements.

For ac circuits, we use moving iron (MI) type ammeters and voltmeters. The deflecting torque in such instruments is proportional to the square of the effective value (or rms value) of the current. Hence, their scale is non-uniform.

(2) *Wattmeter* A wattmeter is an instrument that measures the power (both dc and ac) going to an electrical load. It has two coils—called *current coil* and *potential* (or *pressure*) *coil*. The terminals of the current coil are marked as M and L ; and those of the pressure coil as V_1 and V_2. The terminals M and V_1 are joined together to make a common terminal. The current coil is connected in series with the load and the pressure coil across the load, as shown in Fig. 1.14.

Sometimes, the wattmeter can give negative reading. In such a case, the connections of either the current coil or the pressure coil are to be reversed and the reading is to be treated as negative.

(3) *Tachometer* It is used for the measurement of rotating speed in rpm (revolutions per minute) of a machine. The tapered shaft of the tachometer is inserted into the tapered hole in the shaft of the machine. The reading of the tachometer is proportional to the speed of rotation of its shaft. A tachometer can be either analog type or digital type.

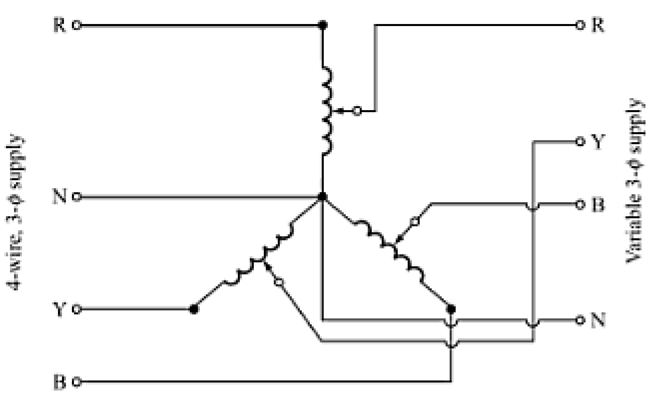


Fig. 1.13 *Variable 3-φ supply.*

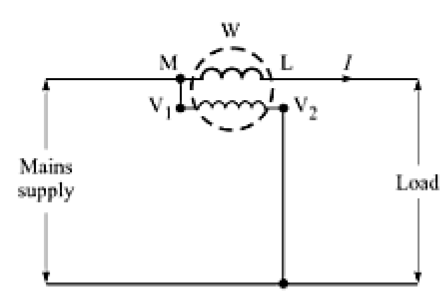


Fig. 1.14 *Wattmeter connections.*

(4) *Rheostat* It is a variable resistance made up of a closely wound wire of high resistivity (such as nickel-chromium-iron alloy) over a circular insulating (asbestos or porcelain) tube. These are available both in single-tube and double-tube configurations. A rheostat is specified in terms of its resistance and the maximum current it can carry. Normally, it is 1000 Ω, 1.2 A and 100 Ω, 5 A. Rheostats are used as variable resistances and as potential dividers (as in Fig. 1.11).

(5) *Loading Devices* Commonly used loading devices are (*i*) lamp bank, and (*ii*) loading rheostats. A *lamp bank* consists of a number of 230-V lamps (100 W, 60 W, 40 W, etc.) suitably connected and controlled by switches to provide different loads. A *loading rheostat* consists of a number of identical resistive elements, suitably connected in series, parallel and combinations thereof.

OHM'S LAW

2

OBJECTIVES

After completing this Chapter, you will be able to:

- *State and explain Ohm's law, in both the macroscopic and microscopic form.*
- *State the meaning of short circuit and open circuit.*
- *Determine equivalent resistance of a network having series and parallel combination of resistances.*
- *Develop an ability to apply voltage divider and current divider concepts in solving the circuits.*
- *Use star-delta transformations in solving complicated networks.*
- *Use principle of duality in solving electrical networks.*
- *Read the values of resistors from the colour code printed on their body.*
- *Calculate the resistance of a given specimen at any temperature.*

2.1 OHM'S LAW

This is the most fundamental law in Electrical Engineering. It states that *the potential difference between the two ends of a conductor is directly proportional to the current flowing through it, provided its temperature and other physical parameters remain unchanged.* That is,

$$V \propto I \quad \text{or} \quad V = RI \tag{2.1}$$

The constant of proportionality R is called the ***resistance*** of the conductor. Its unit is ***ohm*** (Ω). The unit ohm is defined as the resistance which permits a flow of one ampere of current when a potential difference of one volt is applied to the resistance.

There is another way of stating Ohm's law,

$$I \propto V \quad \text{or} \quad I = GV \tag{2.2}$$

The constant of proportionality G is called the ***conductance*** of the conductor. Comparison of Eq. 2.2 with Eq. 2.1 shows that the conductance is the reciprocal of the resistance,

$$G = \frac{1}{R} \tag{2.3}$$

The SI unit of conductance is ***siemens*** (S).

Ohm's Law in Graphical Form

Ohm's law is presented in graphical form in Fig. 2.1. The voltage is shown as independent variable (cause) and current as dependent variable (effect). The slope of the line is the reciprocal of resistance ($1/R$), and is called conductance. A conductor showing straight line v-i characteristics (as in Fig. 2.1) is said to have a *linear* resistance.

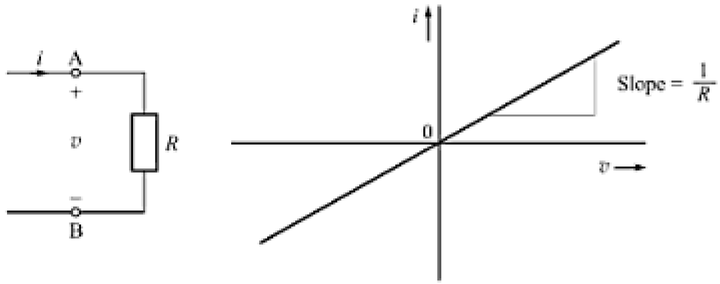


Fig. 2.1 *Ohm's law in graphical form.*

Short Circuit and Open Circuit

These two are special resistances. A ***short circuit*** ($R = 0$) permits current to flow ($i \neq 0$) without any resulting voltage ($v = 0$). As shown in Fig. 2.2, the characteristic of a short circuit is a vertical line. An ***open circuit*** ($R = \infty$) permits voltage ($v \neq 0$) with no current ($i = 0$). Since power going into a resistance is $p = vi$, no power is required for the short circuit or the open circuit.

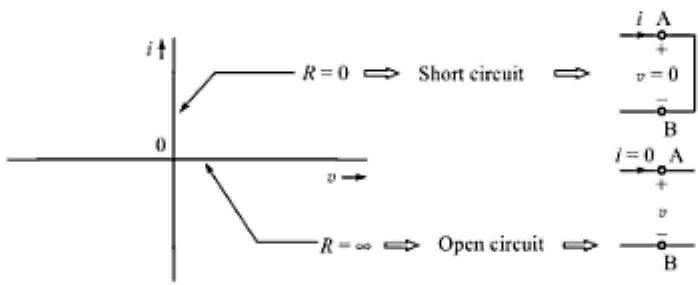


Fig. 2.2 *The characteristics of a short circuit ($R = 0$) and open circuit ($R = \infty$).*

Switches

An ideal switch is also a special resistance. It can be changed from a short circuit to an open circuit to turn an electrical device ON or OFF. Ideal switches receive no electrical energy from the circuit. Different types of switches used in electric circuits are shown in Fig. 2.3. Figure 2.3*a* shows a *single-pole, single-throw* switch in its open (OFF) state. Figure 2.3*b* shows a *single-pole, double-throw* switch which switches one input line between two output lines. Figure 2.3*c* shows a *double-pole, single-throw* switch; the dashed line indicates mechanical coupling between the two components of the switch.

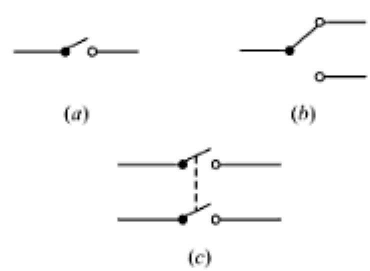


Fig. 2.3 *Different types of switches.*

2.2 RESISTANCE

The flow of current through a conductor is analogous to the flow of traffic on a road. The congestion of traffic on a road depends on how narrow and how long the road is. Similarly, the opposition to the flow of current through a conductor (Fig. 2.4) depends on how narrow is its cross-section and how long is its length. In other words, the electrical resistance of a conductor is directly proportional to its length (L) and inversely proportional to its area of cross-section (A),

$$R \propto \frac{L}{A} \quad \text{or} \quad R = \rho \frac{L}{A} \tag{2.4}$$

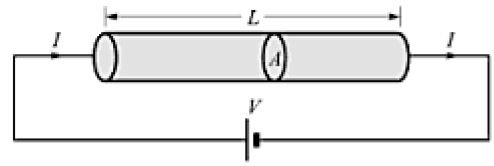


Fig. 2.4 *Current I flowing through a conductor.*

Here, ρ is a constant of proportionality and is known as the ***resistivity*** of the material. The unit of resistivity is ***ohm metre*** (Ω m).

EXAMPLE 2.1

A conducting wire has a resistance of 5 Ω. What is the resistance of another wire of the same material but having half the diameter and four times the length?

Solution The ratio of the resistance of the second wire to that of the first wire is

$$\frac{R_2}{R_1} = \frac{\rho(L_2/A_2)}{\rho(L_1/A_1)} = \frac{L_2}{L_1} \times \frac{A_1}{A_2} = \frac{L_2}{L_1} \times \frac{\pi d_1^2/4}{\pi d_2^2/4} = \frac{L_2}{L_1} \times \left(\frac{d_1}{d_2}\right)^2 = \frac{4}{1} \times \left(\frac{2}{1}\right)^2 = 16$$

Hence, $R_2 = 16R_1 =$ **80 Ω.**

EXAMPLE 2.2

A copper wire has a resistance of 10 Ω. The wire is drawn so that its length increases three times. What is the resistance of the new wire?

Solution If the length of the wire is made three times by drawing it, its area of cross-section must decrease three times, as the volume of the wire remains the same in the drawing process. The ratio of the second wire to that of the first wire is

$$\frac{R_2}{R_1} = \frac{\rho(L_2/A_2)}{\rho(L_1/A_1)} = \frac{L_2}{L_1} \times \frac{A_1}{A_2} = \frac{3}{1} \times \frac{3}{1} = 9$$

Hence, the resistance of the second wire is $R_2 = 9R_1 = 9 \times 10 =$ **90 Ω.**

Microscopic Form of Ohm's Law

By definition, current density is the current per unit area. Thus,

$$J \equiv \frac{I}{A} = \frac{(V/R)}{A} = \frac{V}{RA} = \frac{V}{(\rho L/A)A} = \frac{(V/L)}{\rho} = \frac{E}{\rho}$$

where, $E = V/L$ is the electric field at a point in the conductor. The above equation can be written as

$$E = \rho J \tag{2.5}$$

This is often referred to as microscopic form of Ohm's law.

Series Combination of Resistances

Two or more resistances are said to be connected in ***series***, if *same* (*not* merely equal) current flows through them. Figure 2.5*a* shows three resistances connected in series with a voltage source of emf V. Figure 2.5*b* shows its equivalent circuit, in which the three resistances are replaced by a single resistance R_s.

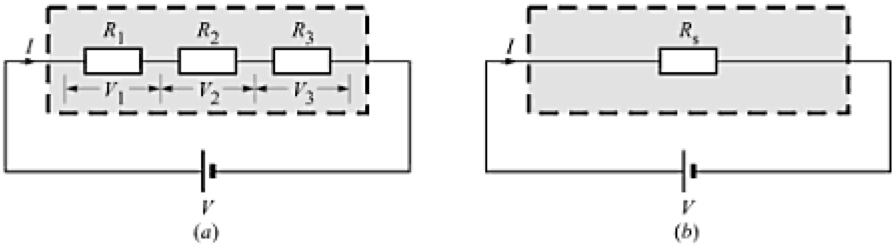


Fig. 2.5 *Resistances in series.*

The same current I flows through the three resistances. The applied voltage V must be equal to the sum of the three individual voltages, V_1, V_2, and V_3. That is,

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) \tag{2.6}$$

From the equivalent circuit of Fig. 2.5*b*, we can write

$$V = IR_s \tag{2.7}$$

From Eqs. 2.6 and 2.7, it can be seen that

$$R_s = R_1 + R_2 + R_3$$

Thus, *the equivalent resistance of a number of resistances connected in series is equal to the sum of individual resistances.* In general, for n resistances in series, we can write

$$R_s = R_1 + R_2 + R_3 + \ldots R_n = \sum_{j=1}^{n} R_j \tag{2.8}$$

Obviously, if n identical resistances, each of resistance R, are connected in series, the equivalent resistance will simply be nR. That is,

$$\boxed{R_{sn} = nR} \tag{2.9}$$

Parallel Combination of Resistances

Two or more resistances are said to be connected in ***parallel***, if *same* (*not* merely equal) voltage exists across them. Figure 2.6*a* shows three resistances in parallel, connected across a voltage source of emf V. Figure 2.6*b* shows its equivalent circuit, in which the three resistances are replaced by a single resistance R_p.

Same voltage V appears across all the three resistances. Hence, the three currents are given as

$$I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2} \quad \text{and} \quad I_3 = \frac{V}{R_3}$$

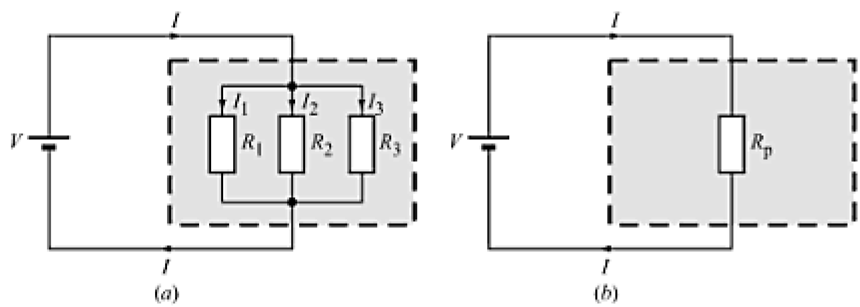


Fig. 2.6 *Resistances in parallel.*

Since the total current I entering the combination divides into I_1, I_2 and I_3, we have

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad (2.10)$$

From the equivalent circuit of Fig. 2.6*b*, we can write

$$I = \frac{V}{R_p} \quad (2.11)$$

Comparing Eqs. 2.10 and 2.11, we get

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus, *the reciprocal of equivalent resistance of a number of resistances connected in parallel is equal to the sum of the reciprocals of the individual resistances.* In general, for n resistances in parallel, we can write

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \frac{1}{R_n} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (2.12)$$

Obviously, if n identical resistances, each of resistance R, are connected in parallel, the equivalent resistance will simply be R/n. That is,

$$\boxed{R_{pn} = \frac{R}{n}} \quad (2.13)$$

In case ***only two resistances*** are connected in parallel, the equivalent resistance is given as

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad (2.14)$$

or

$$\boxed{R_p = \frac{R_1 R_2}{R_1 + R_2}} \quad (2.15)$$

For two resistances in parallel, using Eq. 2.15 is much more convenient than using Eq. 2.14.

Voltage Divider

The concept of voltage divider is very useful in analysing electric circuits. Consider the circuit of Fig. 2.7, in which two resistances R_1 and R_2 are connected in series with a voltage source V. The current I is given as

$$I = \frac{V}{R_1 + R_2}$$

Therefore, the voltage V_1 across resistance R_1 is given as

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} R_1$$

or

$$V_1 = V \frac{R_1}{R_1 + R_2} \quad (2.16)$$

Similarly, the voltage across R_2 is

$$V_2 = V \frac{R_2}{R_1 + R_2} \quad (2.17)$$

Thus, we find that *the voltage appearing across one of the series resistances is the total voltage times the ratio of its resistance to the total resistance.*

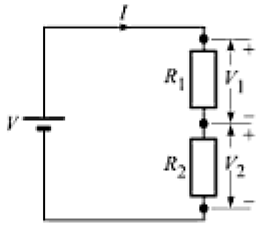


Fig. 2.7 *Illustration of voltage divider.*

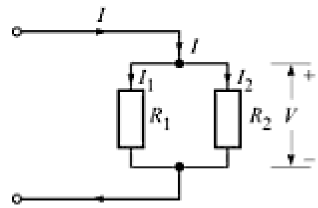


Fig. 2.8 *Illustration of current divider.*

Current Divider

Like voltage divider, the concept of current divider is also useful in analysing circuits. Consider the circuit in Fig. 2.8, in which two resistances R_1 and R_2 are connected in parallel. Same voltage V appears across both the resistances. The total current I entering the combination divides into I_1 and I_2, as shown. Therefore, we have

$$V = I_1 R_1 \quad \text{and} \quad V = I_2 R_2 \quad \text{or} \quad I_2 = \frac{I_1 R_1}{R_2}$$

$$I = I_1 + I_2 = I_1 + I_1 \frac{R_1}{R_2} = I_1 \left(\frac{R_1 + R_2}{R_2} \right)$$

or

$$I_1 = I \frac{R_2}{R_1 + R_2} \tag{2.18}$$

Similarly, the current through R_2 is given as

$$I_2 = I \frac{R_1}{R_1 + R_2} \tag{2.19}$$

Thus, we find that *the current through one of the two parallel resistors is the total current times the ratio of the* ***other*** *resistance to the sum of resistances.*

EXAMPLE 2.3

Using voltage divider technique, determine the voltage across the four resistances in the circuit of Fig. 2.9*a*.

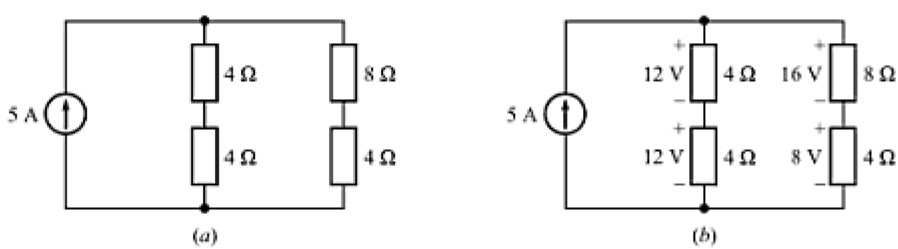


Fig. 2.9

Solution The voltage across the two parallel combinations divides independently in the two paths. The equivalent resistance seen by the current source is

$$R_p = (4 + 4) \parallel (8 + 4) = \frac{8 \times 12}{8 + 12} = 4.8\ \Omega$$

Hence, the total voltage across the parallel combination is $5 \times 4.8 = 24$ V. This voltage equally divides between the two resistors in the first branch. In the second branch, the same voltage divides in the ratio of 8 : 4. The results are shown in Fig. 2.9*b*.

EXAMPLE 2.4

Using the voltage divider and current divider techniques, determine the unknown currents through and voltages across the resistances in the circuit of Fig. 2.10.

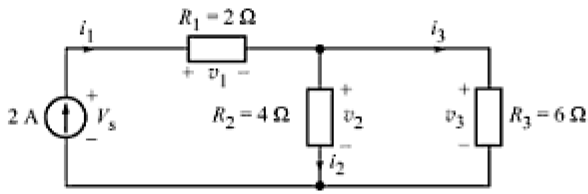


Fig. 2.10

Solution We first combine resistances to simplify the circuit. Thus, R_2 and R_3 can be replaced by their parallel equivalent $R_p = 4 \parallel 6 = 2.4\ \Omega$ (Fig. 2.11*a*). The 2-Ω resistance is now combined with 2.4 Ω to give 4.4 Ω (Fig. 2.11*b*). Clearly, the voltage across the current source is $V_s = 2 \times 4.4 = 8.8$ V.

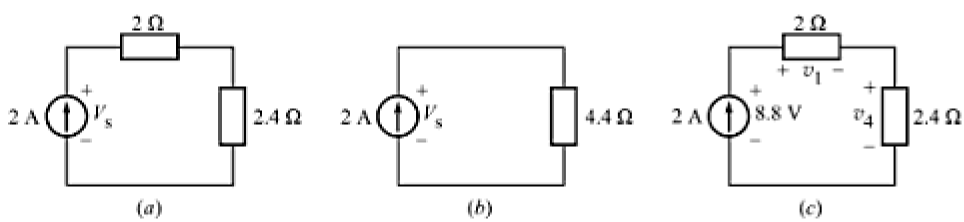


Fig. 2.11

We now restore the original circuit. Figure 2.11*c* is the same as Fig. 2.11*a*, but now we know that a voltage of 8.8 V is applied across the series combination of the 2 Ω and 2.4 Ω. The voltages across these resistances have been marked as v_1 and v_4. We can now use voltage divider technique to find these voltages. You may object to it and say, "It is a current source and not a voltage source which you are dividing." True, but the voltage created by the current source divides in a series circuit. It does not matter whether we have a 2-A current source producing 8.8 V or an 8.8-V voltage source producing 2 A—the circuit will respond in the same way. We are quite justified to divide the 8.8 V, and get

$$v_1 = 8.8 \times \frac{2}{2 + 2.4} = \mathbf{4\ V} \quad \text{and} \quad v_4 = 8.8 \times \frac{2.4}{2 + 2.4} = \mathbf{4.8\ V}$$

Voltage v_4 is the same as the voltages v_2 and v_3 in the original circuit (Fig. 2.10). Obviously, current $i_1 = \mathbf{2\ A}$, and according to Ohm's law,

$$i_2 = \frac{4.8\ \text{V}}{4\ \Omega} = \mathbf{1.2\ A} \quad \text{and} \quad i_3 = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

Comments on Series and Parallel Combinations

Consider the simple circuit of Fig. 2.12*a*, and ask yourself, "Are the voltage source V_s and the resistance R in series or in parallel?" The answer is clearly "both". The two elements carry the same current I, and hence they are in series. They have the same voltage V_s across them, and hence they are in parallel.

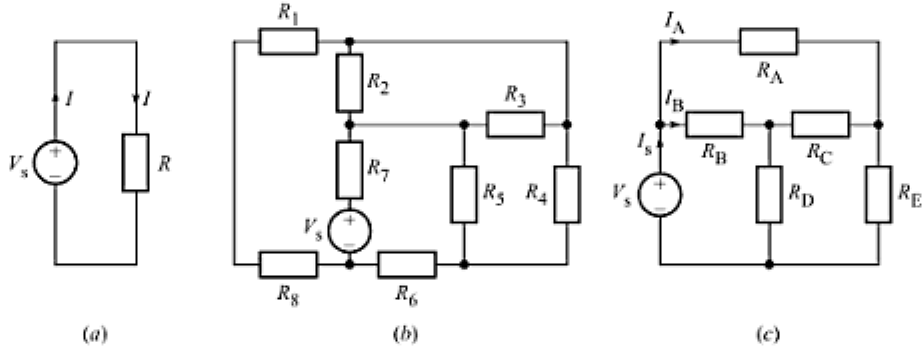


Fig. 2.12 *Circuits for some useful comments on series and parallel combinations.*

Often, the circuits are drawn in such a way so that it becomes difficult to spot the series and parallel combinations. Consider, for example, Fig. 2.12*b*. Which resistances are in series and which resistance are in parallel? The circuit has only two resistances R_1 and R_8 in series. **Note** that these two resistances are physically placed parallel to each other, but electrically they are in series. The circuit has only two resistances R_2 and R_3 in parallel. No other combinations of resistances are in series or in parallel.

A resistance (or any other element) need not be in series or in parallel with other resistance (or element). For example, the resistances R_5 and R_4 in Fig. 2.12*b* are neither in series nor in parallel with any other resistance. The resistances R_5, R_4 and R_3 make what is called a ***delta-connection***. In Fig. 2.12*c*, there are no resistances that are in series or in parallel with any other resistance. However, the resistances R_B, R_C and R_D make what is called a ***star-connection***.

There is an important ***caution*** about the use of current divider. Many students apply this concept where it is not applicable. For example, consider the circuit of Fig. 2.12*c*. The circuit contains no resistances in series or in parallel. *Without the resistances connected in parallel, there is no way that the concept of current divider can be applied.* However, many students take a quick look at resistances R_A and R_B and try to apply current division, writing an ***incorrect*** equation such as

$$\cancel{I_A = I_s \frac{R_B}{R_A + R_B}}$$

Remember, parallel resistances must be connected between the same pair of points (called ***nodes***).

Star and Delta Connections

There are two ways in which three resistances can be connected across three points of a network. One arrangement, shown in Fig. 2.13*a*, is called ***star*** or ***wye*** (Y) connection. The other, shown in Fig. 2.13*b*, is called ***delta*** (Δ) or ***mesh*** connection. In some cases, a network can be solved *readily* by means of star-delta transformation. Star-to-delta transformation means finding R_1, R_2 and R_3 in terms of R_A, R_B and R_C. Similarly, delta-to-star transformation means finding R_A, R_B and R_C in terms of R_1, R_2 and R_3.

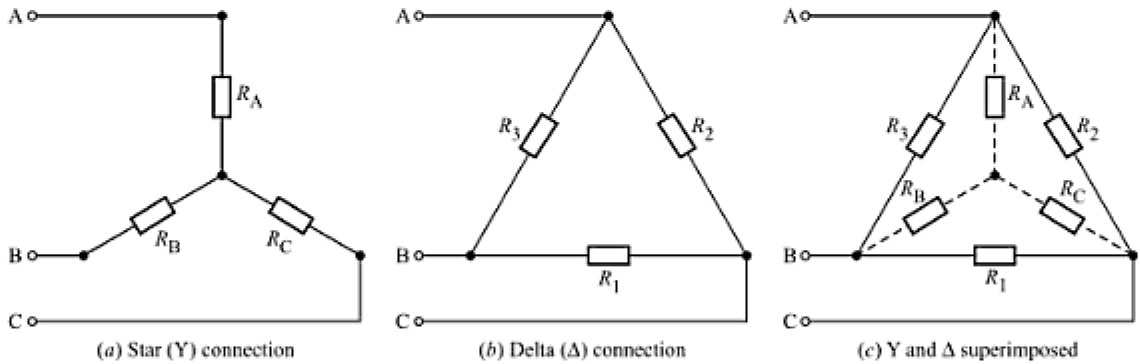


Fig. 2.13 *Three resistances connected across three points.*

Figure 2.13*c* shows the two arrangements superimposed. **Note** that the resistance opposite to R_A has been named as R_1, that opposite to R_B as R_2 and that opposite to R_C as R_3. This has been deliberately done so that the star-delta transformation formulae become easy to remember.

Delta-to-Star Transformation If the two arrangements in Fig. 2.13 are to be equivalent, the resistance between any pair of terminals (AB, BC or CA) in the two circuits has to be the same, when the third line is left open. Keeping terminal A open, we equate the resistance between terminals B and C in the two circuits, to get

$$R_B + R_C = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \tag{2.20}$$

Similarly, we can write

$$R_C + R_A = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3}$$

and

$$R_A + R_B = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

Adding above three equations and dividing by 2, we get

$$R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} \tag{2.21}$$

Subtracting Eq. 2.20 from this equation gives

$$\boxed{R_A = \frac{R_2R_3}{R_1 + R_2 + R_3}} \tag{2.22}$$

Similarly, we get

$$R_B = \frac{R_3R_1}{R_1 + R_2 + R_3} \tag{2.23}$$

and

$$R_C = \frac{R_1R_2}{R_1 + R_2 + R_3} \tag{2.24}$$

Equations 2.22 to 2.24 are a set of equations which transforms a delta connection into its equivalent star connection.

These relationships may be remembered thus: *the equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.*

NOTE

For a symmetrical delta-connection, $R_1 = R_2 = R_3 = R_\Delta$ (say). Then, the equivalent resistance of the symmetrical star-connection is simply given as

$$R_Y = \frac{R_\Delta R_\Delta}{R_\Delta + R_\Delta + R_\Delta} = \frac{R_\Delta}{3} \tag{2.25}$$

MNEMONICS

Remembering the above relation becomes easy if you recall that the same relation is obtained for the equivalent resistance of parallel combination of three resistances.

Star-to-Delta Transformation To get the reverse transformation, we multiply Eq. 2.22 and Eq. 2.23 so as to get

$$R_A R_B = \frac{R_1 R_2 R_3^2}{(R_1 + R_2 + R_3)^2} \tag{2.26}$$

Similarly, we can get

$$R_B R_C = \frac{R_1^2 R_2 R_3}{(R_1 + R_2 + R_3)^2} \tag{2.27}$$

$$R_C R_A = \frac{R_1 R_2^2 R_3}{(R_1 + R_2 + R_3)^2} \tag{2.28}$$

Adding Eqs. 2.26 to 2.28 gives

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3^2 + R_1^2 R_2 R_3 + R_1 R_2^2 R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3} = \frac{R_2 R_3}{R_1 + R_2 + R_3} R_1 = R_A R_1 \tag{2.29}$$

Therefore,

$$\boxed{R_1 = R_B + R_C + \frac{R_B R_C}{R_A}} \tag{2.30}$$

Similarly,

$$R_2 = R_C + R_A + \frac{R_C R_A}{R_B} \tag{2.31}$$

and

$$R_3 = R_A + R_B + \frac{R_A R_B}{R_C} \tag{2.32}$$

These relationships may be remembered thus: *the equivalent delta resistance connected between two terminals is equal to the sum of the two star resistances connected to those terminals plus the product of the same two star resistances divided by the third star resistance.*

EXAMPLE 2.5

Calculate the effective resistance between points A and B for the combinations of resistances, given in Fig. 2.14.

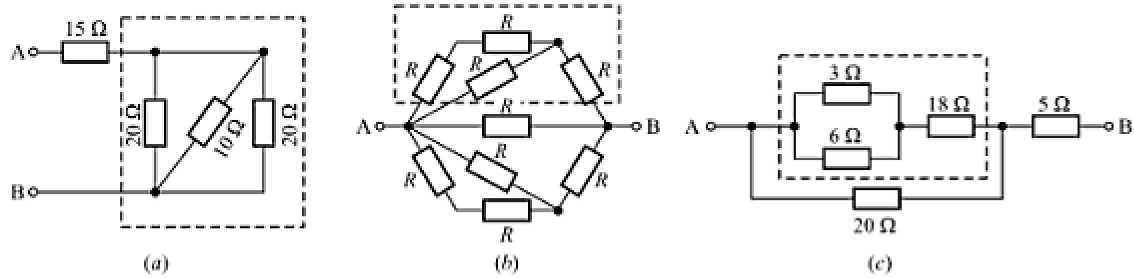


Fig. 2.14 *Combinations of resistances.*

Solution

(*a*) The resistance R_p of the network within the dotted box is given by

$$\frac{1}{R_p} = \frac{1}{20} + \frac{1}{10} + \frac{1}{20} = \frac{1+2+1}{20} = \frac{4}{20} = \frac{1}{5} \quad \text{or} \quad R_p = 5\ \Omega$$

Therefore,

$$R_{AB} = 15\ \Omega + (20\ \Omega \parallel 10\ \Omega \parallel 20\ \Omega) = 15\ \Omega + 5\ \Omega = \mathbf{20\ \Omega}$$

Alternatively, we can have much *easier* and *quick* solution. Just view the network within the dotted box as parallel combination of two resistances of 20 Ω each, which is then paralleled with a resistance of 10 Ω. That is,

$$R_{AB} = 15\ \Omega + (20\ \Omega \parallel 20\ \Omega) \parallel (10\ \Omega)$$
$$= 15\ \Omega + (10\ \Omega) \parallel (10\ \Omega) = 15\ \Omega + 5\ \Omega = \mathbf{20\ \Omega}$$

Above calculation can be done orally in just one minute!

(*b*) The resistance of the network within dotted box is

$$R_1 = [(R+R) \parallel R] + R = \frac{2}{3}R + R = \frac{5}{3}R$$

By symmetry, the resistance of the lower network will also be the same, i.e.,

$$R_2 = \frac{5}{3}R$$

Thus, the entire network between points A and B is a parallel combination of three resistances, R_1 (= $5R/3$), R and R_2 (= $5R/3$). The combination of R_1 and R_2 is simply $5R/6$. Thus,

$$R_{AB} = (5R/6) \parallel R = \frac{(5R/6)\,R}{(5R/6)+R} = \frac{5R^2/6}{11R/6} = \frac{\mathbf{5}}{\mathbf{11}}\boldsymbol{R}$$

(*c*) The resistance of the network within dotted box is

$$R_1 = (3\ \Omega \parallel 6\ \Omega) + 18\ \Omega = 2\ \Omega + 18\ \Omega = 20\ \Omega$$

∴ $$R_{AB} = (20\ \Omega) \parallel (20\ \Omega) + 5\ \Omega = 10\ \Omega + 5\ \Omega = \mathbf{15\ \Omega}$$

EXAMPLE 2.6

Determine the currents I_1, I_2 and I_3 flowing in the parallel branches in the network shown in Fig. 2.15.

Solution Effective resistance connected across the battery is given as

$$R_{eff} = 2 + \frac{1}{(1/12)+(1/20)+(1/30)} + 2 = 2 + 6 + 2 = 10\ \Omega$$

∴ $$I = \frac{V}{R_{eff}} = \frac{100}{10} = 10\ \text{A}$$

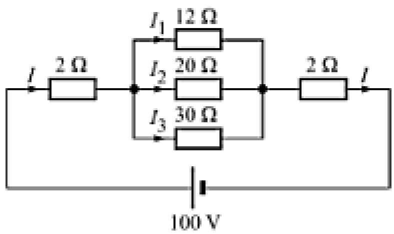


Fig. 2.15

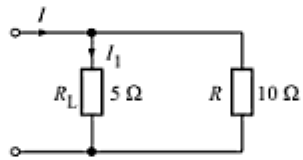


Fig. 2.16

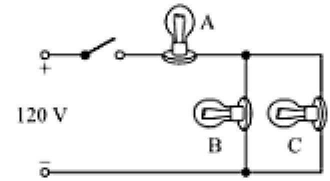


Fig. 2.17

The voltage across the three resistances connected in parallel has to be the same. Therefore, applying Ohm's law, we have

$$12I_1 = 20I_2 = 30I_3 \quad \Rightarrow \quad I_2 = \frac{12}{20}\,I_1 = 0.6I_1 \quad \text{or} \quad I_3 = \frac{12}{30}\,I_1 = 0.4I_1$$

The total current delivered by the source must be equal to the sum of the currents flowing in the three parallel branches. Thus,

$$10 = I_1 + I_2 + I_3 = I_1 + 0.6I_1 + 0.4I_1 = 2I_1 \quad \Rightarrow \quad I_1 = \mathbf{5\ A}$$

$$\therefore \qquad I_2 = 0.6I_1 = \mathbf{3\ A} \qquad \text{and} \qquad I_3 = 0.4I_1 = \mathbf{2\ A}$$

EXAMPLE 2.7

Calculate the supply current I in the network given in Fig. 2.16, if the 5-Ω resistance dissipates energy at the rate of 20 W.

Solution Power dissipated by 5-Ω resistance is given as

$$P = I_1^2 R_L \quad \text{or} \quad 20 = I_1^2 \times 5 \quad \Rightarrow \quad I_1^2 = \frac{20}{5} = 4 \quad \text{or} \quad I_1 = 2\ \text{A}$$

The concept of current divider gives

$$I_1 = I \times \frac{R}{R + R_L} \quad \text{or} \quad 2 = I \times \frac{10}{10+5} = \frac{2}{3}I \quad \Rightarrow \quad I = \mathbf{3\ A}$$

EXAMPLE 2.8

Three 60-W, 120-V light bulbs are connected across a 120-V power line as shown in Fig. 2.17. Find (*a*) the voltage across each bulb, and (*b*) the total power dissipated in the three bulbs.

Solution

(*a*) The resistance of each bulb is given as

$$R = \frac{V^2}{P} = \frac{120^2}{60} = 240\ \Omega$$

The combined resistance of bulbs B and C is $R_{BC} = 240/2 = 120\ \Omega$. Therefore, using potential divider, the voltage across B (and also A) is given as

$$V_B = V_C = 120 \times \frac{120}{240 + 120} = \mathbf{40\ V}; \quad \text{and} \quad V_A = 120 - 40 = \mathbf{80\ V}$$

(*b*) $P = P_A + P_B + P_C = \dfrac{(80)^2}{240} + \dfrac{(40)^2}{240} + \dfrac{(40)^2}{240} = 26.67 + 6.67 + 6.67 = \mathbf{40\ W}$

EXAMPLE 2.9

As the variable resistor R in Fig. 2.18 varies from a short circuit to an open circuit, it is desired that the equivalent resistance also vary between 30 Ω and 75 Ω. (*a*) Design R_1 and R_2 to accomplish this result. (*b*) Find R to give $R_{eq} = (30 + 75)/2$ Ω.

Solution

(*a*) *Minimum* value of R_{eq} is obtained when $R = 0$ (i.e., a short circuit, because the parallel combination of R_2 and R is reduced to zero). *Maximum* value of R_{eq} is obtained when R is an open circuit. Hence,

$$R_1 = 30 \quad \text{and} \quad R_1 + R_2 = 75 \quad \Rightarrow \quad R_1 = \mathbf{30\ \Omega} \quad \text{and} \quad R_2 = \mathbf{45\ \Omega}$$

(*b*) Required value of R_{eq} is $(30 + 75)/2\ \Omega = 52.5\ \Omega$. It means that the parallel combination of R_2 and R should have a value,

$$R_p = 52.5 - R_1 = 52.5 - 30 = 22.5\ \Omega$$

This value is exactly half of $R_2 = 45\ \Omega$. Hence, the value of R should also be **45 Ω**.

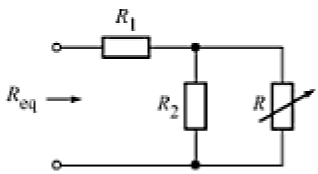


Fig. 2.18

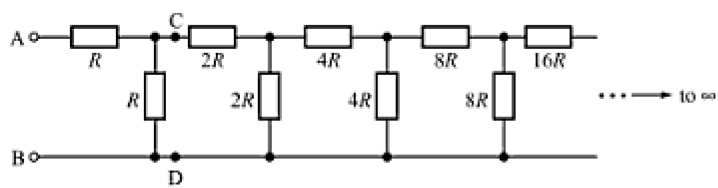


Fig. 2.19

EXAMPLE 2.10

Calculate the equivalent resistance between terminals A and B in terms of the resistance R, for the infinite ladder network of Fig. 2.19.

Solution If we remove the first two resistances (i.e., the first rung of the ladder), the remaining circuit across the terminals C and D has an equivalent resistance, which must be double of the original ladder. If the equivalent resistance across A-B is R_x, we can replace the remaining ladder across C-D by a single resistance $2R_x$. Thus,

$$R_x = R + (R \parallel 2R_x) \quad \text{or} \quad R_x = R + \frac{(R)(2R_x)}{R + 2R_x} \quad \text{or} \quad 2R_x^2 - 3RR_x - R^2 = 0$$

$$\Rightarrow \qquad R_x = \frac{3R \pm \sqrt{9R^2 + 8R^2}}{4} = \mathbf{1.78}\boldsymbol{R} \qquad \text{(ignoring negative value)}$$

EXAMPLE 2.11

A π-section of resistances is given in Fig. 2.20*a*. Convert this π-section in to its equivalent T-section, as shown in Fig. 2.20*b*.

Solution Note that the π-section is nothing but a delta (Δ) connection of three resistances R_1, R_2 and R_3. Similarly, the T-section is same as a star (Y) connection of three resistances R_A, R_B and R_C. Note that in Fig. 2.20*a*, we have marked the terminals according to our standard convention. Terminal A is opposite to resistance R_1, terminal B is opposite to resistance R_2, and so on. Therefore, for the required conversion, we use Eqs. 2.22 to 2.24, to get

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{9 \times 6}{3 + 9 + 6} = \frac{54}{18} = 3\ \Omega$$

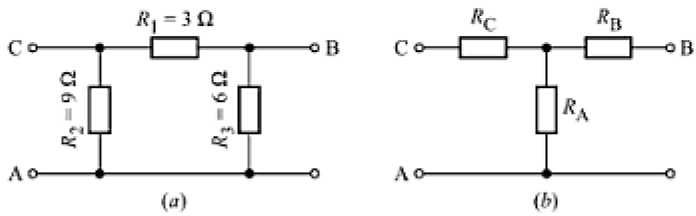


Fig. 2.20

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 6}{3+6+9} = \frac{18}{18} = 1.0\ \Omega$$

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{3 \times 9}{3+6+9} = \frac{27}{18} = 1.5\ \Omega$$

EXAMPLE 2.12

Two coils connected in parallel across 100-V dc supply, take 10 A current from the supply. Power dissipated in one coil is 600 W. What is the resistance of each coil?

Solution The effective resistance of the two coils in parallel is

$$R_{\text{eff}} = \frac{100}{10} = 10\ \Omega$$

Power in one coil is given as

$$600 = \frac{100^2}{R_1} \quad \Rightarrow \quad R_1 = \frac{10000}{600} = \mathbf{16.67\ \Omega}$$

Since the two coils are connected in parallel, we should have

$$10 = \frac{16.67 \times R_2}{16.67 + R_2} \quad \Rightarrow \quad R_2 = \mathbf{25\ \Omega}$$

EXAMPLE 2.13

An electric boiler draws 12-A current at 115 V for a period of 6 hours. If electrical energy costs Rs. 2.50 per kWh, determine the cost of the boiler operation.

Solution Total energy consumed is

$$W = 115\ \text{V} \times 12\ \text{A} \times 6\ \text{h} = 8280\ \text{Wh} = 8.28\ \text{kWh}$$

Therefore, the cost of the boiler operation is

$$\text{Cost} = \text{Energy} \times \text{Rate} = 8.28\ \text{kWh} \times 2.50\ \text{Rs/kWh} = \textbf{Rs. 20.70}$$

EXAMPLE 2.14

A toaster rated at 1000 W, 240 V is connected to a 220-V supply. Will the toaster be damaged? Will its rating be affected?

Solution The resistance, and the current rating of the toaster are

$$R = \frac{V^2}{P} = \frac{240^2}{1000} = 57.6\ \Omega \quad \text{and} \quad I_{\text{rating}} = I_{\max} = \frac{P}{V} = \frac{1000}{240} = 4.167\ \text{A}$$

When the toaster is connected to 220-V supply, the current drawn is

$$I = \frac{V}{R} = \frac{220}{57.6} = 3.82 \text{ A}$$

This current being less than the current rating, the **toaster will not be damaged**. The power consumed is

$$P_1 = 220 \text{ V} \times 3.82 \text{ A} = 840.4 \text{ W}$$

Thus, the power rating is **less** than its original power rating.

2.3 DUALITY

Two circuits are said to be ***dual*** of each other, if the *voltage* equations for the one have the same mathematical form as the *current* equations for the other. In dual circuits, the form of equations remain the same, if following pair of terms are interchanged:

Current ⇔ Voltage
Series ⇔ Parallel
Resistance ⇔ Conductance
Short circuit ⇔ Open circuit
Closed switch ⇔ Open switch

Thus, the two forms of Ohm's law, namely, $V = RI$ and $I = GV$, are dual of each other. Similarly, the concept of voltage divider is dual of the concept of current divider. Note that Eqs. 2.18 and 2.19 can be re-written, in terms of the two conductances G_1 (= $1/R_1$) and G_2 (= $1/R_2$), as

$$I_1 = I\frac{G_1}{G_1 + G_2} \quad \text{and} \quad I_2 = I\frac{G_2}{G_1 + G_2}$$

which have the same form as Eqs. 2.16 and 2.17.

The ***principle of duality*** is very useful in electrical engineering.

2.4 CONDUCTORS AND RESISTORS

A conductor is used to conduct current from one point to another in an electric circuit. Silver is the best conductor followed by copper and aluminum. Silver, being very costly, is rarely used for this purpose. Metallic conductors in the shape of bars, tubes, wires, or sheets are commonly used. Bus-bars are solid thick conductors to carry large currents (hundreds of amperes). Thin bus-bars are used in electronic circuits and computers to carry information signals.

A wire may be single solid or may have a number of solid wires (called *strands*) twisted together to increase flexibility. Wires are either bare (as in overhead transmission lines) or covered with some insulation (as in cables). Wires meant to carry electronic signals are protected from external magnetic fields by covering them with a metallic braid. Such wires are called *shielded wires.*

The property of a conductor to carry current is called its *conductance*, expressed in ***siemens*** (S). Ideally, a good conductor should have *infinite* conductance (or, equivalently, its *resistance* must be *zero*).

Resistors

Often, we deliberately introduce resistance at some point in an electrical or electronic circuit. The physical device that does this job is called a ***'resistor'***. A resistor can be wire wound or carbon moulded.

Wire Wound Resistor A wire wound resistor consists of a resistance wire (made of constantan, manganin, or nichrome) wound on a heat-resisting (ceramic) tube. The ends of the wire are brought out to suitable terminals for connecting it to external circuit. Wire wound resistor may be either fixed or variable (such as rheostats and potentiometers). These resistors have resistances from a fraction of an ohm to thousands of ohm. The power rating varies from a fraction of a watt to several kilowatts.

Carbon Moulded Resistors Electronic circuits need resistors of quite high value (say, of the order of megaohms). The cost of wire wound resistors of such high value is prohibitive. Since carbon has high resistivity, by powdering carbon and mixing it with a suitable binding material we can make resistors of high value. Such carbon moulded resistors are coated with some insulating material and baked to hard finish. Copper leads are embedded at each end. The available resistance values range from a few ohms to tens of megaohms. The power rating is from $^1/_4$ watts to a few watts.

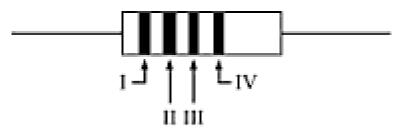


Fig. 2.21 *Colour coded resistor.*

The size of such resistors is quite small. It becomes difficult to imprint their values on their surface. Therefore, it is usual practice to indicate the value by making four colour bands on its body (Fig. 2.21), using a standard colour code given in Table 2.1.

The colour bands are always read from left to right from the end that has the bands closest to it. The first two bands indicate the first two significant figures of resistance value (in ohms) The third band indicates the decimal multiplier (or the number of zeros that follow the second figure). The last band stands for the tolerance (in per cent) about the value indicated by the first three bands. It is a measure of the precision with which the resistor was made by the manufacturer. In case the fourth band is not present, the tolerance is taken as ±20 %.

EXAMPLE 2.15

A resistor has a colour band sequence : Yellow, Violet, Orange and Gold, Find the range in which its value must lie so as to satisfy the manufacturer's tolerance.

Table 2.1 *Resistance colour code*.*

Colour	*Number*	*Multiplier*	*Tolerance(%)*
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold	–	10^{-1}	5
Silver	–	10^{-2}	10
No colour	–	–	20

* MNEMONICS

As an aid to remember the sequence of colour codes given above, the student can memorise one of the following (all the capital letters stand for colours):

(*a*) Bill Brown Realised Only Yesterday Good Boys Value Good Work

(*b*) Bye Bye Rosie Off You Go Bristrol Via Great Western

(*c*) B B ROY of Great Britain had a Very Good Wife.

Solution Using Table 2.1, we have

1st Band	2nd Band	3rd Band	4th Band
Yellow	Violet	Orange	Gold
4	7	10^3	$\pm 5\ \% = 47\ \text{k}\Omega \pm 5\ \%$

Now 5 % of 47 kΩ = $\frac{47 \times 10^3 \times 5}{100}\ \Omega = 2.35\ \text{k}\Omega$

Therefore, the resistance should be within the range 47 kΩ ± 2.35 kΩ or between **44.6 kΩ** and **49.35 kΩ.**

EXAMPLE 2.16

A resistor has a colour band sequence: Grey, Blue, Gold and Gold. What is the range in which its value must be so as to satisfy the manufacturer's tolerance?

Solution The specification of the resistor can be found by using the colour coding table as follows:

1st Band	2nd Band	3rd Band	4th Band
Gray	Blue	Gold	Gold
8	6	10^{-1}	$\pm 5\ \% = 86 \times 0.1\ \Omega \pm 5\ \%$
			$= 8.6\ \Omega \pm 5\ \%$

$$5\ \%\text{ of } 8.6\ \Omega = \frac{8.6 \times 5}{100} = 0.43\ \Omega$$

The resistance should lie somewhere between the values (8.6 – 0.43) Ω and (8.6 + 0.43) Ω or **8.17 Ω** and **9.03 Ω**.

2.5 VARIATION OF RESISTANCE WITH TEMPERATURE

The resistance R of a material changes as its temperature changes. A rise in temperature increases the molecular movement within the material. As a result the drift of free electrons through the material is impeded. In other words the resistance of the material increases. Most conductors (pure metals) show this characteristic. For a moderate range of temperature, the change in resistance is usually proportional to the change in temperature. The ratio of the change in resistance per degree change in temperature to the resistance at some definite temperature, adopted as standard, is termed *the temperature coefficient of resistance* and is designated as α. Table 2.2 gives the value of α at 0 °C for different materials.

In some materials such as semiconductors, electrolytes, etc. a rise in temperature makes more free electrons (or other charge-carriers) available to contribute towards the conduction of current. This increased availability of free electrons offsets the impeding effect of the enhanced molecular movements within the material. The net effect is a decrease in resistance of material with a rise in its temperature. Such materials are said to have *negative temperature coefficient of resistance.*

Assume that the resistance of a conductor at a standard temperature T_0 (usually, T_0 is taken as 0 °C) is R_0 and at T_1 is R_1. Assuming that the variation of R with rise in temperature is linear, we can write

$$R_1 = R_0[1 + \alpha_0 (T_1 - T_0)] \tag{2.33}$$

where α_0 is the temperature coefficient of resistance at the standard temperature. Similarly, at temperature T_2, the resistance is

$$R_2 = R_0[1 + \alpha_0 (T_2 - T_0)] \tag{2.34}$$

Table 2.2 *Temperature coefficients of some materials.*

Material	*Temperature coefficient* (ohms/°C/ohm) at 0 °C	*Material*	*Temperature coefficient* (ohm/°C/ohm) at 0 °C
Advance metal	*	Manganin	*
Aluminium	0.004 20	Mercury	0.000 88
Antimony	0.003 88	Monel metal	0.002 08
Bismuth	0.004 35	Nichrome	0.000 44
Brass (annealed)	0.002 08	Nichrome II	0.000 16
Calorite	*	Nickel	0.006
Constantan	*	Platinum	0.003 7
Copper		Platinum-iridium	0.00 1 2
Annealed		Silver	0.004 11
International			
Standard	0.004 26	Steel	
Hard drawn	0.004 13	Hard	0.00 16
Pure	0.004 10	Soft	0.004 58
German silver	0.000 36	Tantalum	0.003 3
Gold	0.003 65	Tin	0.004 58
		Tungsten	0.004 9
Iron	0.006 18	Zinc	0.004 0
Lead	0.004 66		

* Nearly zero

Eliminating R_0 from the above two equations, we get

$$\frac{R_1}{1+\alpha_0(T_1-T_0)} = \frac{R_2}{1+\alpha_0(T_2-T_0)} \tag{2.35}$$

If we put $T_0 = 0$ °C, we get

$$\frac{R_1}{1+\alpha_0 T_1} = \frac{R_2}{1+\alpha_0 T_2} \tag{2.36}$$

Obviously, similar relationship exists for resistivity ρ of the material.

EXAMPLE 2.17

The resistance of a transmission line is 126 Ω at 20 °C. Determine the resistance of the line at –35 °C. The temperature coefficient of the material of transmission line is 0.004 26 ohm/°C/ohm, at 0 °C.

Solution Given: $R_1 = 126\ \Omega$, $T_1 = 20$ °C, $T_2 = -35$ °C, $\alpha_0 = 0.004\,26$.

Using Eq. 2.36, we get

$$\frac{R_1}{1+\alpha_0 T_1} = \frac{R_2}{1+\alpha_0 T_2}$$

$$\frac{126}{1+0.00426\times 20} = \frac{R_2}{1+0.00426(-35)}$$

$$R_2 = \frac{0.8509}{1.0852}\times 126 = \mathbf{98.8\ \Omega}$$

EXAMPLE 2.18

The resistance of copper winding of a motor at room temperature of 20 °C is 3.42 Ω. After an extended operation of the motor at full load, the winding resistance increases to 4.22 Ω. Find the temperature rise. Given that the temperature coefficient of copper at 0 °C is 0.004 26 ohm/°C/ohm.

Solution Given: $R_1 = 3.42\ \Omega$, $T_1 = 20\ °C$, $R_2 = 4.22\ \Omega$, $\alpha_0 = 0.00426$

Using Eq. 2.36,

$$\frac{R_1}{1+\alpha_0 T_1} = \frac{R_2}{1+\alpha_0 T_2}$$

or $$\frac{3.42}{1+0.004\,26\times 20} = \frac{4.22}{1+0.004\,26 T_2}$$

or $$3.42\,(1+0.00426\,T_2) = 4.22\,(1+0.00426\times 20)$$

or $$T_2 = 79.6\ °C$$

$\therefore$ $$\text{The temperature rise} = T_2 - T_1 = 79.6 - 20 = \mathbf{59.6\ °C}$$

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 2.19

One metre long metallic wire is broken into two unequal parts P and Q. Part P of the wire is uniformly extended into another wire R. Length of R is twice the length of P and the resistance of R is equal to that of Q. Find (*a*) the ratio of the resistances of P and R, and (*b*) the ratio of the lengths of P and Q.

Solution Let the length of P be x. Then, the length of Q will be $y = 1 - x$. If k is the resistance per metre of the metallic wire, the resistances of the two parts are

$$R_P = kx \qquad \text{and} \qquad R_Q = k(1-x)$$

When part P is extended to twice its length to make wire R, the area of cross-section reduces to half (as the volume remains the same). Since the resistance of a wire is directly proportional to its length and inversely proportional to its area of cross-section, the resistance of wire R will be 4 times that of P. That is,

$$R_R = 4R_P = 4kx$$

Given that $R_R = R_Q$, or $4kx = k(1-x) \Rightarrow x = 1/5$. Therefore, $y = 1 - x = 4/5$.

(*a*) $\dfrac{R_P}{R_R} = \dfrac{kx}{4kx} = \dfrac{1}{4} = \mathbf{1:4}$; (*b*) $\dfrac{l_P}{l_Q} = \dfrac{x}{y} = \dfrac{1/5}{4/5} = \dfrac{1}{4} = \mathbf{1:4}$

EXAMPLE 2.20

Reduce the networks, given in Fig. 2.22, to a single resistance between terminals A and B.

Solution

(*a*) The resistance between terminals A and B is

$$R_{AB} = (5 \parallel 6) + (4 \parallel 3) = \frac{5\times 6}{5+6} + \frac{4\times 3}{4+3} = 2.7 + 1.7 = \mathbf{4.4\ \Omega}$$

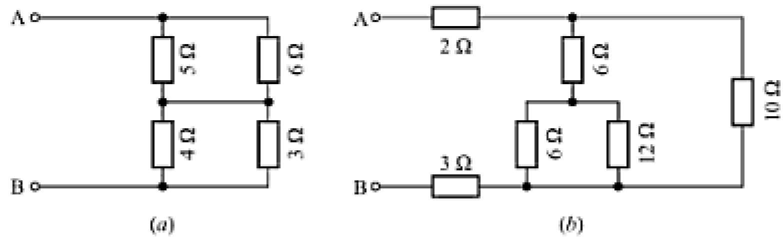


Fig. 2.22

(*b*) The resistance between terminals A and B is

$$R_{AB} = 2 + [\{6 + (6 \parallel 12)\} \parallel 10] + 3 = 2 + [\{6 + 4\} \parallel 10] + 3 = 2 + [10 \parallel 10] + 3$$
$$= 2 + 5 + 3 = \mathbf{10\ \Omega}$$

EXAMPLE 2.21

Twelve identical wires, each of resistance 12 Ω, are arranged to form the edges of a cube as shown in Fig. 2.23*a*. A current of 20 mA enters into the cube at the corner A, and leaves it at the diagonally opposite corner G. Calculate the potential difference between these two corners.

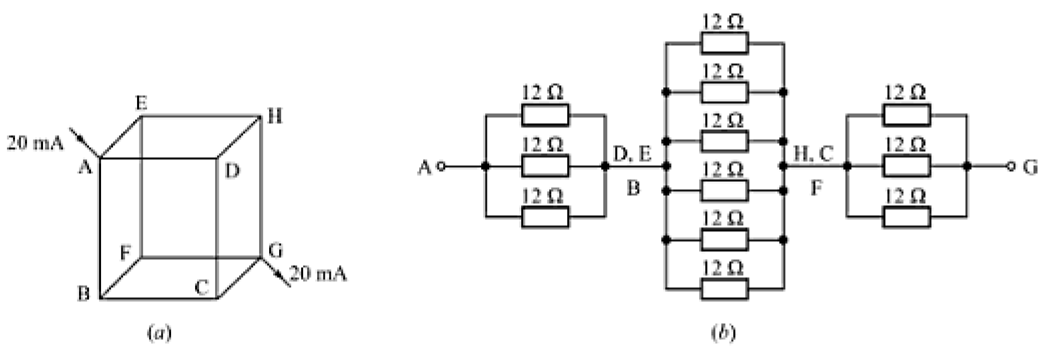


Fig. 2.23

Solution Due to symmetry of the network, the current of 20 mA will equally divide into three branches AD, AE, and AB. Therefore, the points D, E, and B will be at the same potential. Similarly, equal currents will be flowing in the branches HG, CG and FG, and the points H, C and F will be at the same potential. If the points at the same potential are joined together, the current distribution in different branches of the network does not change. Thus, the equivalent circuit of the given network is given in Fig. 2.23*b*. We have three sets of parallel combinations of equal resistances. The effective resistance of the first and third set is $R_{p1} = R_{p3} = 12/3 = 4\ \Omega$ and that of the second set is $R_{p2} = 12/6 = 2\ \Omega$.

Therefore, the effective resistance across the corners A and G is

$$R_{AG} = R_{p1} + R_{p2} + R_{p3} = 4\ \Omega + 2\ \Omega + 4\ \Omega = \mathbf{10\ \Omega}$$

Using Ohm's law, the potential difference across the corners A and G is given as

$$V_{AG} = IR_{AG} = 20\text{ mA} \times 10\ \Omega = \mathbf{0.2\ V}$$

EXAMPLE 2.22

Figure 2.24*a* shows a network of 9 resistances. The number on each resistance represents its value in ohms. Find the resistance across the points E and F.

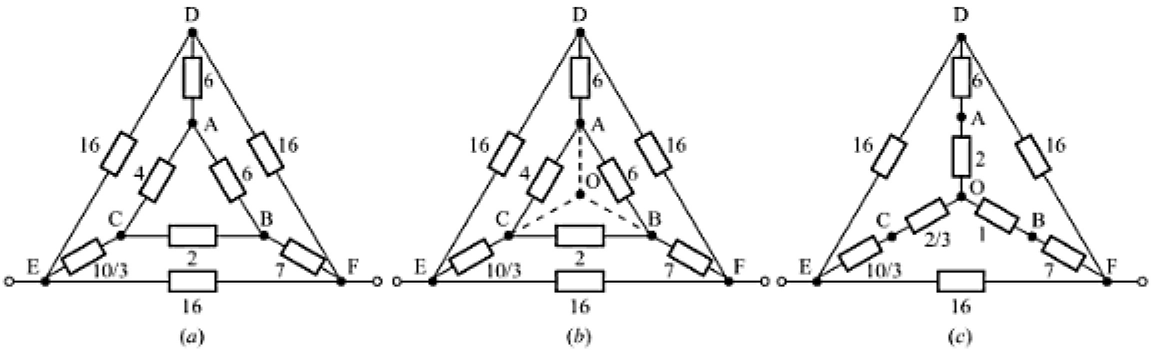


Fig. 2.24

Solution The delta connection between the points A, B and C is first converted into its equivalent star connection (Fig. 2.24*b*), creating a new node O,

$$R_{AO} = \frac{4 \times 6}{2+4+6} = \frac{24}{12} = 2\ \Omega$$

$$R_{BO} = \frac{2 \times 6}{2+4+6} = \frac{12}{12} = 1\ \Omega$$

$$R_{CO} = \frac{2 \times 4}{2+4+6} = \frac{8}{12} = \frac{2}{3}\ \Omega$$

The network of Fig. 2.24*a* can now be drawn as in Fig. 2.24*c*. Reducing the series combination of two resistances into a single resistance gives us the network of Fig. 2.25*a*.

Now, converting the star into delta gives the network of Fig. 2.25*b*. Combining the parallel resistances, we get the network of Fig. 2.25*c*. The equivalent resistance between the points E and F is simply the parallel combination of the two branches,

$$R_{EF} = \frac{8 \times (8 + 32/3)}{8 + (8 + 32/3)} = \frac{8 \times 56}{80} = \mathbf{5.6\ \Omega}$$

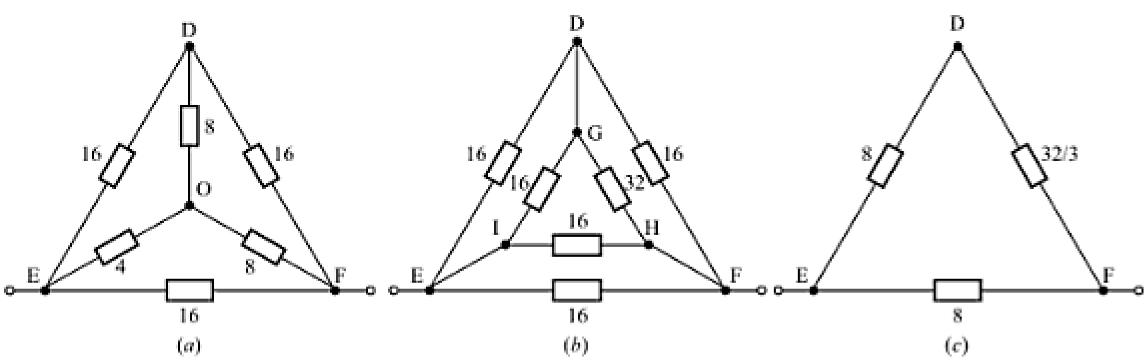


Fig. 2.25

EXAMPLE 2.23

Determine the current drawn from the 5-V battery in the network shown in Fig. 2.26*a*.

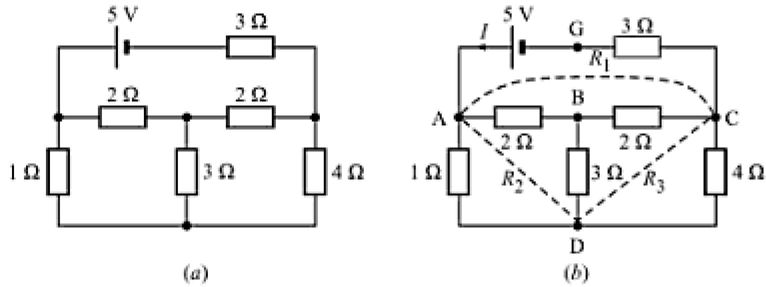


Fig. 2.26

Solution Current I drawn from the battery can be calculated easily, if we know the equivalent resistance R_{eq} of the network between the points A and G (Fig. 2.26*b*). The star connection between points A, C and D can be converted into its equivalent delta connection (shown by dotted lines), using Eqs. 2.30 to 2.32,

$$R_1 = \frac{2\times 3 + 3\times 2 + 2\times 2}{3} = \frac{16}{3}\ \Omega$$

$$R_2 = \frac{2\times 3 + 3\times 2 + 2\times 2}{2} = \frac{16}{2} = 8\ \Omega$$

$$R_3 = \frac{2\times 3 + 3\times 2 + 2\times 2}{2} = \frac{16}{2} = 8\ \Omega$$

The resulting network is shown in Fig. 2.27*a*. *It is important to visualize here that the point B is lost during the process of star-to-delta transformation.* Now, combining the parallel branches, we get the network of Fig. 2.27*b*. Combining the two series resistances, we get the network of Fig. 2.27*c*. We now combine the two parallel resistances to give an equivalent resistance of

$$\frac{\frac{16}{3}\times\frac{32}{9}}{\frac{16}{3}+\frac{32}{9}} = \frac{32}{15}\ \Omega$$

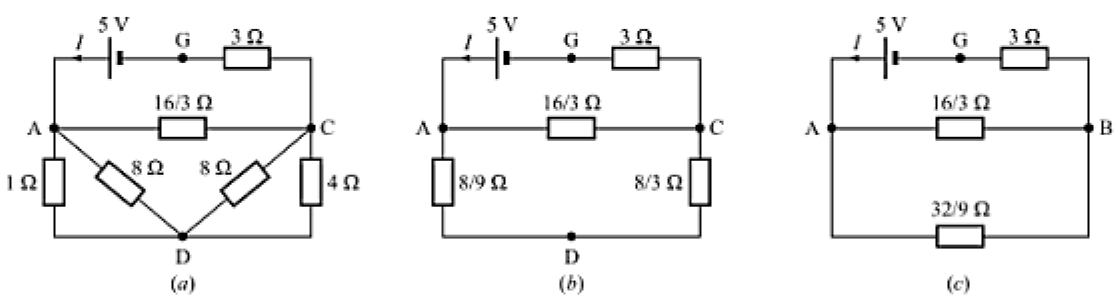


Fig. 2.27

Thus, the network reduces to a simple circuit of Fig. 2.28*a*. Finally, the two series resistances can be combined to get equivalent resistance of 77/15 Ω (Fig. 2.28*b*). The current I drawn from the battery is

$$I = \frac{5}{77/15} = \frac{75}{77} = \mathbf{0.974\ A}$$

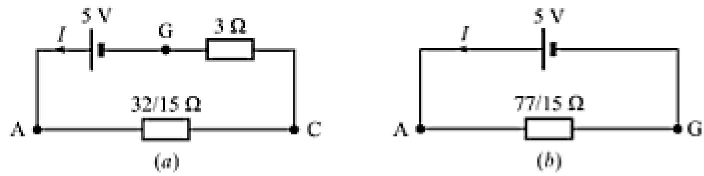


Fig. 2.28

NOTE

It is observed that during the network reduction/simplification process some points in the original network are lost. Hence, care should be taken during this process that no point of ultimate relevance is lost.

EXAMPLE 2.24

In the network shown in Fig. 2.29, the total power dissipated is 488 W. Determine the current flowing in each resistance and the pd between A and B.

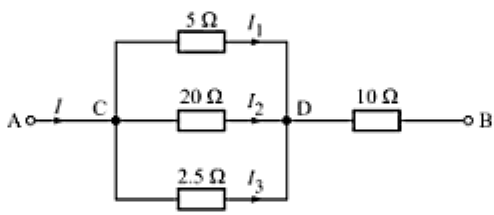


Fig. 2.29

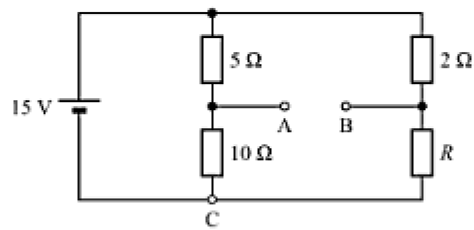


Fig. 2.30

Solution The equivalent resistance of the parallel combination is given by

$$\frac{1}{R_{CD}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{2.5} = 0.2 + 0.05 + 0.4 = 0.65\ \text{S} \quad \Rightarrow \quad R_{CD} = \frac{1}{0.65} = 1.54\ \Omega$$

$$\therefore \qquad R_{AB} = 1.54 + 10 = 11.54\ \Omega$$

Total voltage V across A and B is given by

$$P = \frac{V^2}{R} \quad \Rightarrow \quad V = \sqrt{PR} = \sqrt{488 \times 11.54} = \mathbf{75\ V}$$

$$\therefore \qquad I = \frac{V}{R} = \frac{75}{11.54} = \mathbf{6.5\ A}$$

$$\therefore \qquad V_{CD} = V - 10I = 75 - 10 \times 6.5 = 10\ \text{V};\ I_1 = \frac{10}{5} = \mathbf{2\ A};\ I_2 = \frac{10}{20} = \mathbf{0.5\ A};\ I_3 = \frac{10}{2.5} = \mathbf{4\ A}$$

EXAMPLE 2.25

In the circuit shown in Fig. 2.30, voltage $V_{AB} = 5$ V. Find the value of resistance R.

Solution Applying voltage divider concept,

$$V_{AC} = 15 \times \frac{10}{10+5} = 10\ \text{V}; \qquad \therefore \quad V_{BC} = V_{BA} + V_{AC} = -V_{AB} + V_{AC} = -5 + 10 = 5\ \text{V}$$

Again applying voltage divider concept to the second branch,

$$V_{BC} = 15 \times \frac{R}{2+R}; \quad \text{or} \quad 5 = \frac{15R}{2+R} \quad \text{or} \quad 10 + 5R = 15R \quad \Rightarrow \quad R = 1\ \Omega$$

EXAMPLE 2.26

In the circuit shown in Fig. 2.31*a*, determine (*a*) the current supplied by the 100-V source, and (*b*) the voltage across 6-Ω resistance.

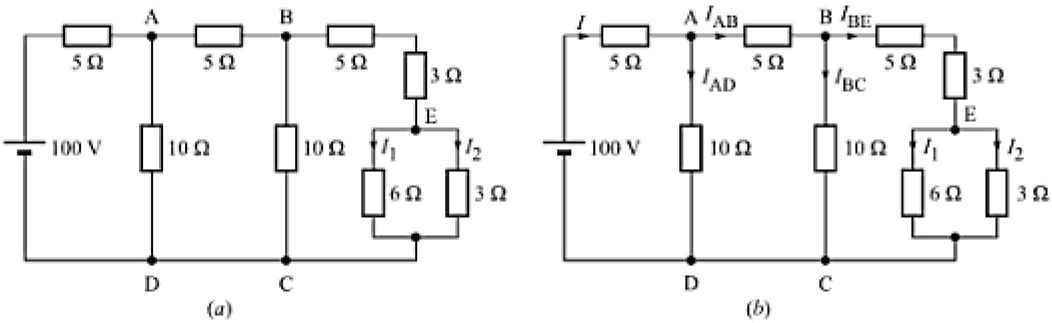


Fig. 2.31

Solution

(*a*) $R_{BC} = 10 \parallel \{5 + 3 + (6 \parallel 3)\} = 10 \parallel \{5 + 3 + 2\} = 10 \parallel 10 = 5\ \Omega$

$R_{AD} = 10 \parallel (5 + 5) = 5\ \Omega$; $\quad \therefore \quad R_{total} = 5 + 5 = 10\ \Omega$

Therefore, the total current supplied by the 100-V source is

$$I = \frac{100}{10} = \mathbf{10\ A}$$

(*b*) We use the concept of current divider to find the current I_1 through 6-Ω resistance. At A, the current of 10 A divides into two parts—I_{AD} and I_{AB} (see Fig. 2.31*b*). Note that the current I_{AB} depends not only on R_{AB}, but the total resistance on the right of point A (which is 10 Ω). Thus, the two currents will be equal. That is, $I_{AD} = 5$ A and $I_{AB} = 5$ A.

Again at B, current I_{AB} divides equally into $I_{BC} = 2.5$ A and $I_{BE} = 2.5$ A. The current I_{BE} divides into I_1 and I_2. Thus,

$$I_1 = 2.5 \times \frac{3}{6+3} = \frac{2.5}{3}\text{A}; \qquad \therefore \quad V_6 = I_1 \times 6 = \frac{2.5}{3} \times 6 = \mathbf{5\ V}$$

EXAMPLE 2.27

A resistance of *R* ohms is connected in series with a parallel combination of 8 ohms, 12 ohms and 24 ohms. The total power dissipated in the circuit is 80 W, when the applied voltage is 20 V. Calculate *R*.

Solution The equivalent resistance of the parallel combination is given by

$$\frac{1}{R_p} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{3+2+1}{24} = \frac{1}{4}; \quad \therefore R_p = 4\ \Omega$$

The total resistance of the circuit is $R_{total} = R + 4$ ohms. Power dissipated is given as

$$P = \frac{V^2}{R_{total}} \quad \text{or} \quad R_{total} = \frac{V^2}{P} = \frac{20^2}{80} = \frac{400}{80} = 5\ \Omega; \quad \Rightarrow \quad R = R_{total} - R_p = 5 - 4 = \mathbf{1\ \Omega}$$

EXAMPLE 2.28

A combination of two resistances R_1 and R_2 connected in parallel across a 100-V supply takes 10 A current from the mains. Determine the resistance R_2, if the power dissipated in R_1 is 600 W.

Solution The power dissipated in R_1 is given as

$$P = \frac{V^2}{R_1} \quad \Rightarrow \quad R_1 = \frac{V^2}{P} = \frac{100^2}{600} = \frac{100}{6}\ \Omega$$

$$\therefore \quad I_1 = \frac{V}{R_1} = \frac{100}{100/6} = 6\text{ A}; \quad I_2 = I - I_1 = 10 - 6 = 4\text{ A}; \quad R_2 = \frac{V}{I_2} = \frac{100}{4} = \mathbf{25\ \Omega}$$

EXAMPLE 2.29

A diesel electric generating set supplies an output of 50 kW. The calorific value of the fuel is 12 500 kcal/kg. If overall efficiency of the unit is 35 %, calculate (*a*) the quantity of oil needed per hour, and (*b*) the electrical energy generated per tonne of the fuel. Given that 1 kWh = 860 kcal.

Solution

$$\text{Input power} = \frac{\text{Output power}}{\text{Efficiency}} = \frac{50\text{ kW}}{0.35} = 142.9\text{ kW}$$

$$\therefore \quad \text{Heat energy per hour} = 142.9\text{ kWh} = 142.9 \times 860\text{ kcal}$$

(*a*) Fuel needed per hour $= \dfrac{142.9 \times 860\text{ kcal}}{12500\text{ kcal/kg}} = \mathbf{9.833\ kg}$

(*b*) The power output of the generator is 50 kW. In one hour, the energy generated is 50 kWh, and the fuel needed is 9.833 kg. Thus, the energy generated per tonne (= 1000 kg) of fuel is

$$= \frac{50\text{ kWh}}{9.833} \times 1000 = \mathbf{5085\ kWh}$$

EXAMPLE 2.30

Two heaters A and B are connected in parallel across a supply voltage. Heaters A and B produce 500 kcal in 20 minutes and 1000 kcal in 10 minutes, respectively. The resistance of heater A is 10 ohms. (*a*) Calculate the resistance of heater B. (*b*) If the two heaters are connected in series across the same supply voltage, how much heat will be produced in 5 minutes.

Solution

(*a*) The amounts of heat produced by the two heaters are

$$H_A = \frac{V^2}{R_A} \times t \quad \text{or} \quad 500 \times 4.2\text{ kJ} = \frac{V^2 \times (20 \times 60\text{ s})}{10}\text{ J} = \frac{V^2 \times (20 \times 60\text{ s})}{10 \times 1000}\text{ kJ} \qquad (i)$$

$$\text{and} \quad H_B = \frac{V^2}{R_B} \times t \quad \text{or} \quad 1000 \times 4.2\text{ kJ} = \frac{V^2 \times (10 \times 60\text{ s})}{R_B}\text{ J} = \frac{V^2 \times (10 \times 60\text{ s})}{R_B \times 1000}\text{ kJ} \qquad (ii)$$

Dividing (*i*) by (*ii*), we get $0.5 = 2R_B/10 = R_B/5 \quad \Rightarrow \quad R_B = \mathbf{2.5\ \Omega}$

(*b*) The series resistance, $R = 10 + 2.5 = 12.5\ \Omega$ and time $t = 5 \times 60$ s. The heat produced in five minutes is given as

$$H = \frac{V^2}{R} \times t = \frac{V^2 \times (5 \times 60 \text{ s})}{12.5 \times 1000} \text{ kJ} \qquad (iii)$$

Dividing (*iii*) by (*i*), we get

$$\frac{H}{500 \times 4.2} = \frac{10 \times 1000}{12.5 \times 1000} \times \frac{5 \times 60}{20 \times 60} \text{ kJ} \quad \text{or} \quad H = (500 \times 4.2) \times \frac{1}{5} \text{ kJ} = \mathbf{420\ kJ = 100\ kcal}$$

EXAMPLE 2.31

The domestic power load in a house comprises the following:
8 lamps of 100 W each, 3 fans of 80 W each, 1 refrigerator of 1/2 hp, 1 heater of 1000 W.

(*a*) Calculate the total current taken from the supply of 230 V.

(*b*) Calculate the energy consumed in a day, if on an average only a quarter of the above load persists all the time.

Solution

(*a*) The total load is given as

S. No.	*Item*	*Load*
1.	8 lamps of 100 W each	$8 \times 100 = 800$ W
2.	3 fans of 80 W each	$3 \times 80 = 240$ W
3.	1 refrigerator of 1/2 hp	$1 \times 1/2$ hp $= 1/2 \times 746$ W $= 373$ W
4.	1 heater of 1000 W	$1 \times 1000 = 1000$ W
	Total load	$= 2413$ W

$\therefore$ Current taken from the supply, $I = \dfrac{P}{V} = \dfrac{2413}{230} \approx$ **10.5 A**

(*b*) Energy consumed per day = 2413 W × 1/4 × 24 = 14478 Wh = **14.478 kWh**

SUMMARY

TERMS OF CONCEPTS

- ***Ohm's law*** states that the potential difference between the two ends of a conductor is directly proportional to the current flowing through it, provided its temperature and other physical parameters remain unchanged.
- The reciprocal of *resistance* (measured in Ω) is called *conductance* (measured in S).
- A ***short circuit*** ($R = 0$) permits current to flow ($i \neq 0$) without any resulting voltage ($v = 0$). An ***open circuit*** ($R = \infty$) permits voltage ($v \neq 0$) with no current ($i = 0$).
- ***Resistance*** is a measure of the opposition to the flow of charge through a load.
- Loads are ***series*** connected if the same current flows through each of them.
- Loads are ***parallel*** connected if the same potential difference is applied to each of them.
- The resistances of resistors can be identified by a colour code system.

IMPORTANT FORMULAE

- ***Ohm's Law:*** $V = RI$ or $I = GV$

- $R = \rho \dfrac{L}{A}$, where ρ is the ***resistivity*** of the material, measured in Ω m.
- Resistances in series: $R_s = R_1 + R_2 + R_3$; if n equal resistances, each R ohms, are connected in series, their equivalent resistance is nR ohms.
- Resistances in parallel: $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$; if n equal resistances, each R ohms, are connected in parallel, their equivalent resistance is R/n ohms.
- ***Voltage divider***: $V_1 = V\dfrac{R_1}{R_1 + R_2}$; ***Current divider***: $I_1 = I\dfrac{R_2}{R_1 + R_2}$
- ***Delta-to-star***: $\boxed{R_A = \dfrac{R_2 R_3}{R_1 + R_2 + R_3}}$; ***Star-to-delta***: $\boxed{R_1 = R_B + R_C + \dfrac{R_B R_C}{R_A}}$
- Variation of resistance with temperature: $R_1 = R_0[1 + \alpha_0(T_1 - T_0)]$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	Ohm's law can be expressed as: 'The current in a conductor is given as the ratio of the voltage across it to its conductance'.	☐	☐	
2.	Potential difference can be expressed as voltage rise or voltage drop.	☐	☐	
3.	The temperature coefficient of resistance (α) of a substance depends on its temperature.	☐	☐	
4.	A cassette player requires three 1.5-V batteries. If one battery is inserted backward, the net voltage developed will be 3 V.	☐	☐	
5.	A battery charger supplies 5 A current into a 12.6-V car battery. In this process, the power supplied to the car battery is 63 W.	☐	☐	
6.	A 1000-Ω resistor is rated at 5 W. Maximum current that can be permitted to flow through it is 70.7 A.	☐	☐	
7.	Two circuit elements are said to be connected in series when same current flows through them.	☐	☐	
8.	Two resistors are said to be in parallel if they share the same current.	☐	☐	
9.	If five resistors of conductance G are connected in parallel, their equivalent resistance will be $5/G$.	☐	☐	
10.	In a current divider consisting of three resistors, the resistances are in the ratio 1 : 2 : 3. The largest resistance will carry 18.2 % of the total current.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** True
6. False	**7.** True	**8.** False	**9.** False	**10.** True

REVIEW QUESTIONS

1. State and explain Ohm's law. Define resistivity of a material.

2. Explain what is meant by 'conventional current'. How does it differ from the current due to electron-flow in a conductor?

3. What is meant by the terms: (*a*) a short circuit, and (*b*) an open circuit.

4. Explain the meaning of *single-pole double-throw* and *double-pole double-throw* switches.

5. Describe the resistance parameter from three different points of view: circuit, energy and geometric.

6. How does the resistance of a conductor vary with its temperature?

7. Why do we use colour bands to indicate the values of the resistors used in electronic circuits?

8. What do the third and fourth colour bands on a resistor signify?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. The specific resistance ρ depends upon

(*a*) the material, the area of cross-section and the length of the conductor
(*b*) the area of cross-section and the length of the conductor only
(*c*) the area of cross-section of the conductor only
(*d*) the nature of the material of the conductor only

2. The resistance of a conductor increases when

(*a*) its length increases
(*b*) its area increases
(*c*) both its length and area increase
(*d*) its resistivity is kept constant

3. On increasing its temperature, the resistance of a conductor made of a metal

(*a*) decreases (*b*) increases
(*c*) remains constant (*d*) varies either way

4. The temperature coefficient of a conductor is defined as

(*a*) increase in its resistance per degree celsius
(*b*) increase in its resistance per kelvin
(*c*) increase in its resistance per ohm per degree celsius
(*d*) decrease in its resistance per degree celsius per ohm

5. The 'ampere second' could be the unit of

(*a*) conductance (*b*) power
(*c*) energy (*d*) charge

6. The polarity of voltage drop across a resistor is determined by

(*a*) the value of the resistor
(*b*) the value of current through the resistor
(*c*) the direction of current through the resistor
(*d*) the polarity of the source

7. If 110 V is applied across a 220-V, 100-W bulb, the power consumed by it will be

(*a*) 100 W (*b*) 50 W
(*c*) 25 W (*d*) 12.5 W

8. A resistance of 10 Ω is connected across a supply of 200 V. When another resistance of R ohms is connected in parallel with the above 10-Ω resistor, the current drawn from the supply doubles. The value of R is

(*a*) 5 Ω (*b*) 10 Ω
(*c*) 20 Ω (*d*) 40 Ω

9. Three resistances each of R ohms are connected in star. Its equivalent delta will comprise three resistances each of value

(*a*) $3R$ ohms (*b*) $2R$ ohms
(*c*) $R/3$ ohms (*d*) $R/2$ ohms

10. The current in a 5-Ω resistor connected in a network is 2 A. If this resistor is replaced by another of 10 Ω, the current in the branch will be

(*a*) more than 2 A (*b*) less than 2 A
(*c*) 2 A (*d*) 1 A

11. Three resistances each of R ohms are connected in delta. Its equivalent star will comprise three resistances each of value
   (a) $3R$ ohms (b) $2R$ ohms
   (c) $R/3$ ohms (d) $R/2$ ohms
12. A current of 10 A enters a parallel combination of two resistances of 2 Ω and 3 Ω. Then the currents through these resistances will be
   (a) 4 A, 6 A (b) 6 A, 4 A
   (c) 2 A, 8 A (d) 5 A, 5 A
13. Sixteen resistances each of 32 Ω are connected in parallel. The equivalent resistance of this combination is
   (a) 2 Ω (b) 16 Ω
   (c) 32 Ω (d) 16 × 32 Ω
14. Three resistors of 4 Ω, 6 Ω and 9 Ω are connected in parallel in a network. Maximum power will be consumed by
   (a) 4-Ω resistor (b) 6-Ω resistor
   (c) 9-Ω resistor (d) all resistors
15. The minimum number of resistors required to form a series-parallel combination is
   (a) one (b) two
   (c) three (d) four

ANSWERS

1. *d*	2. *a*	3. *b*	4. *c*	5. *d*	6. *c*	7. *c*	8. *b*	9. *a*	10. *b*
11. *c*	12. *b*	13. *a*	14. *a*	15. *c*					

PROBLEMS

(A) SIMPLE PROBLEMS

1. Two cubes of different materials measuring k metres and $2k$ metres on one side have equal resistances between any two faces. Find the ratio of their resistivities. **[Ans.** 2 : 1 **]**
2. Find the effective resistance between terminals A and B for the networks given in Fig. 2.32.
   [Ans. (a) 2 Ω; (b) $10R/9$; (c) 12 Ω; (d) 5 Ω; (e) $10R/3$**]**
3. The resistance of two coils is 25 ohms when connected in series, and 6 ohms when connected in parallel. Determine the individual resistances of the two coils. **[Ans.** 15 Ω , 10 Ω**]**
4. Determine the current I_x flowing through the 3-Ω resistor in the circuit of Fig. 2.33. **[Ans.** 4/3 A**]**

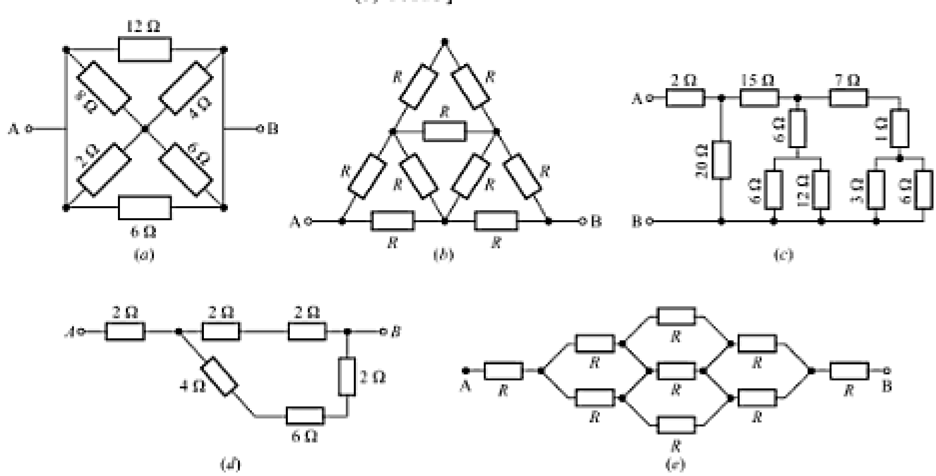


Fig. 2.32

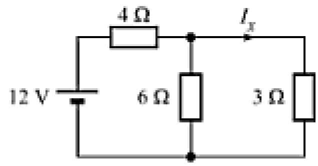


Fig. 2.33

5. When two resistances R_1 and R_2 are connected in parallel, they dissipate four times the power that they dissipate when they are connected in series with the same ideal source of emf. If $R_1 = 3\ \Omega$, find R_2. **[Ans.** 3 Ω**]**

6. If two electric bulbs, each designed to operate with a power of 500 W in 220-V line, are put in series in a 110-V line, what will be the power dissipated by each bulb? **[Ans.** 31.25 W**]**

7. A resistance of 8 ohms is connected in series with a parallel combination of 12 ohms and 24 ohms. The whole circuit is connected across a 100-V supply. Find (*a*) the current drawn from the supply, (*b*) the voltage across 8-ohm resistance, and (*c*) the currents flowing in 12-ohm and 24-ohm resistances.

[Ans. (*a*) 6.25 A; (*b*) 50 V; (*c*) 4.17, 2.08 A**]**

8. Determine I, I_1, I_2, V_{ab}, and V_{bc} in the network of Fig. 2.34. **[Ans.** 3 A, 1 A, 2 A, 6 V, 12 V**]**

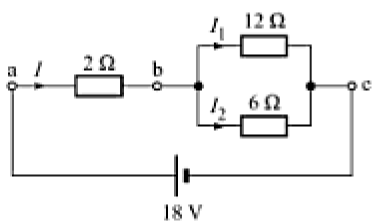


Fig. 2.34

9. Determine the voltage across and current through each resistor in the network of Fig. 2.35.

[Ans. 10 A, 7.5 A, 2.5 A, 50 V, 30 V **]**

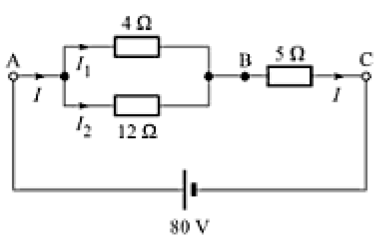


Fig. 2.35

10. Determine the total resistance between points A and B in the network shown in Fig. 2.36.

[Ans. 3 Ω**]**

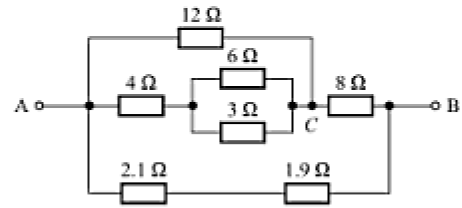


Fig. 2.36

11. For the circuit shown in Fig. 2.37, find (*a*) the equivalent resistance between points A and B, (*b*) the current and power supplied by the battery.

[Ans. (*a*) 8 Ω; (*b*) 6 A, 288 W**]**

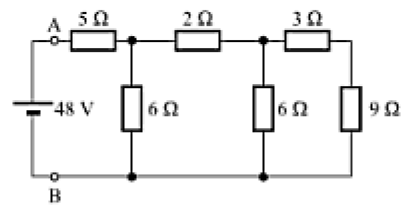


Fig. 2.37

12. Determine the current in 15-kΩ resistance in the circuit shown in Fig. 2.38. **[Ans.** 13.6 μA**]**

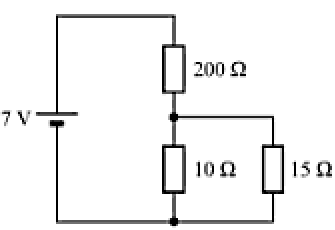


Fig. 2.38

(B) TRICKY PROBLEMS

13. Calculate the current drawn from a 12-V supply with internal resistance 0.5 Ω by the infinite ladder network, each resistance being 1 ohm, in Fig. 2.39. [**Ans.** 3.71 A]

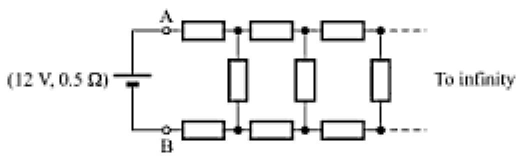


Fig. 2.39

14. Determine the value of resistance R, if the power dissipated in 10-ohm resistance is 360 W in the circuit of Fig. 2.40. [**Ans.** 36 Ω]

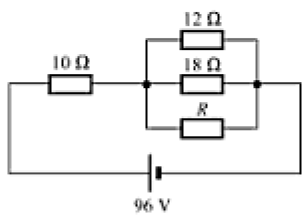


Fig. 2.40

15. The current in the 6-ohm resistor of the network shown in Fig. 2.41 is 2 A. Find the currents in all other resistors and the voltage across the network.

[**Ans.** 3.5 A, 1.5 A, 2.5 A, 1 A, 46 V]

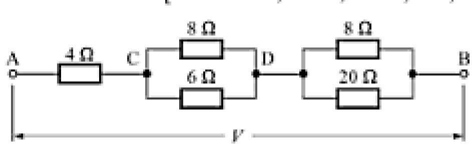


Fig. 2.41

16. Find the equivalent resistance between points A and B in the network shown in Fig. 2.42.

[**Ans.** 4 Ω]

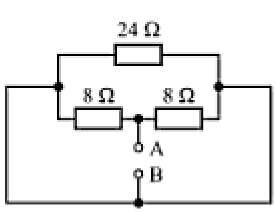


Fig. 2.42

17. In the network shown in Fig. 2.43, calculate (*a*) the current in the other resistances, (*b*) the value of the unknown resistance X, and (*c*) the equivalent resistance between points A and B.

[**Ans.** (*a*) 2 A, 1 A, 2 A; (*b*) 15 Ω; (*c*) 3 Ω]

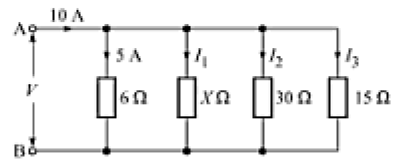


Fig. 2.43

18. Find the voltage V for the circuit shown in Fig. 2.44. [**Ans.** 310 V]

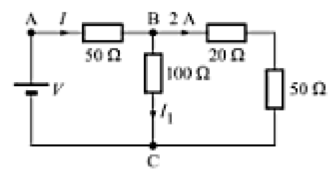


Fig. 2.44

19. In the circuit shown in Fig. 2.45, find (*a*) R and V_S, and (*b*) the power delivered by the source V_S.

[**Ans.** (*a*) 3 Ω and 8 V; (*b*) 16 W]

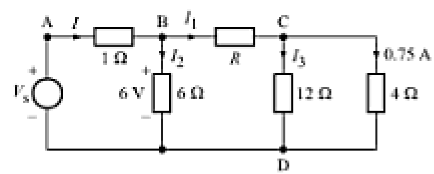


Fig. 2.45

20. In the circuit shown in Fig. 2.46, determine (*a*) the equivalent resistance between points A and B, (*b*) the total current, and (*c*) the power delivered to the 16-Ω resistance.

[**Ans.** (*a*) 28 Ω ; (*b*) 3.5714 A; (*c*) 12.75 W]

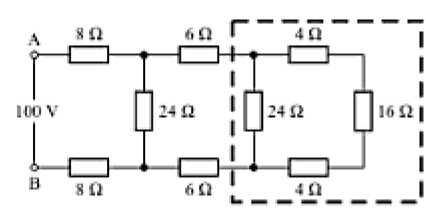


Fig. 2.46

21. In the circuit shown in Fig. 2.47, find (*a*) the current in 15-Ω resistance, (*b*) the voltage across 18-Ω resistance, and (*c*) the power dissipated in 7-Ω resistance.

[**Ans.** (*a*) 6.33 A; (*b*) 18 V; (*c*) 77.6 W]

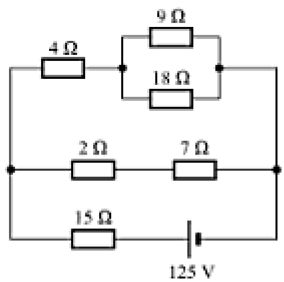


Fig. 2.47

22. If the total power dissipated in the network shown in Fig. 2.48 is 16 W, calculate the value of *R* and the total current. [**Ans.** 6 Ω; 2 A]

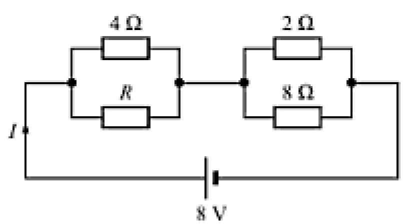


Fig. 2.48

23. The equivalent resistance of four resistances joined in parallel is 20 ohms. The current flowing through them are 0.6 A, 0.3 A, 0.2 A and 0.1 A, respectively. Find the value of each resistance.

[**Ans.** 40 Ω, 80 Ω, 120 Ω, 240 Ω]

24. A 10-Ω resistance is in series with a parallel combination of 15-Ω resistance and 5-Ω resistance. If the current in 5-Ω resistance is 6 A, what total power is dissipated in the three resistances?

[**Ans.** 880 W]

25. A resistance of 10 ohms is connected in series with a combination of two resistances arranged in parallel, each of 20 ohms. Calculate the value of resistance which should be shunted across the parallel combination so that the total current drawn by the circuit is 1.5 A with applied voltage of 20 V. [**Ans.** 5 ohms]

26. A resistance of *R* ohms is connected in series with a parallel circuit, consisting of 15-ohm and 10-ohm resistances. When a voltage of 20 V is applied across the circuit, a total power of 40 watts is dissipated. Find *R*. [**Ans.** 4 ohms]

27. The circuit shown in Fig. 2.49 represents a battery-charger charging a battery. (*a*) Find the power into the battery being charged. (*b*) What is the time it would take to impart 1 kilocoulomb charge to the battery? (*c*) What energy is given to the battery in this period of time?

[**Ans.** (*a*) 44.8 W; (*b*) 286 s; (*c*) 12.8 kJ]

28. In the circuit of Fig. 2.50, determine (*a*) the voltage v_{ab}, (*b*) the current *i*, and (*c*) the sum of powers into R_1, R_2 and the 2-V battery.

[**Ans.** (*a*) 9 V; (*b*) –15 A; (*c*) 84 W]

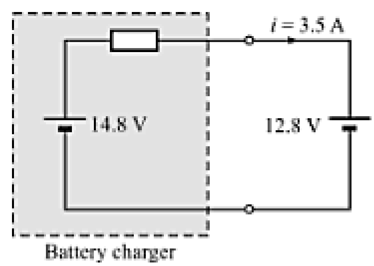


Fig. 2.49

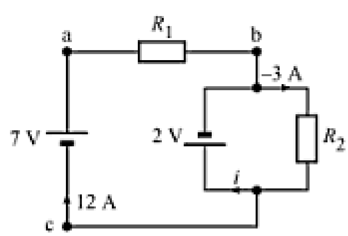


Fig. 2.50

(C) CHALLENGING PROBLEMS

29. Determine the equivalent resistance at the terminals A-B of the networks given in Fig. 2.51.

[**Ans.** (*a*) 4 Ω; (*b*) 36 Ω; (*c*) 18.75 Ω]

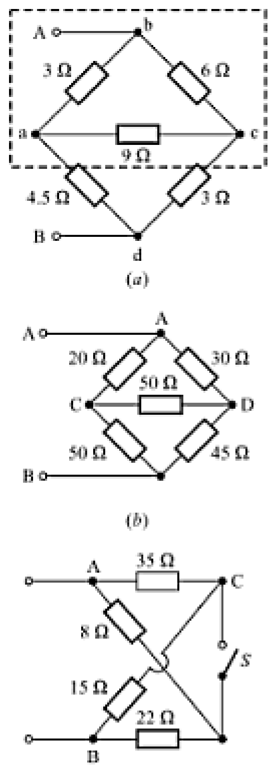


Fig. 2.51

30. A car radio designed to operate from a 6.3-V system uses 4.5 A of current, as shown in Fig. 2.52. (*a*) What resistance *R* should be placed in series with this radio if it is to be used in a 12.6-V system? (*b*) What should be the power rating of this resistance?

[**Ans.** 1.4 Ω, Min. 28.35 W]

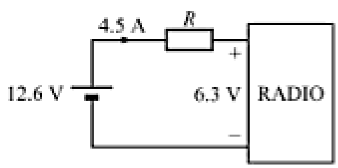


Fig. 2.52

31. (*a*) Find the equivalent resistance of the parallel combination shown in Fig. 2.53. (*b*) If 10 A current enters the parallel combination at point *a*, what is the current flowing downward in the 6-Ω resistance? (*c*) What is the current flowing upward in the 2-Ω resistance? (*d*) What is the voltage V_{ab} for this current?

[**Ans.** (*a*) 1.09 Ω; (*b*) 1.82 A; (*c*) –5.45 A; (*d*) 10.9 V]

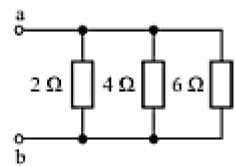


Fig. 2.53

32. (*a*) Two series resistances are to work as a voltage divider, with the smaller receiving 30 % of the total voltage. What are the resistors, given that their equivalent resistance is 200 Ω?

(*b*) If both resistors are 2-W resistors, what is the maximum total voltage the voltage divider can handle without exceeding this rating for either resistor?

[**Ans.** (*a*) 60 Ω and 140 Ω; (*b*) 23.9 V]

33. For the circuits shown in Fig. 2.54, find the indicated unknowns using the concept of voltage and current dividers.

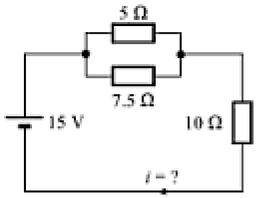
[**Ans.** (*a*) 1.15 A; (*b*) 20 V, 0.16 A; (*c*) 0.8686 V; (*d*) –5.0 V]

34. An electric kettle is required to heat 0.6 kg of water from 10 °C to boiling point in 5 minutes, the supply voltage being 230 V. The efficiency of the kettle is 78 %. Calculate (*a*) the resistance of the heating element, and (*b*) the cost of the energy consumed at Rs. 2.30 per kWh. Assume 1 kcal = 4200 J.

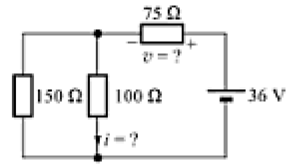
[**Ans.** (*a*) 54.72 Ω; (*b*) 20 p]

35. An electric kettle contains 1.2 kg of water at 20 °C. It takes 20 minutes to raise the temperature to 100 °C. Assuming the heat loss due to radiation, and heating the kettle to be 60 kJ, find the current taken by the kettle from the supply of 230 V. Assume specific heat capacity of water to be 4190 J/kg .

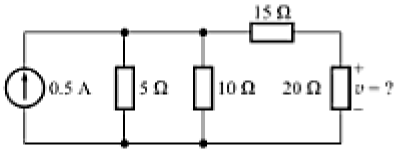
[**Ans.** 1.675 A]



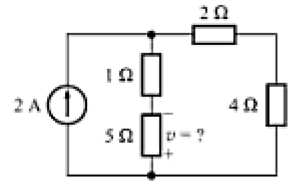
(*a*)



(*b*)



(*c*)



(*d*)

Fig. 2.54

36. The resistance of a coil embedded in a large transformer is 12 ohms at 25 °C. After the transformer has been in operation for several hours, the resistance is found to be 13.4 ohms. If $\alpha = 393 \times 10^{-5}$ at 20 °C, what is the temperature of the core of the transformer? [**Ans.** 53.54 °C]

EXPERIMENTAL EXERCISE 2.1

V-R CHARACTERISTICS OF LAMPS

Objectives

1. To plot voltage versus resistance characteristics of a tungsten filament lamp.
2. To plot voltage versus resistance characteristics of a carbon filament lamp.
3. To study the effect of temperature variation on the resistance of the two filaments.

Apparatus Single-phase AC power supply 220 V; Variac 0-270 V, 15 A; Voltmeter (MI type) 0-250 V; Ammeter (MI type) 0-1 A; Tungsten filament lamp 230 V, 100 W; Carbon filament lamp 230 V, 100 W.

Circuit Diagram The circuit diagram is shown in Fig. 2.55.

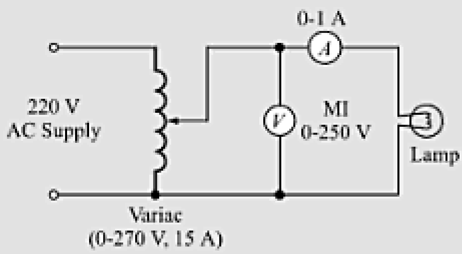


Fig. 2.55 *Experimental circuit.*

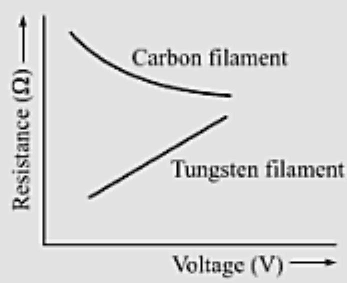


Fig. 2.56 *Variation of resistance with applied voltage.*

Brief Theory When current flows through the filament of the lamp, the electrical energy is converted into heat energy. The filament gets heated up and its temperature rises. The temperature is almost proportional to the applied voltage. Hence, the effect of voltage variation on resistance is similar to the effect of temperature variation on resistance.

If R_0 is the resistance of a filament at a standard temperature T_0 (0 °C), its resistance R_1 at a temperature T_1 is given as

$$R_1 = R_0 + \alpha_0 (T_1 - T_0)R_0$$

where α_0 is called *temperature coefficient of resistance* of the material at 0 °C.

The resistance of tungsten increases with temperature (α_0 positive), but that of carbon decreases with temperature (α_0 negative).

Procedure

1. Make connections as given in Fig. 2.55, using tungsten filament lamp.
2. Adjust the Variac for 100 V and take voltmeter and ammeter readings.
3. Increase the voltage up to 220 V in steps of 20 V, and after waiting for 2-3 minutes take readings of the voltmeter and ammeter.
4. Repeat the above steps for the carbon filament lamp.
5. Switch off the supply.

Observations

S. No.	*For tungsten lamp*			*For carbon lamp*		
	V (in V)	I in (A)	$R = V/I$ (in Ω)	V (in V)	I (in A)	$R = V/I$ (in Ω)
1						
2						
3						
4						
5						

Calculations For each set of readings of the voltmeter and ammeter, calculate the resistance of the lamp using the relation,

$$R = \frac{V}{I}$$

Graphs Plot the graphs of resistance versus applied voltage for both the lamps, preferably on the same graph paper (as in Fig. 2.56).

Results

1. The variation of resistance with the applied voltage (and hence with the temperature) is shown in the graph, for both lamps.
2. The resistance of the tungsten filament is seen to increase with the voltage.
3. The resistance of the carbon filament is seen to decrease with the voltage.

Precautions

1. Before switching on the supply, the zero reading of the voltmeter and ammeter should be checked.
2. On changing the voltage, wait for about 2 minutes so that the temperature of the filament is stabilized before taking the readings.

Viva-Voce

1. Can you name some materials having positive temperature coefficient of resistance?
   Ans.: Copper, Aluminium, Silver, etc.
2. Can you name some materials having negative temperature coefficient of resistance?
   Ans.: Germanium, Silicon, Gallium arsenide, etc.
3. Why is the filament of the lamp made of tungsten? Why not of copper?
   Ans.: For emitting light, the temperature of the filament should be raised high (more than 3000 K). Copper will melt at such high temperature.
4. Can you name some materials whose resistance is not much affected by rise in temperature?
   Ans.: Manganin and constantan. That is why these materials are used in making rheostats.
5. What is the effect of temperature on the resistance of insulating materials?
   Ans.: Their resistance decreases with rise in temperature.
6. Name a material that is bad conductor of electricity but good conductor of heat.
   Ans.: Mica.
7. Can you think of some application of mica, in which this property is used?
   Ans.: Yes, it is used in electric iron press.

NETWORK ANALYSIS

3

OBJECTIVES

After completing this Chapter, you will be able to:

- *Describe the three basic passive components—resistor, capacitor and inductor—from circuit viewpoint, energy viewpoint and geometric viewpoint.*
- *Derive the expressions for the equivalent inductance for series and parallel combinations.*
- *Derive the expressions for the equivalent capacitance for series and parallel combinations.*
- *Draw the V-I characteristics and circuit symbol of (a) ideal and practical voltage sources, and (b) ideal and practical current sources.*
- *Transform a practical voltage source into a practical current source and vice versa.*
- *To determine a single equivalent source if a number of practical voltages sources and current sources are connected in series-parallel combination.*
- *Describe different types of controlled or dependent sources.*
- *Apply Kirchhoff's voltage law* (KVL) *and Kirchhoff's current law* (KCL) *to solve an electrical circuit.*
- *Perform loop-current analysis of any general electrical network.*
- *Perform mesh analysis of a planar network.*
- *Perform node-voltage analysis of any general electrical network.*
- *Perform nodal analysis of a network containing only independent current sources.*
- *Select a more convenient method of analysis of a given network.*

3.1 NETWORK COMPONENTS

A network can be modelled in terms of (*a*) the ***interconnection*** of *elements*, *components* or *devices*, (*b*) a set of ***input signals***, and (*c*) a set of ***output signals***. The description of a network is written in terms of ***network variables***. These network variables are '***current through the components***' and '***voltage across the components***'. There are two kinds of components or devices in a network: (*i*) ***active***, and (*ii*) ***passive***. An active device supplies energy to the passive devices. The active device is also called a '***source***', and the passive device is called a '***load***'. There are *three* basic passive devices or components:

(*i*) Resistor (having *resistance*, R), measured in ohms (Ω),
(*ii*) Capacitor (having *capacitance*, C), measured in farads (F), and
(*iii*) Inductor (having *inductance*, L), measured in henrys (H).

The *energy received* by a passive component is

(*i*) Dissipated as heat, as in case of a resistance, or
(*ii*) Stored in it, in the form of:
  (*a*) Electric field, as in the case of a capacitance, or
  (*b*) Magnetic field, as in the case of an inductance.

There are *three* different ***points of view*** you can look at a passive component:

(*i*) *Circuit Viewpoint* We describe the component in terms of voltage-current relationship. This view point explains its behaviour when connected in a circuit.

(*ii*) *Energy Viewpoint* We describe the component in terms of energy dissipated by it or stored in it.

(*iii*) *Geometric Viewpoint* We describe the component in terms of its geometrical dimensions and the properties of the material used.

The Resistance (R)

The resistance element was discussed in detail in last Chapter. From ***circuit viewpoint***, a resistance R is an element that satisfies Ohm's law:

$$v = Ri \quad \text{or} \quad R = \frac{v}{i} \tag{3.1}$$

From ***energy viewpoint***, a resistance R is described by

$$W = i^2 Rt \quad \text{or} \quad R = \frac{W}{i^2 t} \tag{3.2}$$

From ***geometric viewpoint***, a resistance R is described as

$$R = \rho \frac{L}{A} \tag{3.3}$$

where, ρ is the resistivity of the material.

The Capacitance (C)

It exhibits the property of storing energy in an electric field. Its influence in an electric circuit is felt only when there occurs a change in potential difference across its terminals. It plays a significant role at the instant when a circuit is switched ON or OFF. At the time of switching ON, as there is no voltage across the capacitor*, it behaves as a ***short-circuit***. When the capacitor is fully charged, its voltage remains constant and the current i reduces to zero. It then behaves like an ***open-circuit***.

(*i*) *Circuit Viewpoint* From ***circuit view point***, a capacitance C is an element that satisfies the relation,

$$i = C\frac{dv}{dt} \quad \text{or} \quad C = \frac{i}{dv/dt} \tag{3.4}$$

It shows that the unit of capacitance is 'ampere-second/volt', also 'coulomb/volt'. This unit is given a name ***farad*** (F). The above relationship has been represented graphically in Fig. 3.1*a*. Figure 3.1*b* shows circuit symbol of a capacitor.

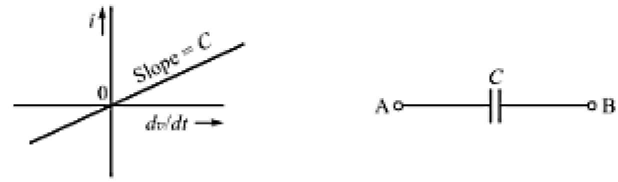
There are two important properties of a capacitor. Both these properties can be derived from Eq. 3.4:

(*i*) *No current flows through a capacitor, if the voltage across it remains constant.*
(*ii*) *The voltage across a capacitor cannot change instantaneously.*

---

* The physical device is called *capacitor* (or *condenser*), whereas its property is called *capacitance*. However, the two terms are often used interchangeably.

If the voltage is constant ($dv = 0$), Eq. 3.4 dictates that the current has to be zero ($i = 0$). If the voltage across the capacitor has to change instantaneously (i.e., for finite dv, we must have $dt = 0$), the current would have to be infinite which is a physical impossibility.



(*a*) Graphical representation. (*b*) Circuit symbol.

Fig. 3.1 *The capacitance.*

(ii) Energy Viewpoint It can be shown that the energy stored in a capacitor is

$$W = 1/2Cv^2 \tag{3.5}$$

Therefore, from energy point of view, a capacitor can be described in terms of energy stored as

$$C = \frac{2W}{v^2} \tag{3.6}$$

(iii) Geometric Viewpoint Consider, for example, a parallel plate capacitor. If A is the area of each plate separated by a distance d and ε is the *permittivity* of the material between the two plates, the capacitance of such a device is given as

$$C = \varepsilon\frac{A}{d} \tag{3.7}$$

Often, we write $\varepsilon = \varepsilon_r\,\varepsilon_0$, where ε_r is the relative permittivity (a pure number) and ε_0 is the permittivity of free space ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m). Note that Eq. 3.7 gives the value of the capacitance C in terms of geometric configuration of the capacitor.

The Inductance (L)

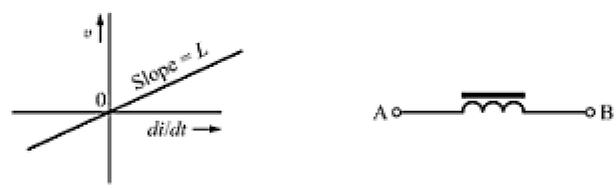
It exhibits the property of storing energy in a magnetic field. Its influence in an electric circuit is felt only when there occurs a change in current through the circuit.

(i) Circuit Viewpoint The behaviour of an inductance can be easily understood when we consider it from circuit point of view. The voltage v appearing across an inductor bears a definite relationship with the current i through the circuit,

$$v = L\frac{di}{dt} \quad \text{or} \quad L = \frac{v}{di/dt} \tag{3.8}$$

The constant L is called *inductance*. Above equation shows that the unit of inductance is 'volt-second/ampere' or 'Vs/A'. This unit is given the name ***henry*** (H). The inductance has been represented graphically in Fig. 3.2*a*. Figure 3.2*b* shows circuit symbol of an inductance.

The circuit element which exhibits the property given by Eq. 3.8 is called *inductor*. Ideally, an inductor has no resistance. But in practice, since an inductor is made by putting a conducting wire in a coil form, it has a little resistance. If the value of the inductance L is independent of current i, it is said to be a *linear inductance*.



(*a*) Graphical representation. (*b*) Circuit symbol.

Fig. 3.2 *The inductance.*

There are two important properties of an inductor. Both these properties can be derived from Eq. 3.8:

(*i*) *No voltage appears across an inductor, if the current through it remains constant.*
(*ii*) *The current through an inductor cannot change instantaneously.*

If the current is constant ($di = 0$), Eq. 3.8 dictates that the voltage has to be zero ($v = 0$). If the current through the inductor has to change instantaneously (i.e., for finite di, we must have $dt = 0$), the voltage would have to be infinite which is a physical impossibility.

An inductor has a property of opposing change in current. This property can be looked upon as the property of ***inertia***. An inductor plays a significant role at the instant when a circuit is switched ON or OFF. At the time of switching ON, the current i in the circuit is zero (though it tends to change very fast with time), it behaves as an ***open-circuit***. When a constant current flows through the circuit, it behaves as a ***short-circuit***.

(*ii*) *Energy Viewpoint* It can be shown that the energy stored in an inductor is

$$W = 1/2 L i^2 \tag{3.9}$$

Therefore, from energy point of view, an inductor can be described in terms of energy stored as

$$L = \frac{2W}{i^2} \tag{3.10}$$

(*iii*) *Geometric Viewpoint* It can be shown that the inductance of a coil of N turns having an area of cross-section A and a length l is given as

$$L = \frac{\mu N^2 A}{l} \tag{3.11}$$

where μ is the *permeability* of the material of the core. Often, we write $\mu = \mu_r \mu_0$, where μ_r is the relative permeability (a pure number) and μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m). Note that Eq. 3.11 gives the value of the inductance L in terms of geometric configuration of the inductor.

IMPORTANT NOTE

Comparison of Eq. 3.4 with Eq. 3.8 shows that inductance (L) is the ***dual*** of capacitance (C).

EXAMPLE 3.1

Find the value of capacitance parameter in each of the following cases:

(*i*) Two flat parallel plates are separated by 0.1 mm thick layer of mica having a relative permittivity of 10 and having a total area of 0.113 m^2.
(*ii*) A voltage of 100 V yields an energy storage of 0.05 J in an electric field.
(*iii*) A voltage increases linearly from 0 to 100 V in 0.1 s causing a current flow of 5 mA.

Solution

(*i*) Using Eq. 3.7, we get

$$C = \varepsilon \frac{A}{d} = \varepsilon_r \varepsilon_0 \frac{A}{d} = \frac{10 \times 8.854 \times 10^{-12} \times 0.113}{0.1 \times 10^{-3}} = \mathbf{0.1\ \mu F}$$

(*ii*) Using Eq. 3.6, we get

$$C = \frac{2W}{v^2} = \frac{2 \times 0.05}{(100)^2} = \mathbf{10\ \mu F}$$

(*iii*) Using Eq. 3.4, we get

$$C = \frac{i}{dv/dt} = \frac{5 \times 10^{-3}}{100/0.1} = \mathbf{5\ \mu F}$$

EXAMPLE 3.2

Find the inductance of the coil in each of the following:

(*i*) A current of 0.2 A yields an energy storage of 0.2 J in a magnetic field.
(*ii*) A current increases linearly from zero to 0.1 A in 0.2 s producing a voltage of 10 V.
(*iii*) A current of 0.1 A increases at the rate of 0.5 A/s producing a power of 2.5 W.

Solution

(*i*) Using Eq. 3.10, we get

$$L = \frac{2W}{i^2} = \frac{2 \times 0.2}{0.2 \times 0.2} = \mathbf{10\ H}$$

(*ii*) Using Eq. 3.8, we get

$$L = \frac{v}{di/dt} = \frac{10}{0.1/0.2} = \mathbf{20\ H}$$

(*iii*) We know that instantaneous power is given as $p = iv$. Therefore,

$$p = iL\frac{di}{dt}$$

or

$$L = \frac{p}{i\,di/dt} = \frac{2.5}{0.1 \times 0.5} = \mathbf{50\ H}$$

3.2 SERIES AND PARALLEL COMBINATIONS

The rules for combining inductances are the same as those for resistances. That is, the equivalent inductance of n ***inductances connected in series*** is given as

$$L_s = L_1 + L_2 + L_3 \ldots + L_n = \sum_{i=1}^{n} L_i \tag{3.12}$$

Same way, the equivalent inductance of n ***inductances connected in parallel*** is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots + \frac{1}{L_n} = \sum_{i=1}^{n} \frac{1}{L_i} \tag{3.13}$$

Since the capacitance (C) is ***dual*** of inductance (L), the rules for combining capacitances will be ***dual*** of those for inductances. Hence, the equivalent capacitance of n ***capacitances connected in series*** is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i} \tag{3.14}$$

Same way, the equivalent capacitance of n ***capacitances connected in parallel*** is given as

$$C_p = C_1 + C_2 + C_3 \ldots + C_n = \sum_{i=1}^{n} C_i \tag{3.15}$$

EXAMPLE 3.3

Three inductors are connected as shown in Fig. 3.3. Given that $L_1 = 2L_2$. Find L_1 and L_2 such that the equivalent inductance of the combination is 0.7 H.

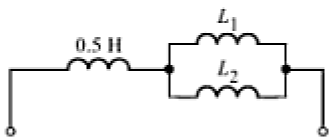


Fig. 3.3

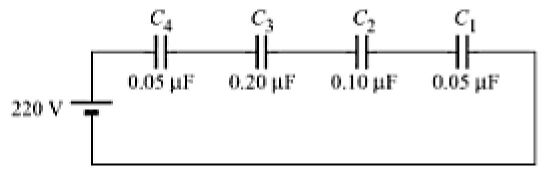


Fig. 3.4

Solution The equivalent inductance is given as

$$L_{eq} = 0.5 + \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad 0.7 = 0.5 + \frac{L_1 L_2}{L_1 + L_2} \quad \Rightarrow \quad \frac{(2L_2)L_2}{(2L_2) + L_2} = 0.2$$

Hence, L_2 = **0.3 H** and L_1 = **0.6 H**.

EXAMPLE 3.4

Determine the equivalent capacitance of the network shown in Fig. 3.4 and the voltage drop across each capacitor.

Solution The total capacitance (in μF) is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{0.05} + \frac{1}{0.10} + \frac{1}{0.20} + \frac{1}{0.05} = 55$$

$$\therefore \qquad C_s = \frac{1}{55} = \mathbf{0.0182\ \mu F}$$

The charge transferred to each capacitor is

$$Q = C_s V = 0.0182 \times 10^{-6} \times 220 = 4 \times 10^{-6}\ \text{C}$$

$$\therefore \qquad V_1 = \frac{Q}{C_1} = \frac{4 \times 10^{-6}}{0.05 \times 10^{-6}} = \mathbf{80\ V} \qquad V_2 = \frac{Q}{C_2} = \frac{4 \times 10^{-6}}{0.10 \times 10^{-6}} = \mathbf{40\ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{4 \times 10^{-6}}{0.20 \times 10^{-6}} = \mathbf{20\ V} \qquad V_4 = \frac{Q}{C_4} = \frac{4 \times 10^{-6}}{0.05 \times 10^{-6}} = \mathbf{80\ V}$$

EXAMPLE 3.5

Two capacitors are charged as shown in Fig. 3.5. After the switch is closed, what voltage exists across each capacitor ?

Solution Since the voltage polarities across the two capacitors are same, on closing the switch the total charge available for redistribution is Q = 400 μC + 200 μC = 600 μC. Since, the two capacitors are connected in parallel, the equivalent capacitance, C = 2 μF + 10 μF = 12 μF. Hence the voltage across the parallel combination is given as

$$V = \frac{Q}{C} = \frac{600 \times 10^{-6}}{12 \times 10^{-6}} = \mathbf{50\ V}$$

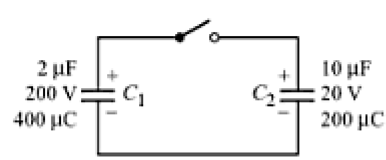


Fig. 3.5

EXAMPLE 3.6

A series combination of two capacitances $C_1 = 2\ \mu F$ and $C_2 = 8\ \mu F$ is connected across a dc supply of 300 V. Determine (*a*) the charge, (*b*) the voltage, and (*c*) the energy stored in each capacitor.

Solution

(*a*) ***Charge***: The equivalent capacitance of the series combination is given as

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 8}{2 + 8} = 1.6\ \mu F$$

When capacitances are connected in series, the charge on each capacitor is same. Hence, the charge on each capacitor is given as

$$Q = CV = (1.6\ \mu F) \times (300\ V) = \mathbf{480\ \mu C}$$

(*b*) ***Voltage***: The voltages across the two capacitors are given as

$$V_1 = \frac{Q}{C_1} = \frac{480\ \mu C}{2\ \mu F} = \mathbf{240\ V} \quad \text{and} \quad V_2 = \frac{Q}{C_2} = \frac{480\ \mu C}{8\ \mu F} = \mathbf{60\ V}$$

(*c*) ***Energy Stored***:

$$W_1 = (1/2)C_1V_1^2 = (1/2)\,(2 \times 10^{-6})\,(240)^2 = 57.6 \times 10^{-3}\ J = \mathbf{57.6\ mJ}$$
$$W_2 = (1/2)C_2V_2^2 = (1/2)\,(8 \times 10^{-6})\,(60)^2 = 14.4 \times 10^{-3}\ J = \mathbf{14.4\ mJ}$$

EXAMPLE 3.7

In the network of Fig. 3.6*a*, determine the value of capacitance C such that the equivalent capacitance between point A and B is 1 μF.

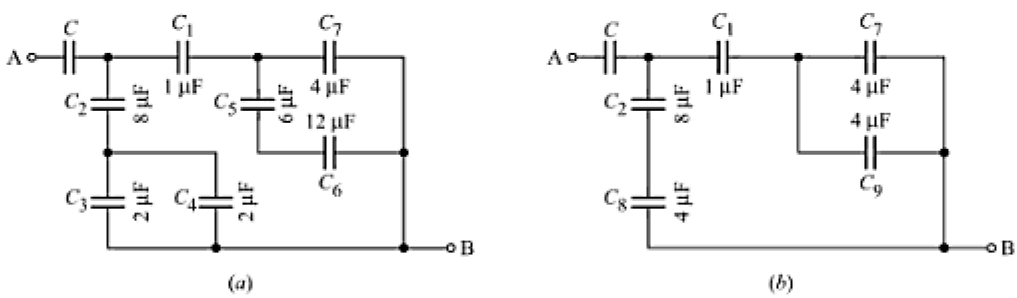


Fig. 3.6

Solution Capacitances C_3 and C_4 are in parallel. Therefore, their equivalent capacitance $C_8 = 2\ \mu F + 2\ \mu F = 4\ \mu F$. Capacitances C_5 and C_6 are in series. Therefore, their equivalent capacitance C_9 is $(6 \times 12)/(6 + 12) = 4\ \mu F$. The given network reduces to that shown in Fig. 3.6*b*. Combining C_2 and C_8 gives $C_{10} = 8/3\ \mu F$. Combining C_7 and C_9 gives $C_{11} = 8\ \mu F$. Thus, the network reduces to that shown in Fig. 3.7*a*.

Capacitances C_1 and C_{11} are combined to give $C_{12} = 8/9\ \mu F$ (Fig. 3.7*b*). Capacitances C_{10} and C_{12} are combined to give $C_{13} = 32/9\ \mu F$ (Fig. 3.7*c*). Since the required equivalent capacitance between point A and B is 1 μF, we must have

$$\frac{1}{1} = \frac{1}{C} + \frac{9}{32} \quad \text{or} \quad \frac{1}{C} = \frac{23}{32} \quad \text{or} \quad C = \frac{32}{23} = \mathbf{1.39\ \mu F}$$

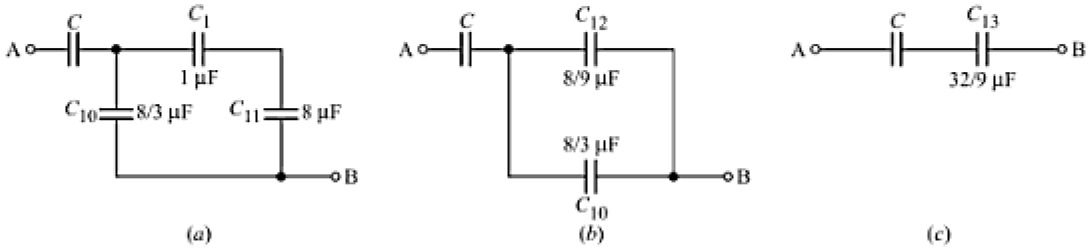


Fig. 3.7

3.3 ENERGY SOURCES

An electric circuit must have one or more energy sources. Just as we can view Ohm's law with two perspectives ($V = RI$ or $I = GV$), similarly an energy source could be taken as a *voltage source* or a *current source*. Let us first see what an *ideal voltage source* and an *ideal current source* mean.

Ideal Voltage Source

It is defined as an energy source whose terminal voltage (V) is independent of the output current (I). It means its terminal voltage is independent of the load resistance (R_L) connected to it. For any value of R_L, right from zero (i.e., a short circuit) to infinity (i.e., an open circuit), the terminal voltage remains constant. The mathematical definition of an ideal dc voltage source is described by Fig. 3.8*a*, and its graphical characteristic is shown in Fig. 3.8*b*. **Note** that the source determines the voltage, but the current is determined by the load. Figure 3.9 shows the circuit symbols for different types of *ideal voltage sources.*

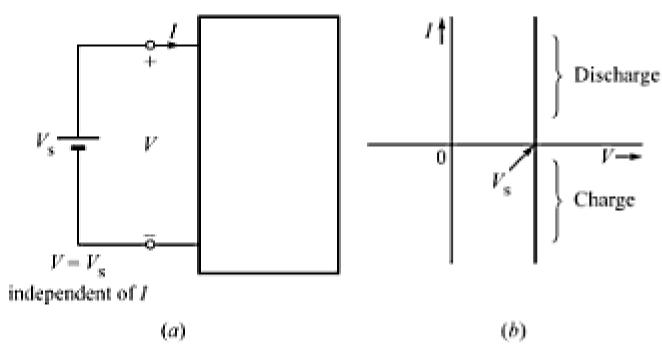


Fig. 3.8 *Ideal voltage source.*

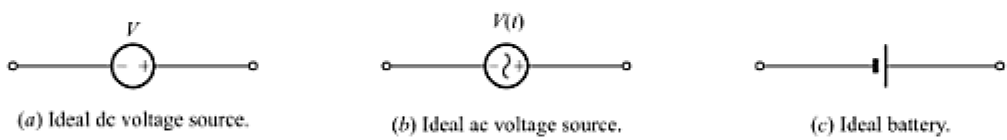


Fig. 3.9 *Symbols for different types of ideal voltage sources.*

Ideal Current Source

It is defined as a source which delivers a constant current, independent of its output voltage. Thus, the output current of such a source remains unchanged for R_L varying from zero (i.e., a short circuit) to infinity (i.e., an open circuit). The mathematical definition of an ideal dc current source is described by Fig. 3.10*a*, and its graphical characteristic is shown in Fig. 3.10*b*. **Note** that the source determines the current, but the voltage is determined by the load. Figure 3.11 shows the circuit symbols for different types of *ideal current sources*.

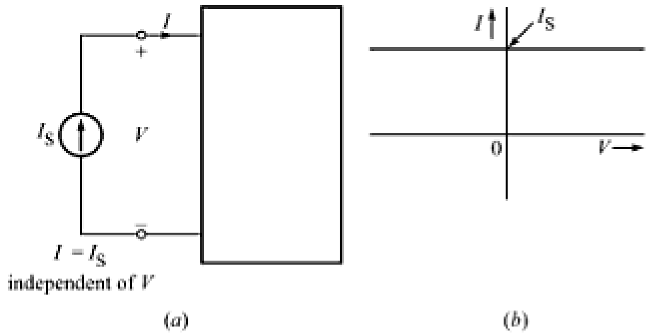


Fig. 3.10 *Ideal current source.*

Note that practically no such device as an *ideal voltage source* or an *ideal current source* exists. A source would meet the requirement of an ideal source only if it could supply an infinite amount of power. Practically, it is impossible. However, the concept of ideal sources helps us to describe the characteristics of practical sources.

Fig. 3.11 *Symbols for different types of ideal current sources.*

Practical Voltage Source

Consider a load resistance R_L connected to the terminals A-B of a practical voltage source such as a battery (Fig. 3.12*a*). What happens when we increase the load* on the source (by decreasing the value of the load resistance R_L)? Our experience is that the terminal voltage V_L decreases. To account for this fact, a *practical voltage source* is modelled as *an ideal voltage source in series with a resistance* (as shown in the dotted box in Fig. 3.12*a*). This resistance is named as '*internal resistance*' or '*source resistance*', R_{SV}. The voltage of this ideal source is called the ***electromotive force (emf)*** of the practical voltage source.

Note that the resistance R_{SV} has no physical existence. Point C inside the source is not accessible. This internal resistance is merely introduced to account for the non-ideality of the practical voltage source.

The linear relationship between V_L and I_L is given as

$$V_L = V_{SV} - R_{SV} I_L$$

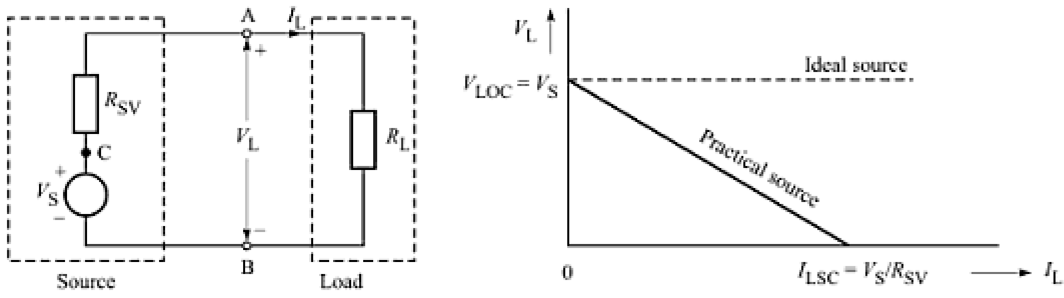
This characteristic line is plotted in Fig. 3.12*b*. The figure also shows the characteristic line (dotted) for an ideal voltage source. The open-circuit voltage (V_{LOC}) and short-circuit current (I_{LSC}) are given as

$$V_{LOC} = V_S \tag{3.16}$$

and

$$I_{LSC} = \frac{V_S}{R_{SV}} \tag{3.17}$$

* In electrical engineering, the term 'load' means the 'load current (I_L)', and **not** the 'load resistance (R_L)'.



(*a*) Circuit model. (*b*) Characteristics.

Fig. 3.12 *A practical voltage source.*

Practical Current Source

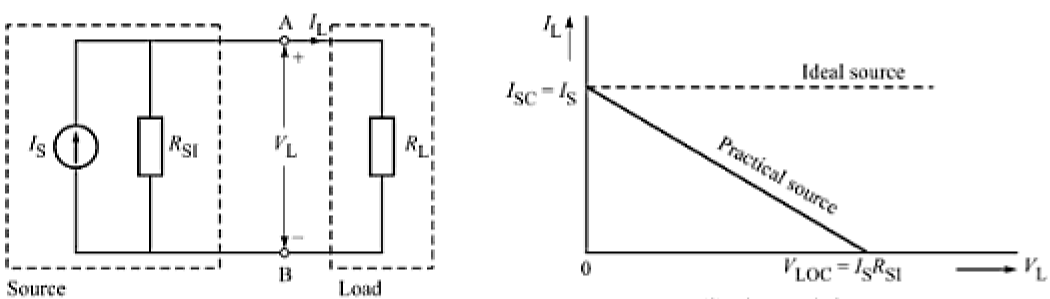
A *practical current source* is modelled as *an ideal current source in parallel with a resistance* (as shown in the dotted box in Fig. 3.13*a*). This resistance R_{SI} is called the *internal resistance*. (**Note** that a practical current source is *dual* of a practical voltage source.) The characteristic of a practical current source is described by the straight line

$$I_L = I_S - \frac{V_L}{R_{SI}}$$

This characteristic line is plotted in Fig. 3.13*b*. The figure also shows the characteristic of an ideal current source. The open-circuit voltage (V_{LOC}) and short-circuit current (I_{LSC}) are given as

$$V_{LOC} = R_{SI} I_S \tag{3.18}$$

$$I_{LSC} = I_S \tag{3.19}$$

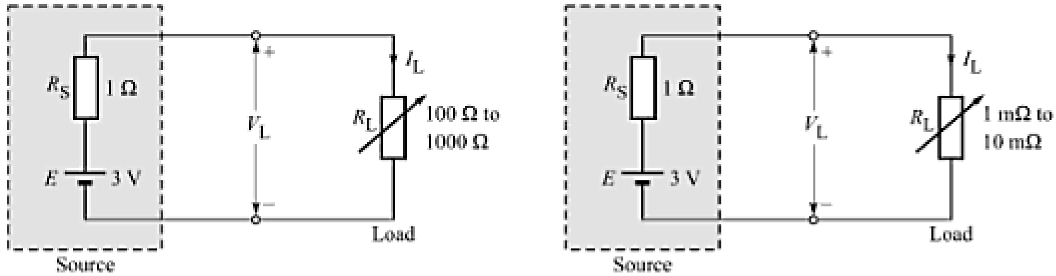


(*a*) Circuit model. (*b*) Characteristics.

Fig. 3.13 *A practical current source.*

EXAMPLE 3.8

A battery of emf 3 V and internal resistance of 1 Ω is used to supply power to a variable load resistance R_L of range (*a*) 100 Ω to 1000 Ω; (*b*) 1 mΩ to 10 mΩ, as shown in Fig. 3.14. Determine in the two cases, the % change in V_L and I_L.



(*a*) R_L = 100 Ω to 1000 Ω. (*b*) R_L = 1 mΩ to 10 mΩ.

Fig. 3.14 *A battery supplies power to a variable load.*

Solution

(*a*) For the extreme values of R_L, the terminal voltage (V_L) and current (I_L) are given as

$$I_{L1} = \frac{3}{(100+1)} = \mathbf{0.0297\ A}; \qquad V_{L1} = E - I_{L1}R_i = 3 - 0.0297 \times 1 = \mathbf{2.9703\ V}$$

$$I_{L2} = \frac{3}{(1000+1)} = \mathbf{0.002997\ A}; \qquad V_{L2} = E - I_{L2}R_i = 3 - 0.002997 \times 1 = \mathbf{2.9970\ V}$$

$$\therefore \quad \%\text{ change (a decrease) in } I_L = \frac{0.0297 - 0.002997}{0.0297} \times 100\ \% = \mathbf{89.9\ \%}$$

$$\therefore \quad \%\text{ change (an increase) in } V_L = \frac{2.9970 - 2.9703}{2.9703} \times 100\ \% = \mathbf{0.89\ \%}$$

(*b*)
$$I_{L1} = \frac{3}{(0.001+1)} = \mathbf{2.997\ A}; \qquad V_{L1} = E - I_{L1}R_i = 3 - 2.997 \times 1 = \mathbf{0.003\ V}$$

$$I_{L2} = \frac{3}{(0.01+1)} = \mathbf{2.970\ A}; \qquad V_{L2} = E - I_{L2}R_i = 3 - 2.970 \times 1 = \mathbf{0.03\ V}$$

$$\therefore \quad \%\text{ change (a decrease) in } I_L = \frac{2.997 - 2.970}{2.997} \times 100\ \% = \mathbf{0.9\ \%}$$

$$\therefore \quad \%\text{ change (an increase) in } V_L = \frac{0.03 - 0.003}{0.003} \times 100\ \% = \mathbf{900\ \%}$$

COMMENTS

On varying the load, in (*a*) the percentage change in I_L is large (almost 90 %), but the percentage change in V_L is quite small (less than 1 %). In the other instance, (*b*) the percentage change in I_L is quite small (less than 1 %), but the percentage change in V_L is large (900 %). In both cases, the load changes by the same order (10 times).

It is more appropriate to view the source in case (*a*) as a *practical voltage source*, since its characteristic approaches to an ideal voltage source. Similarly, in case (*b*) the source is more appropriately treated as a *practical current source*. An energy source can be treated either as a voltage source or as a current source, depending upon which type suits better for the analysis of the network. Thus, the transformation of one type of source into another type is frequently needed while analysing networks.

Source Transformation

A practical voltage source can be *transformed* into a practical current source, and vice versa. Two sources would be equivalent if they produce identical value of I_L and V_L when they are connected to the same load R_L, whatever be its value. The two equivalent sources should also provide the same open-circuit voltage and the same short-circuit current. Equating Eq. 3.16 to Eq. 3.18, we get

$$V_S = R_{SI} I_S$$

Equating Eq. 3.17 to Eq. 3.19, gives

$$I_S = \frac{V_S}{R_{SV}}$$

From the above two equations, it follows that

$$R_{SV} = R_{SI} = R_S \text{ (say)} \qquad \text{and} \qquad V_S = R_S I_S \tag{3.20}$$

where, R_S would represent the internal resistance of either of the sources.

As shown in Fig. 3.15, the two practical sources will be equivalent with respect to the load terminals. However, they are not equivalent internally.

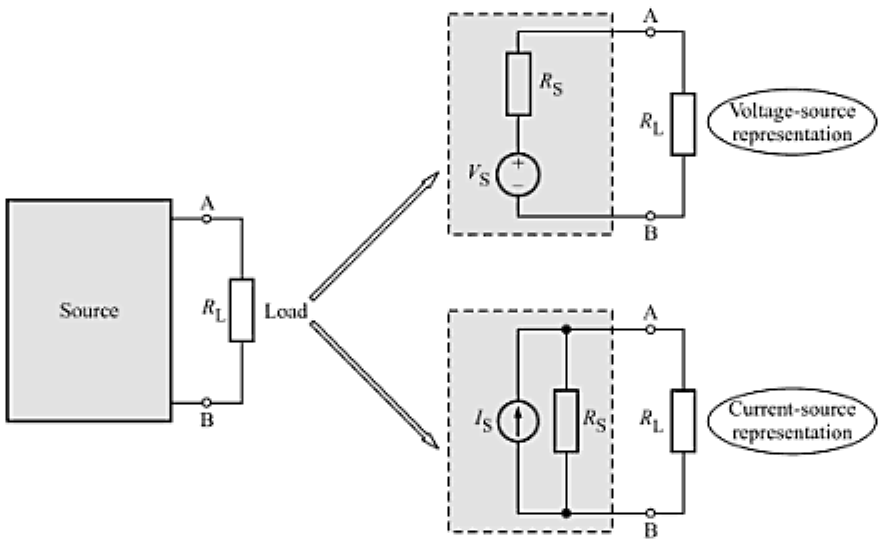


Fig. 3.15 *A source connected to a load can be treated either as a voltage source or as a current source.*

EXAMPLE 3.9

In the circuit of Fig. 3.16*a*, replace the practical current source by its equivalent voltage source and check whether you get same I_L and V_L in the two cases. Also find the power delivered by the ideal part of the sources in the two cases.

Solution Using Eq. 3.20, the internal resistance and the voltage of the equivalent voltage source are given as

$$R_S = 2\ \Omega \qquad \text{and} \qquad V_S = R_S I_S = 2 \times 3 = 6\text{ V}$$

The equivalent voltage source is shown in Fig. 3.16*b*. Let us now find I_L, V_L and source power P_S in the two cases.

Case I: $I_L = 3 \times \frac{2}{2+4} = \mathbf{1\ A}$; $V_L = 1 \times 4 = \mathbf{4\ V}$ and $P_S = I_S^2 R = 3^2 \times (2 \parallel 4) = \mathbf{12\ W}$

Case II: $I_L = \frac{6}{2+4} = \mathbf{1\ A}$; $V_L = 6 \times \frac{4}{2+4} = \mathbf{4\ V}$ and $P_S = \frac{V^2}{R} = \frac{6^2}{2+4} = \mathbf{6\ W}$

Thus, we find that the two energy sources are equivalent as regards the terminal relationships. But, the total power delivered by the ideal part of the two sources is different.

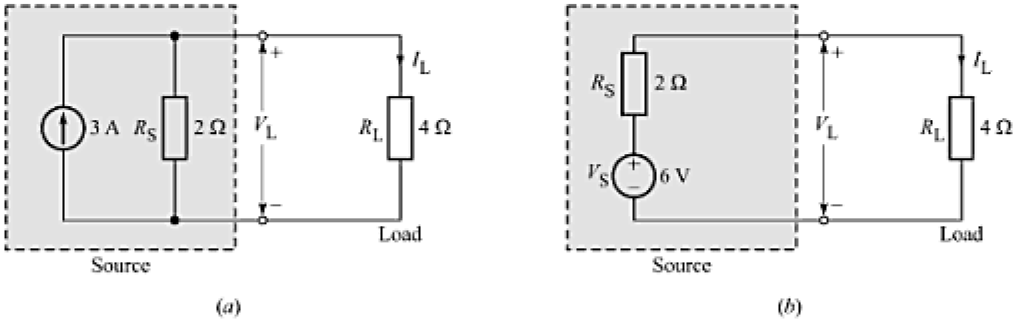


Fig. 3.16 *Equivalent sources.*

EXAMPLE 3.10

Find the currents through the two resistors in the circuit of Fig. 3.17. Then transform the current source and 2-Ω resistor to an equivalent voltage source and again find the currents through the two resistors. Compare the results.

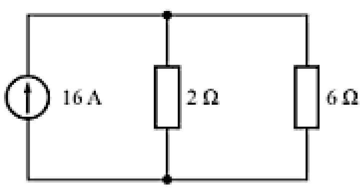


Fig. 3.17

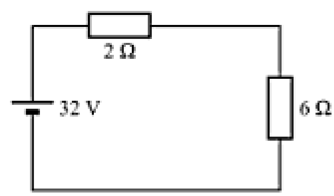


Fig. 3.18

Solution By current division rule,

$$I_{2\,\Omega} = 16 \times \frac{6}{6+2} = \mathbf{12\ A} \qquad \Rightarrow \qquad I_{6\,\Omega} = 16 - I_{2\,\Omega} = 16 - 12 = \mathbf{4\ A}$$

Now, transforming the current source gives a voltage source of 16 × 2 = 32 V in series with a 2-Ω resistor, as shown in Fig. 3.18. In this circuit, the current through both the resistors is the same and is given as

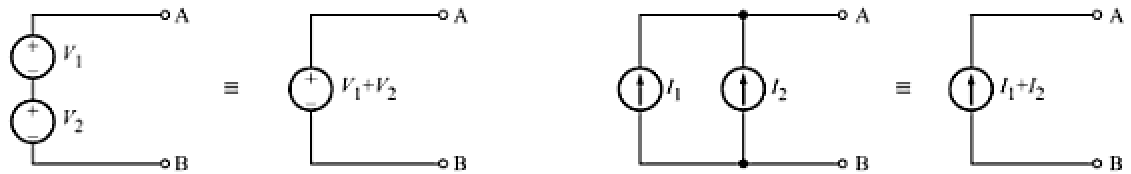
$$I_{2\,\Omega} = I_{6\,\Omega} = \frac{32}{2+6} = \mathbf{4\ A}$$

Note that the current in 6-Ω resistor is the same as for the original circuit, but the current in 2-Ω resistor is different. This result illustrates the fact that although a transformed source produces the same voltages and currents in the circuit exterior to the source, but the voltage and current inside the source usually change.

3.4 COMBINATION OF SOURCES

Ideal Sources

Two *ideal voltage sources* connected in ***series*** can be replaced by a single equivalent ideal voltage source by adding the voltages of the two sources, as shown in Fig. 3.19*a*. Its *dual* statement is that two *ideal current sources* connected in ***parallel*** can be replaced by a single equivalent ideal current source by adding the currents of the two sources, as shown in Fig. 3.19*b*.

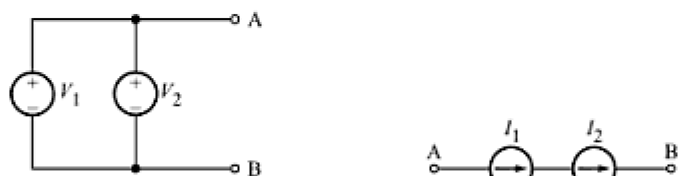


(*a*) Ideal voltage sources.

(*b*) Ideal current sources.

Fig. 3.19 ***Valid*** *combination of two ideal sources.*

Two *ideal voltage sources* (of different values) cannot be connected in parallel. This connection contradicts the definition of ideal voltage source. Similarly, the *dual* statement would be that two *ideal current sources* (of different values) cannot be connected in series. Thus, the combinations of ideal sources shown in Fig. 3.20 are invalid and such connections are not permitted. However, two ideal voltage sources of same value can be connected in parallel. In such a case, we must have $V_1 = V_2 = V$. Similarly, its dual is also true. Two ideal current sources of same value can be connected in series. In such a case, we must have $I_1 = I_2 = I$.



(*a*) Ideal voltage sources in parallel.

(*b*) Ideal current sources in series.

Fig. 3.20 ***Invalid*** *combination of two ideal sources.*

Practical Sources

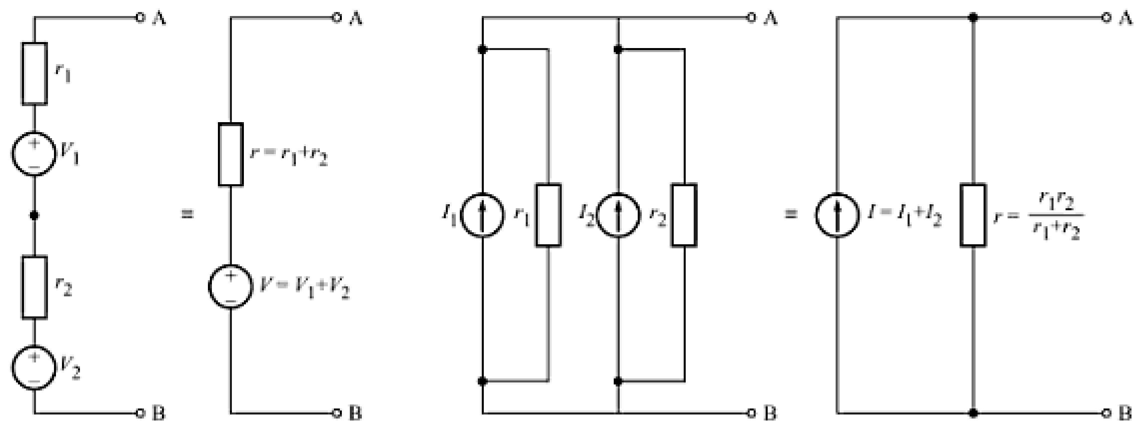
Practical sources can be combined in whatever way we like. Two ***practical voltage sources connected in series*** can be combined **directly**, as shown in Fig. 3.21*a*. The voltage and the internal resistance of the equivalent voltage source are given as

$$V = V_1 + V_2 \qquad \text{and} \qquad r = r_1 + r_2 \tag{3.21}$$

Similarly, the *dual* is true for ***practical current sources.*** Two ***practical current sources connected in parallel*** can be combined **directly**, as shown in Fig. 3.21*b*. The current and the internal resistance of the equivalent current source are given as

$$I = I_1 + I_2 \qquad \text{and} \qquad r = r_1 \parallel r_2 = \frac{r_1 r_2}{r_1 + r_2} \tag{3.22}$$

In case, two practical voltage sources are connected in parallel, they are first transformed into their equivalent practical current sources and then combined directly, as explained above. Similarly, when two practical current sources are connected in series, they are first transformed into their equivalent practical voltage sources and then combined directly, as explained above. The example given below explains the above process.



(*a*) Practical voltage sources.

(*b*) Practical current sources.

Fig. 3.21 ***Direct*** *combination of two ideal sources.*

EXAMPLE 3.11

The network shown in Fig. 3.22*a* has four voltage sources connected across points A and B. Reduce this network to a single voltage source, by using source transformation.

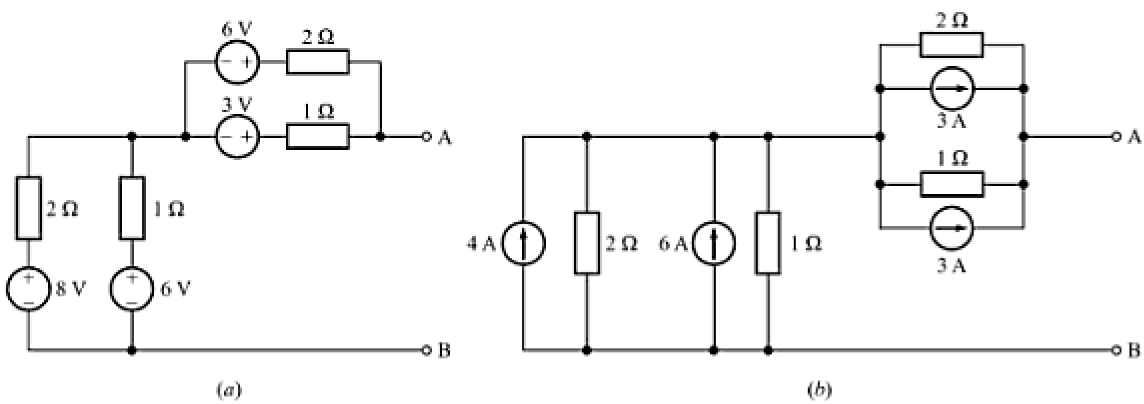


Fig. 3.22

Solution First, all the voltage sources are transformed into their corresponding equivalent current sources, to give the network of Fig. 3.22*b*. The current sources in parallel are then directly combined to get the network of Fig. 3.23*a*. Next, the current sources are transformed into their corresponding equivalent voltage sources, to give the network of Fig. 3.23*b*. Finally, the two voltage sources in series are combined directly to give the network of Fig. 3.23*c*.

Thus, the network of Fig. 3.22*a* is simply equivalent to a single voltage source of 32/3 V with internal resistance of 4/3 Ω.

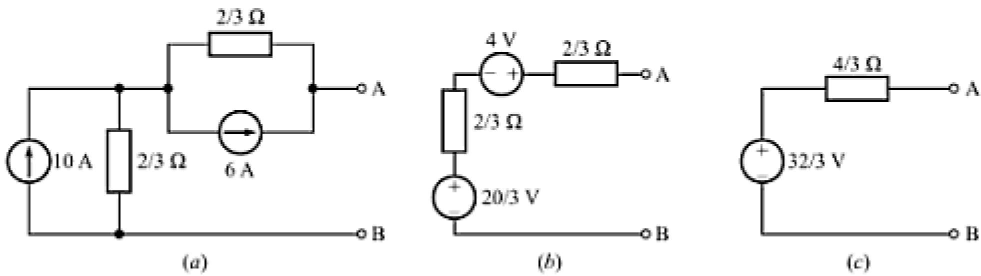


Fig. 3.23

EXAMPLE 3.12

We take a ***benchmark example**** of a circuit given in Fig. 3.24*a*. Using source transformation, we shall determine the voltage v across 3-Ω resistor

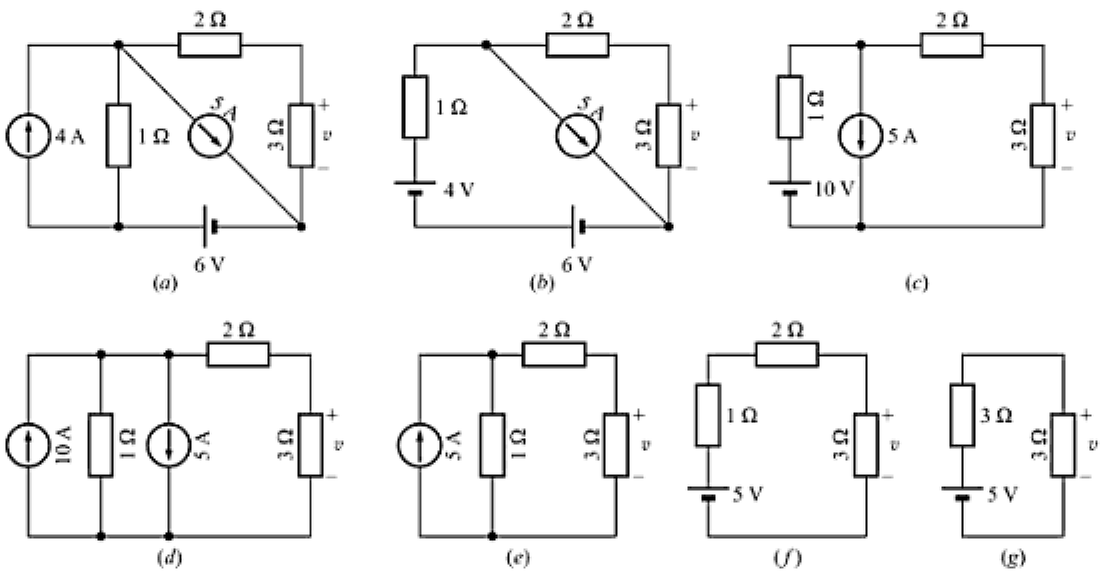


Fig. 3.24

Solution First, we transform the 4-A current source in parallel with 1-Ω resistor into a voltage source of 4 V in series with 1-Ω resistor (Fig. 3.24*b*). The two voltage sources are combined to give a 10-V source in series with a 1-Ω resistor, as shown in Fig. 3.24*c*. We again transform this 10-V voltage source into a current source of 10 A in parallel with 1-Ω resistor, as shown in Fig. 3.24*d*. The two current sources, having opposite current-directions, are combined to give a 5-A current source (Fig. 3.24*e*). Transforming this current source into voltage source (Fig. 3.24*f*), and then combining the two resistances we get a single voltage source of 5 V in series with a 3-Ω resistor (Fig. 3.24*g*). Finally, using voltage divider, v = **2.5 V.**

* We call this our "**benchmark example**" because we will solve this problem by different methods presented in this Book.

3.5 TYPES OF SOURCES

From different view points, energy sources can be classified as given below.

(1) Voltage source *or* Current source
(2) Ideal source *or* Practical source
(3) DC source *or* AC source
(4) Independent source *or* Dependent (controlled) source

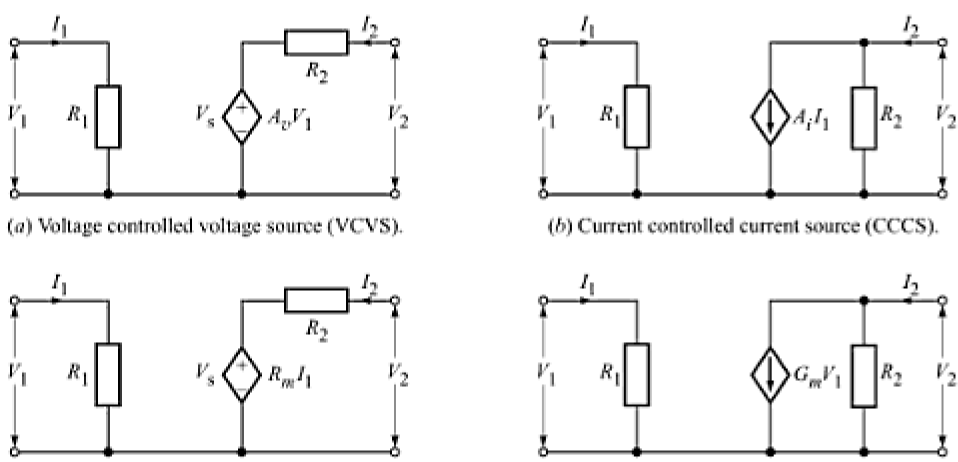
We already know what is meant by a *voltage source* or a *current source*, and an *ideal source* or a *practical source*. A *dc source* (such as a battery) supplies dc power, whereas an *ac source* (such as the power supply in our homes) supplies ac power.

The value of an ***independent source*** does not depend on the current through or the voltage across any other element of the network. Such sources are represented by a small circle. On the other hand, the value of ***dependent sources*** depends on some other current or voltage. These are also known as ***controlled sources***. Such sources are represented by a diamond shape. We find application of the controlled sources in electronics, while making the small-signal or ac equivalent of an electronic device (such as a bipolar junction transistor or a field-effect transistor). These devices are used in amplifiers, having a pair of *input terminals* and a pair of *output terminals*. As shown in Fig. 3.25, there are four kinds of controlled (or dependent) sources:

(*a*) *Voltage Controlled Voltage Source* (VCVS) The proportionality constant A_v is called *voltage ratio* (having no units).

(*b*) *Current Controlled Current Source* (CCCS) The proportionality constant A_i is called *current ratio* (having no units).

(*c*) *Current Controlled Voltage Source* (CCVS) The proportionality constant R_m is called *mutual resistance* (with units of Ω).



(*a*) Voltage controlled voltage source (VCVS). (*b*) Current controlled current source (CCCS).
(*c*) Current controlled voltage source (CCVS). (*d*) Voltage controlled current source (VCCS).

Fig. 3.25 *Dependent or controlled sources.*

***(d) Voltage Controlled Current Source* (VCCS)** The proportionality constant G_m is called *mutual conductance* (with units of S).

EXAMPLE 3.13

For the circuit of Fig. 3.26, determine (*a*) the value of current I, (*b*) the power absorbed by the dependent source, and (*c*) the resistance "seen" by the independent voltage source.

Solution

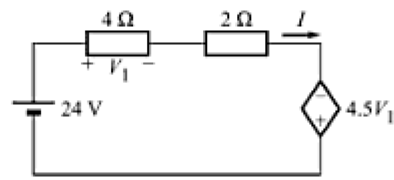


Fig. 3.26

(*a*) Applying Ohm's law to the 4-Ω resistor, gives $V_1 = 4I$. Therefore, the value of dependent voltage source is $4.5V_1 = 4.5(4I) = 18I$. By applying KVL, we get

$$24 - 4I - 2I + 18I = 0 \quad \Rightarrow \quad I = \mathbf{-2\ A}$$

The minus sign shows that the 2-A current flows opposite to the reference direction shown.

(*b*) For the dependent source, the current and voltage reference do not confirm to a passive element, and hence the power absorbed is

$$P = -(4.5V_1)(I) = -4.5(4I)(I) = -18I^2 = -18(-2)^2 = \mathbf{-72\ W}$$

The negative sign indicates that the dependent source is supplying power instead of absorbing it.

(*c*) The resistance "seen" by the source is equal to the ratio of the source voltage to the current going out of its positive terminal:

$$R = \frac{24}{I} = \frac{24}{-2} = \mathbf{-12\ \Omega}$$

The negative sign of the resistance is a result of the action of the dependent source. It indicates that the remainder of the circuit supplies power to the independent source of 24 V. Actually, it is the dependent source alone that supplies this power, as well as to the two resistors.

3.6 KIRCHHOFF'S LAWS

The two laws given by *Gustav Robert Kirchhoff* (1824-1887) form the fundamental principles used in writing circuit equations. These laws relate to the *topology* (i.e., the way the circuit elements are connected) of the circuit. The laws do not depend on the nature of the elements of the circuit.

(1) Kirchhoff's Current Law (KCL)

It states that *the algebraic sum of currents meeting at a junction in a circuit is zero.* If there are k number of branches meeting at a junction (also called a ***node***), then

$$\sum_{j=1}^{k} I_j = 0 \tag{3.23}$$

KCL can be stated in another way. *The sum of currents flowing from a junction is equal to the sum of currents flowing towards it.* Note that this law is just a restatement of *principle of conservation of charge.* Since, charges cannot accumulate at a junction, the amount of charge entering it at an instant must be the same as the amount of charge leaving it.

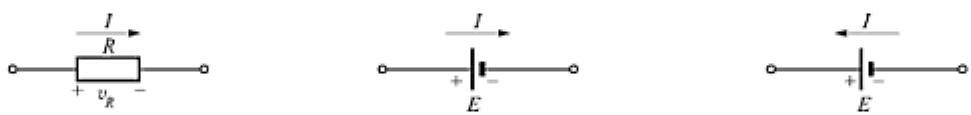
(2) Kirchhoff's Voltage Law (KVL)

It states that *at any instant the algebraic sum of voltages around a closed loop or circuit is zero.* For a closed loop having k elements,

$$\sum_{j=1}^{k} V_j = 0 \tag{3.24}$$

This law is just a restatement of *principle of conservation of energy*. The charges travelling around a closed loop transfer energy from one element to another, but do not receive energy themselves on the average. It means if you move a hypothetical test charge around a complete loop, the total energy exchanged would add up to zero.

***Applying* KVL** There are different ways to apply KVL in a closed loop of a circuit. Often, students commit error while applying KVL. The *first* step is to mark the voltage polarity (+ and –) across each element in the closed loop. In a resistance, the polarity depends upon the assumed reference direction of current. As shown in Fig. 3.27*a*, the end into which the current enters is marked +. **Note** that, as shown in Figs. 3.27*b* and *c*, the polarity of the voltage (emf) across a battery does not depend upon the assumed direction of current. The polarity of emf is fixed, but the current can flow through it in any direction. When the current flows into the positive terminal of the battery (Fig. 3.27*b*), it acts as a *load*. But, if the current flows away from the positive terminal, the battery acts as a *source* (Fig. 3.27*c*).



(*a*) A resistance. (*b*) A battery as a *load*. (*c*) A battery as a *source*.

Fig. 3.27 *Polarity of voltages.*

The *next* step is to go round the selected loop, and add up all the voltages with + or – signs. Any one of the following two rules can be followed:

(*i*) *Rule* 1 While travelling, if you meet a *voltage rise*, write the voltage with *positive* sign; if you meet a *voltage drop*, write the voltage with *negative* sign.

(*ii*) *Rule* 2 While travelling, write the voltage with positive sign if + is encountered first; write the voltage with negative sign if – is encountered first.

The above two rules give opposite signs to the voltages, which is immaterial as you are taking the algebraic sum. Whichever rule you follow, remain consistent throughout. In this book, we shall be following Rule 1. Applying this rule is easy, as it has a strong analogy with the physical height (altitude) of a place. When you travel up a slope, the height (above sea level) increases. When you travel down a slope, the height or altitude decreases.

EXAMPLE 3.14

Find the voltage V_{ab} across the open-circuit in the circuit shown in Fig. 3.28.

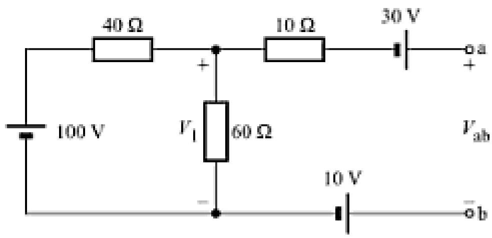


Fig. 3.28

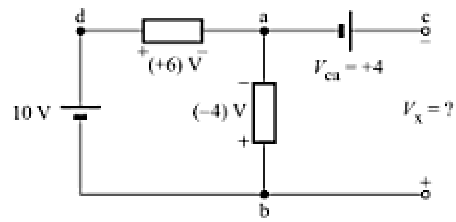


Fig. 3.29

Solution As the terminals a-b are open, no current flows through 10-Ω resistance. Assuming a current I (in the clockwise direction) in the left loop, we write the KVL equation going clockwise around the loop starting from the bottom of 100-V battery,

$$+100 - 40I - 60I = 0 \quad \Rightarrow \quad I = 1\text{ A}$$

We can now calculate the voltage V_1 across the 60-Ω resistance,

$$V_1 = IR = 1 \times 60 = 60\text{ V}$$

Note that we could find this voltage easily by just applying the voltage divider concept. We can determine V_{ab} by writing KVL equation around the rightmost loop. What loop ? There is indeed a loop even though no path exists for current. Thus, KVL must be satisfied even when there is no path for current. Let us start at point b and go counterclockwise,

$$-10 + V_1 + 0 \times 10 + 30 - V_{ab} = 0 \quad \text{or} \quad -10 + 60 + 0 + 30 - V_{ab} = 0 \quad \Rightarrow \quad V_{ab} = \mathbf{80\ V}$$

EXAMPLE 3.15

The circuit of Fig. 3.29 has mixed reference-direction convention. The voltages are marked with both the +/– and the subscript notation. Determine the unknown voltage v_x and v_{cd}.

Solution Let us confirm that the given voltages satisfy KVL. We write a KVL equation going clockwise around the loop starting from the bottom of the battery,

$$+(10) - (+6) + (-4) = 0$$

In this equation, the signs outside the parentheses come from the reference directions marked and the signs inside come from the numerical values of the voltages. We note that the KVL is satisfied.*

Now, we write KVL equation for the rightmost loop (even if it has no path for the current to flow), going from b to a to c to b,

$$-(-4) + (+4) + v_x = 0 \quad \Rightarrow \quad v_x = \mathbf{-8\ V}$$

Determining v_{cd} means to find the voltage of point c with respect to point d. The easiest way to do this is to stand at point d and walk towards point c and note down how high in voltage we have gone. Thus,

$$v_{cd} = -(+4) + (+6) = +\mathbf{2\ V}$$

Solving Simultaneous Equations Using Cramer's Rule

There exists a systematic method of solving simultaneous equations. Let us consider, for example, following equations in three variables, x, y and z, written in matrix form,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

* What would it mean if KVL were not satisfied around this loop ? It would mean that the author or the printer had made an error, because a circuit in which KVL is not satisfied is not a circuit at all— it is **nonsense**, like 1+3 = 5.

Cramer's rule gives the solution of the above equations as

$$x = \frac{\Delta_a}{\Delta}; \qquad y = \frac{\Delta_b}{\Delta}; \qquad \text{and} \qquad z = \frac{\Delta_c}{\Delta}$$

Here, the determinants are given as

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix};$$

$$\Delta_a = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; \quad \Delta_b = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \text{and} \quad \Delta_c = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Evaluating Determinants

There are two ways for evaluating the value of a determinant. The **first method**, which is in common use, is to expand it in terms of its minors. To do this, we select any row j or any column k, multiply each element in that row or column by its minor and by $(-1)^{j+k}$, and then add the products. The minor of the element appearing in row j and column k is the determinant obtained when row j and column k are removed; it is indicated by Δ_{jk}.

The **second method,** which is applicable to a third-order determinant (the one that we mostly come across), is much more convenient. We repeat the first two columns to the right of the third column. We then take the sum of the products of the numbers on the diagonals indicated by downward arrows, and subtract from this the sum of the products of the numbers on the diagonals indicated by upward arrows.

For **example,** to understand these two methods, we evaluate the determinant

$$\begin{vmatrix} 2 & -3 & 4 \\ 6 & 10 & 8 \\ 7 & -5 & 9 \end{vmatrix}$$

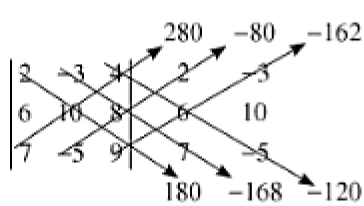
First Method

$$\Delta = (2) \times \begin{vmatrix} 10 & 8 \\ -5 & 9 \end{vmatrix} - (6) \times \begin{vmatrix} -3 & 4 \\ -5 & 9 \end{vmatrix} + (7) \times \begin{vmatrix} -3 & 4 \\ 10 & 8 \end{vmatrix}$$

$$= (2) \times [10 \times 9 - (-5) \times 8] - (6) \times [(-3) \times 9 - (-5) \times 4] + (7) \times [(-3) \times 8 - (10) \times 4]$$

$$= 2 \times (90 + 40) - (6) \times (-27 + 20) + 7 \times (-24 - 40) = 260 + 42 - 448 = \mathbf{-146}.$$

Second Method



$$\Delta = (180 - 168 - 120) - (280 - 80 - 162) = (-108) - (38) = \mathbf{-146}$$

EXAMPLE 3.16

Using KCL and KVL, determine the currents I_x and I_y in the network shown in Fig. 3.30.

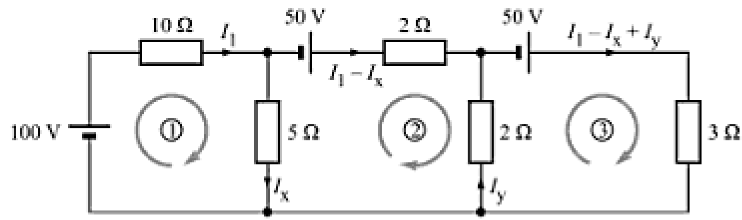


Fig. 3.30

Solution Using KCL, the currents in other branches are marked as shown in Fig. 3.30. Writing KVL equations for the loops 1, 2 and 3, we get

$$100 - 10I_1 - 5I_x = 0 \quad \Rightarrow \quad 5I_x + 0I_y + 10I_1 = 100$$
$$5I_x + 50 - 2(I_1 - I_x) + 2I_y = 0 \quad \Rightarrow \quad 7I_x + 2I_y - 2I_1 = -50$$
$$-2I_y + 50 - 3(I_1 - I_x + I_y) = 0 \quad \Rightarrow \quad 3I_x - 5I_y - 3I_1 = -50$$

These equations can be written in matrix form,

$$\begin{bmatrix} 5 & 0 & 10 \\ 7 & 2 & -2 \\ 3 & -5 & -3 \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_1 \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \\ -50 \end{bmatrix}; \quad \Delta = \begin{vmatrix} 5 & 0 & 10 \\ 7 & 2 & -2 \\ 3 & -5 & -3 \end{vmatrix} = 5(-6-10) - 7(0+50) + 3(0-20) = -490$$

$$\Delta_1 = \begin{vmatrix} 100 & 0 & 10 \\ -50 & 2 & -2 \\ -50 & -5 & -3 \end{vmatrix} = 1900; \quad \Delta_2 = \begin{vmatrix} 5 & 100 & 10 \\ 7 & -50 & -2 \\ 3 & -50 & -3 \end{vmatrix} = -250$$

$$\therefore \qquad I_x = \frac{\Delta_1}{\Delta} = \frac{1900}{-490} = \mathbf{-3.87\ A}; \qquad \text{and} \qquad I_y = \frac{\Delta_2}{\Delta} = \frac{-250}{-490} = \mathbf{0.51\ A}$$

The negative sign on I_x implies that the actual current flows in the direction opposite to that given in the figure.

NOTE

Using calculator (Casio *fx*-991ES), we get the above results just by keying in the coefficients of the three simultaneous equations.

EXAMPLE 3.17

Using Kirchhoff's laws, determine R_1, R_2, I_1, I_2 and I_3 in the circuit of Fig. 3.31.

Solution Applying KCL at nodes B and C, we get

$$I_1 + I_2 = 20 \qquad \text{or} \qquad I_1 + I_2 + 0I_3 = 20 \qquad (i)$$

and

$$I_3 = 30 + I_2 \qquad \text{or} \qquad 0I_1 - I_2 + I_3 = 30 \qquad (ii)$$

Applying KVL to the outer loop ABCDEFGHA, we get

$$-0.1I_1 + 0.3I_2 + 0.2I_3 - 120 + 110 = 0 \quad \text{or} \quad I_1 - 3I_2 - 2I_3 = -100 \qquad (iii)$$

Using the calculator, we get the solution of the above equations as

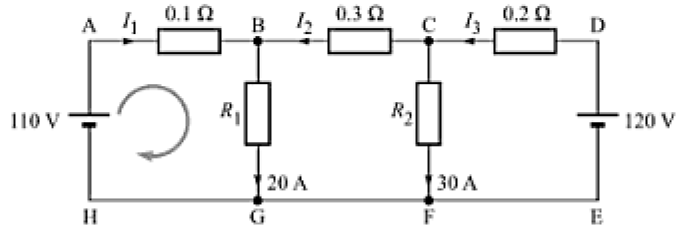


Fig. 3.31

$$I_1 = \mathbf{10\ A};\quad I_2 = \mathbf{10\ A}\quad \text{and}\quad I_3 = \mathbf{40\ A}$$

Now, applying KVL to the loop ABGHA, we get

$$-0.1I_1 - 20R_1 + 110 = 0 \quad \Rightarrow \quad R_1 = \frac{110 - 0.1I_1}{20} = \mathbf{5.45\ \Omega}$$

Applying KVL to the loop CDEFC, we get

$$0.2I_3 - 120 + 30R_2 = 0 \quad \Rightarrow \quad R_2 = \frac{120 - 0.2I_3}{30} = \mathbf{3.74\ \Omega}$$

3.7 LOOP-CURRENT ANALYSIS

It is a general method of analysing a network. It can be applied to any network, howsoever complicated it may be. The variables are currents and the equations are based on KVL. We shall present the method by analysing the circuit of Fig. 3.32*a*, to find the voltage across the 2-Ω resistance. We give below step-by-step procedure.

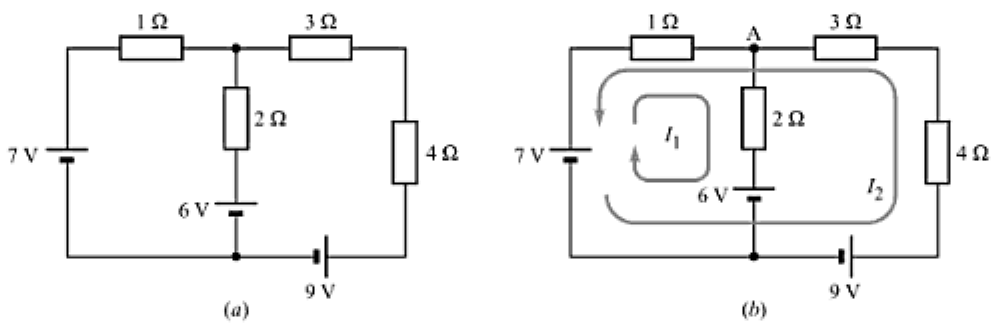


Fig. 3.32

***Step* 1** Recognize the *independent loops** and label the loop currents. A *loop* is a closed path and a *loop current* is the current that flows in the closed path following the loop. In the given circuit, we recognize two independent loops and mark two loop currents I_1 and I_2, as shown in Fig. 3.32*b*.

***Step* 2** Write KVL equations for each loop in the special form:

$$\Sigma \text{ Voltage drops across resistors in loop} = \Sigma(+ \text{ or } -) \text{ voltage sources in loop}$$

* An independent loop does not pass through a current source. Furthermore, it is more convenient to apply loop-current method to a network that contains no current sources. Hence, in case a network contains current source, convert all the current sources to their equivalent voltage sources before attempting to apply loop-current method.

Use + if the voltage source aids the loop current for that loop and – if the voltage source opposes the loop current for that loop.

Step 3 Solve for the loop currents and compute currents or voltages required.

Note that when we write one KVL equation for each loop and we have one loop current for each loop, we always get the same number of equations and unknowns. Following the above procedure, the two equations are

$$(I_1 - I_2)\,(1) + I_1(2) = +7 - 6 \qquad \text{and} \qquad I_2(4) + I_2(3) + (I_2 - I_1)\,(1) = +9 - 7$$

The first term in the first equation is the voltage drop across 1-Ω resistance. Net current flowing through this resistance in the direction of the loop is $I_1 - I_2$. The second term is the voltage drop across 2-Ω resistance. The current flowing through this resistance in the direction of the loop is I_1. The first term on the right side has a + sign, because the 7-V source tends to force the loop current in the same direction as the loop current. The second term has a – sign, because the 6-V source tends to oppose I_1. The second equation has been written the same way.

These two equations can be solved to get $I_1 = 0.435$ A and $I_2 = 0.304$ A. We can now compute the voltage across the 2-Ω resistance, as $I_1 \times 2 = 0.435 \times 2 = 0.870$ V, with + polarity at the top.

Applications The loop-current method is useful when we wish to determine only one current or voltage. In such a case, we can define all the unknown loop currents to avoid that path except one loop current. The resulting equation can then be solved for that one loop current.

***Have we Ignored* KCL** In loop-current method, we solve for the currents by writing only KVL equations. Is the KCL ignored? The answer is 'no'. While defining the loop currents, the KCL is automatically satisfied. The KCL applies to *branch currents,* which flow from one point to another. The loop currents flow in complete loops. If a loop current flows *into* a given node, it flows *out* of it also. For example, if we wrote

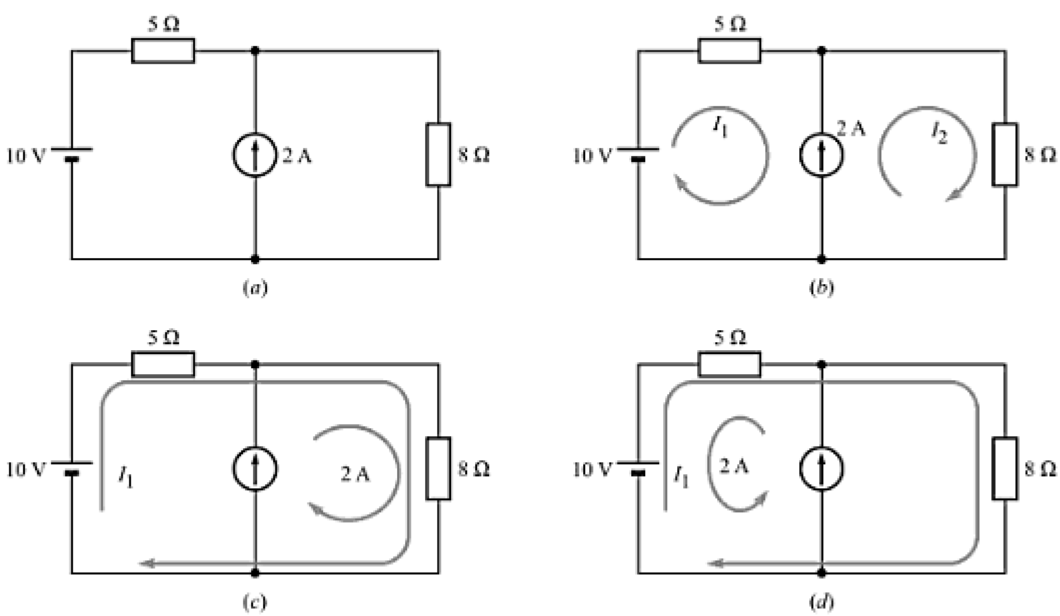


Fig. 3.33

KCL for node A in the circuit of Fig. 3.32b, each of the two loop currents, I_1 and I_2, contribute two equal and opposite terms, adding up to zero.

Counting Independent Loops The circuit of Fig. 3.33a appears to have two loops. But, these two loops are not independent. Suppose that we had marked the two loop currents I_1 and I_2 in the standard way, as shown in Fig. 3.33b. Then, by definition of a current source we must have

$$I_2 - I_1 = 2\text{ A}$$

Thus, we find that these two currents are not independent. If one is known, the other is no longer unknown. The values of these two currents are *constrained* by the above relation.

We identify independent loops by turning OFF all sources. By 'turning OFF a voltage source' means to 'short circuit it', and by 'turning OFF a current source' means to 'open circuit it'. When we turn OFF both the sources in Fig. 3.33a, we are left with one loop containing two resistances. Thus, we have only one independent loop, requiring one unknown and one KVL equation.

Suppose that we are interested to determine the current through 5-Ω resistance in Fig. 3.33a. We select the unknown loop current I_1 passing through 5-Ω resistance (but not through the current source) and a known loop current of 2 A defined to flow in the right loop, as shown in Fig. 3.33c. We now have only one unknown and so we need only one equation. This, we get by writing KVL around the loop of I_1,

$$10 - 5I_1 - 8(I_1 + 2) = 0 \quad \Rightarrow \quad I_1 = \mathbf{-0.462\ A}$$

EXAMPLE 3.18

Using loop-current analysis, determine the current through 8-Ω resistance in the circuit of Fig. 3.33a.

Solution Our aim is to determine current through 8-Ω resistance. Therefore, we should select the unknown loop current I_1 passing through 8-Ω resistance (but not through the current source) and a known loop current of 2 A to flow in the left loop, as shown in Fig. 3.33d.

Now, writing KVL equation around the loop of I_1, we get

$$10 - 5(I_1 - 2) - 8I_1 = 0 \quad \Rightarrow \quad I_1 = \mathbf{1.538\ A}$$

Thus, the current through 8-Ω resistance is **1.538 A.**

EXAMPLE 3.19

Consider the ***benchmark example*** of Fig. 3.24a (redrawn in Fig. 3.34a), and solve it by using loop-current analysis.

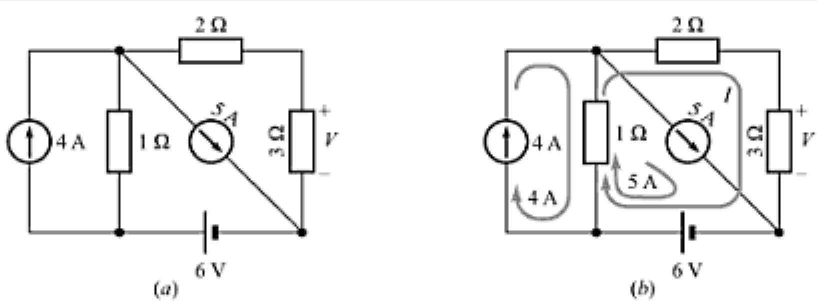


Fig. 3.34

Solution We note that the given circuit has one independent loop and two constrained loops. Our aim is to determine the voltage across 3-Ω resistance. So, we should select the unknown loop current I passing through 3-Ω resistance (but not through any current source). The two known loop currents of 4 A and 5 A are marked to flow in the two loops as shown in Fig. 3.34*b*. Writing KVL equation around the loop of I, we get

$$-2I - 3I + 6 - 1 \times (I + 5 - 4) = 0 \quad \Rightarrow \quad I = \frac{5}{6}\text{ A}$$

Therefore, the unknown voltage $v = 3I =$ **2.5 V**.

3.8 MESH ANALYSIS

In circuit terminology, a ***loop*** is any closed path. A ***mesh*** is a special loop, namely, the smallest loop one can have. In other words, a mesh is a loop that contains no other loops. In the fuller sense of loop-current analysis, we can define loop current with great freedom. A loop current can go wherever we wish. The only limitations are that we must define the correct number of loop currents, and that we must go through each resistance with at least one loop current.

In *mesh analysis*, we define a mesh current for each mesh so that the above guidelines are automatically met. However, the application of mesh analysis is restricted only to ***planar*** networks. A network is said to be planar, if it can be drawn on a sheet of paper without crossing lines. Although at first glance, the network of Fig. 3.35*a* appears to be *nonplanar*. But a closer look at it shows that it can be redrawn as in Fig. 3.35*b*, and hence it is a planar network. However, the network of Fig. 3.35*c* is definitely nonplanar.

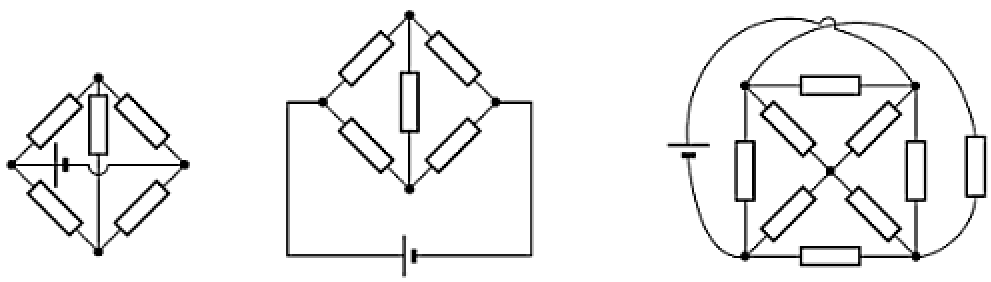


Fig. 3.35

Once a network has been clearly drawn in planar form (as in Fig. 3.35*b*), it often has the appearance of a multi-paned window. Each pane in the window may be considered to be a mesh. The mesh analysis not only tells us the minimum number of unknown currents to be taken, but it also ensures that the equations obtained by writing KVL equations for each mesh are independent.

To understand the beauty of mesh analysis, let us consider the circuit of Fig. 3.36*a*, redrawn in Fig. 3.36*b*. It has two meshes and the two mesh currents* I_1 and I_2 have been marked (both in clockwise direction), as shown in Fig. 3.36*b*. Let us write the KVL equations for these two meshes,

$$7 - 1 \times I_1 - 2 \times (I_1 - I_2) - 6 = 0 \qquad \text{and} \qquad 6 - 2 \times (I_2 - I_1) - 3 \times I_2 - 4 \times I_2 - 9 = 0$$

* We define **mesh current** as a current that flows only around the perimeter of a mesh.

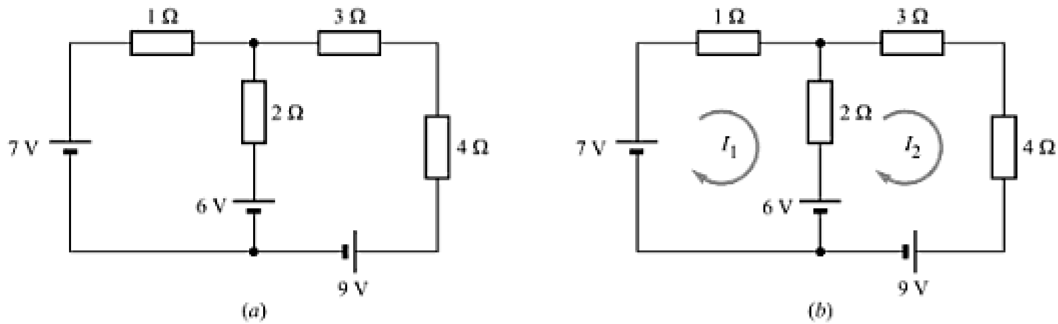


Fig. 3.36

or $$(1 + 2)I_1 - 2I_2 = 1 \tag{3.25}$$

and $$-2I_1 + (2 + 3 + 4)I_2 = -3 \tag{3.26}$$

Let us write the above two equations in ***matrix form***,

$$\begin{bmatrix} (1+2) & -2 \\ -2 & (2+3+4) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 7-6 \\ 6-9 \end{bmatrix} \tag{3.27}$$

or $$\begin{bmatrix} 3 & -2 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

or $$\mathbf{RI} = \mathbf{E} \tag{3.28}$$

where $\mathbf{R} = \begin{bmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 9 \end{bmatrix}$ is a square matrix, and is called ***resistance matrix***.

$\mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ is a column matrix, and is called ***mesh current matrix***.

$\mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ is a column matrix, and is called ***source matrix***.

Resistance Matrix It is a square matrix. The elements on its principal diagonal (such as R_{11}, R_{22}, … etc.) are called ***self-resistances*** and have positive values. Thus, R_{11} is the self-resistance of the mesh 1, and it is equal to the sum of all the resistances in the mesh 1. The elements off the principal diagonal are called ***mutual resistances***, and have zero or negative values. Thus, R_{12} is the mutual resistance between mesh 1 and mesh 2. It represents the total resistance common between these two meshes. If there is no resistance common to two meshes, their mutual resistance will be zero. The matrix is symmetrical about the principal diagonal (i.e., $R_{12} = R_{21}$, … etc).

Source Matrix It is a column matrix. Its each element is the algebraic sum of the voltage sources that forces the current in the same direction as the mesh current.

Procedure for Mesh Analysis

The beauty of mesh analysis lies in the fact that KVL equations in the matrix form (i.e., in the form of Eq. 3.27) *can be written directly by mere* ***inspection*** *of the network*. Yes, it is possible, even for a multi-mesh network. The symmetry and sign pattern of the matrices minimizes the chances of committing errors while writing the KVL equations. This method is, however, limited to *those planar networks which contain only independent voltage sources.* Most of the networks that we deal with are of this category. The procedure is outlined below.

1. Make sure that the given network is planar.
2. Make sure that the network contains only independent voltage sources. If there is a practical current source, convert it into an equivalent practical voltage source.
3. Assuming that the network has m meshes, assign a mesh current in each mesh, $I_1, I_2, I_3, \ldots I_m$, ***all in clockwise*** (or in counterclockwise) direction.
4. Write matrix equations directly by inspection of the network, keeping in mind the nature of the elements of resistance matrix and source matrix.
5. Solve the equations to determine the unknown mesh currents, either using Cramer's rule, or using a scientific calculator.

Let us apply the above procedure to the network of Fig. 3.36*a*, and check whether we obtain the same equations as Eq. 3.27. The resistances of 1 Ω and 2 Ω are included in mesh 1. Hence, $R_{11} = 1 + 2 = 3$. The resistance common to mesh 1 and mesh 2 is 2 Ω. Therefore, $R_{12} = R_{21} = 2$. These are entered with negative sign in the resistance matrix. The self resistance of the mesh 2 is $R_{22} = 2 + 3 + 4 = 9$. The first element of the source matrix represents the net source voltage that forces the current I_1 around mesh 1. Since 7-V source aids current I_1 and 6-V source opposes current I_1, we have $E_1 = +7 - 6 = 1$. In mesh 2, source of 6 V aids current I_2 and source of 9 V opposes it. Hence, we have $E_2 = +6 - 9 = -3$. Thus, we find that the mesh equations can be written directly by inspection of the network.

How to Handle Current Sources

We cannot incorporate current sources into our normal mesh analysis. If a circuit has current sources, a modest extension of the standard procedure is needed. There are three possible methods.

First Method One simple method is, if possible, to first transform the current sources into voltage sources. This reduces the number of meshes by 1 for each current source. Apply the standard procedure of mesh analysis to determine the assumed mesh currents. Go back to the original circuit, and get additional equations, one for each current source, by expressing the source currents in terms of assumed mesh currents. Solve these equations to get all the mesh currents. Let us apply this method to find mesh current in the circuit of Fig. 3.37*a*.

The three mesh currents are marked. We convert the 13-A current source in parallel with 5-Ω resistor into an equivalent 65-V voltage source in series with 5-Ω resistor, as shown in Fig. 3.37*b*. This reduces the number of meshes to two. We can write the mesh equations in the matrix form just by inspection,

$$\begin{bmatrix} 9 & -5 \\ -5 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 52 \end{bmatrix}$$

Using Cramer's rule, we get

$$I_1 = \mathbf{5\ A} \qquad \text{and} \qquad I_2 = \mathbf{7\ A}$$

We now go back to the original circuit of Fig. 3.37*a*. Obviously, the current through the current source is

$$I_2 - I_3 = 13\text{ A} \quad \Rightarrow \quad I_3 = I_2 - 13 = 7 - 13 = \mathbf{-6\ A}$$

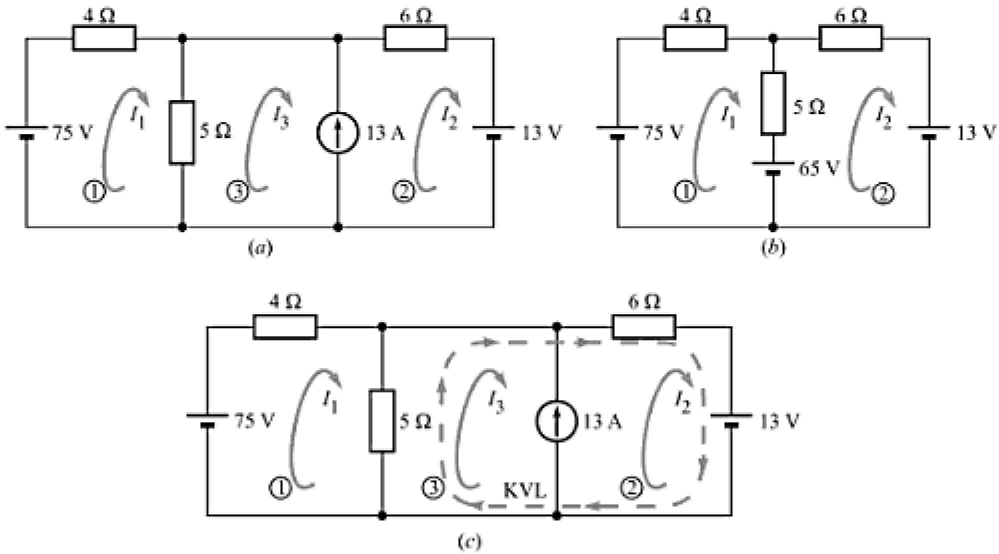


Fig. 3.37

Second Method We can assign unknown voltages to each current source, apply KVL around each mesh and then relate the source currents to the assumed mesh currents. This is generally a difficult approach.

Third Method A better approach is to use the so-called ***supermesh method***. We create a *supermesh* from two meshes that have a current source as a common element; the current source is in the interior of the supermesh. Thus, the number of meshes is reduced by 1 for each current source present. If the current source lies on the perimeter of the circuit, then the single mesh in which it is found is ignored. The KVL is then applied to the meshes and supermeshes, using the assumed mesh currents. Let us apply this method to the circuit of Fig. 3.37*a*.

In this circuit, we find that the 13-A current source is in the common boundary of meshes 3 and 2. Ignoring this current source leads to a supermesh comprising the 5-Ω and 6-Ω resistors and the 13-V source (as shown by the dotted path in Fig. 3.37*c*). Going along the dotted arrow, the KVL equation for this supermesh is

$$-5(I_3 - I_1) - 6I_2 - 13 = 0 \quad \text{or} \quad 5I_1 - 6I_2 - 5I_3 = 13 \tag{i}$$

The KVL equation for mesh 1 is

$$9I_1 + 0I_2 - 5I_3 = 75 \tag{ii}$$

We have only two equations for three unknowns. The third equation is obtained by applying KCL to either node of the current source, or simply, by noting that the current up through the current source in terms of the mesh currents is $I_2 - I_3$. This current must, of course, be equal to the 13 A of the source. Thus, we have

$$0I_1 + I_2 - I_3 = 13 \tag{iii}$$

In the matrix form, these three equations are

$$\begin{bmatrix} 5 & -6 & -5 \\ 9 & 0 & -5 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 75 \\ 13 \end{bmatrix}$$

The solutions of these equations are the same as before: I_1 **= 5 A**, I_2 **= 7 A** and I_3 **= –6 A.**

EXAMPLE 3.20

Apply mesh analysis to determine current drawn from the source in the network of Fig. 3.38.

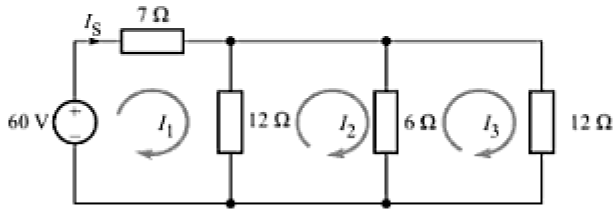


Fig. 3.38

Solution There are three independent meshes, for which the three mesh currents I_1, I_2, and I_3 are all marked with clockwise directions. The current I_S drawn from the source is same as I_1. To determine this current, we write the mesh equations in matrix form by inspection of the circuit,

$$\begin{bmatrix} 7+12 & -12 & 0 \\ -12 & 12+6 & -6 \\ 0 & -6 & 6+12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

Using calculator, we get $I_S = I_1 =$ **6 A**

EXAMPLE 3.21

Apply mesh analysis to determine current through 7-Ω resistance in the network of Fig. 3.39.

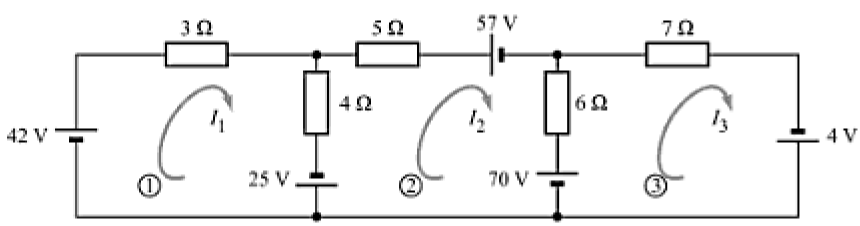


Fig. 3.39

Solution The given network is a planar network having independent voltage sources. It has three meshes for which the mesh currents I_1, I_2, and I_3 are all marked with clockwise directions. By inspection, the matrix equation is written as

$$\begin{bmatrix} 3+4 & -4 & 0 \\ -4 & 4+5+6 & -6 \\ 0 & -6 & 6+7 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 42+25 \\ -25-57-70 \\ 70+4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 7 & -4 & 0 \\ -4 & 15 & -6 \\ 0 & -6 & 13 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 67 \\ -152 \\ 74 \end{bmatrix}$$

Using calculator, we get the current through **7-Ω** resistance as $I_{7\,\Omega} = I_3 =$ **2 A**

NOTE

The *mesh analysis* is quite similar to *loop analysis*. The principle difference is that the current paths selected in loop analysis are not necessarily meshes. Also, there is no convention regarding the direction of loop currents; they can be a mixture of clockwise or counterclockwise directions.

For loop analysis, no current source needs to be transformed to a voltage source. But each current source should have only one loop current flowing through it so that the loop current is known. Also, then KVL is *not* applied to this loop because the current source voltage is not known.

If the current through only one component is desired, the loops should be selected such that only one loop current flows through this component. Then, we are to solve for only one current. In contrast, for mesh analysis, finding current through an interior component requires solving for two mesh currents, requiring more labour.

3.9 NODE-VOLTAGE ANALYSIS

This method is *dual* of the loop-current analysis. It is based on KCL and can be applied to any network. We write KCL equations at all nodes except one, in such a way that current variables are never formally defined. This is done by expressing the currents in terms of "node voltages". The procedure for node-voltage analysis consists of following simple steps.

1. Define a ***reference node*** (also called ***ground node*** or ***datum node***). Often, the node with the most wires is chosen as the reference node. Mark the reference node with *r*.
2. Count and label the independent nodes. Voltages on these nodes are referenced positive with respect to the ground node.
3. Label the independent nodes.
4. Write KCL in a special form,

   Σ Currents leaving the node in resistors = Σ Currents entering the node from current sources.

The procedure is explained with the help of following example.

EXAMPLE 3.22

Using node-voltage analysis, find the voltage across the 3-Ω resistor in the circuit of Fig. 3.40*a*.

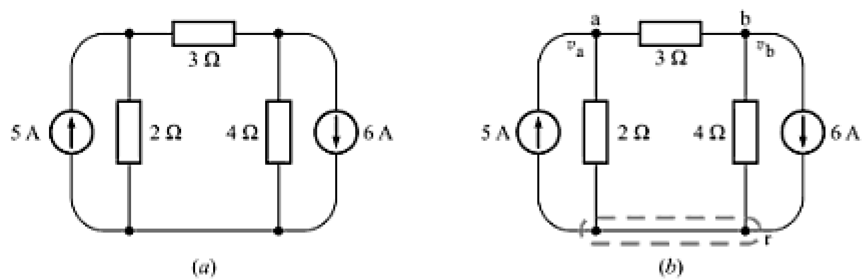


Fig. 3.40

Solution The circuit has three nodes. In this case, we chose the node at the bottom as reference node. We mark it as *r*, as shown in Fig. 3.40*b*. The remaining two nodes, labeled as a and b are the independent nodes. Let the voltage of these nodes be v_a and v_b, respectively. We write KCL equations in terms of these node voltages. For node a, the KCL equation is

$$\frac{v_a - (0)}{2} + \frac{v_a - v_b}{3} = +(5) \qquad (i)$$

The left side of the above equation represents the current leaving node a in the two resistors connected directly to node a. The right side represents the current entering from the 5-A source. The current flowing through 2-Ω resistor is merely the voltage v_a divided by the resistance between node a and reference node. We have written zero (0) for the voltage of the reference node as a reminder. The current from a to b through 3-Ω resistor is simply the voltage at a, minus the voltage at b, divided by the resistance between a and b. Similarly, we can write KCL for node b,

$$\frac{v_b - v_a}{3} + \frac{v_b - (0)}{4} = -(+6) \qquad (ii)$$

Note that the first term on the left side of the above equation is just the negative of the second term of Eq. (*i*). This is so because we are now expressing the current referenced in the opposite direction. Note also that the current source term on the right side has a negative sign, because this current is *leaving* the node. The above two equations can be solved, by using calculator, to get

$$v_a = 2.44 \text{ V} \qquad \text{and} \qquad v_b = -8.89 \text{ V} \qquad (iii)$$

We can now calculate the primary unknown, the voltage v_{ab} across the 3-Ω resistor,

$$v_{ab} = v_a - v_b = 2.44 - (-8.89) = \mathbf{11.3\ V}$$

How to Handle Voltage Sources

Ordinarily, we cannot incorporate voltage sources into our normal node-voltage method. If the circuit has voltage sources, there are techniques to handle them. If one terminal of a voltage source with a series resistance is grounded (as in the circuit of Fig. 3.41), the KCL equation can be written in terms of this voltage.

Difficulty arises, if a circuit contains floating* voltage sources. One simple way out is to first transform the voltage sources into current sources (see Example 3.42). There is another way which uses the concept of *constrained node* or *supernode*. This method is especially suitable for the circuits having a floating voltage source with no series resistance.

Supernode The two ends of a voltage source cannot make two independent nodes, since one node-voltage can be determined in terms of the other node-voltage and the source voltage. Hence, we treat these end nodes and the voltage source together as a '*supernode*'. The supernode is usually indicated by the region enclosed by a dotted line. The KCL is then applied to both nodes at the same time. This is certainly possible. If the total current leaving one node is zero and the total current leaving the other node is zero, then the total current leaving the combination of two nodes is also zero. The procedure of using the concept of supernode is made clear in Example 3.23.

Counting Independent Nodes An *independent node* is a node whose voltage cannot be derived from the voltage of another node. Independent nodes in a network can be counted by turning OFF all sources and first counting all the nodes separated by resistors. The number of independent nodes is one less than this number.

EXAMPLE 3.23

Apply node-voltage analysis to the circuit of Fig. 3.41 to determine the current in 12-Ω resistor.

Solution The given network has two nodes. Node 2 is marked as reference node. Hence, there is only one independent node (node 1). We assign voltage V_1 to this node. Writing KCL equation for node 1, we get

$$I_1 + I_2 + I_3 = 0 \quad \text{or} \quad \frac{V_1 - (0)}{12} + \frac{V_1 - 60}{7} + \frac{V_1 - (0)}{4} = 0 \quad \Rightarrow \quad V_1 = 18 \text{ V}$$

* A voltage source is floating if its neither terminal is connected to ground.

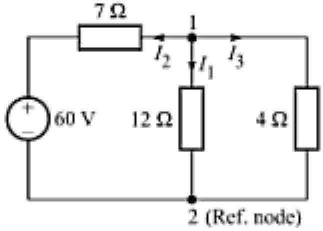


Fig. 3.41

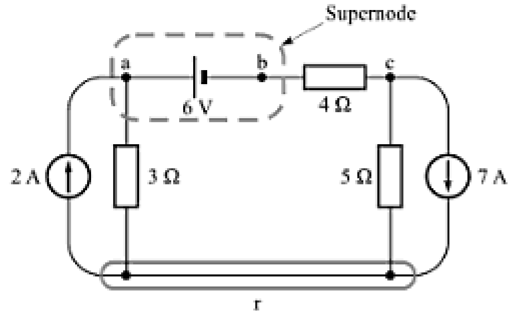


Fig. 3.42

Once the voltage at node 1 is known, we can easily determine the current in any branch. The current I_1 in 12-Ω resistor is

$$I_1 = \frac{V_1 - 0}{12} = \frac{18}{12} = \mathbf{1.5\ A}$$

EXAMPLE 3.24

Determine the current through 4-Ω resistor in the circuit of Fig. 3.42.

Solution Here, we have one node constrained to another independent node rather than to the reference node. The voltages at node a and b are unknown, but they are not independent. If we knew either of them, we could determine the other. Obviously, the two node voltages are related as

$$v_a - v_b = 6 \quad \text{or} \quad v_a - v_b + 0v_c = 6 \qquad (i)$$

We can therefore treat the two constrained nodes a and b, as a ***supernode***. Now, writing KCL for this supernode, we get

$$\frac{v_a}{3} + \frac{v_b - v_c}{4} = 2 \quad \text{or} \quad 0.33v_a + 0.25v_b - 0.25v_c = 2 \qquad (ii)$$

Applying KCL to node c gives

$$\frac{v_c}{5} + \frac{v_c - v_b}{4} = -7 \quad \text{or} \quad 0v_a - 0.25v_b + 0.45v_c = -7 \qquad (iii)$$

Above equations can be written in the matrix form as

$$\begin{bmatrix} 1 & -1 & 0 \\ 0.33 & 0.25 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ -7 \end{bmatrix}$$

We solve the above equations using calculator to get

$$v_b = -8.77\ \text{V} \qquad \text{and} \qquad v_c = -20.43\ \text{V}$$

Finally, the current through 4-Ω resistor is

$$\frac{v_b - v_c}{4} = \frac{-8.77 - (-20.42)}{4} = \mathbf{2.9125\ A}$$

EXAMPLE 3.25

Consider the ***benchmark example*** of Fig. 3.24*a* (redrawn in Fig. 3.43*a*), and solve it by using node-voltage analysis.

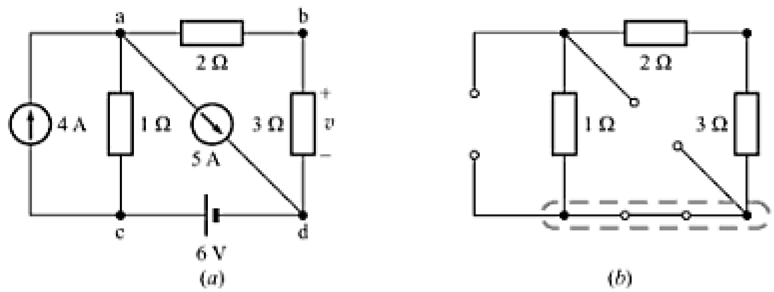


Fig. 3.43

Solution Nodes c and d are constrained to one another. To find the number of independent nodes, we turn OFF the sources to get the circuit of Fig. 3.43*b*. There are three nodes, two of which are independent. However, if we add the two series resistors to make a 5-Ω resistor we will have only one independent node (node a), and hence we will have to solve only one equation. The unknown voltage across 3-Ω resistor can then be determined by applying voltage divider rule. Taking node d as reference, we write KCL equation for node a as

$$\frac{v_a-(6)}{1}+\frac{v_a-(0)}{5}=+4-5 \quad\Rightarrow\quad v_a=\frac{6-1}{1.2}=4.17\text{ V}$$

Using the voltage divider, the voltage across 3-Ω resistor is

$$v=4.17\times\frac{3}{2+3}=\mathbf{2.5\ V}$$

3.10 NODAL ANALYSIS

This method is *dual* of mesh analysis. It can be applied to a network that contains only independent current sources. Therefore, if a network contains practical voltage sources, we should first convert them into their equivalent practical current sources. We can then proceed with nodal analysis, and write the KCL equations in the following matrix form, just by inspection of the network.

$$\begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} I_{S1} \\ I_{S2} \end{bmatrix} \quad \text{or} \quad \mathbf{GV}=\mathbf{I}_S \tag{3.29}$$

Here, the square matrix **G** is called ***conductance matrix***, the column matrix **V** is called ***node-voltage matrix***, and the column matrix $\mathbf{I}_S$ is called ***node-current source matrix***.

G_{11} = ***self-conductance*** *of node* 1,
and is the sum of all conductances connected to node 1.

$G_{12}=G_{21}$ is the ***mutual-conductance*** between *node* 1 and *node* 2,
and is sum of all conductances connected between *node* 1 and *node* 2.

I_{S1} = algebraic sum of source currents entering node 1.

I_{S2} = algebraic sum of source currents entering node 2.

Note that all elements on the principal diagonal of matrix **G** are positive and all elements off-diagonal are negative (or zero), just as in the resistance matrix **R** occurring in mesh-analysis. The method of nodal analysis is made clear in the following Example.

EXAMPLE 3.26

Determine the current through 5-Ω resistor in the circuit of Fig. 3.44*a*.

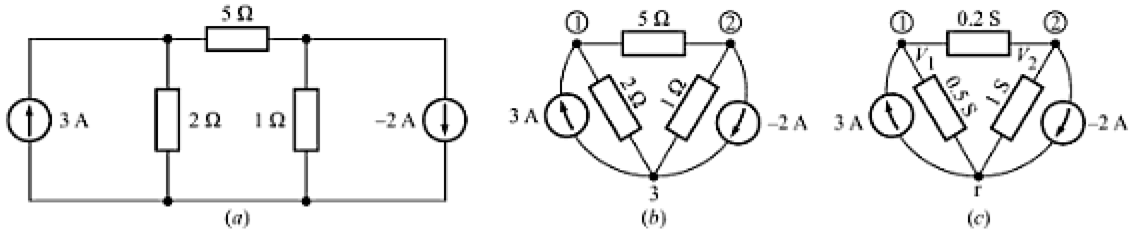


Fig. 3.44

Solution The given network is redrawn in Fig. 3.44*b* to emphasize the three nodes. Node 3 is taken as reference node r. The remaining two nodes are the independent nodes, for which we are to write nodal-analysis equations by inspection. Since we are going to use conductances, it is better to first convert all resistances into conductances (in siemens), as in Fig. 3.44*c*. The conductances connected to node 1 are 0.5 S and 0.2 S. Hence, $G_{11} = 0.5 + 0.2 = 0.7$ S. Similarly, $G_{22} = 1 + 0.2 = 1.2$ S. Only one conductance (= 0.2 S) is connected between node 1 and node 2. Hence, $G_{12} = G_{21} = 0.2$ S. The current source of 3 A is entering node 1. Hence, $I_{S1} = 3$ A. Current *leaving* node 2 is –2 A. Hence, $I_{S2} = -(-2) = 2$ A. Thus,

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Using calculator, we get the solution of the above matrix equation as

$$V_1 = 5 \text{ V} \qquad \text{and} \qquad V_2 = 2.5 \text{ V}$$

Now, the current through 5-Ω resistor is given as

$$I = \frac{V_1 - V_2}{5} = \frac{5 - 2.5}{5} = \mathbf{0.5\ A}$$

3.11 CHOICE OF METHOD OF ANALYSIS

The primary consideration in choosing a suitable method of analysis is to answer the question, "How many equations must be solved?" It is possible for a circuit to have more independent nodes than loops, or vice versa. In general, if a network has b branches and n nodes, it will have n – 1 independent nodes and b – (n – 1) independent loops. Obviously, we should choose the method that requires fewer equations to be solved. However, if the circuit is *nonplanar*, then there is no choice; only the nodal analysis can be applied.

There are some fine points that help us to choose right method. If there are many current sources, nodal analysis is favoured. On the other hand, if many voltage sources are there, loop-current analysis might be easier. If the unknown is a voltage, nodal analysis might be the best, but if the unknown is a current, using loop analysis might be more efficient.

If there is only one source and the circuit is not too complicated, the method of voltage and current dividers is favoured.

EXAMPLE 3.27

Find the current I in the circuit shown in Fig. 3.45.

Solution This circuit has two independent loops but only one independent node (if we combine the series resistors). Thus, we favour the nodal analysis. Choosing the node at the bottom as reference node, we designate the voltage at the top node as V. The single nodal equation is

$$\frac{V-10}{2}+\frac{V-0}{1+3}+\frac{V-8}{6}=0 \quad \Rightarrow \quad V=6.91\text{ V}$$

$$\therefore \qquad I=\frac{6.91}{1+3}=\mathbf{1.73\ A}$$

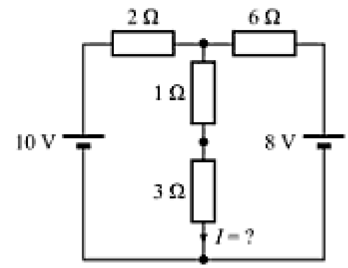


Fig. 3.45

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 3.28

A series combination of two capacitances C_1 and C_2 is connected across a 200-V dc supply, and it is found that the potential difference across C_1 is 120 V. This pd increases to 140 V, when a 3-μF capacitor is connected in parallel with C_2. Determine the capacitances C_1 and C_2.

Solution

Case I (Fig. 3.46*a*): The charge on each capacitor is *same*. That is,

$$C_1V_1 = C_2V_2 \quad \text{or} \quad 120C_1 = 80C_2 \quad \text{or} \quad C_1 = 2C_2/3 \qquad (i)$$

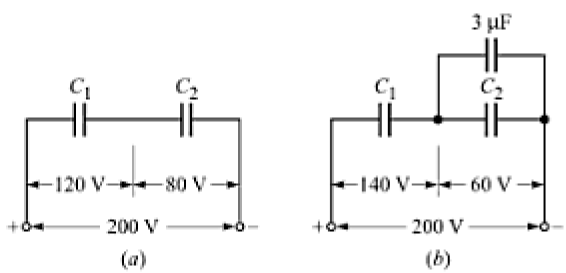


Fig. 3.46

Case II (Fig. 3.46*b*): The capacitance of 3 μF in parallel with C_2 gives an equivalent capacitance of $(C_2 + 3)$ μF. This is in series with C_1. Again, the charge on these two must be the same,

$$140C_1 = 60(C_2 + 3) \qquad (ii)$$

Solving (*i*) and (*ii*) gives

$$C_1 = \mathbf{3.6\ \mu F} \qquad \text{and} \qquad C_2 = \mathbf{5.4\ \mu F}$$

EXAMPLE 3.29

Calculate the current in each branch of the network of Fig. 3.47.

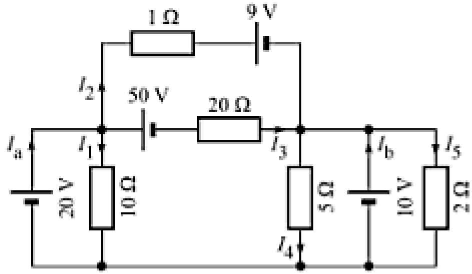


Fig. 3.47

Solution

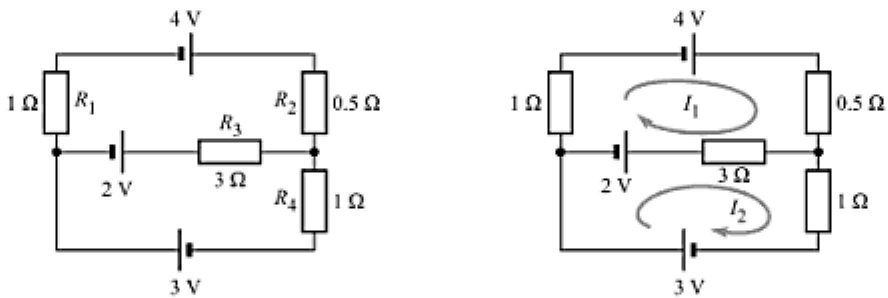
$$I_1 = \frac{20}{10} = \mathbf{2\ A}; \quad I_4 = \frac{10}{5} = \mathbf{2\ A}; \quad I_5 = \frac{10}{2} = \mathbf{5\ A};$$

$$I_2 = \frac{20-(10+9)}{1} = \mathbf{1\ A}; \quad I_3 = \frac{20-50-10}{20} = \mathbf{-2\ A};$$

$$I_a = I_1 + I_2 + I_3 = 2 + 1 + (-2) = \mathbf{1\ A}; \quad I_2 + I_3 + I_b = I_4 + I_5 \ \Rightarrow \ I_b = 2 + 5 - 1 - (-2) = \mathbf{8\ A}$$

EXAMPLE 3.30

Determine the currents in various resistances of the network of Fig. 3.48*a*.



(*a*) (*b*)

Fig. 3.48

Solution There are two independent meshes. For writing mesh equations, we assign the two mesh currents I_1 and I_2 in the same (clockwise) direction, as shown in Fig. 3.48*b*. Writing mesh equations by inspection,

$$\begin{bmatrix} 1+0.5+3 & -3 \\ -3 & 3+1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4-2 \\ 2+3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 4.5 & -3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Solving the above, we get $I_1 = 2.55$ A, and $I_2 = 3.167$ A. Therefore,

Current through R_1 = Current through $R_2 = I_1 = \mathbf{2.55\ A}$

Current through $R_3 = I_1 - I_2 = 2.55 - 3.167 = \mathbf{-0.617\ A}$

Current through $R_4 = I_2 = \mathbf{3.167\ A}$

EXAMPLE 3.31

In the circuit shown in Fig. 3.49, the cells E_1 and E_2 have emf of 4 V and 8 V and internal resistance of 0.5 Ω and 1 Ω, respectively. Calculate the current in each resistor and potential difference across each cell.

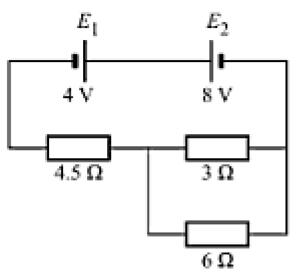


Fig. 3.49

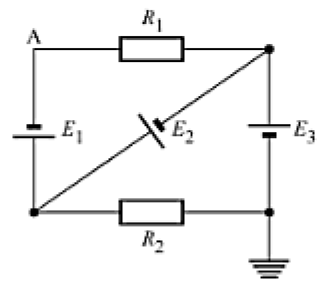


Fig. 3.50

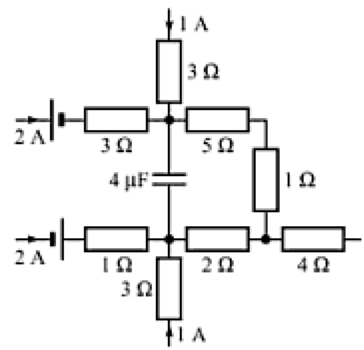


Fig. 3.51

Solution The 3-Ω resistance and 6-Ω resistance can be combined to give a 2-Ω resistance. Therefore, in the single loop obtained, the total resistance is $R_t = 2 + 4.5 + 1 + 0.5 = 8$ Ω, and the net emf driving the current anticlockwise is $E = E_2 - E_1 = 8 - 4 = 4$ V. Thus, the loop current,

$$I = \frac{E}{R_t} = \frac{4}{8} = \mathbf{0.5\ A}$$

This is the current through 4.5-Ω resistance. Using current divider, the current in 3-Ω resistance and 6-Ω resistance is, respectively,

$$I_{3\,\Omega} = 0.5 \times \frac{6}{6+3} = \mathbf{0.333\ A} \qquad \text{and} \qquad I_{6\,\Omega} = 0.5 \times \frac{3}{6+3} = \mathbf{0.166\ A}$$

The pd across cells E_1 and E_2 are

$$V_1 = E_1 + Ir_1 = 4 + 0.5 \times 0.5 = \mathbf{4.25\ V} \qquad \text{and} \qquad V_2 = E_2 - Ir_2 = 8 - 0.5 \times 1 = \mathbf{7.5\ V}$$

Note that the pd of cell E_1 is more than its emf. This is so because this cell is working as a load, and is getting charged. The cell E_2 is working as a source and is getting discharged.

EXAMPLE 3.32

For the circuit shown in Fig. 3.50, $R_1 = 5$ Ω, $R_2 = 9$ Ω, $E_1 = 8$ V, $E_2 = 6$ V and $E_3 = 4$ V. Find the potential of the point A.

Solution To determine the potential of the point A (with respect to the ground), we stand at ground, traverse a path to point A and see how high in potential we reach. The best alternative would be to go across E_3, then across E_2, and then finally across E_1, because all these voltages are known. Hence,

$$V_A = E_3 + E_2 - E_1 = 4 + 6 - 8 = \mathbf{2\ V}$$

EXAMPLE 3.33

A part of a circuit in steady state is shown in Fig. 3.51. Calculate the energy stored in the capacitor.

Solution In steady state, no current flows through a capacitor. Applying KCL to the node at the top of the capacitor, we get the current through the 5-Ω resistance as 1 + 2 = 3 A. The same current flows through the 1-Ω resistance. The current through the 2-Ω resistance can be determined by applying KCL to the node at the bottom of the capacitor, as 2 – 1 = 1 A. We can now calculate the pd across the capacitor as

$$V = 3 \times 5 + 3 \times 1 + 1 \times 2 = 20 \text{ V}$$

Therefore, the energy stored in the capacitor is

$$W = \frac{1}{2}CV^2 = \frac{1}{2}(4\ \mu\text{F})\ (20\ \text{V})^2 = \mathbf{800\ \mu J}$$

EXAMPLE 3.34

In the circuit shown in Fig. 3.52*a*, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V, $R = r_1 = r_2 = r_3 = 1\ \Omega$. (*a*) Find the potential difference between points A and B, and the current through each branch. (*b*) If r_2 is short-circuited and the point A is connected to the point B, find the current through E_1, E_2 and E_3 and the resistance R.

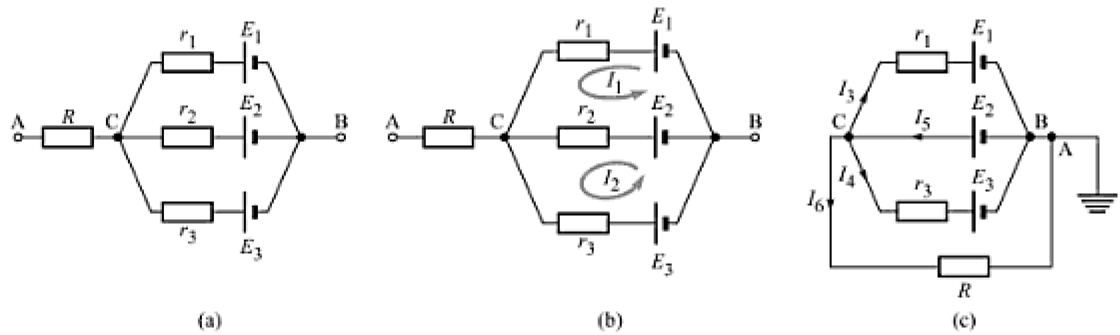


Fig. 3.52

Solution

(*a*) We assign the loop currents I_1 and I_2, as shown in Fig. 3.52*b*. Writing the two KVL equations for the two loops, we get

$$E_1 - I_1 r_1 - (I_1 - I_2) r_2 - E_2 = 0 \quad \Rightarrow \quad 3 - I_1 - I_1 + I_2 - 2 = 0 \quad \Rightarrow \quad -2I_1 + I_2 + 1 = 0$$
$$E_2 - (I_2 - I_1) r_2 - I_2 r_3 - E_3 = 0 \quad \Rightarrow \quad 2 - I_2 + I_1 - I_2 - 1 = 0 \quad \Rightarrow \quad -2I_2 + I_1 + 1 = 0$$

Solving these two equations, we get $I_1 = I_2 = 1$ A. Thus,

Current through $r_1 = I_1 = \mathbf{1\ A}$; Current $r_2 = I_1 - I_2 = 1 - 1 = \mathbf{0}$; and
Current through $r_3 = I_2 = \mathbf{1\ A}$; Current through $R = \mathbf{0\ A}$ (no closed path)

The pd across A and B = $E_2 + (I_1 - I_2) \times r_2 + 0 \times R = E_2 + 0 \times r_2 + 0 \times R = \mathbf{2\ V}$

(*b*) On short-circuiting r_2 and connecting point A to point B, the circuit changes to that shown in Fig. 3.52*c*. It has two nodes C and B. The voltage $V_{CB} = E_2 = 2$ V, irrespective of the currents in other branches. If node B is taken as reference (i.e., grounded), the voltage of node C is $V_C = V_{CB} = 2$ V. The currents in other branches can easily be found by applying Ohm's law,

$$I_3 = \frac{V_C - E_1}{r_1} = \frac{2-3}{1} = \mathbf{-1\ A}; \quad I_4 = \frac{V_C - E_3}{r_3} = \frac{2-1}{1} = \mathbf{1\ A}; \quad I_6 = \frac{V_C}{R} = \frac{2}{1} = \mathbf{2\ A}$$

The current in branch containing E_2 can be found by applying KCL at node C,

$$I_5 = I_3 + I_4 + I_6 = (-1) + 1 + 2 = \mathbf{2\ A}$$

EXAMPLE 3.35

Determine the voltage drop V_{ab} across the open circuit in Fig. 3.53. Also, state which resistors have no effect on the result obtained.

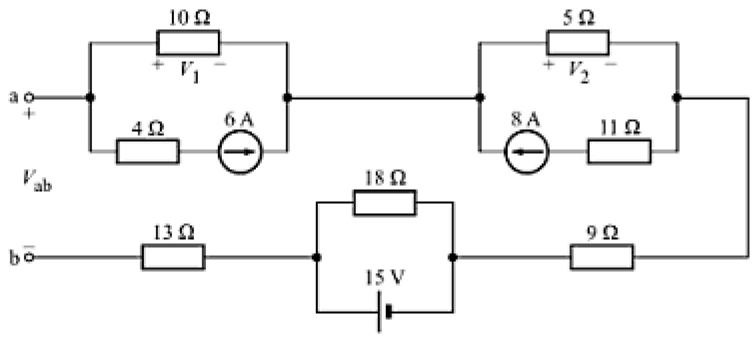


Fig. 3.53

Solution No current flows through 13-Ω resistance and 9-Ω resistance, as there is no closed path for the current to flow. The source current of 6-A flows in its loop and is not affected by the resistances connected in series. The pd across 10-Ω resistance is $6 \times 10 = 60$ V, irrespective of the value of 4-Ω resistance. Similarly, the pd across 5-Ω resistance is $5 \times 8 = 40$ V, irrespective of the value of 11-Ω resistance. The pd across the parallel combination of 15-V source and 18-Ω resistance remains 15 V, irrespective of the value of the 18-Ω resistance. Thus, **4-Ω, 11-Ω, 9-Ω, 18-Ω** and **13-Ω** resistances will have no effect on the voltage drop V_{ab}.

Note that we can conclude from above that ***a resistance connected in series with an ideal current source can be ignored and is redundant to the circuit.*** Similarly, ***a resistance connected in parallel with an ideal voltage source can be ignored and is redundant to the circuit.***

$$V_{ab} = -15 + 5 \times 8 - 6 \times 10 = -15 + 40 - 60 = \mathbf{-35\ V}$$

EXAMPLE 3.36

Find V_1 in the circuit of Fig. 3.54.

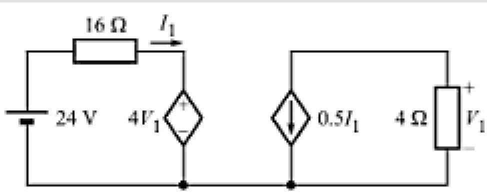


Fig. 3.54

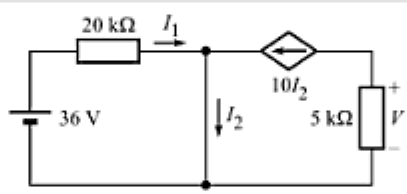


Fig. 3.55

Solution In the output loop, the current $0.5I_1$ flows through 4-Ω resistance, from bottom to top. Therefore, we have

$$V_1 = -(0.5I_1) \times 4 = -2I_1$$

Writing KVL equation for the input loop, and using the above result, we get

$$24 - 16I_1 - 4V_1 = 0 \quad \text{or} \quad 24 - 16I_1 - 4(-2I_1) = 0 \quad \Rightarrow \quad I_1 = 3\text{ A}$$

$$\therefore \quad V_1 = -2I_1 = -2 \times 3 = \mathbf{-6\ V}$$

EXAMPLE 3.37

Determine the voltage V in the circuit shown in Fig. 3.55.

Solution Applying KCL at the upper node, $I_1 - I_2 + 10I_2 = 0 \;\Rightarrow\; I_1 = -9I_2$.

Now, writing KVL equation for the left loop,

$$36 - (20\text{ k}\Omega) \times I_1 = 0 \quad \text{or} \quad 36 - (20\text{ k}\Omega) \times (-9I_2) = 0 \;\Rightarrow\; I_2 = -0.2\text{ mA}$$

Since a current of $10I_2$ is flowing from bottom to top through 5-kΩ resistance, we have

$$\therefore \quad V = -10I_2 \times (5\text{ k}\Omega) = -10 \times (-0.2\text{ mA}) \times (5\text{ k}\Omega) = \mathbf{10\ V}$$

EXAMPLE 3.38

Find the currents i_1 and i_2 in the network of Fig. 3.56.

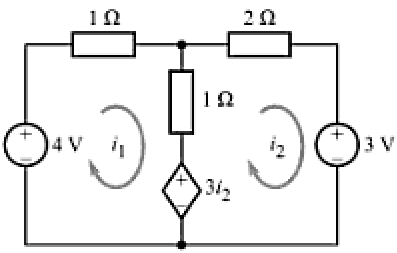


Fig. 3.56

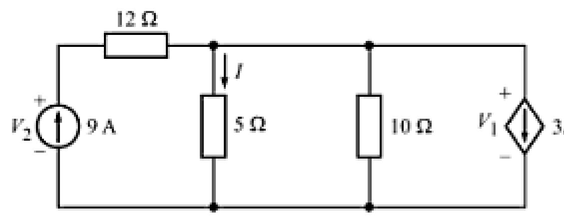


Fig. 3.57

Solution Writing loop equation for the output loop gives

$$3i_2 - (i_2 - i_1) \times 1 - i_2 \times 2 - 3 = 0 \;\Rightarrow\; i_1 = \mathbf{3\ A}$$

Writing loop equation for the input loop, we get

$$4 - i_1 \times 1 - (i_1 - i_2) \times 1 - 3i_2 = 0 \;\Rightarrow\; i_1 + i_2 = 2 \;\Rightarrow\; i_2 = \mathbf{-1\ A}$$

EXAMPLE 3.39

Determine the voltage V_1 and V_2 in the circuit of Fig. 3.57.

Solution The network has two nodes. Taking the bottom node as reference, we can write nodal voltage equation for the top node, in terms of its voltage V_1,

$$9 = I + \frac{V_1}{10} + 3I = 4I + \frac{V_1}{10}; \quad \text{where} \quad I = \frac{V_1}{5}.$$

$$\therefore \quad 9 = 4 \times \frac{V_1}{5} + \frac{V_1}{10} = \frac{9V_1}{10} \;\Rightarrow\; V_1 = \mathbf{10\ V}$$

Now, For the left loop,

$$V_2 - 12 \times 9 = V_1 \;\Rightarrow\; V_2 = V_1 + 108 = 10 + 108 = \mathbf{118\ V}$$

EXAMPLE 3.40

Find current I in the circuit of Fig. 3.58, by applying nodal-voltage analysis.

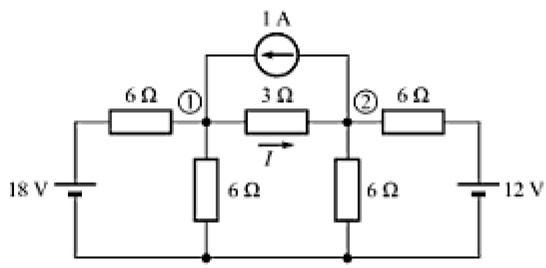


Fig. 3.58

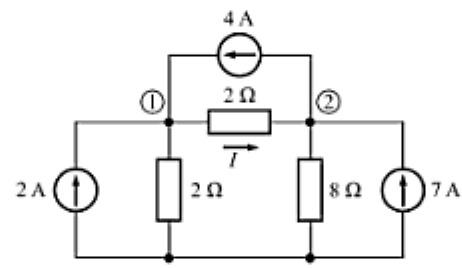


Fig. 3.59

Solution Let the two nodal-voltages be V_1 and V_2. Writing KCL equations at the two nodes,

$$\frac{V_1-18}{6}+\frac{V_1}{6}+\frac{V_1-V_2}{3}=1 \quad\Rightarrow\quad 2V_1-V_2=12 \qquad (i)$$

$$\frac{V_2-12}{6}+\frac{V_2}{6}+\frac{V_2-V_1}{3}+1=0 \quad\Rightarrow\quad 4V_2-2V_1=6 \qquad (ii)$$

Solving these two equations, we get V_1 = 9V and V_2 = 6 V.

$$\therefore \qquad I=\frac{V_1-V_2}{3}=\frac{9-6}{3}=\mathbf{1\ A}$$

EXAMPLE 3.41

Find current I in the circuit of Fig. 3.59, by node analysis.

Solution Let V_1 and V_2 be the voltages of the independent nodes 1 and 2. Writing the node equations by inspection, we get

$$\begin{bmatrix}\frac{1}{2}+\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{8}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}4+2\\ 7-4\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}1 & -0.5\\ -0.5 & 0.625\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}6\\ 3\end{bmatrix}$$

Solving the above, we get V_1 = 14 V and V_2 = 16 V.

$$\therefore \qquad I=\frac{V_1-V_2}{2}=\frac{14-16}{2}=\mathbf{-1\ A}$$

EXAMPLE 3.42

Determine the current through 2-S resistor in the circuit of Fig. 3.60*a*.

Solution Apparently, the circuit has 7 nodes. But a careful inspection indicates that it has only 4 nodes, as shown in Fig. 3.60*b*. The bottom node is made reference node r, remaining three are assigned voltages V_1, V_2 and V_3. The node analysis equation in matrix form are written directly as

$$\begin{bmatrix}(4+3) & -3 & -4\\ -3 & (3+2+1) & -2\\ -4 & -2 & (4+2+5)\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix}=\begin{bmatrix}(-3)+(-8)\\ -(-3)\\ -(-25)\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}7 & -3 & -4\\ -3 & 6 & -2\\ -4 & -2 & 11\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\end{bmatrix}=\begin{bmatrix}-11\\ 3\\ 25\end{bmatrix}$$

Solving the above for V_2 and V_3 only, we get V_2 = 2 V and V_3 = 3 V. Therefore, the current I through 2-S resistor is

$$I=G(V_2-V_3)=2(2-3)=\mathbf{-2\ A}$$

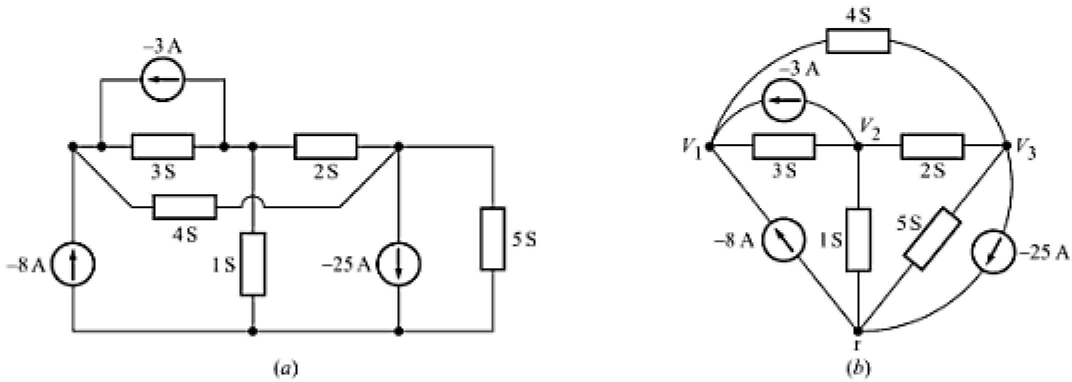


Fig. 3.60

EXAMPLE 3.43

Find the node voltages in the circuit shown in Fig. 3.61*a*.

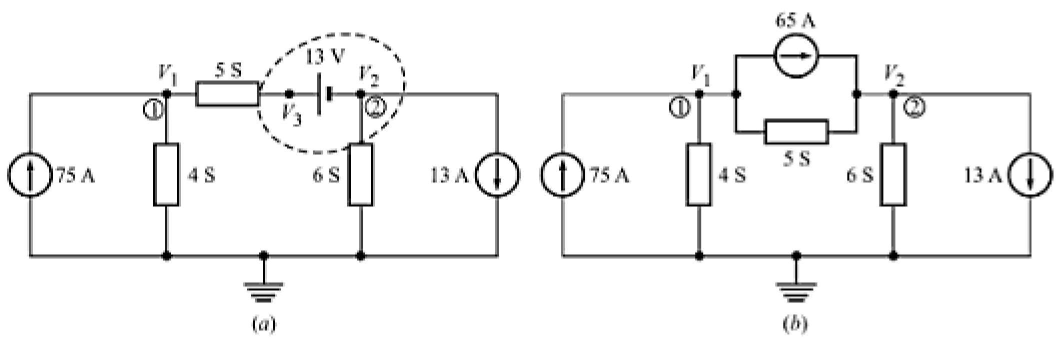


Fig. 3.61

Solution ***First Method*** One approach is to transform the 13-V source and series 5-S resistor to an equivalent current source of 65 A and a parallel resistor of 5 S, as shown in the circuit of Fig. 3.61*b*. Now, we can write the nodal equations in matrix form for the two nodes just by inspection,

$$\begin{bmatrix} 9 & -5 \\ -5 & 11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 52 \end{bmatrix}$$

Solving these equations, we get

$$V_1 = \mathbf{5\ V} \qquad \text{and} \qquad V_2 = \mathbf{7\ V}$$

Now, from the original circuit shown in Fig. 3.61*a*, we find that the 13-V source makes V_3 more negative than V_2 by 13 V. Thus, $V_3 = V_2 - 13 = 7 - 13 = \mathbf{-6\ V}$.

Second Method Another approach is to use the concept of supernode. The voltage source is enclosed in a region by a dotted line, as shown in Fig. 3.61*a*. The KCL is then applied to this closed surface:

$$6V_2 + 5(V_3 - V_1) = -13 \qquad (i)$$

The KCL equation for node 1 is

$$9V_1 - 5V_3 = 75 \qquad (ii)$$

Since, there are three unknowns, we need another independent equation. This is obtained from the voltage drop across the voltage source,

$$V_2 - V_3 = 13 \qquad (iii)$$

The solutions are, of course, the same :

$$\mathbf{V_1 = 5\ V,\ V_2 = 7\ V} \quad \text{and} \quad \mathbf{V_3 = -6\ V}$$

In general, for the supernode approach, the KCL equations must be augmented with KVL equations, the number of which is equal to the number of the floating voltage sources.

SUMMARY

TERMS AND CONCEPTS

- There are three types of basic passive components in electric circuits—*R*, *L* and *C*. There are three different points of view you can look at a passive component : (*i*) Circuit view point, (*ii*) Energy view point, and (*iii*) Geometric view point.
- No current flows through a ***capacitor***, if the voltage across it remains constant. The voltage across a ***capacitor*** cannot change instantaneously.
- No voltage appears across an ***inductor***, if the current through it remains constant. The current through an ***inductor*** cannot change instantaneously.
- At the time of switching ON a circuit, a capacitor behaves as a short-circuit. Under steady state, it behaves as an open-circuit.
- At the time of switching ON a circuit, an inductor behaves as an open-circuit. Under steady state, it behaves as a short-circuit.
- Capacitance (*C*) is ***dual*** of inductance (*L*).
- *Series* combination and *parallel* combination of inductances follow the same rule as for resistances.
- *Series* combination and *parallel* combination of capacitances follow the inverse rule as for resistances.
- An ***ideal voltage source*** has terminal voltage (*V*) which is independent of the output current (*I*).
- An ***ideal current source*** delivers a constant current (*I*), independent of its output voltage (*V*).
- A practical source can be viewed as a practical voltage source or a practical current source, with $V_S = R_S I_S$.
- There are four ways to classify sources : (*i*) either a voltage source or a current source, (*ii*) either an ideal source or a practical source, (*iii*) either a dc source or an ac source, and (*iv*) either an independent source or a dependent source.
- There are four kinds of controlled or dependent sources : (*i*) VCVS, (*ii*) CCCS, (*iii*) CCVS, and (*iv*) VCCS.
- **Kirchhoff's Current Law (KCL)** states that *the algebraic sum of currents meeting at a junction in a circuit is zero.*
- **Kirchhoff's Voltage Law (KVL)** states that *at any instant the algebraic sum of voltages around a closed loop or circuit is zero.*
- The ***loop-current analysis*** is a general method and can be applied to any network. The variables are currents and the equations are based on KVL.
- The ***mesh analysis*** can be applied to only *planar network* (i.e., a network which can be drawn on a sheet of paper without crossing lines).
- In mesh analysis, the equations can be written in matrix form directly by inspection of the network.

- The ***node-voltage analysis*** is dual of loop current analysis, and can be applied to any network. The variables are node-voltages and the equations are based on KCL.
- If the network has only independent current sources, we can apply nodal analysis and write the equation in matrix form directly by inspection of the network.
- In a network, the number of independent loops is b – (n – 1), whereas the number of independent nodes is n – 1. We should adopt the method which requires fewer equations to solve.
- A resistance connected in series with an ideal current source can be ignored and is redundant to the circuit.
- A resistance connected in parallel with an ideal voltage source can be ignored and is redundant to the circuit.

IMPORTANT FORMULAE

- The ***resistance*** parameter (R), from three viewpoints, is expressed as
  $R = \dfrac{v}{i}$; $R = \dfrac{W}{i^2 t}$; and $R = \rho \dfrac{L}{A}$
- The ***capacitance*** parameter (C), from three viewpoints, is expressed as
  $C = \dfrac{i}{dv/dt}$; $C = \dfrac{2W}{v^2}$ and $C = \varepsilon \dfrac{A}{d}$ (for a parallel plate capacitor)
- The ***inductance*** parameter (L), from three viewpoints, is expressed as
  $L = \dfrac{v}{di/dt}$; $L = \dfrac{2W}{i^2}$ and $L = \dfrac{\mu N^2 A}{l}$ (for a solenoid)
- For ***transforming*** a practical voltage source to a practical current source, or vice-versa,
  $V_S = R_S I_S$ and $R_{SV} = R_{SI} = R_S$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	If V_{ab} = +2 V and V_{cb} = –1 V, then V_{ca} = –3 V.	☐	☐	
2.	In an electrical circuit, an active component supplies energy whereas a passive component always dissipates energy.	☐	☐	
3.	When a capacitor in a circuit is fully charged, it behaves as a short-circuit.	☐	☐	
4.	A source of electrical energy can work as an ideal current source only if its internal resistance is zero.	☐	☐	
5.	If two ideal voltage sources having emf 2 V and 4 V are connected in parallel, the net emf across the parallel combination will be 3 V.	☐	☐	
6.	A battery of emf 10 V is connected in a circuit. A current of 2 A leaves its negative terminal. The power supplied by this battery is 20 W.	☐	☐	
7.	In a current-controlled voltage-source, the units of the current-multiplier will be ohms.	☐	☐	
8.	Normally, the loop-current method is applied to a network that contains no current sources.	☐	☐	
9.	If a network has n nodes, the number of independent nodes will be n + 1.	☐	☐	
10.	A resistance connected in parallel with an ideal voltage source can be removed altogether without affecting the remaining circuit.	☐	☐	

Your Score

ANSWERS

1. True	**2.** False	**3.** False	**4.** False	**5.** False
6. False	**7.** True	**8.** True	**9.** False	**10.** True

REVIEW QUESTIONS

1. What are the different points of view a passive component connected in a circuit can be looked at? Describe the capacitance parameter from these points of view.
2. Explain how an inductor behaves in a circuit when (*a*) it is just switched ON, and (*b*) the circuit has reached its steady state.
3. Explain how a capacitor behaves in a circuit when (*a*) it is just switched ON, and (*b*) the circuit has reached its steady state.
4. How can you justify that the voltage across a capacitor cannot change instantaneously?
5. Derive an expression for finding the equivalent capacitance of a parallel combination of three capacitances.
6. What do you understand by 'an ideal voltage source' and 'an ideal current source'? Draw their circuit symbols.
7. Derive the equation for converting a voltage source into a current source and vice versa.
8. State and explain Kirchhoff's voltage law. Derive it from the law of conservation of energy.
9. State and explain Kirchhoff's current law. How does it follow from the conservation of charge?
10. Explain the loop-current method of solving a network. Can it be applied to a network that is not planar?
11. How does the mesh-analysis differ from the loop-current analysis?
12. Explain the nodal voltage method of solving a network. How are the nodal equations written?
13. Differentiate between branch current and mesh current.
14. Define the terms: loop, mesh, node, planar circuit, graph of a circuit.
15. Explain how you will decide which method of solving a given circuit is more suitable.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. A capacitor opposes
   (*a*) the change in voltage
   (*b*) the change in current
   (*c*) the change in both voltage and current
   (*d*) none of these
2. The energy stored in a capacitor is given by
   (*a*) C^2V (*b*) $C^2V/2$
   (*c*) $Q^2/2C$ (*d*) CV
3. An ideal voltage source should have
   (*a*) infinite internal resistance
   (*b*) zero internal resistance
   (*c*) terminal voltage in proportion to current
   (*d*) terminal voltage in proportion to load
4. If the area of the plates of a capacitor is increased two times, then its capacitance
   (*a*) increases four times
   (*b*) decreases four times
   (*c*) increases two times
   (*d*) decreases two times
5. If the voltage across the plates of a 1-F capacitor is increased by 2 V, then the charge on the plates will
   (*a*) increase by 0.5 C
   (*b*) decrease by 0.5 C
   (*c*) decrease by 2 C
   (*d*) increase by 2 C
6. If the number of turns of a coil is doubled, its inductance
   (*a*) decreases two times
   (*b*) decreases four times
   (*c*) increases two times
   (*d*) increases four times

7. When a number of capacitors are connected in series, the total capacitance is
   (*a*) greater than any of the capacitors
   (*b*) greater than the largest capacitor
   (*c*) smaller than the smallest capacitor
   (*d*) none of the above
8. If ordinate represents voltage and abscissa the current, the characteristics of an *ideal current source* would be represented by
   (*a*) a vertical line
   (*b*) a horizontal line
   (*c*) a diagonal line at 45° to either axis extending from the first to the third quadrant
   (*d*) a diagonal line at 45° to either axis extending from the fourth to the second quadrant
9. Kirchhoff's current law states that
   (*a*) the sum of currents in a series circuit is zero
   (*b*) the sum of currents in a parallel circuit is zero
   (*c*) the sum of currents in a series-parallel circuit is zero
   (*d*) the sum of currents entering a node is equal to the sum of currents leaving the node
10. Kirchhoff's voltage law states that
   (*a*) the sum of emfs in a closed loop is zero
   (*b*) the sum of voltage drops and emfs in a closed loop is zero
   (*c*) the total voltage drop in a series circuit is always finite
   (*d*) the sum of potential differences across each element in a circuit is zero.
11. The number of equations required to analyze a given network by loop-current method is equal to
   (*a*) the number of independent loops
   (*b*) the number of nodes
   (*c*) the number of branches
   (*d*) none of these
12. The number of equations required to analyze a given network by nodal analysis is equal to
   (*a*) the number of independent loops
   (*b*) the number of nodes
   (*c*) the number of branches
   (*d*) one less than the number of nodes

ANSWERS

1. *a*	**2.** *c*	**3.** *b*	**4.** *c*	**5.** *d*	**6.** *d*	**7.** *c*	**8.** *a*	**9.** *d*	**10.** *b*
11. *a*	**12.** *d*								

PROBLEMS

(A) SIMPLE PROBLEMS

1. If a 12-V car battery has a 0.04-Ω internal resistance. What is the battery terminal voltage when the battery delivers 40 A? **[Ans.** 10.4 V]
2. If a 12-V car battery has a 0.1-Ω internal resistance. What terminal voltage causes a 4-A current to flow into the positive terminal? **[Ans.** 12.4 V]
3. Find the voltage V_{ab} across the open circuit in the circuit shown in Fig. 3.62. **[Ans.** 80 V]
4. For the circuit shown in Fig. 3.63, Find V_1, V_{ad}, V_{bc} and $V_{ac} + V_{ce}$. **[Ans.** 9 V, –3 V, 4 V, 2 V]

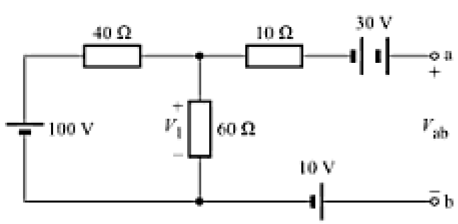


Fig. 3.62

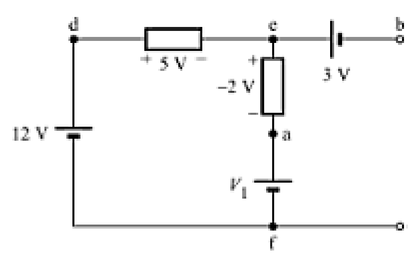


Fig. 3.63

5. For the circuit shown in Fig. 3.64, determine the unknown currents using KCL.
[Ans. $I_5 = 2$ A, $I_3 = 1.5$ A**]**

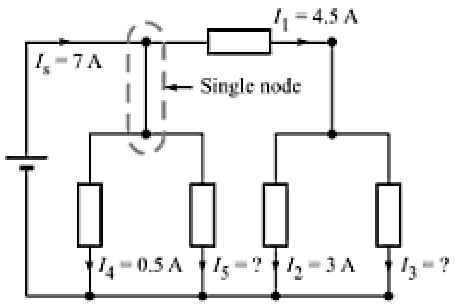


Fig. 3.64

6. For the circuit shown in Fig. 3.65, determine the current in the 10-Ω resistor for $R = 6$ Ω, using node-voltage analysis. **[Ans.** 0 A**]**

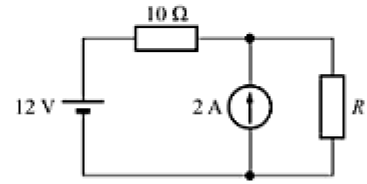


Fig. 3.65

7. Determine the voltages at nodes B and C in the network shown in Fig. 3.66.
[Ans. $V_B = V_C = 114$ V**]**

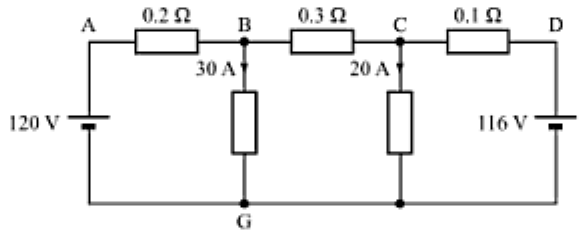


Fig. 3.66

(B) TRICKY PROBLEMS

8. Using KCL, determine i and i_{ab} in the circuit shown in Fig. 3.67. **[Ans.** –15 A, 6 A**]**

9. Use KCL, KVL and Ohm's law to find v_2, v_3, i, R_1 and R_3, in the circuit of Fig. 3.68.
[Ans. $i = -0.6$ A, $v_2 = -2$ V, $v_3 = 15$ V, $R_1 = 1.79$ Ω, $R_3 = 3$ Ω**]**

10. Apply mesh analysis and determine the loop-currents I_1, I_2 and I_3 in the circuit shown in Fig. 3.69.
[Ans. $I_1 = 4.5$ A, $I_2 = -2.5$ A, $I_3 = 0.5$ A**]**

11. Using loop-current analysis, find the loop currents in the circuit shown in Fig. 3.70.
[Ans. $I_1 = -0.25$ A, $I_2 = -4.75$ A, $I_3 = -4$ A**]**

12. Find the loop currents in the network shown in Fig. 3.71, by using the loop-current method.
[Ans. $I_1 = 4.9$ A, $I_2 = 1.36$ A, $I_3 = 3$ A**]**

13. Determine the current through the 3-Ω resistor in the circuit shown in Fig. 3.72, by using mesh analysis. **[Ans.** $I_{3\,\Omega} = -1.35$ A**]**

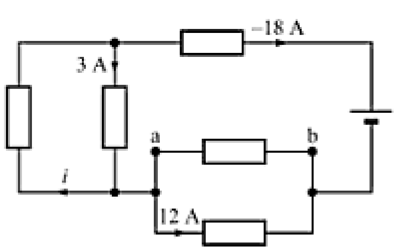


Fig. 3.67

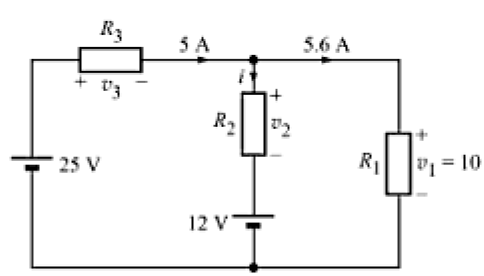


Fig. 3.68

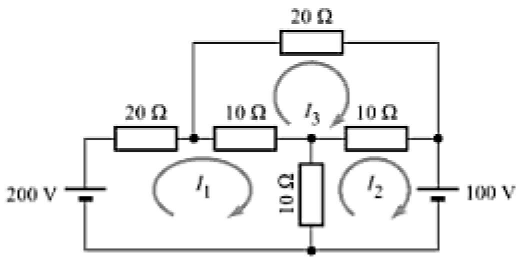


Fig. 3.69

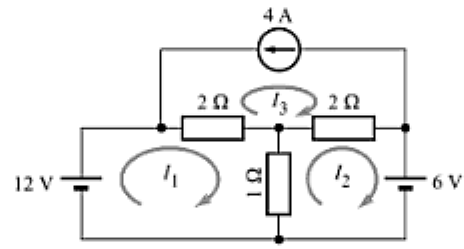


Fig. 3.70

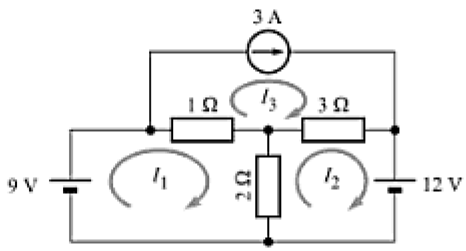


Fig. 3.71

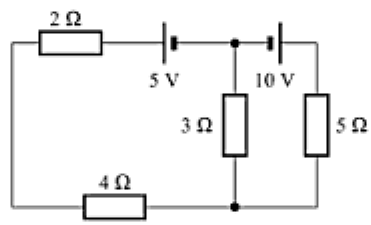


Fig. 3.72

14. Determine the current through the 1-Ω resistor in the circuit shown in Fig. 3.73, by using mesh analysis. [**Ans.** 1.39 A]

15. Transform the voltage source within the dotted box in the circuit shown in Fig. 3.74 into its equivalent current source and then apply nodal analysis to determine voltage at node b. [**Ans.** 7.27 V]

16. Using source transformation, reduce the network shown in Fig. 3.75 to a single loop circuit and then determine current through the 20-Ω resistor. [**Ans.** 1.375 A]

17. Find the current downward in the 50-Ω resistor in the circuit shown in Fig. 3.76, by first solving for v_b using node-voltage analysis. [**Ans.** 0.0448 A]

18. For the circuit shown in Fig. 3.77, solve for V_a and V_b using node-voltage analysis, and then determine the voltage across the three resistors. [**Ans.** $V_{ar} = -6.557$ V, $V_{br} = +4.393$ V, $V_{ab} = -10.95$ V]

19. Using nodal analysis, determine the voltages of nodes 1 and 2, for the circuit shown in Fig. 3.78. [**Ans.** 19.43 V, 18.28 V]

20. By applying nodal analysis to the circuit of Fig. 3.79, determine the currents I_1 and I_2. [**Ans.** 4.75 A, 4.25 A]

21. Find the current I by using node-voltage analysis for the circuit given in Fig. 3.80. [**Ans.** −2.255 A]

22. Using node-voltage analysis, determine the current I in the 15-Ω resistor of the network shown in Fig. 3.81. [**Ans.** 4 A]

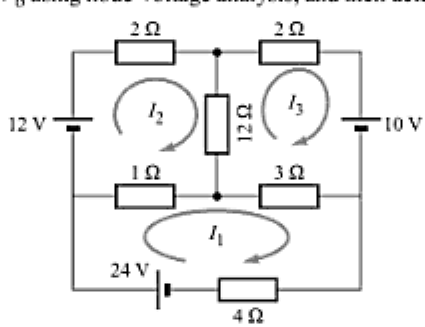


Fig. 3.73

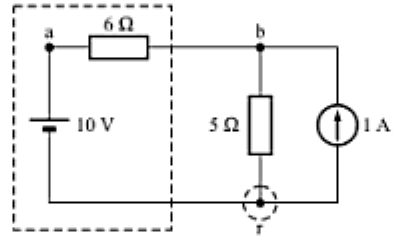


Fig. 3.74

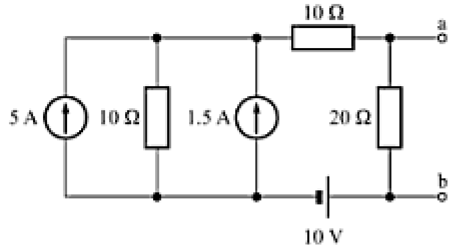


Fig. 3.75

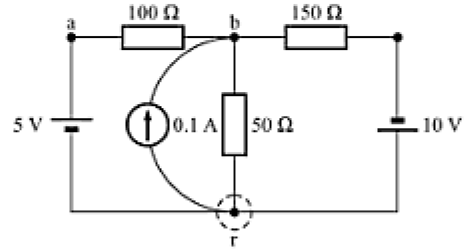


Fig. 3.76

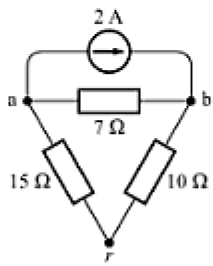


Fig. 3.77

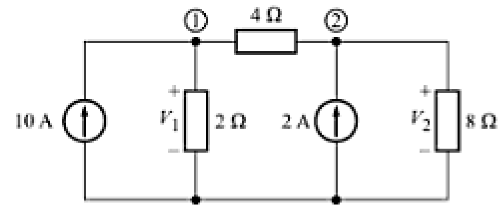


Fig. 3.78

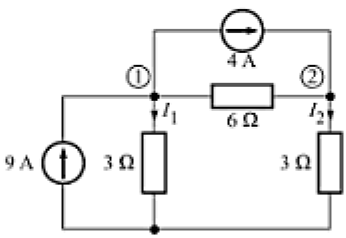


Fig. 3.79

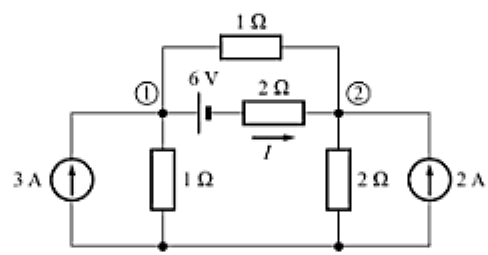


Fig. 3.80

(C) CHALLENGING PROBLEMS

23. Two batteries having emfs of 10 V and 7 V, and internal resistances of 2 Ω and 3 Ω respectively, are connected in parallel across a load of resistance 1 Ω. Calculate (*i*) the current supplied by each battery, (*ii*) the current through the load, and (*iii*) the voltage across the load.

[**Ans.** (*i*) 3 A and 1 A; (*ii*) 4 A; (*iii*) 4 V]

24. For the circuit shown in Fig. 3.82, determine currents I_1 to I_5 by using node-voltage analysis.

[**Ans.** I_1 = 1.25 A, I_2 = 3.75 A, I_3 = –2.5 A, I_4 = 5 A, I_5 = –7.5 A]

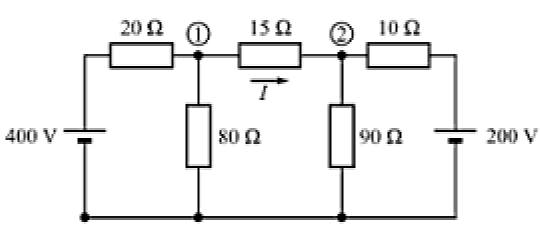


Fig. 3.81

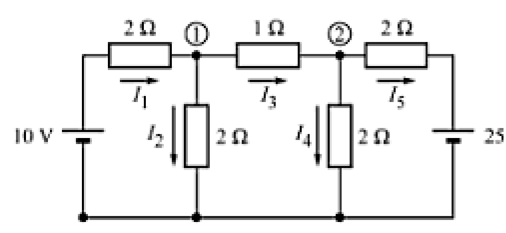


Fig. 3.82

EXPERIMENTAL EXERCISE 3.1

KIRCHHOFF'S CURRENT LAW

Objective To verify Kirchhoff's Current Law (KCL).

Apparatus DC power supply 220 V; Three ammeters (MC type) 0-5 A; Three rheostats 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 3.83.

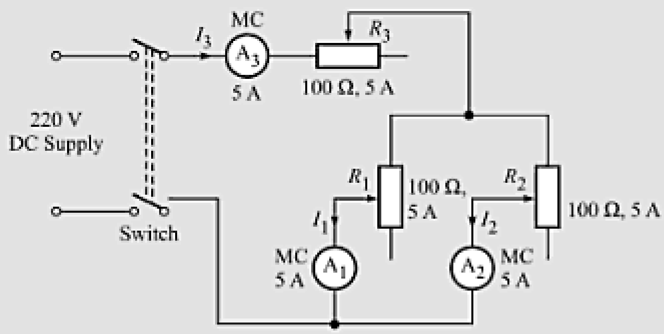


Fig. 3.83

Brief Theory Kirchhoff's Current Law states that at any instant the algebraic sum of currents meeting at a junction in a circuit is zero. In other words, the sum of currents flowing away from the junction is equal to the sum of currents flowing towards the junction. For n branches meeting at a junction, KCL can be stated as

$$\sum_{j=1}^{n} I_j = 0$$

Procedure

1. Make connections as given in Fig. 3.83.
2. Set all the three rheostats to their maximum value.
3. Switch ON the supply.
4. Note the readings of the ammeters A_1, A_2 and A_3, giving the values of the currents I_1, I_2 and I_3 in the circuit.
5. Change the settings of rheostats so as to get different readings in the ammeters. Note the readings.
6. Repeat step 5 for five different settings of the rheostats.
7. Switch OFF the supply.

Observations

S. No.	I_1 (*in* A)	I_2 (*in* A)	I_3 (*in* A)	$I_1 + I_2$ (*in* A)
1.				
2.				
3.				
4.				
5.				

Calculations For each set of settings of the rheostats, add the readings in columns 2 and 3, and record it in column 5 of the table.

Results

1. For each set of readings, we find that the value of I_3 in column 4 is nearly the same as that of $(I_1 + I_2)$ in column 5.
2. Kirchhoff's current law (KCL) is, therefore, verified.

Precautions

1. Before switching on the supply, the zero reading of the ammeters should be checked.
2. The terminals of the rheostats should be connected properly.
3. While setting the rheostats, care should be taken that the current I_3 as recorded by the ammeter A_3 does not exceed 5 A.

Viva-Voce

1. In an ac circuit consisting of resistances, inductances and capacitances, the three currents meeting at a junction were found to have the values $I_3 = 4.5$ A, $I_1 = 2.8$ A and $I_2 = 3.2$ A. Here, $I_1 + I_2 = 6$ A, whereas $I_3 = 4.5$ A. Therefore, will you say that KCL is not applicable?
Ans.: No. KCL is applicable to both the dc as well as ac circuits. However, in ac circuits, we consider *phasor sum* rather than *algebraic sum*.

2. What do you mean by *algebraic sum*?
Ans.: In algebraic sum, the sign of the quantity is given due consideration. If a current flowing towards a junction is assigned a 'positive' sign, then the current leaving the junction must be assigned a 'negative' sign.

3. Is KCL based on the principle of conservation of energy?
Ans.: No. It is based on the principle of conservation of charge. Since the accumulation of electric charge at a junction is not possible, the amount of charge entering a junction must be the same as the amount of charge leaving the junction.

4. What do you mean by the term 'node' in reference to an electric circuit?
Ans.: A node is a point in a circuit where more than two elements are joined together.

EXPERIMENTAL EXERCISE 3.2

KIRCHHOFF'S VOLTAGE LAW

Objectives To verify Kirchhoff's Voltage Law (KVL).

Apparatus DC power supply 220 V; One ammeter (MC type) 0-10 A; Two voltmeters (MC type) 0-300 V; Two rheostats 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 3.84.

Brief Theory Kirchhoff's Voltage Law states that at any instant the algebraic sum of voltages around a loop or a closed circuit is zero. This statement simply tells us that if we start form a point in an electric circuit and go around a loop so as to come back to the same point, the net potential rise (or potential drop) is zero. For a loop having k elements, the KVL can be stated as

$$\sum_{j=1}^{k} V_j = 0$$

where V_j represents the voltage drop (or voltage rise) of jth element in the loop.

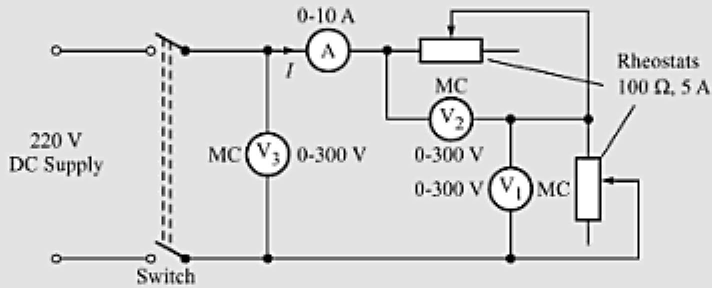


Fig. 3.84

Kirchhoff's voltage law can also be stated like this: In any closed circuit or a loop, the algebraic sum of the products of current and resistance in each of the conductors is equal to the algebraic sum of the emfs.

Procedure

1. Make connections as given in Fig. 3.84.
2. Set both the rheostats to their maximum value.
3. Switch ON the supply.
4. Note the readings of the ammeter A, and the three voltmeters V_1, V_2 and V_3.
5. Change the settings of the rheostats so as to get different readings in the voltmeters V_1 and V_2. Note the readings.
6. Repeat step 5 for five different settings of the rheostats.
7. Switch OFF the supply.

Observations

S. No.	I (*in* A)	V_1 (*in* V)	V_2 (*in* V)	V_3 (*in* V)	$V_1 + V_2$ (*in* V)
1.					
2.					
3.					
4.					
5.					

Calculations For each set of settings of the rheostats, add the voltages V_1 and V_2 recorded in columns 3 and 4, and record the same in column 6 of the table.

Results

1. For each set of readings, we find that the value of V_3 in column 5 is nearly the same as that of ($V_1 + V_2$) in column 6.
2. Kirchhoff's Voltage Law (KVL) is, therefore, verified.

Precautions

1. Before switching ON the supply, the zero reading of the ammeter and voltmeters should be checked.
2. The terminals of the rheostats should be connected properly.
3. While setting the rheostats, care should be taken that the current I as recorded by the ammeter A does not exceed 5 A, the current rating of the rheostats.

Viva-Voce

1. While applying KVL to an ac circuit, will you find the algebraic sum of the voltage drops across each element in a loop?
   Ans.: No. In ac circuits, we consider *phasor sum* rather than *algebraic sum*.
2. What do you mean by the term '*loop*'?
   Ans.: A loop is a closed path in an electrical circuit. That is, if we start from one point, we should be able to come back to the same point by traversing the loop.
3. Is a loop different from a mesh?
   Ans.: Yes. A mesh is a special loop that has no loop within it. Thus, all meshes are loops, but all loops are not meshes.
4. In using KVL, is it necessary that a voltage drop is written with a negative sign?
   Ans.: No. It depends upon the sign convention chosen. We may take voltage drop as positive and voltage rise as negative, or vice versa. Both ways, we get the same result.

NETWORK THEOREMS

4

OBJECTIVES

After completing this Chapter, you will be able to:

- *State and apply 'superposition theorem' to solve a circuit containing more than one source.*
- *State and apply 'Thevenin's theorem' to solve a circuit.*
- *State the benefits of 'Thevenin's theorem'.*
- *State and apply 'Norton's theorem' to solve a circuit.*
- *State, prove and apply 'maximum power transfer theorem'.*
- *Find a single equivalent voltage source for a number of voltage sources connected in parallel using 'Millman's theorem'.*
- *State and prove 'reciprocity theorem'.*
- *State and prove 'Tellegen's theorem'.*
- *State the importance of 'Tellegen's theorem'.*

4.1 INTRODUCTION

Simple circuits can be solved by using Ohm's law, Kirchhoff's laws, voltage divider, current divider, series and parallel combination of sources and resistors, etc. Special techniques, known as *network theorems* and *network reduction methods,* have been developed which drastically reduce the labour of solving a more complicated network. The network theorems provide simple conclusions and good insight into the problems. Some of these have universal applications, whereas some are limited to the networks containing linear* elements only.

4.2 SUPERPOSITION THEOREM

This theorem states that *the response in a linear circuit at any point due to multiple sources can be calculated by summing the effects of each source considered separately, all other sources being made inoperative* (or *turned* OFF). The theorem is applicable only to a linear network (containing independent and/or dependent sources).

How to Make a Source Inoperative

When a *voltage source* is made inoperative or turned OFF, no voltage drop exists across its terminals but *a current can still* flow through it. Hence, it acts like a *short-circuit* (Fig. 4.1*a*).

* A linear element obeys Ohm's law, e.g., resistance, inductance, etc. Semiconductor diodes and transistors are not linear elements.

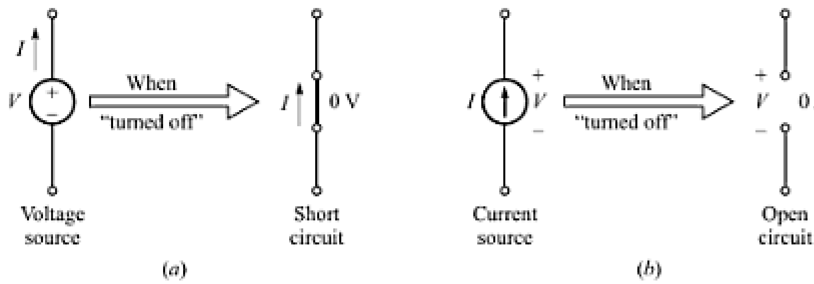


Fig. 4.1 *"Turning off" the sources.*

Similarly, when a *current source* is made inoperative or turned off, no current flows through it but a voltage can appear across its terminals. Hence, it acts like an *open-circuit* (Fig. 4.1*b*).

By making a source *inoperative* or turned OFF means that the voltage source is replaced by a short-circuit and the current source is replaced by an open-circuit.

EXAMPLE 4.1

Using superposition theorem, find the current I in the circuit shown in Fig. 4.2*a*.

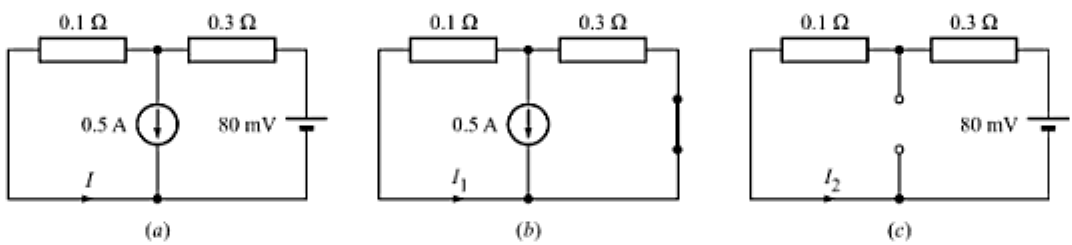


Fig. 4.2 *Principle of superposition illustrated.*

Solution Let us first consider the response I_1 due to the 0.5-A current source, and turn OFF the 80-mV voltage source by shorting it (Fig.4.2*b*). Applying current divider, we get

$$I_1 = -0.5 \times \frac{0.3}{0.1 + 0.3} = 0.375 \text{ A}$$

Next, consider the voltage source, and turn OFF the current source by opening it (Fig.4.2*c*). Ohm's law gives

$$I_2 = \frac{80 \times 10^{-3}}{0.1 + 0.3} + 0.2\text{A}$$

By the principle of superposition, the total current is given as

$$I = I_1 + I_2 = -0.375 + 0.2 = -\mathbf{0.175A}$$

Note that in redrawing the circuit for each source, we are always careful to mark the response current in the original direction and also assign a suitable subscript to indicate that we are not working with the original variables. This prevents the possibility of committing errors when we add the individual currents.

EXAMPLE 4.2

Use superposition theorem to find current I_x in the network given in Fig. 4.3.

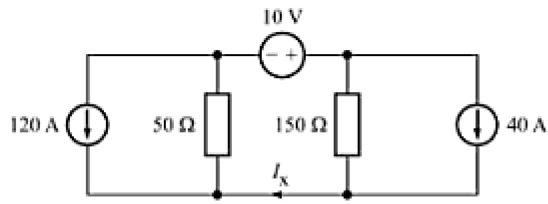


Fig. 4.3

Solution Consider first the voltage source alone. Both current sources are made inoperative by open-circuiting them (Fig. 4.4*a*). Current I_1 is

$$I_1 = \frac{10}{50 + 150} = 0.05 \text{ A}$$

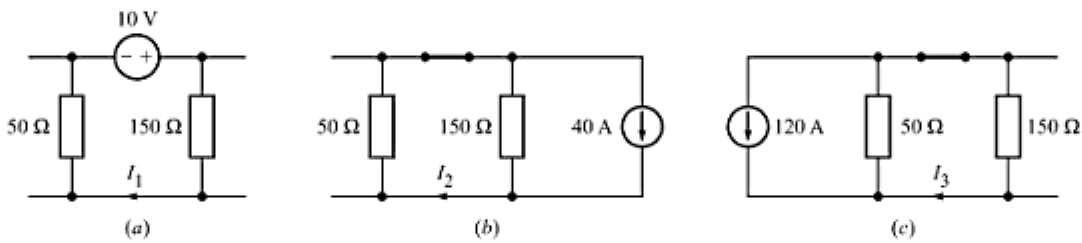


Fig. 4.4

Next consider the 40-A current source alone (Fig. 4.4*b*). Applying current divider, the current I_2 is given as

$$I_2 = 40 \times \frac{150}{50 + 150} = 30 \text{ A}$$

Lastly consider the 120-A current source alone (Fig. 4.4*c*). The current I_3 is

$$I_3 = -120 \times \frac{50}{50 + 150} = -30 \text{ A}$$

Applying the principle of superposition, we get

$$I_x = I_1 + I_2 + I_3 = 0.05 + 30 - 30 = \mathbf{0.05\ A}$$

EXAMPLE 4.3

Consider our benchmark example (Fig. 3.24*a*) discussed in Example 3.11, wherein we had calculated the voltage across 3-Ω resistor as 2.5 V, by using source transformation. The same circuit is shown in Fig. 4.5*a*. We again find voltage v across 3-Ω resistor by applying the principle of superposition.

Solution Consider first the **4-A current source**, while turning OFF the remaining two sources. The turned-OFF 5-A current source is replaced by an open-circuit and the turned-OFF 6-V voltage source is replaced by a short-circuit, as shown in Fig. 4.5*b*. We find that the current of 4 A divides into two parallel paths. Therefore, using current divider, we get

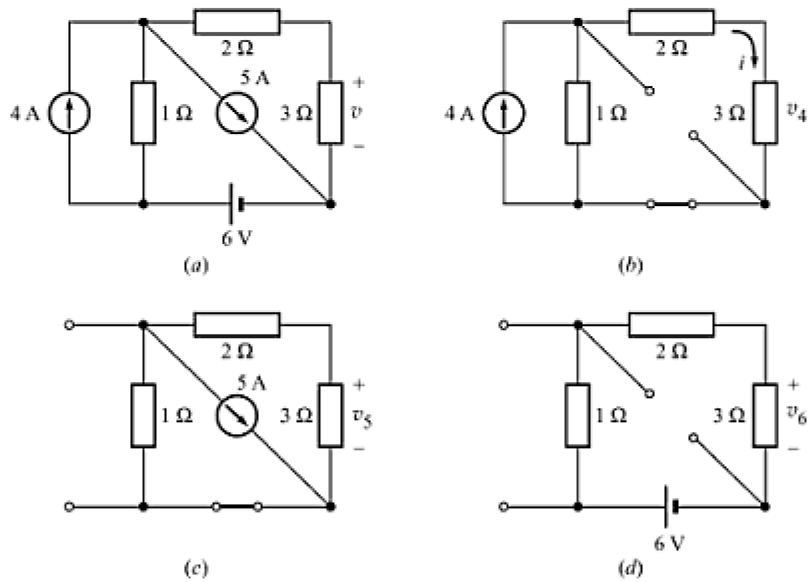


Fig. 4.5

$$i = 4 \times \frac{1}{1 + (2 + 3)} = \frac{2}{3} \text{A}$$

Thus, voltage v_4 across 3-Ω resistor due to 4-A current source is

$$V_4 = i \times R = (2/3 \text{ A}) \times (3\Omega) = 2.0\text{V}$$

The polarity of this voltage is the same as the original polarity markings for v. Hence, this contribution to the final summation will appear with a + sign.

Next consider **5-A current source,** with 4-A and 6-V sources turned OFF (Fig. 4.5*c*). From the perspective of the 5-A source, the 2-Ω and 3-Ω resistances are in series and together in parallel with the 1-Ω resistor. Using current-divider, the voltage v_5 across 3-Ω resistor due to 5-A current source is given as

$$v_5 = \left[-5 \times \frac{1}{1 + (2 + 3)} A \right] \times (3\Omega) = -2.5 \text{ V}$$

Note that the actual polarity of this voltage v_5 is opposite to the polarity marked on v. Hence, the minus sign.

We now consider **6-V voltage source,** with 4-A and 5-A sources turned OFF (Fig.4.5*d*). The three resistances are connected in series. The resulting voltage v_6 is calculated by voltage divider, as

$$v_6 = 6 \times \frac{3}{1 + 2 + 3} = 3.0 \text{ V}$$

The polarity of this voltage is same as that of the original voltage v. Hence, the sign of v_6 will be positive in the final summation.

Using principle of superposition, we now obtain the total voltage v across 3-Ω resistor, as

$$v = +v_4 + v_5 + v_6 = +2.0 - 2.5 + 3.0 = +\mathbf{2.5\ V}$$

EXAMPLE 4.4

For the circuit shown in Fig. 4.6*a*, find the value of I_s to reduce the voltage across 4-Ω resistor to zero.

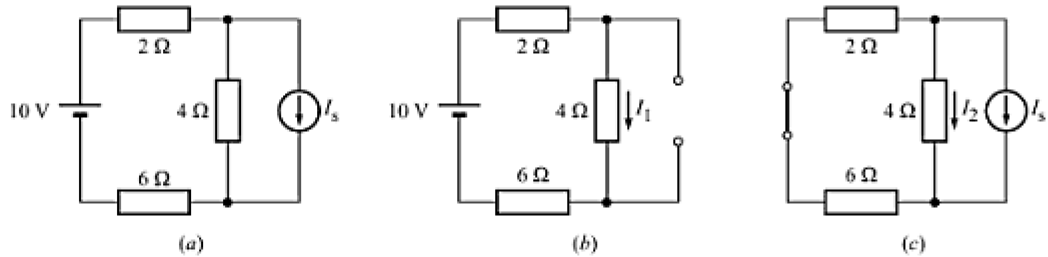


Fig. 4.6

Solution To solve this problem, we apply the principle of superposition. The current I_1 (from top to bottom) in 4-Ω resistor due to 10-V source in Fig. 4.6*b* is

$$I_1 = \frac{10}{2+4+6} = \frac{5}{6}\text{ A}$$

The current I_2 (from top to bottom) in 4-Ω resistor due to current source I_s in Fig. 4.6*c* is

$$I_2 = -I_s \times \frac{2+6}{2+6+4} = -\frac{2}{3}I_s$$

The voltage across 4-Ω resistor can be zero, only if the current through this resistor is zero. That is,

$$I_1 + I_2 = 0 \quad \text{or} \quad \frac{5}{6} + \left(-\frac{2}{3}I_s\right) = 0 \quad \Rightarrow \quad I_s = \frac{5\times 3}{6\times 2} = \mathbf{1.25\ A}$$

4.3 THEVENIN'S THEOREM

This theorem was first proposed by a French telegraph engineer M.L. Thevenin in 1883. Often, we need to find the response (*current*, *voltage* or *power*) in a single load resistance in a network. Thevenin's theorem enables us to do this without solving the entire network. It is specially very helpful and time-saving when we wish to find the response for different values of the load resistance.

Thevenin's Theorem states that *it is possible to simplify any linear circuit containing independent and dependent voltage and current sources, no matter how complex, to an equivalent circuit with just a single voltage source and a series resistance, between any two points of the circuit.*

Procedure

The procedure to apply Thevenin's theorem to a network will be explained in steps, by taking an example. Consider the circuit shown in Fig. 4.7*a*. Suppose that we are interested to find the current in resistor R_2. We proceed as follows.

1. Designate the resistor R_2 as "load" (Fig. 4.7*b*).
2. Pull out the load resistor and enclose the remaining network within a dotted box (Fig. 4.7*c*).
3. Temporarily remove the load resistor R_2, leaving the terminals A and B open (Fig. 4.7*d*).

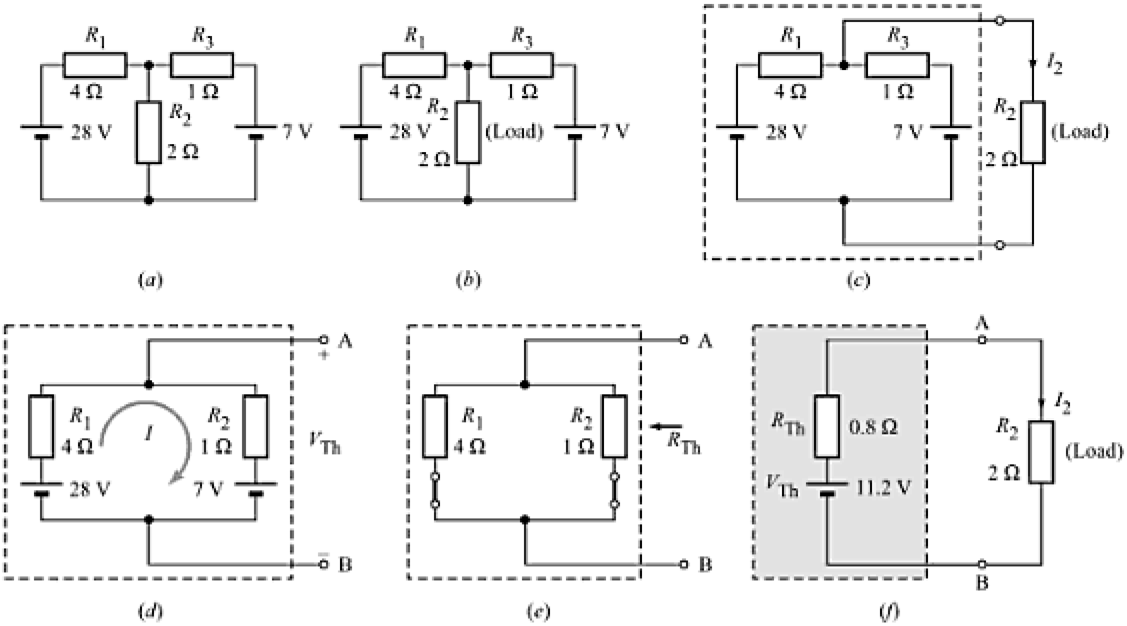


Fig. 4.7

4. Find the ***open-circuit voltage*** across the terminals A-B as follows:

$$I = \frac{28 - 7}{4 + 1} = \frac{21}{5} = 4.2\text{A}; \quad V_{AB} = 7 + 4.2 \times 1 = \mathbf{11\ .2V}$$

5. This is called ***Thevenin voltage***, $V_{Th} = V_{AB} = 11.2$ V.
6. Turn OFF all the sources in the circuit (Fig. 4.7*e*) and find the resistance between terminals *A* and B. This is the ***Thevenin resistance,*** R_{Th}. Thus,

$$R_{Th} = 1\ \Omega \parallel 4\ \Omega = \frac{1 \times 4}{1 + 4} = 0.8\ \Omega$$

7. The circuit within the dotted box (Fig. 4.7*c*) is replaced by the ***Thevenin's equivalent***, consisting of a voltage source V_{Th} in series with a resistor R_{Th}, as shown in Fig. 4.7*f*.
8. The load resistor R_2 is again connected to Thevenin's equivalent (Fig. 4.7*f*) forming a single-loop circuit, and the current I_2 through this resistor is easily calculated as

$$I_2 = \frac{V_{Th}}{R_{Th} + R_2} = \frac{11.2}{0.8 + 2} = \mathbf{4\ A} \tag{4.1}$$

Important Comment on Thevenin's Equivalent

The equivalent circuit replaces the circuit within the box only for the effects *external* to the box. We can no longer ask questions about the circuit in box. For example, if we are interested in the current in R_1 (= 4 Ω) or the total power consumed by the resistors R_1 and R_3 in the box of Fig. 4.7*c*, the equivalent circuit of Fig. 4.7*f* is just useless.

Benefits of Thevenin's Theorem

The example of the circuit considered above was simple. With a little more effort, we could have computed this result directly from the original circuit of Fig. 4.7*a*. You may wonder why solve it by this roundabout method. It is because we get following benefits from the concept of Thevenin's theorem.

1. The theorem enables us to derive the same simple equation as Eq. 4.1 no matter how complicated the original circuit is. There could have been hundreds of sources and thousands of resistors in the original circuit. We still would have reduced the circuit to two parameters, Thevenin's voltage V_{Th} and Thevenin's resistance R_{Th}.
2. Thevenin's equivalent circuit leads to one of the most useful ideas of electrical engineering, namely, the idea of "*output impedance*" of a circuit.
3. We gain the freedom to ask many additional questions such as: What value of R_2 makes the voltage across it 20 V? What value of R_2 draws the maximum power from the circuit? The answer to the first question is that no value of R_2 will give 20 V, as V_{Th} is only 11.2 V. The second question leads us to an interesting and important result, named as *maximum power transfer theorem*, which we shall soon see.

EXAMPLE 4.5

By using Thevenin's equivalent of the circuit within the dotted box, determine the voltage across the load resistor R_L in Fig. 4.8*a*.

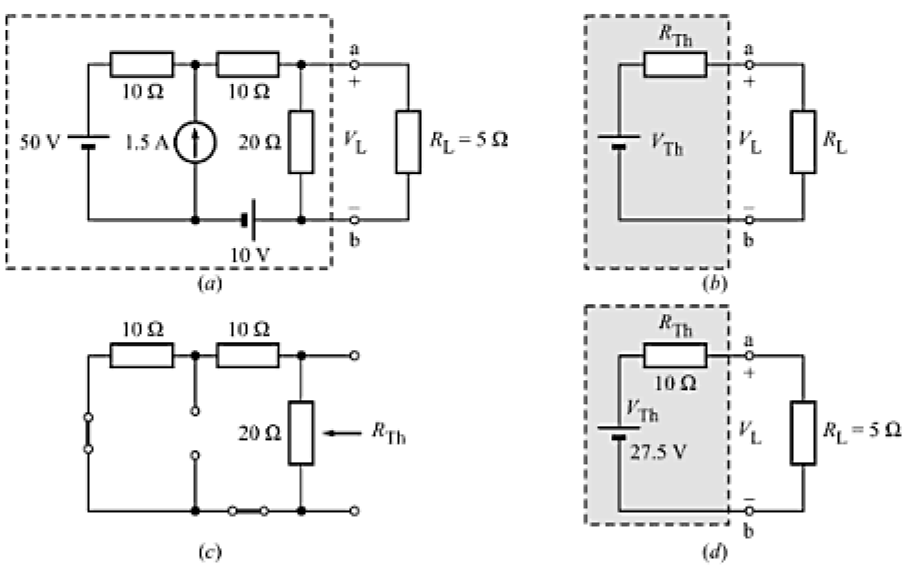


Fig. 4.8

Solution We replace the circuit in the box with a simpler circuit shown in Fig. 4.8*b*. V_{Th} is the open-circuit voltage between terminals a and b with R_L removed. We use superposition principle to find the current in the 20-Ω resistor and then the open-circuit voltage. With 1.5-A current source turned OFF, the current (downward) through 20-Ω resistor due to the two voltage sources is

$$I_1 = \frac{50 - 10}{10 + 10 + 20} = 1.0 \text{ A}$$

The current (downward) due to the 1.5-A current source acting alone is

$$I_2 = 1.5 \times \frac{10}{10 + (10 + 20)} = 0.375 \text{ A}$$

Adding these two currents, we get the current with all sources ON as $I = 1.375$ A, and the voltage across 20-Ω resistor, which is same as the open-circuit voltage or Thevenin's voltage, as

$$V_{Th} = 1.375 \times 20 = 27.5 \text{ V with + at the top.}$$

The polarity is important because we require that the two circuits in the box in Fig. 4.8*a* and *b* be fully equivalent as seen by the load.

To compute R_{Th}, we turn OFF all three sources within the box, as shown in Fig. 4.8*c*, and get

$$R_{Th} = 20\ \Omega \parallel (10\ \Omega + 10\ \Omega) = 10\ \Omega$$

Thus, as far as R_L is concerned, the circuit of Fig. 4.8*a* reduces to the equivalent circuit of Fig. 4.8*d*. Using voltage divider, we can now easily determine the voltage across R_L,

$$V_L = 27.5 \times \frac{5}{5 + 10} = \mathbf{9.17\ V}$$

EXAMPLE 4.6

Let us again consider our ***benchmark example*** (Fig. 3.24*a*) to determine voltage across 3-Ω resistor by applying Thevenin's theorem.

Solution We treat the 3-Ω resistor as load and enclose the remaining circuit within a box (Fig. 4.9*a*). Thevenin voltage V_{Th} is the open-circuit voltage between terminals *a* and *b* (with R_L removed). This can be calculated by source transformation. The 4-A current source in parallel with 1-Ω resistance is transformed into a 4-V voltage source in series

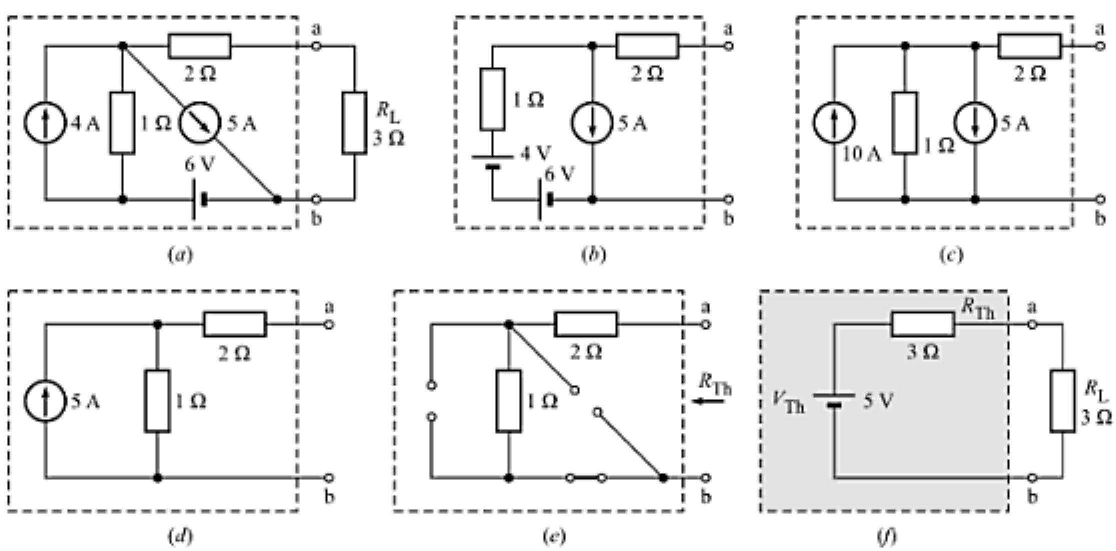


Fig. 4.9

with 1-Ω resistance (Fig. 4.9*b*). Combining 4-V and 6-V voltage sources and then converting it into current source, we get the circuit of Fig. 4.9*c*. Combining the two current-sources, we get the circuit of Fig. 4.9*d*. From this, $V_{Th} = 5 \times 1 = 5$ V, with the + at the top.

To compute R_{Th}, we turn OFF all the sources in the circuit within box and get the circuit of Fig. 4.9*e*. Thus, $R_{Th} = 3\ \Omega$. Thevenin's equivalent circuit is shown in Fig. 4.9*f*. Using voltage divider, the voltage across the load is given as

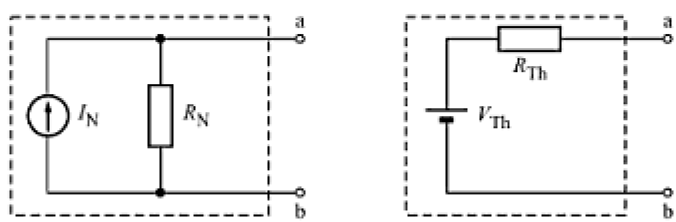
$$V_L = 5 \times \frac{3}{3+3} = \mathbf{2.5\ V}$$

Note that we got the same result in Example 4.3.

4.4 NORTON'S THEOREM

An American engineer named E.L. Norton came up with an equivalent circuit similar to Thevenin's equivalent circuit. He stated that *a two terminal linear network containing independent voltage and current sources may be replaced by* ***an equivalent current source I_N*** *in parallel with* ***a resistance R_N***. The procedure for determining the Norton's equivalent circuit is as follows.

1. Short-circuit the two terminals of the network and determine the current through this short-circuit. This is Norton's current I_N.
2. Turn OFF all the sources in the network and determine the resistance between the two terminals. This is Norton's resistance R_N. However, this is the same resistance R_{Th} that we found in Thevenin's equivalent. Therefore, we may say,
$$R_N = R_{Th} = Req \text{ (say)} \tag{4.2}$$
3. Norton's equivalent circuit is current source I_N in parallel with resistance R_N, as shown in Fig. 4.10*a*.



(*a*) Norton's equivalent circuit (*b*) Thevenin's equivalent circuit

Fig. 4.10

Relationship between Thevenin's and Norton's Equivalent Circuits

If two circuits are equivalent to the same circuit, they must be equivalent to each other. Thus, the two circuits of Fig. 4.10 are equivalent. The open-circuit voltage between terminals *a* and *b* in the circuit of Fig. 4.10*a* is obviously $I_N R_N$. This must be same as V_{Th} of Fig. 4.10*b*. That is,

$$V_{Th} = I_N R_N \tag{4.3}$$

This equation is very useful. It can be applied in the laboratory to find the ***output impedance*** of a source. In practice, it may not be possible to get inside a circuit and turn OFF internal sources. But we can measure the open-circuit output voltage and the current that would flow when the output terminals are shorted. From these values, the output resistance can be computed,

$$R_o = \frac{V_{oc}}{I_{sc}} = \frac{V_{Th}}{I_N} \tag{4.4}$$

EXAMPLE 4.7

Obtain Norton's equivalent circuit with respect to the terminals A-B for the circuit of Fig. 4.11*a* and then determine the value of current that would flow through a load resistor of 5 Ω if it were connected across terminals AB.

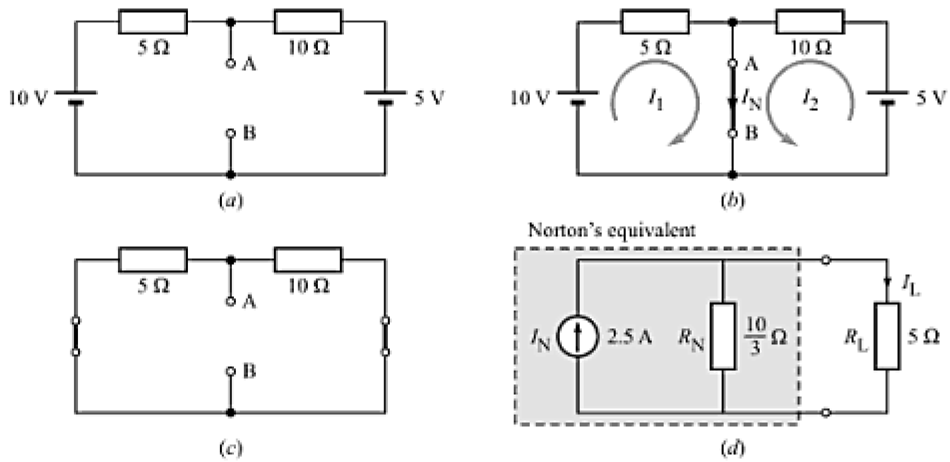


Fig. 4.11

Solution When terminals A-B are shorted (Fig.4.11*b*), the current I_N is given as

$$I_N = I_1 + I_2 = \frac{10}{5} + \frac{5}{10} = \mathbf{2.5\ A}$$

Turning OFF the sources (Fig.4.11*c*), the two resistances of 5 Ω and 10 Ω appear in parallel and the equivalent resistance is given as

$$R_N = \frac{5 \times 10}{5 + 10} = \frac{10}{3}\ \Omega$$

Hence, the Norton's equivalent of the given circuit is as shown within dotted box in Fig. 4.11*d*. Using current divider, the current through 5-Ω resistance connected across A-B is given as

$$I_L = 2.5 \times \frac{(10/3)}{(10/3) + 5} = \mathbf{1\ A}$$

4.5 MAXIMUM POWER TRANSFER THEOREM

This theorem states that *maximum power is absorbed from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load.*

Proof of the Theorem As per Thevenin's theorem, a two-terminal network can be converted into a single voltage source V_{Th} in series with resistance R_{Th} (Fig.4.11). Let us find such a value of R_L that consumes maximum power from the network. The power consumed by the load resistance is given as

$$P = I^2 R_L \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2} \tag{4.5}$$

Power P becomes maximum when differential of P with respect to R_L reduces to zero. That is,

$$\frac{dP}{dR_L} = \frac{(R_{Th} + R_L)^2 \times V_{Th}^2 - V_{Th}^2 R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \quad \Rightarrow \quad R_L = R_{Th} \tag{4.6}$$

Impedance Match When the load resistance is made equal to the output impedance of the circuit, we say the *impedance match* has been done.

Importance of Maximum Power Transfer Often we deal with small amount of power in electronics. We wish to make full use of the power that is available. Obtaining the maximum power out of a circuit is then important. For example, we connect a 'rabbit ears' antenna to the TV set to receive power from radio waves originating at a transmitter miles away. The antenna does not collect much power. So, the TV receiver is designed to make maximum use of the power provided by the antenna.

Optimisation In practice, we always have some constraints. When a best possible design is made under the constraints, we say the design is the *optimum*. Thus, the TV receiver input is *optimized* when its input impedance is matched to the output impedance of the antenna because this gives maximum power to the receiver.

Available Power The maximum power that can be extracted from a circuit is called the *available power*, P_{avl}, and is given by

$$P_{avl} = P\,(\text{when } R_L = R_{Th}) = I^2{}_1 R_L$$

$$= \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L = \left[\frac{V_{Th}}{R_{Th} + R_{Th}}\right]^2 \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}} \tag{4.7}$$

Note that a circuit or a source with low output impedance can supply large amount of power to an external load.

EXAMPLE 4.8

The open-circuit voltage of a standard car-battery is 12.6 V, and the short-circuit current is approximately 300 A. What is the available power from the battery?

Solution From Eq. 4.4, we determine the output impedance of the battery,

$$R_o = \frac{V_{oc}}{I_{sc}} = \frac{12.6}{300} = 0.042\ \Omega$$

Therefore, the available power from Eq. 4.7 is

$$P_{avl} = \frac{V_{Th}^2}{4R_{Th}} = \frac{V_{oc}^2}{4R_o} = \frac{(12.6)^2}{4 \times 0.042} = \mathbf{945\ W}$$

EXAMPLE 4.9

A stereo sound system is operated from a battery, which is made from 8 dry cells connected in series. Each cell has an emf of 1.5 V and an internal resistance of 0.75 Ω. How much is the available power from this battery?

Solution The open-circuit voltage of the battery is $V_{oc} = 8 \times 1.5 = 12$ V, and output impedance of the battery is $R_o = 8 \times 0.75 = 6\ \Omega$. Therefore, the available power is

$$P_{avi} = \frac{V_{oc}^2}{4R_o} = \frac{(12)^2}{4 \times 6} = \mathbf{6W}$$

Comment Compare the available power in above two Examples. Though, the two sources have almost the same emf, but the car-battery can supply much larger power.

EXAMPLE 4.10

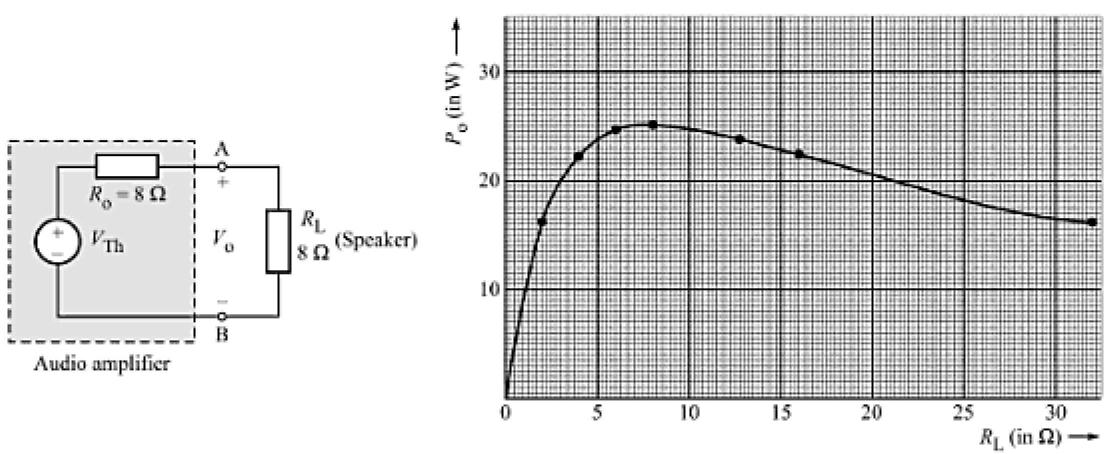
A sound system is designed to give 25 watts into an 8-Ω speaker. (*a*) What would be the voltage provided by this amplifier to the speaker? (*b*) What power will be delivered to the load, if (*i*) the load is a short-circuit, (*ii*) the load is an open-circuit? (*c*) If different speakers of load resistance 2 Ω, 4 Ω, 6 Ω, 8 Ω, 16 Ω and 32 Ω are used in the system, calculate the power delivered to the speaker in each case, and draw a curve showing the variation of the output power P_o with load resistance R_L.

Solution

(*a*) We assume that the speaker is represented by 8-Ω resistance, and that the speaker and the amplifier output impedances are matched ($R_o = 8\ \Omega$). Across the output terminals A-B, the audio amplifier can be represented by Thevenin's equivalent as shown in Fig. 4.12*a*. Using Eq. 4.7, Thevenin's voltage is given by

$$25 = \frac{V_{Th}^2}{4 \times 8} \quad \Rightarrow \quad V_{Th} = 28.28 \text{ V}$$

The voltage V_o across the speaker will be half of this value, as the voltage V_{Th} divides equally between R_o and R_L. Thus, V_o = **14.14 V.**



(*a*) Representing the audio amplifier by Thevenin's equivalent.

(*b*) Maximum power is delivered to the load when $R_L = R_o$.

Fig. 4.12

(*b*) (*i*) If the load is a short-circuit ($R_L = 0$), the output voltage V_o would be zero, and hence the output power,

$$P_o = V_o I_L = 0 \times I_L = 0$$

(*ii*) If the load is an open-circuit ($R_L = \infty$), the load current I_L would be zero, and hence the output power,

$$P_o = V_o I_L = V_o \times 0 = \mathbf{0}$$

(*c*) Using Eq. 4.5, we calculate the output power P_o for different values of R_L as given below.

Speaker Resistance, R_L (in W)	0	2	4	6	8	16	32	∞
Output power, P_o (in W)	0	16	22.22	24.49	25	22.22	16	0

The graph showing the variation of the output power P_o with load resistance R_L is shown in Fig. 4.12*b*.

Note that, as expected, maximum power (25 W) is extracted from the audio amplifier when speaker resistance is 8 Ω.

4.6 MILLMAN'S THEOREM

This theorem is helpful in finding a single equivalent voltage source for a number of voltage sources connected in parallel. It states that *a number of parallel voltage sources $V_1, V_2, V_3 \ldots, V_n$ with internal resistances $R_1, R_2, R_3 \ldots, R_n$, respectively* (as shown in Fig. 4.13*a*), *can be replaced by a single voltage source V in series with equivalent resistance R* (as shown in Fig. 4.13*b*), *as follows.*

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm V_3 G_3 \ldots \pm V_n G_n}{G_1 + G_2 + G_3 \ldots + G_n} \tag{4.8}$$

and

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 \ldots + G_n} \tag{4.9}$$

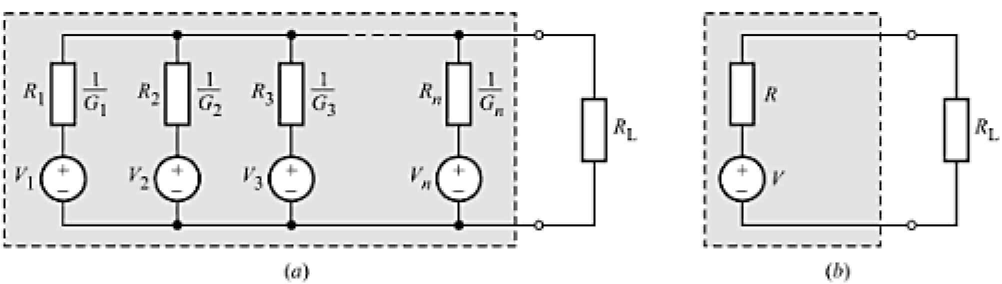


Fig. 4.13 *Millman's Theorem.*

4.7 RECIPROCITY THEOREM

This theorem states that *in a linear bilateral network, if a voltage source V in a branch* A *produces a current I in any other branch* B, *then the same voltage source V acting in the branch* B *would produce the same current I in branch* A. In other words, it simply states that V and I are interchangeable. The ratio V/I is known as the ***transfer resistance.*** The theorem is illustrated in following Example.

EXAMPLE 4.11

In the network of Fig. 4.14*a*, find the current in branch B due to the voltage source of 36 V in branch A. Now transfer the voltage source to branch B (as in Fig. 4.14*b*) and find the current in branch A. Is the reciprocity theorem established? Also, determine the transfer resistance from branch A to branch B.

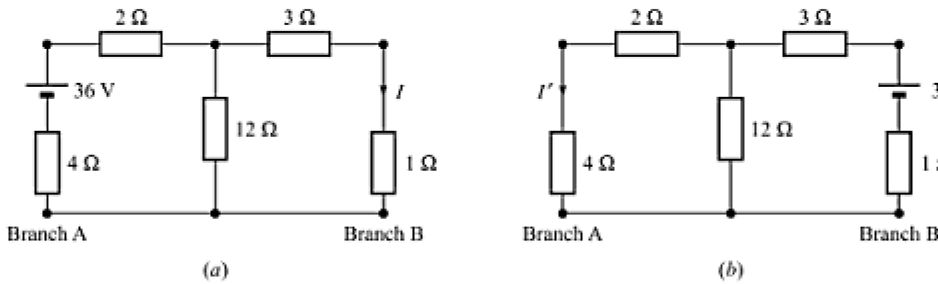


Fig. 4.14

Solution For the network of Fig. 4.14*a*, the equivalent resistance for the voltage source is

$$R_{eq} = 2 + [12 \parallel (3 + 1)] + 4 = 2 + 3 + 4 = 9\ \Omega$$

The current supplied by the voltage source = 36/9 = 4 A. Using current divider, the current I in branch B is given as

$$I = 4 \times \frac{12}{12 + 4} = \mathbf{3\ A}$$

Now, for the network of Fig. 4.14*b*, the equivalent resistance for the voltage source is

$$R_{eq} = 3 + [12 \parallel (2 + 4)] + 1 = 3 + 4 + 1 = 8\ \Omega$$

The current supplied by the voltage source = 36/8 = 4.5 A. Using current divider, the current I' in branch A is given as

$$I' = 4.5 \times \frac{12}{12 + 6} = \mathbf{3\ A}$$

We find that the two currents I and I' have the same value 3 A. Thus, the reciprocity theorem is established. The transfer resistance is given as

$$R_{tr} = \frac{V}{I} = \frac{36}{3} = \mathbf{12\ \Omega}$$

4.8 TELLEGEN'S THEOREM

It was given by D.H. Bernard Tellegen in 1952. It simply states that *the sum of instantaneous power delivered to each branch of a network is zero,*

$$\sum_{k=1}^{b} V_k\, I_k = 0 \tag{4.10}$$

The only requirement (or limitation) is that the voltages V_k satisfy KVL and that the currents I_k satisfy KCL.

The power of the theorem lies in the fact that the voltages V_k and the currents I_k are arbitrary except for the Kirchhoff constraints. The theorem is illustrated in Example 4.12.

EXAMPLE 4.12

Consider the network shown in Fig. 4.15. It has six branches. Arbitrary reference directions have been selected for all branch currents, and corresponding branch voltage is indicated with current entering the plus terminal. Select a set of branch voltages that satisfy KVL. Select two different sets of branch currents, each satisfying KCL. Show that both the sets of branch currents along with the set of branch voltages satisfy Eq. 4.10.

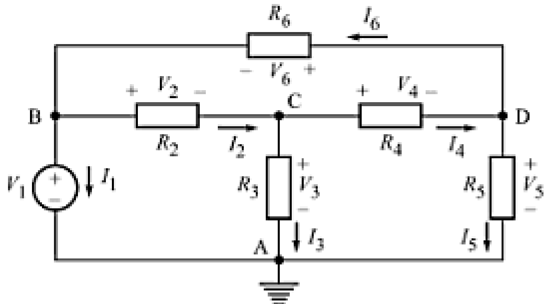


Fig. 4.15

Solution Let us arbitrarily choose $V_1 = 4$ V and $V_2 = 2$ V. Applying KVL around the loop ABCA, we see that we must have $V_3 = 2$ V. Similarly, around loop ACDA, if we choose $V_4 = 3$ V, then as per KVL we are required to let $V_5 = -1$ V. Now, around the loop BCDB, the values we have selected for V_2 and V_4 require that $V_6 = -5$ V. This set of branch voltages is listed in Table 4.1.

Next, we apply KCL successively to nodes B, C and D. At node B, we arbitrarily let $I_1 = 2$ A, and $I_2 = 2$ A. This selection requires that $I_6 = 4$ A. At node C, we arbitrarily select $I_3 = 4$ A, and then it is required that $I_4 = -2$ A. At node D, I_4 and I_6 have already been selected, so that as per KCL we must let $I_5 = -6$ A. This set of branch currents is also listed in Table 4.1.

Table 4.1 *Arbitrary set of branch voltages and branch currents.*

			Branches			
Items	*1*	*2*	*3*	*4*	*5*	*6*
V_k (in V)	4	2	2	3	–1	–5
I_k (in A)	2	2	4	–2	–6	4
$V_k I_k$ (in W)	8	4	8	–6	6	–20
I_k' (in A)	–1	3	2	1	–1	2
$V_k I_k'$ (in W)	–4	6	4	3	1	–10

For each branch, we multiply V_k and I_k, and put the result in the third row of Table 4.1. Adding all the entries in this row, we get

$$\sum_{k=1}^{6} V_k\, I_k = 8 + 4 + 8 - 6 + 6 - 20 = 0$$

This illustrates the Tellegen's theorem. To confirm that Eq. 4.10 is always satisfied, let us take another set of arbitrary currents I_k', satisfying KCL at each node, as indicated in 4^{th} row of Table 4.1. We again find for each branch the product $V_k I_k'$ and put the products in 5^{th} row. Summing up all the entries in the 5^{th} row, we get

$$\sum_{k=1}^{6} V_k I_k' = -4 + 6 + 4 + 3 + 1 - 10 = 0$$

***Proof of Tellegen's Theorem*:** We will again consider the circuit shown in Fig. 4.15 and note that the things that we have done for this network will lead to the same results in general. We will now make use of node-to-datum voltages and currents with double subscript to indicate direction. For example, for the branch 2, we will write

$$V_2 I_2 = (V_B - V_C)I_{BC}$$

Summing a similar product for each of the six branches, we get

$$\sum_{k=1}^{6} V_k I_k = V_B I_{AB} + (V_B - V_C)I_{BC} + V_C I_{CA} + (V_C - V_D)I_{CD} + V_D I_{DA} + (V_D - V_B) I_{DB}$$

This equation is next rearranged by factoring with respect to the node-to-datum voltages,

$$\sum_{k=1}^{6} V_k I_k = V_B (I_{AB} + I_{BC} - I_{DB}) + V_C (-I_{BC} + I_{CA} + I_{CD}) + V_D (-I_{CD} + I_{DA} + I_{DB})$$

$$= V_B \text{ (KCL at node B)} + V_C \text{ (KCL at node C)} + V_D \text{ (KCL at node D)}$$

Each product term in above equation vanishes because each KCL summation is equal to zero. Although, we have shown the validity of Tellegen's theorem for a specific example, the procedure used is identical to that used for a general network.

Note that the theorem depends only on the two Kirchhoff's laws. Therefore, it is applicable to a very general class of networks composed of elements that are linear or nonlinear, passive or active, time-invariant or time-varying. This generality is one of the reasons that makes the Tellegen's theorem a powerful tool.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 4.13

Using Thevenin's theorem, calculate the current I_3 in the circuit of Fig. 3.47.

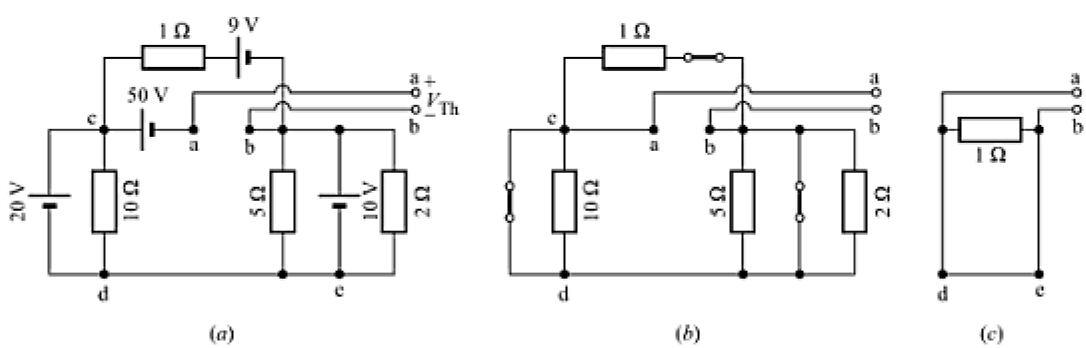


Fig. 4.16

Solution The current I_3 flows in 20-Ω resistance. Hence, we treat this resistance as the load resistance and remove it (Fig. 4.16*a*). To determine, V_{Th}, we write KVL equation for the loop bacdeb,

$$V_{Th} + 50 - 20 + 10 = 0 \quad \Rightarrow \quad V_{Th} = -40 \text{ V}$$

To determine R_{Th}, we short circuit all the voltage sources (Fig. 4.16*b*). Its equivalent circuit is given in Fig. 4.16*c*. Obviously, $R_{Th} = 0\ \Omega$. Hence,

$$I_3 = \frac{V_{Th}}{R_L + R_{Th}} = \frac{-40}{20+0} = \mathbf{-2\ A}$$

EXAMPLE 4.14

Using Thevenin's Theorem, determine the current I_2 in the circuit of Fig. 3.47.

Solution The current I_2 flows in 1-Ω resistance. Hence, we treat this resistance as the load resistance and remove it (Fig. 4.17*a*). To determine, V_{Th}, we write KVL equation for the loop bacdefb,

$$V_{Th} - 20 + 10 + 9 = 0 \quad \Rightarrow \quad V_{Th} = 1\text{V}$$

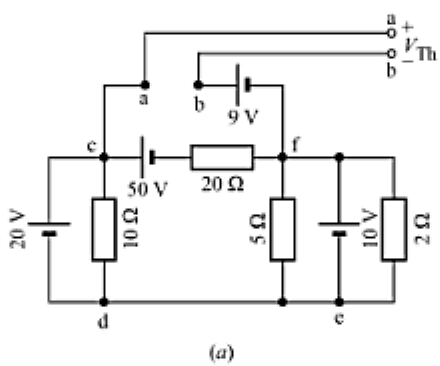


Fig. 4.17

To determine R_{Th}, we short circuit all the voltage sources (Fig. 4.17*b*). Its equivalent circuit is given in Fig. 4.17*c*. Clearly, $R_{Th} = 0\ \Omega$. Hence,

$$I_2 = \frac{V_{Th}}{R_L + R_{Th}} = \frac{1}{1+0} = \mathbf{1\ A}$$

EXAMPLE 4.15

Determine the current through the resistance $R = 0.5\ \Omega$ in the circuit shown in Fig. 4.18, using superposition theorem.

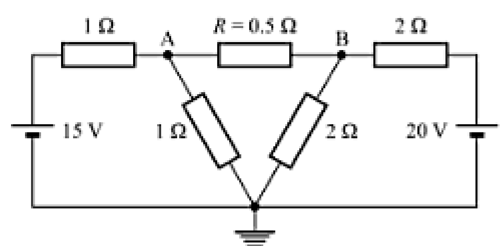


Fig. 4.18

Solution Let us first find the response due to 15-V source, by turning OFF 20-V source. The circuit reduces to that shown in Fig. 4.19*a*. The 2-Ω and 2-Ω resistances come in parallel and the circuit reduces to that shown in Fig. 4.19*b*. The circuit is further simplified to that shown in Fig. 4.19*c*. The current I is given as

$$I = \frac{15}{1+0.6} = 9.375 \text{ A}$$

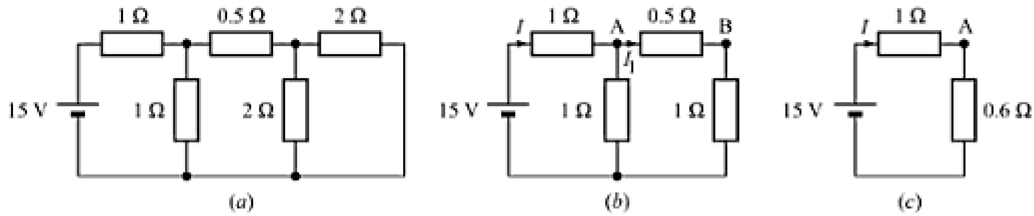


Fig. 4.19

From Fig. 4.19*b*, we get the current I_1 through *R* due to 15-V source as

$$I_1 = I \times \frac{1}{1 + (0.5 + 1)} = 9.375 \times \frac{1}{2.5} = 3.75 \text{ (from A to B)}$$

Now, consider 20-V source alone (by turning OFF 15-V source). The circuit reduces to that shown in Fig. 4.20*a*. It can be reduced to that shown in Fig. 4.20*b*, and then to that shown in Fig. 4.20*c*. From this circuit, we get

$$I' = \frac{20}{(2/3) + 2} = 7.5 \text{ A}$$

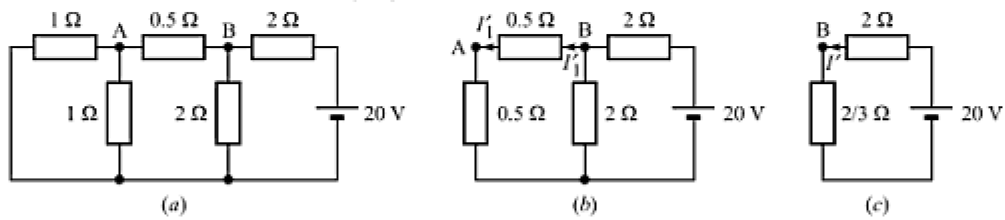


Fig. 4.20

From Fig. 4.20*b*, we get the current I'_1 through *R* due to 20-V source as

$$I'_1 = I' \times \frac{2}{2 + (0.5 + 0.5)} = 7.5 \times \frac{2}{3} = 5 \text{ A (from B to A)}$$

Hence, by superposition theorem, the net current through *R* (from A to B) is

$$I_{net} = I_1 - I'_1 = 3.75 - 5 = -\mathbf{1.25 A} \text{ (from A to B)} = \mathbf{1.25 A} \text{ (from B to A)}$$

EXAMPLE 4.16

Find the voltage across the 5-Ω resistance in the network shown in Fig.4.21, using Thevenin's theorem.

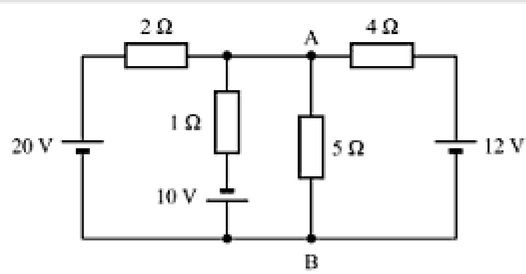


Fig. 4.21

Solution The first step is to remove the 5-Ω resistance (Fig.4.22*a*), and to find the open-circuit voltage V_{oc} across A-B. Using node-voltage analysis, for node C we can write

$$\frac{V_{oc}-20}{2}+\frac{V_{oc}+10}{1}+\frac{V_{oc}-12}{4}=10 \quad \Rightarrow \quad V_{oc}=1.714\text{V}$$

Thus, $V_{Th} = V_{oc} = 1.714$ V.

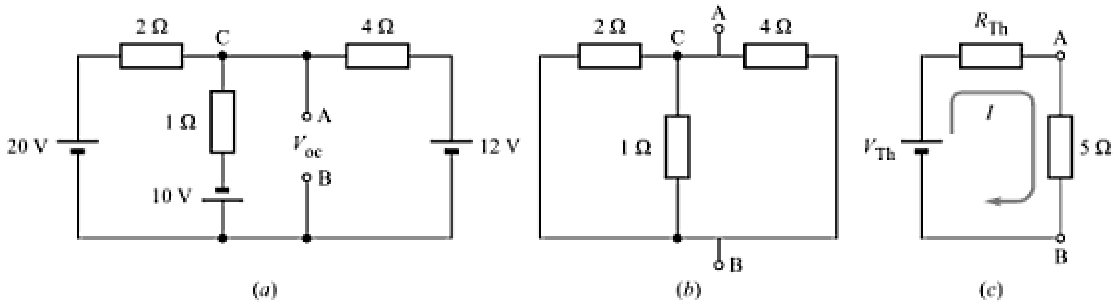


Fig. 4.22

Next, to determine R_{Th} we turn OFF the sources (Fig. 4.22*b*),

$$R_{Th} = 2\ \Omega \,||\, 1\ \Omega \,||\, 4\ \Omega = 0.571\ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 4.22*c*. Therefore, using voltage divider, the voltage across 5-Ω resistance is

$$V_{AB} = V_{TH} \times \frac{5}{5+0.571} = 1.714 \times \frac{5}{5.571} = \mathbf{1.54\ V}$$

EXAMPLE 4.17

Draw Norton's equivalent circuit across terminals A-B in the circuit shown in Fig. 4.23, and hence find the current that would flow through a 2-Ω resistance when connected across the terminals A-B.

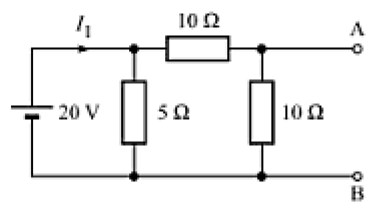


Fig. 4.23

Solution We short-circuit the terminals A-B, and then determine the current I_{sc} through this short (Fig. 4.24*a*). Due to the short-circuit, no current would flow through 10-Ω resistance connected across the short. Hence, this resistance can be removed from the circuit (Fig. 4.24*b*). We find that the 10-Ω resistance and 5-Ω resistance are in parallel, and both are in parallel with 20-V source. Hence, the current through 10-Ω resistance is

$$I_{sc} = \frac{20}{10} = 2\text{A}$$

Thus, $I_N = I_{sc} = 2$ A.

To find R_N, we turn OFF the voltage source (Fig.4.24*c*). The circuit reduces to that shown in Fig. 4.24*d*. Thus,

$$R_N = 10\ \Omega \,||\, 10\ \Omega = 5\ \Omega$$

Norton's equivalent is drawn in Fig. 4.24*e*, with $I_N = 2$ A and $R_N = 5\ \Omega$. The 2-Ω resistance is now connected across A-B, and the current I through this resistance is found by using current-divider,

$$I = I_N \times \frac{R_N}{R_N+2} = 2 \times \frac{5}{5+2} = 1.43\text{A}$$

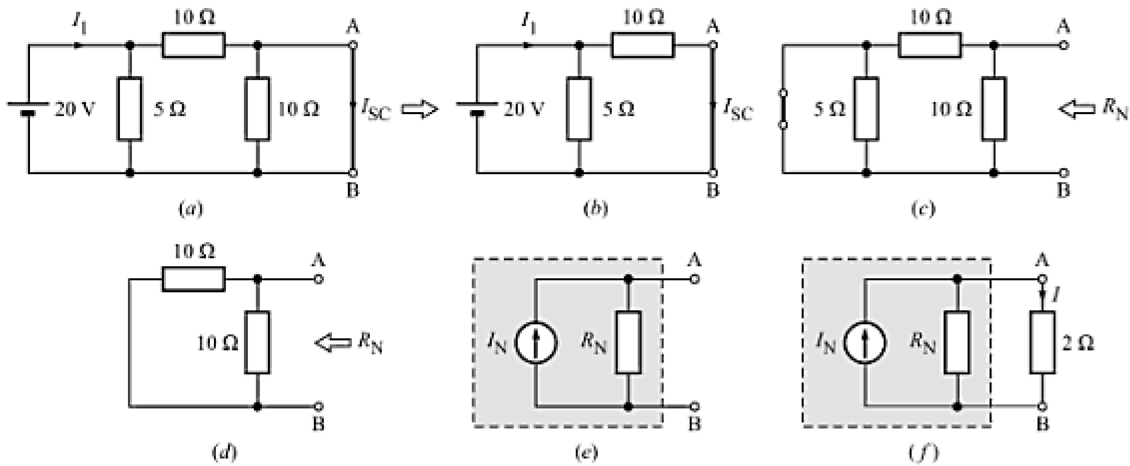


Fig. 4.24

EXAMPLE 4.18

Using Norton's theorem, determine the power consumed by 2-W resistor R_L in the network shown in Fig. 4.25.

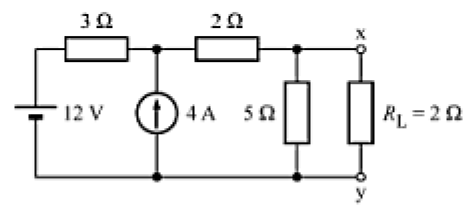


Fig. 4.25

Solution First step is to remove the load resistor R_L and determine Norton's equivalent across terminals x-y. To determine I_N, we short circuit terminals x-y (Fig. 4.26*a*) and find the current I_{sc} through this short-circuit. With x-y shorted, no current would flow through 5-Ω resistance; and hence this resistance can be removed from the circuit (Fig. 4.26*b*). We now apply node-voltage analysis, to determine the voltage at node 1,

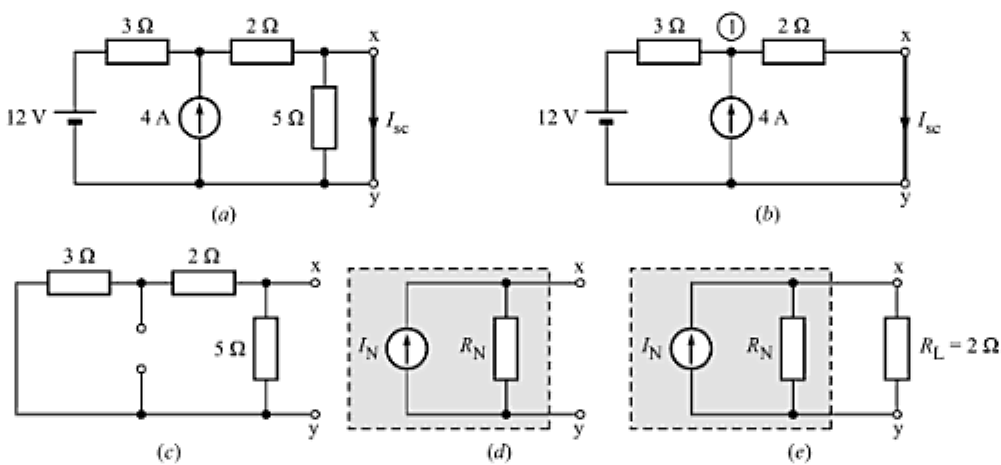


Fig. 4.26

$$\frac{V_1 - 12}{3} + \frac{V_1}{2} = 4 \quad \Rightarrow \quad V_1 = 9.6 \text{ V} \qquad \therefore \quad I_N = I_{SC}\,\frac{V_1}{2} = \frac{9.6}{2} = 4.8 \text{ A}$$

To determine R_N, we turn OFF the sources (Fig. 4.26*c*),

$$R_N = (3 + 2) \parallel 5 = 2.5\ \Omega$$

Norton's equivalent is shown in Fig. 4.26*d*. We now connect the load R_L across *x-y*, and get current I through this load using current divider,

$$I = I_N \times \frac{R_N}{R_N + R_L} = 4.8 \times \frac{2.5}{2.5 + 2} = 2.67\text{A}$$

$$\therefore \qquad P = I^2 R_L = (2.67)^2 \times 2 = \mathbf{14.257\ W}$$

EXAMPLE 4.19

Again consider the circuit shown in Fig. 4.8*a* solved in Example 4.5, and determine the voltage across the load resistance $R_L = 5\ \Omega$, using Norton's theorem.

Solution To determine the Norton's equivalent, we first remove the load from the given circuit and short-circuit the terminals a-b (Fig. 4.27*a*). To find current I_N, we first transform the 50-V voltage source to its equivalent current source (Fig. 4.27*b*), combine the two current sources of 5 A and 1.5 A, and then transform the resulting current source of 6.5 A back to voltage source (Fig. 4.27*c*). The short-circuit effectively removes the 20-Ω resistor from the circuit, so that

$$I_N = \frac{65 - 10}{10 + 10} = 2.75 \text{ A}$$

Norton's resistance R_N is same as Thevenin's resistance R_{Th} determined in Example 4.5 as 10 Ω. Hence, $R_N = 10\ \Omega$. Norton's equivalent with R_L connected across a-b is shown in Fig. 4.27*d*, from which we get

$$I_L = I_N \times \frac{R_N}{R_N + R_L} = 2.75 \times \frac{10}{10 + 5} = 1.833 \text{ A}$$

$$V_L = I_L R_L = 1.833 \times 5 = \mathbf{9.17\ V}$$

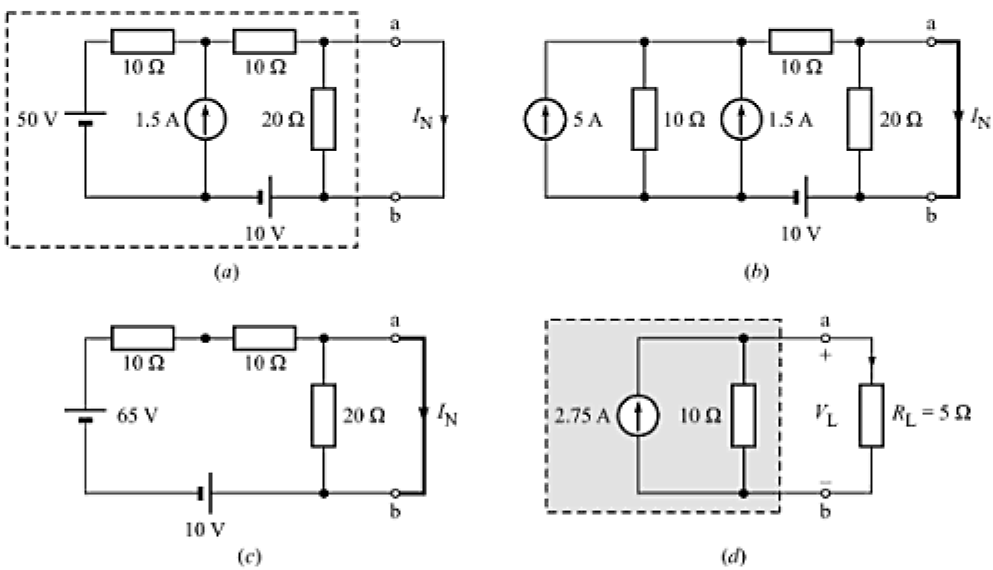


Fig. 4.27

EXAMPLE 4.20

A voltmeter is connected across the 500-kΩ resistor in the circuit shown in Fig. 4.28*a*. Determine the reading of the voltmeter, if (*a*) it is assumed to be ideal, (*b*) it has an input resistance of 10 MΩ.

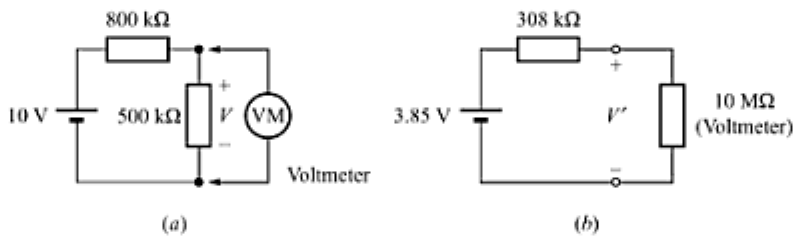


Fig. 4.28

Solution

(*a*) If the voltmeter is ideal, with infinite input impedance, it does not *load* the circuit at all. The reading of the voltmeter would be

$$V = (10\text{V}) \times \frac{500\,\text{k}\Omega}{500\,\text{k}\Omega + 800\,\text{k}\Omega} = \mathbf{3.85\ V}$$

(*b*) If the voltmeter has its own input resistance, it would load the circuit and would change the voltage to be measured. To determine the reading of the voltmeter in such a case, Thevenin's theorem is very helpful. We first find the Thevenin's equivalent across the terminal where we are going to connect the voltmeter. Thevenin's voltage is the open circuit voltage, that is, $V_{\text{Th}} = 3.85$ V, and Thevenin's resistance is

$$R_{\text{Th}} = 800\ \text{k}\Omega \parallel 500\ \text{k}\Omega = 308\ \text{k}\Omega$$

Thevenin's equivalent is drawn in Fig. 4.28*b*, from which the voltmeter reading is given as

$$V' = (3.85\ \text{V}) \times \frac{10\,\text{M}\Omega}{10\,\text{M}\Omega + 308\,\text{k}\Omega} = 3.85 \times 0.97 = \mathbf{3.73\ V}$$

The loaded voltage, which is what the voltmeter will indicate, proves to be **3 %** lower than the unloaded voltage of 3.85 V, which is what we are seeking to measure with the voltmeter. **Note** that, in practice, we can correct for loading error if we know the *output resistance* of the circuit and the *input resistance* of the voltmeter.

EXAMPLE 4.21

For the circuit shown in Fig. 4.29*a*, determine the following:

(*a*) Replace the circuit in the dotted box by Thevenin's equivalent circuit.

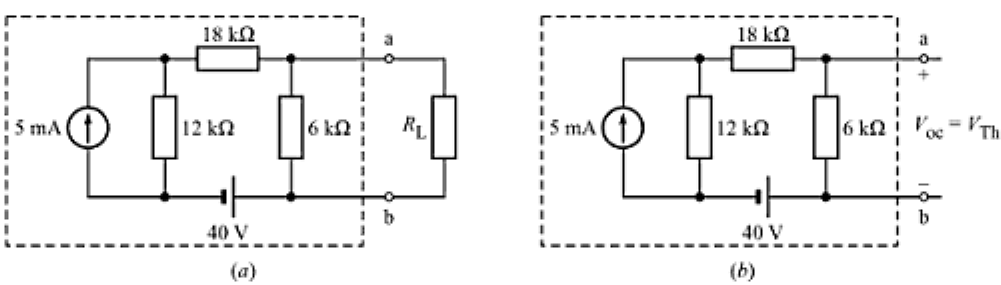


Fig. 4.29

(*b*) Find v_{ab} for $R_L = 3\ k\Omega$.
(*c*) What value of R_L receives maximum power from the circuit?
(*d*) What value of R_L makes the current in the 6-kΩ resistor to be 0.1 mA?

Solution

(*a*) First, we remove R_L and then find $V_{oc} = V_{Th}$ across a-b (Fig. 4.29*b*). We transform the 5-mA current source to voltage source (Fig. 4.30*a*). From the single-loop circuit, using voltage-divider, we get

$$V_{Th} = V_{oc} = (60 - 40) \times \frac{6\,k\Omega}{6\,k\Omega + (12 + 18)\,k\Omega} = \mathbf{3.33\ V}$$

To determine the Thevenin's resistance, we turn OFF all sources (Fig. 4.30*b*) in the given circuit, and get

$$R_{Th} = (6\ k\Omega) \parallel (12\ k\Omega + 18\ k\Omega) = \mathbf{5\ k\Omega}$$

The Thevenin's equivalent circuit is shown in Fig. 4.30*c*.

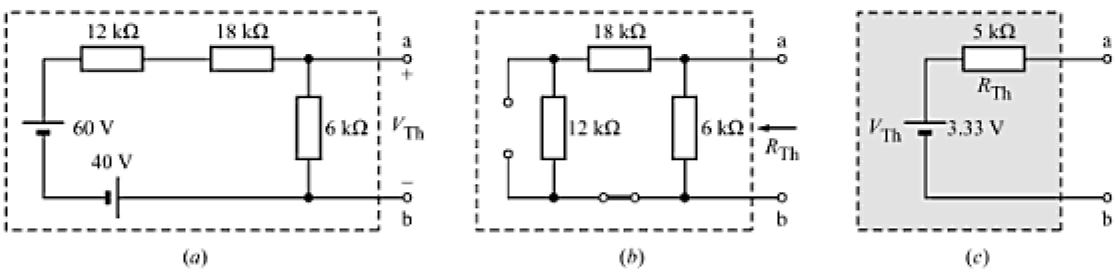


Fig. 4.30

(*b*) Connecting $R_L = 3\ k\Omega$ across a-b, voltage v_{ab} is obtained by using voltage divider as

$$V_{ab} = V_{Th} \times \frac{R_L}{R_L + R_{Th}} = 3.33 \times \frac{3}{3+5} = \mathbf{1.25\ V}$$

(*c*) According to maximum power transfer theorem, the required value of $R_L = R_{Th} = \mathbf{5\ k\Omega}$.

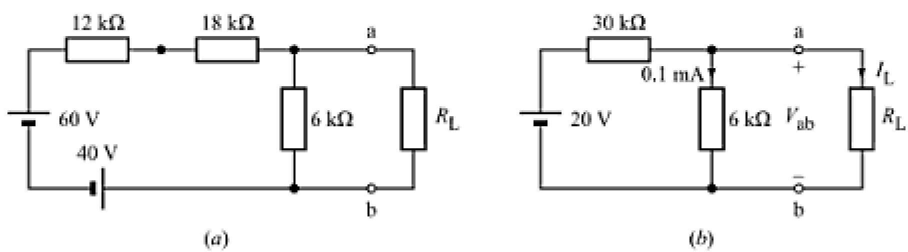


Fig. 4.31

(*d*) We use the equivalent circuit of Fig. 4.30*a*, and connect load resistance R_L (Fig. 4.31*a*). Its simplified circuit is shown in Fig. 4.31*b*. Since the current through 6-kΩ resistor is required to be 0.1 mA, the voltage

$$V_{ab} = IR = (0.1\ mA) \times (6\ k\Omega) = 0.6\ V$$

$$\therefore \quad V_{30k\Omega} = 20 - 0.6 = 19.4\ V \text{ and } I_{30k\,\Omega} = \frac{V_{30\,k\Omega}}{30\,k\Omega} = \frac{19.4\,V}{30\,k\Omega} = 0.647\ mA$$

$$\therefore \quad \text{Applying KCL: } I_L = 0.647 - 0.1 = 0.547\ mA \quad \Rightarrow \quad R_L = \frac{V_{ab}}{I_L} = \frac{0.6\,V}{0.547\,mA} = \mathbf{1.1\ k\Omega}$$

EXAMPLE 4.22

For the circuit shown in Fig. 4.32*a*, determine the following:

(*a*) Find the value of R to receive maximum power from the circuit.
(*b*) For the value of R in part (*a*), find the power supplied by the 12-V source.

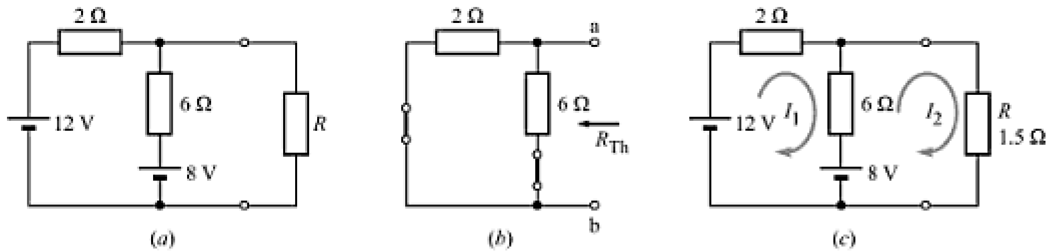


Fig. 4.32

Solution

(*a*) To receive maximum power, the value of resistance R should be equal to the output resistance of the remaining circuit, which is the same as the Thevenin's resistance. We remove R, turn OFF all the sources (Fig. 4.32*b*), and then determine R_{Th},

$$R_{Th} = 2\Omega \parallel 6\Omega = \mathbf{1.5\ \Omega}$$

Thus, when $R = R_{Th} = \mathbf{1.5\ \Omega}$, it would receive maximum power.

(*b*) We adopt mesh analysis to find current supplied by 12-V source. With $R = 1.5\ \Omega$, let us mark the two mesh currents, I_1 and I_2, as shown in Fig. 4.31*c*, and write the mesh equations by inspection, in matrix form,

$$\begin{bmatrix} 8 & -6 \\ -6 & 7.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \Rightarrow \quad I_1 = 3.25\ \text{A}$$

Therefore, the power supplied from the 12-V source is

$$P = VI = 12 \times 3.25 = \mathbf{39\ W}$$

EXAMPLE 4.23

For the circuit shown in Fig. 4.33*a*, find I_s such that the current in the 120-Ω resistor is zero.

Solution Applying superposition theorem, first we find current I_1 through 120-Ω resistor due to 12-V source working alone (Fig. 4.33*b*), and then current I_2 due to current source I_s working alone (Fig. 4.33*c*),

$$I_1 = \frac{12}{80+120} = 0.6\ \text{A and } I_2 = I_s \times \frac{80}{80+120} = 0.4 I_s$$

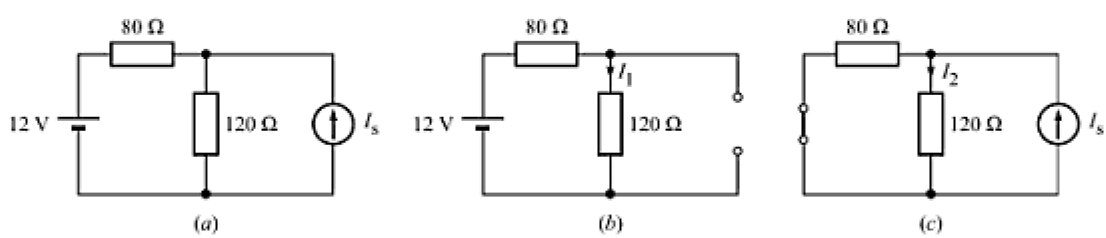


Fig. 4.33

Since the net current through 120-Ω resistor is required to be zero, hence we must have

$$I_1 + I_2 = 0 \text{ or } 0.06\text{ A} + 0.4 I_s = 0 \quad \Rightarrow \quad I_s = -\frac{0.06}{0.4} = -0.15\text{ A}$$

EXAMPLE 4.24

Obtain the Thevenin's equivalent of the circuit shown in Fig. 4.34*a*.

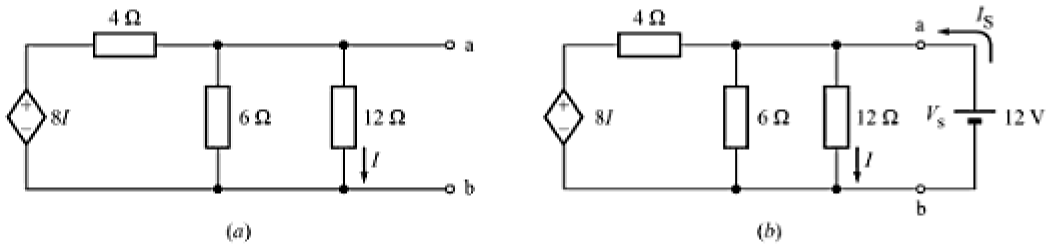


Fig. 4.34

Solution The circuit contains no independent source. So, the current I in 12-Ω resistor will be zero, and hence the dependent voltage source $8I$ will also be zero. Obviously, Thevenin's voltage V_{Th} = **0 V**. To determine R_{Th}, we apply a known voltage source (say, 12 V) across terminals a-b (Fig. 4.34*b*), and find current I_s supplied to the circuit.

$$I = \frac{12\text{ V}}{12\Omega} = 1\text{ A} \quad \Rightarrow \quad \text{Voltage of the dependent source} = 8I = 8\text{ V}$$

Applying KCL to node a,

$$I_S = \frac{12}{12} + \frac{12}{6} + \frac{12-8}{4} = 4\text{ A}; \qquad \therefore \quad R_{Th} = \frac{V_S}{I_S} = \frac{12}{4} = \mathbf{3\ \Omega}$$

SUMMARY

TERMS AND CONCEPTS

- Most circuit problems can be solved by applying Kirchhoff's Laws to produce simultaneous equations; the solution of these equations is often unnecessarily difficult.
- ***Superposition theorem*** states that the response (current or voltage) at any point in a linear network due to multiple sources can be determined by summing the effects of each source considered separately, while all other sources being turned OFF.
- ***Thevenin's theorem*** states that it is possible to simplify any linear circuit containing independent and dependent , voltage and current sources, no matter how complex, to an equivalent circuit with just a single voltage source and a series resistance connected to a load.
- Thevenin's theorem leads to an important concept of '***output impedance***' of a circuit.
- ***Norton's theorem*** states that a two terminal linear network containing independent and dependent, voltage and current sources can be replaced by ***an equivalent current source I_N*** in parallel with ***a resistance R_N.***
- ***Maximum power transfer theorem*** states that maximum power is absorbed from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load.
- When the load resistance is made equal to the output resistance of the circuit, we say that the ***impedance match*** has been done.
- ***Millman's theorem*** helps us in converting a number of practical parallel voltage sources into a single practical voltage source.

- ***Reciprocity theorem*** states that in a linear bilateral network, if a voltage source V in a branch A produces a current I in any other branch B, then the same voltage source V acting in the branch B would produce the same current I in branch A
- ***Tellegen's theorem*** states that *the sum of instantaneous power delivered to each branch of a network is zero.*

IMPORTANT FORMULAE

- Relationship between Thevenin's and Norton's equivalent:

$$V_{Th} = I_N R_N \quad \text{and} \quad R_{Th} = R_N$$

- The ***output resistance*** can be computed by $R_o = \dfrac{V_{oc}}{I_{sc}} = \dfrac{V_{Th}}{I_N}$
- For maximum power transfer, $R_L = R_{Th}$
- The maximum power that can be extracted from a circuit is given as

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

- ***Tellegen's theorem***: $\sum_{k=1}^{b} V_k I_k = 0$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The superposition theorem can be applied to any network, howsoever complex, containing any number of linear and nonlinear elements.	☐	☐	
2.	The defining voltage-current equation for an inductance is *linear.*	☐	☐	
3.	By 'turning OFF' a current source means 'open-circuiting' its terminals.	☐	☐	
4.	For a given network, the value of Thevenin's resistance R_{Th} is same as that of Norton's resistance R_N.	☐	☐	
5.	When a load is matched with the source, the efficiency of the source is 100 %.	☐	☐	
6.	Thevenin's theorem is especially useful in such applications as determining the load for an electronic circuit which will result in maximum average power delivery to the load.	☐	☐	
7.	Thevenin's and Norton's theorems are valid for a load containing any kind of elements, linear or nonlinear, time-varying or time-invariant.	☐	☐	
8.	Reciprocity theorem means that the interchange of an *ideal* voltage source in one branch and an *ideal* ammeter in another branch will not change the reading of the ammeter.	☐	☐	
9.	When the resistance of a voltmeter connected across two terminals of a circuit has the same value as the Thevenin's resistance, it gives the correct reading.	☐	☐	
10.	It is possible to directly derive Norton's equivalent from the Thevenin's equivalent of a circuit.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** True	**5.** False
6. True	**7.** True	**8.** True	**9.** False	**10.** True

REVIEW QUESTIONS

1. State superposition theorem and illustrate its use to solve a network.

2. Explain why superposition theorem is not applicable to a network containing nonlinear elements.

3. State and explain Thevenin's theorem, by means of a simple example.

4. What are the advantages of Thevenin's theorem ?

5. How will you convert Thevenin's equivalent circuit into Norton's equivalent circuit and vice-versa ?

6. State and explain the importance of the maximum power transfer theorem.

7. State Tellegen's theorem. Give some of the implications of this theorem.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly :

1. Thevenin's theorem is based on the idea of
(*a*) an equivalent current source
(*b*) an equivalent source of emf
(*c*) an equivalent power source
(*d*) an equivalent resistance

2. Norton's theorem is based on the idea of
(*a*) an equivalent current source
(*b*) an equivalent source of emf
(*c*) an equivalent power source
(*d*) an equivalent resistance

3. In a circuit, maximum power is transferred to the load when
(*a*) the total circuit resistance is equal to the load resistance
(*b*) the internal resistance of the source of emf is infinite
(*c*) the total series resistance is equal to the equivalent resistance of the parallel combination of resistances
(*d*) the internal resistance of the source is equal to the load resistance

4. The superposition theorem is used when a circuit contains
(*a*) a single voltage source
(*b*) a single current source
(*c*) a number of sources
(*d*) passive elements only

5. When a circuit has a large number of branches and a large number of unknown currents, the circuit can easily be solved by applying
(*a*) Kirchhoff's voltage law
(*b*) Kirchhoff's current law
(*c*) reciprocity theorem
(*d*) loop-current Method

6. The superposition theorem is essentially based on the concept of
(*a*) duality (*b*) reciprocity
(*c*) linearity (*d*) nonlinearity

7. While Thevenizing a circuit between two terminals, V_{Th} equals to
(*a*) the short-circuit terminal voltage
(*b*) the open-circuit terminal voltage
(*c*) the emf of the battery nearest to the terminals
(*d*) the net voltage available in the circuit

8. While calculating Thevenin's resistance R_{Th}, all the current sources in the circuit are
(*a*) treated in parallel with other voltage sources
(*b*) converted into equivalent voltage sources
(*c*) replaced by open-circuits
(*d*) replaced by short-circuits

ANSWERS

1. *b*	**2.** *a*	**3.** *d*	**4.** *c*	**5.** *d*	**6.** *c*	**7.** *b*	**8.** *c*

PROBLEMS

(A) SIMPLE PROBLEMS

1. Determine the current I_x through the 10-Ω resistor in Fig. 4.35, using superposition theorem. **[Ans.** – 1 A]

2. Determine the current through R = 3 Ω resistor (from A to B) in the circuit shown in Fig. 4.36, using superposition theorem. **[Ans.** –0.5 A]

3. Determine the current through 4-Ω resistor (from A to B) in the circuit shown in Fig. 4.37, using superposition theorem. **[Ans.** 0.516 A]

4. Find the current through 4-Ω resistor in the circuit shown in Fig. 4.38, using superposition theorem. **[Ans.** 2.916 A]

5. Find the current through 20-Ω resistor in the circuit shown in Fig. 4.39, using superposition theorem. **[Ans.** 0.714 A]

6. For the circuit shown in Fig. 4.40, determine the current in 50-Ω resistor with the reference direction shown, using the principle of superposition. **[Ans.** 0.0455 A]

7. Find the current I in 10-Ω resistor in the circuit shown in Fig. 4.41, using Thevenin's theorem. **[Ans.** –2 A]

8. Find Thevenin's equivalent at terminals a-b of the network shown in Fig. 4.42. **[Ans.** 100 V, 4 Ω]

9. By applying Thevenin's theorem, find the voltage across the resistance R in the circuit shown in Fig. 4.43. **[Ans.** 12.24 V]

10. Solve the above problem, by applying Norton's theorem. **[Ans.** 12.24 V]

11. Find the current in R_L in the circuit of Fig. 4.44 by using Thevenin's theorem, and then verify your results by using loop current method. **[Ans.** 0.625 A]

12. Solve Prob. 2, by using Thevenin's theorem. **[Ans.** –0.5 A]

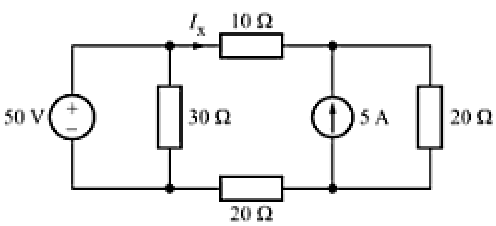


Fig. 4.35

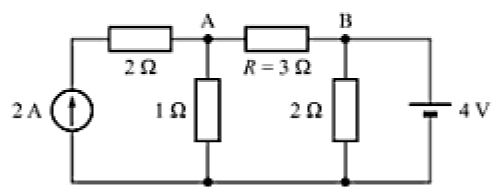


Fig. 4.36

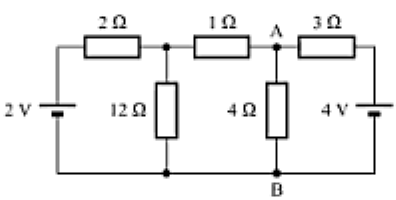


Fig. 4.37

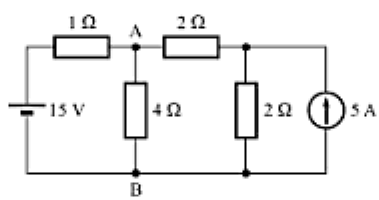


Fig. 4.38

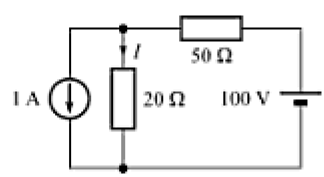


Fig. 4.39

13. Solve Prob. 3, by using Thevenin's theorem. [**Ans.** 0.516 A]

14. Find the current in the 3-Ω resistor of the circuit shown in Fig. 4.45, using Thevenin's theorem. [**Ans.** 5.143 A]

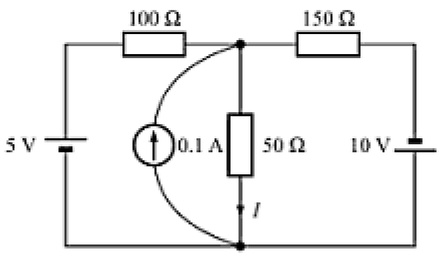


Fig. 4.40

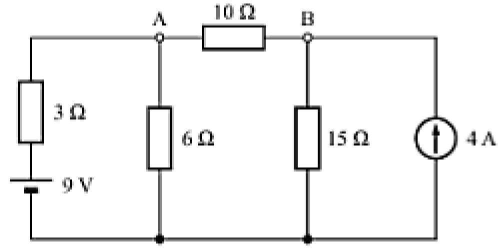


Fig. 4.41

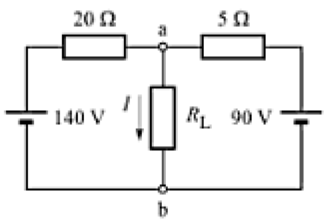


Fig. 4.42

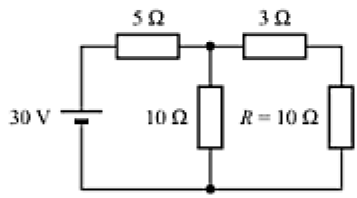


Fig. 4.43

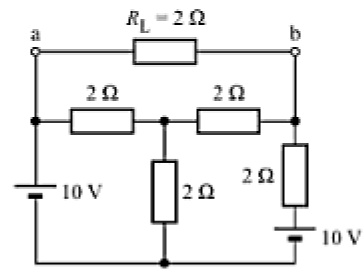


Fig. 4.44

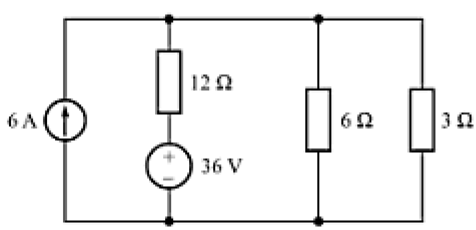


Fig. 4.45

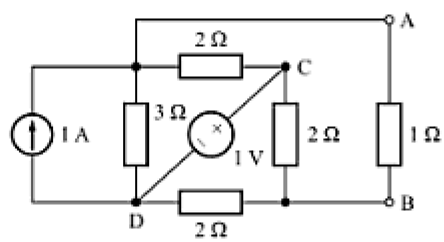


Fig. 4.46

(B) TRICKY PROBLEMS

15. In the circuit shown in Fig. 4.46, determine the current through 1-Ω resistor connected across A-B, using superposition theorem. Verify the result using Thevenin's theorem. [**Ans.** 406.25 mA]

16. Find Thevenin's equivalent circuit for a dc power supply that has a 30-V terminal voltage when delivering 400 mA and a 27-V terminal voltage when delivering 600 mA. [**Ans.** 36 V, 15 Ω]

17. Determine the current through the 3-Ω resistor in the circuit of Fig. 4.47, by using Thevenin's theorem. [**Ans.** 3 A]

18. Determine the current through 6-Ω resistor connected across A-B in the circuit of Fig. 4.48, by using Thevenin's theorem. [**Ans.** 1 A]

19. In the circuit shown in Fig. 4.49, determine (*a*) the value of *R* so that the load of 20 Ω draws maximum

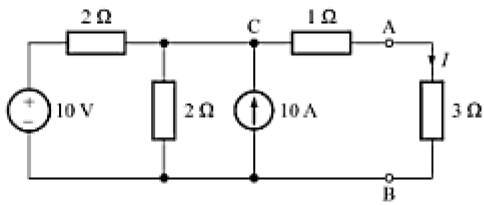


Fig. 4.47

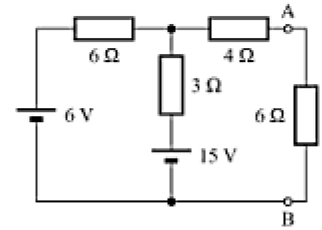


Fig. 4.48

power, and (*b*) the value of the maximum power drawn by the load. [**Ans.** (*a*) 30 Ω, (*b*) 180 W]

20. Using Thevenin's theorem, calculate the current that would flow in a 76-Ω resistor connected between terminals A and B of the circuit shown in Fig. 4.50. [**Ans.** 10 mA]

21. A Wheatstone bridge is shown in Fig. 4.51. The galvanometer connected across B-D has a resistance of 20 Ω. Using Thevenin's theorem, compute the current through this galvanometer.
[**Ans.** 5.12 mA]

22. For the circuit shown in Fig. 4.52, refer to Fig. 4.52. (*a*) Find the current in 3-Ω resistor. (*b*) What resistance, replacing the 3-Ω resistor, would draw one-half the current in part (*a*) ?
[**Ans.** (*a*) 0.667 A; (*b*) 8.4 Ω]

23. For the circuit shown in Fig. 4.53, find the value of *V* to make the current in the 5-Ω resistor to be zero. [**Ans.** 30 V]

24. A circuit, as shown in Fig.4.53*a*, has a variable load and ideal meters to monitor load voltage and current. The table in Fig. 4.54*b* shows partial results of a series of tests. Fill in the blanks in the table with the missing data.
[**Ans.** 0 V, 0.0333 A, 12.85 V, 0 A]

25. Find the current in the 9-Ω resistor of the circuit shown in Fig. 4.55, using Thevenin's theorem.
[**Ans.** 2 A]

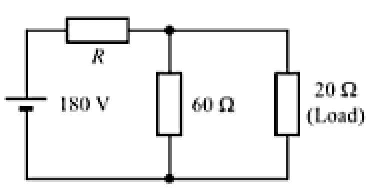


Fig. 4.49

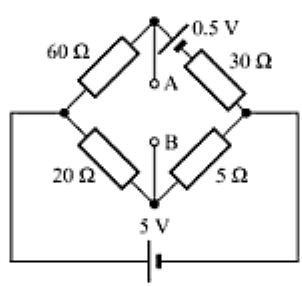


Fig. 4.50

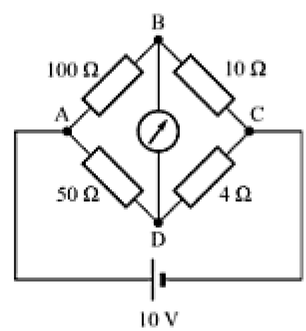


Fig. 4.51

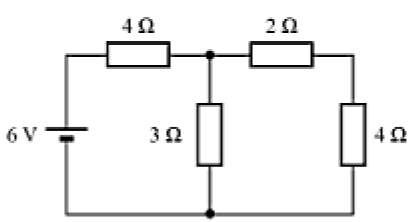


Fig. 4.52

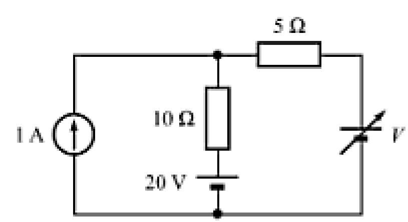
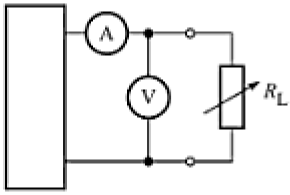


Fig. 4.53



(*a*)

V	I	R_L
	0.15 A	0
10 V		300 Ω
		∞

(*b*)

Fig. 4.54

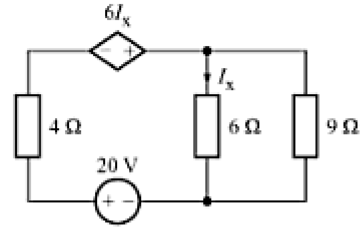


Fig. 4.55

(C) CHALLENGING PROBLEMS

26. Develop a Thevenin's equivalent circuit for the part of the circuit shown in the box in Fig. 4.56. Use this equivalent circuit to find current I, as shown.

[**Ans.** – 0.0625 A]

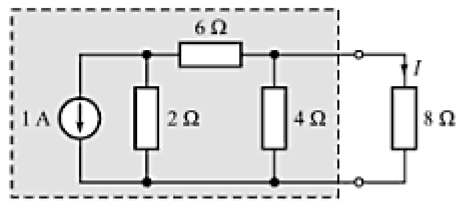


Fig. 4.56

27. A student is testing a circuit containing batteries and resistors. The output voltage is 6.26 V when measured with a good (assumed ideal) voltmeter, but 6.05 V when a 600-Ω resistor is connected across the output terminals. What current would result if the output terminals were short-circuited ?

[**Ans.** 0.301 A]

EXPERIMENTAL EXERCISE 4.1

SUPERPOSITION THEOREM

Objectives To verify superposition theorem.

Apparatus Two DC power supplies 12 V and 6 V, Three ammeters (MC type) 0-2 A, Three rheostats 100 W, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 4.57.

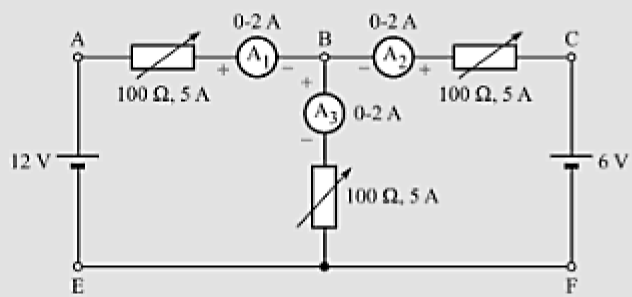


Fig. 4.57

Brief Theory The theorem of superposition states that in a linear network containing several sources (including dependent sources) the response (i.e., the current or voltage) at a point in the network equals the sum of the responses of each source working alone with all other sources made inoperative.

Procedure

1. Make connections as given in Fig. 4.57, keeping the rheostats at their maximum value.
2. Set the three rheostats such that the readings in the three ammeters are within their range.
3. Note the readings of the three ammeters.
4. Without changing the settings of rheostats, disconnect the 12-V source and short-circuit the point A to point E. Note the readings of the three ammeters.
5. Next, replace the 12-V source at its place. Disconnect the 6-V source and short-circuit the point C to F. Again, note the readings of the three ammeters.
6. Repeat steps 2 to 5 for four different settings of the rheostats.
7. Disconnect the circuit.

Observations and Calculations

S. No.	*With both sources*			*With 6-V source* (A)			*With 12-V source* (B)			A+B		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
1.												
2.												
3.												
4.												
5.												

Calculations Add algebraically the readings of the corresponding ammeters in step 4 (i.e., column A) and in step 5 (i.e., column B), and put the sum in the last column (A + B).

Results

1. The sums recorded as A + B in the last column is found almost the same as the readings of the three ammeters in the first column (corresponding to step 3).
2. The superposition theorem is, therefore, verified.

Precautions

1. Before connecting the dc supply, the zero reading of the ammeters should be checked.
2. The terminals of the rheostats should be connected properly.
3. While setting the rheostats, care should be taken that the currents recorded by the ammeters do not exceed 5 A, the current rating of the rheostats.
4. The direction of currents should be correctly identified by noting the + and – terminals of the ammeters.

Viva-Voce

1. Under what conditions is the superposition theorem applicable ?
Ans. : The elements of the network should be ***linear*** and ***bilateral***.

2. What do you mean by the term '*linear*' ?
Ans. : It means that the *V-I* characteristic of the element is a straight line.

3. Q. : What do you mean by '*bilateral*' ?
Ans. : An element which behaves in exactly same manner if connected in the reverse order is said to be bilateral.

4. **Q. :** Can the superposition theorem be applied to determine 'power' in an element of a circuit ?
Ans. : No, not directly, because, 'power' is not a linear function of current or voltage. However, we can first determine the current or the voltage using superposition theorem and then calculate the power

EXPERIMENTAL EXERCISE 4.2

THEVENIN'S THEOREM

Objectives To verify Thevenin's theorem.

Apparatus 220-V dc variable supply, One variac (0-270 V) 15 A, One ammeter (MC type) 0-5 A, One voltmeter (MC Type) 0-250 V, Four rheostats 100 W, 5 A.

Circuit Diagram The circuit diagrams are shown in Fig. 4.58.

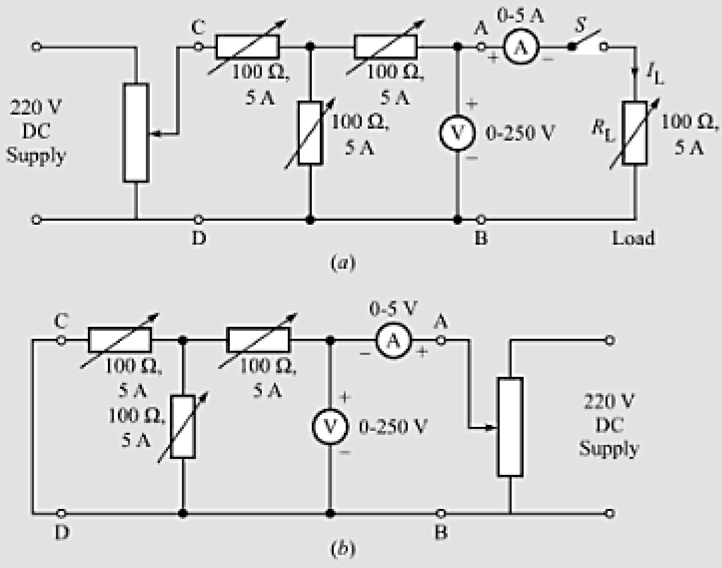


Fig. 4.58

Brief Theory Sometimes, our object of analyzing an electrical circuit is to determine the response (i.e., the current, or the voltage, or the power consumed) in a single resistance. In such a case, this resistance can be treated as a *load resistance*. The remainder of the network can be replaced by a simple *equivalent circuit containing a voltage source in series with a resistance*, using Thevenin's theorem. Thevenin's theorem specially becomes very useful and time-saving, if we are to find the response for different values of the load resistance. Thevenin's theorem may be stated as follows:

1. Any two terminals A-B of a network consisting of linear passive and active elements can be replaced by a simple equivalent circuit containing a voltage source V_{Th} in series with a resistance R_{Th}.
2. The Thevenin's equivalent voltage source V_{Th} is equal to the open-circuit voltage between the terminals A-B (i.e., under the condition when no external resistance is connected across A-B) caused by the active elements of the network.

3. The Thevenin's equivalent resistance R_{Th} is the equivalent resistance looking back into the network across the terminals A-B, when all its sources are made inactive.

According to Thevenin's theorem, the actual current I_L through a load resistance R_L connected across the terminals A-B in the original network is given as

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} \qquad (i)$$

Procedure

1. Make connections as given in Fig. 4.58*a*, keeping the rheostats at their maximum value. Keep the switch S closed and switch on the supply. Set the four rheostats such that the reading in the ammeter is within its range.
2. Note the readings I_2 and V_2 of the ammeter and voltmeter, respectively. The *actual* load current is same as I_2 (i.e., $I_{La} = I_2$). The load resistance R_L is given as

$$R_L = \frac{V_2}{I_2} \qquad (ii)$$

3. Open the switch *S*, and again take the reading V_3 of the voltmeter. This is the open-circuit voltage between terminals A-B and gives the Thevenin's equivalent voltage V_{Th}.
4. Without changing the settings of rheostats, connect the circuit as shown in Fig. 4.58*b*. Note the readings I_4 and V_4 of the ammeter and voltmeter, respectively. Thevenin's equivalent resistance R_{Th} is given as

$$R_{Th} = \frac{V_4}{I_4} \qquad (iii)$$

5. Calculate the current through the load, using Thevenin's theorem, as

$$I_{Lc} = \frac{V_{Th}}{R_L + R_{Th}} \qquad (iv)$$

6. For another setting of the rheostats, repeat steps 2 to 5.
7. Disconnect the circuit.

Observations and Calculations

S. No.	*Step 2*			*Step 3*		*Step 4*		
	V_2	I_2 (= I_{La})	R_L *Eq. (ii)*	V_3 (= V_{Th})	V_4	I_4	R_{th} *Eq. (iii)*	I_{Lc} *Eq. (iv)*
1.								
2.								

Results

1. From the Table, it is observed that the actual value of load current I_{La} in column 2 (under Step 2) tallies with the calculated value of the load current I_{Lc} in the last column.
2. The Thevenin's theorem is, therefore, verified.

Precautions

1. Before connecting the dc supply, the zero readings of the ammeter and voltmeter should be checked.
2. The terminals of the rheostats should be connected properly.
3. While setting the rheostats, care should be taken that the currents recorded by the ammeters do not exceed 5 A, the current rating of the rheostats.

Viva-Voce

1. How do you find the Thevenin's equivalent resistance ?
   Ans. : We first remove the load resistance from the terminals *A-B* and then make all the sources inactive. The resistance measured between terminals *A-B* is then the Thevenin's equivalent resistance.
2. What do you mean by making the sources inactive ?
   Ans. : It means that sources are replaced by their internal resistances. The voltage source is short-circuited and the current source is open-circuited.
3. Do you know another network theorem that serves the same purpose as Thevenin's theorem ?
   Ans. : Yes, it is called Norton's theorem.
4. What is the difference between the two ?
   Ans. : In Thevenin's theorem we find the equivalent voltage source, but in Norton's theorem we find the equivalent current source.

EXPERIMENTAL EXERCISE 4.3

MAXIMUM POWER TRANSFER THEOREM

Objectives To verify maximum power transfer theorem.

Apparatus 12-V supply, One ammeter (MC type) 0-2 A, Two voltmeters (MC Type) 0-15 V, Two rheostats 100 W, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 4.59.

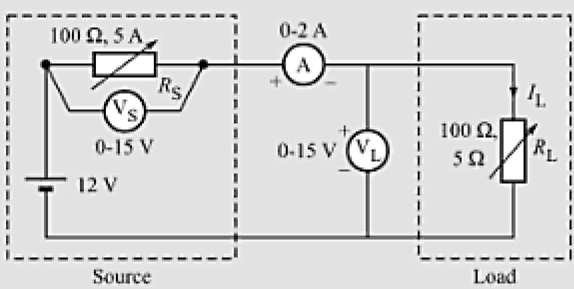


Fig. 4.59

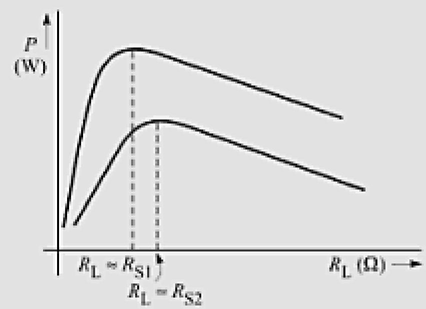


Fig. 4.60

Brief Theory The maximum power transfer theorem states that the power transferred from the source to the load is maximum when the load resistance is equal to the source resistance, i.e.,

$$R_L = R_S$$

Procedure

1. Connect the circuit as shown in Fig. 4.59.
2. Keep the setting of the rheostat R_L at its maximum position.
3. Adjust rheostat R_S such that the reading of the ammeter is around the middle of its scale.
4. Note the readings I_L and V_S of the ammeter and the voltmeter across the rheostat R_S, respectively. Calculate the value of R_S, as

$$R_S = \frac{V_S}{I_L} \qquad (i)$$

5. Reduce the resistance of the rheostat R_L in steps and for each setting note the readings of I_L and V_L of the ammeter and the voltmeter across the rheostat R_L, respectively. Calculate the value of R_L, as

$$R_L = \frac{V_L}{I_L} \tag{ii}$$

6. Also, for each setting of the rheostat R_L calculate the power consumed by the load, as

$$P = V_L I_L \tag{iii}$$

7. For another different setting of the rheostat R_S, repeat steps 4 to 6.
8. Draw the graphs for the two values of R_S.

Observations and Calculations

S. No.	(*a*) *For* R_{s1} = *Eq.* (*i*)				(*b*) *For* R_{s2} = *Eq.* (*i*)			
	V_L	I_L	R_L *Eq.* (*ii*)	P_L *Eq.* (*iii*)	V_L	I_L	R_L *Eq.* (*ii*)	P_L *Eq.* (*iii*)
1.								
2.								
3.								
4.								
5.								

Graph The graphs for the two values of R_S is drawn in Fig. 4.60.

Results

1. From the graphs given in Fig. 4.59, it is observed that as the value of R_L is increased the power P increases, becomes maximum, and then decreases.
2. Also, it is seen that for lower value of R_S the maximum power is greater.
3. The value of R_L for which the power becomes maximum is almost same as that of corresponding R_S.
4. The maximum power transfer theorem is, therefore, verified.

Precautions

1. Before connecting the dc supply, the zero readings of the ammeter and voltmeters should be checked.
2. The terminals of the rheostats should be connected properly.
3. While setting the rheostats, care should be taken that the currents recorded by the ammeters do not exceed 2 A, the current rating of the rheostats.

Viva-Voce

1. Can you give some reason why the value of R_L for which maximum power transfer takes place is a little greater than that of resistance R_S ?

Ans. : The dc supply itself has some internal resistance. Therefore, the actual resistance of our source is slightly more than the resistance of the rheostat R_S.

2. From a source of open-circuit voltage 10 V and of resistance 5 W, how much maximum power can be drawn ?

Ans. : For drawing maximum power, the condition is that $R_L = R_S$. Thus,

$$P_{\max} = I^2{}_L R_L = \left(\frac{E}{R_S + R_L}\right)^2 R_L = \frac{E^2}{4R_S} = \frac{10^2}{4 \times 5} = \mathbf{5\ W}$$

3. Is the maximum power transfer theorem applicable to ac circuits ?
   Ans. : Yes, but for ac circuits the condition for maximum power transfer is that the load impedance should be equal to the complex conjugate of the source impedance.
4. Is it always possible to operate a circuit at maximum power transfer conditions ?
   Ans. : No. For example, the domestic and industrial power supplies have very low source resistance so as to provide good voltage regulation. If the load resistance is also made so low, undesirably high currents would flow.
5. Then, where do you make use of this theorem ?
   Ans. : This theorem is utilized in electronic communication circuits, where the signal power is quite low. In such cases, the load is properly matched with the source so that maximum signal power is transferred to the load.

SUPPLEMENTARY EXERCISES A

A.1 Solved Problems
A.2 Practice Problems

PART A: DC CIRCUITS

Assemblage of

- Chapter 2: Ohm's Law
- Chapter 3: Network Analysis
- Chapter 4: Network Theorems

NOTE

This set of Exercises provides practice to those who wish to attain a higher standard of learning the basic principles of Electrical Engineering. **The real key to success is practice.**

A.1. SOLVED PROBLEMS

PROBLEM A-1

The single loop circuit in Fig. A-1 has a current source of I amperes connected to an independent voltage source of 8 V and a current-controlled voltage source whose voltage in volts is equal to twice the current flow in amperes through it. Determine the power P_1 absorbed by the independent voltage source and the power P_2 absorbed by the dependent voltage source for (*a*) $I = 4$ A, (*b*) $I = -3$ A, and (*c*) $I = 5$ mA.

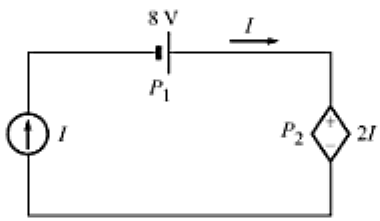


Fig. A-1

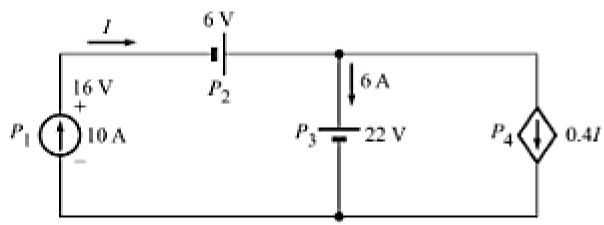


Fig. A-2

Solution We find that the reference arrow for current I is directed away from the positive terminal of the 8-V source. Hence, the formula for power absorbed has a negative sign : $P_1 = -8I$. For the dependent source, we find that the voltage and current references corresponds to the passive sign convention. Hence, $P_2 = 2I(I) = 2I^2$. Therefore,

(*a*) $P_1 = -8I = -8(4) = -$ **32 W** and $P_2 = 2I^2 = 2(4)^2 =$ **32 W.** The negative sign for P_1 shows that independent source delivers power instead of absorbing it. Furthermore, the independent source of 8 V is delivering power of 32 W and the dependent source is absorbing power of 32 W. Hence, the current source is neither absorbing nor delivering any power.

(*b*) $P_1 = -8I = -8(-3) =$ **24 W** and $P_2 = 2I^2 = 2(-3)^2 =$ **18 W.** The power absorbed by the dependent source is still positive, in spite of the reversal of current-flow. It is so because as the current is reversed, the polarity of the voltage also reverses. The actual current flow is still into the actual positive terminal.

(*c*) $P_1 = -8I = -8(5 \times 10^{-3}) = -$ **40m W** and $P_2 = 2I^2 = 2(5 \times 10^{-3})^2 =$ **50 μW** .

PROBLEM A-2

Show that the total power absorbed by components is equal to the total power delivered in the circuit of Fig. 2.

Solution Using passive sign convention, let us calculate the power absorbed by each component :

$$P_1 = -16(10) = \mathbf{-160\ W}; \qquad P_2 = -6(10) = \mathbf{-60\ W};$$

$$P_3 = 22(6) = \mathbf{132\ W}; \qquad \text{and} \qquad P_4 = 22(0.4 \times 10) = 22(4) = \mathbf{88\ W.}$$

The negative sign indicates that it is power delivered and not power absorbed. Thus,

$$\text{Power absorbed, } P_a = 132 + 88 = 220 \text{ W}$$

$$\text{Power delivered, } P_d = 160 + 60 = 220 \text{ W}$$

The two are seen to be equal.

PROBLEM A-3

In the circuit shown in Fig. A-3*a*, determine the voltage v_x.

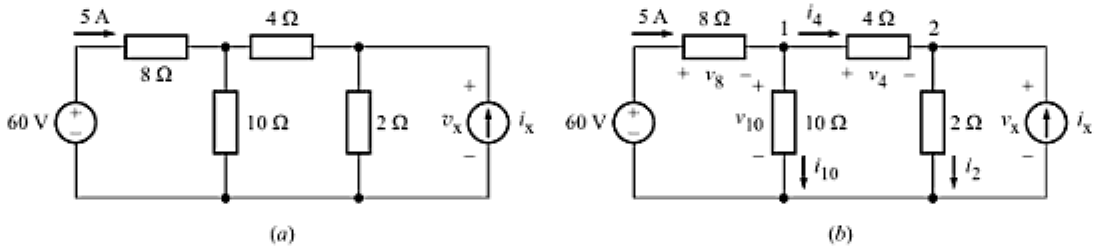


Fig. A-3

Solution Let us first label all the currents and voltages on the elements in the circuit (Fig. A-3*b*). Applying KVL in the first loop, we get

$$60 - v_8 - v_{10} = 0 \qquad \text{or} \qquad 60 - (5\text{ A} \times 8\ \Omega) - v_{10} = 0 \quad \Rightarrow \quad v_{10} = 20\text{ V}$$

$$\therefore \qquad i_{10} = \frac{20\text{ V}}{10\ \Omega} = 2\text{ A}$$

We note that v_x appears across the 2-Ω resistor as well as across the current source i_x. Now applying KVL in the central loop, we get

$$v_{10} - v_4 - v_x = 0$$

Writing KCL for the node 1, we get

$$5 = i_4 + i_{10} = \frac{v_4}{4} + 2 \quad \Rightarrow \quad v_4 = 12\text{ V}$$

Substituting this value of v_4 and knowing that $v_{10} = 20$ V, we get

$$20 - 12 - v_x = 0 \quad \Rightarrow \quad v_x = \mathbf{8\ V}$$

PROBLEM A-4

Determine the voltage, current, and power associated with each element in the circuit of Fig. A-4*a*.

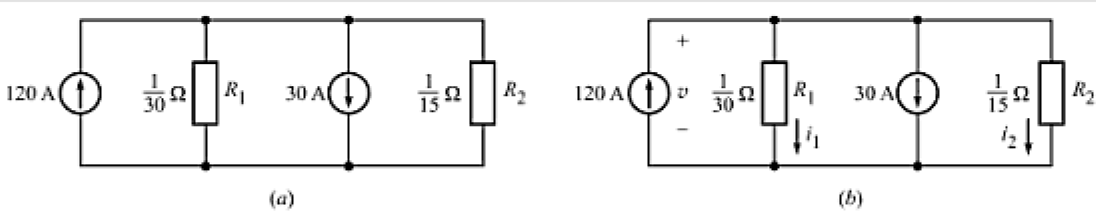


Fig. A-4

Solution Let us first mark voltage v with an arbitrary polarity, as shown in Fig. A-4*b*. Also, we mark currents i_1 and i_2 in the two resistors in conformance with the passive sign convention. Next, we apply KCL at the upper node to get

$$-120 + \frac{v}{1/30} + 30 + \frac{v}{1/15} = 0 \qquad \Rightarrow \quad v = \mathbf{2\ V}$$

$$\therefore \qquad i_1 = \frac{v}{1/30} = \mathbf{60\ A} \quad \text{and} \quad i_2 = \frac{v}{1/15} = \mathbf{30A}$$

The absorbed power in each element is

$$P_{120A} = vi = 2(-120) = \mathbf{-240\ W}, \qquad P_{30A} = vi = 2(30) = \mathbf{60\ W},$$
$$P_{R1} = vi_1 = 2(60) = \mathbf{120\ W}, \quad \text{and} \quad P_{R2} = vi_2 = 2(30) = \mathbf{60\ W}$$

PROBLEM A-5

Determine the value of v and the power delivered by the independent current source in the circuit of Fig. A-5.

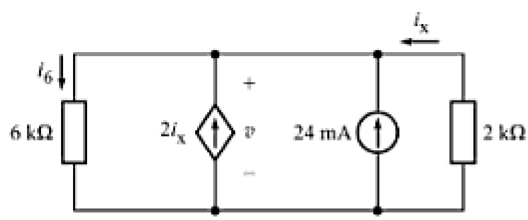


Fig. A-5

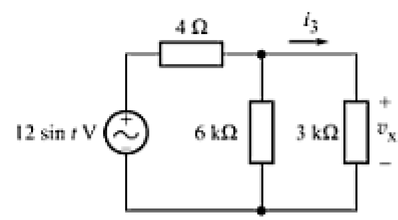


Fig. A-6

Solution Applying KCL for the upper node, we get

$$i_6 - 2\,i_x - 0.\,024 - i_x = 0$$

Applying Ohm's law to each resistor,

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

$$\Rightarrow \qquad \frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0 \quad \Rightarrow \quad v = 600 \times 0.024 = \mathbf{14.4\ V}$$

and

$$P_{24mA} = iv = (24\text{ mA})\,(14.4\text{ V}) = \mathbf{345.6\ m\ W}$$

PROBLEM A-6

Determine v_x and i_3 in the circuit of Fig. A-6.

Solution Since v_x appears across the parallel combination of the 6-Ω and 3-Ω resistors, we can simplify the circuit by combining them without losing v_x.

$$6\ \Omega \,||\, 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Now, applying the principle of voltage division, we get

$$v_x = (12 \sin t) \times \frac{2}{4 + 2} = \mathbf{4 \sin \mathit{t}\ V}$$

Now, the current i_3 can easily be determined as follows.

$$i_3 = \frac{v_x}{3\Omega} = \mathbf{\frac{4}{3}\ \sin \mathit{t}\ A}$$

PROBLEM A-7

In the circuit of Fig. A-7, calculate (*a*) the voltage of the dependent source, (*b*) the power supplied by the dependent source, and (*c*) the power dissipated by the 15-Ω resistor.

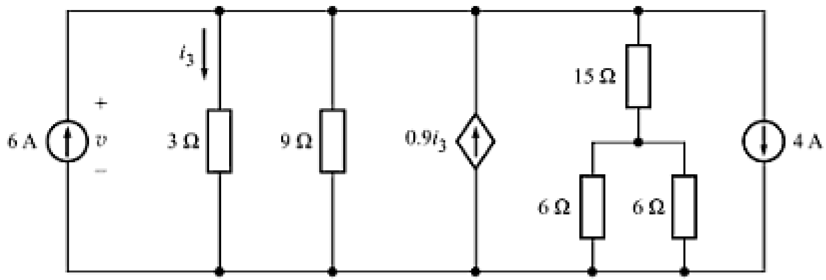


Fig. A-7

Solution

(*a*) The voltage across the dependent source is same as that across the 3-Ω resistor, v. To determine this voltage, we leave the dependent source alone and simplify the remaining circuit. The two current sources can be combined into a single 2-A source. Two 6-Ω resistors are parallel and can be replaced by a single 3-Ω resistor (6 Ω || 6 Ω = 3 Ω). This 3-Ω is now combined with the series 15-Ω resistor to give a single 18-Ω resistor. The resulting circuit is given in Fig. A-8*a*.

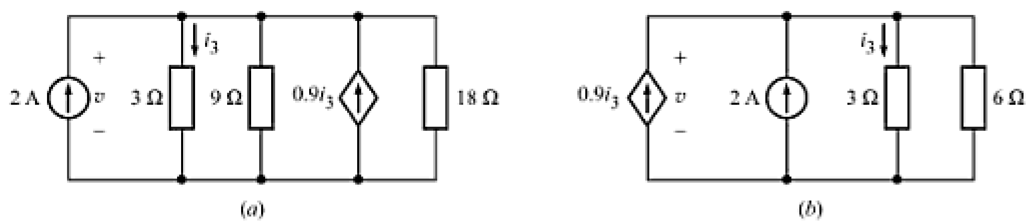


Fig. A-8

After this simplification, you might be tempted to combine 3-Ω, 9-Ω and 18-Ω resistors. However, doing so loses i_3, which is required to find the value of the controlled source. Hence, we further simplify the circuit by combining only 9-Ω and 18-Ω resistors, as shown in Fig. A-8*b*.

Applying KCL to the top node of Fig. A-8*b*, we get

$$-0.9\,i_3 - 2 + i_3 + \frac{v}{6} = 0$$

Applying Ohm, s law to the3-Ω resistor branch, we have v = 3i3. Putting this value of v in above equation, we get

$$-0.9\,i_3 - 2 + i_3 + \frac{3i_3}{6} = 0 \quad \Rightarrow \quad i_3 = \frac{10}{3}\text{ A}$$

Thus, the voltage across the dependent source (which is the same as the voltage across the 3-Ω resistor) is

$$v = 3\,i_3 = 3 \times \frac{10}{3} = \mathbf{10\ V}$$

(*b*) The power supplied by the dependent source is

$$P_{\text{dep}} = v \times 0.9\,i_3 = 10 \times 0.9 \times \frac{10}{3} = \mathbf{30\ W}$$

(*c*) To determine the power dissipated by the 15-Ω resistor, we must return to the original circuit. This resistor is in series with an equivalent 3-Ω resistor, and a voltage of 10 volts appears across this branch of total resistance 18

Ω. Therefore, the current through the 15-Ω resistor is

$$i_{15} = \frac{10}{18} = \frac{5}{9}\text{ A}$$

$$\therefore \qquad P_{15\Omega} = (5/9)^2 \times 15 = \mathbf{4.63\ W}$$

PROBLEM A-8

What is the maximum voltage that can be applied across the series combination of a 150-Ω, 2-W resistor and a 100-Ω, 1-W resistor without exceeding the power rating of either resistor?

Solution From $P = I^2 R$, the maximum safe current for the 150-Ω resistor is

$$I_1 = \sqrt{P/R} = \sqrt{2/150} = 0.115\text{ A}$$

The maximum safe current for the 100-Ω resistor is

$$I_2 = \sqrt{P/R} = \sqrt{1/100} = 0.1\text{ A}$$

The maximum current cannot exceed the lesser of these two currents. Hence, the maximum voltage that can be applied is

$$V_{max} = I(R_1 + R_2) = 0.1 \times (150 + 100) = \mathbf{25\ V}$$

PROBLEM A-9

For the circuit of Fig. A-9, (*a*) calculate the current I and the power absorbed by the dependent source, and (*b*) determine the resistance "seen" by the independent voltage source.

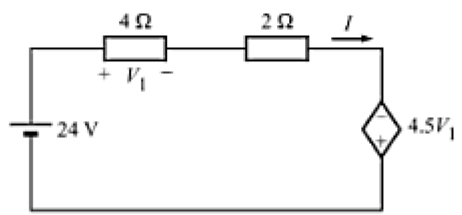


Fig. A-9

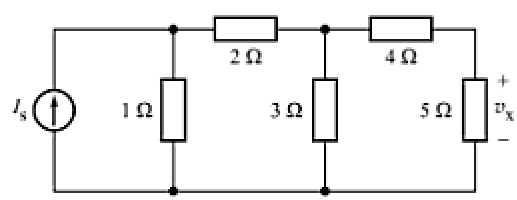


Fig. A-10

Solution

(*a*) Applying Ohm's law to the 4-Ω resistor, we get $V_1 = 4I$. Now, writing KVL equation, we get

$$24 - 4I - 2I + 4.5V_1 = 0 \quad \text{or} \quad 24 - 4I - 2I + 4.5(4I) = 0 \quad \Rightarrow \quad I = 24/(-12) = \mathbf{-2\ A}$$

The negativedicates that the 2-A current actually flows counterclockwise, opposite to the reference direction for I.

Since the current and voltage references for the dependent source are not the same as the passive sign convention, the formula for power absorbed has a negative sign:

$$P = -4.5V_1(I) = -4.5(4I)(I) = -18I^2 = -18(-2)^2 = \mathbf{-72\ W}$$

The negative sign here means that the dependent source is supplying power instead of absorbing it.

(*b*) The resistance "seen" by the source is equal to the ratio of the source voltage to the current flowing away from the positive terminal of the source. Thus,

$$R = \frac{24}{I} = \frac{24}{-2} = \mathbf{-12\ \Omega}$$

The negative sign here indicates that the remaining circuit supplies power to the independent source. Actually, it is the dependent source alone that supplies this power, as well as to the two resistors.

PROBLEM A-10

In the circuit of Fig. A-10, (*a*) if $v_x = 10$ V, find I_s; (*b*) if $I_s = 50$ A, find v_x; (*c*) calculate the ratio v_x/I_s.

Solution Let us mark the currents and nodes in the circuit as shown in Fig. A-11*a*.

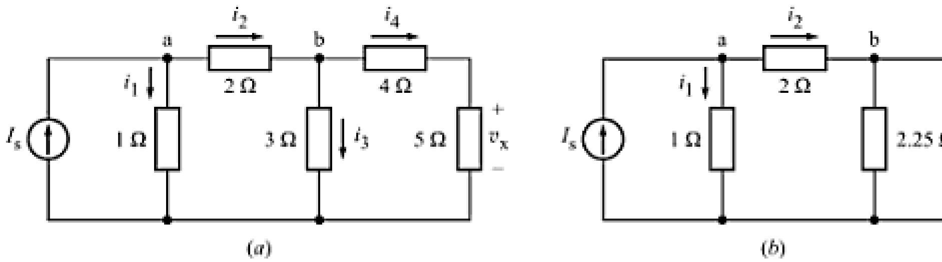


Fig. A-11

(*a*) Given: $v_x = 10$ V. Therefore, $i_4 = (10 \text{ V}/5\ \Omega) = 2$ A Thus, using Ohm's law,

$$v_b = i_4(4+5) = 2(4+5) = 18 \text{ V} \quad \Rightarrow \quad i_3 = \frac{v_b}{3} = \frac{18}{3} = 6 \text{ A}$$

Using KCL: $i_2 = i_3 + i_4 = 6 + 2 = 8$ A

Using Ohm's law: $v_a - v_b = i_2 \times 2 \quad \Rightarrow \quad v_a = 18 + 8 \times 2 = 34 \text{ V} \quad \Rightarrow \quad i_1 = 34/1 = 34$ A

Again, using KCL: $I_s = i_1 + i_2 = 34 + 8 = \mathbf{42\ A}$

(*b*) To be able to use the current divider principle, we first simplify the circuit on the right of node b. The series combination of 4 Ω and 5 Ω is in parallel with 3 Ω. Hence, the equivalent resistance from node b to ground is

$$R_{eq} = (4+5) \parallel 3 = \frac{9 \times 3}{9+3} = 2.25\ \Omega$$

The simplified circuit is shown in Fig. A-49*b*. From this circuit, it is obvious that

$$i_2 = (50 \text{ A}) \frac{1}{1 + (2 + 2.25)} = 9.52 \text{ A}$$

This current again divides into i_3 and i_4 (Fig. A-49*a*). Thus,

$$i_4 = (9.52 \text{ A}) \frac{3}{3 + (4+5)} = 2.38 \text{ A}$$

Finally, we can now find the voltage v_x as

$$v_x = i_4(5\ \Omega) = (2.38 \text{ A})\ (5\ \Omega) = \mathbf{11.90\ V}$$

(*c*) For the first case, the ratio R_t is

$$R_t = \frac{v_x}{I_s} = \frac{10 \text{ V}}{42 \text{ A}} = \mathbf{0.238\ \Omega}$$

For the second case, the ratio R_t is

$$R_t = \frac{v_x}{I_s} = \frac{11.9 \text{ V}}{50 \text{ A}} = \mathbf{0.238\ \Omega}$$

In the two cases, the ratio v_x/I_s is found to be the same. It is not just a coincidence. This ratio has a fixed value, and it is called ***transfer resistance*** of the circuit.

PROBLEM A-11

In the circuit of Fig. A-12, find voltage V_1.

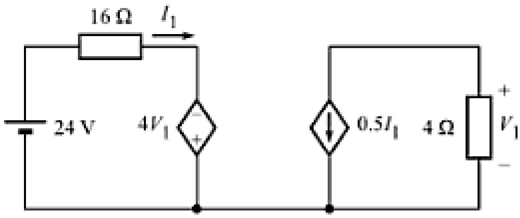


Fig. A-12

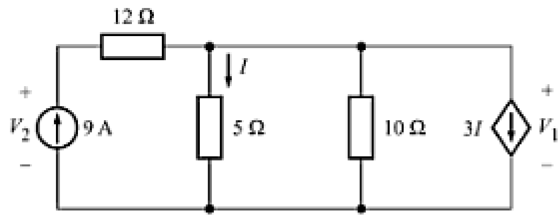


Fig. A-13

Solution We observe that no current flows in the single wire connecting the two parts of the circuit. Applying KVL to the left-hand half of the circuit, we get

$$24 - 16 I_1 - 4 V_1 = 0 \tag{i}$$

Applying Ohm's law to the right-hand half of the circuit, we get

$$V_1 = -0.5 I_1 (4) = -2 I_1 \quad \text{or} \quad I_1 = -0.5 V_1 \tag{ii}$$

Substituting this value of I_1 in Eq (i), gives

$$24 - 16(-0.5 V_1) - 4 V_1 = 0 \quad \Rightarrow \quad V_1 = \mathbf{-6\ V}$$

PROBLEM A-12

Determine voltage V_1 and V_2 in the circuit of Fig. A-13.

Solution We observe that the dependent current source is $3I$. Hence, we first find the controlling current I in terms of V_1, using Ohm's law: $I = V_1/5$. Thus, the dependent current directed source is $3I = (V_1/5) = 0.6 V_1$, directed downward. Applying KCL at the top node, we get

$$\frac{V_1}{5} + \frac{V_1}{10} \, 0.6 V_1 = 9 \quad \Rightarrow \quad V_1 = \mathbf{10\ V}$$

Finally, applying KVL around the outside loop gives

$$V_2 - 12(9) - V_1 = 0 \quad \text{or} \quad V_2 = 12(9) + 10 = \mathbf{118\ V}$$

Note that the 12-Ω resistor has no effect on V_1, but it does have an effect on V_2.

PROBLEM A-13

Determine the power absorbed by the 5-Ω resistor in the circuit of Fig. A-14a.

Solution Let us first redraw the given circuit in a simpler way as in Fig. A-14b. Let us now try to find the voltage of the upper node with respect to the lower node in terms of v_1. The current through the 2-Ω resistor is $v_1/2$. Therefore, the voltage across the series combination of 1-Ω and 2-Ω resistors is $(v_1/2.)(1 + 2) = 1.5 v_1$. Applying KCL at the upper node, we get

$$-5 + \frac{1.5 v_1}{3} - 5 v_1 + \frac{1.5 v_1}{5} = 0 \quad \text{or} \quad 0.5 v_1 - 5 v_1 + 0.3 v_1 = 5$$

$$\Rightarrow \qquad v_1 = -\frac{5}{4.2}\ \text{V} = -1.19\ \text{V}$$

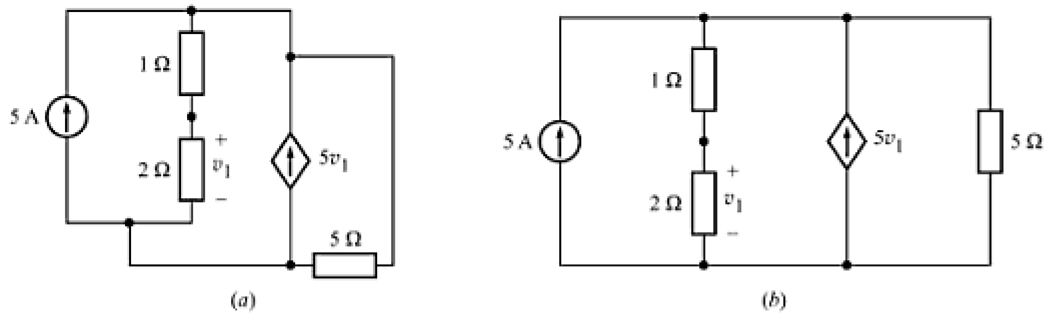


Fig. A-14

The current through the 5-Ω resistor is

$$i = \frac{1.5v_1}{5} = \frac{1.5 \times (-1.19)}{5} = -0.357 \text{ A}$$

Hence, the power absorbed by the 5-Ω resistor is

$$P = i^2 R = (-0.357)^2 (5) = \mathbf{0.637\ W}$$

PROBLEM A-14

In the network shown in Fig. A-15, (*a*) if $R = 80\ \Omega$, find R_{eq}; (*b*) if $R_{eq} = 80\ \Omega$, find R; (*c*) if $R = R_{eq}$, find R.

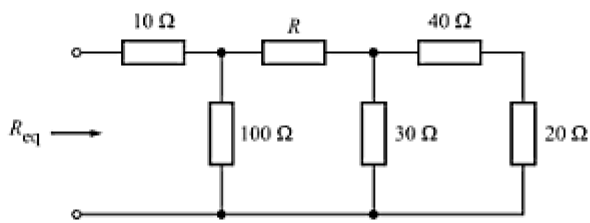


Fig. A-15

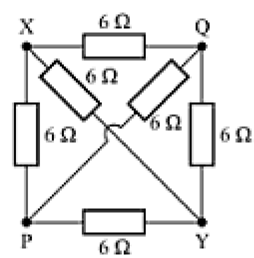


Fig. A-16

Solution

(*a*) If $R = 80\ \Omega$, the equivalent resistance is

$$R_{eq} = 10 + [100 \,||\, \{80 + (30 \,||\, (40 + 20))\}]$$

$$= 10 + \left[100 \,||\, \left\{80 + \frac{30 \times 60}{30 + 60}\right\}\right] = 10 + [100 \,||\, \{80 + 20\}] = 10 + [100 \,||\, 100]$$

$$= 10 + [50] = 60\ \Omega$$

(*b*) If $R_{eq} = 80\ \Omega$, we have

$$80 = 10 + 100 \,||\, (R + 20) \quad \text{or} \quad 80 = 10 + \frac{100\,(R + 20)}{100 + (R + 20)}$$

Dividing by 10, we get

$$\frac{10\,(R + 20)}{100 + (R + 20)} = 8 - 1 \quad \text{or} \quad 10R + 200 = 7R + 840 \quad \text{or} \quad 3R = 640$$

$$\Rightarrow \qquad R = \frac{640}{3} = \mathbf{213.3\ \Omega}$$

(c) If $R = R_{eq}$, we have

$$R = 10 + 100 \| (R + 120), \quad \text{or} \quad R = 10 + \frac{100R + 2000}{R + 120}$$

Multiplying through out by $(R + 120)$, we get

$$R^2 + 120R = 10R + 1200 + 100R + 2000 \quad \text{or} \quad R^2 + 10R - 3200 = 0$$

$$\Rightarrow \qquad R = \frac{-10 \pm \sqrt{100 + 12800}}{2} = 51.78\ \Omega \quad \text{or} \quad -61.79\ \Omega$$

Since a resistance cannot have negative value, $R = \mathbf{51.79\ \Omega}$

PROBLEM A-15

Six resistors, each of 6 Ω, are connected as shown in Fig. A-16. Determine the equivalent resistance across any two diagonal points.

Solution Let us take X-Y at the two diagonal points across which we shall find the equivalent resistance of the network. We shall discuss below three methods of doing it.

First Method We convert the delta-connection (represented by dotted lines) across points PYQ into its star-equivalent (see Fig. A-17*a*). This introduces an additional node N as the star-point. Using standard conversion formula, we find that a delta with 6 Ω as each side is converted into 2 Ω as each leg of the star.

To see clearly the inter-connectivity of different elements, we redraw the network as in Fig. A-17*b*. It is obvious that between X and N, there are two parallel branches each having 6- Ω and 2-Ω resistors in series. Point N is connected to point Y through a 2-Ω resistor. One more 6-Ω resistor is connected between X and Y. Hence, the equivalent circuit is as shown in Fig. A-17*c*. Thus,

$$R_{XY} = 6 \| \{8 \| 8) + 2\} = 6 \| \{4 + 2\} = \mathbf{3\ \Omega}$$

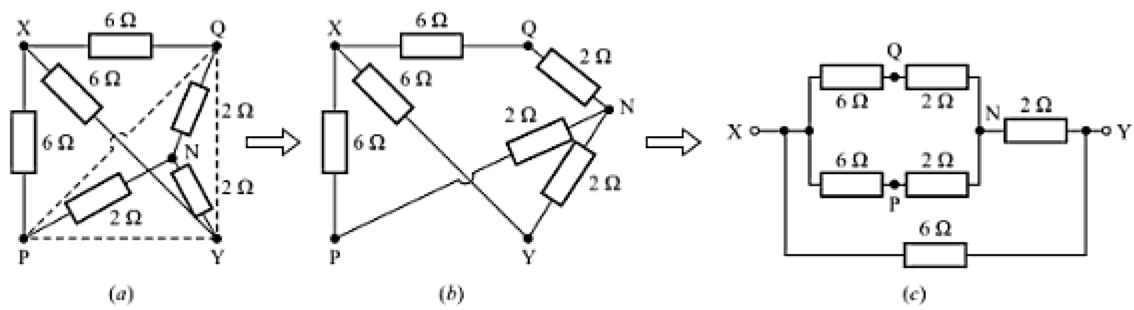
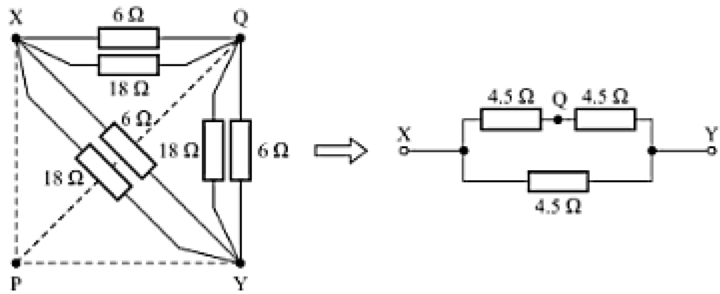


Fig. A-17

Second Method We convert the star-connection (represented by dotted lines) across points XYQ into its delta-equivalent (see Fig. A-18*a*). Using standard conversion formula, we find that a star with 6 Ω as each leg is converted into 18 Ω as each side of the delta. The 18 Ω and 6 Ω in parallel gives 4.5 Ω. Therefore, the equivalent network between X and Y is as shown in Fig. A-18*b*. Thus,

$$R_{XY} = 4.5 \| (4.5 + 4.5) = \mathbf{3\ \Omega}$$



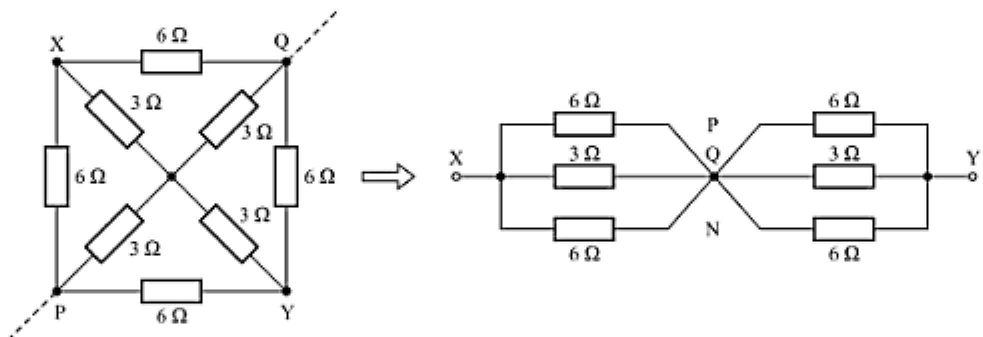
(a) (b)

Fig. A-18

Third Method We break the two 6-Ω resistors in the centre of the network, each into a series combination of two 3-Ω resistors (Fig. A-19*a*). The network is now seen to be *symmetrical* about the dotted line along P-Q. If a battery is connected across X-Y, due to symmetry, all points along the dotted line (such as points Q, N and P), shall be at the same potential. Hence, all these points can be shorted without affecting the equivalent resistance of the network. The resulting network then simplifies to that shown in Fig. A-19*b*. It is now obvious that

$$R_{XY} = 2 \times \{3 \,\|\, (6 \,\|\, 6)\} = 2 \times \{3 \,\|\, 3\} = 2 \times 1.5 = \mathbf{3\ \Omega}$$

Note Obviously, the third method is simplest of all.



(a) (b)

Fig. A-19

PROBLEM A-16

Find voltage V_1 in the circuit of Fig. A-20, using voltage division twice.

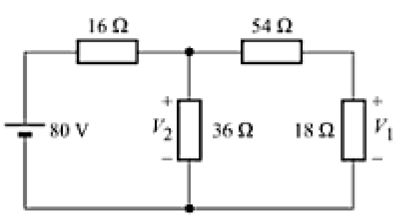


Fig. A-20

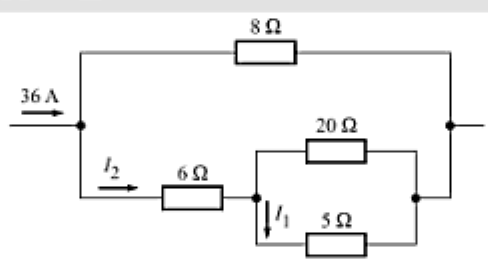


Fig. A-21

Solution It is obvious that V_1 can be found by voltage division if V_2 is known. Therefore, first we determine V_2 by voltage division of the source voltage of 80 V. For this, we need the equivalent resistance to the right of 16-Ω resistor. This resistance is

$$R_{eq} = \frac{(36)\,(54+18)}{36+54+18} = 24\ \Omega$$

By applying the voltage division principle, we get

$$V_2 = 80 \times \frac{24}{24+16} = 48\ \text{V}$$

Note A common mistake committed by students is to ignore the loading effect of the resistors to the right of the V_2 node.

Again by voltage division, we get

$$V_1 = 48 \times \frac{18}{18+54} = \mathbf{12\ V}$$

PROBLEM A-17

Find current I_1 in the circuit of Fig. A-21, using current division twice.

Solution Obviously, the current I_1 can be found by current division, if we know current I_2. For this, we need the equivalent resistance of the bottom three branches:

$$R_{eq} = 6 + \frac{(20)\,(5)}{20+5} = 10\ \Omega$$

By current division principle, we get

$$I_2 = 36 \times \frac{8}{8+10} = 16\ \text{A} \qquad \text{and} \qquad I_1 = 16 \times \frac{20}{20+5} = \mathbf{12.8\ A}$$

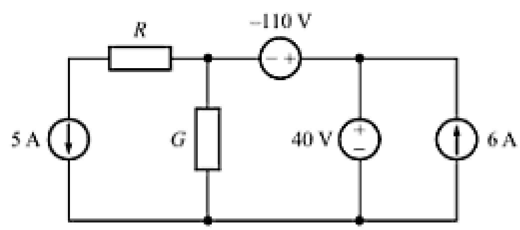


Fig. A-22

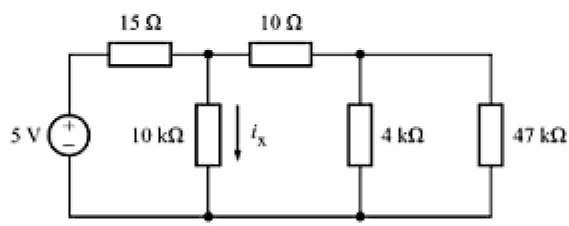


Fig. A-23

PROBLEM A-18

Find R and G in the circuit of Fig. A-22 if the 5-A source supplies 100 W and the 40-V source supplies 500 W.

Solution Voltage across 5-A source,

$$V_{5A} = \frac{P}{I} = \frac{100}{5} = 20\ \text{V} \quad \text{(lower terminal '+' marked)}$$

Current through 40-V source,

$$I_{40V} = \frac{P}{V} = \frac{500}{40} = 12.5\ \text{A} \quad \text{(upwards)}$$

Current through 110-V source,

$$I_{110V} = 12.5 + 6 = 18.5 \text{ A} \quad \text{(towards left)}$$

$$\therefore \quad I_G = I_{110V} - 5 = 18.5 - 5 = \text{A} \quad \text{(downwards)}$$

$$\text{Voltage across } G,\ V_G = 40 - (-110) = 150 \text{ V}$$

Thus,
$$G = \frac{I_G}{V_G} = \frac{13.5 \text{ A}}{150 \text{ V}} = 0.09S = \mathbf{90\ mS}$$

Now, the voltage across R,

$$V_R = V_{5A} + V_G = 20 + 150 = 170 \text{ V}$$

Thus,
$$R = \frac{V_R}{I_R} = \frac{170}{5} = \mathbf{34\ \Omega}$$

PROBLEM A-19

For the circuit shown in Fig. A-23, determine i_x and compute the power absorbed by the 15-kΩ resistor.

Solution Equivalent resistance as faced by the 5-V source,

$$R_{eq} = 15 + [10 \| \{10 + (4 \| 47)\}] = 15 + [10 \| 13.686] = 20.78 \text{ k}\Omega$$

$$\text{Current supplied by the source, } i = \frac{5 \text{ V}}{20.78 \text{ k}\Omega} = 240.6\ \mu\text{A}$$

$$\text{Current, } i_x = (240.6\ \mu\text{A})\frac{13.686}{10 + 13.686} = \mathbf{139\ \mu A}$$

$$\text{Power, } P_{15\Omega} = (24.6\ \mu\text{A})^2\,(15 \text{ k}\Omega) = \mathbf{868\ \mu W}$$

PROBLEM A-20

Find the power absorbed by each element of the circuit shown in Fig. A-24*a*.

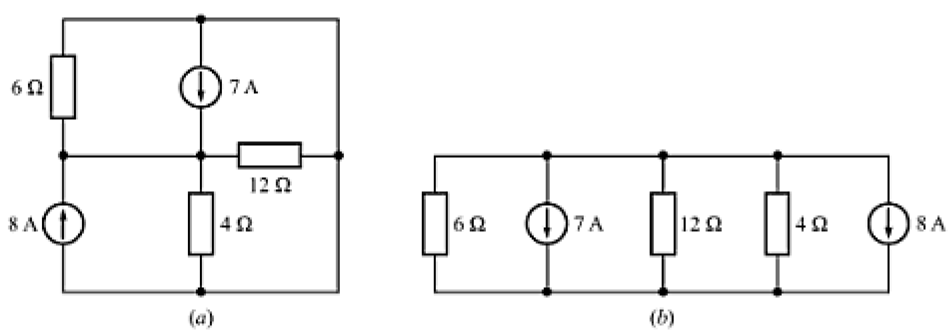


Fig. A-24

Solution The given circuit can be redrawn in the simple way as in Fig. A-24*b*. It is seen to be a single-node-pair circuit. If V is the voltage of the upper node with respect to the lower node, we can write the KCL equation as

$$\frac{V}{6} + 7 + \frac{V}{12} + \frac{V}{4} + 8 = 0 \quad \text{or} \quad V\left(\frac{1}{6} + \frac{1}{12} + \frac{1}{4}\right) = -5 \quad \Rightarrow \quad V = -30 \text{ V}$$

$$\therefore \quad P_{6\Omega} = \frac{V^2}{6} = \frac{(-30)^2}{6} = \mathbf{150\ W}; \quad P_{7A} = VI = -30\,(7) = -\mathbf{210\ W};$$

$$P_{12\Omega} = \frac{V^2}{12} = \frac{(-30)^2}{12} = \mathbf{75\ W}; \quad P_{4\Omega} = \frac{V^2}{4} = \frac{(-30)^2}{4} = \mathbf{225\ W}$$

and $$P_{8A} = VI = -30(8) = \mathbf{-240\ W}$$

Comment: The sum of powers absorbed by all the elements of a circuit must add up to zero: 150 W + (–210 W) + 75 W + (– 240 W) = 0

PROBLEM A-21

Nine resistors, R_1 to R_9, are connected in a network as shown in Fig. A-25*a*. Determine the current delivered by a 30-V source when connected across nodes A and B of the network.

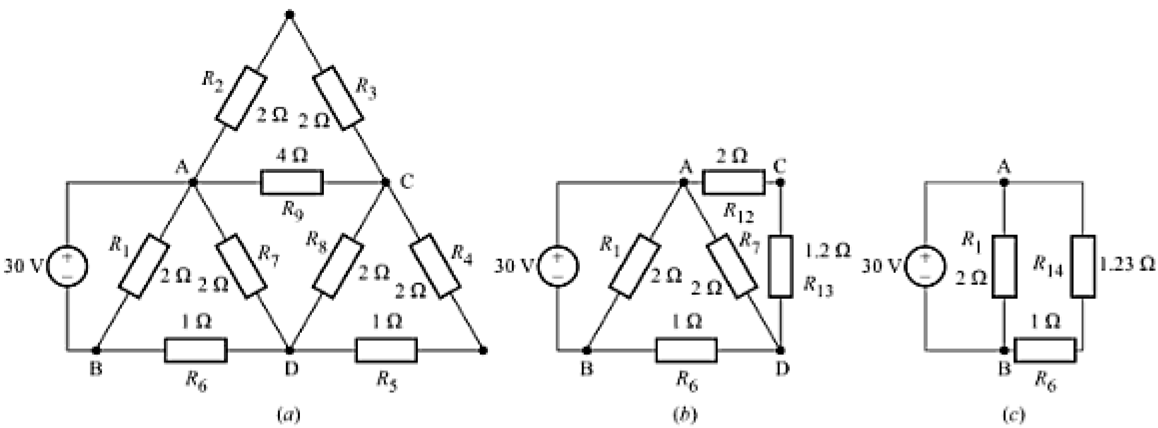


Fig. A-25

Solution The network is simplified by series and parallel combinations. The series combinations of resistors R_2 and R_3 is in parallel with R_9. Thus, as shown in Fig. A-25*b*, these three resistors can be replaced by a single resistor R_{12} across the nodes A and C:

$$R_{12} = (2 + 2) || 4 = 2\ \Omega$$

Similarly, the three resistors R_4, R_5 and R_8 can be replaced by a single resistor R_{13} across the nodes A and C:

$$R_{13} = (2 + 1) || 2 = 1.2\ \Omega$$

Now, we can replace the three resistors R_{12}, R_{13} and R_7 by a single resistor R_{14} across the nodes A and D, as shown in Fig. A-63*c*

$$R_{14} = (2 + 1.2) || 2 = 1.23\ \Omega$$

Further, the three resistors R_{14}, R_6 and R_1 can be replaced by a single resistor across the nodes A and B to give total resistance:

$$R_{AB} = (1 + 1.23) || 2 = 1.054\ \Omega$$

Finally, the current delivered by the 30-V source is given as

$$I = \frac{V}{R_{AB}} = \frac{30}{1.054} = \mathbf{28.46\ A}$$

PROBLEM A-22

The circuit of Fig. A-26 has two current sources and six resistors. Determine the output voltage V_o.

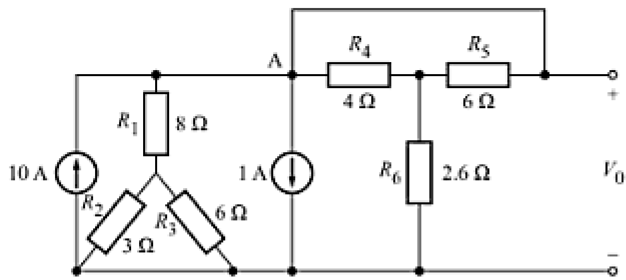


Fig. A-26

Solution At first sight, the circuit appears to have two star-connections of resistors. To solve the circuit, a student is tempted to go in for star-delta conversions. However, if you look at the circuit with proper perspective by redrawing it as shown in Fig. A-27*a*, it can easily be solved by series-parallel combinations of resistors.

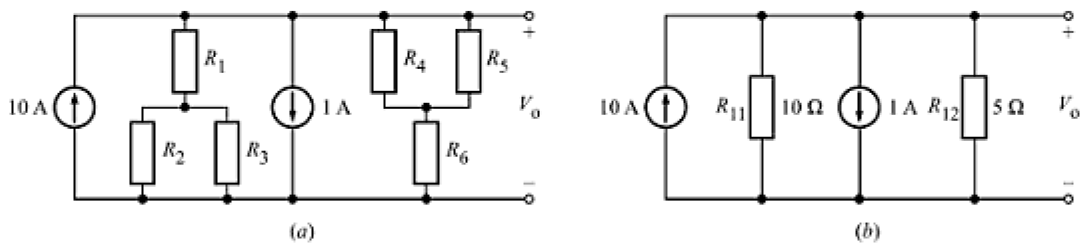


Fig. A-27

We find that the parallel combination of R_2 and R_3 is in series with R_1. Similarly, the parallel combination of R_4 and R_5 is in series with R_6. Thus,

$$R_{11} = R_1 + (R_2 \parallel R_3) = 8 + \frac{3\times 6}{3+6} = 10\ \Omega \qquad \text{and} \qquad R_{12} = R_6 + (R_4 \parallel R_5) = 2.6 + \frac{4\times 6}{4+6} = 5\ \Omega$$

The circuit (Fig. A-27*b*) simply has a single-node-pair. Applying KCL, gives

$$-10 + \frac{V_0}{10} + 1 + \frac{V_0}{5} = 0 \quad \text{or} \quad V_0\left(\frac{1}{10}+\frac{1}{5}\right) = 9 \quad \Rightarrow \quad V_o = \mathbf{30\ V}$$

PROBLEM A-23

For the circuit of Fig. A-28, determine the node-to-reference voltages.

Solution The circuit has five nodes and four different types of sources. We select the central node as reference, and assign v_1 to v_4 to the remaining nodes. We mark ***supernode*** about each *voltage* source, as shown in figure. We now need to write KCL equations only at node 2 and at the supernode containing the dependent voltage source. No equation is needed for node 1, since it is obvious that $v_1 = -12$ V. The KCL equations at node 2,

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14$$

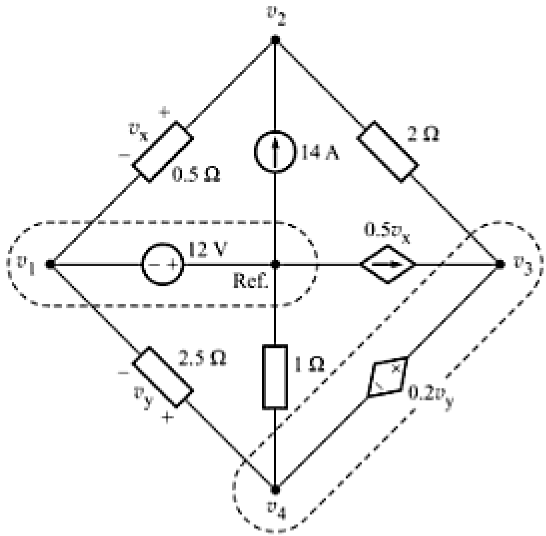


Fig. A-28

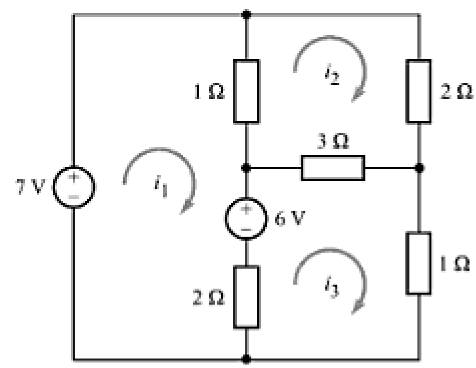


Fig. A-29

and at the 3-4 supernode,

$$\frac{v_3 - v_2}{2} + \frac{v_4 - v_1}{2.5} + \frac{v_4}{1} = 0.5\,v_x$$

Next, we relate the source voltages to the node voltages,

$$v_3 - v_4 = 0.2\,v_y \qquad \text{and} \qquad v_4 - v_1 = v_y$$

Finally, we express the dependent current source in terms of the node voltages,

$$0.5\,v_x = 0.5\,(v_2 - v_1)$$

Eliminating v_x and v_y from the above equations, we get

$$-2\,v_1 + 2.5\,v_2 - 0.5\,v_3 = 14$$
$$01\,v_1 - v_2 + 0.5\,v_3 + 1.4\,v_4 = 0$$
$$v_1 = -12$$
$$0.2\,v_1 + v_3 - 1.2\,v_4 = 0$$

Solving these equations, we get

$$v_1 = \mathbf{-12\ V;} \quad v_2 = \mathbf{-4\ V;} \quad v_3 = \mathbf{0\ V;} \qquad \text{and} \qquad v_4 = \mathbf{-2\ V}$$

PROBLEM A-24

Figure A-29 shows a three-mesh circuit. Use mesh analysis to determine the three mesh currents.

Solution The three mesh currents are assigned clockwise directions, as shown in Fig. A-29. We can either apply KVL around the three meshes, or to save time and avoid errors, we can directly write the mesh equations by inspection as follows:

$$\begin{bmatrix} 1+2 & -1 & -2 \\ -1 & 1+2+3 & -3 \\ -2 & -3 & 2+3+1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7-6 \\ 0 \\ 6 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

Using a scientific calculator, we can easily solve the above equations to get

$$i_1 = \mathbf{3\ A}; \quad i_2 = \mathbf{2\ A}; \qquad \text{and} \qquad i_3 = \mathbf{3\ A}$$

PROBLEM A-25

For the circuit shown in Fig. A-30*a*, use repeated source transformation to obtain a single mesh circuit, and then find the current *I*.

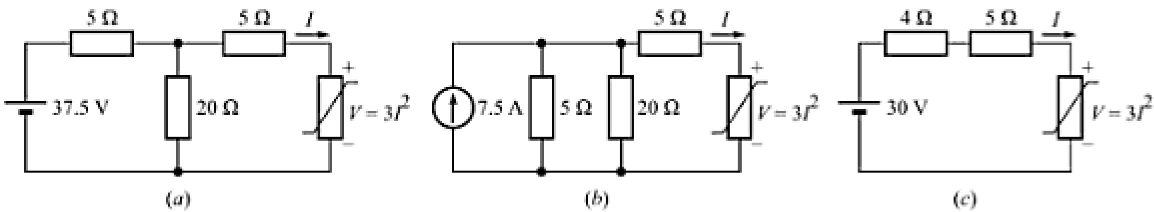


Fig. A-30

Solution First we transform the voltage source into a current source. The resulting current source is 37.5/5 = 7.5 A in parallel with a 5-Ω resistor (see Fig. A-30*b*). The 5-Ω resistor and 20-Ω resistor can be combined into a (5 × 20)/(5 + 20) = 4-Ω resistor. Next, the 7.5-A current source in parallel with 4-Ω resistor is transformed into a voltage source of 7.5 × 4 = 30 V in series with a 4-Ω resistor (Fig. A-30*c*). For this single mesh circuit, the KVL equation is $3I^2 + 9I - 30 = 0$, which can be solved to get

$$I = 2\ \text{A} \quad \text{or} \quad -5\ \text{A}$$

Negative current is physically not possible, since the circuit has a single battery and the current must flow out of its positive terminal. Hence, I = **2 A.**

PROBLEM A-26

Obtain the mesh currents and hence the voltage drop across 6-Ω resistor in the circuit of Fig. A- 31.

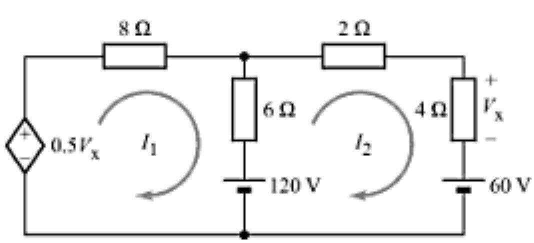


Fig. A-31

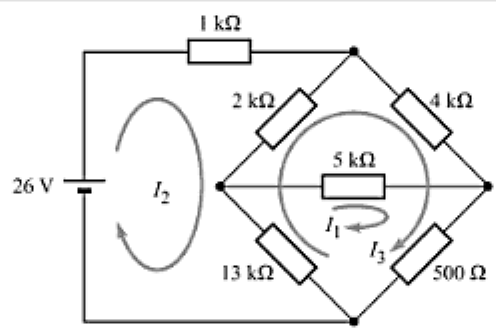


Fig. A-32

Solution Before we write KVL equations for the two meshes, we express the controlling quantity V_x in terms of the mesh current I_2. Using Ohm's law, $V_x = 4I_2$. The voltage of the dependent source is then $0.5V_x = 0.5(4I_2) = 2I_2$. **Note** that because of the presence of the dependent source we cannot write the mesh equations by inspection. Thus, writing

KVL equations for the two meshes, we have

$$(8 + 6)I_1 - 6I_2 - 2I_2 = -120$$

and $$(6 + 2 + 4)I_2 = 120 - 60$$

In matrix form, these simplify to

$$\begin{bmatrix} 14 & -8 \\ -6 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -120 \\ 60 \end{bmatrix}$$

Using a calculator, the above matrix equation can be solved to give

$$\mathbf{I_1 = -8\ A} \qquad \text{and} \qquad \mathbf{I_2 = 1\ A}$$

PROBLEM A-27

Use loop analysis to find the current flowing from left to the right through the 5-kΩ resistor in the circuit of Fig. A-32.

Solution Since the circuit has three meshes, we need three loops. Only one loop current should flow through 5-kΩ resistor so that only one current needs to be solved for. We can select the path of other two loop currents as shown. For convenience, let us take kΩ as the unit of resistance and mA as the unit of current. The KVL equations for the three loops are

$$13(I_1 + I_3 - I_2) + 5I_1 + 0.5(I_1 + I_3) = 0 \quad \text{or} \quad 18.5I_1 - 13I_2 + 13.5I_3 = 0$$
$$1(I_2) + 2(I_2 - I_3) + 13(I_2 - I_3 - I_1) = 26 \quad \text{or} \quad -13I_1 + 16I_2 - 15I_3 = 26$$
$$13(I_1 + I_3 - I_2) + 2(I_3 - I_2) + 4I_3 + .5(I_1 + I_3) = 0 \quad \text{or} \quad 13.5I - 15I_2 + 19.5I_3 = 0$$

Using a calculator, we get

$$\mathbf{I_1 = 2\ mA}$$

PROBLEM A-28

Use the technique of mesh analysis to determine the three mesh-currents in the circuit of Fig. A-33*a*.

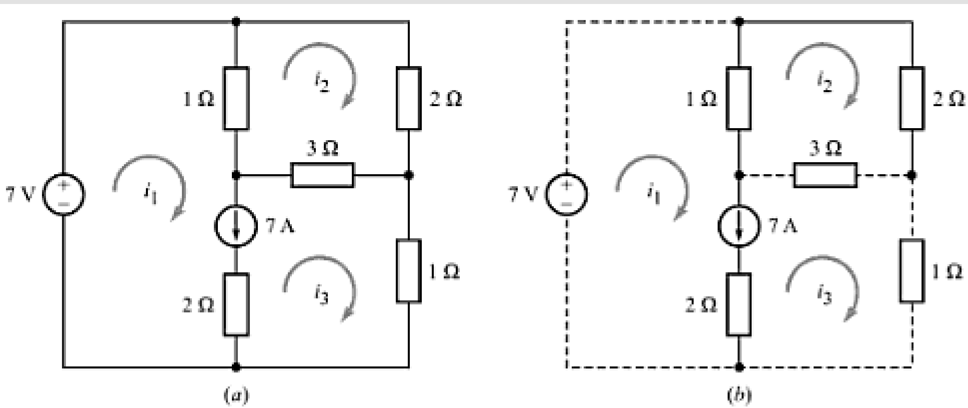


Fig. A-33

Solution In addition to 7-V source, the circuit also has a 7-A independent current source in the common boundary of the meshes 1 and 3. For mesh analysis, we use the concept of *supermesh*. The mesh currents i_1, i_2 and i_3 are assigned as shown in Fig. A-33*b*. Ignoring the current source, a supermesh is created (indicated by broken line) whose interior is that of meshes 1 and 3 as shown. Applying KVL to this loop,

$$1(i_1 - i_2) + 3(i_3 - i_2) + 1\,i_3 = 7 \quad \text{or} \quad i_1 - 4i_2 + 4i_3 = 7$$

Applying KVL around mesh 2,

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \quad \text{or} \quad -i_1 + 6i_2 - 3i_3 = 0$$

The third equation is obtained by relating the source current to the assumed mesh currents,

$$i_1 - i_3 = 7$$

Solving the three equations, we get

$$\mathbf{i_1 = 9\ A}, \quad \mathbf{i_2 = 2.5\ A}, \qquad \text{and} \qquad \mathbf{i_3 = 2\ A}$$

PROBLEM A-29

Use the technique of node analysis to determine the node voltage v_1 in the circuit of Fig. A-34.

Solution In addition to current sources, the circuit also has a 22-V independent voltage source between the nodes 2 and 3. For node analysis, we treat node 2, node 3 and the voltage source together as a *supernode* (indicated by the region enclosed by the broken line). Applying KCL to this supernode,

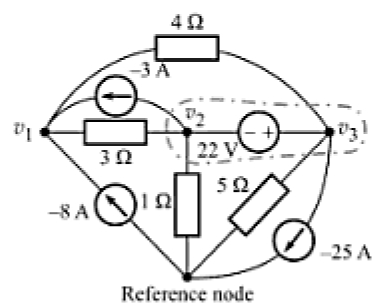


Fig. A-34

$$\frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} + \frac{v_2}{1} = -(-3) - (-25)$$

or $$-0.5833\,v_1 + 1.3333\,v_2 + 0.45 = 28$$

KCL equation at node 1 is

$$\frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4} = -8 - 3$$

or $$0.5833\,v_1 - 0.3333\,v_2 + 0.2500\,v_3 = -11$$

Since we have three unknowns, we need one more equation. This is obtained from the fact that there is a 22-V voltage source between node 2 and node 3:

$$v_2 - v_3 = 22$$

Solving these three equations, we get

$$v_1 = \mathbf{1.071\ V}$$

PROBLEM A-30

Use mesh analysis to find the three unknown currents in the circuit of Fig. A-35.

Solution We note that the 15-A current source is located on the perimeter of the circuit, and hence clearly i_1 = **15 A**. It also means that we may eliminate mesh 1 from consideration. Then, the dependent current source is on the perimeter of the modified network. Therefore, we should avoid writing any equation for mesh 3. Only mesh 2 remains, so we apply KVL:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \quad \text{or} \quad 6i_2 - 3i_3 = 15 \quad \Rightarrow \quad 2i_2 - i_3 = 5$$

The controlling voltage v_x is

$$v_x = 3(i_3 - I_2)$$

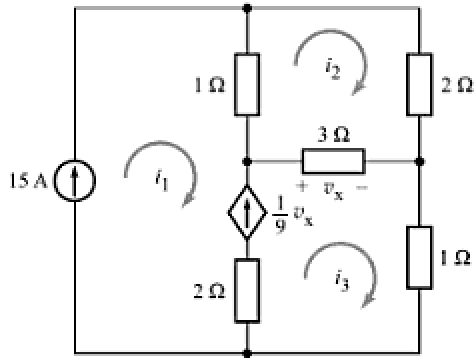


Fig. A-35

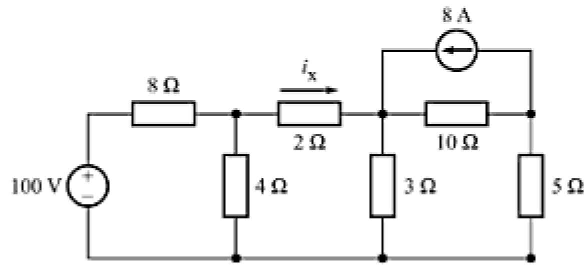


Fig. A-36

The dependent current source being common between mesh 1 and 3, we have

$$\frac{v_x}{9} = i_3 - i_1 \quad \text{or} \quad \frac{3(i_3 - i_2)}{9} = i_3 - i_1 \quad \Rightarrow \quad i_2 + 2\,i_3 = 45$$

Solving above two equations, we get i_2 = **11 A,** and i_3 = **17 A.**

PROBLEM A-31

A planar circuit with four meshes is given in Fig. A-36. Determine the current i_x by (*a*) mesh analysis, and (*b*) nodal analysis.

Solution

(*a*) *By mesh analysis*: The circuit has four distinct meshes (as shown in Fig. A-37*a*). It is obvious that $i_4 = -8$ A; we therefore need to write three more equations. Writing KVL equations for meshes 1, 2 and 3:

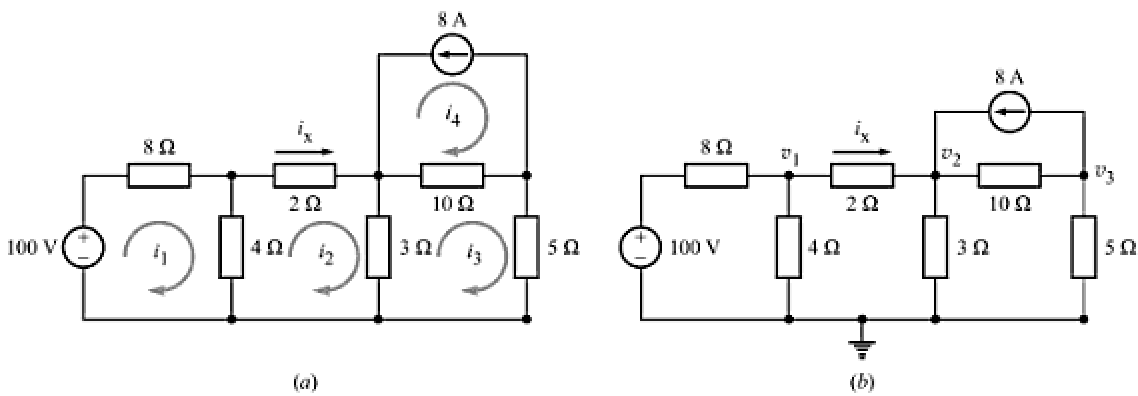


Fig. A-37

$$8\,i_1 + 4\,(i_1 - i_2) = 100 \quad \text{or} \quad 12\,i_1 - 4\,i_2 = 100$$
$$4\,(i_2 - i_1) + 2\,i_2 + 3\,(i_2 - i_3) = 0 \quad \text{or} \quad -4i_1 + 9i_2 - 3\,i_3 = 0$$
$$3\,(i_3 - i_2) + 10\,(i_3 + 8) + 5\,i_3 = 0 \quad \text{or} \quad -3\,i_2 + 18\,i_3 = -80$$

Solving these equations for i_2, we get

$$i_x = i_2 = \mathbf{2.79\ A}$$

(*b*) *By nodal analysis*: Since the largest number of branches are incident on the bottom node, we choose this as the reference node, as shown in Fig. A-37*b*. This leaves us with four nodes. However, since the voltage of the node between 100-V source and 8-Ω resistor is clearly known to be 100 V, we label the remaining nodes with voltages v_1, v_2, and v_3.

Applying KCL to these three nodes, we write

$$\frac{v_1 - 100}{8} + \frac{v_1}{4} + \frac{v_1 - v_2}{2} = 0 \quad \text{or} \quad 0.85\,v_1 - 0.5\,v_2 = 12.5$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + \frac{v_2 - v_3}{10} = 8 \quad \text{or} \quad -0.5\,v_1 + 0.9333\,v_2 - 0.1\,v_3 = 8$$

$$\frac{v_3 - v_2}{10} + \frac{v_3}{5} = -8 \quad \text{or} \quad -0.1\,v_2 + 0.3\,v_3 = -8$$

Solving these equations, we get

$$v_1 = 25.89\text{ V} \quad \text{and} \quad v_2 = 20.31\text{ V}$$

Thus, by Ohm's law, we get

$$i_x = \frac{v_1 - v_2}{2} = \mathbf{2.79\ A}$$

Note that for this particular problem, the mesh analysis proves to be simpler.

PROBLEM A-32

Use mesh analysis to find the power absorbed by the dependent voltage source in the circuit of Fig. A-38.

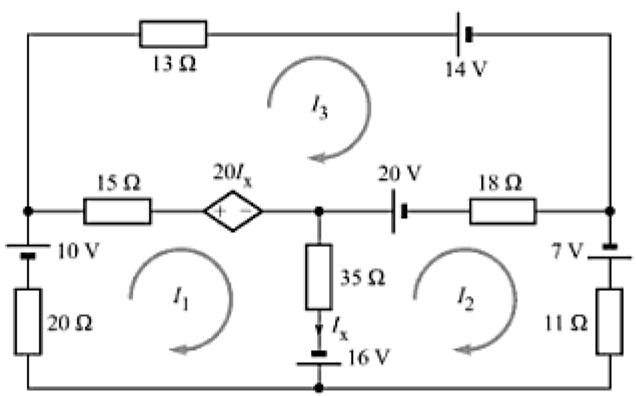


Fig. A-38

Solution The controlling quantity for the dependent source is $I_x = I_1 - I_2$. Hence the voltage of the dependent source is

$$20\,I_x = 20\,(I_1 - I_2)$$

In writing the mesh equations by applying KVL, we shall take the above value for the dependent voltage source,

$$20\,I_1 + 15\,(I_1 - I_3) + 20\,(I_1 - I_2) + 35\,(I_1 - I_2) = 10 + 16 \quad \text{or} \quad 90\,I_1 - 55\,I_2 - 15\,I_3 = 26$$

$$35(I_2 - I_2) + 18(I_2 - I_3) + 11 I_2 = -16 - 20 + 7 \quad \text{or} \quad -35 I + 64 I_2 - 18 I_3 = -29$$

$$13 I + 18(I_3 - I_2) + 15(I_3 - I_1) - 20(I_1 - I_2) = -14 + 20 \quad \text{or} \quad -35 I_1 + 2 I_2 + 46 I_3 = 6$$

In the matrix form, the above equations can be written as

$$\begin{bmatrix} 90 & -55 & -15 \\ -35 & 64 & -18 \\ -35 & 2 & 46 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 26 \\ -29 \\ 6 \end{bmatrix}$$

Note that due to the presence of the dependent voltage source, the resistance matrix has lost its symmetry. We could not have written the equations in matrix form just by inspection.

The solutions of the above equations are $i_1 = 0.148$ A, $I_2 = -0.3$ A, and $I_3 = 0.256$ A.

Finally, the power absorbed by the dependent voltage source is equal to the product of its voltage and the current flowing into its positive terminals:

$$P = [20(I_1 - I_2)](I_1 - I_3) = [20(0.148 + 0.3)](0.148 - 0.148 - 0.256) = \mathbf{-0.968\ W}$$

PROBLEM A-33

Consider the circuit of Fig. A-39*a*. Determine the maximum *positive* current to which the source I_x can be set before any resistor exceeds its power rating and overheats.

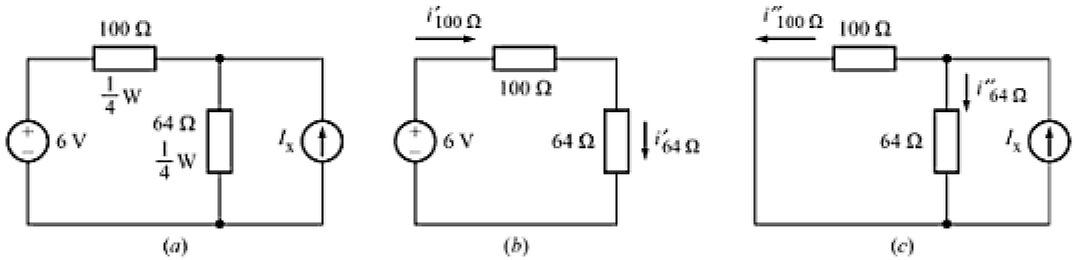


Fig. A-39

Solution Both the resistors are rated to a maximum of ¼ W = 0.25 W. If the circuit permits this value to be exceeded, excessive heating will occur. Since the 6-V battery cannot be changed, we should find an equation involving I_x and the maximum current through each resistor.

Since the power rating of each resistor is 0.25 W, the maximum current the 100-Ω resistor can tolerate is

$$i_{100\Omega(\max)} = \sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{0.25}{100}} = 50 \text{ mA}$$

Similarly, the maximum current the 64-Ω resistor can tolerate is

$$i_{64\Omega(\max)} = \sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{0.25}{64}} \quad 62.5 \text{ mA}$$

Next, to solve the problem we could apply mesh or nodal analysis. However, applying superposition principle gives us slight edge, since we are primarily interested in the effect of the current source. First, we find the contribution of the 6-V source to the currents in the two resistors by redrawing the circuit as in Fig. A-39*b*,

$$i_{100\Omega} = i_{64\Omega} = \frac{6}{100 + 64} = 36.59 \text{ mA}$$

Since this current is less than the maximum permissible current of either resistor, we are sure that the 6-V source acting alone will not pose any overheating problem.

To find the contribution of current source I_x, we consider the circuit of Fig. A-39*c*. We note that $i'_{100\,\Omega}$ is *oposite* in direction to $i'_{100\,\Omega}$, but $i''_{64\Omega}$ *adds* to $i'_{64\Omega}$. Therefore, the currents that can safely be contributed by the source I_x to the two resistors, respectively, are

$$i''_{100\Omega(\text{safe})} = i_{100\,\Omega(\text{max})} - (-i_{100\,\Omega}) = 50 - (-36.59) = 86.59 \text{ mA}$$

and
$$i''_{64\Omega(\text{safe})} = i_{64\Omega(\text{max})} - (i_{64\Omega}) = 6.25 - (36.59) = 25.91 \text{ mA}$$

The current I_x must be limited to a value that does not produce currents greater than the above safe values for the two resistors. The limiting value of I_x for the 100-Ω resistor is given by

$$i''_{100\Omega(\text{safe})} = I_{x(\text{lim})100\,\Omega}\left(\frac{64}{100+64}\right)$$

or
$$I_{x(\text{lim})100\,\Omega} = i''_{100\Omega(\text{safe})}\left(\frac{100+64}{64}\right) = 86.59\left(\frac{100+64}{64}\right) = 221.9 \text{ mA}$$

Similarly, the limiting value of I_x for the 64-Ω resistor is given by

$$I_{x(\text{lim})64\Omega} = i''_{64\Omega(\text{safe})}\left(\frac{100+64}{100}\right) = 25.91\left(\frac{100+64}{100}\right) = 42.49 \text{ mA}$$

Obviously, if none of the resistors is to be overheated, I_x must be less than **42.49 mA**. If the value of I_x is increased, the 64-Ω resistor will overheat long before the 100-Ω resistor does.

Note that originally we found that the 100-Ω resistor has a smaller maximum current. So, it might appear reasonable to expect it to limit I_x. However, because I_x *opposes* the current sent by 6-V source through the 100-Ω resistor but *adds* to the current sent by 6-V source through the 64-Ω resistor, it turns out that it is 64-Ω resistor which sets the limit on I_x.

PROBLEM A-34

Use superposition principle to determine the current i_x in the circuit of Fig. A-40*a*.

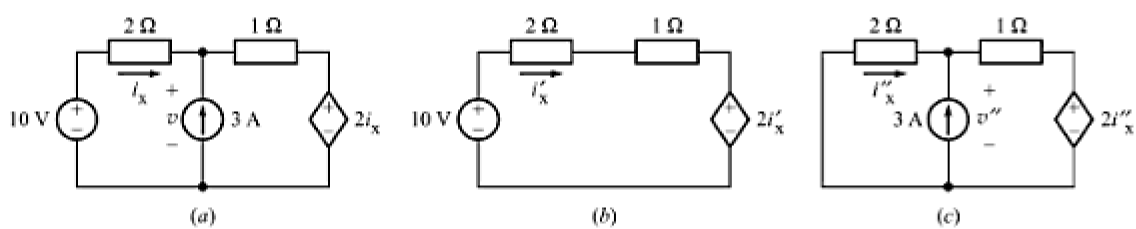


Fig. A-40

Solution The circuit has two independent sources and one dependent source. It is due to the independent sources that the dependent source has a non-zero value. Thus, we need to apply the superposition principle for the independent sources only. First, we consider the 10-V source acting alone (as in Fig. A-40*b*), and then the 3-A source acting alone (as in Fig. A-40*c*).

Writing the single-mesh equation for the circuit of Fig. A-40*b*,

$$2\,i'_x + 1\,i_x + 2\,i'_x = 10 \quad \Rightarrow \quad i'_x = 2\text{A}$$

Next, we write the single-node equation for the circuit of Fig. A-40*c*,

$$\frac{v''}{2} + \frac{v'' - 2i''_x}{1} = 3$$

and relate the dependent-source-controlling quantity to v'':

$$v'' = 2(-i''_x)$$

Solving above two equations, we get $i''_x = -0.6$A. Thus, the final result is

$$i_x = i'_x + i''_x = 2 + (-0.6) = \mathbf{1.4\ A}$$

PROBLEM A-35

Find the Thevenin's equivalent of the circuit of Fig. A-41.

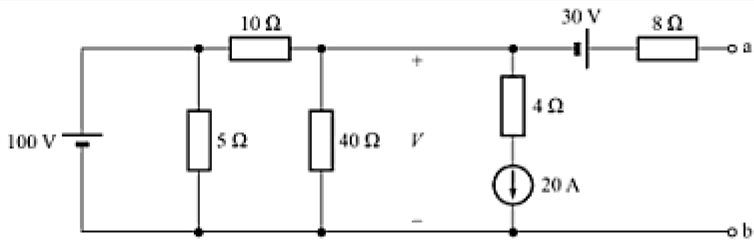


Fig. A-41

Solution Writing the single-node equation,

$$\frac{V-100}{10} + \frac{V}{40} = -20 \quad \Rightarrow \quad V = -80\ \text{V}$$

Since 8-Ω resistor has no current, Thevenin's voltage is given as

$$V_{Th} = V + 30 = -80 + 30 = \mathbf{-50\ V}$$

Note that the 5-Ω resistor (in parallel with the 100-V source) and the 4-Ω resistor (in series with the 20-A source) have no effect on V_{Th}.

For calculating R_{Th}, we turn off all the sources in the given circuit to get the circuit of Fig. A-42*a*. Here, the voltage source is replaced by a short circuit and the current source by an open circuit. Thevenin's resistance is given as

$$R_{Th} = R_{ab} = 8 + 40 \| 10 = \mathbf{16\ \Omega}$$

Note again that the 5-Ω resistor (in parallel with the 100-V source) and the 4-Ω resistor (in series with the 20-A source) have no effect on R_{Th}.

Fig. A-80*b* shows the Thevenin's equivalent circuit.

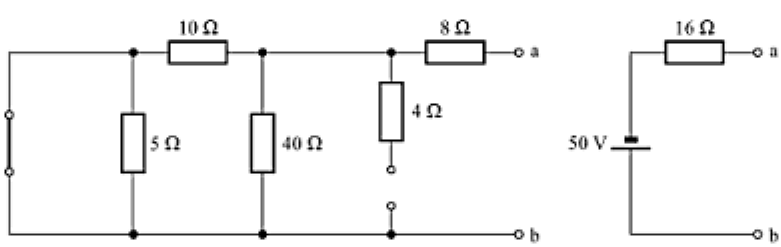


Fig. A-42

PROBLEM A-36

Compute the current through 2-Ω resistor in Fig. A-43*a*, by making use of source transformations to simplify the circuit to a single-loop circuit.

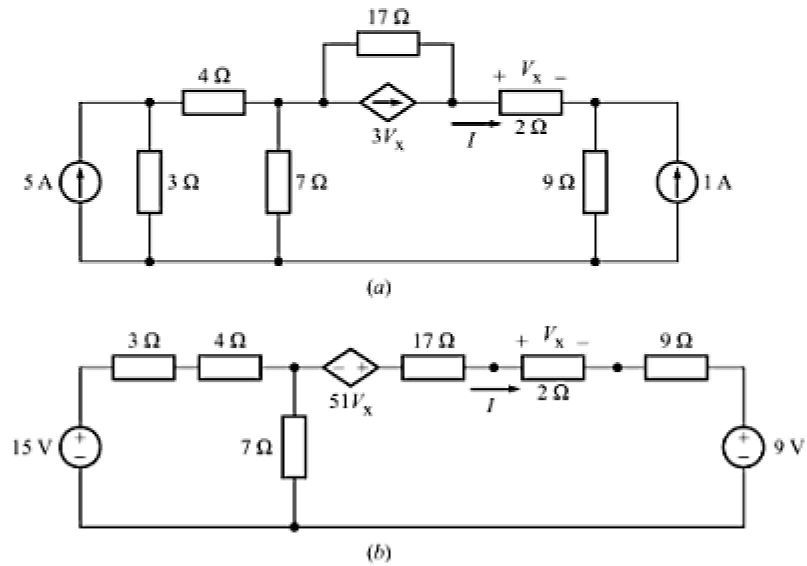


Fig. A-43

Solution We first convert each current source into a voltage source (Fig. A-43*b*). We can then combine resistances to simplify the circuit. However, we must be careful to retain the 2- Ω resistor for two reasons. First, the dependent-source controlling voltage V_x appears across this resistor. Second, we desire to calculate the current flowing through it. Thus, combining 17-Ω and 9-Ω resistors, we get a single 26-Ω resistor; and combining 3-Ω and 4-Ω resistors, we get a single 7-Ω resistor. Now, transforming the 15-V source in series with 7-Ω resistor into its equivalent current source, we get the circuit of Fig. A-44*a*.

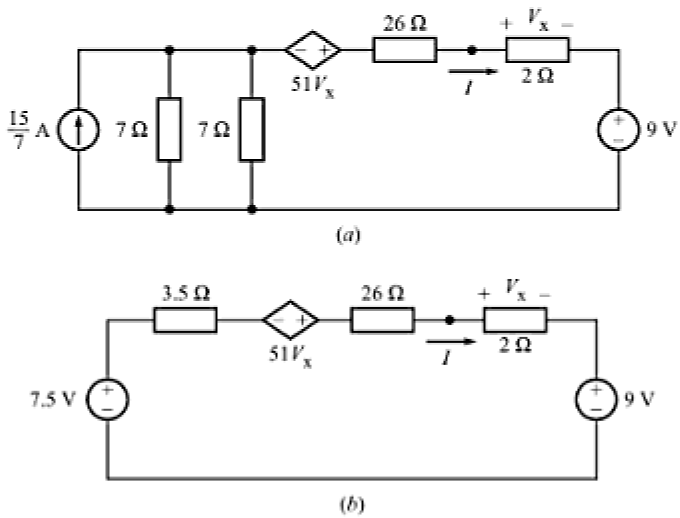


Fig. A-44

Next, we can combine the two 7-Ω resistors (in parallel) into a single 3.5-Ω resistor. This resistor can now be used to transform 15/7-A source into a 7.5-V source to get a single loop circuit (Fig. A-44*b*). We note that the controlling voltage $V_x = (2)I = 2I$. Applying KVL, we get

$$7.5 - 3.5I + 51V_x - 26I - 2I - 9 = 0 \quad \text{or} \quad -1.5 - 31.5I + 51(2I) = 0 \quad \Rightarrow \quad I = \mathbf{21.28\ mA}$$

PROBLEM A-37

Find the Thevenin's equivalent of the circuit shown in Fig. A-45*a*.

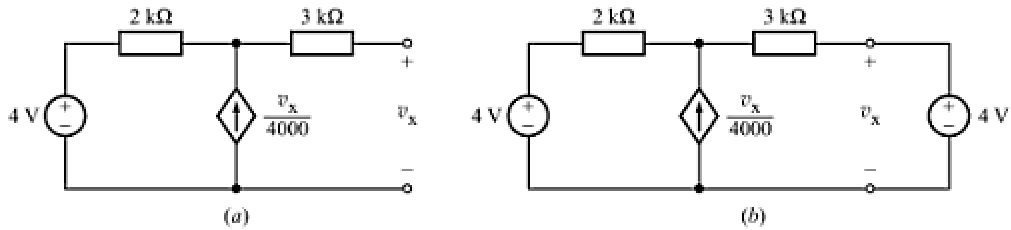


Fig. A-45

Solution Let us first find Thevenin's equivalent voltage V_{Th}. We note that the voltage V_{Th} is the same as the open-circuit voltage $V_{oc} = v_x$. With terminals open-circuited, the dependent source current must pass through the 2-kΩ resistor, since no current can flow through the 3-kΩ resistor. Applying KVL around the outer loop, we get

$$v_x + (3 \times 10^3)(0) - (2 \times 10^3)\left(\frac{v_x}{4000}\right) - 4 = 0 \quad \Rightarrow \quad v_x = 8\ \text{V}$$

Hence, $V_{Th} = v_x = \mathbf{8\ V}$

Next, we find Thevenin's equivalent resistance R_{Th}. Here, because of the presence of the dependent source, we cannot find R_{Th} directly by first making the circuit inactive and then finding the resistance combination. Some special technique is needed:

First Method We connect a suitable voltage source across the terminals and find the resulting current through it. The ratio of this voltage to the current gives us R_{Th}. Here, it is convenient to connect a 4-V source (Fig. A-45*b*). We then have v_x = 4 V and the dependent current source becomes

$$\frac{v_x}{4000} = \frac{4}{4000} = 1\ \text{mA}$$

The 4-V source in series with 2-kΩ resistor can be transformed into an equivalent 2-mA current source in parallel with 2-kΩ resistor (as shown in Fig. A-46*a*). Combining the two current sources and transforming it to an equivalent voltages source, we get the circuit of Fig. A-46*b*. The current I is given as

$$I = \frac{(6-4)\ \text{V}}{(2+3)\ \text{k}\Omega} = \frac{2}{5}\ \text{mA} = 0.4\ \text{mA}$$

Thus, R_{Th} is given as

$$R_{Th} = \frac{4\ \text{V}}{0.4\ \text{mA}} = \mathbf{10\ k\Omega}$$

Second Method A much simpler way is to find the short-circuit current I_{sc}, so that R_{Th} is given by the ratio of V_{oc} to I_{sc}. Upon short-circuiting the output terminals in Fig. A-45*a*, it is obvious that $v_x = 0$ and consequently the dependent current source is dead. Hence,

$$I_{sc} = \frac{4\ \text{V}}{(2+3)\ \text{k}\Omega} = 0.8\ \text{mA}; \quad R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{V_{Th}}{I_{sc}} = \frac{8\ \text{V}}{0.8\ \text{mA}} = \mathbf{10\ k\Omega}$$

The Thevenin's equivalent is shown in Fig. A-46*c*.

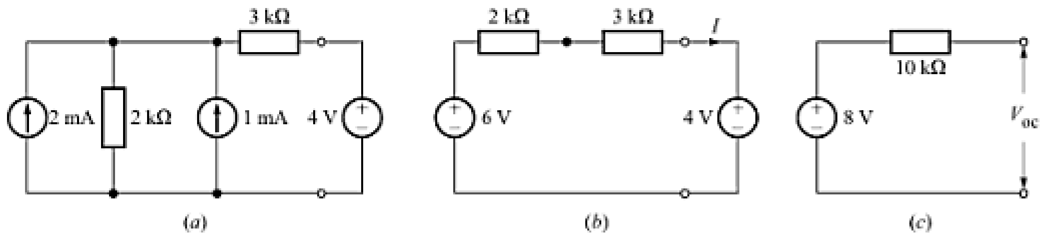


Fig. A-46

PROBLEM A-38

Find the Thevenin's equivalent of the circuit of Fig. A-47*a*.

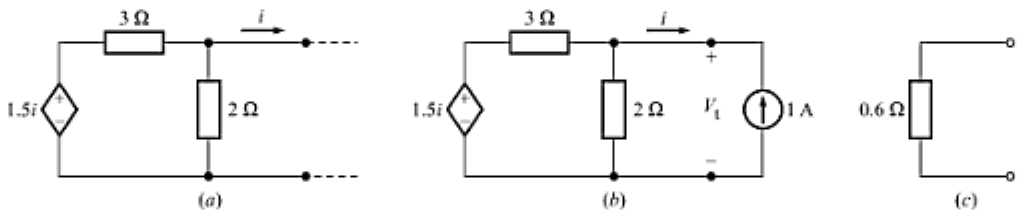


Fig. A-47

Solution The given circuit has a dependent source but no independent source. Since the terminals are already open-circuited, $i = 0$. Consequently, the dependent source is dead and hence $V_{Th} = V_{oc} = \mathbf{0}$.

Next, we find R_{Th}. Since the circuit has no independent source, both V_{oc} and I_{sc} are zero. Hence, we cannot determine R_{Th} by taking the ratio of V_{oc} and I_{sc}. Let us, therefore, be a little tricky. We apply a 1-A source externally (as in Fig. A-47*b*), and find the resulting voltage V_t across the terminals. The Thevenin's resistance R_{Th} is then given by the ratio $V_t/1$. Applying nodal analysis, we get

$$\frac{V_t - 1.5i}{3} + \frac{V_t}{2} = 1 \quad \text{or} \quad \frac{V_t - 1.5(-1)}{3} + \frac{V_t}{2} = 1 \quad \Rightarrow \quad V_t = 0.6 \text{ V}$$

$$\therefore \qquad R_{Th} = \frac{V_t}{1} = \frac{0.6}{1} = \mathbf{0.6\ \Omega}$$

Thevenin's equivalent, shown in Fig. A-47*c*, is just a resistor of 0.6 Ω.

PROBLEM A-39

In the circuit of Fig. A-48, what resistor R_L will absorb maximum power and what is this power?

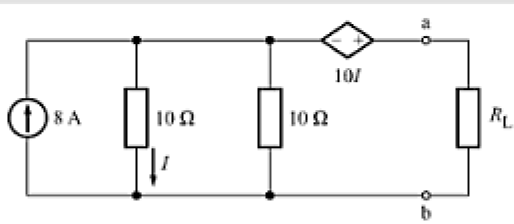


Fig. A-48

Solution We first obtain the Thevenin's equivalent of the portion of the circuit to the left of terminals a and b. Replacing R_L by an open circuit and then applying current division, we get

$$I = 8 \times \frac{40}{40 + 10} = 6.4 \text{ A}$$

Consequently, the dependent voltage source becomes $10I = 64$ V. Now, applying KVL, we get

$$V_{Th} = V_{oc} = V_{ab} = 10I + 64 = 10 \times 6.4 + 64 = 128 \text{ V}$$

Thevenin's resistance R_{Th} can be conveniently determined by the short-circuit current approach. When a short-circuit is placed across terminals a and b, all the components of the circuit of Fig. A-48 become in parallel. Therefore, the voltage drop (from top to bottom) across the 10-Ω resistor must be equal to voltage across the dependent voltage source. That is,

$$10I = -10I$$

This is possible only if $I = 0$. Consequently, the voltage drop across both resistors must be zero. It means that all of the 8 A of the current source must flow through the short circuit. That is, $I_{sc} = 8$ A. Therefore,

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{128}{8} = 16 \ \Omega$$

Thus, for maximum power absorption, $R_L = R_{Th} = \mathbf{16\ \Omega}$. Finally, the maximum power absorbed is

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(128)^2}{4 \times 16} = \mathbf{256\ W}$$

PROBLEM A-40

Refer to the circuit shown in Fig. A-49. (*a*) Find current *I* by using a Δ-Y transformation. (*b*) What resistor *R* replacing the 20-Ω resistor causes the bridge to be balanced? Also, what is the current *I* then?

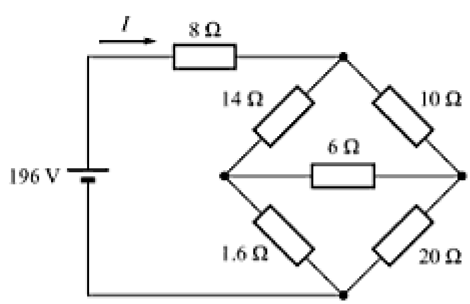


Fig. A-49

Solution

(*a*) We transform the top Δ to a Y. Figure A-50*a* shows the top Δ enclosing a Y as a memory aid for this transformation. All three Y formulae have the same denominator:

$$D = 14 + 10 + 6 = 30 \ \Omega$$

The numerators are the product of the adjacent Δ resistors :

$$R_A = \frac{10 \times 14}{30} = 4.67 \ \Omega; \quad R_B = \frac{14 \times 6}{30} = 2.8 \ \Omega; \quad R_c = \frac{6 \times 10}{30} = 2 \ \Omega$$

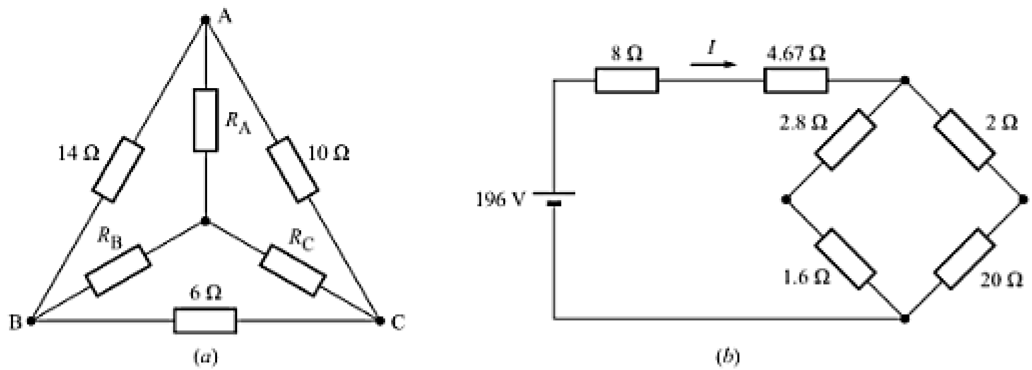


Fig. A-50

With this transformation, the given circuit simplifies to that shown in Fig. A-50*b*. Note that in this circuit all the resistors are in series-parallel. Thus,

$$I = \frac{196}{8 + 4.67 + (2.8 + 1.6) \parallel (2 + 20)} = \mathbf{12\ A}$$

(*b*) We know that for the balance of the Wheatstone bridge, the product of opposite arms must be equal. That is

$$R \times 14 = 1.6 \times 10 \quad \Rightarrow \quad R = \frac{16}{14} = \mathbf{1.14\ \Omega}$$

Under balance condition, the centre arm of the bridge carries no current. Hence, we can consider this as an open circuit, so that the bridge becomes a series-parallel arrangement. The current I is then

$$I = \frac{196}{8 + (14 + 1.6) \parallel (10 + 1.14)} = \mathbf{13.5\ A}$$

PROBLEM A-41

Using a Δ-Y transformation, find the currents I_1, I_2, and I_3 for the circuit shown in Fig. A-51.

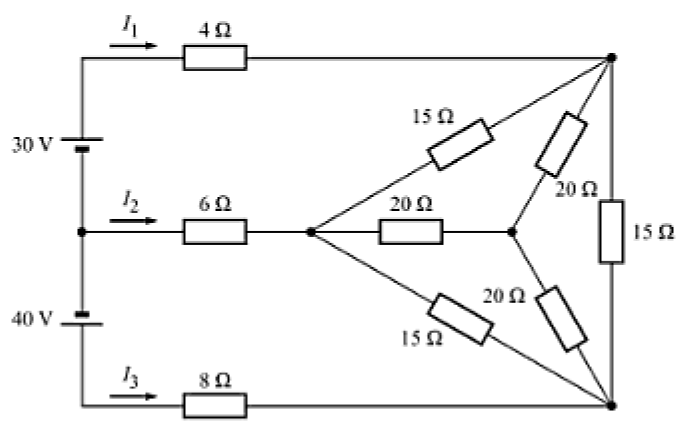


Fig. A-51

Solution The Δ of 15-Ω resistors transforms to a Y of 15/3 = 5-Ω resistors. These resistors are in parallel with the Y of 20-Ω resistors, as shown in Fig. A-52*a*.

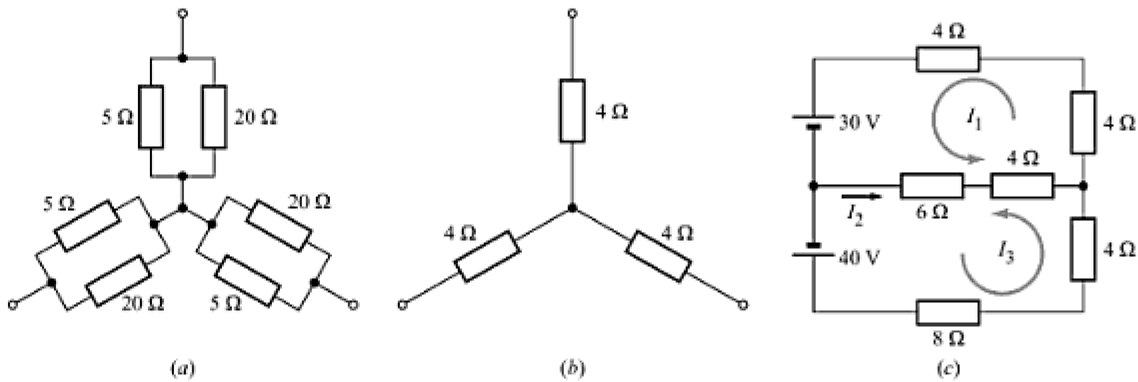


Fig. A-52

Note that it is not obvious that the resistors of the two Y-configurations are in parallel. In fact, they would not be in parallel if all the resistances of each Y were not having the same value. However, when the resistances have the same value, as here, an analysis will show that the centre nodes of the two Y-configurations are at the same potential, just as if a wire were connected between them.

Comment What would you do to solve such a problem, if all the resistances of each Y were not having the same value? In such case, we would transform the Y into a Δ. Then the resistors of the two Δ configurations are necessarily in parallel. Using the parallel combinations, we would find a single Δ, which would again be converted to a Y.

Let us now proceed with the solution. The two Y's can be reduced to a single Y shown in Fig. A-52*b*, in which each Y resistance is 5||20 = 4 Ω. Replacing the Δ-Y combination in Fig. A-51 by this Y, we get a circuit shown in Fig. A-90*c*. Writing KVL equations for the two loops, we get

$$18I_1 + 10I_3 = 30 \qquad \text{and} \qquad 10I_1 + 22I_3 = 40$$

Solving these, we get I_1 = **0.88 A** and I_3 = **1.42 A**. Then, KCL applied to the right hand node gives $I_2 = -I_1 - I_3 =$ **–2.3 A**.

A. 2. PRACTICE PROBLEMS

(A) SIMPLE PROBLEMS

A-1. Find the energy stored in a 12-V car battery rated at 650 Ah. **[Ans.** 28.08 MJ]

A-2. If a 0.25-A current flowing through a light bulb for 4 s causes it to give off 240 J of light and heat energy, what must be the voltage drop across the light bulb? **[Ans.** 240 V]

A-3. Find the power absorbed by each element in the circuit of Fig. A-53.

[Ans. (from left to right) –56 W, 16 W, 60 W, –160 W, –60 W]

A-4. The circuit of Fig. A-54 shows a voltage source of *V* volts connected to a current source of *I* amperes. Find the power absorbed by the voltage source for

(*a*) $V = 2$ V, $I = 4$ A;

(*b*) $V = 3$ V, $I = -2$ A;

(*c*) $V = -6$ V, $I = -8$ A.

[Ans. (*a*) 8 W; (*b*) –6 W; (*c*) 48 W]

A-5. Determine the resistivity of platinum if a cube of 1 cm along each edge has a resistance of

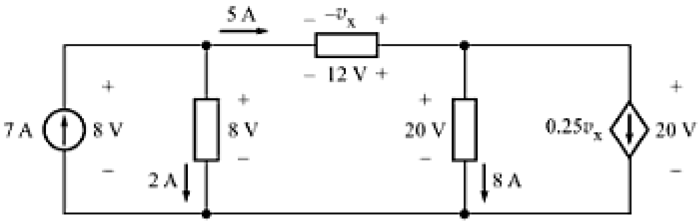


Fig. A-53

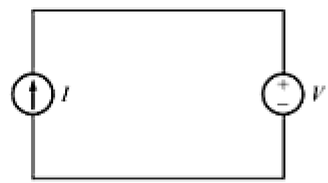


Fig. A-54

10 μΩ across opposite faces. **[Ans.** 0.1 mΩm]

A-6. In the circuit shown in Fig. A-55, if $V_x = 1$ V and $V_R = 9$ V, calculate (*a*) the power absorbed by the element *A*, (*b*) the power supplied by each of the two sources. **[Ans.** (*a*) 45 W; (*b*) 5 W, 40 W]

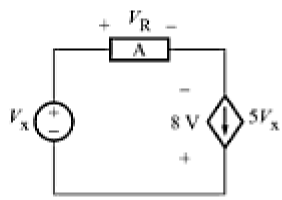


Fig. A-55

A-7. For the circuit shown in Fig. A-56, if $v_2 = 1000 i_2$ and $i_2 = 5$ mA, determine v_S. **[Ans.** – 1 mV]

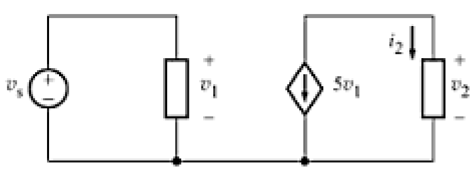


Fig. A-56

A-8. In the circuit of Fig. A-57, if $v_{s1} = 120$ V, $v_{s2} = 30$ V, $R_1 = 30$ Ω, and $R_2 = 30$ Ω, compute the power absorbed by each component.

[Ans. $P_{120V} = -240$ W, $P_{30V} = 60$ W, $P_{30\Omega} = 120$ W, $P_{15\Omega} = 60$ W]

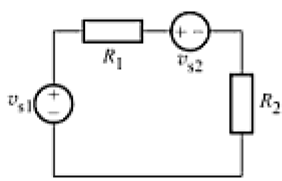


Fig. A-57

A-9. Determine the current *i* and the power delivered by the 80-V source in the circuit of Fig. A-58. **[Ans.** 3 A, 240 W]

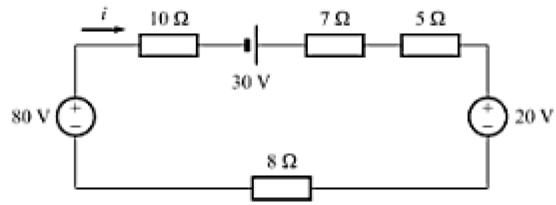


Fig. A-58

A-10. Determine the current *i* in the circuit of Fig. A-59. **[Ans.** –0.333 A]

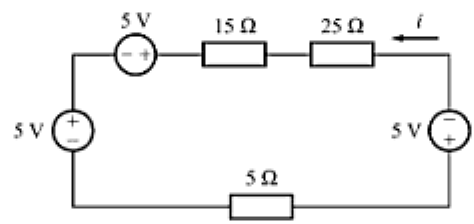


Fig. A-59

A-11. Find the internal resistance of a 2-kW water heater that draws 8.33 A current. **[Ans.** 28.8 Ω]

A-12. Find the voltage v in the circuit given in Fig. A-60.

[Ans. 12.73 V]

A-13. Find the current and unknown voltages in the circuit shown in Fig. A-61.

[Ans. 0.3 A, 3 V, 4.5 V, – 1.8 V, 2.4 V, –3.3 V]

A-14. Find the voltage V_{ab} in the circuit shown in Fig. A-61. **[Ans.** 5.7 V]

A-15. A string of Deepawali lights consists of forty 3-W,

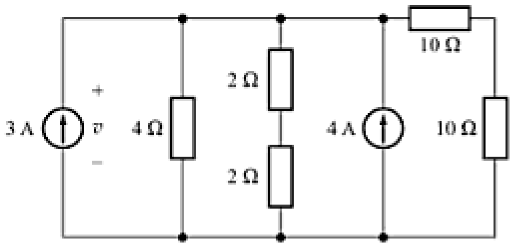


Fig. A-60

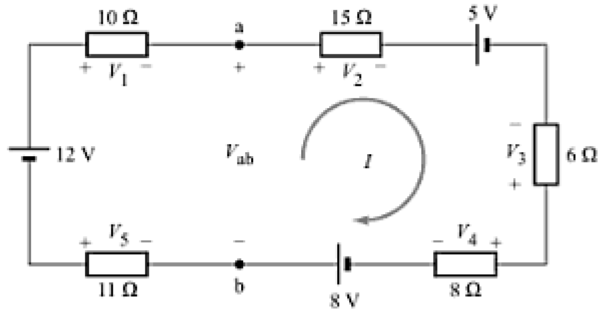


Fig. A-61

6-V bulbs connected in series. What current flows when the string is plugged into a 240-V socket, and what is the net hot resistance of each bulb.

[**Ans.** 0.5 A, 12 Ω]

A-16. A resistance *R* in series with an 8-Ω resistor absorbs 100 W when the two are connected across 60-V line. Determine the unknown resistance *R*.

[**Ans.** 16 Ω or 4 Ω]

A-17. In a series circuit, a current flows from the positive terminal of a 180-V source through two resistors—one has 30 Ω resistance and the other of which has 45 V across it. Find the current and the unknown resistance. [**Ans.** 4.5 A, 10 Ω]

A-18. Three parallel resistors have a total conductance of 2 mS. If two of the resistances are 1 kΩ and 5 kΩ, find the third resistance. [**Ans.** 1.25 kΩ]

A-19. Refer to the circuit of Fig. A-62.

(*a*) Determine the resistance *R* that results in the 12-V source delivering 3.6 mW to the circuit.

(*b*) Determine the resistance *R* that results in the 25-kΩ resistor absorbing 1 mW.

(*c*) Replace the resistor *R* with a voltage source such that no power is absorbed by either resistor.

[**Ans.** (*a*) $R = 0$; (*b*) $R = 20$ kΩ; (*c*) 12 V]

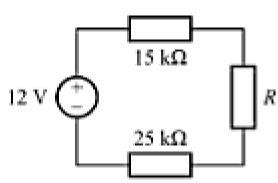


Fig. A-62

A-20. Find the voltages across the various components in the circuit of Fig. A-63. [**Ans.** 5 V, 2 V, 3 V]

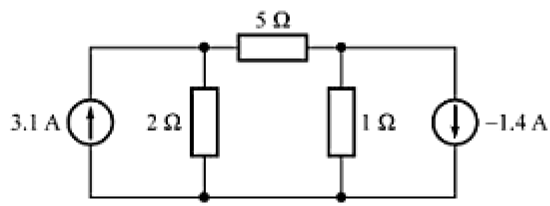


Fig. A-63

A-21. Determine the current in 4-Ω resistor in the circuit of Fig. A-64. [**Ans.** 0.83 A]

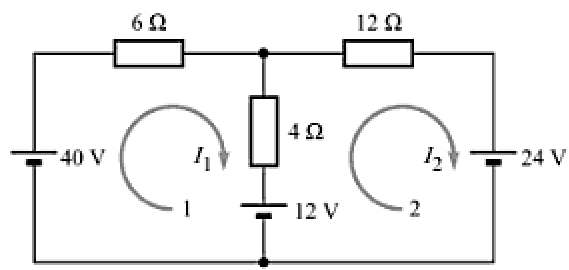


Fig. A-64

A-22. Use the technique of mesh analysis to determine i_1 and i_2 in the circuit of Fig. A-65, and hence find the power absorbed by the 3-Ω resistor.

[**Ans.** 12 W]

A-23. Determine the mesh currents in the circuit of Fig. A-66. [**Ans.** 5 A, −8 A, 2 A]

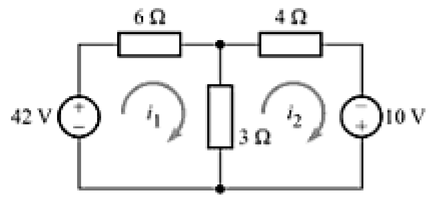


Fig. A-65

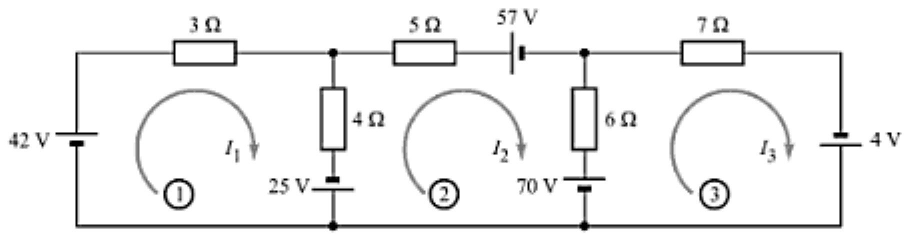


Fig. A-66

A-24. Two 12-V batteries are being charged from a 16-V generator with an internal resistance of 2 Ω. The internal resistances for the two batteries are 0.8 Ω and 0.5 Ω. Determine the currents flowing into the positive terminals of the batteries.
[Ans. 0.667 A, 1.07 A]

A-25. Find the node voltages in the circuit shown in Fig. A-67. **[Ans.** $V_1 = 8.77\ V$, $V_2 = 7.62$ V]

A-26. Find the mesh currents in the circuit shown in Fig. A-68. **[Ans.** $I_1 = 3$ A, $I_2 = -8$ A, $I_3 = 7$ A]

A-27. Use repeated source transformation to obtain current I in the circuit of Fig. A-69. **[Ans.** 2 A]

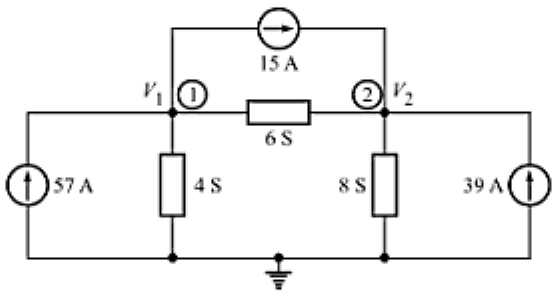


Fig. A-67

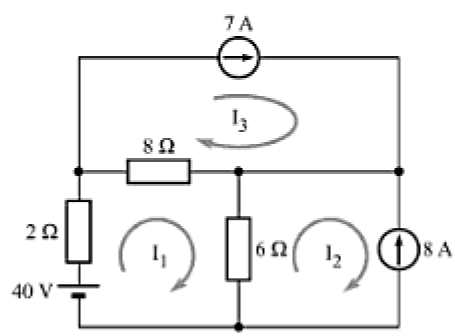


Fig. A-68

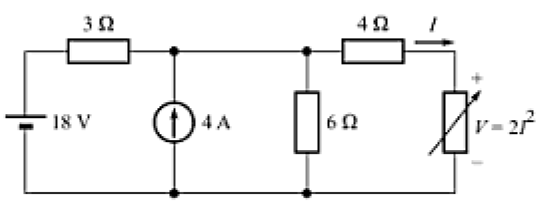


Fig. A-69

A-28. Solve for the mesh currents in the circuit of Fig. A-70. **[Ans.** $I_1 = 5$ mA, $I_2 = -2$ mA]

A-29. Determine the mesh currents in the circuit of Fig. A-70, if the 24-V source is replaced by a source of – 1 V. **[Ans.** $I_1 = 7$ mA, $I_2 =$ mA]

A-30. For the circuit of Fig. A-71, use superposition theorem to find the current i_x in 15-Ω resistor.
[Ans. 0.66 A]

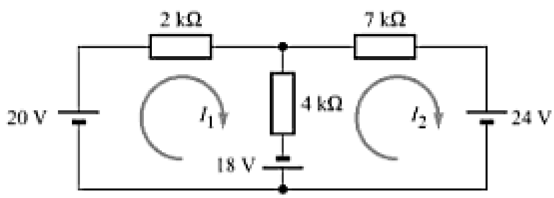


Fig. A-70

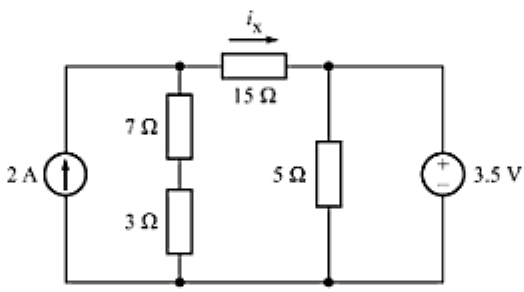


Fig. A-71

A-31. What resistor draws a current of 5 A when connected across terminals *a* and *b* of the circuit of Fig. A-72. **[Ans.** 6 Ω**]**

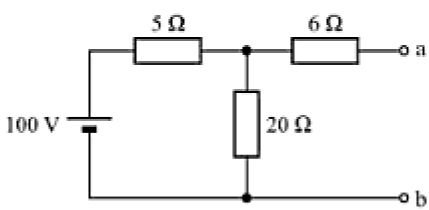


Fig. A-72

A-32. Compute current *I* in the circuit of Fig. A-73 after transforming the 9-mA current source into an equivalent voltage source. **[Ans.** 3.307 mA**]**

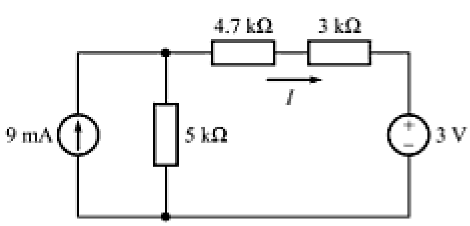


Fig. A-73

A-33. For the circuit of Fig. A-74, compute the current I_x through the 47-kΩ resistor after transforming the 5-V source into an equivalent current source.

[Ans. 192 μA**]**

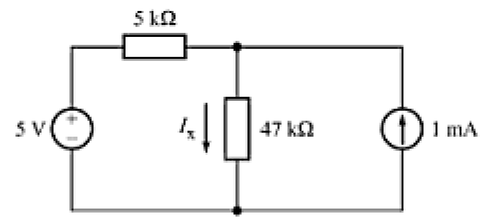


Fig. A-74

A-34. Find the voltage *V* across the 1-MΩ resistor in the circuit of Fig. A-75, by repeatedly using source transformations. **[Ans.** 27.23 V**]**

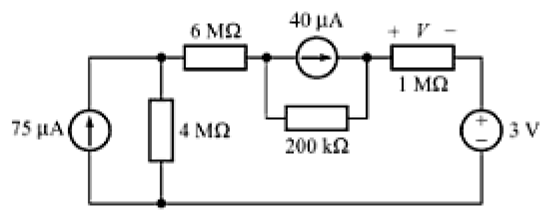


Fig. A-75

A-35. Using repeated source transformations, determine the Norton's equivalent of network inside the dotted box in the circuit of Fig. A-76, and hence find the current that would flow in R_L if (*a*) R_L = 15 Ω, (*b*) R_L = 10 Ω, and (*c*) R_L = 5 Ω.

[Ans. I_N = 1 A, R_N = 5 Ω, (*a*) 0.25 A, (*b*) 0.333 A, (*c*) 0.5 A**]**

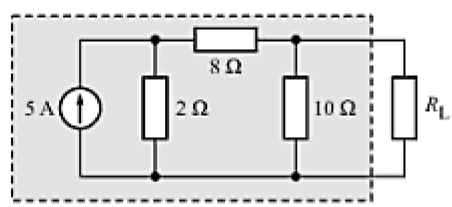


Fig. A-76

A-36. Use Thevenin's theorem to determine the current $I_{2\Omega}$ in the circuit of Fig. A-77.

[Ans. 260.8 mA**]**

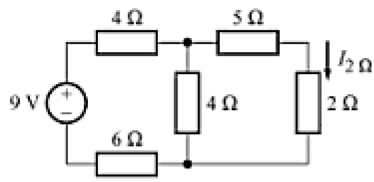


Fig. A-77

A-37. Find Thevenin's and Norton's equivalents for the network faced by 1-kΩ resistor of Fig. A-78 (i.e., treating 1-kΩ resistor as load).

[**Ans.** 8 V, 1.6 mA, 5 kΩ]

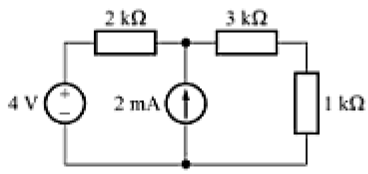


Fig. A-78

A-38. Find the Thevenin's and Norton's equivalents of the circuit of Fig. A-79.

[**Ans.** –7.855 V, –3.235 mA, 2.429 kΩ]

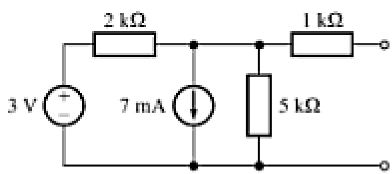


Fig. A-79

A-39. Find the Thevenin's equivalent of the circuit shown in Fig. A-80. [**Ans.** 12 V, 12 Ω]

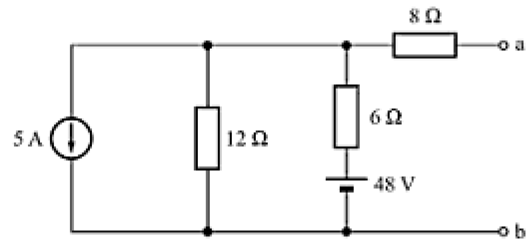


Fig. A-80

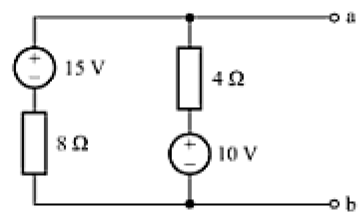


Fig. A-81

A-40. Convert the circuit shown in Fig. A-81 to a single voltage source in series with a single resistor.

[**Ans.** 35/3 V, 8/3 Ω]

(B) TRICKY PROBLEMS

A-41. For the circuit shown in Fig. A-82, determine the power absorbed P_1, P_2 and P_3 for (*a*) $I = 2$ A, and (*b*) $I = -3$ A.

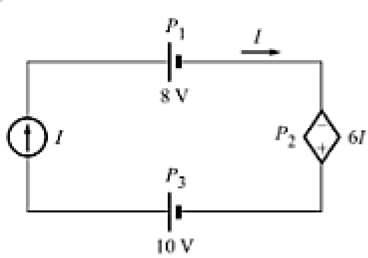


Fig. A-82

[**Ans.** (a)$P_1 = 16$ W, $P_2 = -24$ W, $P_3 = -20$ W;
(b) $P_1 = -24$ W, $P_2 = -54$ W, $P_3 = 30$ W]

A-42. A dc circuit shown in Fig. A-83 has a voltage source V, a current source I and many resistors. In an experiment, it was found that a particular resistor R dissipates 4 W when the voltage source V alone is active, and it dissipates 9 W when the current source I alone is active. Determine the power dissipated by R when both sources are active.

[**Ans.** 25 W]

A-43. In the circuit shown in Fig. A-84, determine v_x.

[**Ans.** 12.8 V]

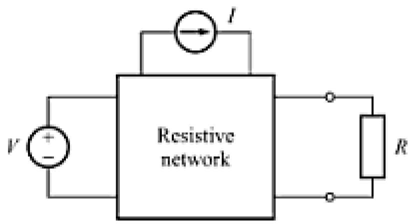


Fig. A-83

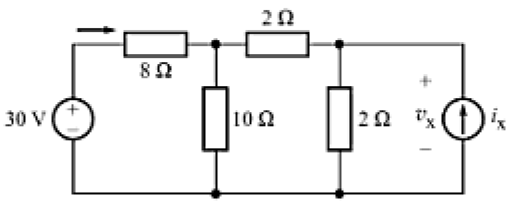


Fig. A-84

A-44. Determine voltage v in the circuit of Fig. A-85.

[**Ans.** 50 V]

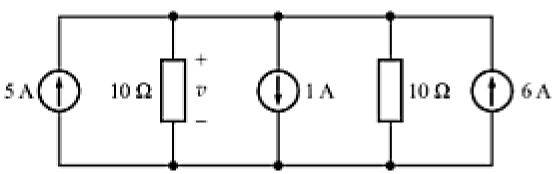


Fig. A-85

A-45. A short circuit across a current source draws 20 A. When the current source has an open circuit across it, the terminal voltage is 600 V. Find the internal resistance of the source. [**Ans.** 30 Ω]

A-46. A short circuit across a current source draws 15 A. If a 10-Ω resistor across the source draws 13 A, what is the internal resistance of the source?

[**Ans.** 65 Ω]

A-47. Resistors R_1, R_2 and R_3 are in series with a 100-V source. The total voltage drop across R_1 and R_2 is 50 V, and that across R_2 and R_3 is 80 V. Find the three resistances, if the total resistance is 50 Ω.

[**Ans.** 10 Ω, 15 Ω and 25 Ω]

A-48. In the circuit of Fig. A-86, find i_1, i_2 and v_3 using resistance combination method and current division. [**Ans.** 100 mA, 50 mA, 0.8 V]

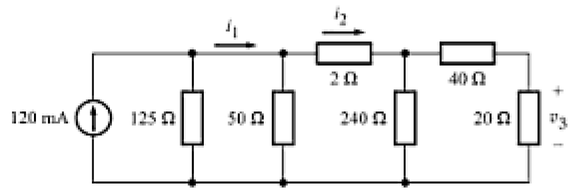


Fig. A-86

A-49. Refer to the circuit of Fig. A-87. (*a*) If $i_x = 5$ A, find v_1 and i_y. (*b*) If $v_1 = 3$ V, find i_x and i_y.

[**Ans.** (*a*) 2.5 A, 25 V; (*b*) 0.6 A, 0.3 A]

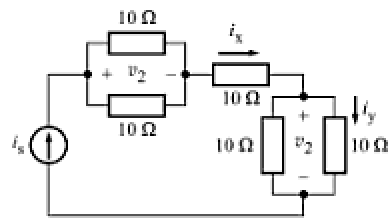


Fig. A-87

A-50. Find the total resistance R_T of the resistor ladder network given in Fig. A-88. [**Ans.** 34 Ω]

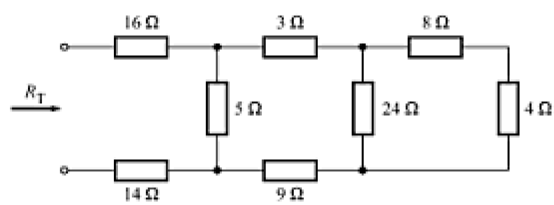


Fig. A-88

A-51. In the circuit shown in Fig. A-89, find the total resistance R_T with the terminals a and b (*a*) open-circuited, and (*b*) short-circuited.

[**Ans.** (*a*) 45.5 Ω; (*b*) 33 Ω]

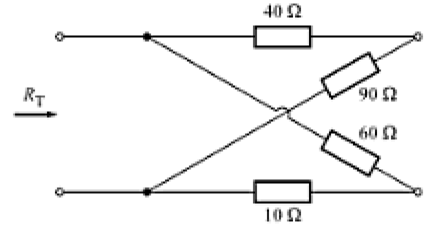


Fig. A-89

A-52. A 12-V battery with a 0.3-Ω internal resistance is to be charged from a 15-V source. If the charging current should not exceed 2 A, what is the minimum resistance of a series resistor which will limit the current to this safe value? **[Ans.** 1.2 Ω**]**

A-53. Find the total resistance R_T of the resistor ladder network shown in Fig. A-90. **[Ans.** 26.6 kΩ**]**

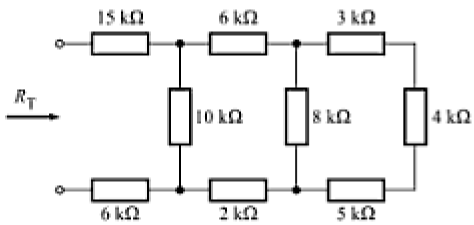


Fig. A-90

A-54. In the circuit shown in Fig. A-91, find the total resistance R_T with the terminals *a* and *b* (*a*) open-circuited, and (*b*) short-circuited.

[Ans. (*a*) 18.2 Ω; (*b*) 18.1 Ω**]**

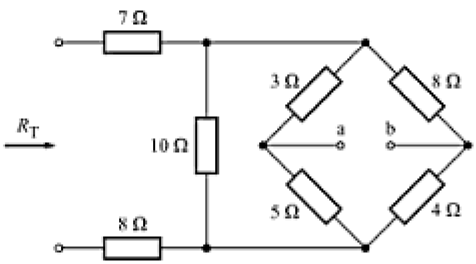


Fig. A-91

A-55. The equivalent resistance of three parallel resistors is 10 Ω. If two of the resistors have resistances of 40 Ω and 60 Ω, determine the resistance of the third resistor. **[Ans.** 17.2 Ω**]**

A-56. Calculate voltage V_1 in the circuit of Fig. A-92.

[Ans. 96 V**]**

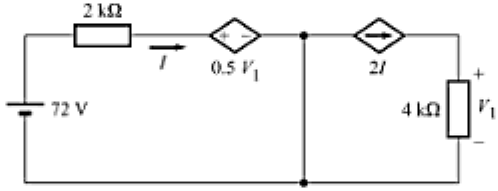


Fig. A-92

A-57. Find R_1 and R_2 in the circuit of Fig. A-93.

[Ans. 20 Ω and 5 Ω**]**

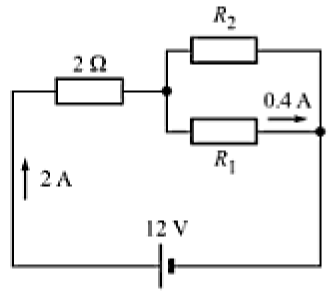


Fig. A-93

A-58. Use voltage division twice to find the voltage V in the circuit of Fig. A-94.

[Ans.: 36 V**]**

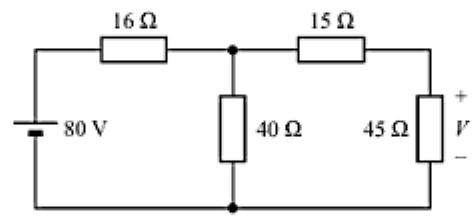


Fig. A-94

A-59. In the circuit of Fig. A-95, use current division twice to find the current I in the load resistor R_L for (*a*) R_L = 0 Ω, (*b*) R_L = 5 Ω, (*c*) R_L = 25 Ω.

[Ans. (*a*) 16 A; (*b*) 9.96 A; (*c*) 3.96 A**]**

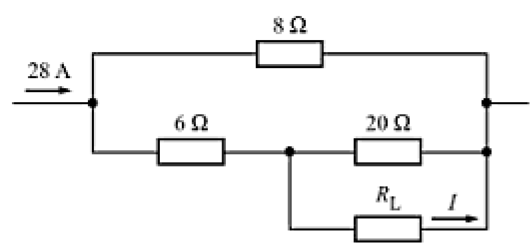


Fig. A-95

A-60. Find i_1 in the circuit of Fig. A-96.

[Ans. 1.75 A**]**

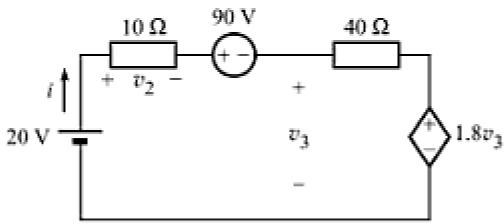


Fig. A-96

A-61. Find the power absorbed by each of the resistors in the circuit of Fig. A-97.

[**Ans.** $P_{2.5\Omega}$ = 250 W, $P_{30\Omega}$ =187.5 W, $P_{6\Omega}$ = 337.5 W, $P_{5\Omega}$ =180 W, $P_{20\Omega}$ = 45 W.]

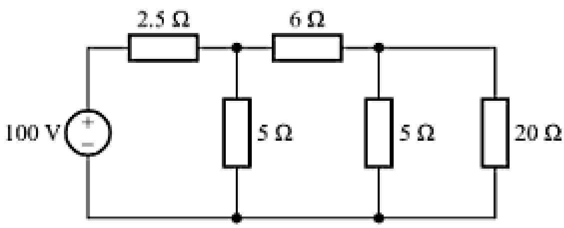


Fig. A-97

A-62. In the circuit of Fig. A-98, find the current through 10-Ω resistor. [**Ans.** 7.142 A]

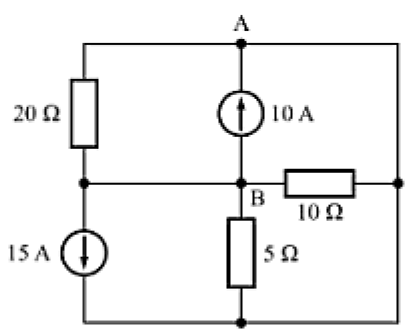


Fig. A-98

A-63. In the circuit of Fig. A-99, (*a*) find R_1 and R_2 such that I_1 = 1 A and I_2 = 5 A, and (*b*) find R_2 such that I_1 = 0.

[**Ans.** (*a*) R_1 = 20 Ω, R_2 = 8 Ω; (*b*) R_2 = 2 Ω]

A-64. For the circuit of Fig. A-100, find the voltage across each current source. [**Ans.** 5.375 V, 0.375 V]

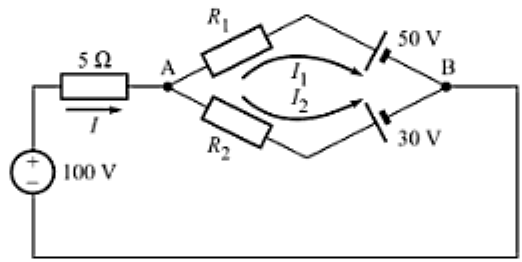


Fig. A-99

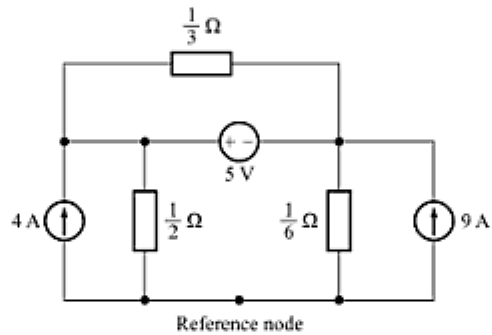


Fig. A-100

A-65. Determine the node voltages V_1 and V_2 in the circuit of Fig. A-101. [**Ans.** 3.38 V, −5 V]

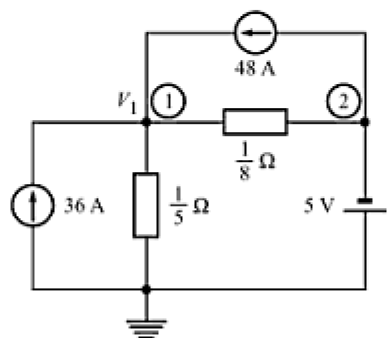


Fig. A-101

A-66. Determine the current I_x in the circuit of Fig. A-102. [**Ans.** −4.86 mA]

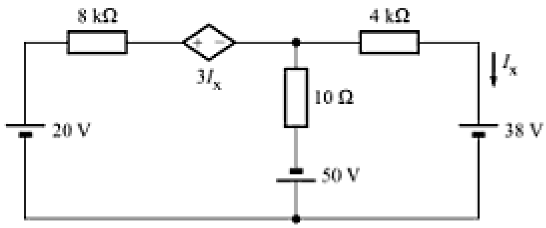


Fig. A-102

A-67. Find the input resistance R_{in} in the circuit of Fig. A-103. **[Ans.** 33.3 Ω]

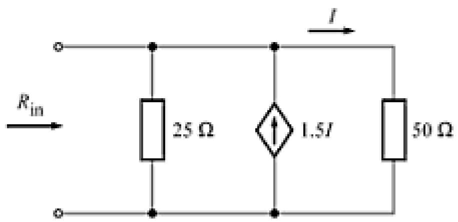


Fig. A-103

A-68. Find the Thevenin's equivalent for the network of Fig. A-104. **[Ans.** –502.5 mV, –100.5 Ω]

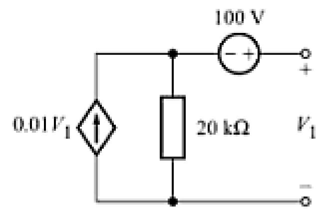


Fig. A-104

A-69. Use superposition principle to find the power absorbed by the 12-Ω resistor in the circuit shown in Fig. A-105. **[Ans.** 685 W]

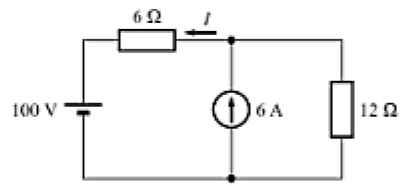


Fig. A-105

(C) CHALLENGING PROBLEMS

A-70. A wire, 50 m in length and 2 mm^2 in cross section, has a resistance of 0.56 Ω. A 100-m length of the same wire has a resistance of 2 Ω at the same temperature. Find the diameter of this wire.

[Ans. 1.19 mm]

A-71. Compute the power absorbed by each element in the circuit of Fig. A-106.

[Ans. $P_{120V} = -960$ W, $P_{30\Omega} = 1920$ W, $P_{dep} = -1920$ W, $P_{15\Omega} = 960$ W]

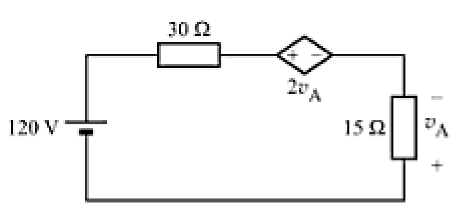


Fig. A-106

A-72. Determine i_A, i_B, and i_C in the circuit of Fig. A-107.

[Ans. 32 turns]

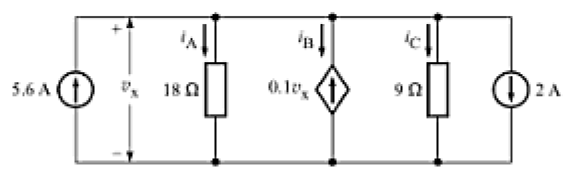


Fig. A-107

A-73. A wire-wound resistor is to be made from a 0.2-mm diameter constantan wire wound around a cylinder of 1-cm diameter. How many turns are needed for a resistance of 50 Ω. (Take resistivity of constantan = 49×10^{-8} Ωm) **[Ans.** 32 turns]

A-74. Determine the power absorbed by the 47-kΩ resistor in the circuit of Fig. A-108.

[Ans. 18.12 μW]

A-75. A resistor in series with a 100-Ω resistor absorbs 80 W when the two are connected across a 240-V line. Find the unknown resistance.

[Ans. 20 Ω or 500 Ω]

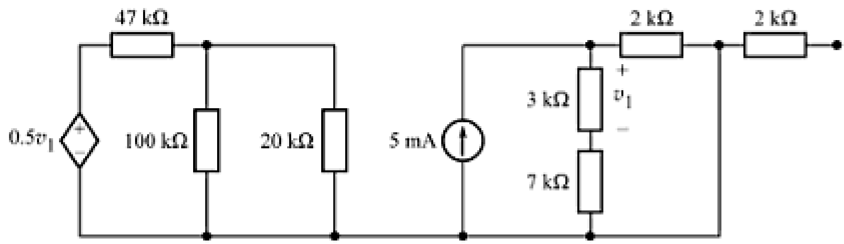


Fig. A-108

A-76. The light bulb shown in the circuit of Fig. A-109. has a rating of 120-V, 60-W. What must be the supply voltage V_s for the light bulb to operate at rated values. [**Ans.** 285 V]

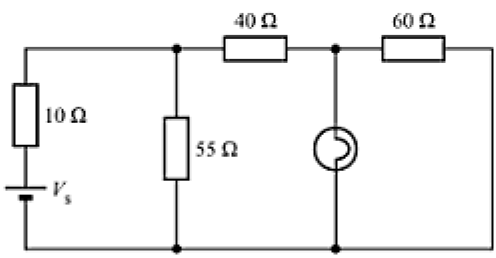


Fig. A-109

A-77. Find the current I in the circuit of Fig. A-110. [**Ans.** 4 A]

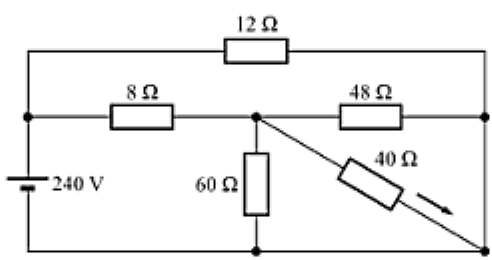


Fig. A-110

A-78. Use repeated current division to find I in the circuit of Fig. A-111. [**Ans.** 4 mA]

A-79. (*a*) Use Ohm's law and Kirchhoff's laws to evaluate all the currents and voltages in the circuit of Fig. A-112. (*b*) Calculate the power absorbed by each of the five circuit elements and show that the sum is zero.

[**Ans.** (*a*) $v_1 = v_2 = 60$ V, $v_3 = 15$ V, $v_4 = v_5 = 45$ V, $i_1 = 27$ A, $i_2 = 3$ A, $i_3 = 24$ A, $i_4 = 15$ A, $i_5 = 9$ A; (*b*) $P_1 = -1.62$ kW, $P_2 = 180$ W, $P_3 = 360$ W, $P_4 = 675$ W, $P_5 = 405$ W, $\Sigma P_{abs} = 0$.]

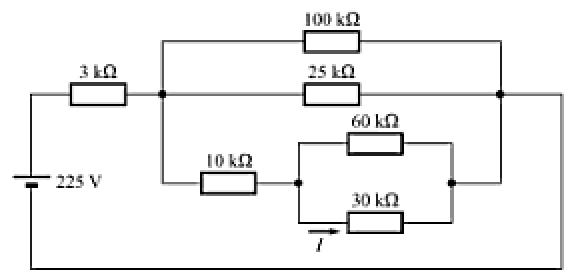


Fig. A-111

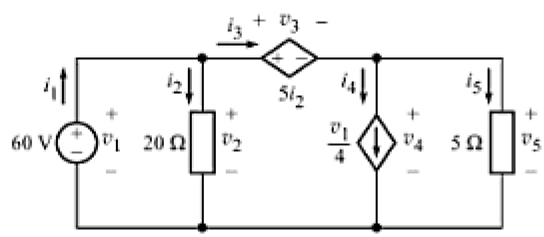


Fig. A-112

A-80 Figure A-113 shows a field-effect-transistor (FET) circuit. Without knowing the current-voltage relationship for the device, we can still solve this circuit since it obeys Ohm's law and Kirchhoff's laws. (*a*) Determine V_{DS}, if $I_D = 1.5$ mA. (*b*) Determine V_{GS}, if $I_D = 1$ mA and $V_G = 3$ V.

[**Ans.** (*a*) 1.5 V; (*b*) 1 V]

A-81. Use both resistance and source combinations, as well as current division, in the circuit of Fig. A-114 to find the power absorbed by the 3-Ω, 12-Ω and 15-Ω resistors.

[**Ans. $P_{3\Omega}$ = 54.95 W; $P_{12\Omega}$ = 0; $P_{15\Omega}$ =175.86 W**]

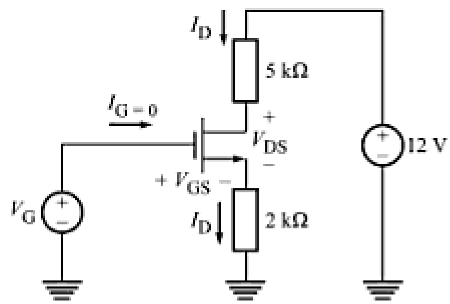


Fig. A-113

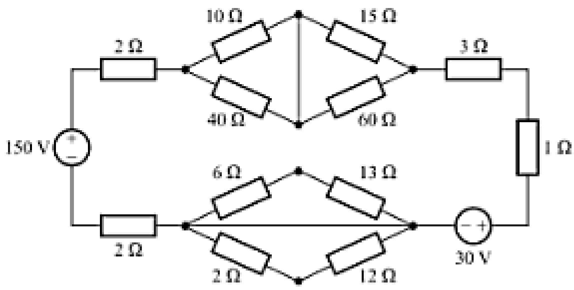


Fig. A-114

A-82. Find the power absorbed by each circuit element of Fig. A-15, if the control for the dependent source is (*a*) $0.8i_x$, and (*b*) $0.8i_y$.

[**Ans.** (*a*) P_{5A} **= –1389 W,** P_{10ms} **= 771.7W,** P_{40ms} **= 3086.9 W,** P_{dep} **= –2469.5W]** (*b*) P_{5A} = –776 **W,** P_{10ms} **= 240.8 W,** P_{40ms} **= 963.5 W,** P_{dep} **= –428.3 W]**

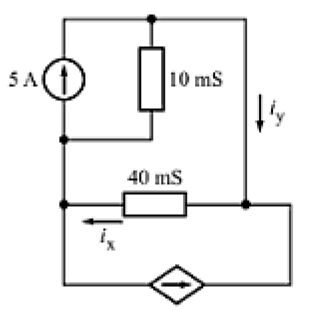


Fig. A-115

A-83. In the circuit of Fig. A-116, if $g_m = 25 \times 10^{-3}$ S and $v_s = 10 \cos 5t$ mV, find $v_o(t)$.

[**Ans.** $v_o(t) = -248.5 \cos 5t$ mV]

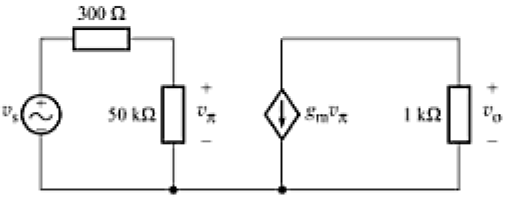


Fig. A-116

A-84. Determine the current i_1 in the circuit of Fig. A-117.

[**Ans.** –1.93 A]

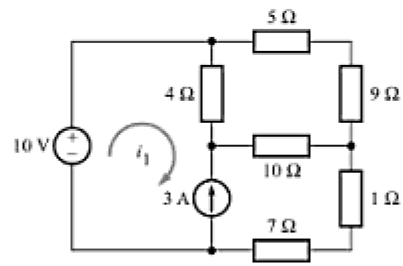


Fig. A-117

A-85. Determine the mesh currents in the circuit of Fig. A-118.

[**Ans.** $I_1 = 2$ mA, $I_2 = -3$ mA, $I_3 = 4$ mA]

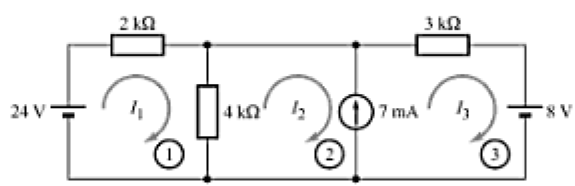


Fig. A-118

A-86. For the circuit of Fig. A-119, (*a*) use nodal analysis to determine v_1 and v_2, and (*b*) compute the power absorbed by the 6-Ω resistor.

[**Ans.** (*a*) 58.85 V, 64.4 V (*b*) 537 W]

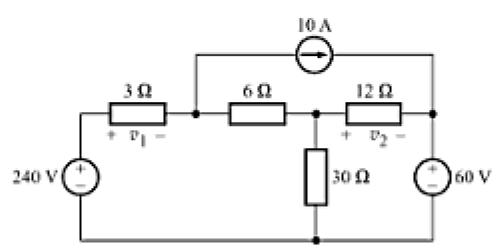


Fig. A-119

A-87. Use nodal analysis to obtain the value of v_x in the circuit of Fig. A-120. **[Ans.** –38.69 V]

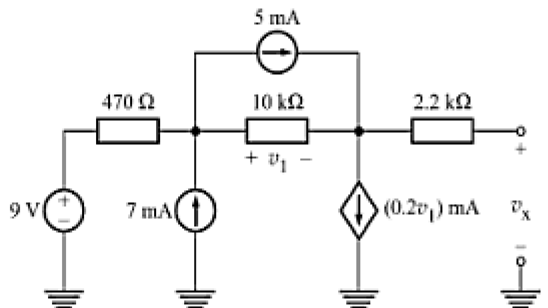


Fig. A-120

A-88. Use superposition principle to obtain the voltage across each current source in the circuit of Fig. A-121. **[Ans.** v_1 = 11.147 V; v_2 = –1.394 V]

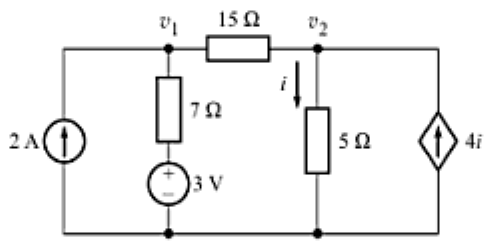


Fig. A-121

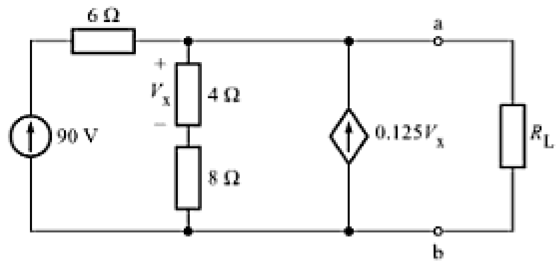


Fig. A-122

A-89. In the circuit of Fig. A-122, what resistor R_L will absorb maximum power and what is this power? **[Ans.** 4.8 Ω, 270 W]

ELECTROMAGNETISM

5

OBJECTIVES

After completing this Chapter, you will be able to:

- *Determine the direction of magnetic field due to electric current, by using right-hand thumb rule.*
- *Write expressions for magnetic field due to (a) a long straight wire, (b) a circular loop, (c) a solenoid, and (d) a toroid.*
- *Determine the magnitude and direction (by using Fleming's left hand rule) of force on a current-carrying conductor kept in a magnetic field.*
- *Determine the value of torque experienced by a current-carrying coil kept in a magnetic field.*
- *Define SI unit of current.*
- *State Faraday's laws of electromagnetic induction.*
- *Explain the three methods of producing induced emf.*
- *Determine the direction of induced emf using Fleming's right-hand rule.*
- *Find the value of emf induced in a coil when it is moved through magnetic field.*

5.1 INTRODUCTION

For several centuries, electric and magnetic forces were thought to be completely different, with nothing in common. It is now realised that these forces are very closely related to each other. These forces are as basic as the gravitational force, pervading *all* matter, and determining their properties.

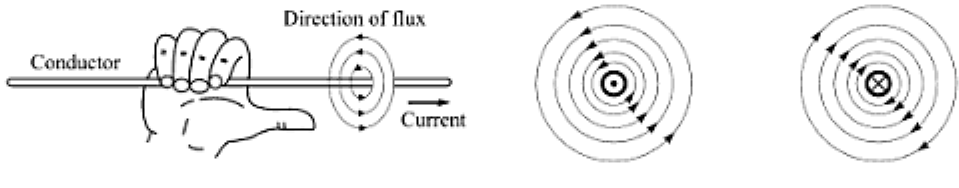
In a rough sense, electricity and magnetism give rise to each other. In 1820, Hans Christian Oersted, a Danish physicist, discovered that a steady electric current produces a magnetic field. In early 1830s, Faraday in England and Henry in USA found that a time-varying magnetic field produces an electric field, i.e., an electromotive force (emf). About forty years later (in 1873) Maxwell argued that the converse relation must also be necessarily true. That is, a time dependent electric field must produce a magnetic field. One of the consequences of this mutual relation is the electromagnetic wave, in the form of radio and light waves.

Electromagnetism is the study of the interaction between electric and magnetic fields and forces.

5.2 MAGNETIC FIELD DUE TO ELECTRIC CURRENT

When a dc current flows through a wire, a magnetic field is set up in its vicinity. Figure 5.1*a* shows a straight conductor through which a steady state (dc) current is flowing from left to right. The magnetic field lines will be in the form of concentric circles around the wire. The direction of this field is conveniently given by the ***right-hand thumb rule***. It says, "Stretch the thumb of your right hand along the current, the curl (natural bend) of fingers gives the direction of the magnetic field."

In some situations, the conductor is placed perpendicular to the paper. The conductor is represented by a small circle and the direction of current is then shown by putting a dot (•) or a cross (×). As a convention*, (•) represents a current coming out of the plane (see Fig. 5.1*b*) and (×) represents a current entering the paper (Fig. 5.1*c*). In Fig. 5.1*b*, the thumb is stretched upward and hence the magnetic field is anti-clockwise. In Fig. 5.1*c*, the thumb is stretched downward and hence the magnetic field is clockwise.



(*a*) Current from left to right. (*b*) Current coming out of the paper. (*c*) Current entering into the paper.

Fig. 5.1 *Magnetic field due to current in a conductor.*

Magnetic Field due to a Coil

The direction of magnetic field produced at the centre of a current carrying coil is also given by *right-hand thumb rule*. But here the role of current and magnetic field is exchanged: "If you bend the fingers of the right hand pointing in the direction of current flow, the thumb points in the direction of magnetic field lines."

Thus, in Fig. 5.2*a*, the current flow is clockwise, hence the magnetic field points downward. In Fig. 5.2*b*, the current flow is anticlockwise; hence the magnetic field is upward. A coil thus acts like a small flat magnet. In Fig. 5.2*a*, the upper surface of the coil can be identified as South Pole of the magnet, whereas in Fig. 5.2*b*, it is North Pole.

(*a*) Single loop (Clockwise current). (*b*) Single loop (Anticlockwise current).

Fig. 5.2 *Direction of magnetic field due to current in a coil.*

Magnetic Field Strength

The strength of magnetic field at a point produced by a current-flow in a wire or in a coil is directly proportional to the current and is inversely proportional to its distance. If the current is doubled, the magnetic field is also doubled. If there is no current, there is no magnetic field. If the direction of current is reversed, the magnetic field also reverses its direction.

* The same convention is used to represent the direction of any vector, such as magnetic field or electric field. One way of remembering the convention is to imagine an arrow pointing in the direction of the vector. If the vector points out of the paper, you see the head of the arrow, namely, the • (dot). If the vector points into the paper, you see the tail of the arrow, namely, the × (cross).

The magnetic field strength is denoted by B and is measured in tesla (T) after the great Yugoslav inventor and scientist Nikola Tesla (1856–1943).

Magnetic Field due to a Long Straight Wire The magnetic field at a distance of x from a long straight wire carrying current I is given as

$$B = \frac{\mu_0 I}{2\pi x} \tag{5.1}$$

where, μ_0 is the permeability of free space (or air) with a value $4\pi \times 10^{-7}$ Tm/A.

Magnetic Field due to a Circular Loop The magnetic field at the centre of a circular loop of radius r, carrying current I is given as

$$B = \frac{\mu_0 I}{2r} \tag{5.2}$$

Magnetic Field due to a Solenoid A coil may also have more than one turn. Figure 5.3*a* shows a ***solenoid*** in which a wire is wound closely in the form of a helix. The wire is coated with an insulating material so that the adjacent turns are electrically insulated from each other. Generally, the length of the solenoid is large compared to its radius. The magnetic field flux produced by each turn tends to link up and the net field pattern is very similar to that of a bar magnet. By applying right-hand thumb rule, we find that the left end of this solenoid is N-pole and right end is S-pole**. The solenoids produce strong magnetic field for such applications as relays, transformers and circuit breakers.

The field strength at the centre of a solenoid having n turns per unit length and carrying current I is given as

$$B = \mu_0 nI \tag{5.3}$$

The magnetic field at either end of the solenoid has half the strength as that in the middle. That is,

$$B_{\text{end}} = \frac{\mu_0 nI}{2} \tag{5.4}$$

EXAMPLE 5.1

A long solenoid is formed by winding 20 turns/cm. What current is necessary to produce a magnetic field of 20 mT inside the solenoid?

Solution The magnetic field inside the solenoid is $B = \mu_0 nI$, where n is the number of turns per metre. Therefore, the required current is

$$I = \frac{B}{\mu_0 n} = \frac{20 \times 10^{-3}}{4\pi \times 10^{-7} \times (20 \times 10^2)} = \mathbf{8.0\ A}$$

EXAMPLE 5.2

A solenoid 50 cm long has four layers of windings of 350 turns each. The radius of the lowest layer is 1.4 cm. If the current carried is 6.0 A, estimate the magnitude of magnetic field (*a*) near the centre of the solenoid on its axis and off its axis, (*b*) near its ends on its axis, and (*c*) far outside the solenoid near its axis.

Solution Since the magnetic field inside the solenoid is quite uniform, its strength near the centre on its axis and off its axis is same and is given by $B = \mu_0 nI$. Note that the radius of the solenoid does not enter in this equation. Therefore,

** The poles of a solenoid can easily be identified by seeing the shape of the letters 'N' and 'S'. As shown in Fig. 5.3*a*, the two ends of these letters point in the direction of current in the coil.

to get n, simply multiply the number of turns per layer by the number of layers and divide the product by the length of the solenoid,

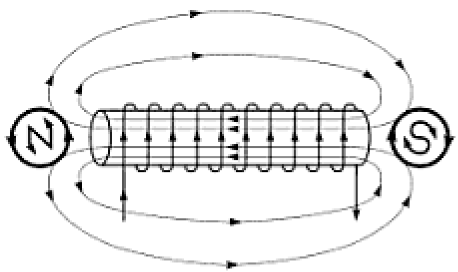
$$n = \frac{350 \times 4}{0.50} = 2800 \text{ m}^{-1}$$

$\therefore$ $$B = \mu_0 n I = 4\pi \times 10^{-7} \times 2800 \times 6.0 = \mathbf{2.1 \times 10^{-2}\ T}$$

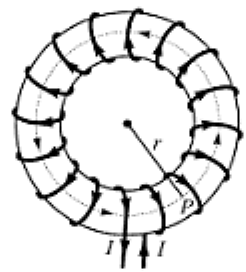
(*b*) Magnetic field strength at the end of the solenoid is

$$B = \frac{\mu_0 n I}{2} = \frac{2.1 \times 10^{-2}}{2} = \mathbf{1.05 \times 10^{-2}\ T}$$

(*c*) The magnetic field far outside the solenoid near its axis is **negligible,** compared to the internal field.



(*a*) A solenoid.



(*b*) A toroid.

Fig. 5.3 *Magnetic field due to coils of many turns.*

Magnetic Field due to a Toroid If a solenoid is bent in a circular shape and the ends are joined, we get a *toroid*, as shown in Fig. 5.3*b*. Alternatively, one can start with a non-conducting ring and wind a conducting wire on it. If r is the radius of the toroid, and it has a total number of n turns, then the magnetic field inside the toroid at any point (such as P) due to current I is given by

$$B = \frac{\mu_0 n I}{2\pi r} \tag{5.5}$$

To increase the magnetic field B, a magnetic material such as iron is introduced inside the cylindrical or toroidal region enclosed by the conducting windings.

5.3 FORCE ON CURRENT-CARRYING CONDUCTOR

Figure 5.4*a* shows a small conductor of length $d\mathbf{l}$ placed perpendicular to the magnetic field $\mathbf{B}$. If it carries a current I, it experiences a force given by

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \tag{5.6}$$

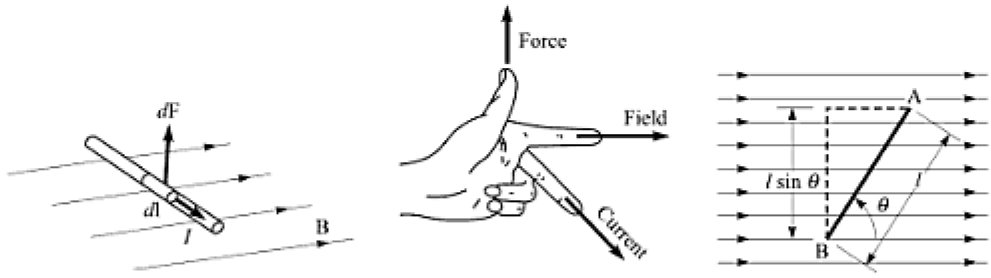
This is the magnetic analog of the electric force experienced by a charge q placed in an electric field $\mathbf{E}$, given as $\mathbf{F} = q\mathbf{E}$. We notice that the force is proportional to I, dl and B, and is perpendicular to both $d\mathbf{l}$ and $\mathbf{B}$.

For the condition shown in Fig. 5.4*a*, the angle between the length vector $d\mathbf{l}$ and the field $\mathbf{B}$ is 90°. Hence, the expression of Eq. 5.6 reduces to

$$dF = I dl\, B$$

For a conductor of length l, carrying current I placed perpendicularly in a magnetic field of strength B, the force on the conductor is

$$F = IBl \tag{5.7}$$



(*a*) Force *d*F on a small element *d*l carrying current *I*.
(*b*) Fleming's left-hand rule gives direction of force.
(*c*) Conductor placed at an angle *θ*.

Fig. 5.4 *Force on a current-carrying conductor.*

If the current-carrying conductor is placed at an angle θ to the magnetic field (see Fig. 5.4*c*), its effective length is $l \sin \theta$, and hence the force experienced by the conductor reduces to

$$F = IBl \sin \theta \tag{5.8}$$

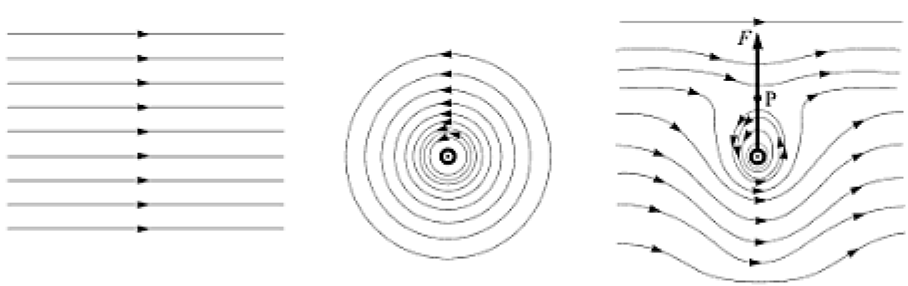
Obviously, if the conductor is placed along the field B, the angle $\theta = 0$, and the force on the conductor too reduces to zero.

Fleming's Left-hand Rule

A well known and convenient way of finding the direction of force on a current-carrying conductor placed in a magnetic field is Fleming's left-hand rule. It says, "Stretch the first finger, the central finger and the thumb of your left hand in mutually perpendicular directions. Now, adjust your wrist so that the first finger points in the direction of magnetic field and the central finger to the direction of current, as shown in Fig. 5.4*b*. The thumb then points in the direction of force on the conductor."

Why a Force is Produced on the Conductor

The reason why a current-carrying conductor placed in a uniform magnetic field (Fig. 5.5*a*) experiences a force is the interaction between the two magnetic fields. A conductor carrying current (say, coming out of the plane of the paper) produces counterclockwise field lines, as shown in Fig. 5.5*b*. This magnetic field disturbs the symmetry of the uniform field **B**. Figure 5.5*c* shows the resulting magnetic field lines. The uniform field



(*a*) Uniform magnetic field.
(*b*) Field due to current.
(*c*) Resultant magnetic field.

Fig. 5.5 *Interaction between two magnetic fields produces force on the current carrying conductor.*

and the field due to the straight wire are in the same direction in the lower-half, and in the opposite direction in the upper-half. At the point P shown, the two exactly cancel each other. The net field at P is zero.

To find the direction of force experienced by the conductor, we can imagine the field-lines to be like stretched rubber strings. The field lines in Fig. 5.5*c* are seen to act as a catapult which exerts an upward force on the conductor.

5.4 TORQUE EXPERIENCED BY A COIL

Consider a rectangular coil MNOP of length a and breadth b, carrying a steady current I. Suppose that it is placed in a uniform magnetic field of strength B, such that the normal to the coil makes an angle θ with the direction of field B, as shown in Fig. 5.6*a*. The conductor MP (and also conductor ON) makes an angle of $90° - \theta$ with the direction of the magnetic field. The force F_2 experienced by this conductor is given by Eq. 5.8, as

$$F_2 = IBb \sin(90° - \theta) = IBb \cos \theta$$

The conductor ON also experiences the same amount of force. The forces on MP and ON are of same magnitude and work along the same line in opposite directions. Therefore, no torque is produced by these forces.

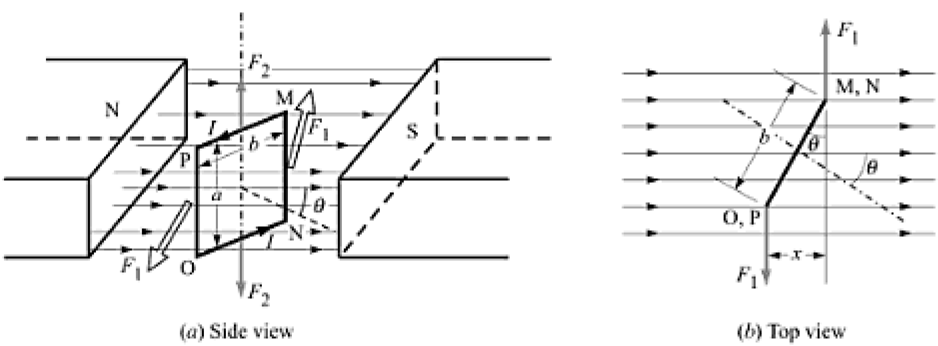


Fig. 5.6 *Torque experienced by a current-carrying coil placed in magnetic field.*

The conductors MN and OP remain perpendicular to the magnetic field. The force F_1 experienced by each of these conductors is given by Eq. 5.7, as $F_1 = IBa$. In case the coil has n turns of wire, the force becomes

$$F_1 = IBan$$

Note that the forces on conductors MN and OP are equal in magnitude and work in opposite directions, but they do not act along the same line. The perpendicular distance between these forces can be found from the top view of the coil, shown in Fig. 5.6*b*, as

$$x = b \sin \theta$$

Therefore, the torque experienced by the coil is

$$\tau = F_1 x = (IBan) \times (b \sin \theta) = BInab \sin \theta = BInA \sin \theta \tag{5.9}$$

where, $A = ab$ is the area of the coil. In electrical machines and measuring instruments, the magnetic field is made radial by introducing a core and suitably shaping the magnetic poles. This way, the angle θ remains 90° for any orientation of the coil. In such case, the torque on the coil becomes

$$\tau = BInA \tag{5.10}$$

It is **important to note** that the torque on a coil depends, besides other things, on its area and not on its shape. It means that a coil having a circular shape or any other irregular shape, but the same area as A, will experience the same amount of torque as given by Eqs. 5.9 and 5.10.

5.5 FORCE BETWEEN PARALLEL CONDUCTORS

Consider two parallel conductors carrying currents I_1 and I_2, separated by a distance r as shown in Fig. 5.7*a*. French scientist Andre Ampere performed a number of experiments and found that

$$F \propto \frac{I_1 I_2}{r}$$

The force is found to be attractive when the direction of currents in the two conductors is the same and repulsive when it is opposite. This fact can be physically explained on the basis of interaction of the magnetic fields produced by the two currents. In case of similar direction of the two currents (Fig. 5.7*b*), the flux lines of the resultant field envelope both the conductors to produce an attraction. The currents in opposite directions (Fig. 5.7*c*) produce a field pattern to keep them away from each other.

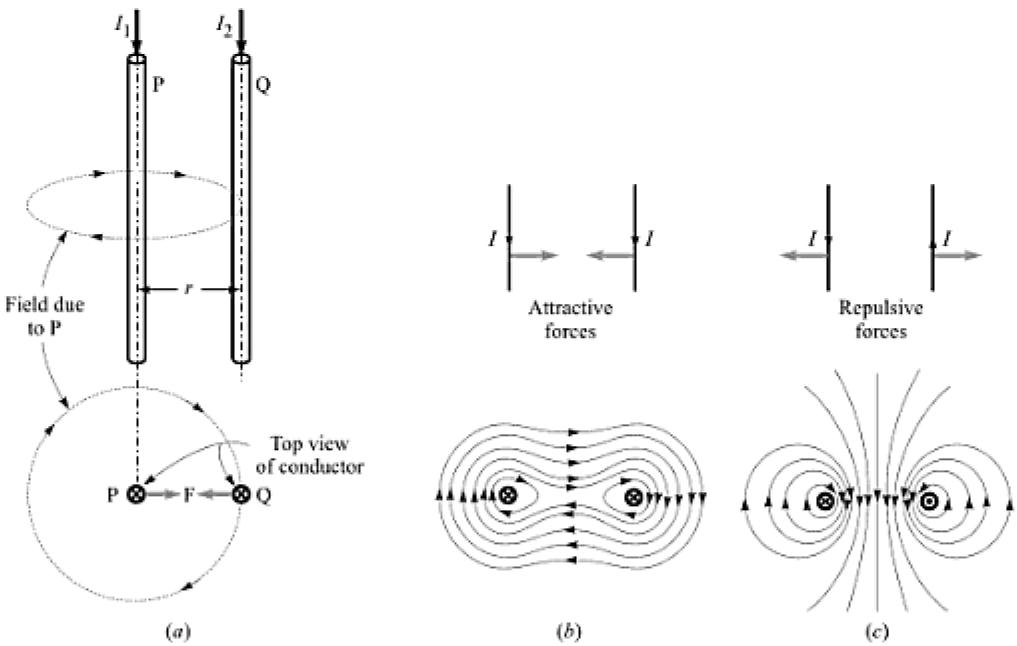


Fig. 5.7 *Two current-carrying conductors experience a force.*

The exact expression for the force can be found by considering that the conductor P (Fig. 5.7*a*) produces at the site of conductor Q a magnetic field given by (Eq. 5.1)

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Due to this field, the conductor B experiences a force BI_2 per metre length (see Eq. 5.7). Thus, the magnitude of the mutual force per unit length between the two infinitely long conductors is given as

$$F = BI_2 = \frac{\mu_0 I_1 I_2}{2\pi r} \tag{5.11}$$

Unit of Current The SI unit of current is based on Eq. 5.11. In this equation, if the two currents are 1 ampere each, and the separation between the two conductors is 1 m, the force will be

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{\mu_0}{2\pi} = 2 \times 10^{-7}\ \text{N}$$

Thus, one ***ampere*** of current is *defined as the current which, when flowing in each of two infinitely long parallel conductors situated in vacuum and separated by one metre, produce on each of the conductors a force of* 2×10^{-7} *newton per metre length.*

EXAMPLE 5.3

Two long straight parallel wires, kept in air 2 m apart, carry currents of 80 A and 30 A in the same direction. Calculate the force between them and specify its nature.

Solution $F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 80 \times 30}{2\pi \times 2} = \mathbf{2.4 \times 10^{-4}\ N\,m^{-1}}$

Since the two currents are in the same direction, the force will be **attractive.**

EXAMPLE 5.4

Two straight wires A and B of lengths 10 m and 12 m carrying currents of 4.0 A and 6.0 A, respectively, in opposite directions lie parallel to each other at a distance of 3.0 cm. Estimate the force on a 15 cm section of wire B near its centre.

Solution Since the ratio of length of the wires to the separation between them is large (more than 300), we can use Eq. 5.11 to calculate the force per unit length of wire B,

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 4 \times 6}{2\pi \times 0.03} = 16 \times 10^{-5}\ \text{N m}^{-1}$$

Therefore, net force on 15 cm (= 0.15 m) section of wire B is

$$F_{\text{net}} = F \times l = 16 \times 10^{-5} \times 0.15 = 2.4 \times 10^{-5} = \mathbf{24\ \mu N}$$

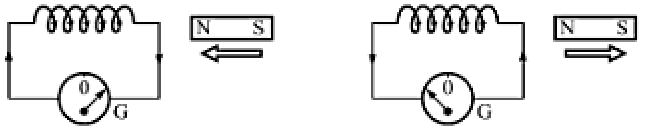
Since the currents are in opposite directions, the force F_{net} is *repulsive*. It means that its direction is normal to wire A away from it.

5.6 ELECTROMAGNETIC INDUCTION

On August 29, 1831, a British scientist Michael Faraday (1791–1867) made the great discovery of obtaining an electric current with the aid of magnetic field. After a variety of experiments, he found that a moving magnetic field does indeed give rise to an emf.

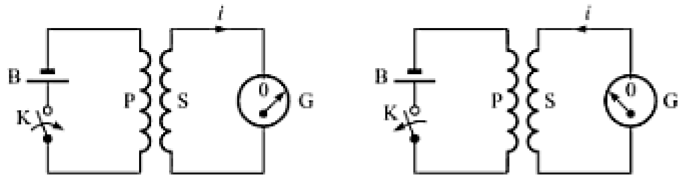
Faraday observed that when a bar magnet is suddenly brought close to a coil, the galvanometer G showed a deflection (Fig. 5.8*a*). As the magnet was moved away quickly, the galvanometer again showed a deflection, but this time in opposite direction (Fig. 5.8*b*). The deflection lasted as long as the magnet was in *motion*.

In another experiment, Faraday's apparatus consisted of two circuits placed close to each other, as shown schematically in Fig. 5.8*c*. One circuit, which we shall call the *primary* circuit, consists of a battery B, a coil P



(*a*) Deflection when the magnet is moved closer.

(*b*) Deflection when the magnet is moved away.



(*c*) Deflection when the key is closed .

(d) Deflection when the key is opened.

Fig. 5.8 *Faraday's experiments on electromagnetic induction.*

of many turns of wire, and a key K for closing and opening the circuit. The other circuit, called the *secondary* circuit, consists of a coil S, and a galvanometer G.

Faraday observed that when the key K in the primary circuit was closed, the galvanometer G in the secondary circuit gave momentary deflection and then returned to its zero position. When the key was opened (Fig. 5.8*d*), there was another momentary deflection of the galvanometer. But this time the deflection was in the opposite direction to the original deflection. In both cases, the current in the secondary exists for only a short time, roughly the time during which the current in the primary circuit is changing.

Faraday's Laws of Electromagnetic Induction

Law I An induced emf is established in a circuit whenever the magnetic field linking that circuit is changed.

Law II The magnitude of the induced emf is equal to the rate of change of the magnetic flux linking the circuit. Stated mathematically,

$$e = \frac{\Phi_2 - \Phi_1}{\Delta t} \quad \text{or} \quad e = \frac{d\Phi}{dt}$$

If a coil has N turns, emf is induced in each turn. The total emf induced in the coil then becomes

$$e = N\frac{d\Phi}{dt} \tag{5.12}$$

Later, Lenz described the direction or the sense of the induced emf.

Lenz's Law The direction of the induced current is such that it opposes (through its magnetic action) the very cause which produced the induced current.

For example, if the magnetic field through a circular loop is increasing, the induced current in the loop has a direction that produces a magnetic field opposing the original field. In fact, the Lenz's law is just an expression of the general *law of conservation of energy.* In Fig. 5.8*a*, the N-pole of the bar magnet is being pushed towards the coil. The induced emf and hence the current through the coil must be such that the right-hand end of the coil becomes a N-pole, so that work has to be done to push the magnet against the coil. It is this mechanical work which causes a current to flow in the coil against its resistance R. *The mechanical work done is converted to electrical energy which in turn produces i^2R loss in the coil in the form of heat energy.*

In view of Lenz's law, Eq. 5.12 is modified as

$$e = -N\frac{d\Phi}{dt} \tag{5.13}$$

5.7 METHODS OF PRODUCING INDUCED EMF

There are three requirements to producing an induced emf in a conductor, namely, (*i*) the presence of a conductor, (*ii*) the presence of a magnetic field, and (*iii*) the linking or cutting of the magnetic flux lines by the conductor. Following are three methods by which this can be achieved.

(1) By using a stationary conductor, a stationary electromagnet and varying the magnetic flux by supplying ac current to the electromagnet. As we shall see in Chapter 13, this principle is used in transformers.

(2) By using a stationary conductor or a moving permanent magnet (or an electromagnet). As we shall see in Chapter 14, this principle is used in large ac generators and motors.

(3) By using a stationary permanent magnet (or an electromagnet such as shown in Fig. 5.3*a*, fed by a dc current) and a moving conductor. As we shall see in Chapter 16, this principle is used in all dc generators and motors.

The emf induced by variation of magnetic flux due to ac current (method 1) is termed as '***statically induced emf***'. We will discuss statically induced emf in Chapter 7. The emf induced by the relative motion between the conductor and the magnetic field (methods 2 and 3) is termed as '***motional emf*** or ***dynamically induced emf***.

5.8 DYNAMICALLY INDUCED EMF

Consider a closed loop of wire of area A placed in a magnetic field. If the field is uniform (constant in space but not necessarily with time) and perpendicular to the surface of the loop (as in Fig. 5.9*a*), the flux Φ (measured in webers) passing through the loop is given as $\Phi = BA$. However, if the normal to the plane of the loop makes an angle θ with the magnetic field (Fig. 5.9*b*), the effective area of the loop reduces to $A\cos\theta$ and the flux passing through the loop becomes

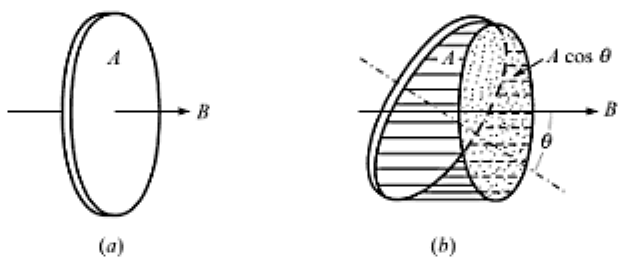


Fig. 5.9 *Magnetic flux passing through a loop of wire.*

$$\Phi = BA\cos\theta \tag{5.14}$$

This shows that as per Faraday's laws, represented by Eq. 5.14, induced emf may be produced in the coil by any of the following three methods:

(i) *By changing the magnetic field B*,

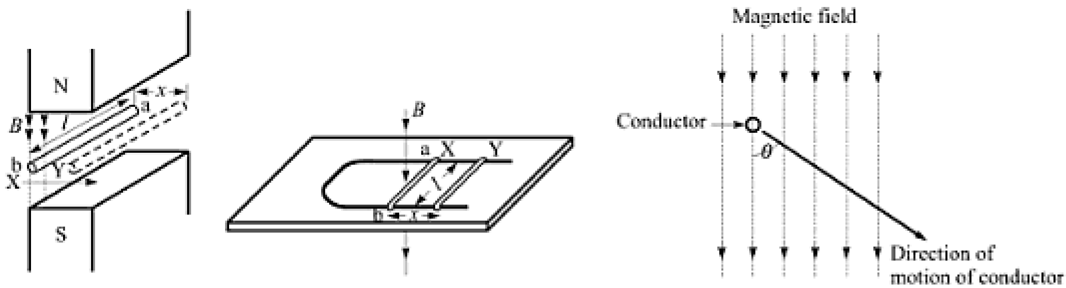
(ii) *By changing the area A of the coil*, and

(iii) *By changing the relative orientation of B and A* (i.e., *by changing the angle θ*).

(i) Induced emf by Changing B This is illustrated in Fig. 5.8*a* and *b*. Change in B through the coil is accomplished by relative motion of the loop and the magnet. For this, it does not matter whether the magnet moves or the coil moves or both move. When the magnet and the coil come closer, B increases and an induced emf is produced.

(ii) Induced emf by Changing A Consider a conductor ab in position X kept in a magnetic field B (Fig. 5.10*a*). Let l be its effective length in the magnetic field. The conductor can be considered to be a part of a *single-turn coil*, as shown in Fig. 5.10*b*. It slides over U-shaped conducting rails situated in the magnetic field. If the conductor moves by a distance x (from position X to position Y) in time t, its average linear velocity *perpendicular* to the flux is $v = x/t$, and the change in area of the coil is clearly lx. The induced emf in the conductor is given as

$$e = -\frac{d\Phi}{dt} = -\frac{Blx}{t} = -Bl\left(\frac{x}{t}\right) = -Blv \tag{5.15}$$



(*a*) (*b*) (*c*)

Fig. 5.10 *Induced emf produced by the motion of the conductor in a magnetic field.*

In case the conductor moves at an angle θ with the magnetic field (Fig. 5.10*c*), the component of its velocity v in a direction perpendicular to the magnetic field is $v \sin\theta$. The induced emf then becomes

$$e = -Blv \sin\theta \tag{5.16}$$

Since this induced emf is produced by a moving conductor (through a magnetic field), it is called ***motional emf***.

(iii) Induced emf by Changing θ Consider a coil MNOP rotating at an angular velocity ω about its axis in a magnetic field B, as shown in Fig. 5.11*a*. The axis of rotation is perpendicular to the magnetic field. At an instant when the normal to the plane of the coil makes an angle θ with the field (Fig. 5.11*b*), the flux passing through the coil is given by Eq. 5.14. If we measure time t from the instant when the coil is perpendicular to the magnetic field, i.e., $\theta = 0$ at $t = 0$, then at any instant,

$$\Phi = BA\cos\theta = BA\cos\omega t$$

From Faraday's laws, the induced emf in a single-turn coil is given as

$$e = -\frac{d\Phi}{dt} = -\frac{d\,(BA\cos\omega t)}{dt} = BA\omega\sin\omega t$$

If the coil has N turns, the net induced emf is given as

$$e = -N\frac{d\Phi}{dt} = NBA\omega\sin\omega t = E_m\sin\omega t \tag{5.17}$$

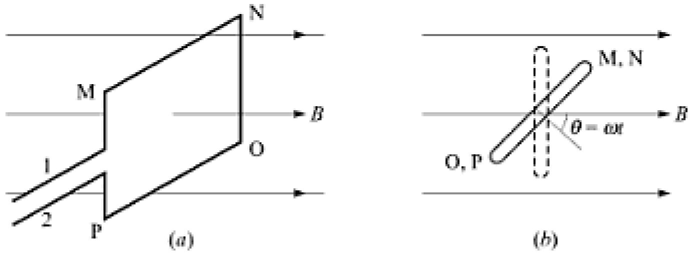


Fig. 5.11 *Induced emf by rotating a coil in magnetic field.*

The induced emf is seen to be varying sinusoidally with time. This is the principle of working of a *dynamo* or a *generator*.

It is **important to note** that the induced emf in a coil depends, besides other things, on its area and not on its shape. It means that the induced emf in a coil having a circular shape or any other irregular shape, but the same area as A, will be the same as given by Eq. 5.17.

Fleming's Right-hand Rule

Although the direction of motional emf could be determined by Lenz's law, it is found more convenient to use *Fleming's right-hand rule*, as illustrated in Fig. 5.12. It says, "Stretch the first finger, the central finger and the thumb of your right hand in mutually perpendicular directions. Now, adjust your wrist so that the first finger points in the direction of magnetic field and the thumb to the direction of motion. The central finger then points in the direction of induced current (or emf)."

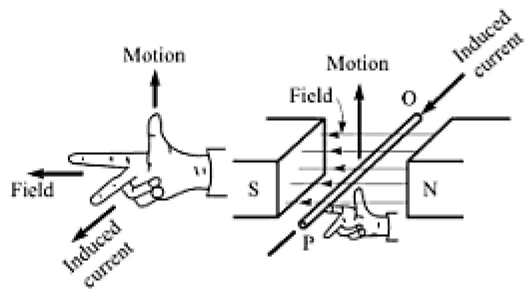


Fig. 5.12 *Illustration of Fleming's right-hand rule.*

IMPORTANT NOTE

In both the rules—Fleming's right-hand rule and Fleming's left-hand rule—the first finger, the central finger and the thumb represent the same quantities. It may therefore be helpful to associate

First finger	with	**F**ield Flux;
Central finger	with	**C**urrent; and
thu**M**b	with	**M**otion of the conductor

Often, students get confused which rule to apply where. In electrical engineering, we come across two types of situations. One may be called *generator action*, and the other *motor action*. In generator action, the induced emf is the result when a conductor is moved in a magnetic field (this is what happens in a dynamo). Whereas, in motor action, the motion of a conductor is the result when a current is passed through the conductor placed in a magnetic field (this is what happens in an electric fan).

You can easily remove the confusion by noting that it is your right hand that (usually) *generates* most of the things (like writing, painting, tightening of a screw, etc). Thus, for the sake of remembering, the right-hand rule can be associated with the *generator* action of the right hand. The other rule (i.e., Fleming's left-hand rule) then applies to the motor action.

5.9 ELECTROMAGNETIC INDUCTION AND LORENTZ FORCE

A charge q moving with a velocity **v** in a magnetic field **B** experiences a magnetic force called ***Lorentz force***, given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \tag{5.18}$$

The phenomenon of motional emf can be explained through Lorentz force. Consider a metal rod PQ being dragged in with velocity **v** inside a magnetic field, as shown in Fig. 5.13*a*. Free electrons present in the rod also move with velocity **v**, and hence experience Lorentz force perpendicular to both **B** and **v**. According to right-hand screw rule (for determining the direction of cross product of two vectors), the force **F** on a positive charge will be upward, from P to Q (Fig. 5.13*b*). Electrons, being negatively charged, experience force $\mathbf{F}_e$ downward from Q to P. Because of this force, the electrons accumulate at the end P, providing it negative polarity. The other end Q deprived of electrons, becomes positively charged.

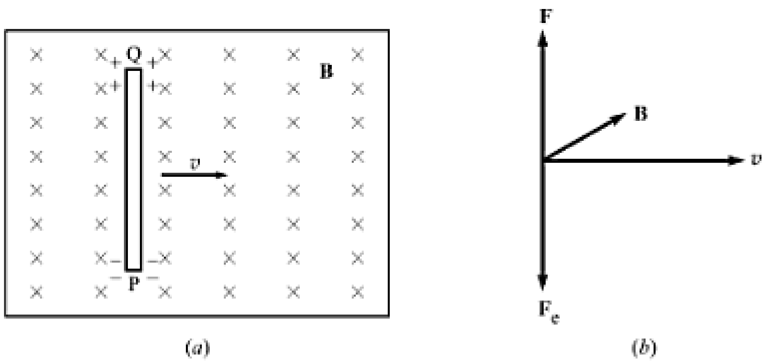


Fig. 5.13 *A metal rod PQ dragged inside a magnetic field.*

As the charges accumulate at the ends of the rod, an electric field is developed. This field in turn produces a force on the electrons in a direction opposite to the Lorentz force. When enough charges accumulate, this force cancels the Lorentz force and the free electrons stop drifting. If E is the electric field produced due to accumulation of charges, the electrostatic force on an electron is eE (e is the charge on an electron). From Eq. 5.18, the Lorentz force is evB. Thus, we must have

$$eE = evB \quad \text{or} \quad E = vB \tag{5.19}$$

This electric field exists along the length l of the rod PQ. Hence, the electric potential difference between P and Q is

$$V_{QP} = l \times E = l \times vB = Blv \tag{5.20}$$

This is the emf induced in the rod. Note that the same result (Eq. 5.15) was obtained on the basis of Faraday's laws.

Loop Moved in a Magnetic Field

Consider a loop PQRS pulled away from a magnetic field B (Fig. 5.14). So long as the loop is fully inside the magnetic field, the flux linked with it remains unchanged. Hence, no emf is induced. In fact, the conductors QR and PS do not cut across the magnetic field at all, and hence no emf is induced in them. However, the conductors PQ and RS do cut across the magnetic field, and hence emf is induced in them. But these two emfs, being equal in magnitude and opposite in polarity while going round the loop, cancel each other and hence no net emf exists in the loop.

When the loop is partly inside and partly outside the magnetic field (as in Fig. 5.14), the net flux linkage with the loop changes with time. Hence, an emf is induced. It is only the conductor PQ of length l where the motional emf Blv is induced. If R is the resistance of the loop, the current in the loop due to this emf is $I = Blv/R$.

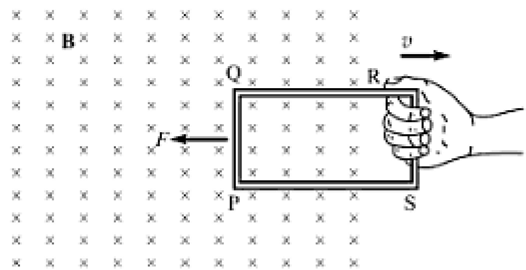


Fig. 5.14 *A loop pulled away from a magnetic field.*

As the loop is pulled away (as shown in Fig. 5.14), an equal and opposite force resists this motion (according to Lenz's law and also Newton's law). The work done on the loop appears as heat loss due to the I^2R loss in the loop. Thus, the power supplied in pulling the loop against the force F is

$$P = I^2R = \frac{(Blv)^2}{R^2} \times R = \frac{B^2 l^2 v^2}{R} \tag{5.21}$$

Since, power = force × velocity, the force is given as

$$F = \frac{P}{v} = \frac{B^2 l^2 v}{R} \tag{5.22}$$

EXAMPLE 5.5

A straight conductor, having an active length of 20 cm, is kept in a uniform magnetic field of 0.5 T. Find the emf produced in the conductor when it is moved at a rate of 5 m/s, in following three cases:

(*a*) its motion is parallel to the magnetic field,
(*b*) its motion is perpendicular to the magnetic field, and
(*c*) its motion is at an angle of 30° to the magnetic field.

Solution The magnitude of the induced emf is given as $e = Blv \sin\theta$.

(*a*) Since $\theta = 0$, the induced emf is $e = Blv \sin\theta = Blv \times 0 =$ **0 V**
(*b*) Here, $\theta = 90°$. Hence, $e = Blv \sin\theta = 0.5 \times 0.2 \times 5 \times 1 =$ **0.5 V**
(*c*) Here, $\theta = 30°$. Hence, $e = Blv \sin\theta = 0.5 \times 0.2 \times 5 \times 0.5 =$ **0.25 V**

EXAMPLE 5.6

An aeroplane with a wing span of 52 metres is flying horizontally at 1100 km/h. If the vertical component of earth's magnetic field is 38×10^{-6} T, find the emf generated between the wing-tips.

Solution The speed of the aeroplane is $v = 1100 \text{ km/h} = \frac{1100 \times 1000}{3600} = 305.6$ m/s

Hence, the induced emf is

$$e = Blv \sin\theta = 38 \times 10^{-6} \times 52 \times 305.6 \times 1 = \mathbf{0.604\ V}$$

EXAMPLE 5.7

A conducting rod of 1 m length is placed with its one end at the centre and the other end at the circumference of a circular metallic ring of radius 1 m. It is rotated about an axis passing through the centre of the ring perpendicular to the plane of the ring, with a frequency of 50 rev/s. A uniform magnetic field of strength $B = 1.0$ Wb/m^2, parallel to the axis of the ring is present everywhere. What is the emf developed between the centre and the metallic ring?

Solution There are no steady currents in this example. Only separation of charges takes place in the rotating rod. To find the emf induced in the rod, we imagine a closed loop formed by the rod and a connection between the centre O and a point, say P, on the circumference, with a resistor (Fig. 5.15). If θ is the angle between the rod and the radius OP at time t, the area of the arc formed by the rod and the radius OP is

$$A = \pi R^2 \times \frac{\theta}{2\pi} = \frac{1}{2}R^2\theta$$

where R is the radius of the circle. Hence, the induced emf is

$$e = \frac{d\Phi}{dt} = \frac{d(BA)}{dt} = B \times \frac{dA}{dt} = B \times \frac{d\left(\frac{1}{2}R^2\theta\right)}{dt} = \frac{1}{2}BR^2\frac{d\theta}{dt}$$

$$= \frac{1}{2} \times 1.0 \times (1^2) \times 50 \times 2\pi = \mathbf{157\ V}$$

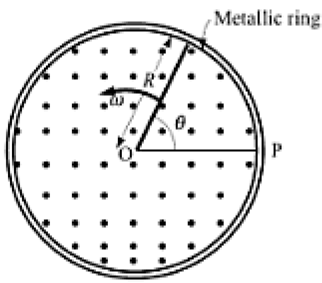


Fig. 5.15

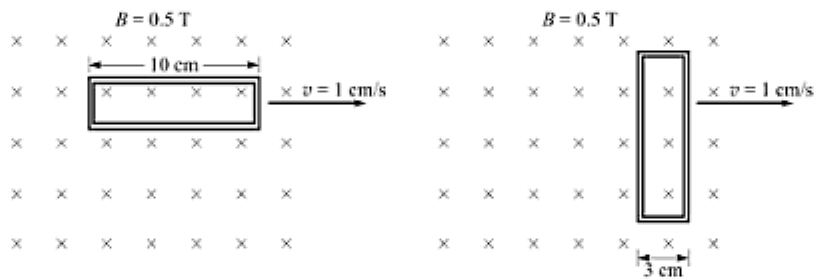


Fig. 5.16

EXAMPLE 5.8

A rectangular loop of sides 10 cm and 3 cm with a small cut is moving with a velocity of 1 cm/s, out of a region of uniform magnetic field of magnitude 0.5 T directed normal to the loop.

(*a*) How much voltage is developed across the cut, if the loop moves in a direction normal to the shorter side (Fig. 5.16*a*)? How long the induced voltage last?
(*b*) How much voltage is developed across the cut, if the loop moves in a direction normal to the longer side (Fig. 5.16*b*)? How long the induced voltage last?
(*c*) Is a force required to pull the loop if it has a cut?
(*d*) Find the force needed to pull the loop if it has no cut, and has a resistance of 1 mΩ.

Solution

(*a*) The induced emf is $e = Blv = 0.5 \times 0.03 \times 0.01 =$ **0.15 mV**

Time for which the induced voltage lasts is the same as the longer side takes to come out of the field. Thus, $t = l/v = (10\text{ cm})/(1\text{ cm/s}) =$ **10 s**

(*b*) The induced emf, $e = Blv = 0.5 \times 0.1 \times 0.01 =$ **0.5 mV**

Now, the time for which the induced voltage last is given as $t = l/v = (3\text{ cm})/(1\text{ cm/s}) =$ **3 s**

(*c*) Because of the gap, no current can flow. Hence, there is no I^2R loss or heat produced. If we neglect friction, **no force** is required to pull the coil.

(*d*) For Fig. 5.16*a*: $F_1 = \dfrac{B^2 l^2 v}{R} = \dfrac{(0.5)^2 \times (0.03)^2 \times 0.01}{1 \times 10^{-3}} = \mathbf{2.25\ mN}$

For Fig. 5.16*b*: $F_1 = \dfrac{B^2 l^2 v}{R} = \dfrac{(0.5)^2 \times (0.1)^2 \times 0.01}{1 \times 10^{-3}} = \mathbf{25\ mN}$

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 5.9

A horizontal overhead line carries a current of 90 A in east-to-west direction. What is the magnitude and direction of the magnetic field due to this current at a point 1.5 m below the line?

Solution The magnitude of the magnetic field is given by Eq. 5.2, as

$$B = \frac{\mu_0 I}{2\pi x} = \frac{4\pi \times 10^{-7} \times 90}{2\pi \times 1.5} = 12 \times 10^{-6}\ \text{T} = \mathbf{12\ \mu T}$$

By applying the right-hand thumb rule, we find that the direction of the magnetic field is from **north-to-south**.

EXAMPLE 5.10

What is the magnitude of the magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of uniform magnetic field of 0.15 T?

Solution Using Eq. 5.8, the force per unit length of the wire is given as

$$F_u = \frac{F}{l} = IB\sin\theta = 8 \times 0.15 \times \sin 30° = \mathbf{0.6\ N\,m^{-1}}$$

EXAMPLE 5.11

A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of uniform horizontal magnetic field of magnitude 0.8 T. What is the magnitude of the torque experienced by the coil?

Solution As shown in Fig. 5.17, the coil MNOP is kept such that the normal to its plane makes an angle of 30° with the uniform magnetic field. The magnetic force F experienced by each of the sides MN and OP is given by Eq. 5.7, as

$$F = IBln = 12 \times 0.8 \times 0.1 \times 20 = 19.2\ \text{N}$$

The normal distance between these two forces is

$$x = l\sin 30° = 0.1 \times 0.5 = 0.05\ \text{m}$$

Therefore, the torque experienced by the coil is

$$\tau = Fx = 19.2 \times 0.05 = \mathbf{0.96\ Nm}$$

EXAMPLE 5.12

A rectangular loop of sides 25 cm and 10 cm carries a current of 15 A. It is placed with its longer side parallel to a long straight conductor carrying a current of 25 A such that the loop and the conductor are in the same plane and the nearer side of the loop is 2.0 cm away from the conductor. What is the net force on the loop?

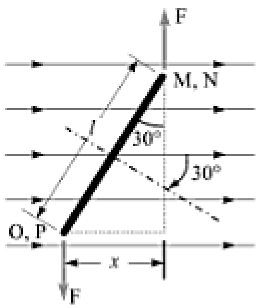


Fig. 5.17

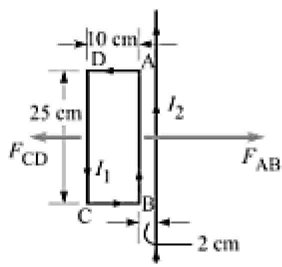


Fig. 5.18

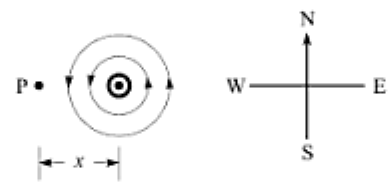


Fig. 5.19

Solution The placement of the rectangular loop near the long straight current-carrying conductor is shown in Fig. 5.18. Parts AD and BC do not experience any force, since these conductors are at right angles to the long straight conductor. The current I_1 in AB and the current I_2 in straight conductor are in the same direction. Hence, the force F_{AB} is *attractive*, and by using Eq. 5.10 its magnitude can be calculated as follows.

$$F_{AB} = F_u \times l = \frac{\mu_0 I_1 I_2}{2\pi r} \times l = \frac{4\pi \times 10^{-7} \times 15 \times 25}{2\pi \times 0.02} \times 0.25 = 93.75 \times 10^{-5}\ \text{N} = 0.9375\ \text{mN}$$

The current I_1 in CD and the current I_2 in straight conductor are in the opposite directions. Hence, the force F_{CD} is *repulsive*, and its magnitude is

$$F_{CD} = F_u \times l = \frac{\mu_0 I_1 I_2}{2\pi r} \times l = \frac{4\pi \times 10^{-7} \times 15 \times 25}{2\pi \times (0.02 + 0.10)} \times 0.25 = 15.625 \times 10^{-5}\ \text{N} = 0.15625\ \text{mN}$$

Therefore, the net force on the loop is

$$F_{net} = F_{AB} - F_{CD} = 0.9375 - 0.15625 = \mathbf{0.78125\ mN}$$

EXAMPLE 5.13

A long vertical wire carrying a current of 10 A in the upward direction is placed in a region where the horizontal component of the earth's magnetic field is 2.0 mT from south to north. Find the location of the point where the resultant magnetic field is zero.

Solution To make the resultant magnetic field zero, the magnetic field produced by the long vertical wire must be equal and opposite to the horizontal component of the earth's magnetic field (= 2.0 mT). This is possible at a point P (see Fig. 5.19) **west** to the wire. If x is the distance of point P from the wire, we have (from Eq. 5.1)

$$B = \frac{\mu_0 I}{2\pi x} = 2.0 \times 10^{-3} \quad \text{or} \quad x = \frac{\mu_0 I}{2\pi \times 2.0 \times 10^{-3}} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 2.0 \times 10^{-3}} = 10^{-3}\ \text{m} = \mathbf{1\ mm}$$

EXAMPLE 5.14

The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart ? Is the force attractive or repulsive ?

Solution Since the length of the wires is 70 cm and their separation is only 1.5 cm, i.e., $l >> r$, we can say that for the given separation the two wires are infinitely long. Hence, using Eq. 5.11, the mutual force per unit length is

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 300 \times 300}{2\pi \times 0.015} = \mathbf{1.2\ N\ m^{-1}}$$

Since the currents in the two wires are in the opposite direction, the force between them will be **repulsive**.

EXAMPLE 5.15

A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of 1.0 T. The field lines make an angle of 60° with the normal to the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

Solution The counter torque required to prevent the coil from moving must be equal (and opposite) to the torque developed, given by Eq. 5.9, as

$$\tau = BInA \sin \theta = 1.0 \times 6.0 \times 30 \times [\pi \times (8 \times 10^{-2})^2] \times \sin 60° = \mathbf{3.1\ Nm}$$

EXAMPLE 5.16

A circular coil of area 300 cm^2 and 25 turns rotates about its vertical diameter with an angular speed of 40 rad/s in a uniform horizontal magnetic field of magnitude 0.05 T. Find the maximum voltage induced in the coil.

Solution Using Eq. 5.17, the maximum value of the emf induced in the coil is given as

$$E_m = NBA\omega = 25 \times 0.05 \times (300 \times 10^{-4}) \times 40 = \mathbf{1.5\ V}$$

EXAMPLE 5.17

A conducting circular loop is placed in a uniform magnetic field $B = 0.02$ T with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of 1.0 mm/s. Find the induced emf in the loop at an instant when the radius is 2 cm.

Solution Let the radius of the loop be r at a time t. The magnetic flux linking with the loop at any instant is given as $\Phi = BA = B \times (\pi r^2)$. Therefore, the emf induced in the loop due to its shrinking is

$$e = \frac{d\Phi}{dt} = B\pi \frac{d(r^2)}{dt} = B\pi \times 2r\frac{dr}{dt} = 0.02 \times \pi \times 2 \times 0.02 \times (1 \times 10^{-3}) = \mathbf{2.5\ \mu V}$$

EXAMPLE 5.18

A semicircular conducting loop ACDA of radius r with centre at point O is placed in a magnetic field B directed into the plane of the paper, as shown in Fig. 5.20*a*. The loop is now made to rotate clockwise with a constant angular velocity ω about an axis passing through point O and perpendicular to the plane of the paper. If the resistance of the loop is R, obtain an expression for the magnitude of the induced current in the loop. Plot a graph between the induced current and time, for two time-periods of rotation.

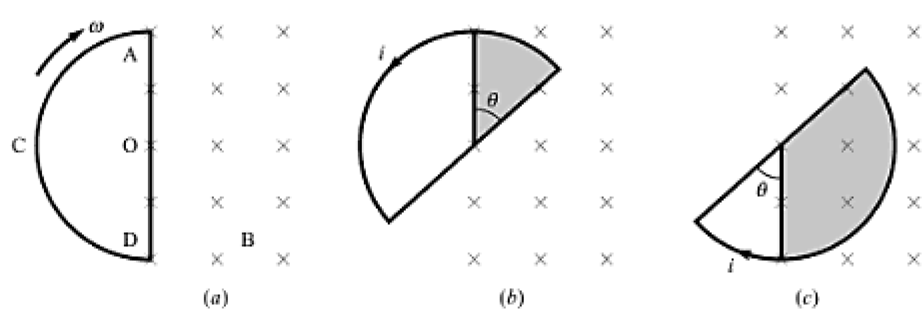


Fig. 5.20 *Semicircular loop in a magnetic field.*

Solution When the loop rotates through an angle $\theta\,(\theta < \pi)$, the area inside the field region is

$$A(t) = \frac{\theta}{\pi}\frac{\pi r^2}{2} = \frac{\theta r^2}{2} = \frac{\omega t\, r^2}{2} = \frac{\omega r^2}{2}t$$

Thus, at any instant the flux linking the loop is

$$\Phi = BA = \frac{B\omega r^2}{2}t$$

$\therefore$ The induced emf, $e = \dfrac{d\Phi}{dt} = \dfrac{B\omega r^2}{2}$

The current through the loop, $i = \dfrac{e}{R} = \dfrac{B\omega r^2}{2R}$

As the flux is increasing (Fig. 5.20*b*), the direction of the induced current is anticlockwise so that the field due to the induced current is opposite to the original field.

After half a rotation, the area of the loop in the field region starts decreasing (Fig. 5.20*c*), and is given by

$$A\,(t) = \frac{\pi r^2}{2} - \frac{\omega r^2}{2}t$$

Hence, the induced current will have the same magnitude but will flow in the opposite direction. The plot for two time-periods is shown in Fig. 5.21.

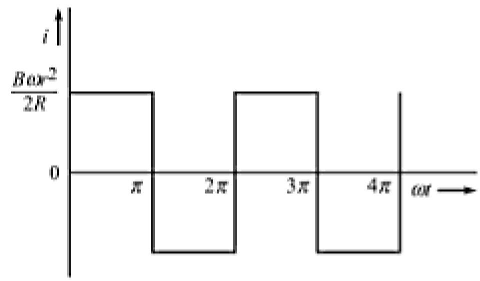


Fig. 5.21

SUMMARY

TERMS AND CONCEPTS

- For a current-carrying straight conductor, the ***right-hand thumb rule*** says, "If you stretch the thumb of your right hand along the current, the bend of fingers gives the direction of the magnetic field."
- For a current-carrying coil, the ***right-hand thumb rule*** says, "Bend the fingers of your right hand along the current, the thumb points in the direction of magnetic field."
- A dot inside a circle represents a current coming out of the plane of the paper, whereas a cross inside a circle means a current entering the plane of paper.
- The magnitude of the magnetic field at a point due to a current-carrying long wire is directly proportional to the current and inversely proportional to its distance.
- The magnitude of the magnetic field at a point due to a current-carrying long wire is directly proportional to the current and inversely proportional to its distance.
- The magnetic field strength (B) is measured in tesla (T), which is also equal to Wb/m^2.
- For both Fleming's right-hand rule and Fleming's left-hand rule, we associate

First finger	with	Field or Flux
Central finger	with	Current
thuMb	with	Motion of the conductor

- Fleming's *right-hand* rule is used to find the direction of induced emf (or current) in a moving conductor placed in a magnetic field (*generator action*).
- Fleming's *left-hand* rule is used to find the direction of force on a current carrying conductor placed in a magnetic field (*motor action*).
- One ***ampere*** of current is defined as that current which, when maintained in each of the two infinitely long parallel conductors situated in vacuum and separated by one metre, produces on each of the conductors a force of 2×10^{-7} newton per metre length.

IMPORTANT FORMULAE

- For an infinitely long straight wire, $B = \dfrac{\mu_0 I}{2\pi x}$.
- At the centre of a circular loop, $B = \dfrac{\mu_0 I}{2r}$.
- For a solenoid: (*a*) at the centre, $B = \mu_0 nI$; (*b*) at either end, $B = \dfrac{\mu_0 nI}{2}$.
- Inside the toroid, $B = \dfrac{\mu_0 nI}{2\pi r}$.
- Force on a straight conductor, $F = IBl \sin\theta$.
- The mutual force between two infinitely long conductors, $F = BI_2 = \dfrac{\mu_0 I_1 I_2}{2\pi r}$.
- Faraday's law, $e = -N\dfrac{d\Phi}{dt}$.
- Flux through a loop, $\Phi = BA\cos\theta$.
- Induced emf in a conductor, $e = -Blv\sin\theta$.
- Induced emf in a coil, $e = NBA\omega\sin\omega t$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The right-hand thumb rule gives the direction of force on a current-carrying conductor placed in a magnetic field.	☐	☐	
2.	The strength of the magnetic field produced at a point by a current-carrying conductor is directly proportional to the current and inversely proportional to the square of its distance from the conductor.	☐	☐	
3.	The unit of magnetic flux is weber (Wb) which is equal to Tm^2.	☐	☐	
4.	The magnetic field at the centre of a circular coil carrying current is inversely proportional to the square of its radius.	☐	☐	
5.	The force experienced by a current-carrying conductor placed at an angle of 90° to the magnetic field is zero.	☐	☐	
6.	Fleming's left-hand rule is used to find the direction of the force experienced by a current-carrying conductor placed in a magnetic field.	☐	☐	
7.	When two parallel conductors carry current in the same direction, the mutual force between them is repulsive.	☐	☐	
8.	Statically produced emf is directly proportional to the rate of change of flux.	☐	☐	
9.	The flux passing through two identical coils are 40 Wb and 80 Wb. The induced emf in the first coil is less than that in the second coil.	☐	☐	
10.	The emf induced in the secondary winding of a transformer is statically induced emf.	☐	☐	

Your Score

ANSWERS

1. False	**2.** False	**3.** True	**4.** False	**5.** False
6. True	**7.** False	**8.** True	**9.** False	**10.** True

REVIEW QUESTIONS

1. What is the magnetic field pattern due to a long current-carrying straight conductor?

2. When a current-carrying conductor is placed in a magnetic field, it experiences a force. How do you find the magnitude and direction of this force?

3. State and explain (*a*) Fleming's left-hand rule, and (*b*) Fleming's right-hand rule.

4. State how you will determine the nature of force between two parallel current-carrying conductors.

5. Sketch the magnetic field around two adjacent parallel current-carrying conductors, when the currents flowing through them are (*a*) in opposite directions, and (*b*) in the same direction.

6. Explain how the unit of current is defined.

7. What is a solenoid? Write the expression for the magnetic field at a point (*a*) inside the solenoid, and (*b*) just at one of its ends.

8. As you move away from the middle of a solenoid, why does the magnetic field decrease?

9. What is the difference between a solenoid and a toroid?

10. Suppose that you are sitting in a room with your back to the wall. Imagine that an electron beam travelling horizontally from the back wall to the front wall is deflected to your right. What is the direction of the magnetic field that exists in the room?

11. What is meant by electromagnetic induction? State and explain Faraday's laws of electromagnetic induction.

12. State Lenz's law. Show, by means of an example, that the Lenz's law and Fleming's right-hand rule give the same direction of induced emf in a circuit.

13. Show that Lenz's law is a consequence of the principle of conservation of energy.

14. One end of a bar magnet is thrust into a coil. It is noted that the induced current in the coil is in clockwise direction as viewed from the front end. Is the end of the bar magnet its N-pole or S-pole?

15. A metallic loop is placed in a nonuniform magnetic field. Will an emf be induced in the loop?

16. Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop? Do the loops attract or repel each other?

17. The battery in the above question is suddenly disconnected. Is a current induced in the other loop? If yes, when does it start and when does it end? Do the loops attract or repel each other?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. An electric charge in uniform motion produces
(*a*) an electric field only
(*b*) a magnetic field only
(*c*) both the electric and magnetic fields
(*d*) no such field at all

2. The unit of magnetic field B is
(*a*) weber (*b*) weber/metre^2
(*c*) newton/metre (*d*) tesla/metre^2

3. The magnetic field at a point due to a current-carrying conductor is directly proportional to
(*a*) the resistance of the conductor
(*b*) the distance from the conductor
(*c*) the thickness of the conductor
(*d*) the current flowing through the conductor

4. Two long parallel wires carrying sinusoidally varying currents in the opposite directions
(*a*) attract each other
(*b*) repel each other

(*c*) do not affect each other
(*d*) get rotated to be perpendicular to each other

5. A wire is placed parallel to the lines of force in a magnetic field and a current flows in this wire. Then
   (*a*) the wire experiences a force in the direction of the magnetic field
   (*b*) the wire does not experience any force
   (*c*) the wire experiences a force in a direction opposite to the magnetic field
   (*d*) the wire experiences a force in a direction perpendicular to the magnetic field

6. The magnetic field of a U-magnet is parallel to this paper with N on the left side. A conductor is placed in the magnetic field so that it is perpendicular to this paper. When a current flows through the conductor out of the paper, it tends to move
   (*a*) upward (*b*) downward
   (*c*) to the right (*d*) to the left

7. Which of the graphs in Fig. 5.22 shows variation of magnetic field B with distance r from a long straight wire carrying a current?

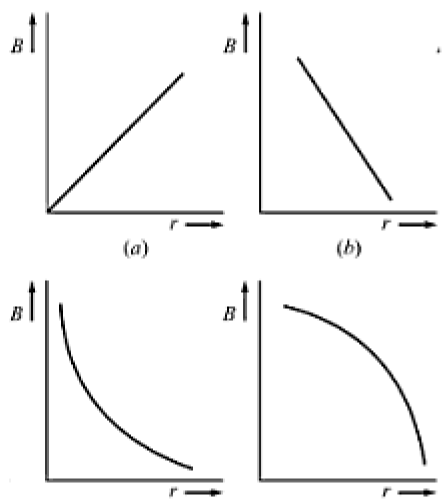


Fig. 5.22

8. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current I is established in the wire as shown in Fig. 5.23, the loop will
   (*a*) rotate about an axis parallel to the wire
   (*b*) move away from the wire
   (*c*) move towards the wire
   (*d*) remain stationary

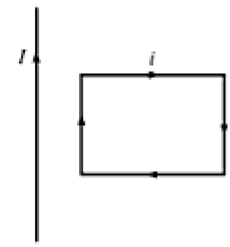


Fig. 5.23

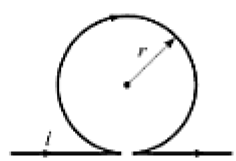


Fig. 5.24

9. An infinitely long straight wire is bent into a shape as shown in Fig. 5.24, the radius of the circular loop being r metre. It carries a current i ampere. Then the magnetic field at the centre of the circular part is
   (*a*) zero (*b*) infinite
   (*c*) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}(\pi + 1)$ (*d*) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}(\pi - 1)$

10. A length of wire carries a steady current. It is bent first to form a circular plane coil of one turn. The same length is now bent more sharply to give a double loop of smaller radius. The magnetic field at the centre of this loop developed due to the same current is
   (*a*) four times its first value
   (*b*) one-fourth of its first value
   (*c*) half of its first value
   (*d*) unaltered

ANSWERS

1. *c*	**2.** *b*	**3.** *d*	**4.** *b*	**5.** *b*	**6.** *a*	**7.** *c*	**8.** *c*	**9.** *d*	**10.** *a*

PROBLEMS

(A) SIMPLE PROBLEMS

1. A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.4 A. What is the magnitude of the magnetic field at the centre of the coil? [**Ans.** 0.314 mT]

2. A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire? [**Ans.** 35 μT]

3. A long straight wire in the horizontal plane carries a current of 50 A in north-to-south direction. Give the magnitude and direction of **B** at a point 2.5 m east of wire. [**Ans.** 4 μT, vertically up]

4. A 3.0-cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is 0.27 T. What is the magnetic force on the wire? [**Ans.** 81 mN]

(B) TRICKY PROBLEMS

5. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre. [**Ans.** 25.1 mT]

6. A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. (*a*) What magnetic field should be set up normal to the conductor so as to make the tension in the wires zero? (*b*) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.)
[**Ans.** (*a*) Horizontal magnetic field of 0.26 T; (*b*) 1.176 N]

7. A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of 400 μT parallel to the current. Find the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire. [**Ans.** 500 μT]

8. A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the voltage developed across the cut if the velocity of the loop is 1 cm/s in a direction normal to (*a*) the longer side, (*b*) the shorter side of the loop? For how long does the induced voltage last in each case?
[**Ans.** (*a*) 0.24 mV lasting 2 s;
(*b*) 0.06 mV lasting 8 s]

(C) CHALLENGING PROBLEMS

9. A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 s. The magnetic flux between the pole pieces is known to be 0.8 mWb. Determine the emf induced in the wire. [**Ans.** 1.6 mV]

10. A uniform magnetic field B exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.02$ T/s. Find the current induced in the frame. The resistivity of copper is 1.7×10^{-8} Ω m.
[**Ans.** 93 mA]

MAGNETIC CIRCUITS 6

OBJECTIVES

After completing this Chapter, you will be able to:

- *State what is meant by magnetomotive force (mmf), magnetic field strength (H), magnetic permeability (μ), reluctance ($\mathcal{R}$) and permeance ($\mathcal{G}$).*
- *Draw analogy between electric and magnetic circuits.*
- *Solve problems related to composite magnetic circuits.*
- *Explain the meaning of leakage and fringing of magnetic flux.*
- *State Kirchhoff's laws for magnetic circuits.*
- *State two reasons for including an air gap in a magnetic circuit.*
- *State a disadvantage of including an air gap in a magnetic circuit.*

6.1 INTRODUCTION

The lines of magnetic flux form *closed loops*. The complete closed path followed by any line or group of lines of magnetic flux is referred to as a ***magnetic circuit***.

An electric circuit provides a path for electric current, whereas a magnetic circuit provides path for magnetic flux. Just as a copper (or aluminium) wire, because of its high conductivity, confines the electric current within itself, similarly, a ferromagnetic material (such as iron or steel), due to its high *permeability*, confines magnetic flux within itself. The knowledge of magnetic circuits is essential to understand the working of electromagnetic devices such as transformers, rotating machines and electromagnetic relays.

6.2 MAGNETOMOTIVE FORCE (MMF)

In an electric circuit, the current is due to the existence of an electromotive force (emf). By analogy, we may say that in a magnetic circuit, the magnetic flux is due to the existence of a ***magnetomotive force (mmf)***. Basically, the magnetic field results from a moving charge (or an electric current). Hence, mmf is caused by a current flowing through one or more turns. The value of the mmf is proportional to the current and the number of turns. It is expressed in *ampere turns* (At), but for the purpose of dimensional analysis, it is expressed in amperes, since the number of turns is dimensionless. The mmf is represented by the symbol $\mathcal{F}$.

Magnetic Field Strength (*H*)

Suppose that a current of I amperes flows through a coil of N turns wound on a toroid of length l metres, as shown in Fig. 6.1. The mmf is the *total* current linked with the magnetic circuit, namely, IN ampere-turns. If the magnetic circuit is homogeneous and of uniform cross-sectional area, *the mmf per metre length of the magnetic circuit* is termed as the ***magnetic field strength***, ***magnetic field intensity***, or ***magnetizing force***. It is represented by symbol H and is measured in ampere-turns per metre (At/m). Thus, for the magnetic circuit of Fig. 6.1,

$$H = \frac{\mathcal{F}}{l} = \frac{IN}{l} \tag{6.1}$$

Magnetic field intensity, H, is independent of the medium. Its value depends only on the number of turns N and the current I flowing in the coil.

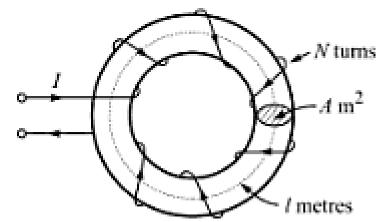


Fig. 6.1 *A magnetic circuit in the form of a toroid.*

Magnetic Permeability (μ)

If the core of the toroid (Fig. 6.1) is vacuum or air (which is nonmagnetic), the magnetic flux density B in the core bears a definite ratio to the magnetic field strength H. This ratio is called *permeability of free space*, and is represented by symbol μ_0. Thus, for vacuum or air,

$$\frac{B}{H} = \mu_0 \quad \text{or} \quad B = \mu_0 H \tag{6.2}$$

The value of the constant μ_0 is $4\pi \times 10^{-7}$ Tm/A.

If the nonmagnetic core of the toroid is replaced by a magnetic material such as iron or steel, the flux produced by the given mmf is greatly increased. As a result, the flux density B also increases many times. In general, we can write

$$B = \mu H$$

where, μ is called the ***permeability*** of the material. *The ratio of the flux density produced in a material to the flux density produced in vacuum by the same magnetic field strength* is termed as the ***relative permeability*** and is denoted by the symbol μ_r. It is simply a numeric, having no units. It signifies the degree to which the material is more permeable to the magnetic flux as compared to free space. Thus, the permeability (μ) of a material can be expressed as a product of μ_0 and its relative permeability, i.e., $\mu = \mu_0 \mu_r$. The permeability of air is same as that of vacuum. That is, for air, $\mu_r = 1$.

The permeability of *diamagnetic materials* is a bit inferior to that of air. The μ_r of diamagnetic materials is, therefore, slightly less than unity (e.g., for copper, $\mu_r = 0.999998$). *Paramagnetic materials* have μ_r slightly greater than unity (e.g., for aluminium, $\mu_r = 1.000008$). Most *ferromagnetic materials* have μ_r in hundreds or in thousands. For some materials as Deltamax, μ_r can be as high as 100 000.

One good thing about ferromagnetic materials is that their μ_r is very high, but one problem is that the values of their μ_r are not constant. It greatly varies with magnetic field strength. It is, therefore, a usual practice to represent the relationship between the flux density B and the magnetic field strength H graphically as in Fig. 6.2. These curves are called *magnetization characteristics* of the ferromagnetic materials.

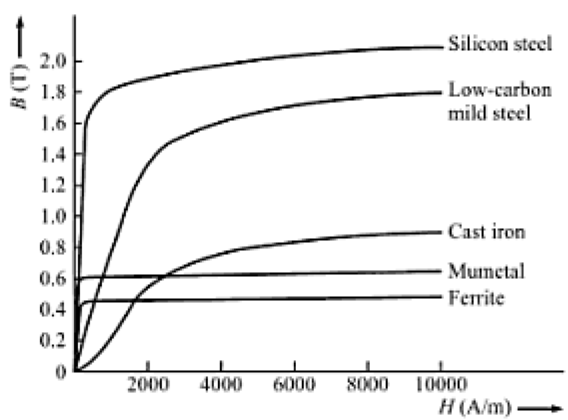


Fig. 6.2 *Magnetization characteristics of ferromagnetic materials.*

Reluctance ($\mathcal{R}$) and Permeance ($\mathcal{G}$)

We know that the electrical *resistance* of a conductor of length l and area of cross-section A is given as

$$R = \rho \frac{l}{A} \quad \text{or} \quad R = \frac{1}{\sigma} \frac{l}{A}$$

where, ρ is the *resistivity* and its reciprocal σ (= $1/\rho$) is the *conductivity* of the material.

Just as the current I in an electric circuit is limited by the presence of resistance of the electric circuit, the flux Φ in a magnetic circuit is limited by the presence of the ***reluctance*** of the magnetic circuit. Thus, the reluctance is a measure of the opposition by a magnetic circuit to the setting up of the flux. It is represented by the symbol $\mathcal{R}$. Thus, by analogy, the reluctance of a specimen of magnetic material is given as

$$\mathcal{R} = \frac{1}{\mu} \frac{l}{A} = \frac{1}{\mu_r \mu_0} \frac{l}{A} \tag{6.3}$$

The reciprocal of reluctance is known as ***permeance*** ($\mathcal{G}$). Thus, permeance in magnetic circuits is analog of conductance (G) in electric circuit.

Note that in magnetic circuits, no quantity analogous to electrical resistivity is defined.

6.3 MAGNETIC CIRCUIT THEORY

A typical magnetic circuit consists of a number of iron paths of specified geometry, connected in series and parallel and having one or more air-gaps. It also has a source of mmf in the form of a coil of specified number of turns carrying a dc current.

Consider a simple magnetic circuit of Fig. 6.1. The coil wound around the toroid has N turns and carries a current of I amperes. The toroid has a cross-sectional area of A metre^2 and a mean circumference of l metres. Then, the value of mmf, $\mathcal{F} = NI$ ampere-turns. Because of this mmf, a magnetic field of strength H is set up throughout the length l of the toroid. According to the definition of H, we should have

$$\mathcal{F} = Hl \tag{6.4}$$

Since, the flux density B is flux per unit area, total flux is given as

$$\Phi = B \times A \tag{6.5}$$

Dividing Eq. 6.5 by 6.4, we get

$$\frac{\Phi}{\mathcal{F}} = \frac{BA}{Hl} = \frac{B}{H} \frac{A}{l} = \mu \frac{A}{l} = \mu_r \mu_0 \frac{A}{l}$$

or

$$\Phi = \frac{\mathcal{F}}{l/(\mu_r \mu_0 A)} \tag{6.6}$$

Similar equation in electrical circuits gives the electric current I as the ratio of emf E to the resistance R. That is,

$$I = \frac{E}{R} \tag{6.7}$$

Comparison of Eq. 6.6 with 6.7 suggests that the quantity $l/(\mu_r \mu_0 A)$ should represent some sort of resistance in magnetic circuits. This quantity is called ***reluctance*** of the magnetic circuit, which we have defined in Eq. 6.3.

Analogy between Electric and Magnetic Circuits

There exists a close analogy between electric and magnetic circuits, as brought out in Table 6.1. Understanding this analogy is helpful while solving magnetic circuits.

Though electric and magnetic circuits are quite similar, there are some **differences** too:

1. Energy must be continuously supplied to an electric circuit to maintain current in it, whereas the magnetic flux once set up, does not need any further supply of energy.
2. The magnetic circuit stores energy in its field whereas the electric circuit immediately releases its energy as heat.
3. In magnetic circuits involving ferromagnetic materials, on increasing the magnetic field strength the flux density increases only till the state of saturation is reached. But in electric circuits, there is no such phenomenon as saturation.

Table 6.1 *Analogy between Electric and Magnetic Circuits*

S. No.	*Electric Circuits*		*Magnetic Circuits*	
	Quantity	*Units*	*Quantity*	*Units*
1.	EMF, E	volts (V)	MMF, $\mathcal{F}$	ampere turn (At)
2.	current, I	ampere (A)	Flux, Φ	weber (Wb)
3.	Resistance, R $R = \frac{1}{\sigma} \cdot \frac{l}{A}$	ohm (Ω)	Reluctance, $\mathcal{R}$ $\mathcal{R} = \frac{1}{\mu_r \mu_0} \cdot \frac{l}{A}$	ampereturn/weber (At/Wb)
4.	Conductivity, σ	siemen/metre (S/m)	Permeability, μ	tesla metre per ampere (Tm/A)
5.	Conductance, G	siemen (S)	Permeance, $\mathcal{G}$	weber/ampere (Wb/A)
6.	Current density, J	ampere/metre^2 (A/m^2)	Flux density, B	weber/metre^2 or tesla (Wb/m^2 or T)
7.	Electric field intensity, $\mathcal{E}$	volt/metre (V/m)	Magnetic field intensity, H	ampere turn per metre (At/m)
8.	**Ohms law** : $I = \frac{E}{R}$		$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$	

EXAMPLE 6.1

A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 60 cm and a uniform cross-sectional area of 500 mm^2. If the current through the coil is 4 A, calculate (*a*) the magnetic field strength, (*b*) the flux density, and (*c*) the total flux.

Solution

(*a*) The magnetic field strength is

$$H = \frac{NI}{l} = \frac{200 \times 4}{0.6} = \mathbf{1333\ A/m}$$

(*b*) The flux density, $B = \mu H = \mu_r \mu_0 H = 1 \times 4\pi \times 10^{-7} \times 1333 = \mathbf{1675\ \mu T}$

(*c*) The total flux, $\Phi = BA = (1675 \times 10^{-6}) \times (500 \times 10^{-6}) = \mathbf{0.8375\ \mu Wb}$

EXAMPLE 6.2

Calculate the magnetomotive force (mmf) required to produce a flux of 0.015 Wb across an air gap of 2.5 mm long, having an effective area of 200 cm^2.

Solution The magnetic flux density, $B = \dfrac{\Phi}{A} = \dfrac{0.015}{200 \times 10^{-4}} = 0.75$ T

Therefore, magnetic field strength for the gap, $H = \dfrac{B}{\mu_0} = \dfrac{0.75}{4\pi \times 10^{-7}} = 597000$ A/m

$\therefore$ mmf required, $\mathcal{F} = Hl = 597000 \times 2.5 \times 10^{-3} = \mathbf{1492\ At}$

EXAMPLE 6.3

A mild-steel ring having a cross-sectional area of 500 mm^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Calculate (*a*) the reluctance of the ring, and (*b*) the current required to produce a flux of 800 μWb in the ring. Assume the relative permeability of mild-steel to be 380.

Solution

(*a*) The reluctance, $\mathcal{R} = \dfrac{1}{\mu_r \mu_0}\dfrac{l}{A} = \dfrac{0.4}{380 \times 4\pi \times 10^{-7} \times 500 \times 10^{-6}} = \mathbf{1.675 \times 10^6\ A/Wb}$

(*b*) The mmf, $\mathcal{F} = \Phi\mathcal{R} = (800 \times 10^{-6}) \times (1.675 \times 10^6) = 1340$ A

$\therefore$ magnetising current, $I = \dfrac{\mathcal{F}}{N} = \dfrac{1340}{200} = \mathbf{6.7\ At}$

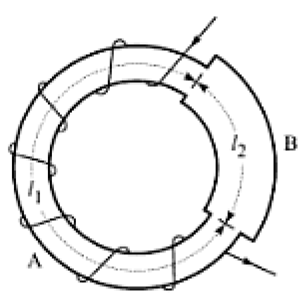
Composite Magnetic Circuit

Consider a magnetic circuit consisting of two specimen of steel, A and B as shown in Fig. 6.3*a*. Let l_1 and l_2 be the mean lengths, A_1 and A_2 be the cross-sectional areas, and μ_1 and μ_2 be the absolute permeabilities of these two specimens, respectively. Then

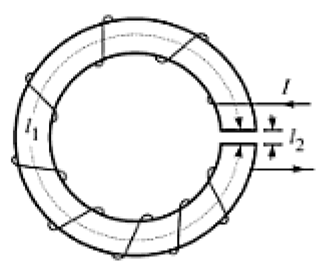
$$\text{Reluctance of } A,\ \mathcal{R}_1 = \frac{l_1}{\mu_1 A_1} \qquad \text{and} \qquad \text{Reluctance of } B,\ \mathcal{R}_2 = \frac{l_2}{\mu_2 A_2}$$

If a coil is wound on core A and if the magnetic flux is assumed to be confined to the steel core, then

$$\text{Total reluctance of the magnetic circuit, } \mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} \tag{6.8}$$



(*a*) Composite magnetic circuit.



(*b*) Steel ring with an air gap.

Fig. 6.3 *Magnetic circuits.*

Now, applying Ohm's law for the magnetic circuit, we get

$$\text{Total flux, } \Phi = \frac{\text{mmf of coil}}{\text{total reluctance}} = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\dfrac{l_1}{\mu_1 A_1} + \dfrac{l_2}{\mu_2 A_2}}$$

Let us consider another magnetic circuit with an air gap, as shown in Fig. 6.3*b*. It has a steel-ring with a small air-gap. Almost all the flux goes straight across the gap from one face to the other. Hence, the area of the gap may be assumed to be the same as that of the ring. If A is the area of cross-section and l_1 and l_2 are the lengths of the core and the air-gap, respectively,

$$\text{Total reluctance, } \mathcal{R} = \frac{l_1}{\mu_1 A} + \frac{l_2}{\mu_0 A} = \frac{1}{\mu_0 A}\left(\frac{l_1}{(\mu_1/\mu_0)} + l_2\right) = \frac{1}{\mu_0 A}\left(\frac{l_1}{\mu_r} + l_2\right) \tag{6.9}$$

Since the relative permeability μ_r (= μ_1/μ_0) of steel is very large (of the order of thousand), the major contribution in the total reluctance $\mathcal{R}$ is by the air-gap, though its length l_2 may be quite small (say, a few millimetres).

Magnetic Leakage and Fringing

Consider the magnetic circuit consisting of a steel ring with air gap (Fig. 6.4*a*). A metal ring dd is placed symmetrically in the air gap. The winding carrying current I is concentrated over short length of the core. In actual practice we find that the entire flux produced by the coil does not pass through the metallic ring dd. Some of the flux lines, such as a, b and c, *leak* through the core and return via short paths through air. This is called ***leakage flux***, since it does not contribute to the ***useful flux*** passing through the metallic ring.

In practice, besides the flux leakage, there is another deviation from the ideal situation. The useful flux passing across the air-gap tends to bulge outward as shown in Fig. 6.4*a*. This happens because the magnetic flux lines tend to repel each other while passing through a nonmagnetic material. This phenomenon is known as ***fringing***. Its effect is to cause a slight increase (say, about 10 %) in the cross-sectional area at the air gap. This results in a decrease of flux density in the air gap.

The distinction between the useful and the leakage fluxes may be more obvious if we consider an electrical machine. For instance, Fig. 6.4*b* shows two poles of a six-pole machine. For simplicity, the armature slots (in the rotor) have been omitted here. As shown, some of the flux lines (marked L) do not enter the armature core and thus do not contribute in generating an emf in the armature winding. These represent *leakage flux*. On the other hand, some of the flux passing between the pole tips and the armature core follows bulged paths (marked F). It is referred to as *fringing flux*. Since this fringing flux is cut by the armature conductors, it forms a part of the useful flux.

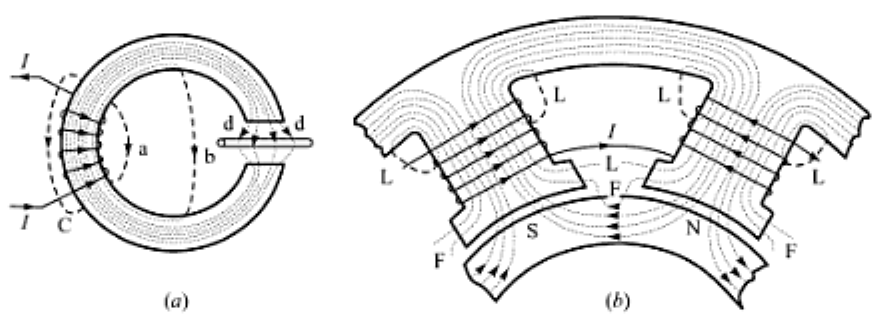


Fig. 6.4 *Magnetic leakage and fringing.*

From Fig.6.4*b* it is seen that the effect of leakage flux is to increase the requirement of the total flux through the exciting winding. We define ***leakage factor*** as follows:

$$\text{Leakage factor, } \lambda = \frac{\text{Total flux through exciting winding}}{\text{Useful flux}}$$

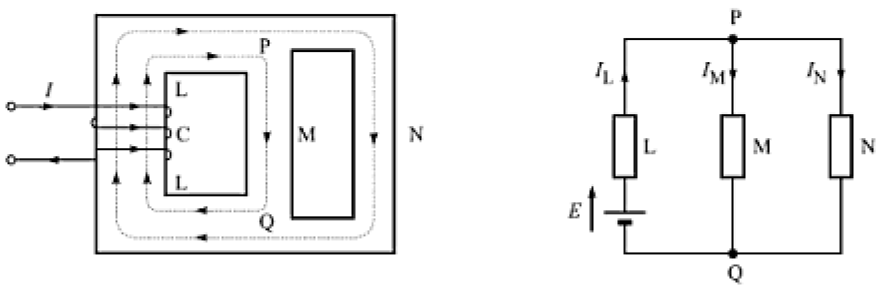
The value of the leakage factor for electrical machines is about 1.15 to 1.25.

Kirchhoff's Laws for Magnetic Circuits

Because of the analogy between electric and magnetic circuits, the Kirchhoff's laws are also applicable in magnetic circuit in the following form.

***Kirchhoff's Flux Law* (*KFL*)** *The total magnetic flux towards a junction is equal to the total magnetic flux away from that junction.* This law follows from the fact that each line of flux is a closed path. Suppose that a ferromagnetic core is arranged as shown in Fig. 6.5*a*. Coil C is wound on limb L and carries a current I. The total magnetic flux produced in limb L divides at point P; some flux passes along limb M and the remaining along limb N. These two parts join again at Q. There is no break or discontinuity in any of the lines of flux at P and Q. Thus, we must have

$$\Phi_L = \Phi_M + \Phi_N$$



(*a*) Magnetic circuit to illustrate Kirchhoff's laws.

(*b*) Electric circuit equivalent to the given magnetic circuit.

Fig. 6.5

***Kirchhoff's Magnetomotive Force Law* (*KML*)** *In a closed magnetic circuit, the algebraic sum of the product of the magnetic field strength and the length of each part of the circuit is equal to the resultant magnetomotive force.* For example, if H_L is the magnetic field strength required for limb L and l_L is the length of the circuit from Q via L to P, and if H_M and l_M are the corresponding values for limb M and H_N and l_N are those for the limb extending from P via N to Q, then

$$\text{Total mmf of the coil C} = H_L l_L + H_M l_M$$

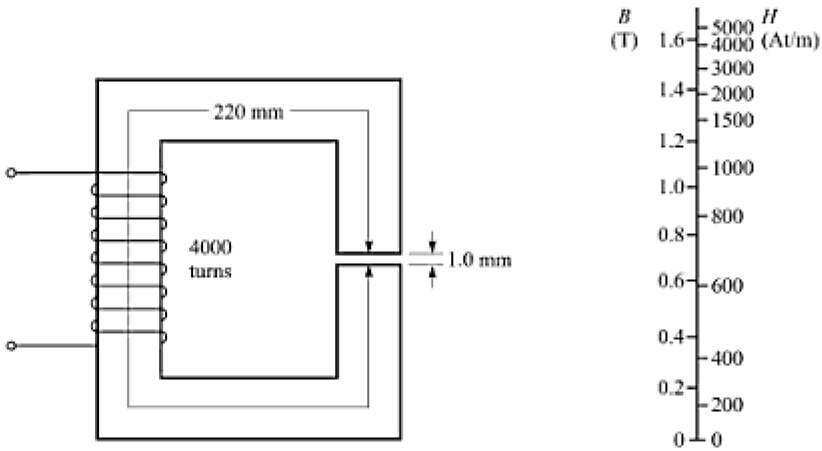
also
$$\text{Total mmf of the coil C} = H_L l_L + H_N l_N$$

and
$$0 = H_M l_M - H_N l_N$$

It may be helpful to compare the magnetic circuit of Fig. 6.5*a* with the corresponding electric circuit of Fig. 6.5*b*.

EXAMPLE 6.4

A magnetic circuit, shown in Fig. 6.6*a* is of uniform cross-section of area 50 mm^2, throughout. The ferromagnetic core has mean length of 220 mm and the air gap has a length of 1.0 mm. The air gap has the same effective cross-sectional area as the ferromagnetic core. A coil of 4000 turns is wound on the core. The magnetic characteristic of the material is given in the nomogram of Fig. 6.6*b*. Estimate the current in the coil to produce a flux density of 0.9 T in the air gap, assuming that all flux passes through both parts of the magnetic circuit.



(*a*) Magnetic circuit for Example 6.4.

(*b*) Magnetic characteristic of the core.

Fig. 6.6

Solution Given : $B = 0.9$ T; $N = 4000$. Using Kirchhoff's second law, we require determining the mmf for both the core and the air gap in order to calculate the total mmf.

For ferromagnetic core : $l_c = 220$ mm $= 0.22$ m; $A_c = 50$ mm$^2 = 50 \times 10^{-6}$ m^2.

From the given nomogram of Fig. 6.6*b*, corresponding to $B = 0.9$ T, $H_c = 820$ A/m.

$\therefore$ MMF for the core, $\mathcal{F}_c = H_c l_c = 820 \times 0.22 = 180$ At

For air gap : $l_g = 1.0$ mm $= 0.001$ m; $A_g = 50$ mm$^2 = 50 \times 10^{-6}$ m^2.

$$H_g = \frac{B}{\mu_0} = \frac{0.9}{4\pi \times 10^{-7}} = 716\,000 \text{ At}$$

$\therefore$ MMF for the air gap, $\mathcal{F}_g = H_g l_g = 716\,000 \times 0.001 = 716$ At

Therefore,

$$\text{Total mmf, } \mathcal{F} = \mathcal{F}_c + \mathcal{F}_g = 180 + 716 = 896 \text{ At}$$

$$\therefore \quad \text{Magnetization current, } I = \frac{\mathcal{F}}{N} = \frac{896}{4000} = \mathbf{0.224\ A}$$

Hints to Solve Magnetic Circuits From the Examples 6.1 to 6.4, we can make following observations about solving problems on magnetic circuits :

1. In most problems, the solution takes a standard form, shown as a chain in Fig. 6.7. You are to start at or near one end of the chain. Just go on doing calculations and move to the other end. In Example 6.4, we started at B and worked down the chain in order to obtain I.

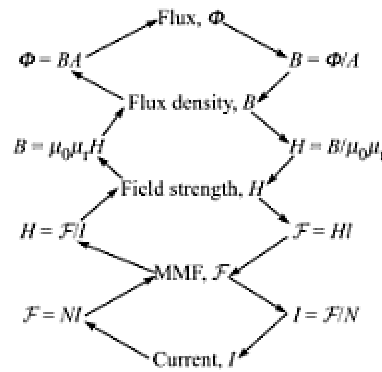


Fig. 6.7 *Steps to solve a problem on magnetic circuit.*

2. The calculation of mmf for an air gap can be shortened by noting that a flux density of 1.0 T in air requires a magnetic field strength of about 800 000 A/m. If you remember this figure, the air gap mmf can be quickly obtained by multiplying it by the required flux density. For instance, in Example 6.4, you would have got the mmf in air gap as 800 000 × 0.9 = 720 000 A/m (whereas the actual value obtained was 716 000 A/m). The small difference would have made a negligible difference in the final result.
3. The reluctances can be treated like resistances in electric circuits. Thus, the series reluctances can be added directly, while parallel reluctances can be combined by the reciprocal method.
4. For the Example 6.4, the relative permeability of the steel core is given as

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.9}{4\pi \times 10^{-7} \times 820} = 873$$

Therefore, the reluctances of steel core and of air gap are

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A} = \frac{0.22}{4\pi \times 10^{-7} \times 873 \times 50 \times 10^{-6}} = 4\,010\,000 \text{ A/Wb}$$

and

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A} = \frac{0.001}{4\pi \times 10^{-7} \times 50 \times 10^{-6}} = 15\,910\,000 \text{ A/Wb}$$

We find that the reluctance of air gap is approximately four times that of steel core, although its length is only 1.0 mm (compared to 220 mm for steel core). Note the predominant effect of the air gap.

5. Had there been no air gap in the core of the Example 6.4, the current required to produce a flux density of 0.9 T would have been only 0.045 A (instead of 0.224 A). This extra demand for current raises the question as to why an air gap should be introduced at all !

Air Gaps in Magnetic Circuits Generally, air gaps appear in magnetic circuits for two purposes. The ***first*** is to permit part of a magnetic circuit to move, for example, in relays and in electrical machines (motors and generators). In a relay, the closing of the air gap permits the movement of the crank which operates the switching contacts. In electrical machines, a cylindrical rotor rotates inside a stator. Without a little air gap, the rotor cannot rotate.

The ***second*** purpose is to make the magnetization characteristic of the circuit more linear. Such characteristic ensures a linear relationship between voltage and current. To see the effect of introducing an air gap on the magnetization characteristic, we plot the flux Φ verses the mmf $\mathcal{F}$. Suppose that we need to establish a flux Φ_1 in the magnetic circuit. With no air gap you need smaller mmf $\mathcal{F}_m$. But with some air gap, you need much larger mmf $\mathcal{F}_m$. To see the advantage derived by introducing an air gap of 0.2 mm and then of 1.0 mm, let us plot Φ verses $\mathcal{F}/\mathcal{F}_m$, as in Fig. 6.8. This graph clearly shows how introducing even a very small air gap makes the characteristic more linear.

Stacking in Magnetic Circuits Due to time-varying flux, eddy currents are induced in the magnetic core. This causes what is called *eddy current loss*[1]. To reduce this loss, the magnetic cores are usually made of thin sheets called ***laminations.*** The laminations are coated with varnish and then stacked together. The

[1] We shall study more about these losses in Chapter 13.

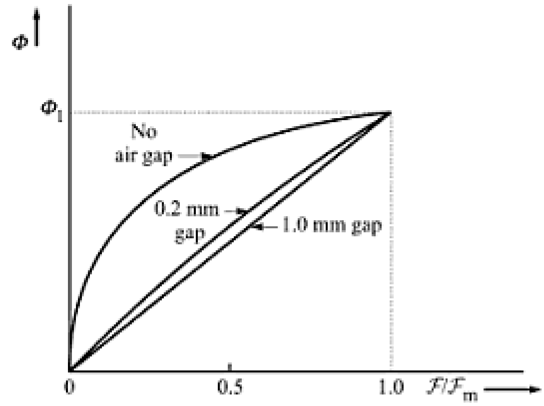


Fig. 6.8 *Effect of air gap on linearization of magnetization characteristic.*

varnish provides insulation from one sheet to the other. The varnished surfaces do not constitute any air gap in the magnetic path. The flux is set up without any difficulty. However, due to the thickness of the varnish surface, the active cross-sectional area is reduced by a small amount (say, 5-10 %). ***Stacking factor*** is defined as the ratio of effective area to the overall area of cross-section. Typically, its value lies in the range 0.9-0.95.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 6.5

A magnetic circuit is made up of low-carbon mild steel laminations shaped as in Fig. 6.9. The width of the core is 40 mm and the core is built to a depth of 50 mm, of which 8 % is taken up by insulation between the laminations. The air gap is 2.0 mm long and the effective area of the gap is 2500 mm^2. The coil is wound with 800 turns. If the leakage factor is 1.2, calculate the magnetizing current required to produce a flux of 0.0025 Wb across the air gap. If required, use the magnetization characteristics of mild steel as given in Fig. 6.2.

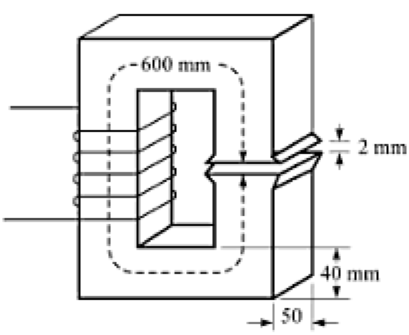


Fig. 6.9

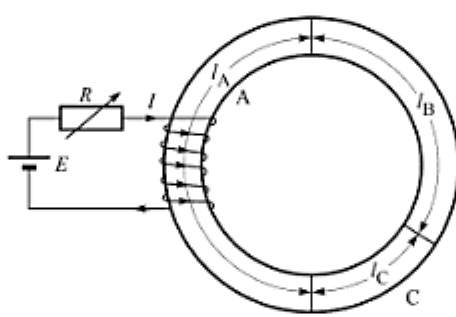


Fig. 6.10

Solution Cross-sectional area of the core, A_c = width × depth = 40 × 50 = 2000 mm^2

$$= 2 \times 10^{-3}\ \text{m}^2$$

For air gap: l_g = 2.0 mm = 2×10^{-3} m; A_g = 2500 mm^2 = 2.5×10^{-3} m^2; $\Phi = 2.5 \times 10^{-3}$ Wb.

$$B_g = \frac{\Phi}{A_g} = \frac{2.5\times10^{-3}}{2.5\times10^{-3}} = 1\ \text{T}; \quad H_g = \frac{B_g}{\mu_0} = \frac{1}{4\pi\times10^{-7}} = 796\ 000\ \text{At/m}$$

$\therefore$ mmf, $\mathcal{F}_g = H_g l_g = 796\ 000 \times 2 \times 10^{-3} = 1592$ At

For the core: Since 8 % is taken up by the insulation between the laminations, the effective area of cross-section (through which the flux passes) is only 92 % of the total area.

$$\therefore \qquad A_c = 2 \times 10^{-3} \times 0.92 = 1.84 \times 10^{-3}\ \text{m}^2$$

Flux through the core, Φ_c = flux in air gap × leakage factor = $2.5 \times 10^{-3} \times 1.2 = 3.0 \times 10^{-3}$ Wb

$$\therefore \qquad B_c = \frac{\Phi_c}{A_c} = \frac{3.0\times10^{-3}}{1.84\times10^{-3}} = 1.63\ \text{T}$$

Corresponding to this value of flux density, the value of magnetic field strength is read from Fig. 6.2 as 4000 At/m (approx). Hence,

$$\text{mmf, } \mathcal{F}_c = H_c l_c = 4000 \times 0.6 = 2400\ \text{At}$$

$$\therefore \qquad \text{Total mmf needed, } \mathcal{F} = \mathcal{F}_g + \mathcal{F}_c = 1592 + 2400 = 3992\ \text{At}$$

Hence, the required magnetizing current, $I = \dfrac{\mathcal{F}}{N} = \dfrac{3992}{800} = \mathbf{5\ A}$

EXAMPLE 6.6

A toroid of uniform cross-sectional area 0.001 m^2 consists of three materials as shown in Fig. 6.10. Material A is silicon steel with length l_A = 0.3 m, material B is low-carbon mild steel with length l_B = 0.2 m, and material C is cast iron with length l_C = 0.1 m. The exciting coil has 100 turns. Determine (*i*) the mmf for setting up a flux of 600 µWb, (*ii*) the current in the coil, and (*iii*) the relative permeability and reluctance of each of the three core materials. If required, use the data given in Fig. 6.2.

Solution

(*a*) Since the cross-sectional area and the flux for the three sections are the same, the flux densities are also same. That is,

$$B_A = B_B = B_C = B = \frac{\Phi}{A} = \frac{600\times10^{-6}}{0.001} = 0.6\ \text{T}$$

For this value of B, the corresponding value of H is found for the three sections of the toroid from Fig. 6.2, as

$$H_A = 20\ \text{At/m}, \qquad H_B = 700\ \text{At/m} \qquad \text{and} \qquad H_C = 2500\ \text{At/m}$$

Hence, by applying Kirchhoff's magnetomotive force law (KML), the total mmf required to set up the flux of 600 µWb is

$$\mathcal{F} = \mathcal{F}_A + \mathcal{F}_B + \mathcal{F}_C = H_A l_A + H_B l_B + H_C l_C = 20 \times 0.3 + 700 \times 0.2 + 2500 \times 0.1$$
$$= 6\ \text{At} + 140\ \text{At} + 250\ \text{At} = \mathbf{396\ At}$$

(*b*) To produce above mmf, the current required in the coil is

$$I = \frac{\mathcal{F}}{N} = \frac{396}{100} = \mathbf{3.96\ A}$$

(*c*) Since $B = \mu H = \mu_r \mu_0 H$, we have $\mu_r = \dfrac{B}{\mu_0 H}$. Therefore, the relative *permeabilities* of the three materials are given as

$$\mu_{rA} = \frac{B}{\mu_0 H_A} = \frac{0.6}{4\pi\times10^{-7}\times20} = \mathbf{23\ 885}; \quad \mu_{rB} = \frac{B}{\mu_0 H_B} = \frac{0.6}{4\pi\times10^{-7}\times700} = \mathbf{682}$$

and
$$\mu_{rC} = \frac{B}{\mu_0 H_C} = \frac{0.6}{4\pi\times10^{-7}\times2500} = \mathbf{191}$$

By applying Ohm's law, the *reluctances* of the three sections are given as

$$\mathcal{R}_A = \frac{\mathcal{F}_A}{\Phi} = \frac{6\text{ At}}{600\ \mu\text{Wb}} = \mathbf{10\,000\ At/Wb}; \quad \mathcal{R}_B = \frac{\mathcal{F}_B}{\Phi} = \frac{140\text{ At}}{600\ \mu\text{Wb}} = \mathbf{233\,333.3\ At/Wb}$$

and
$$\mathcal{R}_C = \frac{\mathcal{F}_C}{\Phi} = \frac{250\text{ At}}{600\ \mu\text{Wb}} = \mathbf{416\,666.6\ At/Wb}$$

EXAMPLE 6.7

A rectangular core made of low-carbon mild steel alloy is shown in Fig. 6.11. The mean length of the core (excluding the air gap) is 40 cm, and the air gap is 1 mm long. The exciting coil has 300 turns. Neglecting leakage and fringing of flux, find the current in the exciting coil to set up a flux of 600 μWb in the air gap. If required, use the magnetization characteristics of Fig. 6.2.

Solution Flux density, $B = \frac{\Phi}{A} = \frac{600 \times 10^{-6}}{4 \times 10^{-4}} = 1.5\text{ T}$

For the air gap:

$$H_g = \frac{B}{\mu_0} = \frac{1.5}{4\pi \times 10^{-7}} = 1.194 \times 10^6\text{ At/m}; \quad \mathcal{F}_g = H_g l_g = 1.194 \times 10^6 \times 1 \times 10^{-3} = 1194\text{ At}$$

For the core: From Fig. 6.2, for a flux density of 1.5 T, the magnetic field strength H for low-carbon mild steel is 3000 At/m.

$$\therefore \quad \mathcal{F}_c = 3000 \times 0.4 = 1200\text{ At}$$

Therefore, the total mmf needed, $\mathcal{F} = \mathcal{F}_c + \mathcal{F}_g = 1200 + 1194 = 2394\text{ At}$

$$\therefore \quad I = \frac{\mathcal{F}}{N} = \frac{2394}{300} = \mathbf{7.98\ A}$$

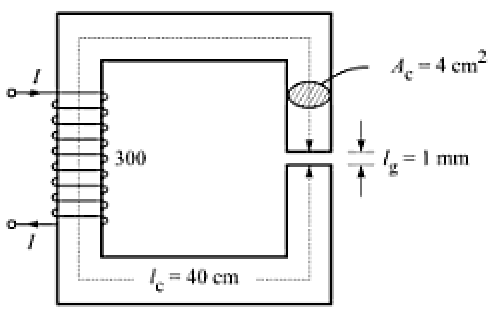


Fig. 6.11

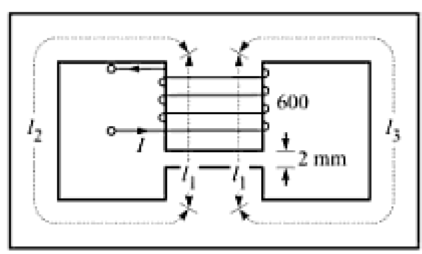


Fig. 6.12

EXAMPLE 6.8

The magnetic circuit shown in Fig. 6.12 has following dimensions: $l_1 = 10$ cm, $l_2 = l_3 = 18$ cm, cross-sectional area of l_1 path = 6.25 cm^2, cross-sectional area of l_2 and l_3 paths = 3 cm^2, length of air gap = 2 mm. Taking the relative permeability of the core material as 800, find the required current in the 600-turn exciting coil so as to establish a flux of 100 μWb in the air gap. Neglect leakage and fringing.

Solution *For air gap*: The flux density, $B_g = \frac{\Phi}{A} = \frac{100 \times 10^{-6}}{6.25 \times 10^{-4}} = 0.16\text{ T}$

$$\therefore \quad H_g = \frac{B_g}{\mu_0} = \frac{0.16}{4\pi \times 10^{-7}} = 127 \times 10^3\text{ At/m}; \quad \mathcal{F}_g = H_g l_g = 127 \times 10^3 \times 0.002 = 254\text{ At}$$

For path l_1: $B_1 = 0.16$ T;

$$\therefore \qquad H_1 = \frac{B_1}{\mu_r \mu_0} = \frac{0.16}{800 \times 4\pi \times 10^{-7}} = 159 \text{ At/m}; \quad \mathcal{F}_1 = H_1 l_1 = 159 \times (10 - 0.2) \times 10^{-2} = 15.582 \text{ At}$$

The two paths l_2 and l_3 are similar, and are in parallel. Therefore, the total flux in path l_1 divides equally between these two paths.

For path l_2: $\Phi_2 = \dfrac{\Phi}{2} = \dfrac{100\ \mu\text{Wb}}{2} = 50\ \mu\text{Wb}$; $\quad B_2 = \dfrac{\Phi_2}{A_2} = \dfrac{50 \times 10^{-6} \text{ Wb}}{3 \times 10^{-4} \text{ m}^2} = 0.167$ T

$$\therefore \qquad H_2 = \frac{B_2}{\mu_r \mu_0} = \frac{0.167}{800 \times 4\pi \times 10^{-7}} = 166 \text{ At/m}; \quad \mathcal{F}_2 = H_2 l_2 = 166 \times 18 \times 10^{-2} = 29.88 \text{ At}$$

Now, considering the loop consisting of the air gap, path l_1 and l_2 and applying KML, we get

The mmf produced by the coil = total mmf drop around the loop

or $$NI = \mathcal{F}_g + \mathcal{F}_1 + \mathcal{F}_2 = 254 + 15.582 + 29.88 = 299.462 \text{ At}$$

or the required current, $I = \dfrac{\mathcal{F}}{N} = \dfrac{299.462}{600} = \mathbf{0.499\ A}$

EXAMPLE 6.9

In the magnetic circuit shown in Fig. 6.13*a*, the limbs *B* and *C* have areas of cross-section 0.01 m^2 and 0.02 m^2, and air gaps of 1.0 mm and 2.0 mm, respectively. If the flux in limb *B* is 1.5 mWb, find the flux in limb *A*, assuming permeability of the iron to be infinite. Also determine the value of the exciting current *I*, if the coil has 500 turns.

Solution Since the permeability of iron is assumed infinite, the reluctance of the iron path is zero. Figure 6.13*b* shows the electrical equivalent circuit of the magnetic circuit, where $\mathcal{R}_1$ and $\mathcal{R}_2$ are the reluctances of the air gaps in limbs *B* and *C*, respectively. The flux Φ produced by the mmf $\mathcal{F}$ divides itself into Φ_1 and Φ_2 in the two parallel paths. The mmf across the two parallel paths must be same. Hence,

$$\mathcal{R}_1 \Phi_1 = \mathcal{R}_2 \Phi_2 \quad \text{or} \quad \frac{l_1}{\mu_0 A_1} \Phi_1 = \frac{l_2}{\mu_0 A_2} \Phi_2$$

$$\Rightarrow \qquad \Phi_2 = \frac{A_2}{A_1} \times \frac{l_1}{l_2} \times \Phi_1 = \frac{0.02 \text{m}^2}{0.01 \text{m}^2} \times \frac{1.0 \text{mm}}{2.0 \text{mm}} \times 1.5 \text{ mWb} = 1.5 \text{ mWb}$$

Applying KFL, we get the total flux,

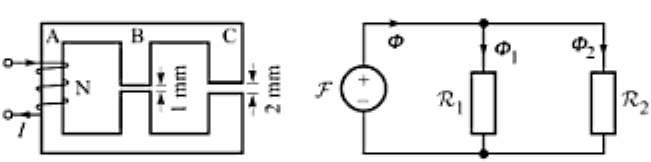
$$\Phi = \Phi_1 + \Phi_2 = 1.5 + 1.5 = \mathbf{3.0\ mWb}$$

The reluctances of the two parallel paths are given as

$$\mathcal{R}_1 = \frac{l_1}{\mu_0 A_1} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.01} = 79\,577 \text{ At/Wb} \quad \text{and} \quad \mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.02} = 79\,577 \text{ At/Wb}$$

Therefore, the net reluctance of the circuit is

$$\mathcal{R} = \mathcal{R}_1 \,||\, \mathcal{R}_2 = \frac{79\,577}{2} = 39\,788.5 \text{ At/Wb}$$



(*a*) (*b*)

Fig. 6.13

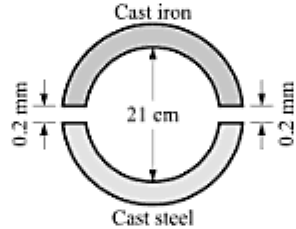


Fig. 6.14

According to Ohm's law, we have

$$\mathcal{F} = \Phi\mathcal{R} \quad \text{or} \quad NI = \Phi\mathcal{R} \quad \Rightarrow \quad I = \frac{\Phi\mathcal{R}}{N} = \frac{3.0 \times 10^{-3} \times 39\,788.5}{500} = \mathbf{238.7\ mA}$$

EXAMPLE 6.10

A ring having a mean diameter of 21 cm and a cross-sectional area of 10 cm^2 is made up of semicircular sections of cast iron and cast steel with each joint having reluctance equal to an air gap of 0.2 mm, as shown in Fig. 6.14. Determine the ampere turns required to produce a flux of 0.8 mWb. The relative permeabilities of cast iron and cast steel are 166 and 800, respectively. Neglect fringing and leakage effects.

Solution The flux density, $B = \dfrac{\Phi}{A} = \dfrac{0.8 \times 10^{-3}}{10 \times 10^{-4}} = 0.8\ \text{T}$

For total air gap: Ampere turns required is given as

$$\mathcal{F}_g = H_g l_g = \frac{B}{\mu_0} \times l_g = \frac{0.8}{4\pi \times 10^{-7}} \times (2 \times 0.2 \times 10^{-3}) = 254.64\ \text{At}$$

For cast iron: Ampere turns required is given as

$$\mathcal{F}_i = H_i l_i = \frac{B}{\mu_r \mu_0} \times \frac{\pi D}{2} = \frac{0.8}{166 \times 4\pi \times 10^{-7}} \times \frac{\pi \times 0.21}{2} = 1265.06\ \text{At}$$

For cast steel: Ampere turns required is given as

$$\mathcal{F}_s = H_s l_s = \frac{B}{\mu_r \mu_0} \times \frac{\pi D}{2} = \frac{0.8}{800 \times 4\pi \times 10^{-7}} \times \frac{\pi \times 0.21}{2} = 262.5\ \text{At}$$

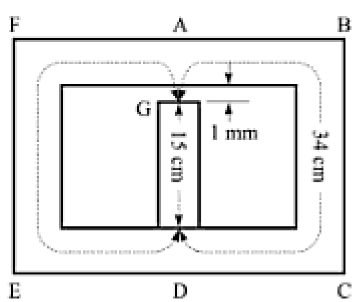
Therefore, total ampere turns required is

$$\mathcal{F} = \mathcal{F}_g + \mathcal{F}_i + \mathcal{F}_s = 254.64 + 1265.06 + 262.5 = \mathbf{1782.2\ At}$$

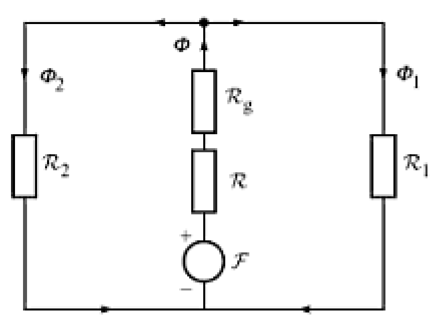
EXAMPLE 6.11

The magnetic circuit shown in Fig. 6.15*a* is made of wrought iron. The central limb has a cross-sectional area of 8 cm^2 and each side-limb has a cross-sectional area of 5 cm^2. Calculate the required ampere turns of the coil wound on the central limb so as to produce a flux of 1 mWb in it. Neglect the magnetic leakage and fringing effects. The magnetization of wrought iron is given by the following table.

Flux density, B (T)	1.00	1.25
Magnetic field strength, H (At/m)	200	500



(*a*) Magnetic circuit.



(*b*) Its electrical equivalent circuit.

Fig. 6.15

Solution Figure 6.15*b* shows the electrical equivalent circuit of the given magnetic circuit. The mmf $\mathcal{F}$ is applied to the central limb, producing a flux Φ in it. $\mathcal{R}$ is the reluctance of the part DG, $\mathcal{R}_g$ is the reluctance of the air gap, $\mathcal{R}_1$ is the reluctance of the part ABCD, and $\mathcal{R}_2$ is the reluctance of the part AFED. Since the parts ABCD and AFED are identical, the reluctances $\mathcal{R}_1$ and $\mathcal{R}_2$ are equal. Hence, the flux Φ equally divides into two parallel branches. Thus, $\Phi_1 = \Phi_2 = \Phi/2$.

$$\text{Flux density in the central limb, } B = \frac{\Phi}{A} = \frac{10^{-3}}{8 \times 10^{-4}} = 1.25 \text{ T.}$$

Corresponding to this value of B, the magnetic field strength H is given in the table as 500 At/m. Therefore, the ampere turns needed for the part DG is

$$\mathcal{F}_{DG} = Hl = 500 \times 0.15 = 75 \text{ At}$$

The ampere turns needed for the air gap is

$$\mathcal{F}_g = H_g l_g = \frac{B}{\mu_0} \times l_g = \frac{1.25}{4\pi \times 10^{-7}} \times 0.001 = 994.7 \text{ At}$$

To determine the applied ampere turns $\mathcal{F}$, we need to consider only one loop, say, ABCDA. The flux through the part ABCD is given as $\Phi_1 = \Phi/2 = 0.5$ mWb. Therefore, the flux density for this part is

$$B_1 = \frac{\Phi_1}{A_1} = \frac{0.5 \times 10^{-3}}{5 \times 10^{-4}} = 1 \text{ Wb/m}^2 = 1 \text{ T}$$

For this value of B, the corresponding value of H is given in the table as 200 At/m. Thus, the ampere turns required for the part ABCD is

$$\mathcal{F}_1 = H_1 l_1 = 200 \times 0.34 = 68 \text{ At}$$

Applying KML to loop ABCDA, the total ampere turns required is

$$\mathcal{F} = \mathcal{F}_{DG} + \mathcal{F}_g + \mathcal{F}_1 = 75 + 994.7 + 68 = \mathbf{1137.7\ At}$$

SUMMARY

TERMS AND CONCEPTS

- A complete closed path followed by the magnetic flux is referred to as a ***magnetic circuit.***
- ***Magnetomotive force*** (***mmf***) in a magnetic circuit is caused by a current flowing in a coil wound over a core. It is represented by symbol $\mathcal{F}$, and is measured in ampere-turns (At).
- A ***magnetic flux*** is created by a magnetomotive force (mmf).
- For a given value of mmf, the magnitude of flux density (B) depends on the material of the electromagnet.
- ***Magnetic field strength*** or ***intensity*** (H) is the mmf per metre length of the magnetic circuit. It is measured in At/m.
- ***Permeability*** (μ) of a material is the ratio of flux density (B) to the magnetic field strength (H).
- ***Relative permeability*** (μ_r) is the ratio of the flux density produced in a material to the flux density produced in vacuum by the same magnetic field strength.
- For ferromagnetic material, μ_r is very high (hundreds or thousands).
- ***Reluctance*** ($\mathcal{R}$) of a magnetic circuit is the ratio of the magnetomotive force to the flux.
- A magnetic circuit shows the phenomenon of ***saturation***. Once the saturation is reached, the flux density does not further increase on increasing the magnetic field strength.
- Magnetic circuits have series and parallel paths, and Kirchhoff's laws in analogous form are obeyed.
- Whole of the magnetic flux produced by a coil is not confined to the core. The flux that completes its path through air is called ***leakage flux***. The ***leakage factor*** is defined as the ratio of the flux through the exciting winding to the useful flux. Its value is about 1.15 to 1.25.

IMPORTANT FORMULAE

- MMF, $\mathcal{F} = NI$ (At)
- Magnetic field strength, $H = \dfrac{\mathcal{F}}{l} = \dfrac{NI}{l}$ (At/m)
- Flux density, $B = \mu H$ (T)
- $\mu = \mu_r \mu_0$, where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.
- Ohm's law : Flux (Φ) = $\dfrac{\text{mmf } (\mathcal{F})}{\text{Reluctance } (\mathcal{R})}$.
- Reluctance, $\mathcal{R} = \dfrac{\mathcal{F}}{\Phi}$

  $\mathcal{R} = \dfrac{1}{\mu}\dfrac{l}{A} = \dfrac{1}{\mu_r \mu_0}\dfrac{l}{A}$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	Permeability of a material is the measure of the ease with which the magnetic domains can be aligned.	☐	☐	
2.	The ratio of the flux density produced in vacuum to the flux density produced in the material by the same magnetic field strength is termed as the relative permeability of the material.	☐	☐	
3.	The electrical analog of the reluctance in a magnetic circuit is called resistance.	☐	☐	
4.	The reciprocal of reluctance in a magnetic circuit is called susceptance.	☐	☐	
5.	The value of the mmf for a magnetic circuit is directly proportional to the conductivity of the core material.	☐	☐	
6.	Energy must be continuously supplied to a magnetic circuit so long as the magnetic flux continues to exist in the circuit.	☐	☐	
7.	The unit of magnetic flux density is A/m^2.	☐	☐	
8.	Ohm's law for a magnetic circuit is expressed as Total flux, $\Phi = \dfrac{\text{mmf of coil } (\mathcal{F})}{\text{total reluctance } (\mathcal{R})}$	☐	☐	
9.	The bulging of magnetic flux in an air gap in a magnetic circuit is called fringing.	☐	☐	
10.	The leakage factor in a magnetic circuit is defined as the ratio of the useful flux to the total flux through the exciting winding.	☐	☐	

Your Score

ANSWERS

1. True	**2.** False	**3.** True	**4.** False	**5.** False
6. False	**7.** False	**8.** True	**9.** True	**10.** False

REVIEW QUESTIONS

1. Draw the analogy between electric and magnetic circuits. State and explain Kirchhoff's laws for magnetic circuits.
2. Explain in what respects a magnetic circuit differs from an electric circuit.
3. What is meant by magnetic leakage and fringing as applied to electrical machines? Define leakage factor.
4. Presence of an air gap in a magnetic circuit puts a heavy demand on the ampere turns required. Then, why do we have to include an air gap in many situations?
5. (*a*) State a disadvantage of including an air gap in a magnetic circuit.
   (*b*) State two reasons for including an air gap in a magnetic circuit.
6. Define the terms: permeability, relative permeability, reluctance, permeance.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. The flux in a magnetic circuit can be compared in an electric circuit to
   (*a*) the voltage (*b*) the current
   (*c*) the resistance (*d*) the inductance
2. The term permeability for a material means
   (*a*) the number of turns on an air core
   (*b*) the mmf required to produce one unit of magnetic flux
   (*c*) the ability of the material to conduct electrical current through it
   (*d*) the ability of the material to conduct magnetic lines of force
3. The equivalent to Ohm's law in a magnetic circuit is
   (*a*) permeability = flux density/magnetic field intensity
   (*b*) reluctance = 1/permeance
   (*c*) mmf = flux × reluctance
   (*d*) flux = mmf × reluctance
4. An air gap is usually inserted in a magnetic circuit to
   (*a*) prevent saturation
   (*b*) increase the flux
   (*c*) increase the mmf
   (*d*) decrease the flux
5. A magnetic circuit requires 800 ampere turns to produce a certain amount of magnetic flux. If the exciting coil of 100 turns has 5 ohms resistance, the voltage to be applied to the exciting coil must be
   (*a*) 40 V (*b*) 20 V
   (*c*) 10 V (*d*) 5 V
6. The relative permeability of a ferromagnetic material is usually
   (*a*) 1 (*b*) less than 1
   (*c*) more than 500 (*d*) more than 10^8

ANSWERS

1. *a* **2.** *d* **3.** *c* **4.** *a* **5.** *a* **6.** *c*

PROBLEMS

(A) SIMPLE PROBLEMS

1. An air cored toroidal coil has 3000 turns and carries a current of 0.1 A. The cross-sectional area of the coil is 4 cm^2 and the mean length of the magnetic circuit is 15 cm. Find the magnetic field strength, the flux density and the total flux within the coil.
   [**Ans.** 2000 A/m, 2.51 mT, 1.004 μWb]

2. Repeat Prob. 1, if the core is made of cast iron. If required, the magnetization characteristic of Fig. 6.2 may be used.
[**Ans.** 2000 A/m, 0.57 T, 228 μWb]

3. What will be the flux density in an air gap 0.1 mm long with a cross-sectional area of 25 cm^2, if an mmf of 300 At is creating the flux?
[**Ans.** 0.376 T]

4. A cast iron ring with a mean diameter of 10 cm and a cross-sectional area of 3 cm^2 has a radial gap of 0.15 cm. Calculate the required current through the coil of 1000 turns wound uniformly over the ring to produce a flux density of 0.6 T in the air gap. Take relative permeability of cast iron as 238.
[**Ans.** 1.345 A]

5. An iron ring with a mean diameter of 50 cm has an air gap of 1 mm and a winding of 200 turns. If the relative permeability of the iron is 400 when a current of 1.25 A flows through the winding, find the flux density. [**Ans.** 0.0637 T]

(B) TRICKY PROBLEMS

6. A ring shaped core is made of a ferromagnetic material having relative permeability of 1000. The core has two sections. The thicker section has a mean length of 10 cm and area of cross-section of 6 cm^2. The thinner section has a mean length of 20 cm and area of cross-section of 4 cm^2. It is required to establish a flux density of 1.5 T in the thicker section. The current through the coil needed to produce desired amount of mmf is not to exceed 0.5 A. Compute the number of turns of the coil.
[**Ans.** 955]

7. An iron ring with a mean length of magnetic path of 20 cm and of small cross-section area has an air gap 1 mm long. A current of 1 A in a coil of 440 turns wound uniformly over the ring produces a flux density of $16\pi \times 10^{-3}$ T. Neglecting leakage and fringing, determine the relative permeability of iron. [**Ans.** 19.9]

8. A ring shaped electromagnet has an air gap 6 mm long and 20 cm^2 in area. The mean length of the iron core is 50 cm and its cross-sectional area is 10 cm^2. Assuming the permeability of iron to be 1800, determine the ampere turns required to set up a flux density of 0.5 Wb/m^2 in air gap.
[**Ans.** 2607.93 At]

9. A circular ring of mean length 4π cm and of cross-sectional area 10 cm^2 has an air gap 0.4π mm long. The relative permeability of iron is 1000. The ring is wound with a coil of 2000 turns. Determine the flux in the air gap, if the coil carries a current of 2 mA. Ignore the leakage and fringing effects.
[**Ans.** 3.64 μWb]

10. An iron ring with a mean length of magnetic path 25 cm and a small cross-sectional area has an air gap 1 mm long. It is wound uniformly with a coil of 540 turns. If a current of 1.5 A in the coil produces a flux density of 180π mT, determine the relative permeability of iron. Neglect the leakage and fringing effects. [**Ans.** 312.5]

11. A ring-shaped electromagnet has an air gap 6.5 mm in length and 22 cm^2 in area. The mean length of the iron-core is 52 cm and its cross-sectional area is 11 cm^2. Assuming the relative permeability of iron to be 1800, calculate the ampere turns required to produce a flux density of 0.55 Wb/m^2 in the air gap. [**Ans.** 3099.2 At]

(C) CHALLENGING PROBLEMS

12. A cast steel ring has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and across the ring, an iron bar of dimensions 18 cm × 3 cm × 0.4 cm is tightly fit with no air gap, as shown in Fig. 6.16. Neglecting flux leakage, determine the ampere turns required to be applied to one-half of the ring so as to produce a flux density of 1.0 T in the other half of the ring. The *B-H* characteristics of the two materials are given in the following table. [**Ans.** 692.685 At]

Quantity	*For cast steel*			*For iron*		
B (T)	1.0	1.1	1.2	1.2	1.4	1.45
H (At/m)	900	1020	1220	590	1200	1650

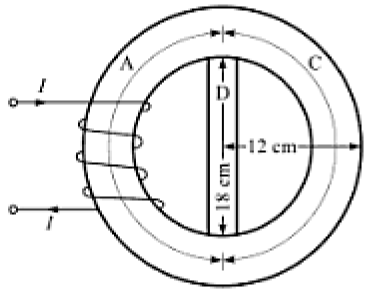


Fig. 6.16

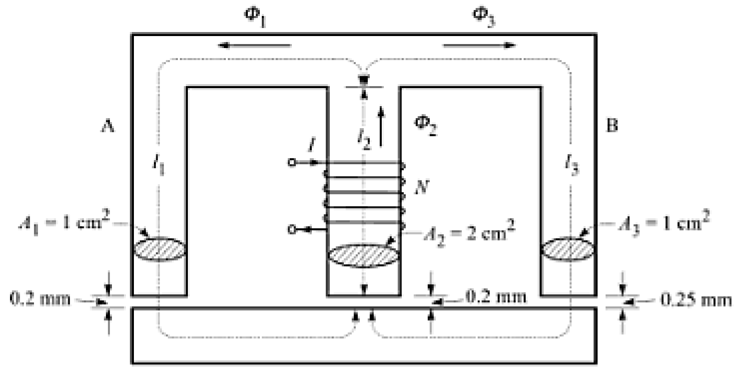


Fig. 6.17

13. The magnetic circuit of Fig. 6.17 is made of cast steel. The central limb having a length of 20 cm and cross-sectional area of 2 cm^2 has a coil of 672 turns wound over it. Each of the outer paths *A* and *B* has mean length of 50 cm and cross-sectional area of 1 cm^2. The three air gaps are 0.2 mm, 0.2 mm and 0.25 mm long, as shown in figure. Determine the required current in the coil so as to establish a flux of 0.75 mWb in the central limb, assuming the relative permeability of the core to be (*a*) $\mu_r = \infty$, and (*b*) $\mu_r = 5000$.

[**Ans.** (*a*) 1.875 A; (*b*) 2.5 A]

SELF AND MUTUAL INDUCTANCES

7

OBJECTIVES

After completing this Chapter, you will be able to:

- *Define 'self inductance' of a coil.*
- *Give a reason why back emf is induced in a coil.*
- *Derive an expression for inductance of a coil in terms of its physical dimensions and the magnetic properties of the core.*
- *Give circuit symbols of different types of inductors.*
- *Define 'mutual inductance' and 'coefficient of coupling' between two coils.*
- *Explain the meaning of the dot convention.*
- *Calculate the net inductance of coupled coils connected in series or in parallel.*
- *Derive an expression for the energy stored in a magnetic field.*
- *Derive an expression for the lifting power of a magnet.*

7.1 SELF INDUCTANCE

We have seen in Chapter 5 that a statically induced emf can be self-induced too. Whenever the current through a coil changes, the flux linked with it also changes. This change in flux induces an emf in the coil. Thus, a coil is capable of inducing an emf in itself by changing the current flowing through it. This property of the coil is known as ***self inductance***. The self-induced emf is directly proportional to the rate of change of current,

$$e \propto \frac{di}{dt} \quad \text{or} \quad e = L\frac{di}{dt} \tag{7.1}$$

where, L is called the *coefficient of self inductance* or simply *inductance*. The SI unit of inductance is henry[1]. It is evident that every coil or even a single conductor has self inductance. However, its presence in the circuit is felt only when the current changes.

An inductor can be linear or nonlinear. For a linear inductor, the magnitude of inductance is independent of the magnitude of current. For such inductors, strict proportionality exists between current and flux. An air-cored inductor is linear. When iron (or any other ferromagnetic material) is used as core, the inductor becomes nonlinear.

Energy Stored in an Inductor If the current through an inductor is made to increase from 0 to I in time t, an emf e is induced in the coil. During this process, the total energy W absorbed by the inductor is given

[1] Named after Joseph Henry (1797–1898).

as

$$W = \int_0^t ei\,dt = \int_0^t \left(L\frac{di}{dt}\right) i\,dt = \int_0^I Li\,di = \frac{Li^2}{2}\Bigg|_0^I = \frac{1}{2}LI^2 \tag{7.2}$$

Inductance from Geometrical Viewpoint Combining Faraday's law of electromagnetic induction (Eq. 5.12) and Eq. 7.1, we get

$$e = N\frac{d\Phi}{dt} = L\frac{di}{dt} \quad \text{or} \quad L = N\frac{d\Phi}{di} \tag{7.3}$$

For a linear inductor, the rate of change of flux with current is constant and Eq. 7.3 can be written as

$$L = N\frac{\Phi}{I} \tag{7.4}$$

But we know from the knowledge of magnetic circuits, that

$$\Phi = \frac{NI}{\mathcal{R}} \qquad \text{and} \qquad \mathcal{R} = \frac{l}{\mu A}$$

So that Eq. 7.4 becomes

$$L = \frac{N^2 \mu A}{l} \tag{7.5}$$

This equation shows that the inductance depends on the physical dimensions of the coil and the magnetic properties of the core-material.

Determination of L The value of self inductance L can be determined by using any one of the three relations given in Eqs. 7.1, 7.4 and 7.5.

EXAMPLE 7.1

The current in a coil decreases from 10 A to 4 A in 0.1 second. If the inductance of the coil is 4 H, find the emf induced in the coil.

Solution Using Eq. 7.1, the emf induced in the coil is

$$e = L\frac{di}{dt} = 4 \times \frac{10-4}{0.1} = \mathbf{240\ V}$$

EXAMPLE 7.2

A coil of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A. Calculate the inductance of the coil. If this current is uniformly reversed in 0.01 s, calculate the emf induced.

Solution Using Eq. 7.4, we have

$$L = N\frac{\Phi}{I} = 150 \times \frac{0.01}{10} = \mathbf{0.15\ H}$$

Now, the induced emf is given by Eq. 7.1 as

$$e = L\frac{di}{dt} = 0.15 \times \frac{10-(-10)}{0.01} = \mathbf{300\ V}$$

EXAMPLE 7.3

A flux of 0.4 mWb is created by a current of 10 A flowing through a 100-turn coil. Calculate the inductance of the coil corresponding to the complete reversal of the current in 0.01 second. Also find the magnitude of the emf induced.

Solution Using Eq. 7.3, we have

$$L = N\frac{d\Phi}{di} = 100 \times \frac{[0.4-(-0.4)]\,\text{mWb}}{[10-(-10)]\,\text{A}} = \mathbf{4\ mH}$$

Now, using Eq. 7.1, the induced emf is

$$e = L\frac{di}{dt} = 4\times10^{-3}\times\frac{10-(-10)}{0.01} = \mathbf{8\ V}$$

EXAMPLE 7.4

An air-cored solenoid with length 30 cm and internal diameter 1.5 cm has a coil of 900 turns wound on it. Estimate its inductance. Also, calculate the amount of energy stored in it when the current through the coil rises from 0 to 5 A.

Solution Cross-sectional area of the solenoid is given as

$$A = \pi r^2 = \pi(0.75\times10^{-2})^2 = 1.77\times10^{-4}\ \text{m}^2$$

Now, using Eq. 7.5, we have

$$L = \frac{N^2\mu_0 A}{l} = \frac{(900)^2\times4\pi\times10^{-7}\times(1.77\times10^{-4})}{0.30} = \mathbf{0.6\ mH}$$

The energy stored in the solenoid is given by Eq. 7.2, as

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\times(0.6\times10^{-3})\times5^2 = \mathbf{7.5\ mJ}$$

EXAMPLE 7.5

An iron rod 20 cm long and 2 cm in diameter is bent into a closed circular ring and is uniformly wound with 3000 turns of wire. When a current of 0.5 A is passed through the coil, the flux density in the iron rod is found to be 1.2 T. Calculate (*a*) the permeability of iron, (*b*) the relative permeability of iron, (*c*) the inductance of the coil, (*d*) the voltage that would be induced in the coil if the current is interrupted so that the flux in the iron rod falls to 10 % of its former value in 0.01 second.

Solution The area of cross-section of the core is

$$A = \pi r^2 = \pi\times(1\times10^{-2})^2 = 3.1416\times10^{-4}\ \text{m}^2$$

(*a*) The magnetic field strength inside the core is

$$H = \frac{NI}{l} = \frac{3000\times0.5}{0.2} = 7500\ \text{At/m}$$

$$\therefore \qquad \mu = \frac{B}{H} = \frac{1.2}{7500} = \mathbf{1.6\times10^{-4}\ Tm/A}$$

(*b*) The relative permeability of iron is given as

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.6\times10^{-4}}{4\pi\times10^{-7}} = \mathbf{127}$$

(*c*) The inductance of the coil is given as

$$L = \frac{N\Phi}{I} = \frac{NBA}{I} = \frac{3000\times1.2\times3.1416\times10^{-4}}{0.5} = \mathbf{2.26\ H}$$

(*d*) The voltage in the coil is

$$e = N\frac{d\Phi}{dt} = 3000 \times \frac{1.2 \times 3.1416 \times 10^{-4}(1-0.1)}{0.01} = \mathbf{101.78\ V}$$

EXAMPLE 7.6

The resistance and inductance of a coil are 3 Ω and 0.1 mH, respectively. What potential difference exists across the terminals of this coil at the instant when the current is 1 A, but increasing at the rate of 10 000 A per second?

Solution The potential difference across the coil will be due to the drop across its resistance as well as the emf induced in the inductance. Thus,

$$V = iR + L\frac{di}{dt} = 1 \times 3 + 0.1 \times 10^{-3} \times \frac{10000}{1} = \mathbf{4\ V}$$

7.2 INDUCTORS

Inductors (also called *chokes*) are available in different types ranging from large high current iron-cored chokes to tiny low current coils. Air-cored coils are wound on a tubular insulating material such as cardboard, fibre, hard rubber, bakelite, etc. Such coils find use in electronic circuits working at high frequencies. Their inductance values are in millihenry (mH) and microhenry (μH) range.

Iron-cored chokes may have laminated iron core or powdered iron cores. These are mainly used at ac power frequency (50 Hz) and at audio frequencies (20 Hz to about 10 kHz). Such chokes have inductance of a few henrys. An inductor can be made variable, by having tapped coils (taps are brought out at several points in the winding), by having a slider (similar to a rheostat), or by having a movable core. Standard symbols of different types of inductor are shown in Fig. 7.1.

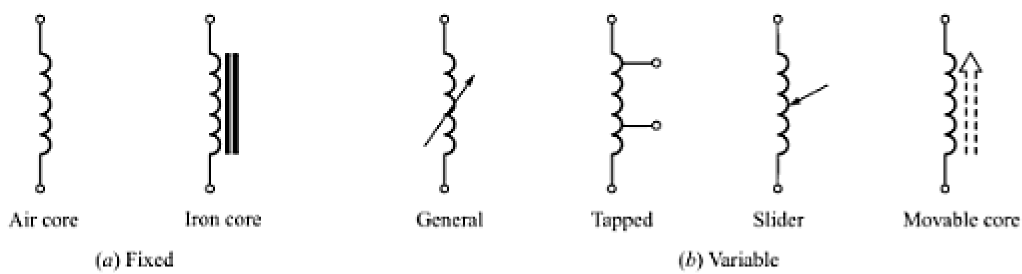


Fig. 7.1 *Standard symbols for different types of inductors.*

7.3 MUTUAL INDUCTANCE

When interchange of energy takes place between two circuits, we say that the two circuits are *mutually coupled*. As illustrated in Fig. 7.2, there can be three ways of coupling two circuits: (*a*) conductive, (*b*) electrostatic, and (*c*) electromagnetic. In Fig. 7.2*a*, resistor R_{12} is the mutual resistance between circuit ① and circuit ②. In Fig.7.2*b*, the two circuits are coupled electrostatically through the mutual capacitor *C*.

Figure 7.2*c* shows magnetic coupling between two circuits. A part of magnetic flux produced by a coil in one circuit interlinks with the coil in other circuit. Energy may be transferred from one circuit to the other through the medium of magnetic flux that is common to both circuits. When current in one coil changes, there

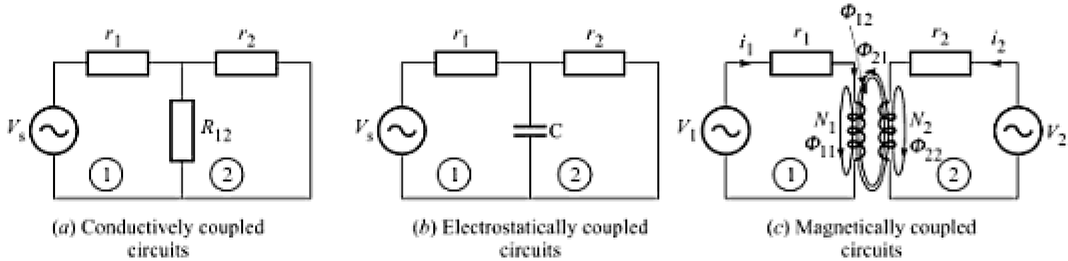


Fig. 7.2 *Mutual coupling between two circuits.*

occurs a change in the flux linking with the other. As a result, there is an induced emf in the other coil. The mutually induced emf e_2 in the second coil is dependent on the rate of change of current i_1 in the first coil. We can, therefore, write

$$e_2 \propto \frac{di_1}{dt} \quad \text{or} \quad e_2 = M\frac{di_1}{dt} \tag{7.6}$$

The constant of proportionality M is called ***coefficient of mutual inductance,*** or simply ***mutual inductance***. Like self inductance (L), the unit of mutual inductance (M) is also henry (H).

Note that mutual inductance is a property which is associated with two or more coils that are physically close together. A circuit element called "mutual inductor" does not exist. Furthermore, mutual inductance is not a property which is associated with a single pair of terminals. Instead, it is defined with reference to two pairs of terminals. The physical device whose operation is based inherently on mutual inductance is called transformer[2].

Magnetic Coupling

Consider two coils placed close together, as in Fig. 7.2*c*. Current i_1 flowing in coil ① establishes a total magnetic flux Φ_1. Only a part of this flux, Φ_{12}, links with the coil ②. The remaining flux Φ_{11} is confined to coil ① itself. Thus, $\Phi_1 = \Phi_{11} + \Phi_{12}$. Similarly, a part Φ_{21} of the total flux Φ_2 produced by the current i_2 links with coil ①. Flux Φ_{22} remains confined to coil ② itself. That is, $\Phi_2 = \Phi_{22} + \Phi_{21}$.

Let N_1 and N_2 be the number of turns in coils ① and ②, respectively. If current i_1 is time-varying, the resulting flux Φ_1 as well as linking flux Φ_{12} is also time-varying. Therefore, from Faraday's law, the emf induced in coil ② is given as

$$e_2 = N_2\frac{d\Phi_{12}}{dt} \tag{7.7}$$

Comparing this equation with Eq. 7.6, the mutual inductance from coil ① to coil ② is given by

$$M_{12}\frac{di_1}{dt} = N_2\frac{d\Phi_{12}}{dt} \quad \text{or} \quad M_{12} = N_2\frac{d\Phi_{12}}{di_1} \tag{7.8}$$

Similarly, we can get the expression for mutual inductance from coil ② to coil ① as

$$M_{21} = N_1\frac{d\Phi_{21}}{di_2} \tag{7.9}$$

[2] We shall study more about transformers in Chapter 13.

If the flux and current are linearly related (i.e., if the permeability of the mutual path remains constant), above two equations can be written as

$$M_{12} = N_2 \frac{\Phi_{12}}{i_1} \tag{7.10}$$

and

$$M_{21} = N_1 \frac{\Phi_{21}}{i_2} \tag{7.11}$$

These equations suggest that the mutual inductance between two coils can be defined as the *ratio of weber turns in one coil to the current through the other coil.* Furthermore, because of the reciprocity of coupling between the two coils, we have

$$M_{12} = M_{21} = M$$

Coefficient of Coupling (k)

It is a measure of how close is the coupling between two coils. It gives an idea of what portion of the flux produced by one coil links with the other coil.

From Eq. 7.5, the self inductances of the two coils can be written as

$$L_1 = \frac{N_1^2 \mu A}{l} \quad \text{and} \quad L_2 = \frac{N_2^2 \mu A}{l} \tag{7.12}$$

The magnetic flux produced in coil ① due to the current i_1 is given as

$$\Phi_1 = \frac{\text{mmf}}{\mathcal{R}} = \frac{N_1 i_1}{l/\mu A} = \frac{N_1 i_1 \mu A}{l} \tag{7.13}$$

The flux that links with the coil ② is only a part of Φ_1. That is, $\Phi_{12} = k\Phi_1$, where $0 \le k \le 1$. Then, the mutual inductance from coil ① to coil ② is given, from Eqs. 7.10 and 7.13, as

$$M_{12} = N_2 \frac{\Phi_{12}}{i_1} = \frac{k\Phi_1 N_2}{i_1} = \frac{kN_1 N_2 \mu A}{l} \tag{7.14}$$

Similarly, the mutual inductance from coil ② to coil ① can be written as

$$M_{21} = N_{21} \frac{\Phi_{21}}{i_2} = \frac{k\Phi_2 N_1}{i_2} = \frac{kN_2 N_1 \mu A}{l} \tag{7.15}$$

Multiplying Eq. 7.14 and 7.15, putting $M_{12} = M_{21} = M$, and using Eqs. 7.12, we get

$$M^2 = k^2 \frac{N_1 N_2 \mu A}{l} \times \frac{N_2 N_1 \mu A}{l} = k^2 \left(\frac{N_1^2 \mu A}{l}\right)\left(\frac{N_2^2 \mu A}{l}\right) = k^2 L_1 L_2$$

$$\therefore \quad k = \frac{M}{\sqrt{L_1 L_2}} \tag{7.16}$$

Thus, the ***coefficient of coupling*** k is defined as the ratio of the mutual inductance M to the square root of the product of inductances of coil ① and coil ②. If the entire flux produced in one coil links with the other, the value of k is unity, and the coils are said to be *tightly coupled* or closely coupled. On the other extreme, if the flux produced in coil does not link with the other coil at all, the value of k is zero. The two coils are then said to be magnetically *isolated.*

EXAMPLE 7.7

A solenoid consists of 2000 turns of wire wound on a length of 70 cm. A search coil of 500 turns having a mean area of 30 cm^2 is placed centrally inside the solenoid. Assuming $k = 1$, calculate (*a*) the mutual inductance, and (*b*) the emf induced in the search coil if the current in the solenoid uniformly changes at a rate of 260 A/s.

Solution

(*a*) Using Eq. 7.14, the mutual inductance is given as

$$M = \frac{kN_1 N_2 \mu A}{l} = \frac{1 \times 2000 \times 500 \times 4\pi \times 10^{-7} \times 30 \times 10^{-4}}{0.70} = \mathbf{5.38\ mH}$$

(*b*) The emf induced is given as

$$e_2 = M\frac{di_1}{dt} = 5.38 \times 10^{-3} \times 260 = \mathbf{1.4\ V}$$

EXAMPLE 7.8

The number of turns in two coupled coils is 600 and 1700, respectively. When a current of 6 A flows in the second coil, the total magnetic flux produced in this coil is 0.8 mWb, and the flux that links with the first coil is only 0.5 mWb. Calculate L_1, L_2, k and M.

Solution Using Eq. 7.4, the self inductance of the second coil is given as

$$L_2 = N_2\frac{\Phi_2}{I_2} = 1700 \times \frac{0.8 \times 10^{-3}}{6} = \mathbf{0.226\ H}$$

The coefficient of coupling, $k = \dfrac{\Phi_{21}}{\Phi_2} = \dfrac{0.5 \times 10^{-3}}{0.8 \times 10^{-3}} = \mathbf{0.625}$

Using Eq. 7.12, $L_1 = L_2 \times \dfrac{N_1^2}{N_2^2} = 0.226 \times \dfrac{(600)^2}{(1700)^2} = \mathbf{0.028\ H}$

Using Eq. 7.16, $M = k\sqrt{L_1 L_2} = 0.625 \times \sqrt{0.028 \times 0.226} = \mathbf{0.05\ H}$

EXAMPLE 7.9

A coil *A* of 1200 turns and another coil *B* of 800 turns lie near each other so that 60 percent of the flux produced in one links with the other. It is found that a current of 5 A in coil *A* produces a flux of 0.25 mWb, while the same current in coil *B* produces a flux of 0.15 mWb. Determine the mutual inductance and coefficient of coupling between the coils.

Solution The inductances of the two coils are given as

$$L_1 = N_1\frac{\Phi_1}{I_1} = 1200 \times \frac{0.25 \times 10^{-3}}{5} = \mathbf{0.06\ H}$$

and

$$L_2 = N_2\frac{\Phi_2}{I_2} = 800 \times \frac{0.15 \times 10^{-3}}{5} = \mathbf{0.024\ H}$$

Consider the case when 5 A current flows through coil *A*. The flux linking with the coil *B* is then given as

$$\Phi_{12} = k\Phi_1 = 0.6 \times 0.25 \times 10^{-3} = 0.15 \times 10^{-3}\ \text{Wb}$$

∴

$$M = N_2\frac{\Phi_{12}}{I_1} = 800 \times \frac{0.15 \times 10^{-3}}{5} = \mathbf{0.024\ H}$$

and

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.024}{\sqrt{0.06 \times 0.024}} = \mathbf{0.6325}$$

7.4 DOT CONVENTION

We have as yet considered only a mutual voltage induced across an *open-circuited* coil. In general, a nonzero current flows in each of the two coils, and a mutual voltage is produced in each coil because of the current in the other coil. *This mutual voltage is present independently of and in addition to any voltage due to self-induction.* In other words, the voltage across the terminals of coil ① is composed of two terms,

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{7.17}$$

Similarly, for coil ②,

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \tag{7.18}$$

The sign of the first term in each of these equations is decided by treating the coil as a passive element, as was explained in Fig. 1.7*a*. That is, in a passive element the current enters into the positively marked terminal of the assumed voltage reference.

Sign of Mutual Voltage

In each of the Eqs. 7.17 and 7.18, the sign of the second term (i.e., the mutual voltage) depends not only on the current directions and the assumed voltage reference, but also on the way *the two coils are wound.*

Consider the physical construction of two mutually coupled coils wound on a cylindrical core, as shown in Fig. 7.3. The direction of each winding is evident. Let us assume that the current i_1 is positive and increasing with time. As per right-hand thumb rule, the flux produced by this current is directed towards right. Since i_1 is increasing with time, the flux, which is proportional to i_1, is also increasing with time. Now, consider the second coil. Let us also think of i_2 as positive and increasing with time. Application of right-hand thumb rule shows that the flux due to i_2 is also directed towards right and is increasing. We find that the assumed directions of currents i_1 and i_2 produce *additive* fluxes. Therefore, the current i_2 induces a voltage (due to mutual induction) in the first coil which has the same sense (i.e., direction) as the self-induced voltage in that coil.

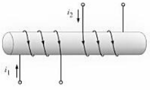


Fig. 7.3 *Two coils wound on a cylindrical core.*

Dot Convention

"Dot convention" is a convenient way of determining the sign of mutual voltage, without going into the physical construction of the two coils. Firstly, the existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown in Fig. 7.4. Secondly, the dot convention makes use of a large dot placed at one end of each of the two coils. The sign of the mutual voltage is then determined as follows:

> *A current **entering** the **dotted** terminal of one coil produces an open-circuit voltage which is positively sensed at the **dotted** terminal of the second coil.*

Thus, in Fig. 7.4*a*, the current i_1 enters the dotted terminal of L_1, then the voltage v_2 is sensed positively at the dotted terminal of L_2, and $v_2 = M\,di_1/dt$. It is important to note that marking of the reference polarity of voltage v_2 is independent of the phenomenon of mutual induction. For example, you may feel more convenient to represent v_2 by marking a positive voltage reference at the *undotted* terminal, as shown in Fig. 7.4*b*; then obviously, $v_2 = -M(di_1/dt)$.

The dot convention as stated above can also be stated in an alternative and equivalent way as follows:

> *A current **entering** the **undotted** terminal of one coil produces an open-circuit voltage which is positively sensed at the **undotted** terminal of the second coil.*

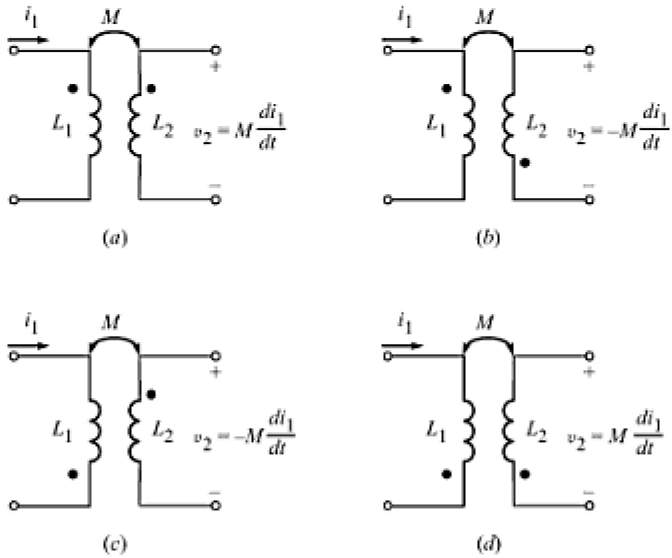


Fig. 7.4 *Dot convention applied to determine the sign of mutually induced voltage.*

This is illustrated in Figs. 7.4*c* and *d*. Also, it is very obvious that there are always two possible locations for the dots, because both dots may always be moved to the other ends of the coils and additive fluxes still result. Thus, Fig. 7.4*a* is equivalent to Fig. 7.4*d*, and Fig. 7.4*b* is equivalent to Fig. 7.4*c*.

7.5 COUPLED COILS IN SERIES

There are two ways of connecting two coupled coils in series. Current flowing in the series combination may produce the two fluxes either in the same direction or in the opposite direction.

Series Aiding Combination

Figure 7.5*a* shows the connection of two coupled coils in series aiding. As per dot convention, the fluxes produced by the two coils are additive. Because of the series connection, the currents in the two coils are same, $i_1 = i_2 = i$. Using Eqs. 7.17 and 7.18, the voltages across the two coils are given as

$$v_1 = L_1\frac{di}{dt} + M\frac{di}{dt} \qquad \text{and} \qquad v_2 = L_2\frac{di}{dt} + M\frac{di}{dt}$$

Hence, the total voltage across the combination is given as

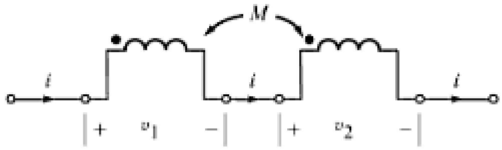
$$v = v_1 + v_2 = \left(L_1\frac{di}{dt} + M\frac{di}{dt}\right) + \left(L_2\frac{di}{dt} + M\frac{di}{dt}\right) = (L_1 + L_2 + 2M)\frac{di}{dt} \tag{7.19}$$

If L_{sa} is the equivalent inductance of the *series aiding combination*, we should have

$$v = L_{sa}\frac{di}{dt} \tag{7.20}$$

Comparing Eq. 7.20 with Eq. 7.19, we get

$$L_{sa} = L_1 + L_2 + 2M \tag{7.21}$$



(a) Series aiding combination (b) Series opposing combination

Fig. 7.5 *Coupled coils in series.*

Series Opposing Combination

In Fig. 7.5*b*, the two coupled coils are connected in series such that their fluxes oppose each other. As a result, the voltage due to mutual induction will have negative sign. Hence,

$$v_1 = L_1\frac{di}{dt} - M\frac{di}{dt} \qquad \text{and} \qquad v_2 = L_2\frac{di}{dt} - M\frac{di}{dt}$$

Hence, the total voltage across the combination is given as

$$v = v_1 + v_2 = \left(L_1\frac{di}{dt} - M\frac{di}{dt}\right) + \left(L_2\frac{di}{dt} - M\frac{di}{dt}\right) = (L_1 + L_2 - 2M)\frac{di}{dt} \tag{7.22}$$

If L_{so} is the equivalent inductance of *the series opposing combination*, we should have

$$v = L_{so}\frac{di}{dt} \tag{7.23}$$

Comparing Eq. 7.23 with Eq. 7.22, we get

$$L_{so} = L_1 + L_2 - 2M \tag{7.24}$$

Note that we can get Eq. 7.24 from Eq. 7.21, just by changing the sign of *M*. This is a general conclusion. If we reverse the terminals of one of the two coupled coils, which is equivalent to moving the dot on one coil to its other end (compare Figs. 7.5*a* and *b*), the sign of the mutual voltage changes. This fact can be used in any circuit involving coupled coils.

Measurement of *M*

The mutual inductance (M) between two coupled coils can be measured by measuring the equivalent inductance L_{sa} of series aiding combination and the equivalent inductance L_{so} of series opposing combination, since from Eqs. 7.21 and 7.24 we have

$$L_{sa} - L_{so} = 4M \quad \text{or} \quad M = \frac{L_{sa} - L_{so}}{4} \tag{7.25}$$

From the values of L_1, L_2 and M, the coefficient of coupling can be calculated using Eq. 7.16.

EXAMPLE 7.10

The net inductance of two similar coupled coils in series aiding and series opposing connections are 1.4 mH and 0.6 mH. Determine the mutual inductance and the coefficient of coupling.

Solution Using Eq. 7.25, we get

$$M = \frac{L_{sa} - L_{so}}{4} = \frac{1.4 - 0.6}{4}\ \text{mH} = \mathbf{0.2\ mH}$$

From Eq. 7.21, we have

$$L_{sa} = L_1 + L_2 + 2M \quad \text{or} \quad (1.4 \text{ mH}) = L_1 + L_2 + 2 \times (0.2 \text{ mH}) \quad \Rightarrow \quad L_1 + L_2 = 1 \text{ mH}$$

Since the two coils are similar, $L_1 = L_2 = 0.5$ mH.

Now, we can determine the coefficient of coupling using Eq. 7.16, as

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.2\,\text{mH}}{\sqrt{(0.5\,\text{mH}) \times (0.5\,\text{mH})}} = \mathbf{0.4}$$

EXAMPLE 7.11

The coefficient of coupling between two coils is 0.6. When the two coils are connected in series such that their fluxes are in the same direction, the net inductance is 1.8 H. However, when connected in series such that their fluxes are in opposite directions, the net inductance is 0.8 H. Determine the mutual inductance and the self-inductances of the two coils.

Solution Using Eqs. 7.21 and 7.24, we get

$$1.8 = L_1 + L_2 + 2M \tag{i}$$

and

$$0.8 = L_1 + L_2 - 2M \tag{ii}$$

Subtracting Eq. (*ii*) from (*i*), we get

$$4M = 1.0 \quad \text{or} \quad M = \mathbf{0.25\ H}$$

Adding Eq. (*i*) and (*ii*), we get

$$2(L_1 + L_2) = 2.6 \quad \text{or} \quad L_1 + L_2 = 1.3 \text{ H} \tag{iii}$$

Now, from Eq. 7.16, we have

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{or} \quad L_1 L_2 = \frac{M^2}{k^2} = \frac{(0.25)^2}{(0.6)^2} = 0.1736 \tag{iv}$$

Using the algebraic relation, $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$, and using Eqs. (*iii*) and (*iv*), we get

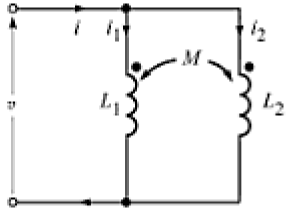
$$L_1 - L_2 = \sqrt{(L_1 + L_2)^2 - 4L_1 L_2} = \sqrt{(1.3)^2 - 4 \times 0.1736} = 0.9978 \tag{v}$$

Solving Eqs. (*iii*) and (*v*), we get

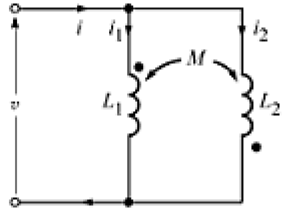
$$L_1 = \mathbf{1.149\ H} \quad \text{and} \quad L_2 = \mathbf{0.151\ H}$$

7.6 COUPLED COILS IN PARALLEL

There are two cases of parallel connection of two coupled coils, namely, parallel aiding (Fig. 7.6*a*) and parallel opposing (Fig. 7.6*b*). In the parallel connection, the currents in the two coils are different, but the voltages across both have to be the same, i.e., $v_1 = v_2 = v$.



(*a*) Parallel aiding combination.



(*b*) Parallel opposing combination.

Fig. 7.6 *Coupled coils in parallel.*

Parallel Aiding Combination

For the two coils in Fig. 7.6*a*, Eqs. 7.17 and 7.18 can be written as

$$v = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{and} \quad v = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

Thus, we should have

$$L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} \tag{7.26}$$

Since $i = i_1 + i_2$, we can replace i_2 by $(i - i_1)$ in the above equation to get

$$L_1 \frac{di_1}{dt} + M\frac{d}{dt}(i - i_1) = L_2 \frac{d}{dt}(i - i_1) + M\frac{di_1}{dt}$$

or
$$(L_1 + L_2 - 2M)\frac{di_1}{dt} = (L_2 - M)\frac{di}{dt}$$

$\Rightarrow$
$$\frac{di_1}{dt} = \frac{L_2 - M}{L_1 + L_2 - 2M}\frac{di}{dt} \tag{7.27}$$

Similarly, by replacing i_1 by $(i - i_2)$ in Eq. 7.26, we can get

$$\frac{di_2}{dt} = \frac{L_1 - M}{L_1 + L_2 - 2M}\frac{di}{dt} \tag{7.28}$$

Using Eqs. 7.27 and 7.28, the voltage across the first coil can written as

$$v = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} = L_1\left(\frac{L_2 - M}{L_1 + L_2 - 2M}\right)\frac{di}{dt} + M\left(\frac{L_1 - M}{L_1 + L_2 - 2M}\right)\frac{di}{dt}$$

$$= \left(\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\right)\frac{di}{dt} \tag{7.29}$$

If L_{pa} is the equivalent inductance of the *parallel aiding combination*, we must have

$$v = L_{pa}\frac{di}{dt} \tag{7.30}$$

Comparing Eq. 7.30 with 7.29, we get

$$L_{pa} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \tag{7.31}$$

Parallel Opposing Combination

On comparing the circuit of Fig. 7.6*b* with that of Fig. 7.6*a*, we find that the only difference is that the dot on the second coil has been moved to its other end. It means that the two coils are connected in parallel such that their fluxes are opposing each other. As a result, the induced emf in one coil due to the variation of current in other coil becomes negative. Therefore, we can get the expression for the equivalent inductance of the *parallel aiding combination* (Fig. 7.6*b*) just by changing the sign of M in Eq. 7.31. Thus, we have

$$L_{po} = \frac{L_1 L_2 - (-M)^2}{L_1 + L_2 - 2(-M)} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \tag{7.32}$$

EXAMPLE 7.12

Two coupled coils, with coefficient of coupling 0.433, have self inductances of 8 H and 6 H. Determine the equivalent inductance of the combination when they are connected in parallel such that (*a*) the mutual induction assists the self-induction, and (*b*) the mutual induction opposes the self-induction.

Solution From Eq. 7.16, the mutual inductance is given as

$$M = k\sqrt{L_1 L_2} = 0.433 \times \sqrt{8 \times 6} = 3 \text{ H}$$

(*a*) From Eq. 7.31, $L_{pa} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \dfrac{8 \times 6 - 3^2}{8 + 6 - 2 \times 3} = \mathbf{4.875\ H}$

(*b*) From Eq. 7.32, $L_{po} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \dfrac{8 \times 6 - 3^2}{8 + 6 + 2 \times 3} = \mathbf{1.95\ H}$

7.7 ENERGY STORED IN MAGNETIC FIELD

When a coil is connected to an electrical energy source, the current gradually increases from zero to its final value I. Due to the current flow in the inductance (L) part of the coil, a magnetic field is established. As given by Eq. 7.2, the energy stored in the inductance as magnetic field is

$$W = \frac{1}{2} L I^2 \tag{7.33}$$

In addition to this energy stored in the magnetic field, some energy is dissipated as heat due to the current flowing in the resistance (R) part of the coil.

Note that once the field is established and the current has attained its steady value, no more energy is required to maintain the magnetic field. On the other hand, whenever the circuit is broken, the magnetic field collapses and the energy stored is used in generating the induced emf or current.

The expression for the energy stored in inductance (given by Eq. 7.33) can be modified by using Eq. 7.5, and putting

$$L = \frac{N^2 \mu A}{l} = \frac{\mu_0 \mu_r A N^2}{l}$$

Hence, the energy stored in the magnetic field is

$$W = \frac{1}{2} L I^2 = \frac{1}{2}\mu_0\mu_r A \frac{N^2 I^2}{l} = \frac{1}{2}\mu_0\mu_r A l \frac{N^2 I^2}{l^2} = \frac{1}{2}\mu_0\mu_r (Al)\left(\frac{NI}{l}\right)^2$$

Since, Al represents the volume of the magnetic field, and NI/l represents the magnetic field intensity (H), we have

$$W = \frac{1}{2}\mu_0\mu_r (Al)\left(\frac{NI}{l}\right)^2 = \frac{1}{2}\mu_0\mu_r H^2 \text{ (Volume)}$$

Thus, the ***energy stored per unit volume*** is given as

$$W_u = \frac{1}{2}\mu_0\mu_r H^2 \tag{7.34}$$

Since, the magnetic flux density, $B = \mu_0\mu_r H$, the above expression can be modified as

$$W_u = \frac{1}{2}\mu_0\mu_r H^2 = \frac{1}{2} BH \tag{7.35}$$

or $$W_u = \frac{B^2}{2\mu_0 \mu_r} \quad \text{joules/m}^3 \tag{7.36}$$

7.8 LIFTING POWER OF A MAGNET

Let us consider two magnetic poles, north and south, one over the other but separated by some distance, as shown in Fig. 7.7. Let A be their area of cross-section, and F be the total force of attraction between them.

Now, suppose that one of the poles (say, N-pole) is pulled apart against this attractive force through a small distance dx. Then, in doing so, the increase in volume of the magnetic field is Adx, and the work done is Fdx. The work done in pulling the poles apart is stored in the magnetic field as additional energy due to increase in volume. Thus,

$$Fdx = \left(\frac{B^2}{2\mu_0}\right)(Adx)$$

Therefore, the force of attraction between the two poles (or the lifting power of the magnet),

$$F = \frac{B^2 A}{2\mu_0} \quad \text{newtons}$$

And, the force per unit area,

$$F_u = \frac{B^2}{2\mu_0} \quad \text{N/m}^2 \tag{7.37}$$

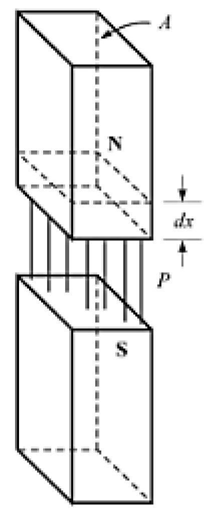


Fig. 7.7

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 7.13

An air cored coil is required to have a length of 2.5 cm and an average cross-sectional area of 2 cm^2. Find the number of turns needed to give an inductance of 400 µH.

Solution We know that the inductance of an air-cored solenoid is given as

$$L = \frac{N^2 \mu_0 A}{l}$$

Therefore, the number of turns needed is given as

$$N = \sqrt{\frac{Ll}{\mu_0 A}} = \sqrt{\frac{400 \times 10^{-6} \times 2.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 2 \times 10^{-4}}} = \mathbf{200}$$

EXAMPLE 7.14

The coefficient of coupling between two coils is 0.75. When a current of 3 A flows in the first coil having 250 turns, the total flux produced in this coil is 4 mWb. When this current is linearly changed from 3 A to zero in 3 ms, the voltage induced in the second coil is 70 V. Determine L_1, L_2, M and N_2.

Solution The self inductance of the first coil is given as

$$L_1 = N_1 \frac{\Phi_1}{i_1} = \frac{250 \times 4 \times 10^{-3}}{3} = \mathbf{0.333\ H}$$

Since the voltage induced in the second coil is given as $v_2 = M\, di_1/dt$, we have

$$70 = M \times \frac{3}{3 \times 10^{-3}} \quad \Rightarrow \quad M = \mathbf{0.07\ H}$$

Since $M = k\sqrt{L_1 L_2}$, the self-inductance of the second coil is given as

$$L_2 = \frac{M^2}{k^2 L_1} = \frac{(0.07)^2}{(0.75)^2 \times 0.333} = \mathbf{0.0262\ H}$$

Since $L \propto N^2$, the number of turns in the second coil is given as

$$N_2 = N_1 \sqrt{\frac{L_2}{L_1}} = 250 \times \sqrt{\frac{0.0262}{0.333}} = \mathbf{70}$$

EXAMPLE 7.15

A coil has 1000 turns enclosing a magnetic circuit of 20 cm^2 in cross-section. With 4-A current in the coil, the flux density is 1.0 Wb/m^2, and with 9-A current, it is 1.4 Wb/m^2. Find the mean value of inductance between these current limits and the induced emf if the current decreases from 9 A to 4 A in 0.05 second.

Solution For the two values of the current, the inductances are

$$L_1 = N_1 \frac{\Phi_1}{i_1} = \frac{N_1 B_1 A}{i_1} = \frac{1000 \times 1.0 \times 20 \times 10^{-4}}{4} = \mathbf{0.5\ H}$$

and

$$L_2 = N_2 \frac{\Phi_2}{i_2} = \frac{N_2 B_2 A}{i_2} = \frac{1000 \times 1.4 \times 20 \times 10^{-4}}{9} = \mathbf{0.31\ H}$$

$\therefore$ Mean value of self inductance, $L = \dfrac{L_1 + L_2}{2} = \dfrac{0.5 + 0.31}{2} = \mathbf{0.405\ H}$

$$\text{Induced emf, } e = L\frac{di}{dt} = 0.405 \times \frac{9-4}{0.05} = \mathbf{40.5\ V}$$

EXAMPLE 7.16

Two coils having 100 and 150 turns, respectively, are wound side by side on a closed iron circuit of cross-section 125 cm^2 and mean length 200 cm. If the relative permeability of the iron is 2000, calculate (*a*) the self inductance of each coil, (*b*) the mutual inductance between them, and (*c*) the emf induced in the second coil if the current in the first coil changes from 0 to 5 A in 0.02 second.

Solution

(*a*) The self inductances of the two coils are given as

$$L_1 = \frac{N_1^2 \mu_0 \mu_r A}{l} = \frac{(100)^2 \times 4\pi \times 10^{-7} \times 2000 \times 125 \times 10^{-4}}{2} = \mathbf{157.1\ mH}$$

and

$$L_2 = \frac{N_2^2 \mu_0 \mu_r A}{l} = \frac{(150)^2 \times 4\pi \times 10^{-7} \times 2000 \times 125 \times 10^{-4}}{2} = \mathbf{353.4\ mH}$$

(*b*) Since the two coils are wound side by side, there is a tight coupling and the coefficient of coupling can be assumed to be 1. The mutual inductance between the coils is then given as

$$M = k\sqrt{L_1 L_2} = 1 \times \sqrt{157.1 \times 353.4} = \mathbf{235.6\ mH}$$

(*c*) The emf induced in the second coil is

$$e_2 = M\frac{di_1}{dt} = 235.6\times10^{-3}\times\frac{5-0}{0.02} = \mathbf{58.9\ V}$$

EXAMPLE 7.17

Two identical coils with terminals T_1 T_2 and T_3 T_4, respectively, are placed side by side. The inductances measured under different sets of connections are as follows:

When T_2 is connected to T_3 and inductance measured between T_1 and T_4 is 4 H.

When T_2 is connected to T_4 and inductance measured between T_1 and T_3 is 0.8 H.

Determine the self inductance of each coil and the coefficient of coupling.

Solution Let L be the inductance of each of the two coils. Then, we must have

For first measurement: $L + L + 2M = 4$ or $L + M = 2$ (*i*)

For second measurement: $L + L - 2M = 0.8$ or $L - M = 0.4$ (*ii*)

Solving Eqs. (*i*) and (*ii*), we get $L = \frac{2+0.4}{2} = \mathbf{1.2\ H}$ and $M = \frac{2-0.4}{2} = \mathbf{0.8\ H}$

$$\therefore \qquad k = \frac{M}{\sqrt{L_1L_2}} = \frac{0.8}{\sqrt{1.2\times1.2}} = \mathbf{0.667} \text{ or } \mathbf{66.7\ \%}$$

EXAMPLE 7.18

A coil of inductance 200 mH is magnetically coupled with another coil of inductance 800 mH. The coefficient of coupling between the two coils is 0.5. Calculate the equivalent inductance of (*a*) series aiding, (*b*) series opposing, (*c*) parallel aiding, and (*d*) parallel opposing combinations.

Solution The mutual inductance between the two coils is given as

$$M = k\sqrt{L_1L_2} = 0.5\times\sqrt{200\times800} = 200\text{ mH}$$

(*a*) *Series aiding*: $L_{sa} = L_1 + L_2 + 2M = 200 + 800 + 2\times200 = \mathbf{1400\ mH}$

(*b*) *Series opposing*: $L_{so} = L_1 + L_2 - 2M = 200 + 800 - 2\times200 = \mathbf{600\ mH}$

(*c*) *Parallel aiding*: $L_{pa} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{200\times800-(200)^2}{200+800-2\times200} = \mathbf{200\ mH}$

(*d*) *Parallel opposing*: $L_{po} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{200\times800-(200)^2}{200+800+2\times200} = \mathbf{85.71\ mH}$

EXAMPLE 7.19

A horse-shoe magnet is made from a bar of wrought iron 45 cm long and cross-sectional area 6 cm^2. Exciting coils of 500 turns are wound on each limb and are connected in series. Determine the exciting current needed for the magnet to lift a load of 60 kg, assuming that the load makes close contact with the magnet. The relative permeability of iron is 800.

Solution The force of attraction needed at each pole,

$$F = \frac{60}{2} = 30\text{ kg} = 30\times9.8\text{ N} = 294\text{ N}$$

Now, total lifting force of the magnet is given as

$$F = \frac{B^2 A}{2\mu_0}$$

Thus, the required magnetic flux density is given as

$$B = \sqrt{\frac{2\mu_0 F}{A}} = \sqrt{\frac{2 \times 4\pi \times 10^{-7} \times 294}{6 \times 10^{-4}}} = 1.11 \text{ Wb/m}^2$$

$$\therefore \qquad H = \frac{B}{\mu_0 \mu_r} = \frac{1.11}{4\pi \times 10^{-7} \times 800} = 1104 \text{ At/m}$$

Now, the length of the iron path = 45 cm = 0.45 m. Therefore, the total ampere turns needed,

$$At = 1104 \times 0.45 = 496.8$$

Thus, the current required,

$$I = \frac{496.8}{2 \times 500} = \mathbf{0.4968\ A}$$

SUMMARY

TERMS AND CONCEPTS

- ***Self inductance*** of a coil arises when an emf is induced in itself by changing the current flowing through it.
- Isolated inductances in series and parallel are added the same way as resistances.
- An inductor may be fixed or variable, iron-cored or air-cored.
- When a magnetic field is set up by an inductor, it stores energy.
- ***Mutual inductance*** is the property of two magnetically coupled coils because of which there is an induced emf in one coil due to change in current in the other coil.
- ***Coefficient of coupling*** (k) is the ratio of flux linkage between primary and secondary coils to the flux produced by primary current.
- The mutual voltage is present independently of and in addition to any voltage due to self-induction.
- Both L and M are measured in henrys (H).

IMPORTANT FORMULAE

- Self-induced emf, $e = L\dfrac{di}{dt}$.
- Energy stored, $W = \dfrac{1}{2}LI^2$.
- EMF induced by mutual inductance, $e_2 = M\dfrac{di_1}{dt}$.
- $L = \dfrac{N^2 \mu A}{l}$.
- $M_{21} = N_1 \dfrac{d\Phi_{21}}{di_2}$.
- Coupling coefficient, $k = \dfrac{M}{\sqrt{L_1 L_2}}$; $0 \le k \le 1$.

- $v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$.
- Two coupled coils in
  - (*i*) Series aiding, $L_{sa} = L_1 + L_2 + 2M$
  - (*ii*) Series opposing, $L_{so} = L_1 + L_2 - 2M$
  - (*iii*) Parallel aiding, $L_{pa} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$
  - (*iv*) Parallel opposing, $L_{po} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$
- The energy stored, $W_u = \dfrac{B^2}{2\mu_0 \mu_r}$.
- The lifting power, $F_u = \dfrac{B^2}{2\mu_0}$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The self inductance of a solenoid is directly proportional to its length.	☐	☐	
2.	From energy point of view, the inductance of a coil is given as $L = (2 \times \text{Energy})/I^2$.	☐	☐	
3.	If all parameters are same, a closed iron-circuit choke has higher inductance than an open iron-circuit choke.	☐	☐	
4.	Inductances in series and in parallel are added the same way as capacitances.	☐	☐	
5.	The inductance of a coil wound on an iron core is inversely proportional to the permeability of the core material.	☐	☐	
6.	Due to mutual inductance between two coils, there is an induced emf in one coil due to a change in current in the other coil.	☐	☐	
7.	The coefficient of coupling between two coils is the ratio of the flux produced in one coil to the flux linking with the other coil.	☐	☐	
8.	The mutual voltage is present independently of and in addition to any voltage due to self-induction.	☐	☐	
9.	The equivalent inductance of a series combination of two coupled coils is $L = L_1 + L_2$.	☐	☐	
10.	When two coupled coils are connected in parallel such that their fluxes aid each other, the equivalent inductance is given as $L_{pa} = \dfrac{L_1 L_2 + M^2}{L_1 + L_2 + 2M}$.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** False
6. True	**7.** False	**8.** True	**9.** False	**10.** False

REVIEW QUESTIONS

1. Explain the term 'self inductance'. Why is the induced emf in a coil called 'back emf' or 'counter emf'?
2. What are the factors on which the inductance of a coil depends? Derive the necessary expression for calculating the inductance.
3. Define and explain self and mutually induced emf.
4. Explain what is meant by self inductance and mutual inductance. Define the units in which each is measured.
5. Explain the concept of mutual inductance. Define coefficient of coupling and derive an expression between self inductances of the two coils, mutual inductance between them and the coefficient of coupling.
6. Prove that the coefficient of mutual inductance M between two coils of self inductances L_1 and L_2 is given by $M = k\sqrt{L_1 L_2}$, where k is the coefficient of coupling between the two coils.
7. Two coils of self inductances L_1 and L_2 are placed side by side so that the mutual inductance between them is M. If they are connected in series aiding, derive the expression for the net inductance of the combination.
8. Derive an expression for the equivalent inductance of two coupled coils connected in parallel such that their fluxes oppose each other.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. The principle of statically induced emf is utilized in
   (*a*) transformer (*b*) motor
   (*c*) generator (*d*) battery
2. The magnitude of statically induced emf depends on
   (*a*) the coil resistance
   (*b*) the flux magnitude
   (*c*) the rate of change of flux
   (*d*) all of the above
3. The unit(s) of inductance is (are)
   (*a*) henry (*b*) Vs/A
   (*c*) Wb/A (*d*) all of the above
4. If an emf of 8 V is induced in a coil of inductance 4 H, the rate of change of current through it must be
   (*a*) 32 A/s (*b*) 0.5 A/s
   (*c*) 2 A/s (*d*) 12 A/s
5. The iron core from an iron-cored coil is removed so that it becomes air-cored. The inductance of the coil will
   (*a*) decrease (*b*) increase
   (*c*) remains the same (*d*) any of the above
6. Two coils with zero coefficient of coupling between them have self inductances of 16 H and 9 H. If two coils are connected in series, the net inductance will be
   (*a*) 5 H (*b*) 25 H
   (*c*) 7 H (*d*) 144/25 H
7. Two coils have inductances of 4 mH and 9 mH, and coefficient of coupling of 0.5. If the two coils are connected in series aiding, the net inductance will be
   (*a*) 7 mH (*b*) 13 mH
   (*c*) 16 mH (*d*) 19 mH
8. A coil having a core of length 10 cm has an inductance of 5 mH. If its core length is doubled, all other quantities remaining the same, the inductance will become
   (*a*) 1.25 mH (*b*) 2.5 mH
   (*c*) 10 mH (*d*) 25 mH
9. The mutual inductance between two closely coupled coils is 100 mH. If the number of turns of one coil is reduced to half and that of the other is doubled, the new value of mutual inductance would be
   (*a*) 400 mH (*b*) 100 mH
   (*c*) 50 mH (*d*) 25 mH
10. The mutual inductance between two magnetically coupled coils depends on
    (*a*) the number of turns of the coils
    (*b*) the cross-sectional area of their common core
    (*c*) the permeability of the core
    (*d*) all of the above

11. The overall inductance of two coils connected in series with mutual inductance aiding self inductance is L_1, and with mutual inductance opposing self inductance is L_2. Then the mutual inductance M is given by
(*a*) $L_1 + L_2$ (*b*) $L_1 - L_2$
(*c*) $0.5(L_1 + L_2)$ (*d*) $0.25(L_1 - L_2)$

12. Two coupled coils connected in series have an equivalent inductance of 16 mH and 8 mH depending on the interconnection. Then, the mutual inductance between the coils is
(*a*) $8\sqrt{2}$ mH (*b*) 12 mH
(*c*) 4 mH (*d*) 2 mH

13. The coupling between two magnetically coupled coils is said to be ideal if the coefficient of coupling is
(*a*) 0 (*b*) 0.25
(*c*) 0.5 (*d*) 1

14. Two magnetically coupled coils with $L_1 = L_2 = 0.6$ H have a coupling coefficient of $k = 0.8$. Their turns ratio N_1/N_2 must be
(*a*) 1 (*b*) 2
(*c*) 3/4 (*d*) 4

15. A 500-turn coil has an inductance of 3.5 mH. If the number of turns are increased to 1000, all other parameters remaining the same, the inductance will be
(*a*) 3.5 mH (*b*) 1.75 mH
(*c*) 7 mH (*d*) 14 mH

ANSWERS

1. *a*	**2.** *c*	**3.** *d*	**4.** *c*	**5.** *a*	**6.** *b*	**7.** *d*	**8.** *b*	**9.** *b*	**10.** *d*
11. *d*	**12.** *d*	**13.** *d*	**14.** *a*	**15.** *d*					

PROBLEMS

(A) SIMPLE PROBLEMS

1. Calculate the resistance and inductance of an air-cored solenoid of mean diameter 1 cm and length 1 m. It has 1000 turns of copper wire of diameter 0.5 mm. The resistivity of copper is given as 1.72×10^{-8} Ωm. [**Ans.** 2.75 Ω, 98.7 μH]

2. Calculate the inductance of an air-cored toroid having a mean diameter of 25 cm and circular cross-sectional area of 6.25 cm^2, wound uniformly with 1000 turns of copper wire. [**Ans.** 1 mH]

3. A toroid has a core of square cross-sectional area of 2500 mm^2 and mean diameter of 250 mm. The core material has a relative permeability of 1000. Determine the number of turns to be wound on the core so as to give an inductance of 1 H. [**Ans.** 500]

4. Two coils have a mutual inductance of 0.3 H. If the current in one coil is varied from 5 A to 2 A in 0.4 s, calculate (*a*) the average emf induced in the second coil, (*b*) the change of flux linked with the second coil assuming that it is wound with 200 turns. [**Ans.** (*a*) 2.25 V; (*b*) 0.01125 Wb/s]

5. The self inductance of a coil of 500 turns is 0.25 H. If 60 % of the flux is linked with a second coil of 10 000 turns, calculate (*a*) the mutual inductance of the two coils, and (*b*) the emf induced in the second coil when current in the first coil changes at the rate of 100 A/s. [**Ans.** (*a*) 3 H; (*b*) 300 V]

6. A primary coil having an inductance of 100 μH is connected in series with a secondary coil of inductance 240 μH, and the total inductance of the combination is measured to be 146 μH. Determine the coefficient of coupling. [**Ans.** 0.626]

7. Two inductances of 15 mH and 25 mH are connected in series such that their fluxes oppose each other. They are so placed that the coefficient of coupling is 0.8. Calculate the total inductance of series combination. [**Ans.** 9 mH]

8. Determine the force required to separate two surfaces with 100 cm^2 of contact area, when the flux density normal to the surfaces is 1 Wb/m^2. [**Ans.** 3979 N]

(B) TRICKY PROBLEMS

9. An iron-core solenoid has a mean length of 0.4 m and cross-sectional area of 80 cm^2. It is wound uniformly with 1200 turns of wire. The relative permeability of the core material is 1000. Determine the self-induced emf in the solenoid, if the current through the coil changes from 2 A to zero in 0.01 s. [**Ans.** 7238 V]

10. An iron ring of 0.15 m diameter and 0.001 m^2 in cross-section with a saw cut 2 mm wide is wound with 300 turns of wire. The gap flux density is 1 T. The relative permeability of iron is 500. Determine the exciting current and inductance. [**Ans.** 7.79 A, 64.18 mH]

11. Find the inductance of a coil in which a current of 0.2 A increasing at the rate of 0.4 A per second represents a power flow of 0.4 W. [**Ans.** 5 H]

12. Two coils A and B are wound on the same iron core. There are 500 turns on A and 3000 turns on B. A current of 5 A through coil A produces a flux of 600 μWb in the core. If this current is reversed in 0.02 s, calculate the average emf induced in coils A and B. [**Ans.** 30 V, 180 V]

13. Two coupled coils have a coefficient of coupling 0.85, and number of turns 100 and 800. With first coil open, a current of 5 A in second coil produces a flux of 0.35 mWb in it. Determine L_1, L_2 and M. [**Ans.** 0.875 mH, 56 mH, 5.95 mH]

14. A solenoid has 800 turns and length of 1.2 m. Its self inductance is 2 mH. A search coil having 50 turns with a mean diameter of 30 mm is placed at the centre of the solenoid. If the self inductance of the search coil is 25 μH, find the mutual inductance and the coefficient of coupling. [**Ans.** 29.6 μH, 0.132]

15. Two coils with a coefficient of coupling of 0.6 between them are connected in series so as to magnetize in the same direction, and then in the opposite direction. The corresponding values of the total inductances are 2.96 H and 1.04 H, respectively. Find the self inductance of the two coils and the mutual inductance between them. [**Ans.** 0.4 H, 1.6 H, 0.48 H]

16. Two coils of self inductances 150 mH and 250 mH and mutual inductance 120 mH are connected in parallel. Determine the equivalent inductance of the combination if (*a*) the mutual flux helps the individual flux, (*b*) the mutual flux opposes the individual flux. [**Ans.** (*a*) 144 mH; (*b*) 36 mH]

(C) CHALLENGING PROBLEMS

17. The number of turns in a coil is 250. When current of 2 A flows in this coil, a flux of 0.3 mWb is established. When this current is reduced to zero in 2 ms, the voltage induced in another coil lying in the vicinity is 63.75 V. If the coefficient of coupling between the two coils is 0.85, find the self-inductances of the two coils, the mutual inductance and the number of turns in the second coil. [**Ans.** 37.5 mH, 150 mH, 63.75 mH, 500]

18. Two identical 1000-turn coils X and Y lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 5 A in X produces in it a flux of 0.05 mWb. Calculate the self inductance of each coil and their mutual inductance. If the current in X changes from 6 A to –6 A in 0.01 s, what is the emf induced in Y? [**Ans.** 0.01 H, 0.01 H, 0.006 H, 7.2 V]

SUPPLEMENTARY EXERCISES

B

PART B: ELECTROMAGNETIC CIRCUITS

Assemblage of

- Chapter 5: Electromagnetism
- Chapter 6: Magnetics Circuits
- Chapter 7: Self and Mutual Inductances

B.1. SOLVED PROBLEMS

PROBLEM B-1

Given the waveform of the current in a 3-H inductor as shown in Fig. B-1*a*, determine the inductor voltage and sketch it.

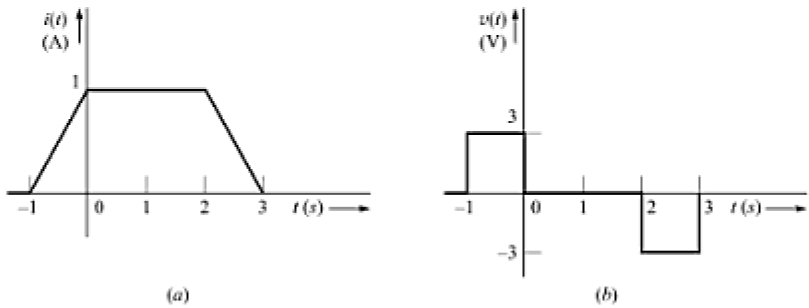


Fig. B-1

Solution Since the current is zero for $t < -1$ s, the voltage is zero in this interval. During the interval $-1\text{ s} < t < 0$, the current *increases* at a linear rate of 1 A/s, and hence the voltage produced is constant and is given by

$$L\frac{di}{dt} = 3 \times 1 = \mathbf{3\ V}$$

During the interval $0 < t < 2$ s, the current is constant at 1 A. That is, $di/dt = 0$, and hence the voltage produced is zero.

During the interval $2\text{ s} < t < 3$ s, the current linearly *decreases* and results in $di/dt = -1$ A/s. Hence the voltage produced is constant and is given by

$$L\frac{di}{dt} = 3 \times (-1) = \mathbf{-3\ V}$$

Fort $t > 3$ s, the current is constant at zero, and hence the voltage $v(t)$ is zero. The complete voltage waveform is shown in Fig. B-1*b*.

PROBLEM B-2

A choke has an inductance of 3 H and a resistance (of the wire) of 0.1 Ω. As shown in Fig. B-2*a*, it is connected to an ac source of $12 \sin(\pi t/6)$ A. (*a*) Find the maximum energy stored in the inductor. (*b*) Calculate how much energy is dissipated in its resistance in the time during which the energy is being stored in and then recovered from the inductor.

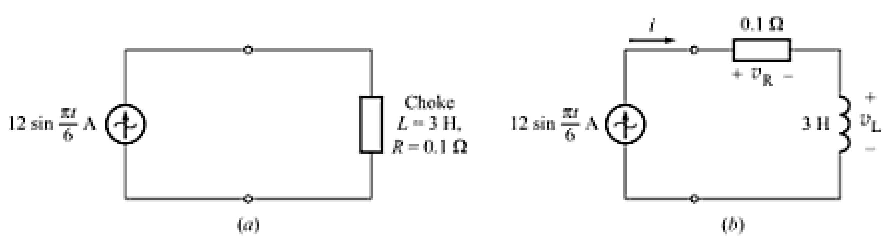


Fig. B-2

Solution

(*a*) The energy stored in the inductor becomes maximum when the current through it becomes maximum. The maximum value of current is 12 A, and occurs at a time when $\sin(\pi t/6) = \pm 1$. Therefore, the maximum energy stored is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2} \times 3 \times (\pm 12)^2 = \mathbf{216\ J}$$

(*b*) Figure B-2*b* shows the circuit representation of the choke. The voltage across the resistor is given by

$$v_R = Ri = 0.1 \times 12\sin(\pi t/6) = 1.2\sin(\pi t/6)\ \text{V}$$

and the voltage across the inductance is given by

$$v_L = L\frac{di}{dt} = 3\frac{d}{dt}\left(12\sin\frac{\pi t}{6}\right) = 3 \times 12 \times (\pi/6)\cos\left(\frac{\pi t}{6}\right) = 6\pi\cos\left(\frac{\pi t}{6}\right)\ \text{V}$$

The current increases from zero to 12 A from $t = 0$ to $t = 3$ s; and again decreases from 12 A to zero in next 3 s. The 216 J of energy stored in the inductor in first 3 s leaves it completely in next 3 s. In these 6 s, the power dissipated in the resistor is given by

$$p_R = Ri^2 = 0.1 \times (12\sin(\pi t/6))^2 = 14.4\sin^2(\pi t/6)\ \text{W}$$

Therefore, the energy converted into heat in the resistor within this 6 s is given as

$$w_R = \int_0^6 p_R\,dt = \int_0^6 14.4\sin^2\frac{\pi}{6}t\,dt = \int_0^6 14.4\left(\frac{1}{2}\right)\left(1 - \cos\frac{\pi}{3}t\right)dt = \mathbf{43.2\ J}$$

PROBLEM B-3

A metal rod wound with 3500 turns is 25 cm long and 2.5 cm in diameter. It is bent into a closed ring. When 0.6 A current is passed through the coil, the flux density inside the metal is found to be 0.45 Wb/m^2. Assuming that there is no flux leakage, find (*a*) the relative permeability of the metal, (*b*) the self-inductance of the coil, and (*c*) the average emf induced in the coil, if on interrupting the current, the value of the flux in the metal is found to be 8 % of its original value in 1.5 ms.

Solution

(*a*) The ampere turns per metre of flux path length is

$$H = \frac{NI}{l} = \frac{3500 \times 0.6}{0.25} = 8400\ \text{At/m}$$

The relative permeability of the metal is given as

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.45}{4\pi \times 10^{-7} \times 8400} = \mathbf{42.63}$$

(*b*) Area of cross-section of the metal ring, $A = \pi r^2 = \pi(2.5 \times 10^{-2}/2)^2 = 4.911 \times 10^{-4}\ \text{m}^2$

∴ Magnetic flux in the ring, $\Phi = BA = 0.45 \times 4.911 \times 10^{-4}\ \text{m}^2 = 2.21 \times 10^{-4}\ \text{Wb}$

∴ Self-inductance, $L = \dfrac{N\Phi}{I} = \dfrac{3500 \times 2.21 \times 10^{-4}}{0.6} = \mathbf{1.289\ H}$

(*c*) The original value of flux, $\Phi_1 = 2.21 \times 10^{-4}$ Wb

The changed value of flux, $\Phi_2 = 0.08 \times 2.21 \times 10^{-4}\ \text{Wb} = 0.1768 \times 10^{-4}$ Wb

∴ Induced emf, $e = -N\dfrac{d\Phi}{dt} = -3500 \times \dfrac{(0.1768 - 2.21) \times 10^{-4}}{0.0015} = \mathbf{474.4\ V}$

PROBLEM B-4

There are 800 turns enclosing a magnetic circuit of cross-section 15 cm^2. A flux density of 0.8 Wb/m^2 is produced when 5-A current flows through the coil. When the current is increased to 10 A, the flux density increases to 1.2 Wb/m^2. (*a*) Find the mean value of inductance between these limits of current. (*b*) If the current uniformly decreases from 10 A to 5 A in 0.04 s, find the value of induced emf.

Solution

(*a*) The mean value of inductance,

$$L = N\frac{d\Phi}{dI} = N\frac{d(BA)}{dI} = NA\frac{dB}{dI}$$

$$= NA\frac{B_2 - B_1}{I_2 - I_1} = 800 \times (15 \times 10^{-4}) \times \frac{1.2 - 0.8}{10 - 5} = 0.096 \text{ H} = \mathbf{96\ mH}$$

(*b*) The induced emf,

$$e = -L\frac{di}{dt} = -0.096 \times \frac{5 - 10}{0.04} = \mathbf{12\ V}$$

PROBLEM B-5

An air-cored solenoid of mean diameter 5 cm and length 80 cm is wound with 1200 turns. Determine its inductance. Also, find how much emf would be induced if the current flowing in the coil is changed from 10 A to –10 A in 30 ms.

Solution We know that the inductance of a solenoid is given as

$$L = \frac{N^2 \mu A}{l} = \frac{N^2 \mu_r \mu_0 A}{l}$$

$$= \frac{(1200)^2 \times 1 \times 4\pi \times 10^{-7} \times [\pi \times (2.5 \times 10^{-2})^2]}{0.8} = 4.44 \times 10^{-3} \text{ H} = \mathbf{4.44\ mH}$$

The induced emf in the coil due to the change in current is given as

$$e = -L\frac{di}{dt} = -4.44 \times 10^{-3} \times \frac{[(-10) - 10]}{0.03} = \mathbf{2.96\ V}$$

PROBLEM B-6

A solenoid consists of 1000 turns of wire wound on a length of 80 cm. A search coil of 400 turns enclosing a mean area of 25 cm^2 is placed centrally in the solenoid. Calculate (*a*) the mutual inductance between the two coils, and (*b*) the emf induced in the search coil when the current in the solenoid is changing uniformly at the rate of 200 A/s.

Solution

(*a*) The flux density at the centre of the solenoid due to a current I is given as

$$B_c = \mu_0 \frac{NI}{l} = 4\pi \times 10^{-7} \times \frac{1000\, I}{0.8} = 15.715 \times 10^{-4} I \text{ T}$$

The flux linking the search coil,

$$\Phi_2 = B_c \times \text{Area} = (15.715 \times 10^{-4} I)\,(25 \times 10^{-4}) = 392.875 \times 10^{-8} I \text{ Wb}$$

Therefore, the mutual inductance between the coils is given as

$$M = \frac{N_2 \Phi_2}{I_1} = \frac{400 \times 392.875 \times 10^{-8} I}{I} = 1.57 \times 10^{-3} \text{ H} = \mathbf{1.57\ mH}$$

(*b*) The emf induced in the search coil due to a change in current in the solenoid,

$$e_2 = -M\frac{di_1}{di} = -1.57 \times 10^{-3} \times 200 = \mathbf{-0.314\ V}$$

PROBLEM B-7

A coil of 250 turns carrying a current of 2 A produces a flux of 0.3 mWb. When the current is reduced to zero in 2 milliseconds, the voltage induced in a nearby coil is 60 V. Calculate the self-inductance of each coil and the mutual inductance between the two coils. Assume the coefficient of coupling to be 0.7.

Solution The self-inductance of the first coil,

$$L_1 = N_1\frac{\Phi}{I_1} = 250 \times \frac{0.3 \times 10^{-3}}{2} = \mathbf{37.5\ mH}$$

When the current in first coil is changed, the induced emf in the second coil is given as

$$e_2 = M\frac{di_1}{dt}$$

Hence, we have

$$60 = M \times \frac{(2-0)}{2 \times 10^{-3}} = M \times 10^3 \quad \Rightarrow \quad M = \frac{60}{10^3} = \mathbf{60\ mH}$$

The self-inductance of the second coil can now be calculated from the relation,

$$M = k\sqrt{L_1 L_2} \quad \Rightarrow \quad L_2 = \left(\frac{M}{k}\right)^2 \frac{1}{L_1} = \left(\frac{60}{0.7}\right)^2 \frac{1}{37.5}\text{mH} = \mathbf{195.9\ mH}$$

PROBLEM B-8

A solenoid of 500 turns is wound on a former of length 1 m and of diameter 3 cm. This is placed co-axially within another solenoid of the same length and number of turns but of diameter 6 cm. If the diameter of the wire used is 1 mm and material is copper, determine the inductance and resistance of each solenoid. (ρ for copper = 0.5 μΩm)

Solution Given: $l_1 = l_2 = 1$ m; $N_1 = N_2 = 500$;

$$A_1 = \pi(1.5 \times 10^{-2})^2 = 7.07 \times 10^{-4}\ \text{m}^2;\ A_2 = \pi(3 \times 10^{-2})^2 = 28.28 \times 10^{-4}\ \text{m}^2$$

The self-inductances of the two solenoids are

$$L_1 = \frac{\mu_0 A_1 N_1^2}{l_1} = \frac{4\pi \times 10^{-7}(7.07 \times 10^{-4})(500)^2}{1} = \mathbf{0.222\ mH}$$

and

$$L_2 = \frac{\mu_0 A_2 N_2^2}{l_2} = \frac{4\pi \times 10^{-7}(28.28 \times 10^{-4})(500)^2}{1} = \mathbf{0.888\ mH}$$

Now, the total lengths of the wire used in the two windings,

$$l_{w1} = \pi d_1 N_1 = \pi \times 3 \times 10^{-2} \times 500 = 47.124\ \text{m}$$

and

$$l_{w2} = \pi d_2 N_2 = \pi \times 6 \times 10^{-2} \times 500 = 94.248\ \text{m}$$

Area of cross-section of the wire,

$$A_{w1} = A_{w2} = \pi r^2 = \pi \times (0.5 \times 10^{-3})^2 = 0.785 \times 10^{-6}\ \text{m}^2$$

Therefore, the resistances of the two solenoids are

$$R_1 = \rho\frac{l_{w1}}{A_{w1}} = 0.5 \times 10^{-6} \times \frac{47.124}{0.785 \times 10^{-6}} = \mathbf{30.0\ \Omega}$$

and

$$R_2 = \rho\frac{l_{w2}}{A_{w2}} = 0.5 \times 10^{-6} \times \frac{94.248}{0.785 \times 10^{-6}} = \mathbf{60.0\ \Omega}$$

PROBLEM B-9

A long solenoid of cross-sectional area 2 cm^2 has primary winding with 25 turns per cm. At the middle of the solenoid is wound a secondary winding of 100 turns. The primary winding is carrying a current of 2 A. The supply to the primary winding is suddenly disconnected so that its current falls to zero in 1 ms. Determine the average emf induced in the secondary winding.

Solution Assuming the length of the solenoid to be l metres, we have

$$N_1 = 2500l,\ N_2 = 100; \quad A = 2\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$$

Therefore, the mutual inductance is given as

$$M = \frac{\mu_0 A N_1 N_2}{l} = \frac{4\pi \times 10^{-7}(2 \times 10^{-4})(2500l)(100)}{l} = \mathbf{62.83\ \mu H}$$

The induced emf in the secondary is given as

$$e_2 = M\frac{di_1}{dt} = 62.83 \times 10^{-6} \times \frac{(2-0)}{1 \times 10^{-3}} = \mathbf{125.66\ mV}$$

PROBLEM B-10

A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring and carrying a current of 3 A. Calculate (*a*) the inductance of the coil, (*b*) the emf induced in the coil when a current of 5 A is switched off so that it uniformly reduces to zero in 1 ms, and (*c*) the mutual inductance between the coils if the second coil of 600 turns is uniformly wound over the first coil.

Solution

(*a*) A current of 3 A in the first coil produces a flux of 0.5 mWb. If the current is reduced to zero, the flux also reduces to zero. Therefore, the self-inductance of the first coil is given as

$$L_1 = N_1\frac{d\Phi}{di_1} = 900 \times \frac{(0.5-0)\times 10^{-3}}{(3-0)} = \mathbf{0.15\ H}$$

(*b*) The induced emf in the first coil is given as

$$e_1 = L_1\frac{di_1}{dt} = 0.15 \times \frac{(5-0)}{(1-0)\times 10^{-3}} = \mathbf{750\ V}$$

(*c*) Since the second coil is wound over the first coil on the ring, we can safely assume that the entire flux produced by the current in the first coil links with the second coil. We know that a current of 3 A produces a flux of 0.5 mWb, therefore the flux produced by a current of 5 A is

$$\Phi = \frac{0.5 \times 5}{3} = 0.83\ \text{mWb}$$

There are following two ways of obtaining the induced emf in the second coil:

$$e_2 = M\frac{di_1}{dt} \qquad \text{and} \qquad e_2 = N_2\frac{d\Phi}{dt}$$

Hence, the mutual inductance between the two coils is given by

$$M\frac{di_1}{dt} = N_2\frac{d\Phi}{dt} \quad \Rightarrow \quad M = N_2\frac{d\Phi}{dt}\cdot\frac{dt}{di_1} = N_2\frac{d\Phi}{di_1}$$

$$\therefore \qquad M = N_2\frac{d\Phi}{di_1} = 600 \times \frac{[(2.5/3)-0]\times 10^{-3}}{(5-0)} = \mathbf{0.1\ H}$$

PROBLEM B-11

The combined inductance of two coils connected in series is either 0.75 H or 0.25 H, depending on the relative directions of the currents in the two coils. If one of the coils, when isolated, has a self-inductance of 0.15 H, calculate (*a*) the mutual inductance, and (*b*) the coefficient of coupling between the coils.

Solution

(*a*) The equivalent inductance of the series aiding combination is given as

$$L_{sa} = L_1 + L_2 + 2M$$

or

$$0.75 = 0.15 + L_2 + 2M \tag{i}$$

And the equivalent inductance of the series opposing combination is given as

$$L_{so} = L_1 + L_2 - 2M$$

or

$$0.25 = 0.15 + L_2 - 2M \tag{ii}$$

Adding Eqs. (*i*) and (*ii*), we get

$$1.0 = 0.3 + 2L_2 \quad \Rightarrow \quad L_2 = 0.35 \text{ H}$$

Thus, from Eq. (*i*) we get

$$0.75 = 0.15 + 0.35 + 2M \quad \Rightarrow \quad M = \mathbf{0.125\ H}$$

(*b*) The coefficient of coupling between the coils,

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.15 \times 0.35}} = \mathbf{0.5455}$$

PROBLEM B-12

Two coils A and B, having 100 and 150 turns respectively, are wound side-by-side on a closed iron circuit of cross-section 125 cm^2 and mean length 2 m. Determine (*a*) the self-inductance of each coil, (*b*) the mutual inductance between the coils, and (*c*) the emf induced in coil B, when the current in coil A is changed from zero to 5 A in 0.02 s. Take the relative permeability of iron as 2000.

Solution

(*a*) The reluctance of the closed iron circuit is given as

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A} = \frac{2}{4\pi \times 10^{-7} \times 2000 \times 125 \times 10^{-4}} = 63.662 \text{ kAt/Wb}$$

The self-inductances of the two coils are

$$L_A = \frac{N_A^2}{\mathcal{R}} = \frac{(100)^2}{63.662 \times 10^3} = \mathbf{0.157\ H} \qquad \text{and} \qquad L_B = \frac{N_B^2}{\mathcal{R}} = \frac{(150)^2}{63.662 \times 10^3} = \mathbf{0.353\ H}$$

(*b*) The mutual inductance between the coils is

$$M = \frac{N_A N_B}{\mathcal{R}} = \frac{100 \times 150}{63.662 \times 10^3} = \mathbf{0.2356\ H}$$

(*c*) The emf induced in coil B is

$$e_B = M\frac{di_A}{dt} = 0.2356 \times \frac{5-0}{0.02} = \mathbf{58.9\ V}$$

PROBLEM B-13

A solenoid A consisting of 800 turns of wire wound on a former of length 90 cm and diameter 3 cm is placed co-axially within another solenoid B of same length and same number of turns but of diameter 6 cm. Find (*a*) the mutual inductance, and (*b*) the coupling coefficient of the arrangement.

Solution Given: $N_a = N_b = N = 800$; $l_a = l_b = l = 0.9$ m

(*a*) The mutual inductance of the arrangement is given by

$$M = \frac{N_a \Phi_a}{I_b} \qquad (i)$$

The flux density at the centre of the solenoid B due to a current I_b is given by

$$B = \frac{\mu_0 N_b I_b}{l_b} \qquad (ii)$$

Assuming uniform flux density throughout, the flux inside the solenoid A is given as

$$\Phi_a = BA_a = \frac{\mu_0 N_b I_b A_a}{l_b} \qquad (iii)$$

Substituting Eq. (*iii*) into Eq. (*i*), we get

$$M = \frac{\mu_0 N_a N_b I_b A_a}{I_b l_b} = \frac{\mu_0 N_a N_b A_a}{l_b} \qquad (iv)$$

Thus, the mutual inductance of the arrangement is given as

$$M = \frac{\mu_0 N_a N_b A_a}{l_b} = \frac{4\pi \times 10^{-7} \times (800)^2 \times [(\pi/4)(0.03)^2]}{0.9} = 0.632 \times 10^{-3} = \mathbf{0.632\ mH}$$

(*b*) The self-inductances of the two solenoids are given as

$$L_a = \frac{\mu_0 N_a^2 A_a}{l_a} \quad \text{and} \quad L_b = \frac{\mu_0 N_b^2 A_b}{l_b} \qquad (v)$$

Using Eqs (*iv*) and (*v*), the coefficient of coupling is given as

$$k = \frac{M}{\sqrt{L_a L_b}} = \frac{\mu_0 N_a N_b A_a}{l_b} \cdot \sqrt{\frac{l_a l_b}{\mu_0 N_a^2 A_a \mu_0 N_b^2 A_b}} = \frac{\mu_0 N^2 A_a}{l} \cdot \sqrt{\frac{l^2}{\mu_0^2 N^4 A_a A_b}}$$

$$= \frac{\mu_0 N^2 A_a}{l} \cdot \frac{l}{\mu_0 N^2 \sqrt{A_a A_b}} = \frac{A_a}{\sqrt{A_a A_b}} = \sqrt{\frac{A_a}{A_b}} = \sqrt{\frac{(\pi/4)(0.03)^2}{(\pi/4)(0.06)^2}} = \frac{0.03}{0.06} = \mathbf{0.5}$$

PROBLEM B-14

An iron ring, having mean diameter of 10 cm, cross-sectional area of 8 cm^2 and relative permeability of 500, is uniformly wound with 300 turns of wire. The iron ring is required to have a flux density of 1.2 Wb/m^2. (*a*) Determine the exciting current, the inductance and the stored energy. (*b*) If an air gap of 2 mm is cut in the ring, determine the above quantities.

Solution

(*a*) The ampere turns per metre of flux path length,

$$H = \frac{B}{\mu_0 \mu_r} = \frac{1.2}{4\pi \times 10^{-7} \times 500} = 1910 \text{ At/m}$$

The length of the flux path, $l_i = \pi D = \pi \times 0.1 = 0.3142$ m

Thus, total ampere turns required, $At = Hl_i = 1910 \times 0.3142 = 600$

Therefore, the exciting current, $I = \dfrac{At}{N} = \dfrac{600}{300} = \mathbf{2\ A}$

The inductance of the coil can be calculated as

$$L = \frac{N\Phi}{I} = \frac{NBA}{I} = \frac{300 \times 1.2 \times (8 \times 10^{-4})}{2} = 0.144 \text{ H} = \mathbf{144\ mH}$$

The stored energy in the magnetic field is

$$W = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.144 \times (2)^2 = \mathbf{0.288\ J}$$

(*b*) After an air gap of 2 mm is cut in the ring, there are two kinds of magnetic paths in series.

The iron path length, $l_i = 0.3142 - 0.002 = 0.3122$ m

The air gap path length, $l_g = 0.002$ m

For the same flux density of 1.2 Wb/m^2, as calculated above, the ampere turns per metre of iron path length required is $H_i = 1910$ At/m. Hence, the total ampere turns required by the iron path is

$$At_i = Hl_i = 1910 \times 0.3122 = 596.3$$

And the ampere turns required by the air gap is

$$At_g = H_g l_g = \frac{B}{\mu_0} l_g = \frac{1.2}{4\pi \times 10^{-7}} \times 0.002 = 1910$$

Therefore, the total ampere turns required,

$$At = At_i + At_g = 596.3 + 1910 = 2506.3$$

Hence, due to the presence of air gap, the exciting current is much higher and is given as

$$I = \frac{At}{N} = \frac{2506.3}{300} = \mathbf{8.35\ A}$$

The inductance, due to the presence of air gap, is much reduced and is given as

$$L = \frac{N\Phi}{I} = \frac{NBA}{I} = \frac{300 \times 1.2 \times (8 \times 10^{-4})}{8.35} = 0.03449 = \mathbf{34.49\ mH}$$

The stored energy,

$$W = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.03449 \times (8.35)^2 = \mathbf{1.202\ J}$$

Note that if the same flux density is maintained, the energy stored increases when an air gap is present in the magnetic path.

PROBLEM B-15

An iron ring of mean length 100 cm and circular cross-sectional area of 10 cm^2 has an air gap of 1 mm and a winding of 100 turns. Assuming the relative permeability of iron as 500, determine the inductance of the coil.

Solution For a magnetic circuit, the mmf $\mathcal{F}$ is equal to the product of the flux Φ produced and the reluctance $\mathcal{R}$ of the magnetic path. That is, $\mathcal{F} = \Phi\mathcal{R}$. Or,

$$NI = \Phi\mathcal{R} \quad \Rightarrow \quad \Phi = \frac{NI}{\mathcal{R}}$$

Therefore, the inductance of the winding is given by

$$L = \frac{N\Phi}{I} = \frac{N}{I}\cdot\frac{NI}{\mathcal{R}} = \frac{N^2}{\mathcal{R}}$$

So, let us first calculate the total reluctance of the given magnetic path,

$$\mathcal{R}_t = \mathcal{R}_i + \mathcal{R}_g = \frac{l_i}{\mu_0\mu_r A} + \frac{l_g}{\mu_0 A} = \frac{1}{\mu_0\mu_r A}(l_i + \mu_r l_g)$$

$$= \frac{1}{4\pi\times 10^{-7}\times 500\times 10\times 10^{-4}}((1-0.001) + 500\times 0.001) = 2.38\times 10^6 \text{ At/Wb}$$

$$\therefore \qquad L = \frac{N^2}{\mathcal{R}} = \frac{(100)^2}{2.38\times 10^6} = \mathbf{4.2\ mH}$$

PROBLEM B-16

A coil of 100 turns is wound on a toroidal core having a reluctance of 10^4 At/Wb. (*a*) When the coil current is 5 A and is increasing at the rate of 200 A/s, determine the energy stored in the magnetic field and the voltage applied across the coil, assuming coil resistance to be zero. (*b*) How are your answers affected if the coil resistance is 2 Ω?

Solution The inductance of the coil is given by

$$L = \frac{N^2}{\mathcal{R}} = \frac{(100)^2}{10^4} = \mathbf{1\ H}$$

(*a*) The energy stored is

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\times 1\times 5^2 = \mathbf{12.5\ J}$$

The voltage applied across the coil is same as the emf induced in the coil,

$$v = e = L\frac{di}{dt} = 1\times 200 = \mathbf{200\ V}$$

(*b*) The energy stored in the coil would remain the same. However, there would be an additional energy loss in the coil due to its resistance, $W_R = I^2R = 5^2\times 2 =$ **50 W**. Furthermore, the voltage across the coil would be increased by $V_R = IR = 5\times 2 = 10$ V. Therefore, the net voltage across the coil would be

$$v = e + V_R = 200 + 10 = \mathbf{210\ V}$$

PROBLEM B-17

A 0.5-m long single-layer solenoid has an effective diameter of 10 cm and is wound with 2500 turns. A small concentrated co-axial coil of diameter 12 cm is wound with 160 turns in the middle of the solenoid. Calculate the mutual inductance between the two coils.

Solution Let I_1 be the current flowing through the solenoid. Then the flux density produced inside the solenoid is

$$B = \mu_0 H = \frac{\mu_0 N_1 I_1}{l_1} = \frac{4\pi\times 10^{-7}\times 2500\, I_1}{0.5} = 6.283\times 10^{-3} I_1 \text{ Wb/m}^2$$

The area of cross-section of the solenoid,

$$A_1 = \frac{\pi}{4}\times(0.1)^2 = 7.85\times 10^{-3} \text{ m}^2$$

Assuming uniform magnetic flux, the total flux inside the solenoid is

$$\Phi_1 = BA_1 = 6.283 \times 10^{-3} I_1 \times 7.85 \times 10^{-3} = 49.32 \times 10^{-6} I_1 \text{ Wb}$$

Now, we know that the field strength outside the solenoid is negligible. Therefore, the flux linking the second coil is only Φ_1, irrespective of its area of cross-section. That is,

$$\Phi_2 = \Phi_1 = 49.32 \times 10^{-6} I_1 \text{ Wb}$$

Hence, the mutual inductance between the two coils is

$$M = \frac{N_2 \Phi_2}{I_1} = \frac{160 \times 49.32 \times 10^{-6} I_1}{I_1} = 7.891 \times 10^{-3} = \mathbf{7.891\ mH}$$

PROBLEM B-18

A horse-shoe electromagnet is made from a bar of wrought iron 1.2 m long and 80 cm^2 cross-section. The relative permeability of wrought iron is 800. It is required to lift a 1190-kg car. If the magnet has two series-connected coils of 1200 turns on each limb, calculate the exciting current needed. Assume that the car makes close contact with the magnet.

Solution The force required to lift the car, $F_T = 1190 \times 9.8 = 11662$ N

The force that has to be exerted by one pole of the electromagnet on the car is

$$F = \frac{11662}{2} = 5831 \text{ N}$$

The force exerted by a pole of the electromagnet is given as

$$F = \frac{B^2 A}{2\mu_0}$$

$$\therefore \quad B = \sqrt{\frac{F \times 2\mu_0}{A}} = \sqrt{\frac{5831 \times 2 \times 4\pi \times 10^{-7}}{80 \times 10^{-4}}} = 1.3535 \text{ Wb}$$

Therefore, the required field intensity is given as

$$H = \frac{B}{\mu_0 \mu_r} = \frac{1.3535}{4\pi \times 10^{-7} \times 800} = 1346.35 \text{ At/m}$$

The total ampere turns needed,

$$At = Hl = 1346.35 \times 1.2 = 1615.62$$

Since both the limbs have coils, the total number of turns, $N = 2 \times 1200 = 2400$

Finally, the exciting current required,

$$I = \frac{At}{N} = \frac{1615.62}{2400} = \mathbf{0.6732\ A}$$

PROBLEM B-19

The following particulars are taken from the magnetic circuit of a relay:

Mean length of the iron circuit	=	20 cm;	Length of air gap	=	2 mm;
Number of turns on the core	=	8000;	Current through the coil	=	50 mA;
Relative permeability of iron	=	500;	Area of the core	=	0.5 cm^2;

Neglecting the leakage, determine the flux density in air gap, and the pull exerted on the armature.

Solution The mmf, $\mathcal{F} = NI = 8000 \times 50 \times 10^{-3} = 400$ At

The total reluctance of magnetic path is given as

$$\mathcal{R} = \mathcal{R}_i + \mathcal{R}_g = \frac{l_i}{\mu_0 \mu_r A} + \frac{l_g}{\mu_0 A} = \frac{1}{\mu_0 \mu_r A}(l_i + \mu_r l_g)$$

$$= \frac{1}{4\pi \times 10^{-7} \times 500 \times 0.5 \times 10^{-4}}(20 \times 10^{-2} + 500 \times 2 \times 10^{-3}) = 3.82 \times 10^{7} \text{ At/Wb}$$

Therefore, the flux, $\Phi = \dfrac{\mathcal{F}}{\mathcal{R}} = \dfrac{400 \text{ At}}{3.82 \times 10^{7} \text{ At/wb}} = 1.047 \times 10^{-5}$ Wb

Hence, flux density, $B = \dfrac{\Phi}{A} = \dfrac{1.047 \times 10^{-5}}{0.5 \times 10^{-4}} = \mathbf{0.2094 \ Wb/m^2}$

The pull on the armature is given as

$$F = \frac{B^2 A}{2\mu_0} = \frac{0.2094 \times 0.5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = \mathbf{4.166 \ N}$$

PROBLEM B-20

The hysteresis loss of iron weighing 12 kg is equivalent to 300 J/m^3/cycle. Find the loss of energy/hour at 50 Hz. The density of iron is 7800 kg/m^3.

Solution The volume of 12 kg iron, $V = \dfrac{m}{d} = \dfrac{12}{7800} = 1.538 \times 10^{-3}$ m^3

∴ Hysteresis loss in iron per cycle = (300 J/m^3/cycle) × (1.538 × 10^{-3} m^3) = 0.4614 J/cycle

∴ Hysteresis loss per hour = (0.4614 J/cycle) × (50 cycles/s) (3600 s)

= 83052 J = **83.052 kJ**

PROBLEM B-21

The hysteresis loop of an iron ring was found to have an area of 10 cm^2 on a scale of 1 cm = 1000 At/m (x-axis) and 1 cm = 0.2 Wb/m^2 (y-axis). The ring has a mean length of 100 cm and a cross-sectional area of 5 cm^2. Compute the hysteresis loss in watts for a frequency of 50 Hz.

Solution Hysteresis loss per cycle,

$$W_{hc} = (10 \text{ cm}^2) \times (1000 \text{ At/m})/\text{cm} \times (0.2 \text{ Wb/m}^2)/\text{cm}$$
$$= 2000 \text{ At-Wb/m}^3 = 2000 \text{ J/m}^3$$

The volume of the ring, V = (1 m) × (5 × 10^{-4} m^2) = 5 × 10^{-4} m^3

Therefore, the hysteresis loss for the ring in watts,

$$W_h = (2000 \text{ J/m}^3/\text{cycle}) \times (50 \text{ cycles/s}) \times (5 \times 10^{-4} \text{ m}^3) = 50 \text{ J/s} = \mathbf{50 \ W}$$

PROBLEM B-22

The relay frame shown in Fig. B-3 is a typical series magnet circuit. If the exciting coil has 100 turns, find the current required to establish a flux of 0.3 mWb. From the *B-H* curve for sheet steel and cast iron, it is found that a flux density of 0.3 Wb/m^2 requires the following magnetising forces:

Sheet steel, H_{ss} = 200 At/m; Cast iron, H_{ci} = 1800 At/m.

Assuming that the relay is open with an air gap of 2 mm, find the exciting current to establish a flux of 0.3 mWb in the air gap.

Solution Here, we have $N = 100$. For air gap,

$$\Phi_g = 0.3 \times 10^{-3}\ \text{Wb};\quad A_g = (3 \times 2)\ \text{cm}^2 = 6 \times 10^{-4}\ \text{m}^2;\quad l_g = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$$

For sheet steel,

$$\Phi_{ss} = 0.3 \times 10^{-3}\ \text{Wb};\quad H_{ss} = 200\ \text{At/m};\quad l_{ss} = (10 + 10 + 9.8)\ \text{cm} = 0.298\ \text{m}$$

For cast iron,

$$\Phi_{ci} = 0.3 \times 10^{-3}\ \text{Wb};\quad H_{ci} = 1850\ \text{At/m};\quad l_{ci} = 10\ \text{cm} = 0.1\ \text{m}$$

The flux density in air gap is given as

$$B_g = \frac{\Phi_g}{A_g} = \frac{0.3 \times 10^{-3}}{6 \times 10^{-4}} = 0.5\ \text{Wb/m}^2$$

Thus, the total ampere turns (or mmf) required,

$$\mathcal{F} = \frac{B_g l_g}{\mu_0} + H_{ss} l_{ss} + H_{ci} l_{ci} = \frac{0.5 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} + 200 \times 0.298 + 1850 \times 0.1$$
$$= 795.8 + 59.6 + 185 = 1040.4\ \text{At}$$

Hence, the exciting current required,

$$I = \frac{\mathcal{F}}{N} = \frac{1040.4}{100} = \mathbf{10.4\ A}$$

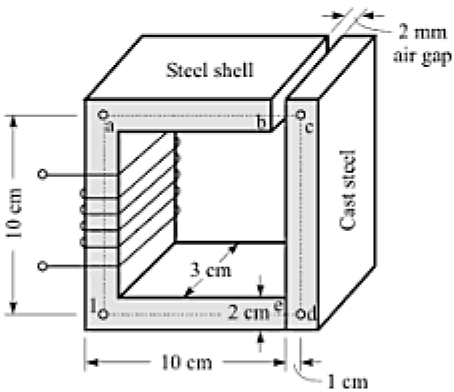


Fig. B-3

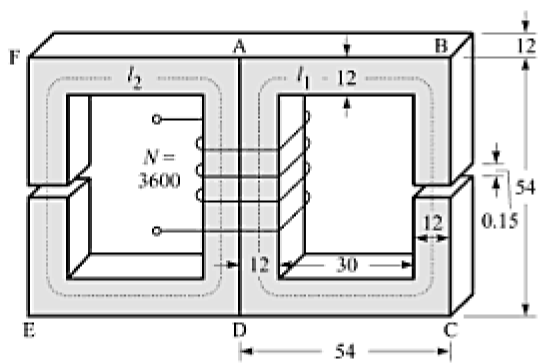


Fig. B-4

PROBLEM B-23

A magnetic circuit has two parallel branches, which are similar, as shown in Fig. B-4. All dimensions are given in cm. The relative permeability of steel at the operating point is 700. A coil of 3600 turns is wound on the central limb. Neglecting leakage and fringing, calculate the flux density in the air gaps, when the coil carries a current of 1.6 A.

Solution Let B be the flux density in the air gap. Since there is no leakage and fringing, the flux density in the side steel-limbs is also B. The area of cross-section of the side limb,

$$A_s = (12 \times 12)\ \text{cm}^2 = 144 \times 10^{-4}\ \text{m}^2$$

Therefore, the flux in the side limb,

$$\Phi_s = A_s B = 144 \times 10^{-4} B\ \text{Wb}$$

Since there are two similar side limbs in parallel, the flux in the central limb,

$$\Phi_c = 2\Phi_s = 288 \times 10^{-4} B\ \text{Wb}$$

The area of cross-section of the central limb,

$$A_c = (12 + 12) \times 12 \text{ cm}^2 = 288 \times 10^{-4} \text{ m}^2$$

Therefore, the flux density in the central limb,

$$B_c = \frac{\Phi_c}{A_c} = \frac{288 \times 10^{-4} B}{288 \times 10^{-4}} = B \text{ Wb/m}^2$$

Now, the mean length of one side steel-limb,

$$l_s = l_{ABCD} = (54 - 6 - 6) + (54 - 6 - 6 - 0.15) + (54 - 6 - 6) = 125.85 \text{ cm} = 1.2585 \text{ m}$$

The mean length of the central limb,

$$l_c = 54 - 6 - 6 = 42 \text{ cm} = 0.42 \text{ m}$$

Applying Kirchhoff's second law around the loop ABCD, the required mmf is

$$\mathcal{F} = \mathcal{F}_c + \mathcal{F}_s + \mathcal{F}_g$$

or

$$\mathcal{F} = H_c l_c + H_s l_s + H_g l_g = \frac{B}{\mu_0 \mu_r} l_c + \frac{B}{\mu_0 \mu_r} l_s + \frac{B}{\mu_0} l_g = \frac{B}{\mu_0 \mu_r}(l_c + l_s + \mu_r l_g)$$

$$= \frac{B}{4\pi \times 10^{-7} \times 700} \times (0.42 + 1.2585 + 700 \times 0.15 \times 10^{-2})$$

$$= \frac{B}{4\pi \times 10^{-7} \times 700} \times (2.7285) \text{ At}$$

But, the mmf provided by the exciting coil is given as

$$\mathcal{F} = NI = 3600 \times 1.6 = 5760 \text{ At}$$

$$\therefore \quad \frac{B}{4\pi \times 10^{-7} \times 700} \times (2.7285) = 5760$$

or

$$B = \frac{5760 \times 4\pi \times 10^{-7} \times 700}{2.7285} = \mathbf{1.857 \text{ Wb/m}^2}$$

PROBLEM B-24

A horse-shoe type relay needs an excitation of 2000 ampere turns to raise the armature, when the air gap is 1.5 mm. The length of the iron path is 50 cm and the area of each pole shoe is 3 cm^2. The relative permeability of iron is 300. Find (*a*) the pull on the armature, and (*b*) the force needed to pull the armature away, when the air gap closes to 0.2 mm, the excitation remaining the same. Neglect fringing and flux leakage.

Solution

(*a*) Let B_g be the flux density in the air gap (as well as in the iron path). Then, the mmf provided by the exciting coil = mmf required by iron path + mmf required by air gap

or

$$\mathcal{F} = \mathcal{F}_i + \mathcal{F}_g$$

or

$$2000 \text{ At} = H_i l_i + H_g l_g = \frac{B_g}{\mu_0 \mu_r} l_i + \frac{B_g}{\mu_0} l_g = \frac{B_g}{\mu_0 \mu_r}(l_i + \mu_r l_g)$$

$$= \frac{B_g}{4\pi \times 10^{-7} \times 300}(0.5 + 300 \times 1.5 \times 10^{-3}) = 2520 B_g \text{ At}$$

$$\therefore \quad B_g = \frac{2000}{2520} = 0.794 \text{ Wb/m}^2$$

Hence, the pull on the armature by each pole, $F = \dfrac{B_g^2 A}{2\mu_0} = \dfrac{(0.794)^2 \times 3 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 75.2 \text{ N}$

The total pull on the armature by both poles, $F_t = 2F = 2 \times 75.2 = \mathbf{150.4 \text{ N}}$

(*b*) With l_g = 0.2 mm, we have

$$2000\text{ At} = \frac{B_g}{\mu_0\mu_r}(l_i + \mu_r l_g)$$

$$= \frac{B_g}{4\pi \times 10^{-7} \times 300}(0.5 + 300 \times 0.2 \times 10^{-3}) = 1484.2 B_g\text{ At}$$

$$\therefore \qquad B_g = \frac{2000}{1484.2} = 1.3475\text{ Wb/m}^2$$

The force needed at each pole to pull the armature away is given as

$$F = \frac{B_g^2 A}{2\mu_0} = \frac{(1.3475)^2 \times 3 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 216.6\text{ N}$$

Due to the two poles, the total force needed $F_t = 2F = 2 \times 216.6 = \mathbf{433.2\ N}$

PROBLEM B-25

The magnetic circuit shown in Fig. B-5 is made of iron having a square cross-section of 3 cm side. It has two parts A and B, with relative permeabilities of 1000 and 1200 respectively, separated by two air gaps, each 2 mm wide. The part B is wound with a total of 1000 turns of wire on the two side limbs and carries a current of 1 A. Calculate (*a*) the reluctance of part A, (*b*) the reluctance of part B, (*c*) the reluctance of the air gaps, (*d*) the total reluctance of the complete magnetic circuit, (*e*) the mmf, (*f*) the total flux, and (*g*) the flux density.

Solution

(*a*) The reluctance of part A,

$$\mathcal{R}_a = \frac{l_a}{\mu_a \mu_{ra} A} = \frac{17 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 3^2 \times 10^{-4}} = \mathbf{0.1503 \times 10^6\ At/Wb}$$

(*b*) The reluctance of part B,

$$\mathcal{R}_b = \frac{l_b}{\mu_0 \mu_{rb} A} = \frac{(17 + 8.5 + 8.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1200 \times 3^2 \times 10^{-4}} = \mathbf{0.2505 \times 10^6\ At/Wb}$$

(*c*) The reluctance of the air gaps,

$$\mathcal{R}_g = 2 \times \frac{l_g}{\mu_0 A} = 2 \times \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 3^2 \times 10^{-4}} = \mathbf{3.537 \times 10^6\ At/Wb}$$

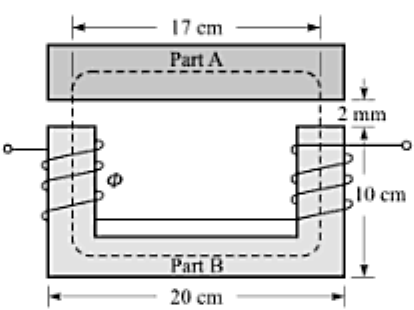


Fig. B-5

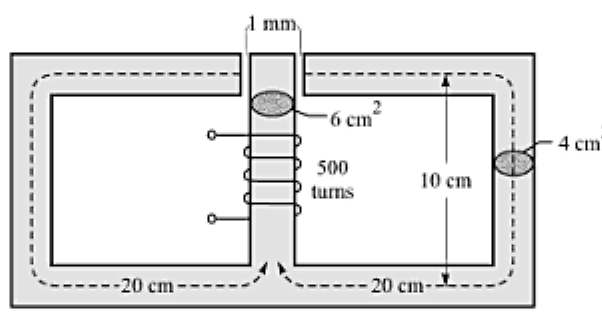


Fig. B-6

(*d*) The total reluctance of the complete magnetic circuit,

$$\mathcal{R}_t = \mathcal{R}_a + \mathcal{R}_b + \mathcal{R}_g = (0.1503 + 0.2505 + 3.537) \times 10^6 = \mathbf{3.938 \times 10^6\ At/Wb}$$

(*e*) The mmf, $\mathcal{F} = NI = 1000 \times 1 = \mathbf{1000\ At}$

(*f*) The total flux, $\Phi = \dfrac{\text{mmf}}{\text{reluctance}} = \dfrac{\mathcal{F}}{\mathcal{R}} = \dfrac{1000}{3.938 \times 10^6} = 0.2539 \times 10^{-3} = \mathbf{0.2539\ mWb}$

(*g*) The flux density, $B = \dfrac{\Phi}{A} = \dfrac{0.2539 \times 10^{-3}}{(3 \times 3) \times 10^{-4}} = \mathbf{0.2821\ Wb/m^2}$

PROBLEM B-26

A magnetic circuit made of wrought iron is shown in Fig. B-6. It has two air gaps, each 1 mm wide. The cross-sectional area of the central limb is 6 cm^2 and of each outer limb is 4 cm^2. If the coil wound on the central limb has 500 turns, calculate the exciting current required to produce a flux of 0.9 mWb in it. The magnetising curve for wrought iron gives

B(Wb/m^2)	1.125	1.5
H(At/m)	500	2000

Solution *For the central limb*:

$$\text{The flux density, } B_1 = \frac{\Phi_1}{A_1} = \frac{0.9 \times 10^{-3}}{6 \times 10^{-4}} = 1.5 \text{ Wb/m}^2$$

From the given table, for this flux density of 1.5 Wb/m^2, the corresponding magnetic field strength H = 2000 At/m. Therefore, the ampere turns required for the central limb,

$$At_1 = H_1 l_1 = 2000 \times 0.1 = 200 \text{ At}$$

For outer limb: Since the two outer limbs have the same dimensions, the flux in the central limb is equally divided in the two. Thus, the flux in the outer limb,

$$\Phi_2 = \frac{1}{2}\Phi_1 = \frac{1}{2} \times 0.9 = 0.45 \text{ mWb}$$

$$\text{The flux density,} \quad B_2 = \frac{\Phi_2}{A_2} = \frac{0.45 \times 10^{-3}}{4 \times 10^{-4}} = 1.125 \text{ Wb/m}^2$$

From the given table, for this flux density of 1.125 Wb/m^2, the corresponding magnetic field strength H = 500 At/m. Therefore, the ampere turns required for the outer limb,

$$At_2 = H_2 l_2 = 500 \times 0.2 = 100 \text{ At}$$

For the air gap: $B_g = B_2 = 1.125$ Wb/m^2; $l_g = 1 \times 10^{-3}$ m. The magnetic field strength,

$$H_g = \frac{B_g}{\mu_0} = \frac{1.125}{4\pi \times 10^{-7}} = 8.952 \times 10^5 \text{ At/m}$$

Therefore, the ampere turns required for the air gap,

$$At_g = H_g l_g = 8.952 \times 10^5 \times 10^{-3} = 895.2 \text{ At}$$

Hence, the total ampere turns required $= 200 + 100 + 895.2 = 1195.2$ At

$$\therefore \quad \text{Exciting current, } I = \frac{At}{N} = \frac{1195.2}{500} = \mathbf{2.39\ A}$$

B. 2. PRACTICE PROBLEMS

(A) SIMPLE PROBLEMS

B-1. A conductor 50 cm long is moved at right angles to a uniform magnetic field of flux density 0.12 Wb/m^2 at a speed of 20 cm/s. Determine the value of emf induced in the conductor. [**Ans.** 12 mV]

B-2. A 100-turns square coil with 10 cm side is rotated at a uniform speed of 1000 rpm about an axis at right angles to a uniform magnetic field of flux density 0.5 T. Find the instantaneous value of the induced emf when the plane of the coil is (*a*) at right angles to the field, (*b*) at 30° to the field, and (*c*) in the plane of the field.

[**Ans.** (*a*) 0 V; (*b*) 45.4 V; (*c*) 52.4 V]

B-3. A transmission line carries a current of 100 A. What would be the magnetic field *B* at a point on the road if the line is 8 m above the road? [**Ans.** 2.5 µT]

B-4. A 1-m long conducting rod rotates with an angular frequency of 400 s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring. [**Ans.** 100 V]

B-5. A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 100 revolutions per minute. If the length of each spoke is 30 cm and the horizontal component of the earth's magnetic field is 2.0×10^{-5} T, find the emf induced between the axis and the outer end of a spoke. Neglect the centripetal force acting on the free electrons of the spoke. [**Ans.** 9.4 µV]

B-6. The coil of an electromagnet has 50 turns and carries a current of 1 A. If the length of the magnetic circuit is 20 cm, find the mmf and the magnetic field strength. [**Ans.** 50 At, 250 At/m]

B-7. A magnetic flux of 900 µWb passing through a coil of 500 turns is reversed in 0.2 s. Find the average value of emf induced in the coil. [**Ans.** 4.5 V]

B-8. A wire of length 80 cm moves at right angles to its length with a speed of 30 m/s in a uniform magnetic field of flux density 1.2 Wb/m^2. Determine the emf induced in the conductor when the direction of motion is (*a*) perpendicular to the field, and (*b*) inclined at 45° to the direction of field.

[**Ans.** (*a*) 28.8 V; (*b*) 20.4 V]

B-9. A coil consisting of 480 turns is placed in a magnetic field of 0.8 mWb. If the coil is moved from this field to another field of 0.3 mWb in 0.08 s, what is the average emf induced in it? If the resistance of the coil is 100 Ω, find the induced current in the coil. [**Ans.** 3 V, 30 mA]

B-10. A square coil of 10 cm side having 100 turns is rotated at 1000 rpm about its axis at 90° to the direction of the magnetic field of flux density 1 Wb/m^2. Calculate the instantaneous value of emf, when the plane of the coil is at (*a*) 30°, and (*b*) 45° to the direction of the flux.

[**Ans.** (*a*) 90.67 V; (*b*) 74 V]

B-11. Calculate the inductance of a toroidal coil of 100 turns wound uniformly on a non-magnetic core ring of mean diameter 140 mm and the cross-sectional area of 750 mm^2. [**Ans.** 21.471 µH]

B-12. A solenoid 80 cm in length and 8 cm in diameter has 4000 turns uniformly wound over it. Calculate (*a*) its inductance, and (*b*) the energy stored in the magnetic field when 2-A current flows through the solenoid. [**Ans.** (*a*) 0.126 H; (*b*) 0.252 J]

B-13. The field winding of a dc electromagnet is wound with 960 turns and has a resistance of 50 Ω. When the exciting voltage is 230 V, the magnetic flux linking the coil is 5 mWb. Calculate the self-inductance of the coil and the energy stored in the magnetic field. [**Ans.** 1.043 H, 11.04 J]

B-14. An iron ring of mean diameter 30 cm having a square cross-section of 2 cm × 2 cm is wound with 400 turns of wire of 2 mm^2 cross-section. Calculate the self-inductance of the coil. Assume $\mu_r = 800$.

[**Ans.** 68.3 mH]

B-15. If a coil of 150-turns carrying a current of 10 A produces a flux of 0.01 Wb. Calculate the inductance of this coil. If this current is steadily reversed in 0.01 second, calculate the emf induced in it.

[**Ans.** 0.15 H, 300 V]

B-16. A coil consists of 750 turns. A current of 10 A in it gives rise to a magnetic flux of 1200 μWb. Calculate the self inductance, the stored energy, and the emf induced if the current is reversed in 0.01 s. [**Ans.** 0.09 H, 4.5 J, 180 V]

B-17. The reluctance of a magnetic circuit is known to be 10^5 At/Wb and its excitation coil has 200 turns. The current in the coil is changing at 200 A/s. Calculate (*a*) the inductance of the coil, (*b*) the voltage induced in the coil, and (*c*) the energy stored in the coil when the instantaneous current at $t = 1$ s is 1 A. Neglect the resistance of the coil.

[**Ans.** (*a*) 0.4 H; (*b*) 80 V; (*c*) 0.2 J]

B-18. Two coils A and B, each having 1200 turns, are placed near each other. With coil B open-circuited, when a current of 5 A flows through the coil A, a flux of 0.2 Wb is produced. Only 30 % of this flux links with coil B. Determine the emf induced in coil B when the current in coil A changes at the rate of 2 A/s. [**Ans.** 28.8 V]

B-19. Two coils are wound side by side on a paper-tube former. An emf of 0.25 V is induced in coil A when the flux linking it changes at the rate of 1 mWb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. Determine the mutual inductance between the coils. [**Ans.** 1.25 mH]

B-20. Two coils A and B have self-inductances of 10 μH and 40 μH, respectively. A current of 2 A in coil A produces a flux linkage of 5 μWb in coil B. Calculate (*a*) the mutual inductance between the coils, (*b*) the coefficient of magnetic coupling, (*c*) the average emf induced in coil B, if the current of 1 A in coil A is reversed at a uniform rate in 0.1 second. [**Ans.** (*a*) 2.5 μH; (*b*) 0.125; (*c*) 50 μV]

B-21. Two identical coils A and B, each having 1000 turns, lie in parallel planes such that 50 % of the flux produced by one links the other. A current of 5 A in coil A produces in it a flux of 50 μWb. Calculate the self-inductance, mutual inductance, and the coefficient of coupling. If the current in coil A changes form 6 A to −6 A in 0.01 s, what will be the magnitude of emf induced in coil B ?

[**Ans.** 0.01 H, 0.005 H, 6.0 V]

B-22. Two 200-turn, air-cored solenoids, 25 cm long have a cross-sectional area of 3 cm^2 each. The mutual inductance between them is 0.5 μH. Find (*a*) the self-inductance, (*b*) the coefficient of coupling.

[**Ans.** (*a*) 60.32 μH; (*b*) 0.0083]

B-23. A solenoid of length 1 m and diameter 10 cm has 5000 turns. Calculate (*a*) the approximate inductance, and (*b*) the energy stored in the magnetic field, when a current of 2 A flows in the solenoid. [**Ans.** (*a*) 0.25 H; (*b*) 0.5 J]

B-24. Two coils A and B, each with 100 turns, are mounted so that a part of the flux produced by one links the other. When the current in coil A is changed from 2 A to −2 A in 0.5 second, an emf of 8 mV is induced in coil B. Calculate (*a*) the mutual inductance between the coils, and (*b*) the flux produced in coil B due to 2-A current in coil A.

[**Ans.** (*a*) 1 mH; (*b*) 20 μWb]

B-25. Two coils A and B are wound side by side on a paper-tube former. An emf of 0.25 V is induced in coil A when the flux linking it changes at the rate of 1 mWb/s. A current of 2 A in coil B causes a flux of 10 μWb to link coil A. Calculate the mutual inductance between the two coils.

[**Ans.** 1.25 mH]

B-26. Two coils A and B, each with 1200 turns, are placed near each other. When coil A is open-circuited and coil B carries a current of 50 A, the flux produced by coil A is 0.2 Wb and 30 % of this flux links with all the turns of coil B. Calculate the induced emf in coil B on open circuit, when the current in coil A is changing at a rate 1 A/s. [**Ans.** 1.44 V]

B-27. Two coils A and B have self inductances of 120 μH and 300 μH, respectively. A current of 1 A in coil A produces a total flux linkage of 100 μWb-turns in coil B. Calculate (*a*) the mutual inductance between the coils, (*b*) the coupling coefficient, and (*c*) the average emf induced in coil B if a current of 1 A in coil A is reversed at a uniform rate in 0.1 s.

[**Ans.** (*a*) 100 μH; (*b*) 0.527; (*c*) 2 mV]

B-28. Determine the force in kilograms required to separate two surfaces with 100 cm^2 contact area when the flux density normal to the surface is 1.0 Wb/m^2. [**Ans.** 406 kg]

B-29. An iron ring 15 cm in diameter and 10 cm^2 in cross-section is wound with 200 turns of wire. For a flux density of 1 Wb/m^2 and a permeability of 500, find the exciting current. [**Ans.** 3.75 A]

B-30. An electromagnet has an air gap of 3 mm. It is required to establish a flux density of 1.257 Wb/m^2 in the air gap. Calculate the ampere turns required.

[**Ans.** 3001 At]

(B) TRICKY PROBLEMS

B-31. A conducting circular loop having a radius of 5.0 cm, is placed perpendicular to a magnetic field of 0.5 T. It is removed from the field in 0.5 s. Find the average emf produced in the loop during this time.

[**Ans.** 7.8 mV]

B-32. A conducting square loop of side 5.0 cm is placed in a magnetic field which varies sinusoidally with time, as $B = B_0 \sin \omega t$, where $B_0 = 0.40$ T and $\omega = 300\ \text{s}^{-1}$. The normal to the coil makes an angle of 60° with the field. Find (*a*) the maximum emf induced in the coil, (*b*) the emf induced at $t = (\pi/900)$ s, and (*c*) the emf induced at $t = (\pi/600)$ s.

[**Ans.** (*a*) 0.15 V; (*b*) 0.075 V; (*c*) zero]

B-33. A 20-cm long conducting rod is set into pure translational motion with uniform velocity of 10 cm/s perpendicular to its length. A uniform magnetic field of magnitude 0.10 T exists in a direction perpendicular to the plane of motion. (*a*) Find the average magnetic force on the free electrons of the rod. (*b*) For what electric field inside the rod, the electric force on a free electron will balance the magnetic force? (*c*) Find the motional emf induced in the rod across its ends.

[**Ans.** (*a*) 1.6×10^{-21} N; (*b*) 0.01 V/m; (*c*) 2 mV]

B-34. The current through a 0.2 H inductor is shown in Fig. B-7. Assuming the passive sign convention, find v_L at t equal to (*a*) 0, (*b*) 2 ms, and (*c*) 6 ms.

[**Ans.** (*a*) 0.4 V, (*b*) 0.2 V, (*c*) –0.267 V]

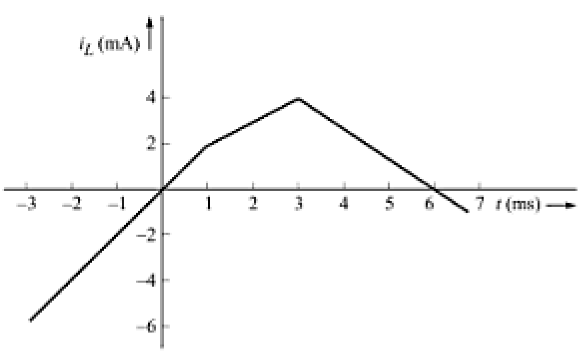


Fig. B-7

B-35. Two identical coils A and B consisting of 1500 turns each lie in parallel planes. A current of 4A flowing in the coil A produces a flux of 0.04 mWb in it, and 70% of this flux links with coil B. (*a*) Calculate the self-inductance of each coil, and the mutual inductance between them. (*b*) If the current in coil A changes from 4 A to –4 A in 20 ms, what will be the emf induced in coil B?

[**Ans.** (*a*) 15 mH, 10.5 mH; (*b*) 4.2 V]

B-36. With reference to Fig. B-8, (*a*) find the value of time at which the inductor is absorbing the maximum power, (*b*) find the value at which it is supplying maximum power, and (*c*) find the energy stored in the inductor at $t = 40$ ms.

[**Ans.** (*a*) 40^- ms; (*b*) 20^+ ms and 40^+ ms; (*c*) 2.5 J]

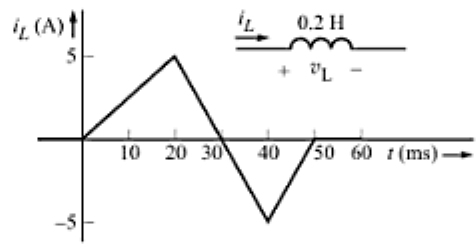


Fig. B-8

B-37. (*a*) If $i_s = 0.4t^2$ A for $t > 0$ in the circuit Fig. B-9*a*, find $v_{in}(t)$ for $t > 0$. (*b*) If $v_s = 40t$ V for $t > 0$ in the circuit of Fig. B-9*b* and $i_L(0) = 5$ A, find $i_{in}(t)$ for $t > 0$. [**Ans.** (*a*) $4t^2 + 4t$ V; (*b*) $4t^2 + 4t + 5$ A]

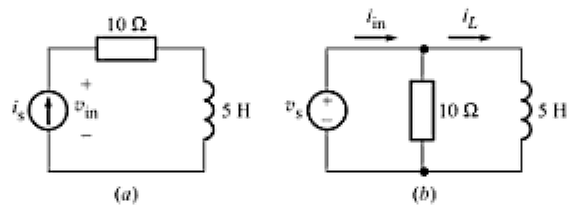


Fig. B-9

B-38. A nonmagnetic ring having a mean diameter of 30 cm and a cross-sectional area of 4 cm² is uniformly wound with two coils A and B one over the other. Coil A has 90 turns and coil B has 240 turns. Calculate the mutual inductance between the two coils. Also, calculate the emfs induced in coil

A and coil B, when a current of 6 A in coil A is reversed in 0.02 s.

[**Ans.** 11.52 μH, 2.592 mV, 6.912 mV]

B-39. An iron rod, 2 cm in diameter and 20 cm long, is bent into a closed ring. It is uniformly wound with 3000 turns of wire. It is observed that when a current of 0.5 A is passed through this coil, a flux of density 0.5 Wb/m^2 is produced. Assuming that all the flux is linked with every turn of the coil, determine (*a*) the *B*/*H* ratio for iron, and (*b*) the inductance of the coil. What voltage would be developed in the coil if the current through the coil is interrupted and the flux in the iron falls to 10 % of its previous value in 0.001 second?

[**Ans.** (*a*) 6.67×10^{-5} H/m; (*b*) 0.94 H; 424 V]

B-40. A coil has 1000 turns enclosing a magnetic circuit of 20 cm^2 cross-section. With 4 A current, a flux density of 1.0 Wb/m^2 is produced and with 9 A current, it is 1.4 Wb/m^2. Find the mean value of the inductance between these current-limits, and the induced emf, if the current falls from 9 A to 4 A in 0.05 seconds. [**Ans.** 0.16 H, 16 V]

B-41. Two coils having 30 and 600 turns, respectively, are wound side by side on a closed iron circuit of area of cross-section of 100 cm^2 and mean length of 200 cm. If the relative permeability of iron is 2000, calculate the mutual inductance between the two coils. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the emf induced in the second coil.

[**Ans.** 0.226 H, 22.6 V]

B-42. Two identical 750-turns coils *A* and *B* lie in parallel planes. A current changing at the rate of 1500 A/s in coil *A* induces an emf of 11.25 V in coil *B*. Determine the mutual inductance of the arrangement. If each coil has a self-inductance of 15 mH, find the flux produced on coil *A* per ampere and the percentage of this flux which links the coil *B*.

[**Ans.** 7.5 mH, 20 μWb/A, 50 %]

B-43. Two coils *A* and *B*, having 12500 and 16000 turns, respectively, lie in parallel planes such that 60 % of the flux produced in coil *A* links coil *B*. It is found that a current of 5 A in coil *A* produces 0.6 mWb flux, while the same current produces in coil *B*, 0.8 mWb flux. Determine (*a*) the mutual inductance, and (*b*) the coefficient of coupling between the two coils. [**Ans.** (*a*) 1.15 H, (*b*) 0.586]

B-44. Two coils, having 30 and 600 turns, respectively, are wound side by side on a closed iron circuit of cross-sectional area 100 cm^2, and mean length of 150 cm. A current in the first coil grows steadily from zero to 10 A in 0.01 s. Find the emf induced in the other coil. The permeability of iron is 2000.

[**Ans.** 301 V]

B-45. A coil of 300 turns and of resistance 10 Ω is wound uniformly over a steel ring of mean circumference 30 cm and cross-sectional area 9 cm^2. It is connected to a supply of 20 V(dc). If the relative permeability of steel is 1500, find (*a*) the magnetising force, (*b*) the reluctance, (*c*) the mmf, and (*d*) the flux.

[**Ans.** (*a*) 2000 At/m; (*b*) 1.7684×10^5 At/Wb; (*c*) 600 At; (*d*) 3.393 mWb]

B-46. An iron ring of 20 cm diameter having cross-sectional area of 100 cm^2 is wound with 400 turns of wire. (*a*) Calculate the exciting current required to establish a flux density of 1 Wb/m^2. (*b*) If the relative permeability of iron is 1000, how much energy is stored? [**Ans.** (*a*) 1.25 A; (*b*) 2500 J]

B-47. Two tightly-coupled (k = 1) coils have a mutual inductance of 32 mH. What is the average emf induced in one, if the current through the other changes from 3 mA to 15 mA in 4 ms? Given that one coil is twice as long as other and it has twice the number of turns as in the other, calculate the inductance of each coil. Neglect leakage.

[**Ans.** 96 mV, 45.25 mH, 22.63 mH]

B-48. The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the two coils. If one of the coils when isolated has a self inductance of 0.2 H, determine (*a*) the mutual inductance, and (*b*) the coefficient of coupling between them.

[**Ans.** (*a*) 0.125 H; (*b*) 0.72]

B-49. Two similar coils have a coupling coefficient of 0.25. When they are connected cumulatively in series, the total inductance is 80 mH. Calculate the self inductance of each coil. Also, calculate the total inductance when the coils are differentially connected in series. [**Ans.** 32 mH, 48 mH]

B-50. An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 Wb/m^2 if the permeability of iron is 1000. What is the value of energy stored?

[**Ans.** 1.25 A, 2.5 J]

B-51. The pole face area of an electromagnet is 0.5 m^2/pole. It has to lift an iron ingot weighing 1000 kg. If the pole faces are parallel to the surface of the ingot at a distance of 1 mm, determine the coil mmf required. Assume permeability of iron to be infinity.

[**Ans.** 250 At]

B-52. Find the pull in kilograms exerted on the plunger of an electromagnet when the total flux uniformly distributed is 0.8 mWb. The diameter of the plunger is 5.08 cm. [**Ans.** 12.8 kg]

B-53. A cast-steel electromagnet has an air gap 3 mm wide and an iron path 40 cm long. Find the number of ampere turns needed to produce a flux density of 0.7 Wb/m^2 in the air gap. It is known that the *B-H* curve for cast steel gives H = 660 At/m corresponding to B = 0.7 Wb/m^2. Neglect leakage and fringing. [**Ans.** 1935 At]

B-54. An iron ring consists of three parts, as shown in Fig. B-10. It has

$$l_1 = 10 \text{ cm}, A_1 = 5 \text{ cm}^2;$$
$$l_2 = 8 \text{ cm}, A_2 = 3 \text{ cm}^2;$$
$$l_3 = 6 \text{ cm}, A_3 = 2.5 \text{ cm}^2;$$

It is wound with a coil of 250 turns. Calculate the current required to produce a flux of 0.4 mWb/m^2 in the ring. Given the relative permeabilities of the three parts as

$$\mu_1 = 2670, \mu_2 = 1050, \text{ and } \mu_3 = 650.$$

[**Ans.** 888.9 mA]

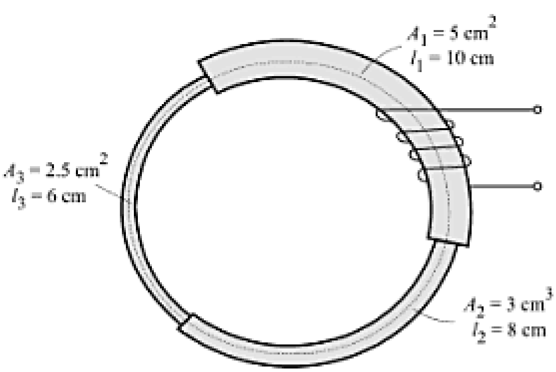


Fig. B-10

B-55. A cast steel ring of mean diameter 21 cm is fitted with a cast steel bar, as shown in Fig. B-11. The magnetising curve for cast steel is given by the table,

B(Wb/m^2)	1	1.1	1.18	1.25	1.33
H(At/m)	900	1050	1200	1450	1650

Calculate the current required in the magnetising coil, having 100 turns, to produce a flux of 1.25 mWb in the cast steel bar. [**Ans.** 6.014 A]

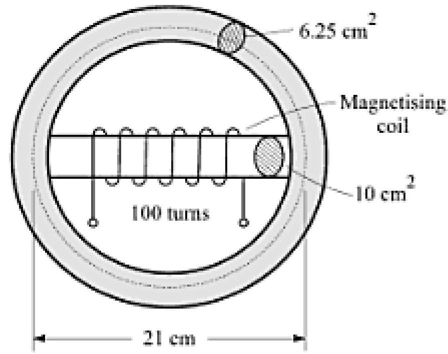


Fig. B-11

B-56. An iron ring of mean length 100 cm with an air gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. Determine the flux density produced when a current of 3 A flows in the winding. Neglect fringing.

[**Ans.** 0.5145 Wb/m^2]

B-57. An iron ring has 100-cm mean diameter, 10-cm^2 cross-section, and 2-mm wide air gap. The relative permeability of the material is 1500. There are 1000 turns of copper wire uniformly wound on it. If it is desired to produce a flux density of 1 Wb/m^2 in the air gap, find (*a*) the reluctance of ring, (*b*) the flux required, (*c*) the mmf required, (*d*) the current in the winding. Neglect leakage and fringing.

[**Ans.** (*a*) 3.257×10^6 At/Wb; (*b*) 1 mWb; (*c*) 3257 At; (*d*) 3.257 A]

B-58. A circular iron ring having a cross-sectional area of 10 cm^2 and a length of 4π cm in iron, has an air gap of 0.4π mm made by a saw cut. The relative permeability of iron is 1000. The ring is wound with a coil of 2000 turns and carries 2-mA current. Determine the air gap flux, neglecting leakage and fringing. [**Ans.** 3.636 μWb]

B-59. An iron ring of mean circumference 1.0 m is uniformly wound with 400 turns of wire. When a current of 1.2 A is passed through the coil, a flux density of 1.15 Wb/m^2 is produced in the iron. Find the relative permeability of iron under these circumstance. **[Ans.** 1907]

B-60. An iron ring of mean length 60 cm has an air gap of 2 mm. It is wound with 300 turns of wire. The relative permeability of iron is 300. Find the flux density when a current of 0.7 A flows through the coil. **[Ans.** 66.1 mWb/m^2]

(C) CHALLENGING PROBLEMS

B-61. A flux density of 0.9 Wb/m^2 is produced by 3500 ampere-turns per metre in a steel ring of diameter 40 cm and cross-sectional area 15 cm^2. The coil on the ring has 440 turns. (*a*) Calculate the exciting current and the self-inductance of the coil. (*b*) If an air gap of 1 cm is cut in the ring for the same flux density, calculate the exciting current needed, and the new value of the self-inductance of the coil.

[Ans. (*a*) 10 A, 59.4 mH; (*b*) 26.2 A, 22.67 mH]

B-62. Two coils A and B, consisting of 50 and 500 turns, respectively, are wound side-by-side on a closed iron circuit of cross-sectional area 80 cm^2 and mean length 120 cm. Calculate (*a*) the mutual inductance between the coils, and (*b*) the emf induced in coil B, if the current in coil A grows steadily from zero to 12 A in 15 ms.

[Ans. (*a*) 0.419 H; (*b*) 335.2 V]

B-63. A dc current of 1 A flowing through a coil of 5000 turns produces a flux of 0.1 mWb. Assuming that whole of this flux links with every turn of the coil; calculate the inductance of the coil. What voltage would be developed across the coil, if the current were interrupted 1 millisecond? What would be the maximum voltage developed across the coil if a 10-μF capacitor were connected across the switch breaking the dc supply?

[Ans. 0.5 H, 500 V, 224 V]

B-64. A flux of 1.5 mWb is produced by a coil of 1100 turns wound on a ring when a current of 3 A flows through it. Calculate (*a*) the inductance of the coil, (*b*) the emf induced in the coil when a current of 10 A is switched off so that it uniformly decreases to zero in 1 ms, and (*c*) the mutual inductance between the coils, if the second coil of 800 turns is uniformly wound over the first coil.

[Ans. (*a*) 0.55 H; (*b*) 5500 V; (*c*) 0.4 H]

B-65. Two coils with a coefficient of coupling of 0.5 are connected in series so as to magnetise (*i*) in the same direction, (*ii*) in the opposite direction. The corresponding values of total inductances in the two cases are 1.9 H and 0.7 H, respectively. Find the mutual inductance between them and the self-inductances of the two coils.

[Ans. 0.3 H, 0.9 H. 0.4 H]

B-66. The total inductance of two coils A and B, when connected in series is 0.5 H or 0.2 H, depending on the relative directions of the current in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate (*a*) the mutual inductance between the two coils, (*b*) the self-inductance of the coil B, (*c*) the coupling coefficient between the two coils, and (*d*) the two possible values of the emf induced in coil A when the current in the series combination decreases at 1000 A/s.

[Ans. (*a*) 0.075 H; (*b*) 0.15 H; (*c*) 0.433; (*d*) 275 V, 125 V]

B-67. A horse-shoe electromagnet is formed out of a bar of wrought iron 45.7 cm long and 6.45 cm^2 in cross-section. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes a close contact with the magnet. The relative permeability of iron is 700. **[Ans.** 0.592 A]

B-68. A horse-shoe type relay requires an excitation of 1800 ampere turns to raise the armature, when the air gap is 1.25 mm. The length of the iron path is 40 cm and the area of each pole shoe is 2 cm^2. The relative permeability of iron is 730. Find (*a*) the pull on the armature, and (*b*) the force needed to pull the armature away, when the air gap closes to 0.1 mm, the excitation remaining the same. Neglect fringing and flux leakage.

[Ans. (*a*) 251.9 N; (*b*) 1940 N]

B-69. An iron ring of 25 cm mean diameter and of circular section 3 cm in diameter has an air gap 1.5 mm wide. It is wound with 700 turns of wire carrying a current of 2 A. If 35 % of total mmf is consumed by the iron path, calculate (*a*) the magnetomotive force, (*b*) the flux density, (*c*) the magnetic flux,

(d) the total reluctance, and (e) the relative permeability of iron.

[**Ans.** (a) 1400 At; (b) 0.7621 Wb/m^2; (c) 0.5387 mWb; (d) 2.599×10^6 At/Wb; (e) 972]

B-70. A circular soft iron ring with a cross-bar of silicon steel fitted into it is shown in Fig. B-12 with the relevant data mentioned. Estimate the required ampere turns to be applied to one-half of the ring to produce a flux density of 1.5 Wb/m^2 in the other half. [**Ans.** 2393 At]

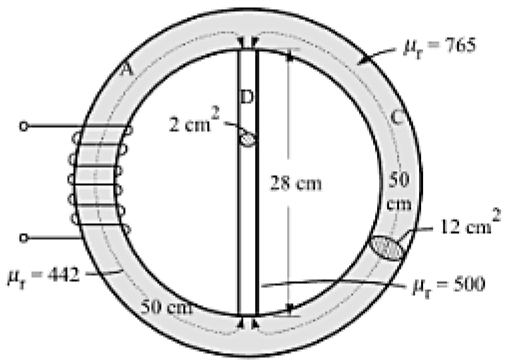


Fig. B-12

B-71. A 680-turn coil is wound on the central limb of the cast steel frame of square cross-section, shown in Fig. B-13. All the dimensions are given in cm. A flux of 1.6 mWb is required in the air gap. Assuming the permeability of cast steel to be 1200, calculate the value of the required exciting current.

[**Ans.** 1.697 A]

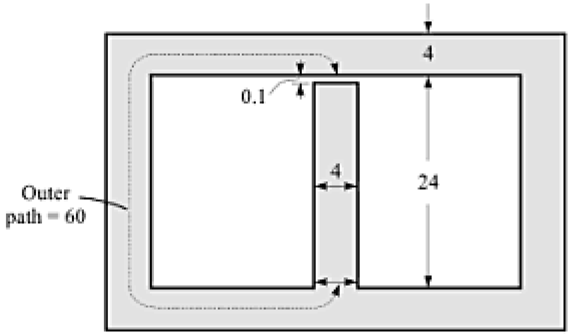


Fig. B-13

B-72. A magnetic core made up of an alloy has the dimensions as shown in Fig. B-14. Calculate the exciting current required to produce a flux of 1 mWb in the air gap, neglecting leakage and fringing. The B-H curve is as follows:

B(T)	1	1.2	1.42	1.513	1.6
H(At/m)	200	400	1792	3300	6000

[**Ans.** 1.65 A]

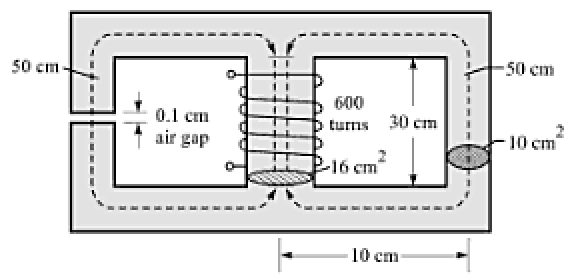


Fig. B-14

B-73. An electromagnet of the form shown in Fig. B-15 is excited by two coils, each having 500 turns. When the exciting current is 0.8 A, the resultant flux density gives the permeability of 1250. Calculate (a) the total reluctance, and (b) the flux produced in 0.5-cm air gap.

[**Ans.** (a) 4.284×10^6 At/Wb; (b) 0.1867 mWb]

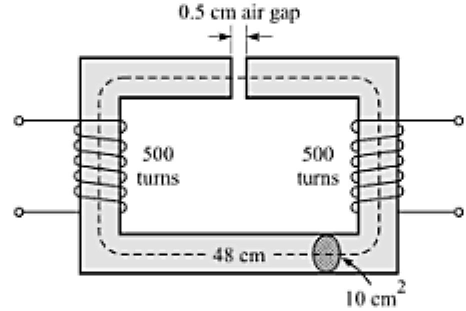


Fig. B-15

DC TRANSIENTS

8

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the difference between 'steady state' and 'transient' response.*
- *Define 'time constant' for a series RL circuit and a series RC circuit.*
- *Derive the necessary equation for the rise and decay of current in an inductive circuit.*
- *Explain why current decays more slowly for large L/R ratio.*
- *Derive the necessary equation for the rise and decay of voltage in a capacitive circuit.*
- *Explain why voltage decays more slowly for large CR.*
- *Write equations for currents and voltages in a single-capacitor RC circuit or in a single-inductor RL circuit, when dc source is switched on or off.*
- *Explain the working of an RC timer circuit.*

8.1 INTRODUCTION

We have seen that both the inductance and capacitance are energy-storing elements. When a network containing these elements is connected to a dc source, energy starts flowing to these elements. The inductor stores energy in the form of magnetic field, and the capacitor in the form of electric field. Initially the rate of flow of energy is high, but as more and more energy is stored, the rate of flow decreases. When maximum possible energy has been stored, the flow of energy stops altogether. We say that the circuit has reached its 'steady state'.

Now, if we switch off the source, or switch over the network to another source, the circuit starts attaining another 'steady state'. The time taken by the circuit to change over from one steady-state condition to another steady-state condition is called *transient time*. In this Chapter, we shall restrict our attention to simple circuits which contain only resistors and inductors or only resistors and capacitors.

The analysis of such circuits is dependent upon the formulation and solution of the integro-differential equations which characterize the circuits. The solution of the differential equation represents the response of the circuit, known as *transient response*.

Several different methods of solving these differential equations exist. This mathematics, however, is not circuit analysis. Our interest lies mainly in the solutions themselves and their meaning and interpretation. We shall try to become sufficiently familiar with the form of response so that we are able to write down answers for new circuits by just plain thinking. An engineer must always remember that these mathematical techniques are only tools with which meaningful and informative answers can be obtained; they do not constitute engineering in themselves.

We shall begin our study of transient analysis by considering the simple *RL* circuit.

8.2 THE SIMPLE *RL* CIRCUIT

Consider the simple series *RL* circuit of Fig. 8.1*a*. The time-varying current is designated as $i(t)$, and we shall assume the value of $i(t) = I_0$ at $t = 0$. You may wonder how there can be a current without any source in the circuit. To know this, consider the circuit of Fig. 8.1*b*, which has a special switch S, called '*make-before-break*'. It closes one circuit before opening the other. Let us assume that the circuit has been in the condition shown in the figure for a long time so as to achieve its steady-state condition. A steady current I_0 has been flowing in the circuit. Since the current I_0 is not changing with time, there is no induced emf in the inductance L. Hence, current flowing through the inductor is

$$I_0 = \frac{V_0}{R_0} \tag{8.1}$$

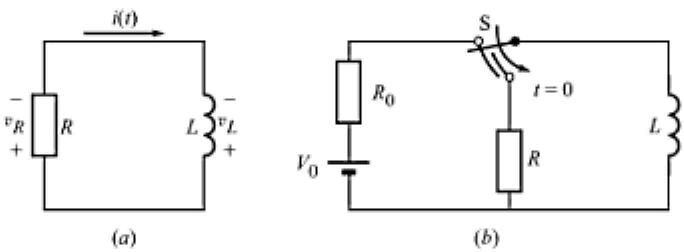


Fig. 8.1 *A series RL circuit with $i(t) = I_0$ at $t = 0$.*

For the circuit of Fig. 8.1*a*, by applying KVL around the loop, we have

$$v_R + v_L = 0 \quad \text{or} \quad Ri + L\frac{di}{dt} = 0 \quad \text{or} \quad \frac{di}{dt} + \frac{R}{L}i = 0 \tag{8.2}$$

Solving this equation means to determine an expression for $i(t)$ which satisfies this equation and also has the value I_0 at $t = 0$. The solution may be obtained by several different methods.

One very direct method of solving a differential equation is to first write the equation in such a way that the variables are separated and then integrating each side of the equation. Thus, we rewrite Eq. 8.2 as

$$\frac{di}{i} = -\frac{R}{L}dt \tag{8.3}$$

Integrating each side and finding the definite integrals between the corresponding limits,

$$\int_{I_0}^{i(t)} \frac{1}{i}di = \int_0^t \left(-\frac{R}{L}\right)dt \quad \text{or} \quad \ln i\Big|_{I_0}^{i} = -\frac{R}{L}t\Big|_0^t \quad \text{or} \quad \ln i - \ln I_0 = -\frac{R}{L}(t-0)$$

or

$$i(t) = I_0 e^{-Rt/L} \tag{8.4}$$

Energy Relations

Let us check the power and energy relations in the circuit of Fig. 8.1*a*. As the time passes, the current exponentially decreases from I_0 to zero. At any instant, the power being dissipated in the resistor,

$$p_R = i^2R = I_0^2Re^{-2Rt/L}$$

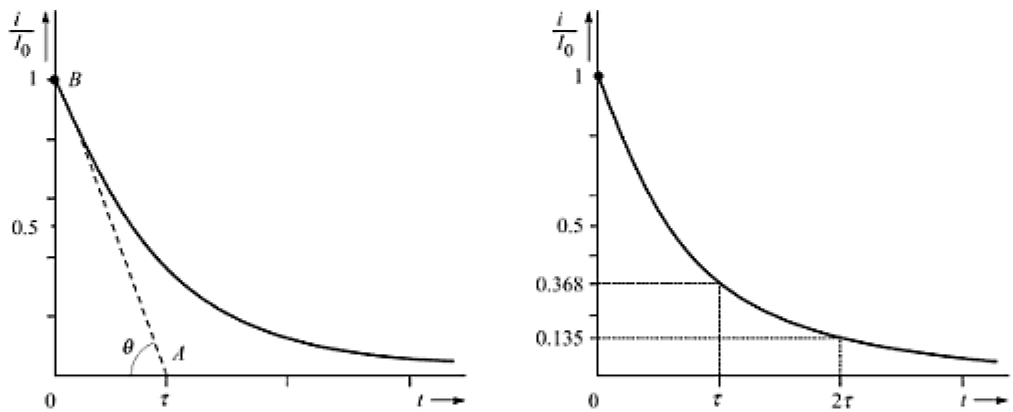
The total energy converted into heat in the resistor is found by integrating the instantaneous power from zero to infinite time,

$$W_R = \int_0^\infty p_R dt = I_0^2 R \int_0^\infty e^{-2Rt/L}\, dt = I_0^2 R\left(-\frac{L}{2R}\right) e^{-2Rt/L}\Big|_0^\infty = \frac{1}{2} L I_0^2$$

This is the result we expect. The total energy stored initially in the inductor is same as given above, and there is no energy stored in the inductor at infinite time. All the initial energy has been dissipated in the resistor.

Nature of the Response

For the series *RL* circuit, we found that the current is given by Eq. 8.4. At zero time, the current has the assumed value I_0. As time increases, the current decreases and approaches zero. The shape of this decaying exponential is seen by plotting normalized current $i(t)/I_0$ versus t, as shown in Fig. 8.2*a*. Since the function we are plotting is $e^{-(R/L)t}$, the curve will not change if R/L does not change. Thus, the same curve is obtained for every series *RL* circuit having the same R/L or L/R ratio.



(*a*) Definition of time constant, τ.

(*b*) Meaning of time constant, τ.

Fig. 8.2 *The shape of the response of a series RL circuit.*

Concept of Time Constant

Suppose that we double the ratio L/R in a series *RL* circuit. Equation 8.4 shows that if t is also doubled, then exponential term $e^{-(R/L)t}$ will be unchanged. In other words, the original response will occur at a later time, and the new curve is obtained by moving each point on the original curve twice as far to the right. It means that with larger L/R ratio, the current takes longer to decay to any fraction of its original value. We might have a tendency to say that the "width" of the curve is doubled, or that the "width" is proportional to L/R. However, we should define our term "width", because the curve extends from $t = 0$ to ∞. Instead, we define '***time constant***' of the circuit as *the time that would be required for the current to drop to zero if it continued to drop at its initial rate.* It is designated by Greek letter τ (*tau*).

The initial rate of decay is given by the slope of line AB drawn tangential to the curve at starting point B.

It is found by evaluating the derivative at zero time,

$$\frac{d}{dt}(i/I_0)\Big|_{t=0} = -\frac{R}{L}e^{-Rt/L}\Big|_{t=0} = -\frac{R}{L} \tag{8.5}$$

Thus, from triangle OAB (Fig. 8.2*a*), we have

$$\tan\theta = \frac{1}{\tau} \quad\Rightarrow\quad \frac{1}{\tau} = \frac{R}{L} \quad \text{or} \quad \tau = \frac{L}{R} \tag{8.6}$$

Since the exponent $-Rt/L$ must be dimensionless, the ratio L/R must have the units of seconds. In terms of time constant τ, the response of the series RL circuit may be written simply as

$$i(t) = I_0 e^{-t/\tau} \tag{8.7}$$

Meaning of Time Constant

An important meaning of time constant τ is obtained by determining the value of $i(t)/I_0$ at $t = \tau$. We have

$$\frac{i(\tau)}{I_0} = e^{-1} = 0.368 \quad \text{or} \quad i(\tau) = 0.368 I_0$$

Thus, in one time constant the response drops to 36.8 % of its initial value. The value of τ can also be determined graphically from this fact, as indicated by Fig. 8.2*b*.

It is interesting to note that in the duration of each time constant, the current decays to 36.8 % of its value at the beginning of this duration. Thus, at the end of second time constant, the value of current will be

$$i(2\tau) = 0.368 i(\tau) = 0.368 \times 0.368 I_0 = 0.1354 I_0$$

Suppose that we are asked, "*How long does it take for the current to decay to zero?*" To find the answer to this question, let us calculate the value of current $i(t)$ at the end of subsequent time constants. We find that $i(3\tau) = 0.0498 I_0$, $i(4\tau) = 0.0183 I_0$, $i(5\tau) = 0.0067 I_0$, … We can now say that *it takes about five time constants for the current to decay to zero*. At the end of this time interval, the current is less than one percent of its original value.

Why Current Decays More Slowly for Large *L/R* Ratio

Let us consider the effect of each element. An increase in L allows greater energy storage for the same initial current. This larger energy requires longer time to be dissipated in the resistor. We may also increase L/R by reducing R. In this case, the power flowing into the resistor is less for the same initial current. Again, a greater time is required to dissipate the stored energy.

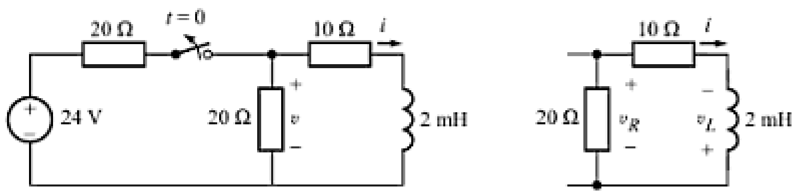
EXAMPLE 8.1

The circuit shown in Fig. 8.3*a* has been in the condition shown for a long time. The switch is opened at $t = 0$. (*i*) Determine the current $i(0^+) = I_0$. (*ii*) Find v_R across 20-Ω resistor at the instant just after the switch is opened. (*iii*) Find v_L across the inductor immediately after the switch is opened.

Solution

(*i*) Under steady-state condition, the voltage drop across an inductor is zero and it behaves as a short-circuit. The equivalent resistance faced by the 24-V source is

$$R_{eq} = 20\ \Omega + (20\ \Omega \parallel 10\ \Omega) = 26.67\ \Omega$$



(*a*) The switch is opened at $t = 0$.   (*b*) After the switch is opened ($t > 0$).

Fig. 8.3 *A series RL circuit.*

The current supplied by the 24-V source, $I = \dfrac{V}{R_{eq}} = \dfrac{24}{26.67} = 0.9$ A

By current division principle, the current through the inductor is

$$I_L = 0.9 \times \frac{20}{20+10} = 0.6 \text{ A}$$

Immediately after the switch is opened, the current remains the same as above. Hence,

$$i(0^+) = I_0 = \mathbf{0.6\ A}$$

(*ii*) The current through the circuit cannot change instantaneously because of the presence of the inductor. So, immediately after the switch is opened, the current flowing in the circuit is I_0. Therefore, the voltage across the 20-Ω resistor is given as

$$v_R = (-I_0)R = -0.6 \times 20 = \mathbf{-12\ V}$$

(*iii*) The current through the inductor cannot change instantaneously. But, immediately after the switch is opened the current starts decaying at the initial rate given by Eq. 8.5. Because of this change, there will be an induced emf in the inductor. This emf in the inductor will have a polarity (as shown in Fig. 8.3*b*) so as to oppose the decay in current. Using Eq. 8.5 the magnitude of the induced emf is given as

$$e = L\frac{di}{dt}\bigg|_{t=0} = L \times \left(I_0 \times \frac{R}{L}\right) = I_0 R = 0.6 \times (20+10) = \mathbf{18\ V}$$

EXAMPLE 8.2

The element values in Fig. 8.1*a* are $R = 0.8\ \Omega$ and $L = 1.6$ H. If $i = 20$ A at $t = -1$ s, find (*a*) $i(0)$, (*b*) the power being absorbed by the inductor at $t = 1$ s, and (*c*) the time at which the energy stored in the inductor is 100 J.

Solution $\tau = \dfrac{L}{R} = \dfrac{1.6}{0.8} = 2$ s. The equation for the instantaneous current can be written as

$$i(t) = I_0 e^{-t/\tau} = I_0 e^{-t/2} \qquad (i)$$

(*a*) The current is given as 20 A at $t = -1$ s. Hence,

$$20 = I_0 e^{-(-1)/2} \quad \text{or} \quad I_0 = \frac{20}{e^{1/2}} = \frac{20}{1.6487} = 12.13 \text{ A}$$

Since I_0 is the value of current at $t = 0$, we have $i(0) = I_0 =$ **12.13 A.**

(*b*) The current through the inductor at $t = 1$ s,

$$i(1) = I_0 e^{-1/2} = 12.13e^{-0.5} = 7.36 \text{ A}$$

Therefore, the power absorbed by the resistor is

$$p(1) = i^2 R = (7.36)^2 \times 0.8 = 43.33 \text{ W}$$

The inductor, by virtue of the emf induced in it, supplies this power to the resistor. Therefore, the power *absorbed* by the inductor is **–43.33 W.**

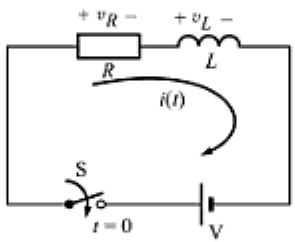
(*c*) If current flowing is $i(t)$, the energy stored is given as

$$W = \frac{1}{2}L\{i(t)\}^2 \quad \text{or} \quad 100 = \frac{1}{2} \times 1.6\{i(t)\}^2 \quad \Rightarrow \quad i(t) = 11.18 \text{ A}$$

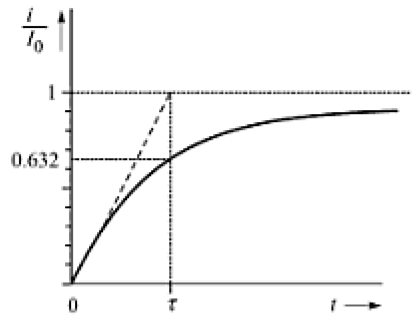
From Eq. (*i*), $$11.18 = 12.13e^{-t/2} \quad \Rightarrow \quad e^{t/2} = 1.085 \quad \text{or} \quad t = \mathbf{0.1632 \text{ s}}$$

Growth of Current in Series *RL* Circuit

Consider the circuit of Fig. 8.4*a*, where a series combination of R and L are connected to a voltage source of emf V through a switch S. Let the switch be closed at $t = 0$. It is evident that before $t = 0$, the current in the circuit is zero. That is, $i(0^-)$[1] $= 0$. Had there been no inductance in the circuit, on closing the switch the current would have jumped to a value $I_0 = V/R$. However, an inductor opposes any change in current. Since the current in an inductor cannot change by a finite amount in zero time, we must have $i(0^+) = 0$. After $t = 0$, the current slowly increases and approaches its steady state value $I_0 = V/R$.



(*a*) The circuit.



(*b*) The response.

Fig. 8.4 *A series RL circuit connected to a voltage source.*

At any time t, after the switch is closed, as per KVL we should have

$$v_L + v_R = V \quad \text{or} \quad L\frac{di}{dt} + iR = V \tag{8.8}$$

To find the response $i(t)$, we shall solve the above differential equation by first separating the variables and then by integration. Thus, Eq. 8.8 can be rewritten as

$$\frac{L}{R}\frac{di}{dt} + i = \frac{V}{R} \quad \text{or} \quad (L/R)\frac{di}{dt} + i = I_0 \quad \text{or} \quad \frac{(L/R)\,di}{(I_0 - i)} = dt$$

Integrating each side of the above equation,

$$\frac{(L/R)\ln(I_0 - i)}{-1} = t + k \quad \text{or} \quad -(L/R)\ln(I_0 - i) = t + k$$

In order to evaluate constant k, we apply initial condition, at $t = 0$, we must have $i = 0$,

$$-(L/R)\ln I_0 = k$$

$\therefore$ $$-(L/R)\ln(I_0 - i) = t - (L/R)\ln I_0 = \quad \text{or} \quad -(L/R)[\ln(I_0 - i) - \ln I_0] = t$$

or $$\ln\left[\frac{I_0 - i}{I_0}\right] = -\frac{Rt}{L} \quad \text{or} \quad \left[\frac{I_0 - i}{I_0}\right] = e^{-Rt/L}$$

[1] The notation $i(0^-)$ means the value of the current at an instant just before the instant $t = 0$. Similarly, $i(0^+)$ represents the value of the current at an instant just after $t = 0$.

$$\Rightarrow \qquad i = I_0(1 - e^{-Rt/L})$$

or
$$i(t) = I_0(1 - e^{-t/\tau}) \tag{8.9}$$

Here, $\tau = L/R$ is the *time constant* of the circuit.

The nature of response of the circuit of Fig. 8.4*a* can be found by plotting $i(t)/I_0$ versus t, as shown in Fig. 8.4*b*. Let us determine the value of $i(t)/I_0$ at $t = \tau$, from Eq. 8.9,

$$\frac{i(\tau)}{I_0} = (1 - e^{-1}) = (1 - 0.368) \quad \text{or} \quad i(\tau) = 0.632 I_0$$

Thus, in one time constant the response rises to 63.2 % of its final value. It is interesting to note that in the duration of each time constant, the current increases by 63.2 % of the difference of its final value and the value at the beginning of this duration. Thus, at the end of second time constant, the value of current will be

$$i(2\tau) = i(\tau) + 0.632[I_0 - i(\tau)] = 0.632 I_0 + 0.632[1 - 0.632]I_0 = 0.864 I_0$$

If we calculate the value of current $i(t)$ at the end of subsequent time constants, we find that $i(3\tau) = 0.9502 I_0$, $i(4\tau) = 0.9817 I_0$, $i(5\tau) = 0.9933 I_0$, … We can now say that *it takes about five time constants for the current to grow to its final steady state value*. At the end of this time interval, the current attains more than 99 % of its final value.

Rate of Growth of Current The initial rate of growth of current is given by the slope of the curve at the origin. It is found by evaluating the derivative of Eq. 8.9 at zero time,

$$\left.\frac{di}{dt}\right|_{t=0} = -\left(-\frac{I_0}{\tau}\right)e^{-t/\tau}\Big|_{t=0} = \frac{I_0}{\tau} = \frac{V}{R}\frac{R}{L} = \frac{V}{L} \tag{8.10}$$

Thus, the smaller the value of L, the faster the current rises to its final value.

EXAMPLE 8.3

A coil having an inductance of 14 H and a resistance of 10 Ω is connected to a dc voltage source of 140 V, through a switch. (*a*) Calculate the value of current in the circuit at an instant 0.4 s after the switch has been closed. (*b*) Once the current reaches its final steady state value, how much time it would take the current to drop to 8 A after the switch is opened?

Solution The time constant of the circuit is $\tau = L/R = 14/10 = 1.4$ s.

(*a*) The final steady state value of the current, $I_0 = \dfrac{V}{R} = \dfrac{140}{10} = 14$ A. Therefore, the value of current at $t = 0.4$ s is given by Eq. 8.9 as

$$i = I_0(1 - e^{-t/\tau}) = 14(1 - e^{-0.4/1.4}) = \mathbf{3.479\ A}$$

(*b*) For decaying current, we use Eq. 8.7. Thus,

$$i(t) = I_0 e^{-t/\tau} \quad \text{or} \quad 8 = 14 e^{-t/1.4} \quad \text{or} \quad e^{-t/1.4} = 0.5714$$

Taking natural log of both sides,

$$-\frac{t}{1.4} = \ln 0.5714 = -0.5596 \quad \Rightarrow \quad t = \mathbf{0.7834\ s}$$

EXAMPLE 8.4

A series combination of 40-mH inductor and an 80-Ω resistor are connected to a dc voltage source of 20 V, as shown in Fig. 8.5*a*. The make-before-break switch S is thrown from position a to position b at $t = 0$. Find the magnitude of the inductor current (*a*) at $t = 0^-$, (*b*) at $t = 0^+$, (*c*) as $t \to \infty$, and (*d*) at $t = 1$ ms.

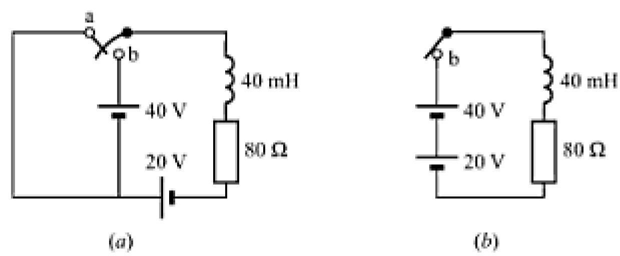


Fig. 8.5

Solution

(*a*) At $t = 0^-$, the current will be the steady state current due to voltage source of 20 V.

$$\therefore \qquad i(0^-) = I_{01} = \frac{V_1}{R} = \frac{20}{80} = \mathbf{0.25\ A}$$

(*b*) At $t = 0^+$, the 20-V and 40-V sources are in series, as shown in Fig. 8.5*b*. But, the current in the inductor cannot change instantaneously.

$$\therefore \qquad i(0^+) = I_{01} = \mathbf{0.25\ A}$$

(*c*) As $t \to \infty$, the current approaches its final steady state value given as

$$I_{02} = \frac{V_1 + V_2}{R} = \frac{20 + 40}{80} = \mathbf{0.75\ A}$$

(*d*) After $t = 0^+$, the current increases from its initial steady-state value I_{01} by an amount decided by the time constant ($\tau = L/R$ = (40 mH)/(80 Ω) = 0.5 ms) of the circuit. Thus,

$$\begin{aligned} i(1\ \text{ms}) &= I_{01} + (I_{02} - I_{01})(1 - e^{-t/\tau}) = 0.25 + (0.75 - 0.25)(1 - e^{-1\times10^{-3}/0.5\times10^{-3}}) \\ &= 0.25 + 0.50(1 - e^{-2}) = \mathbf{0.682\ A} \end{aligned}$$

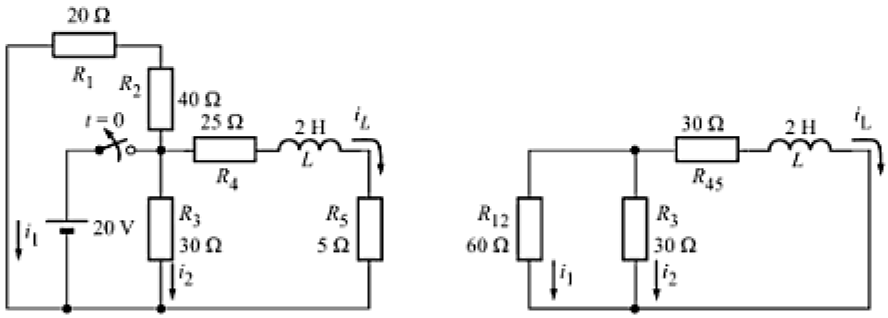
General *RL* Circuit

The results obtained for the simple series *RL* circuit can easily be extended to a circuit containing any number of resistors and one inductor. We fix our attention on the two terminals of the inductor and determine the equivalent resistance across these terminals. The circuit is thus reduced to the simple series case.

While dealing with the general *RL* circuits, you must remember that the current in a resistor may change instantaneously. However, the instantaneous value of current in an inductor is not affected by any sudden change. Once the independent sources are cut off from the circuit, the inductor current decays exponentially as $e^{-t/\tau}$, and every current throughout the circuit must follow the same functional behaviour. This is made clear by considering the inductor current as a source current which is being applied to a resistive network. Every current in the resistive network must have the same time dependence. The procedure is made clear in the Example given below.

EXAMPLE 8.5

In the circuit of Fig. 8.6*a*, the switch is opened at $t = 0$. Find (*a*) $i_L(0^-)$, (*b*) $i_2(0^-)$, (*c*) $i_L(0^+)$, (*d*) i_L(20 ms), and (*e*) i_2(20 ms).



(*a*) The circuit (for $t < 0$). (*b*) The circuit (for $t > 0$).

Fig. 8.6 *A general RL circuit.*

Solution

(*a*) The 20-V source is applied across the inductor branch containing 25 Ω and 5 Ω in series. Hence,

$$i_L(0^-) = I_0 = \frac{20}{25+5} = \mathbf{0.667\ A}$$

(*b*) The 20-V source is applied across R_3. Hence, $i_2(0^-) = \dfrac{20}{30} = \mathbf{0.667\ A}$.

(*c*) Since the current in an inductor cannot change instantaneously,

$$i_L(0^+) = i_L(0^-) = I_0 = \mathbf{0.667\ A}$$

(*d*) At $t = 0$, the switch is opened so that the 20-V source is disconnected. Resistances R_1 and R_2 being in series, make a net resistance $R_{12} = 20 + 40 = 60\ \Omega$. Similarly, resistances R_4 and R_5 are in series and make a net resistance $R_{45} = 25 + 5 = 30\ \Omega$. The equivalent circuit is shown in Fig. 8.6*b*. For determining time constant, we find the equivalent resistance R_{eq} that is connected across the inductor terminals.

$$R_{eq} = R_{45} + (R_{12} \parallel R_3) = 30 + \frac{60 \times 30}{60+30} = 50\ \Omega$$

Time constant, $\tau = \dfrac{L}{R_{eq}} = \dfrac{2}{50} = 40$ ms

The current in the inductor is given as $i_L(t) = I_0 e^{-t/\tau}$. Therefore,

$$i_L(20\text{ ms}) = I_0 e^{-t/\tau} = 0.667 e^{-(20\text{ ms})/(40\text{ ms})} = 0.667 e^{-0.5} = \mathbf{0.405\ A}$$

(*e*) The inductor current i_L acts as source current. By current division,

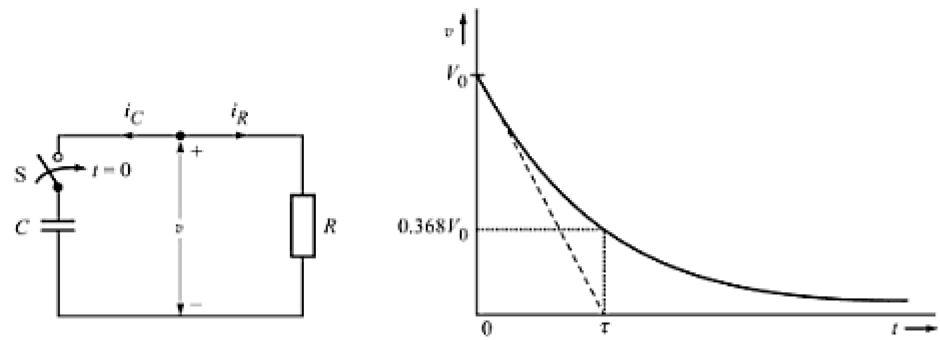
$$i_2(20\text{ ms}) = -i_L(20\text{ ms}) \times \frac{R_{12}}{R_{12}+R_3} = -0.405 \times \frac{60}{60+30} = \mathbf{-0.27\ A}$$

8.3 THE SIMPLE *RC* CIRCUIT

The *RC* circuit has a greater practical importance than the *RL* circuit. If an engineer has a freedom of choice between using a capacitor and using an inductor in designing an electronic circuit, he chooses an *RC* over an *RL* network wherever possible. The reasons for this choice are the smaller losses in a physical capacitor, its lower cost, smaller size, and lighter weight.

Discharging of a Capacitor

Consider a capacitor with an initial stored energy connected across a resistor through a switch, as shown in Fig. 8.7*a*. Let the initial voltage across the capacitor be V_0. Let the switch S be closed at $t = 0$. The charged capacitor acts as a voltage source and forces a current in the circuit. The capacitor keeps discharging and the stored energy keeps dissipating in the resistor. During this process, the voltage v across the capacitor keeps on decreasing.



(*a*) The circuit. (*b*) The voltage response.

Fig. 8.7 *Discharging of a capacitor.*

Both the capacitor and resistor are passive elements. Therefore, as shown in the figure, the current and voltage for them should agree with the passive convention. The total current leaving the node at the top of the circuit diagram must be zero. Therefore,

$$i_C + i_R = 0 \quad \text{or} \quad C\frac{dv}{dt} + \frac{v}{R} = 0 \quad \text{or} \quad \frac{dv}{dt} + \frac{v}{CR} = 0 \tag{8.11}$$

This equation is of the same form as Eq. 8.2, except that i is replaced by v, and L/R by CR. Therefore, the solution of Eq. 8.11 should be of the same form as of Eq. 8.4. Thus, for RC circuit,

$$v = V_0 e^{-t/RC} = V_0 e^{-t/\tau} \tag{8.12}$$

where, $\tau = RC$ is the time constant of the RC circuit. The voltage response of the circuit is shown in Fig. 8.7*b*. At $t = 0$, we obtain from Eq. 8.11, $v = V_0$, which is correct initial condition. As t becomes infinite, the voltage approaches zero. As per definition of the time constant τ, the slope of the curve at $t = 0$ is given as $-V_0/\tau$.

Why Voltage Decays More Slowly for Large *RC*

Larger values of R or C provide larger time constants and slower dissipation of the stored energy. A larger resistance will dissipate a smaller power with a given voltage across it, thus requiring a greater time to convert the stored energy. A larger capacitor stores a larger energy with a given voltage across it, again requiring a greater time to dissipate this initial energy.

Charging of a Capacitor

Consider the circuit of Fig. 8.8*a*, in which a series combination of R and C is connected through a switch S across a voltage source of emf V_0. The capacitor is assumed to have no charge, and therefore the voltage v across it is zero. Now, let the switch be closed at $t = 0$. Since the voltage across a capacitor cannot change instantaneously, we must have

$$v(0^+) = v(0^-) = 0 \tag{8.13}$$

It means that initially the capacitor behaves as a short-circuit. The value of the initial current in the circuit is entirely decided by V_0 and R. That is,

$$i(0^+) = I_0 = \frac{V_0}{R} \tag{8.14}$$

This current also flows through the capacitor, and it starts charging. The voltage v across the capacitor goes on increasing. As v increases, the voltage $v_R(= V_0 - v)$ decreases which results in a decrease in current i. The process continues, till the capacitor is fully charged so that the voltage v becomes the same as the source voltage V_0. The current i finally reduces to zero. That is,

$$v(t \to \infty) = V_0 \qquad \text{and} \qquad i(t \to \infty) = 0$$

The variation of voltage v and current i versus time t is shown in Fig. 8.8*b*.

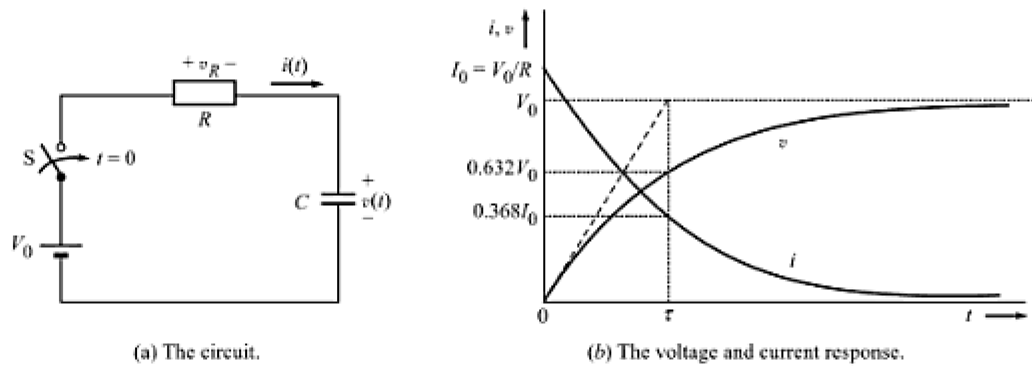


Fig. 8.8 *Charging of a capacitor.*

Response of the Circuit To determine the exact nature of response, we note that at every instant, the KVL must be satisfied,

$$v_R + v = V_0 \quad \text{or} \quad iR + v = V_0 \tag{8.15}$$

Note that the current in the capacitor is the same as in R. Hence, the current i is related to the voltage v across the capacitor by the relation,

$$i = C\frac{dv}{dt}$$

Substituting this value of i in Eq. 8.15, we get

$$RC\frac{dv}{dt} + v = V_0 \tag{8.16}$$

This is the differential equation that gives the response of the circuit. To determine the response, we first separate the variables. Thus, rearranging the terms in Eq. 8.16, we get

$$RC\frac{dv}{(V_0 - v)} = dt$$

Integrating each side of this equation, we get

$$RC\frac{\ln(V_0 - v)}{-1} = t + k \quad \text{or} \quad -RC\ln(V_0 - v) = t + k \tag{8.17}$$

In order to evaluate constant k, we apply initial condition, at $t = 0$, we must have $v = 0$,

$$-RC\ln V_0 = k$$

$$\therefore \qquad -RC\ln(V_0 - v) = t - RC\ln V_0 \quad \text{or} \quad -RC[\ln(V_0 - v) - \ln V_0] = t$$

$$\text{or} \qquad \ln\left[\frac{V_0 - v}{V_0}\right] = -\frac{t}{RC} \quad \text{or} \quad \left[\frac{V_0 - v}{V_0}\right] = e^{-t/RC}$$

$$\Rightarrow \qquad v = V_0(1 - e^{-t/RC})$$

$$v = V_0(1 - e^{-t/\tau}) \tag{8.18}$$

where, $\tau = RC$ is the time constant of the circuit.

The current response of the circuit is obviously given as

$$i(t) = I_0 e^{-t/\tau} \tag{8.19}$$

where I_0 is given by Eq. 8.14.

Note that, as shown in Fig. 8.8*b*, in one time constant τ, the voltage rises to 0.632 times the final value V_0 and the current drops to 0.368 times the initial value I_0.

EXAMPLE 8.6

In Fig. 8.9*a*, the single-pole double-throw switch S has been in position b for a long time so that the 5-μF capacitor is fully discharged. Now, at $t = 0$, the switch is thrown to position a. Determine (*a*) $v(0^+)$, (*b*) $i(0^+)$, (*c*) time constant τ, (*d*) v and i at $t = 15$ ms.

Solution

(*a*) Since the voltage across a capacitor cannot change instantaneously, we have

$$v(0^+) = v(0^-) = \mathbf{0\ V}$$

(*b*) $i(0^+) = I_0 = \dfrac{V_0}{R} = \dfrac{3\text{V}}{1.5\,\text{k}\Omega} = \mathbf{2\ mA}$

(*c*) Time constant, $\tau = RC = (1.5\ \text{k}\Omega)\,(5\ \mu\text{F}) = \mathbf{7.5\ ms}$

(*d*) At $t = 15$ ms: $v = V_0(1 - e^{-t/\tau}) = 3(1 - e^{-(15\text{ ms})/(7.5\text{ ms})}) = 3(1 - 0.1353) = \mathbf{2.594\ V}$

$$i = I_0 e^{-t/\tau} = (2\text{ mA})e^{-(15\text{ ms})/(7.5\text{ ms})} = 2 \times 0.135 = \mathbf{0.27\ mA}$$

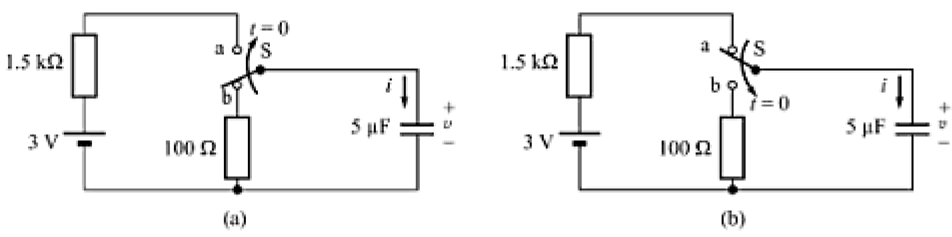


Fig. 8.9 *An RC circuit.*

EXAMPLE 8.7

In Fig. 8.9*b*, the single-pole double-throw switch S has been in position a for a long time so that the 5-μF capacitor is fully charged. Now, at $t = 0$, the switch is thrown to position b. Determine (*a*) $v(0^+)$, (*b*) $i(0^+)$, (*c*) time constant τ, (*d*) v and i at $t = 1.2$ ms.

Solution

(*a*) Since the voltage across a capacitor cannot change instantaneously, we have

$$v(0^+) = v(0^-) = V_0 = \mathbf{3\ V}$$

(*b*) At $t = 0^+$, the capacitor behaves as a voltage source of emf V_0. Hence,

$$i(0^+) = -I_0 = -\frac{V_0}{R} = -\frac{3\text{V}}{100\Omega} = \mathbf{-30\ mA}$$

(*c*) Time constant, $\tau = RC = (100\ \Omega)\ (5\ \mu\text{F}) = \mathbf{0.5\ ms}$

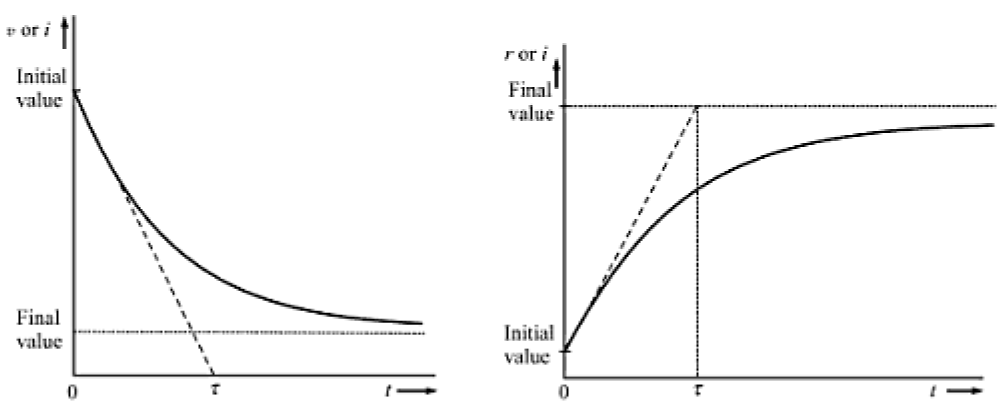
(*d*) At $t = 1.2$ ms : $v = V_0 e^{-t/\tau} = 3e^{-(1.2\text{ ms})/(0.5\text{ ms})} = 3 \times 0.0907 = \mathbf{0.2721\ V}$

$$i = -I_0 e^{-t/\tau} = -(30\text{ mA})e^{-(1.2\text{ ms})/(0.5\text{ ms})} = -30 \times 0.0907\text{ mA} = \mathbf{-2.721\ mA}$$

8.4 DC-EXCITED SINGLE-CAPACITOR *RC* AND SINGLE-INDUCTOR *RL* CIRCUITS

When switches are closed or opened in dc-excited *RL* or *RC* circuits with a single energy-storing element (*C* or *L*), all voltages and currents change *exponentially* from their initial values to their final constant values. These exponential changes for a switching operation at $t = 0$ s, are shown in Fig. 8.10. In Fig. 8.10*a* the initial value is greater than the final value, and in Fig. 8.10*b* the final value is greater. Although we have shown here both initial and final values positive, but in practice both can be negative or one can be positive and the other negative.

The voltages and currents approach their final values asymptotically. It means that they never actually reach them. However, after *five time-constants* they change by 99.3 % of their total change, and so can be considered to be at their final values for most practical purposes.



(*a*) Exponentially decreasing with time. (*b*) Exponentially increasing with time.

Fig. 8.10 *Transients in an RL or RC circuit.*

For a single-capacitor *RC* circuit, the ***time constant*** $\tau = CR_{Th}$; and for a single-inductor *RL* circuit, $\tau = L/R_{Th}$. Here, R_{Th} is Thevenin resistance as "seen" by the capacitor or inductor. If $v(0^+)$ and $i(0^+)$ are *initial values* immediately after switching, and $v(\infty)$ and $i(\infty)$ are *final values*, then the expressions for all the voltages and currents in the circuit for any time *t* are given as

$$\boxed{\begin{aligned} v(t) &= v(\infty) + [v(0^+) - v(\infty)]e^{-t/\tau}\ \text{V} \\ i(t) &= i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}\ \text{A} \end{aligned}} \tag{8.20}$$

Initial and Final Values

By *initial* we mean the instant after a change occurs in the circuit, usually a switch closing or opening. This does not have to be the time origin, although often we define $t = 0$ s as the time of switching action. By *final* we mean the steady-state condition of the circuit, its state after a long period of time.

Sometimes, we may close a switch and then open it before the circuit reaches the final state. The circuit cannot anticipate the second switching action and hence it reacts to the first switching action as if it *would* reach steady state.

Determining Initial Values The initial values always follow from energy considerations. The fundamental principle is that the stored electrical energy in a capacitor and the stored magnetic energy in an inductor must be continuous. From continuity of energy, we conclude that the voltage across a capacitor and the current through an inductor cannot change abruptly. Thus, we always calculate capacitor-voltage (or inductor-current) before the switch is thrown and carry this value over to the moment immediately after the switch is thrown.

A model for an energized capacitor at the instant after the switching action is shown in Fig. 8.11. The capacitor acts as a battery *only* at the first instant. Subsequently, as the current flows through the capacitor, its voltage would change. If the capacitor is uncharged before switching action, it is modelled as a battery of *zero* volt (i.e., a short-circuit). Similarly, an energized inductor is modelled as a current source, as shown in Fig. 8.12. If the inductor current is zero before switching action, it is modelled as a current source of *zero* ampere (i.e., open-circuit).

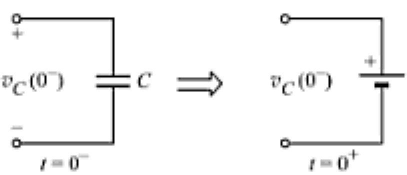


Fig. 8.11

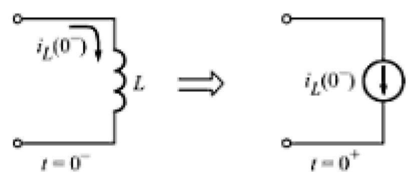


Fig. 8.12

Determining Final Values The final arise out of the eventual steady state of the circuit. All time derivatives must eventually vanish. As a result, the current through a capacitor and the voltage across an inductor must approach zero, as suggested in

$$v_C \rightarrow \text{constant} \quad \Rightarrow \quad i_C = C\frac{dv_C}{dt} \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

$$i_L \rightarrow \text{constant} \quad \Rightarrow \quad v_L = L\frac{di_L}{dt} \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

Thus, a capacitor acts as an open-circuit and an inductor as a short-circuit in establishing final values, as shown in Figs. 8.13 and 8.14, respectively.

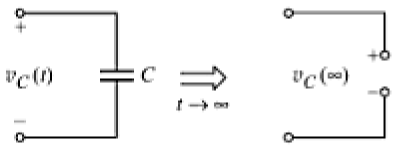


Fig. 8.13

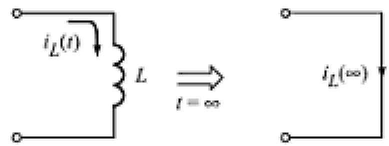


Fig. 8.14

EXAMPLE 8.8

In the circuit shown in Fig. 8.15*a*, the switch is opened at $t = 0$ s. Determine the current $i(t)$ for $t > 0$ s.

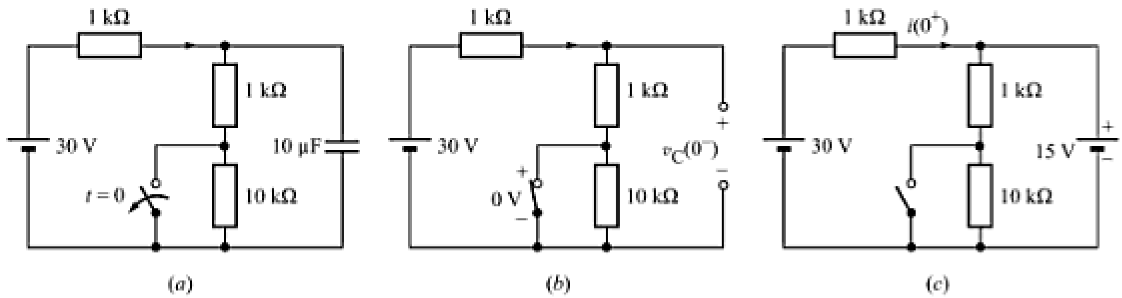


Fig. 8.15

Solution We first calculate the time constant. The Thevenin resistance or the equivalent resistance across the terminals of the capacitor is $R_{Th} = (1\ k\Omega + 10\ k\Omega)\ ||\ (1\ k\Omega) = 917\ \Omega$. Thus,

$$\tau = R_{Th}C = (917\ \Omega) \times (10\ \mu F) = 9.17\ ms$$

At the instant before opening the switch, the circuit is in steady state. Actually, this is the final state from previous actions that established the circuit. Thus, the capacitor acts as an open circuit, as shown in Fig. 8.15*b*. Before opening the switch, the voltage across the capacitor is determined by voltage division,

$$v_C(0^-) = (30\ V) \times \frac{1\,k\Omega}{1\,k\Omega + 1\,k\Omega} = 15\ V$$

At the instant of opening the switch, the capacitor acts as a battery of 15 V, as shown in Fig. 8.15*c*. The initial value of $i(t)$ can be determined from this circuit. Writing KVL around the outer loop containing two voltage sources, we get

$$30 - i(0^+)\,(1\ k\Omega) - 15 = 0 \quad \Rightarrow \quad i(0^+) = 15\ mA$$

A long time (more than 5 time-constants) after the switch is opened, steady state is reached. The capacitor again acts as an open switch. Therefore, the final value $i(\infty)$ is easily derived as

$$i(\infty) = \frac{30\,V}{(1+1+10)\,k\Omega} = 2.5\ mA$$

Having determined the time constant, the initial value and the final value, we can now write the expression for the current, by using Eq. 8.20,

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = \mathbf{2.5 + (15 - 2.5)}e^{-t/9.17\ ms}\ \mathbf{mA}$$

8.5 *RC* TIMERS

An *RC* circuit finds an important application in making a ***timer***, the circuit that measures time. A simple timer consists of a series combination of a switch, a capacitor, a resistor, and a dc voltage source. At the beginning of the time interval to be measured, the switch is closed to start the charging of the capacitor. At the end of the time interval, the switch is opened to stop the charging. The capacitor gets charged to the voltage which is a measure of the time interval. A voltmeter connected across the capacitor can have a scale calibrated in time to give direct reading of the time interval elapsed.

As can be seen from Fig. 8.10*b*, the capacitor voltage changes almost linearly, for times much less than one time constant. This linear change approximation is valid if the time to be measured is one-tenth of a time constant or less.

Why *RC* Circuit is Better Choice than *RL* Circuit

Because of the similarity of the transient response of *RC* and *RL* circuits, it is possible to make *RL* timers. But practically, *RC* timers are considered to be a better choice due to the following reasons:

1. Inductors are not nearly as ideal as capacitors because coils have resistances that are seldom negligible.
2. Inductors are relatively bulky, heavy and difficult to fabricate, especially using integrated-circuit techniques.
3. Inductors are relatively costlier.
4. The magnetic field emanating from the inductors can induce unwanted voltages in other components.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 8.9

The resistance of a coil is 3 Ω and its time constant is 1.8 s. At $t = 0$, a 10-V source is connected to it. Determine (*a*) the current at $t = 1$ s, (*b*) the time at which the current attains half of its final value, and (*c*) the initial rate of growth of current.

Solution Since $\tau = L/R$, $L = \tau R = 1.8 \times 3 = 5.4$ H. Since the current cannot change abruptly in an inductor, we have $i(0^+) = i(0^-) = 0$ A. In the final steady-state, the inductor acts as a short-circuit. The final current is given as

$$i(\infty) = I_0 = \frac{V_0}{R} = \frac{10\text{ V}}{3\Omega} = 3.33\text{ A}$$

From Eq. 8.20,

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = 3.33 + [0 - 3.33]e^{-t/1.8} = 3.33(1 - e^{-t/1.8})\text{ mA}$$

(*a*) The current at $t = 1$ s is given as

$$i(1\text{ s}) = I_0(1 - e^{-t/\tau}) = (3.33\text{ mA})\,(1 - e^{-1/1.8}) = \mathbf{1.424\ A}$$

(*b*) The time is given by

$$\frac{I_0}{2} = I_0(1 - e^{-t/\tau}) \quad\Rightarrow\quad e^{-t/1.8} = 0.5 \quad\Rightarrow\quad t = \mathbf{1.25\ s}$$

(*c*) The initial rate of growth of current is given by Eq. 8.10,

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_0}{L} = \frac{10}{5.4} = \mathbf{1.85\ A/s}$$

EXAMPLE 8.10

When a dc voltage is applied to a series *RL* circuit, after a lapse of 1 second the current reaches a value 0.741 times its final steady-state value. After sufficiently long time, when the current has reached its final value, suddenly the dc voltage source is removed and the terminals of the circuit are shorted. What would be the value of the current after 1 s of this sudden change?

Solution If I_0 is the final steady-state value of the current, at $t = 1$ s, we have

$$i = I_0(1 - e^{-1/\tau}) \quad \text{or} \quad 0.741 I_0 = I_0(1 - e^{-1/\tau}) \quad \Rightarrow \quad e^{-1/\tau} = 0.259$$

During the decay of current, we have $i(t) = I_0 e^{-t/\tau}$. Therefore, at $t = 1$ s, we have

$$i_1 = I_0 e^{-1/\tau} = 0.259 I_0$$

That is, the value of the current in the circuit is **0.259** times the steady-state value.

EXAMPLE 8.11

At $t = 0$, suddenly a dc voltage source of 120 V is applied to a series *RL* circuit having $R = 20\ \Omega$ and $L = 8$ H. Determine (*a*) the current in the circuit at $t = 0.6$ s, and (*b*) the time at which the voltage drops across R and L are same.

Solution The time constant, $\tau = \dfrac{L}{R} = \dfrac{8}{20} = 0.4$ s

The final steady-state current, $I_0 = \dfrac{V_0}{R} = \dfrac{120}{20} = 6$ A

(*a*) At $t = 0.6$ s, the current is given as

$$i(0.6\text{ s}) = I_0(1 - e^{-t/\tau}) = 6(1 - e^{-0.6/0.4}) = \mathbf{4.66\ A}$$

(*b*) The voltage drop across R, $v_R = iR = I_0(1 - e^{-t/\tau})R$

The voltage drop across L, $v_L = L\dfrac{di}{dt} = \dfrac{LI_0}{\tau}e^{-t/\tau} = \dfrac{LI_0 R}{L}e^{-t/\tau} = I_0 e^{-t/\tau}R$

We are to find the time at which these two voltage-drops are same. That is,

$$v_R = v_L \quad \text{or} \quad I_0(1 - e^{-t/\tau})R = I_0 e^{-t/\tau}R \quad \Rightarrow \quad e^{-t/\tau} = 0.5$$

$$\therefore \qquad t = 0.693\tau = 0.693 \times 0.4 = \mathbf{0.277\ s}$$

EXAMPLE 8.12

At $t = 0$, suddenly a dc supply of 30 V is applied to a series *RL* circuit having $R = 12\ \Omega$ and $L = 18$ H. Determine (*a*) the time constant of the circuit, (*b*) the initial rate of change of current, (*c*) the current at $t = 3$ s, (*d*) the energy stored in the magnetic field at $t = 3$ s, and (*e*) the energy lost as heat till $t = 3$ s.

Solution

(*a*) The time constant, $\tau = \dfrac{L}{R} = \dfrac{18}{12} = \mathbf{1.5\ s}$

(*b*) Using Eq. 8.10, the initial rate of change of current is

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_0}{L} = \frac{30}{18} = \mathbf{1.67\ A/s}$$

(*c*) The current at $t = 3$ s is given as

$$i_1 = I_0(1 - e^{-t/\tau}) = \frac{V_0}{R}(1 - e^{-t/\tau}) = \frac{30}{12}(1 - e^{-3/1.5}) = \mathbf{2.16\ A}$$

(*d*) The energy stored in the magnetic field at $t = 3$ s is given as

$$W_1 = \frac{1}{2}Li_1^2 = \frac{1}{2} \times 18 \times (2.16)^2 = \mathbf{42\ J}$$

(*e*) The energy lost as heat is given by the difference of initial energy stored and the energy stored at t = 3 s. Thus, the energy lost,

$$W_{\text{lost}} = W_0 - W_1 = \frac{1}{2}LI_0^2 - 42 = \frac{1}{2} \times 18 \times \left(\frac{30}{12}\right)^2 - 42 = 56.25 - 42 = \mathbf{14.25\ J}$$

EXAMPLE 8.13

The circuit of Fig. 8.16*a* has been in the condition shown for a long time. At $t = 0$, the switch is opened. Determine (*a*) $i(0^+)$, (*b*) $v(0^+)$, (*c*) $v_L(0^+)$, (*d*) i and v at t = 20 µs, and (*e*) i and v at t = 50 µs.

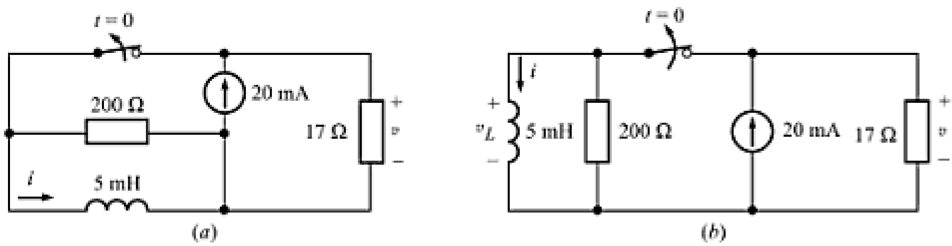


Fig. 8.16 *An LR circuit.*

Solution

(*a*) The equivalent circuit of Fig. 8.16*a* is given in Fig. 8.16*b*. Since the current in an inductance cannot change instantaneously, we must have $i(0^+) = i(0^-)$. When the switch had been closed for long, a constant current I_0 flows through the inductor, and there is no voltage drop across it. It means the inductance acts as a short-circuit. Hence, the entire current of 20 mA from the current source flows through this short rather than dividing itself into two other parallel branches of 200 Ω and 17 Ω. Thus,

$$i(0^+) = i(0^-) = I_0 = \mathbf{20\ mA}$$

(*b*) After the switch is opened, entire 20 mA current flows through 17-Ω resistance. Thus,

$$v(0^+) = iR = (20\text{ mA}) \times (17\ \Omega) = \mathbf{0.340\ V}$$

(*c*) After the switch is opened, the current I_0 (= 20 mA) in the inductor starts decaying through the 200-Ω resistance, so that $i(t) = I_0 e^{-t/\tau}$, with

$$\text{time constant, } \tau = \frac{L}{R} = \frac{5\,\text{mH}}{200\,\Omega} = 25\ \mu\text{s}$$

$$\therefore \qquad v_L(0^+) = -L\frac{di}{dt}\bigg|_{t=0} = -LI_0\left(-\frac{1}{\tau}\right) = \frac{LI_0}{\tau} = \frac{(5\text{ mH})(20\text{ mA})}{25\ \mu\text{s}} = \mathbf{4\ V}$$

(*d*) At t = 20 µs, the value of current i is given as

$$i(t) = I_0 e^{-t/\tau} = (20\text{ mA})e^{-20\,\mu\text{s}/25\,\mu\text{s}} = 20 \times 0.449 = \mathbf{8.98\ mA}$$

Since a constant current flows through 17-Ω resistance, the voltage $v(t) = v(0^+) = \mathbf{0.340\ V}$

(*e*) At t = 50 µs, the value of current i is given as

$$i(t) = I_0 e^{-t/\tau} = (20\text{ mA})e^{-50\,\mu\text{s}/25\,\mu\text{s}} = 20 \times 0.1353 = \mathbf{2.706\ mA}$$

and $\qquad v(t) = v(0^+) = \mathbf{0.340\ V}$

EXAMPLE 8.14

In the circuit of Fig. 8.17*a*, the switch is closed at $t = 0$. Find these currents at t = 5 ms: (*a*) i_L, (*b*) i_x, and (*c*) i_y.

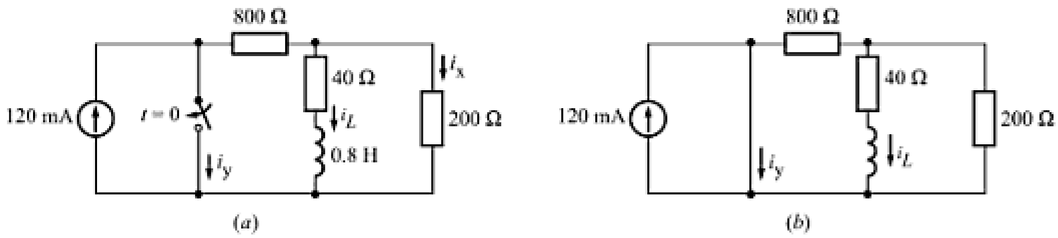


Fig. 8.17

Solution For $t < 0$, under steady-state condition, the inductance acts as a short-circuit. The current 120 mA divides into two parallel branches. Thus,

$$i_L(0^-) = I_{L0} = (120 \text{ mA}) \times \frac{200}{200 + 40} = 100 \text{ mA}$$

(*a*) When switch is closed (Fig. 8.17*b*), the current through the inductor cannot change instantaneously. Hence, $i_L(0^+) = i_L(0^-) = 100$ mA. For $t > 0$, this current acts as a source-current. It decays exponentially through an equivalent resistance given as

$$R_{eq} = 40 + (800 \parallel 200) = 200 \ \Omega$$

The time constant for the decaying circuit, $\tau = L/R_{eq} = 0.8/200 = 4$ ms. Therefore, at $t = 5$ ms, the current in the inductance branch is given as

$$i_L(\text{at } t = 5 \text{ ms}) = I_{L0}e^{-t/\tau} = (100 \text{ mA})e^{-(5 \text{ ms})/(4 \text{ ms})} = \mathbf{28.7 \text{ mA}}$$

(*b*) Due to short-circuit across the current source of 120 mA, no current flows through the 200-Ω resistance branch. The only current in 200-Ω is due to the current i_L (acting as a source current). Hence, by current division,

$$i_x(\text{at } t = 5 \text{ ms}) = -i_L \frac{800}{800 + 200} = -(28.7 \text{ mA}) \times 0.8 = \mathbf{-22.9 \text{ mA}}$$

(*c*) The current in the closed-switch branch flows due to both i_L and current source of 120 mA. Hence, the principle of superposition gives

$$i_y(\text{at } t = 5 \text{ ms}) = i_{y1} + i_{y2} = -i_L \times \frac{200}{200 + 800} + (120 \text{ mA})$$

$$= -(28.7 \text{ mA}) \times 0.2 + (120 \text{ mA}) = \mathbf{114.26 \text{ mA}}$$

EXAMPLE 8.15

After being closed for a very long time, the switch in the circuit of Fig. 8.18*a* is opened at $t = 0$. Find (*a*) $i_L(0^+)$ and $w_L(0^+)$, (*b*) i_L and v_{10} at $t = 1$ s.

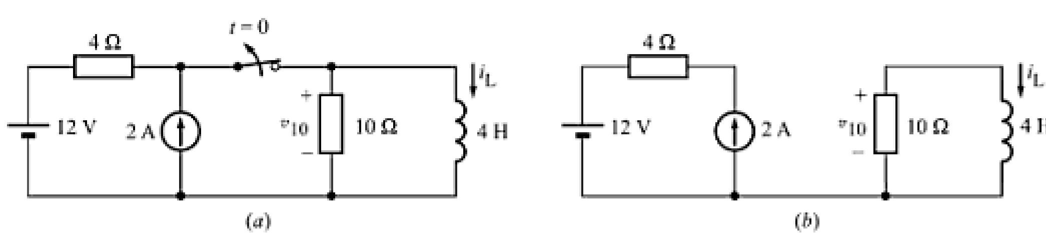


Fig. 8.18

Solution

(*a*) When the switch had remained closed, in the steady-state condition, the inductance behaved as a short-circuit and the current I_{L0} would be due to both the voltage source of 12 V and the current source of 2 A. By principle of superposition, we get

$$I_{L0} = I_{L1} + I_{L2} = \frac{12\text{ V}}{4\ \Omega} + 2\text{ A} = 3\text{ A} + 2\text{ A} = 5\text{ A}$$

Since the current in inductance cannot change instantaneously, we have

$$i_L(0^+) = i_L(0^-) = I_{L0} = \mathbf{5\ A}$$

The energy stored in the inductor is

$$w_L(0^+) = \frac{1}{2}LI_{L0}^2 = \frac{1}{2}\times 4\times 5^2 = \mathbf{50\ J}$$

(*b*) After the switch is opened (Fig. 8.18*b*), the time constant for the decaying current,

$$\tau = \frac{L}{R} = \frac{4}{10} = 0.4\text{ s}$$

At $t = 1$ s, the current in the inductance is given as

$$i_L(t) = I_{L0}e^{-t/\tau} = 5e^{-1/0.4} = \mathbf{0.41\ A}$$

At $t = 1$ s, the voltage across 10-Ω resistance is given as

$$v_{10}(t) = -i_L(t)R = -0.41\times 10 = \mathbf{-4.1\ V}$$

EXAMPLE 8.16

Determine the energy stored in the inductor of the circuit shown in Fig. 8.19 after 20 μs of throwing the switch.

Solution Initial steady-state current in inductance, $I_{L0} = 5$ mA. After the switch is thrown, the current starts decaying, with time constant,

$$\tau = \frac{L}{R} = \frac{5\text{ mH}}{200\ \Omega} = 25\ \mu\text{s}$$

At $t = 20$ μs, the current in the inductance is given as

$$i_L(t) = I_{L0}e^{-t/\tau} = (5\text{ mA})e^{-(20\ \mu\text{s}/25\ \mu\text{s})} = 2.25\text{ mA}$$

Hence, the energy stored in the inductor,

$$w(t) = \frac{1}{2}Li_L^2 = \frac{1}{2}\times(5\text{ mH})\times(2.25\text{ mA})^2 = \mathbf{12.65\ nJ}$$

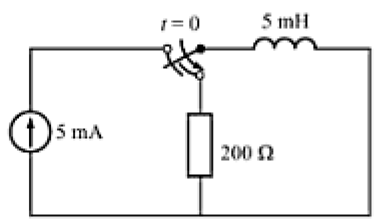


Fig. 8.19

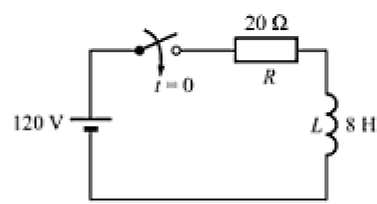


Fig. 8.20

EXAMPLE 8.17

The switch in the circuit of Fig. 8.20 is closed at $t = 0$. Determine (*a*) the current in the circuit at $t = 0.6$ s, and (*b*) the time at which the voltage-drops across R and L are the same.

Solution The final steady-state current, $I_0 = \dfrac{V}{R} = \dfrac{120}{20} = 6$ A

The time constant for the current growth, $\tau = \dfrac{L}{R} = \dfrac{8}{20} = 0.4$ s

Therefore, for $t > 0$, the current in the circuit is given as

$$i(t) = i_L = I_0(1 - e^{-t/\tau}) = 6(1 - e^{-t/0.4}) \text{ A}$$

(*a*) The current in the circuit at $t = 0.6$ s is given as

$$i(0.6 \text{ s}) = 6(1 - e^{-t/0.4}) = 6(1 - e^{-0.6/0.4}) = \mathbf{4.66\ A}$$

(*b*) The voltage across R at any time is given as

$$v_R(t) = iR = 6(1 - e^{-t/0.4})20 = 120(1 - e^{-t/0.4})$$

Whereas the voltage across L at any time is given as

$$v_L(t) = L\frac{di}{dt} = 8\frac{d[6(1 - e^{-t/0.4})]}{dt} = 8 \times 6 \times \frac{1}{0.4}e^{-t/0.4} = 120e^{-t/0.4}$$

Applying the given condition, we get

$$120(1 - e^{-t/0.4}) = 120e^{-t/0.4} \quad \text{or} \quad t = \mathbf{0.277\ s}$$

EXAMPLE 8.18

Determine i_x at $t = -2, 0^-, 0^+, 2$, and 4 ms, in the circuit of Fig. 8.21*a*.

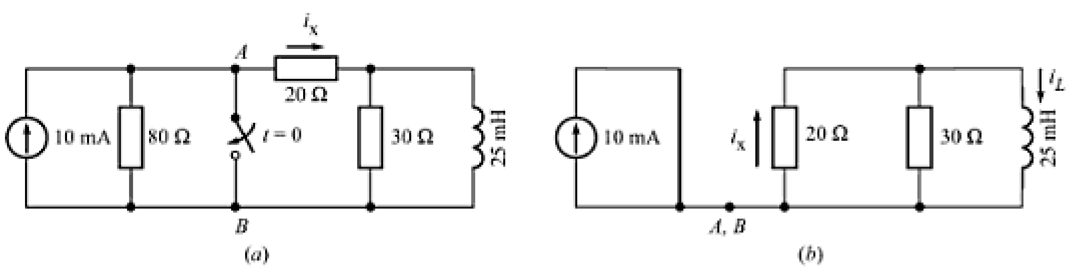


Fig. 8.21

Solution Before the switch is closed, under the steady-state condition the inductor behaves as a short-circuit and the current I_{L0} can be determined by current division,

$$I_{L0} = (10 \text{ mA}) \times \frac{80}{80 + 20} = \mathbf{8\ mA}$$

This is also the current through the inductor for $t < 0$. Therefore,

$$i_x(-2 \text{ ms}) = i_L(0^-) = I_{L0} = \mathbf{8\ mA}$$

Immediately after the switch is closed, the current in the inductor remains the same. That is,

$$i_L(0^+) = i_L(0^-) = I_{L0} = 8 \text{ mA}$$

But, now this works as a current source. For $t > 0$, the equivalent circuit is shown in Fig. 8.21*b*. At $t = 0^+$, by current division, we have

$$i_x(0^+) = i_L(0^+)\frac{30}{30 + 20} = (8 \text{ mA})\frac{30}{30 + 20} = \mathbf{4.8\ mA}$$

The current i_L through the inductor starts decaying through the equivalent resistance of the parallel combination of 20-Ω and 30-Ω,

$$R_{eq} = (20 \parallel 30) = 12\ \Omega \qquad \text{and} \qquad \tau = \frac{L}{R} = \frac{25 \text{ mH}}{12\ \Omega} = 2.08 \text{ ms}$$

So that, for $t > 0$, the current i_L is given by

$$i_L(t) = I_{L0}e^{-t/\tau} = (8 \text{ mA})e^{-t/(2.08 \text{ ms})}$$

Thus, we have

$$i_L(2 \text{ ms}) = (8 \text{ mA})e^{-(2 \text{ ms})/(2.08 \text{ ms})} = 3.05 \text{ mA}$$

and

$$i_L(4 \text{ ms}) = (8 \text{ mA})e^{-(4 \text{ ms})/(2.08 \text{ ms})} = 1.17 \text{ mA}$$

By current division, we have

$$i_x(2 \text{ ms}) = (3.05 \text{ mA})\frac{30}{30+20} = \mathbf{1.83\ mA}$$

and

$$i_x(4 \text{ ms}) = (1.17 \text{ mA})\frac{30}{30+20} = \mathbf{0.702\ mA}$$

EXAMPLE 8.19

Determine $v(0^+)$ and $i(0^+)$ for the circuit of Fig. 8.22.

Solution Before the switch is thrown from a to b (i.e., for $t < 0$), the capacitor has been fully charged to the source voltage, 1.5 V. That is, $v(0^-) = \mathbf{1.5\ V}$. Immediately after the switch is thrown from a to b (at $t = 0^+$), the voltage across the capacitor remains the same, as it cannot change instantaneously. Hence,

$$v(0^+) = V_0 = v(0^-) = \mathbf{1.5\ V}$$

Now, the capacitor starts discharging through the 5-mΩ resistance. Hence, we have

$$i(0^+) = I_0 = \frac{V_0}{R} = \frac{1.5 \text{ V}}{5 \text{ m}\Omega} = 0.3 \text{ kA} = \mathbf{300\ A}$$

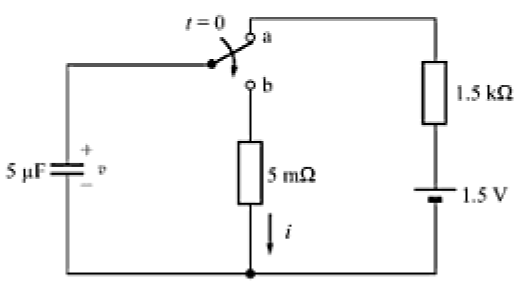


Fig. 8.22

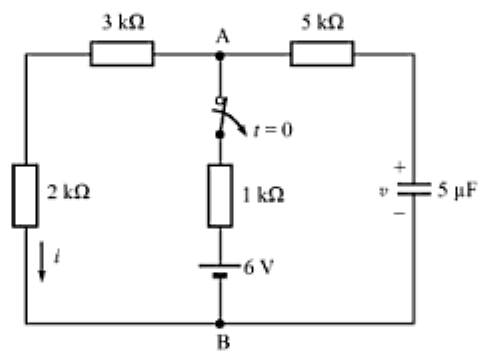


Fig. 8.23

EXAMPLE 8.20

Determine $v(t)$ and $i(t)$ at $t = 0^-$, 0^+, 0.05 s, and 0.10 s, for the circuit shown in Fig. 8.23.

Solution Before the switch is opened, the circuit had attained its steady state condition. The capacitor was fully charged, and hence no current was flowing through it. Therefore, the voltage across the capacitor at $t = 0^-$ is the same as that across points A and B, which can be obtained by voltage division as

$$v(0^-) = V_{AB} = 6 \times \frac{(3+2)}{(3+2)+1} = 5 \text{ V}$$

At $t = 0^-$, the current i is given as

$$i(0^-) = \frac{V}{R} = \frac{6}{(1+3+2)\,\text{k}\Omega} = \mathbf{1\ mA}$$

Since the voltage across the capacitor cannot change instantaneously, we have

$$v(0^+) = V_0 = v(0^-) = \mathbf{5\ V}$$

After the switch is opened, the capacitor acts as a voltage source and starts discharging through the series combination of resistances 5 kΩ, 3 kΩ, and 2 kΩ. Thus,

$$R_{eq} = 5 + 3 + 2 = 10\ \text{k}\Omega \qquad \text{and} \qquad \tau = RC = (10\ \text{k}\Omega) \times (5\ \mu\text{F}) = 0.05\ \text{s}$$

At $t = 0^+$, the current i is given as

$$i(0^+) = I_0 = \frac{V_C}{R_{eq}} = \frac{5\ \text{V}}{10\ \text{k}\Omega} = \mathbf{0.5\ mA}$$

For $t > 0^+$, the voltage v and current i are given by

$$v(t) = V_0 e^{-t/\tau} \qquad \text{and} \qquad i(t) = I_0 e^{-t/\tau}$$

Hence, at $t = 0.05$ s, we have

$$v\,(0.05\ \text{s}) = 5e^{-0.05\ \text{s}/0.05\ \text{s}} = \mathbf{1.834\ V} \qquad \text{and} \qquad i\,(0.05\ \text{s}) = 0.5e^{-0.05\ \text{s}/0.05\ \text{s}} = \mathbf{0.1834\ mA}$$

And at $t = 0.10$ s, we have

$$v\,(0.10\ \text{s}) = 5e^{-0.10\ \text{s}/0.05\ \text{s}} = \mathbf{0.677\ V} \qquad \text{and} \qquad i\,(0.10\ \text{s}) = 0.5e^{-0.10\ \text{s}/0.05\ \text{s}} = \mathbf{0.0677\ mA}$$

EXAMPLE 8.21

Determine $v(0^+)$ and $i(0^+)$ for the circuit of Fig. 8.24*a*.

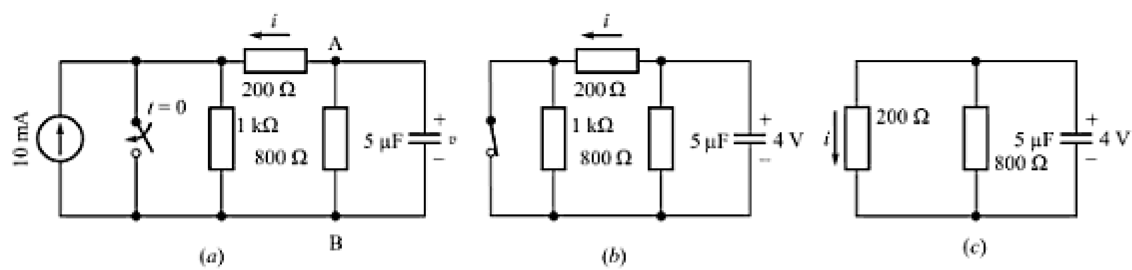


Fig. 8.24

Solution Before the switch is closed, the 10-mA current source forces current into the circuit and charges the capacitor. Once the capacitor is fully charged, no current flows through it. Hence, the steady state voltage across the capacitor is the same as the voltage V_{AB}. The current through branch AB, using current division, is given as

$$I_{AB} = 10\ \text{mA} \times \frac{1000}{1000 + (200 + 800)} = 5\ \text{mA}$$

Thus, the steady state voltage across the capacitor is

$$v(0^-) = V_0 = V_{AB} = I_{AB} \times (800\ \Omega) = (5\ \text{mA}) \times (800\ \Omega) = 4\ \text{V}$$

At $t = 0^+$, the equivalent circuit is shown in Fig. 8.24*b*. Since the voltage across the capacitor cannot change instantaneously, we have

$$v(0^+) = v(0^-) = V_0 = \mathbf{4\ V}$$

The capacitor now acts as a voltage source and supplies current to the circuit. Because of the short due to the closed switch, the circuit changes to that shown in Fig. 8.24*c*. The total discharging current from the capacitor is

$$i_C(0^+) = \frac{4\text{ V}}{(200\ \Omega \| 800\ \Omega)} = \frac{4\text{ V}}{160\ \Omega} = 25\text{ mA}$$

By current division, the current i is given as

$$i(0^+) = (25\text{ mA}) \times \frac{800}{800 + 200} = \mathbf{20\ mA}$$

Alternatively, since the capacitor acts as a voltage source of 4 V at $t = 0^+$, the current i through 200-Ω resistor is given as

$$i(0^+) = \frac{4\text{ V}}{200\,\Omega} = 0.02\text{ A} = \mathbf{20\ mA}$$

EXAMPLE 8.22

In the circuit of Fig. 8.25*a*, the switch is opened at $t = 0$. At $t = 1$ s, determine (*a*) v_C, (*b*) v_R, (*c*) v_{SW}.

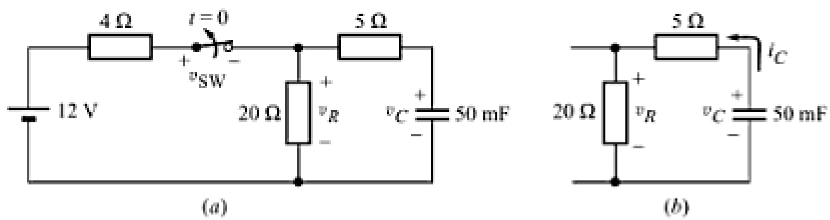


Fig. 8.25

Solution Under steady-state condition, when the capacitor is fully charged no current flows through it. Therefore, by voltage division,

$$v_C(0^-) = v_R(0^-) = 12 \times \frac{20}{20 + 4} = 10\text{ V}$$

Since the voltage across the capacitor cannot change instantaneously, we have

$$v_C(0^+) = v_C(0^-) = V_0 = 10\text{ V}$$

At $t = 0^+$, the equivalent circuit of the given circuit is given in Fig. 8.25*b*. The capacitor starts discharging through an equivalent resistance,

$$R_{eq} = 5 + 20 = 25\ \Omega \quad \text{with} \quad \tau = RC = 25 \times (50 \times 10^{-3}) = 1.25\text{ s}$$

For $t > 0^+$, v_C decays with time as $v_C(t) = V_0 e^{-t/\tau}$. Therefore, at $t = 1$ s, we have

(*a*) $v_C(1\text{ s}) = V_0 e^{-t/\tau} = 10e^{-1/1.25} = \mathbf{4.49\ V}$

(*b*) By potential divider,

$$v_R(1\text{ s}) = v_C \times \frac{20}{20 + 5} = 4.49 \times \frac{20}{20 + 5} = \mathbf{3.59\ V}$$

(*c*) From Fig. 8.16*a*, the voltage across the open switch is given as

$$v_{SW}(1\text{ s}) = 12 - 3.59 = \mathbf{8.41\ V}$$

SUMMARY

TERMS AND CONCEPTS

- The current in an inductor circuit cannot change instantaneously but has to rise or fall exponentially.
- The ***time constant*** (τ) is defined as the time that would be required for the current to rise to its final value

if it continued to rise at its initial rate. In actual practice, the current rises to 63.2 % of its final value in one time-constant

- The time constant of a series *RL* circuit is given as $\tau = L/R$.
- The voltage across a capacitor in a circuit cannot change instantaneously but has to rise or fall exponentially
- ***Charging*** of a capacitor is the process of increasing the charge held in a capacitor.
- ***Discharging*** of a capacitor is the process of reducing the charge held in a capacitor.
- The time constant of a series *RC* circuit is given as $\tau = RC$.
- It takes about five time-constants for the current *i* (in *RL* circuit) and the voltage v_C (in an *RC* circuit) to reach its final steady-state value
- In the calculation of initial values, an *energized inductor* acts as a current source. A special case is an unenergized inductor, which acts as an open circuit.
- In the calculation of final values, an *inductor* acts as a short circuit.
- In the calculation of initial values, an *energized capacitor* acts as a voltage source. A special case is an unenergized capacitor, which acts as a short circuit.
- In the calculation of final values, a *capacitor* acts as an open circuit.

IMPORTANT FORMULAE

- Current growth in an *RL* circuit, $i(t) = I_0(1 - e^{-Rt/L})$, where $I_0 = V_0/R$.
- Current decay in an *RL* circuit, $i(t) = I_0 e^{-t/\tau}$.
- Charging of a capacitor, $v(t) = V_0(1 - e^{-t/\tau})$ and $i(t) = I_0 e^{-t/\tau}$, where $I_0 = V_0/R$.
- Discharging of a capacitor, $v(t) = V_0 e^{-t/\tau}$ and $i(t) = I_0 e^{-t/\tau}$, where $I_0 = V_0/R$.
- The voltages and currents in the *RC* and *RL* circuits,

$$v(t) = v(\infty) + [v(0^+) - v(\infty)]e^{-t/\tau}\text{ V} \quad\text{and}\quad i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}\text{ A.}$$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	When a series *RC* circuit is connected to a dc voltage, the circuit current reaches 63.2 % of its final steady-state value in one time-constant.	☐	☐	
2.	When a resistance *R* is connected to a dc voltage V_0, the current instantaneously rises to the value V_0/R.	☐	☐	
3.	When a series *RL* circuit is switched to a dc voltage V_0, the current at the instant of switching is zero.	☐	☐	
4.	When the value of *L* is in henrys and of *R* is in ohms in a series *RL* circuit, its time constant is *R*/*L* seconds.	☐	☐	
5.	When a series *RL* circuit is switched on to a dc voltage, the initial rate of change of current in the circuit is minimum at $t = 0$.	☐	☐	
6.	When a series *RC* circuit is connected to a dc voltage, the charging current reduces to less than 1 % of its initial value in five time-constants.	☐	☐	
7.	When a series *RL* circuit carrying current I_0 is suddenly open-circuited, the current at $t = 0^+$ is zero.	☐	☐	

8.	When a series *RC* circuit is switched on to a dc voltage, the current is zero at $t = 0^+$.	☐	☐	
9.	During discharging of a capacitor through a resistor, the voltages across *R* and *C* are equal.	☐	☐	
10.	The current flowing in a series *RL* circuit cannot change instantaneously.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** False
6. True	**7.** False	**8.** False	**9.** True	**10.** True

REVIEW QUESTIONS

1. With reference to the rise of current in a series *RL* circuit, answer the following:
   (*a*) What prevents the current from rising to its maximum value instantaneously?
   (*b*) What determines the value of maximum current?
   (*c*) What are the factors that decide how fast the current will reach its maximum value?
   (*d*) Does the supply voltage affect the time taken by the current to reach its final value?

2. Define time constant for a series *RL* circuit and explain its significance.

3. With reference to the energy stored in the inductor, explain why the time constant of a series *RL* circuit increases (*i*) by increasing the value of inductance, and (*ii*) by decreasing the value of resistance.

4. A series *RL* circuit is connected to a dc voltage. (*a*) Does the resistor take energy from the dc supply? If yes, what happens to this energy? (*b*) Does the inductance take energy from the dc supply? If yes, what happens to this energy? (*c*) When the circuit is opened, what happens to the energy taken by (*i*) the resistance, and (*ii*) the inductance?

5. Derive the necessary equation (*a*) for the rise of current with time in an inductive circuit when it is connected to a dc supply, and (*b*) for the decay of current with time in an inductive circuit when it is disconnected from the dc supply.

6. A series *RC* circuit is switched on to a dc voltage V_0 at $t = 0$. Answer the following:
   (*a*) Explain why the current becomes maximum at $t = 0^+$.
   (*b*) Explain why the current decreases with time.
   (*c*) Explain why the voltage across the capacitor does not rise to full value instantaneously.
   (*d*) What is the final value of the voltage across the capacitor?

7. With reference to the energy stored in the electric field in a capacitor, explain why the time constant of a series *RC* circuit increases (*i*) by increasing the value of capacitance, and (*ii*) by increasing the value of resistance.

8. What is the time constant of an *RC* circuit? Show that its units are the same as that of time. What does this quantity signify with regards to capacitor voltage when a capacitor is being charged?

9. Derive the equation for the rise in voltage across a capacitor when it is connected to a dc voltage source.

10. Explain why a high voltage is induced when an inductive circuit is switched off.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. In a series *RL* circuit excited by a battery, the final value of current in the circuit does not depend on
   (*a*) battery emf *E* (*b*) resistance *R*
   (*c*) inductance *L* (*d*) both *E* and *L*
2. In a series *RL* circuit excited by a dc voltage V_0, the initial rate of rise of current is
   (*a*) $\frac{V_0}{L}$ A/s (*b*) $\frac{V_0}{R}$ A/s
   (*c*) $\frac{R}{L}$ A/s (*d*) 0
3. When a circuit is switched off, sparking may occur if the circuit is
   (*a*) highly resistive (*b*) highly inductive
   (*c*) highly capacitive (*d*) all of the above
4. For a series *RC* circuit excited by a dc voltage V_0, the initial current is
   (*a*) zero (*b*) infinity
   (*c*) V_0/R (*d*) V_0/C
5. When a series *RC* circuit is connected to a dc voltage V_0, the final steady state current is
   (*a*) zero (*b*) infinity
   (*c*) V_0/R (*d*) V_0/C
6. A series *RL* circuit is connected to a dc voltage V_0 at $t = 0$. The current in the circuit attains a maximum value at
   (*a*) $t = 0$ (*b*) $t = L/R$
   (*c*) $t = 0.5L/R$ (*d*) $t = \infty$

ANSWERS

1. *c* **2.** *a* **3.** *b* **4.** *c* **5.** *a* **6.** *d*

PROBLEMS

(A) SIMPLE PROBLEMS

1. In a series *RL* circuit, find the ratio: (*a*) $i(2\tau)/i(0)$, (*b*) $i(4\tau)/i(2\tau)$, (*c*) t/τ if $i(t) = 0.1i(\tau)$, and (*d*) t/τ if $i(t) - i(\tau) = 0.1i(0)$.
   [**Ans.** (*a*) 0.1353; (*b*) 0.1353; (*c*) 3.30; (*d*) 0.759]
2. An inductor coil has an inductance of 15 H and a resistance of 10 Ω. It is suddenly connected to a dc supply of 20 V. Calculate (*a*) the time constant of the circuit, (*b*) the initial rate of change of current, (*c*) the current after 2 seconds, (*d*) the rate of change of current after 2 seconds, and (*e*) the energy stored in the magnetic field in 2 seconds.
   [**Ans.** (*a*) 1.5 s; (*b*) 1.33 A/s; (*c*) 1.47 A; (*d*) 0.351 A/s; (*e*) 16.2 J]
3. A circuit having a 100-mH inductance in series with a 500-Ω resistance is connected to a 50-V dc source. Determine the instantaneous values of circuit currents during the period from 0 to 1 ms at an interval of 0.2 ms.
   [**Ans.** 0, 63.2 mA, 86.5 mA, 95 mA, 98.2 mA, 99.3 mA]
4. Determine the current in a series *RL* circuit having $R = 3$ Ω and $L = 12$ H, at $t = 6$ s after a dc supply of 120 V is connected to it. [**Ans.** 31.07 A]
5. A series *RC* circuit has a resistance of 90 kΩ and a capacitance of 25 μF. It is connected across a 160-V battery at $t = 0$. Calculate (*a*) the time constant of the circuit, (*b*) $i(t)$ at $t = 0^+$, 5 s, and ∞.
   [**Ans.** (*a*) 2.25 s; (*b*) 1.778 mA, 0.193 mA, 0]
6. A circuit having a 120-μF capacitor and a 10-kΩ resistance in series is connected across a 230-V dc supply. Determine (*a*) the initial rate of rise of voltage across the capacitor, (*b*) the initial charging current, (*c*) the final charge stored in the capacitor, and (*d*) the ultimate energy stored in the capacitor.
   [**Ans.** (*a*) 191.67 V/s; (*b*) 23 mA; (*c*) 0.0276 C; (*d*) 3.174 J]
7. An 8-μF capacitor is connected in series with a 0.5-MΩ resistor across a 200-V dc supply. Calculate (*a*) the time constant, (*b*) the initial charging current, (*c*) the time taken for the potential difference across the capacitor to grow to 160 V, and (*d*) the current

and pd across the capacitor, 4 s after it is connected to the supply.

[**Ans.** (*a*) 4 s; (*b*) 400 μA; (*c*) 6.44 s; (*d*) 147 μA, 126.4 V]

8. A 20-μF capacitor is charged at a constant current of 5 μA for 10 min. Calculate the final pd across the capacitor and the corresponding charge on it.

[**Ans.** 150 V, 3 mC]

9. A 2-μF capacitor is connected in series with a 1-kΩ resistor. Calculate the values of charging current and capacitor voltage at t = 0, 2, 4, 6, and 8 ms from the instant the circuit is switched on to a 10-V dc supply.

[**Ans.** 10, 3.68, 1.35, 0.5, 0.18 mA; 0, 6.32, 8.65, 9.5, 9.82 V]

10. A 1-μF capacitor has been charged to 100 V. Calculate the value of the resistance that must be connected across this capacitor for discharging it so that the initial discharge current is limited to 1 mA. [**Ans.** 100 kΩ]

11. A 16-μF capacitor is charged to 30 V. How long will it take for its voltage to fall from 8 V to 5 V, if it is discharged through a 1.5-kΩ resistor?

[**Ans.** 11.28 ms]

(B) TRICKY PROBLEMS

12. A capacitor of 10 μF is connected to a dc supply through a resistance of 1.1 MΩ. Calculate the time taken for the capacitor to reach 90 % of its final charge. [**Ans.** 25.33 s]

13. A 3.5-μF capacitor in series with a resistance R is connected through a switch across a 230-V dc supply. A voltmeter is connected across the capacitor. What should be the value of R so that the voltmeter reading is 165 V, 5.65 s after the switch is closed? [**Ans.** 1.281 MΩ]

14. A 2-μF capacitor and a resistance R are connected in series across a 150-V dc supply. A neon lamp which strikes (becomes on) at 90 V is connected across the capacitor. (*a*) What should be the value of R so that the lamp strikes 5 seconds after the switch is closed? (*b*) If R = 1 MΩ, how much time will the lamp take to strike?

[**Ans.** (*a*) 2.73 MΩ; (*b*) 1.83 s]

15. A 10-μF capacitor charged to 40 V is discharged through a 0.2-MΩ resistance. Find (*a*) the time constant of the circuit, (*b*) the time taken by the current to decrease to 50 % of its initial value, and (*c*) the total energy dissipated in the resistor during the transient period, and (*d*) the total energy initially stored in the capacitor.

[**Ans.** (*a*) 2 s; (*b*) 1.4 s; (*c*) 8 mJ; (*d*) 8 mJ]

16. A 12-μF capacitor is allowed to discharge through its own leakage resistance. Using an electronic voltmeter, it is observed that the voltage across the capacitor falls from 120 V to 100 V in 300 s. Calculate the leakage resistance of the capacitor.

[**Ans.** 137 MΩ]

(C) CHALLENGING PROBLEMS

17. For the circuit of Fig. 8.26, determine the steady-state current when the switch S is open. Calculate the value of current at t = 0.8 s after the switch is closed. [**Ans.** 1 A, 1.465 A]

18. In the network of Fig. 8.27, the switch is closed on to position a when t = 0 and then moved to position b when t = 20 ms. Determine the voltage across the capacitor when t = 30 ms. [**Ans.** 1.14 V]

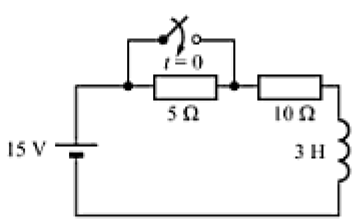


Fig. 8.26

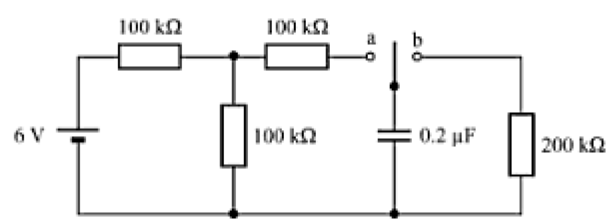


Fig. 8.27

ALTERNATING VOLTAGE AND CURRENT

9

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the requirements for a waveform to be periodic.*
- *Relate the period of a waveform with its frequency.*
- *Define 'phase angle' and 'phase difference' with reference to various sinusoidal voltages and currents in an ac circuit.*
- *State the meaning of average value and effective value of a periodic waveform.*
- *Explain why the effective value is also called rms value.*
- *State the difference among a vector, a complex number and a phasor.*
- *State the difference between apparent power and real power in an ac circuit.*
- *Define power factor.*
- *State the phasor relationship between current and voltage for (a) an inductor, and (b) a capacitor.*

9.1 INTRODUCTION

Today, the vast majority of electrical power is generated, distributed, and consumed in the form of ac power. The term 'ac' means 'alternating current'. Such a current reverses its direction *periodically*. In ac power, the voltages and currents vary with time *sinusoidally*. Such a variation is shown graphically in Fig. 9.1, and is mathematically represented as

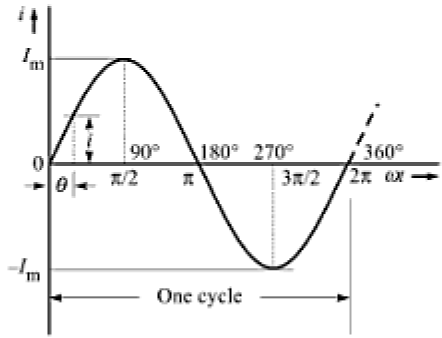
$$i = I_m \sin \omega t \tag{9.1}$$

You may wonder why we should select only the sinusoidal variation. There are many technical and economical reasons for doing this.

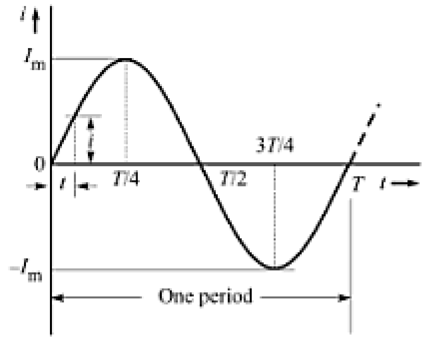
(*i*) A sinusoidal wave, even after repeated differentiation or integration, remains a sinusoidal of the same frequency. Furthermore, the sum or difference of a number of sinusoids of same frequency but of different amplitudes and phase angles is a sinusoid of the same frequency. It is because of this fact that when a sinusoidal voltage is applied to a circuit containing linear passive elements, all currents and voltages in the circuit are also sinusoidal.

(*ii*) Applying sinusoidal voltage to properly designed coils results in a revolving magnetic field, which is capable of doing work. Majority of electric motors used in commercial and industrial applications work on this principle.

(*iii*) Use of sinusoidal voltages underlines the operation of transformers which enable bulk power transmission at high voltages over a long distance.



(a) Current i versus angle ωt.



(b) Current i versus time t.

Fig. 9.1 *A sinusoidal current.*

The best thing about the sinusoidal is that when a sinusoidal current flows through a capacitor, an inductor, or a transformer, the voltage across the element is also a sinusoidal. This is not true for any other waveform. If an electric company generates a sinusoidal waveform of voltage, the waveforms for all its customers are sinusoids too.

9.2 SINUSOIDAL FUNCTIONS—TERMINOLOGY

Sine wave is the basic waveform used in ac circuits. Let us therefore define certain terms associated with sinusoidal waveform, shown in Fig. 9.1.

(1) *Cycle* The values of sine wave repeat after every 2π radians. One complete set of positive and negative values of the function (which goes on repeating) is called a *cycle*. As seen in Fig. 9.1, in one cycle the value of current i increases from zero to a maximum value, decreases through zero to a negative maximum value, and then again increases towards zero.

(2) *Maximum* (*or Peak Value*) It is the maximum value (denoted as I_m), positive or negative, of the quantity. It is also sometimes called the ***amplitude*** of the sinusoid.

(3) *Instantaneous Value* It is the value of the quantity at any instant.

(4) *Time Period* (*or Periodic Time*) It is the duration of time required for the quantity to complete one cycle. It is denoted as T.

(5) *Frequency* It is the number of cycles that occur in one second. It is denoted as f or v. The unit of frequency is hertz (Hz) which is same as cycles/second. Obviously, frequency is the reciprocal of time period,

$$f = \frac{1}{T} \qquad (9.2)$$

The frequency of ac supply in India is 50 Hz, which corresponds to time period, $T = 1/50 = 0.02$ s or 20 ms. In USA, the frequency of ac supply is 60 Hz. AC supply of 400 Hz is used in airborne and some naval applications[1].

[1] Because motors and transformers are smaller and lighter at higher frequency.

(6) *Angular Frequency* The values of a sine function repeat after every 2π radians. Angular frequency, denoted as ω, is equal to the number of radians covered in one second. Its unit is rad/s. Since one cycle covers 2π radians and there are f cycles in one second, the angular frequency is given as

$$\omega = 2\pi f \quad \text{or} \quad \omega = \frac{2\pi}{T} \tag{9.3}$$

(7) *Alteration* It is one-half of the cycle, when it includes either all positive or all negative values.

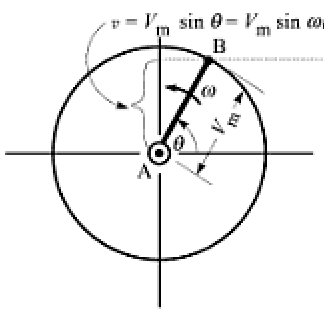
(8) *Phase* It is the fraction of the time-period or cycle that has elapsed since it last passed from the chosen zero position or origin. The phase at time t from the chosen origin is given by t/T.

(9) *Phase angle* It is the equivalent of phase expressed in radians or degrees. It is denoted as θ. Thus, phase angle, $\theta = 2\pi t/T$. As can be seen in Fig. 9.1*a*, the maximum value of sinusoidal current occurs at a phase angle of π/2 radians or 90°.

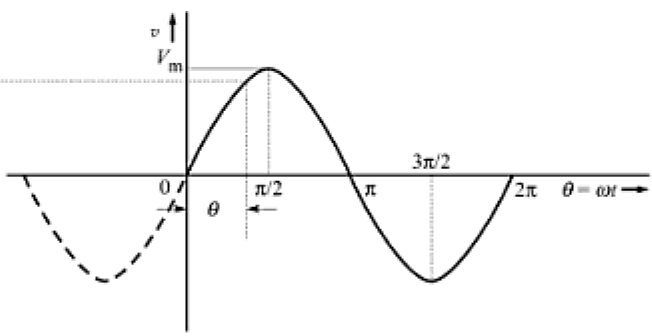
Physical Model for a Sinusoid

In earlier classes, we were introduced to *sines* and *cosines* through the study of a right-angle triangle. The sine of the angle (between the base and hypotenuse) equals to the ratio of the perpendicular to the hypotenuse; the cosine is the ratio of the base to the hypotenuse. There is another way to learn about these functions.

You can get a sinusoidal waveform for an ac voltage by imagining a crank (a bar or a rod) AB, as shown in Fig. 9.2*a*. Let this crank rotate counterclockwise about point A with a uniform angular speed ω. It describes an angle $\theta = \omega t$ in time t, starting from the instant when the crank was horizontal. The vertical projection of the crank is its length times the sine of the angle θ. As the crank rotates, the vertical projection generates a sine wave function of time (see Fig. 9.2*b*). If the length of the crank is V_m, the sinusoidal waveform generated can be expressed as $i = I_m \sin \omega t$



(*a*) A rotating crank.



(*b*) Sinusoidal waveform.

Fig. 9.2

Mathematically, this waveform can be described as a sine function or a cosine function, but we simply call it a ***sinusoid***, or sinusoidal waveform. The term 'sinusoid' merely denotes the type of variation. We have adopted the sine function as the standard mathematical form for a sinusoidal waveform.

Phase and Phase Difference

If the rotating crank had an initial (i.e., at time $t = 0$) counterclockwise displacement of ϕ degrees, the resulting waveform would be as shown in Fig. 9.3*a*. Then the initial instantaneous voltage (at time $t = 0$) becomes $V_0 = V_m \sin \phi$. This shift of the waveform on the time axis or angle axis is called ***phase shift*** and the angle ϕ is called ***phase angle***. The phase is related to the time origin when we use a mathematical description. However, in an ac circuit, what matters are the ***phase differences*** of the various sinusoidal voltages and currents.

We can write a general expression for a sinusoidal voltage by incorporating the phase shift, as

$$v = V_m \sin(\omega t + \phi) \tag{9.4}$$

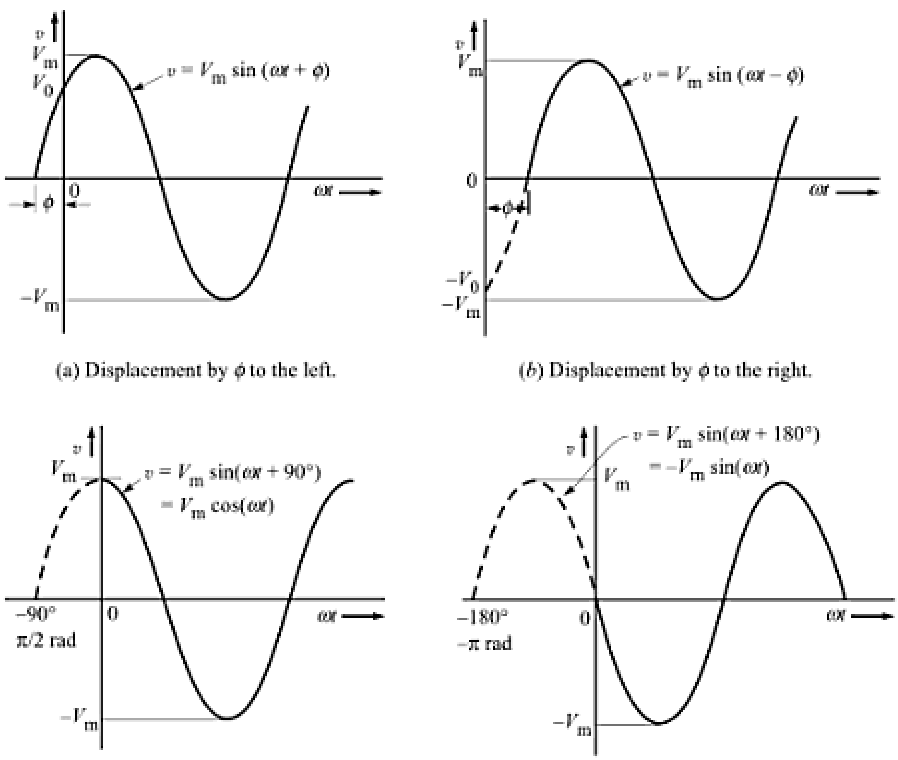
If ϕ is a positive number, the waveform is shifted towards left, and ϕ is known as the *angle of lead* as shown in Fig. 9.3*a*. On the other hand, if ϕ is a negative number, the waveform is shifted towards right, and ϕ is known as the *angle of lag* as shown in Fig. 9.3*b*. In Fig. 9.3*c*, the angle of lead is 90°, and Eq. 9.4 becomes

$$v = V_m \sin(\omega t + 90°) = V_m \cos \omega t$$

The waveform in Fig. 9.3*c* is also seen to be a cosine wave. In Fig. 9.3*d*, the angle of lead is 180°, and Eq. 9.4 becomes

$$v = V_m \sin(\omega t + 180°) = -V_m \sin \omega t$$

Also, as it should be, the waveform in Fig. 9.3*d* is negative of sine wave.



(*a*) Displacement by ϕ to the left.

(*b*) Displacement by ϕ to the right.

(*c*) Displacement by 90° to the left.

(*d*) Displacement by 180° .

Fig. 9.3 *Displaced sinusoidal voltage waveforms.*

Note that the resulting position of the waveform is the same whether the phase shift is 180° (π rad) to the left, or 180° to the right. It means that a phase lead of 180° is the same as the phase lag of 180°. This is so because the sinusoids are generated by a rotating crank. This concept can be extended further. A phase lead of 90° (as in Fig. 9.3*c*) and a phase lag of 270° (= 360° – 90°) is one and the same thing.

NOTE

There are some traditional inconsistencies in electrical engineering which one must tolerate. The mathematical unit for ωt is radians and hence the correct unit for phase should also be radians. However, *it has become normal practice in electrical engineering to express* ωt *in radians and the angle* ϕ *in degrees.*

EXAMPLE 9.1

A sinusoidal voltage is given by $v = 20\sin\omega t$ volts. (*a*) At what angle will the instantaneous value of voltage be 10 V? (*b*) What is the maximum value for the voltage and at what angles?

Solution

(*a*) The angle can be determined by using the given equation,

$$10 = 20\sin\omega t \quad \Rightarrow \quad \theta = \omega t = \sin^{-1}(10/20) = \mathbf{30°}$$

(*b*) The maximum value is $V_m = \mathbf{20\ V}$. This occurs twice in one cycle when $\sin\omega t = \pm 1$, that is at the instants when $\omega t = \mathbf{90°}$ or $\mathbf{270°}$.

EXAMPLE 9.2

The equation for an ac voltage is given as $v = 0.04\sin(2000t + 60°)$ V. Determine the frequency, the angular frequency, and the instantaneous voltage when $t = 160\ \mu s$. What is the time represented by a 60° phase angle?

Solution Comparing the given equation with the general equation given in Eq. 9.3, we get

$$\omega = 2\pi f = \mathbf{2000\ rad/s} \qquad \text{and} \qquad f = \frac{\omega}{2\pi} = \frac{2000}{2\pi} = \mathbf{318.3\ Hz}$$

The instantaneous value of the voltage at $t = 160\ \mu s$ is

$$v = 0.04\sin(2000 \times 160 \times 10^{-6}\ \text{rad} + 60°)\ \text{V} = 0.04\sin(0.32\ \text{rad} + 60°)\ \text{V}$$

$$= 0.04\sin\left(0.32 \times \frac{180°}{\pi} + 60°\right)\ \text{V} = 0.04\sin(18.3° + 60°)\ \text{V} = 0.0392\ \text{V} = \mathbf{39.2\ mV}$$

$$\text{Time period, } T = \frac{1}{f} = \frac{1}{318.3} = 3.14\ \text{ms}$$

Thus, full-cycle of 360° corresponds to 3.14 ms. Therefore, the angle 60° corresponds to

$$t = \frac{60°}{360°} \times 3.14\ \text{ms} = \mathbf{0.52\ ms}$$

EXAMPLE 9.3

A sinusoidal voltage is 20 V peak-to-peak, has a time of 5 ms between consecutive peak and trough, and at $t = 0$ is –3.6 V and decreasing. Find the equation for the instantaneous value of the voltage, and the value of the voltage at $t = 12$ ms.

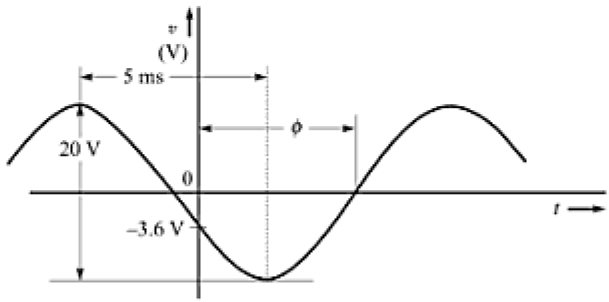


Fig. 9.4

Solution Figure 9.4 shows the sinusoid. The maximum value or peak value is half of the peak-to-peak value. The period is the time between the two consecutive peaks. Thus,

$$V_m = 20/2 = 10\text{ V}; \quad T = 2 \times 5\text{ ms} = 10\text{ ms}; \quad \therefore \quad f = \frac{1}{T} = \frac{1}{10\text{ ms}} = \frac{1}{10 \times 10^{-3}} = 100\text{ Hz}$$

The angular frequency is $\omega = 2\pi f = 2\pi \times 100 = 628.3$ rad/s. Therefore, the sinusoid voltage is of the form

$$v = 10\sin(628.3t + \phi)$$

Now, we are given that at $t = 0$, $v = -3.6$ V. Putting these values, we get

$$-3.6 = 10\sin(\phi) \quad \text{or} \quad \sin\phi = -0.36 \quad \Rightarrow \quad \phi = -158.9° \quad \text{or} \quad 338.9°$$

As can be seen from Fig. 9.4, the given sinusoid is shifted to right by angle ϕ, which is less than 180°. Also, we know that shifting rightward means there is a lag of angle ϕ. Therefore, the equation of the given sinusoidal voltage is

$$v = 10\sin(628.3t - 158.9°)\text{ V}$$

The value of voltage at 12 ms would be

$$v = 10\sin\left(62.8.3 \times 0.012 \times \frac{180°}{\pi} - 158.9°\right)\text{V}$$

$$= 10\sin(432° - 158.9°)\text{ V} = 10\sin(273.1°)\text{ V} = \mathbf{-9.985\ V}$$

EXAMPLE 9.4

An alternating current of frequency 60 Hz has a maximum value of 12 A. (*a*) Write down the equation for its instantaneous value. (*b*) Find the value of current after 1/360 second. (*c*) Find the time taken to reach 9.6 A for the first time.

Solution

(*a*) The angular frequency, $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s. Therefore, the equation for the instantaneous value is given as

$$i = 12\sin 377t\text{ A} \tag{i}$$

(*b*) The value of current at $t = 1/360$ second is

$$i = 12\sin\left(377 \times \frac{1}{360} \times \frac{180°}{\pi}\right)\text{A} = \mathbf{10.4\ A}$$

(*c*) Putting $i = 9.6$ A in Eq. (*i*), we have

$$9.6 = 12\sin\left(377t \times \frac{180°}{\pi}\right) \quad \Rightarrow \quad 377t \times \frac{180°}{\pi} = 53.13°$$

$$\therefore \qquad t = \frac{53.13\pi}{377 \times 180} = \mathbf{2.46\ ms}$$

Average Value

An *average value*, by definition, is the algebraic sum of all the values divided by the total number of values. This is exactly what you do while finding the average marks for a class of students. A waveform is a continuous variation of the value of a quantity with time t (or angle θ), repeated after each cycle. The area under the waveform is found by integration and full cycle is normally taken as 2π radians or T seconds. Thus, the average value V_{av} of the instantaneous voltage v, taken over full cycle is given as

$$V_{av} = \frac{\text{Area under full cycle}}{\text{Length of one cycle}} = \frac{\int_0^{2\pi} v\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} v\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t) \tag{9.5}$$

or

$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt \tag{9.6}$$

Consider the full cycle of a sinusoidal wave shown in Fig. 9.1. It has same area of the positive and negative loops. Hence, the algebraic sum of the two areas is zero. Accordingly, *the average value of a sine wave over a full cycle is zero.* This average value does not convey any useful information about the waveform. Therefore, for a sinusoid, we define average value over half-cycle only. In fact, whenever we talk of the ***average value of a symmetric periodic waveform,*** *it is understood to be determined for only half-cycle, either positive or negative.*

Thus, ***the average value of a sinusoidal current*** i, depicted in Fig. 9.5*a*, is determined as follows.

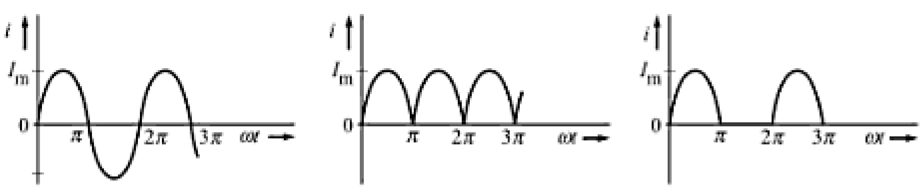
$$I_{av} = \frac{\text{Area under half cycle}}{\text{Length of half cylcle}} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t) = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\, d(\omega t)$$

$$= \frac{I_m}{\pi}[-\cos \omega t]_0^{\pi} = \frac{I_m}{\pi}[-\cos \pi + \cos 0°] = \frac{2I_m}{\pi} = \mathbf{0.637}I_m \tag{9.7}$$

Incidentally, the average value of the current at the output of a full-wave rectifier (Fig. 9.5*b*) is same as above. For this waveform, one cycle is from 0 to π, as it repeats itself after π. Figure 9.5*c* shows the current at the output of a half-wave rectifier. It varies sinusoidally from 0 to π, and remains zero from π to 2π. Therefore, the average value of this current is

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t) = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin \omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}[-\cos \omega t]_0^{\pi} = \frac{I_m}{\pi} = \mathbf{0.318}I_m \tag{9.8}$$



(*a*) Sinusoidal ac current. (*b*) Full-wave rectifier output. (*c*) Half-wave rectifier output.

Fig. 9.5 *Different ac waveforms*

It is obvious that the average value of half-wave rectified current is just half of the average of full-wave rectified current.

Effective or RMS Value

The effective value of an ac current is defined on the basis of its heating effect. Let an ac sinusoidal current $i = I_m \sin \omega t$ flow through a resistor R (Fig. 9.6*a*). Our aim is to find that value of dc current I which gives the same amount of heating, if it flows through a resistor of the same value R (Fig. 9.6*b*). This would be the effective value of the ac sinusoidal current.

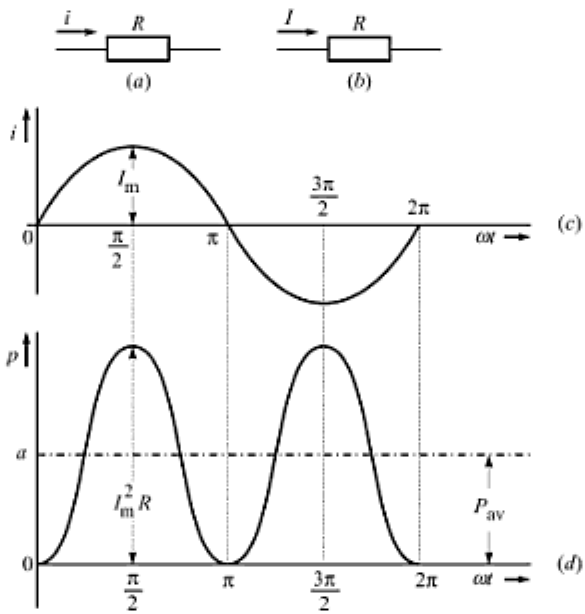


Fig. 9.6 *Power in a resistor: (a) instantaneous current, (b) effective current, (c) current waveform, (d) instantaneous power waveform.*

Figure 9.6*c* shows one cycle of current i. Since, power $p = i^2R$, we can get the variation of instantaneous power with ωt, as shown in Fig. 9.6*d*. Note that two cycles of variation of p are accommodated from 0 to 2π. It means that the power p varies at double the frequency, but it remains positive all the time. It has an average value, P_{av}.

Let us determine the average power, P_{av}. The instantaneous value of the heating power p due to the flow of current i in resistor R is given as

$$p = i^2R = (I_m^2 \sin^2 \omega t)R = I_m^2 R \times \frac{(1 - \cos 2\omega t)}{2} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t \tag{9.9}$$

There is no need to integrate the above expression to find its average value. A closer look shows that the first term in the above expression is constant, (i.e., independent of t). The second term is a cosine function varying with time at a frequency of 2ω. The average value of this term is zero. Hence, the average value of power p is

$$P_{av} = \frac{I_m^2}{2} R$$

The power in resistor R due to dc current I is I^2R. Hence, we put

$$I^2R = \frac{I_m^2}{2} R \quad \text{or} \quad I^2 = \frac{I_m^2}{2}$$

Taking square root, we get the effective value of ac sinusoidal current as

$$I_{eff} = \frac{I_m}{\sqrt{2}} = \mathbf{0.707} I_m \tag{9.10}$$

RMS *Value* Let us examine the procedure of finding the effective value of ac current i. We first found the instantaneous power p absorbed in resistor R, as $p = i^2R$. We then found the average of this power over one cycle, as follows.

$$P_{av} = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T i^2 R\,dt = \left[\frac{1}{T}\int_0^T i^2\,dt\right] R$$

This power, we equated to $I_{eff}^2 R$. Thus, we found that

$$I_{eff}^2 = \frac{1}{T}\int_0^T i^2\,dt \quad \Rightarrow \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\,d(\omega t)} \tag{9.11}$$

The quantity under the square root is the average or mean of the squared function i^2. We therefore conclude that the effective value of an ac wave is the square *root* of the *mean* of the *squared* function. Or, simply I_{eff} is the ***root mean square*** **(rms)** value, which is denoted by symbol I_{rms}.

When we say that the domestic ac power supply is 220 V, it means it is a sinusoidal ac voltage of rms or effective value of 220 V. Its peak value, according to Eq. 9.10, is

$$V_m = \sqrt{2} V_{eff} = \sqrt{2} \times 220 = \mathbf{311\ V}$$

Since it is the effective value of voltage and current which is used in everyday usage, the symbols V and I (without any subscript) are taken to mean the effective values. The subscript 'eff' or 'rms' is used only when there is a possibility of some confusion.

It is interesting to **note** that the rms value is always greater than the average value, except for a rectangular wave. For a rectangular wave, since the heating effect remains constant with time, both the rms and average values are same.

Let us find the rms value of the waveforms shown in Fig. 9.5*b* and *c*. For ***full-wave rectifier*** (Fig. 9.5*b*), one cycle is of only π. Hence, the *rms value* is given as

$$I_{eff} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\,d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t\,d(\omega t)} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} (1-\cos 2\omega t)\,d(\omega t)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \frac{I_m}{\sqrt{2}}$$

It is not surprising that this is the same as the rms value of full sinusoidal ac wave of Fig. 9.5*a*. The two waveforms are the same except that the value of i from π to 2π is positive for the full-wave rectifier output. Squaring positive or negative values of same magnitude gives the same result.

For ***half-wave rectifier*** (Fig. 9.5*c*), the *rms value* is given as

$$I_{\text{eff}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_{\text{m}}^2 \sin^2 \omega t\, d(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0\, d(\omega t)}$$

$$= \sqrt{\frac{I_{\text{m}}^2}{4\pi}\int_0^{\pi} (1-\cos 2\omega t)\, d(\omega t) + 0} = \sqrt{\frac{I_{\text{m}}^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \frac{I_{\text{m}}}{\sqrt{4}} = \frac{I_{\text{m}}}{2}$$

Form Factor and Peak Factor

The ratio of the effective value to the average value is known as the *form factor* of a waveform of any shape (sinusoidal or nonsinusoidal). Thus,

Form factor, $$K_{\text{f}} = \frac{V_{\text{rms}}}{V_{\text{av}}} \tag{9.12}$$

The ***peak factor*** or ***crest factor*** or ***amplitude factor*** of a waveform is defined as the ratio of its peak (or maximum) value to its rms value. Thus,

Peak factor, $$K_{\text{p}} = \frac{V_{\text{m}}}{V_{\text{rms}}} \tag{9.13}$$

Let us calculate these two factors for ***a sinusoidal voltage waveform***,

$$K_{\text{f}} = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{V_{\text{m}}/\sqrt{2}}{2V_{\text{m}}/\pi} = \frac{0.707\,V_{\text{m}}}{0.637\,V_{\text{m}}} = \mathbf{1.11}$$

And $$K_{\text{p}} = \frac{V_{\text{m}}}{V_{\text{rms}}} = \frac{V_{\text{m}}}{V_{\text{m}}/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$$

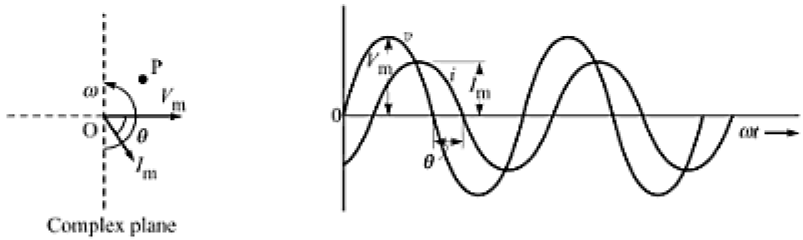
Importance of the Form Factor and Peak Factor The form factor is not of much significance, except when calculating hysteresis loss for different waveforms. An ac magnetizing current with higher form factor produces higher hysteresis loss per cycle. The peak factor of an ac voltage assumes importance while testing the dielectric insulation, since the dielectric stress produced depends on the maximum or peak value of the voltage and not on the rms value.

9.3 CONCEPT OF PHASORS

We have seen in Fig. 9.2 how a rotating crank produces a sinusoidal waveform. We call this rotating crank a *phasor*. Thus, a phasor is just a bar that produces the time-function sinusoid when rotated counterclockwise at proper angular frequency (ω) and projected on the vertical axis.

Figure 9.5*a* shows a voltage phasor of amplitude V_{m} drawn in the reference direction (positive real axis), and a current phasor of amplitude I_{m} drawn at a negative angle θ with the reference direction. When both these phasors rotate counterclockwise, time-function voltage and current sinusoids are produced as shown in Fig. 9.7*b*. These time functions can be represented mathematically as

$$v(t) = V_{\text{m}} \sin \omega t \qquad \text{and} \qquad i(t) = I_{\text{m}} \sin(\omega t - \theta)$$



(*a*) Rotating voltage and current phasors.

(*b*) Waveforms produced.

Fig. 9.7 *Concept of phasors.*

Phasor Diagram It is a diagram containing the phasors of inter-related sinusoidal voltages and currents, with their phase differences indicated. Thus, Fig. 9.7*a* is a phasor diagram showing the amplitudes and phasor relationship of voltage and current. This diagram and the time-function waveforms shown in Fig. 9.7*b* convey the same information about the voltage and current. Obviously, drawing the phasor diagram needs much less effort than drawing the time-varying waveforms. Therefore, the electrical engineers prefer to represent inter-related sinusoidal quantities by phasors rather than by time waveforms.

Suppose an observer is standing at point P in Fig. 9.7*a*. The phasors are rotating counterclockwise at the speed ω radians per second or f revolutions per second. In every revolution, the phasor $\mathbf{V}_m$ passes the observer before the phasor $\mathbf{I}_m$ does. Thus, the phasor $\mathbf{I}_m$ *lags* (or follows) the phasor $\mathbf{V}_m$ by θ degree. The angle θ can also be seen as *time lag* measured in units of time.

Same conclusion can be drawn from Fig. 9.7*b*. Compared to the voltage sine wave, the current sine wave is shifted to the right, or later in time. Therefore, it is correct to describe $I_m \sin(\omega t - \theta)$ as ***lagging*** $V_m \sin \omega t$ by an angle θ, or $V_m \sin \omega t$ as ***leading*** $I_m \sin(\omega t - \theta)$ by an angle θ. The leading or lagging relationship between the two sinusoids should be recognizable both graphically and mathematically.

Note that the phase of two sine waves can be compared only if

1. Both have the same frequency.
2. Both are written with positive amplitude.
3. Both are written as sine functions, or as cosine functions.

Converting Sines to Cosines and Vice Versa

The sine and cosine are essentially the same functions, but with a 90° phase difference. Thus, $\sin \omega t = \cos(\omega t - 90°)$. We can add to or subtract from 360° the argument of any sinusoidal function, without changing the value of the function.

Suppose that we are to compare the phase of $v_1 = V_{m1} \cos(50t + 10°)$ with that of $v_2 = V_{m2} \sin(50t - 30°)$. For doing this, first we must express v_1 as a sine function.

$$v_1 = V_{m1} \cos(50t + 10°) = V_{m1} \sin[(50t + 10°) + 90°] = V_{m1} \sin(50t + 100°)$$

We can now say that v_1 *leads* v_2 by $\phi = 100° - (-30°) = 130°$. It is also correct to say that v_1 *lags* v_2 by 230°, since v_1 may be written as $v_1 = V_{m1} \sin(50t - 260°)$.

Useful Formulae	
$-\sin \omega t = \sin(\omega t \pm 180°)$;	$-\cos \omega t = \cos(\omega t \pm 180°)$
$\pm\sin \omega t = \cos(\omega t \mp 90°)$;	$\pm\cos \omega t = \sin(\omega t \pm 90°)$

EXAMPLE 9.5

Determine the phase difference of the sinusoidal current $i_1 = 4\sin(100\pi t + 30°)$ A with respect to the sinusoidal current $i_2 = 6\sin(100\pi t)$ A in terms of time and draw the phasor diagram to represent the two phasors.

Solution Comparing the expressions of the given sinusoids with the standard form, we get

$$\omega = 2\pi f = 100\pi \quad \Rightarrow \quad f = 50 \text{ Hz} \quad \Rightarrow \quad T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s} = 20 \text{ ms}$$

Thus, the time for full revolution (360°) for either of the phasors is 20 ms. The time taken to cover 30° is

$$t = (20 \text{ ms}) \times \frac{30°}{360°} = \mathbf{1.67 \text{ ms}}$$

The phasor i_1 leads the phasor i_2 by 30° (or 1.67 ms). Taking i_2 as the reference phasor, the phasor i_1 can be drawn 30° leading i_2. The phasor diagram is drawn in Fig. 9.8, in terms of the maximum values of the two sinusoidal currents.

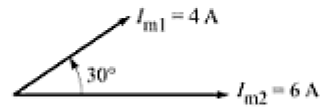


Fig. 9.8 *Phasor diagram.*

9.4 ALGEBRAIC OPERATIONS ON PHASORS

While analyzing an ac circuit, often we need to perform algebraic operations on sinusoids or phasors. For example, suppose that the currents,

$$i_1 = I_{m1}\sin\omega t \qquad \text{and} \qquad i_2 = I_{m2}\sin(\omega t + \theta_2) \tag{9.14}$$

are flowing through two parallel branches of an ac circuit. How do we find the total current i_3 flowing in the parallel combination? Do we simply add I_{m1} and I_{m2} to get I_{m3}, the peak value of i_3? No, in ac circuits we cannot do this. We can find the sum of i_1 and i_2 in following three ways:

1. By using the plots of waveforms.
2. By using trigonometrical identities.
3. By using the concept of phasors.

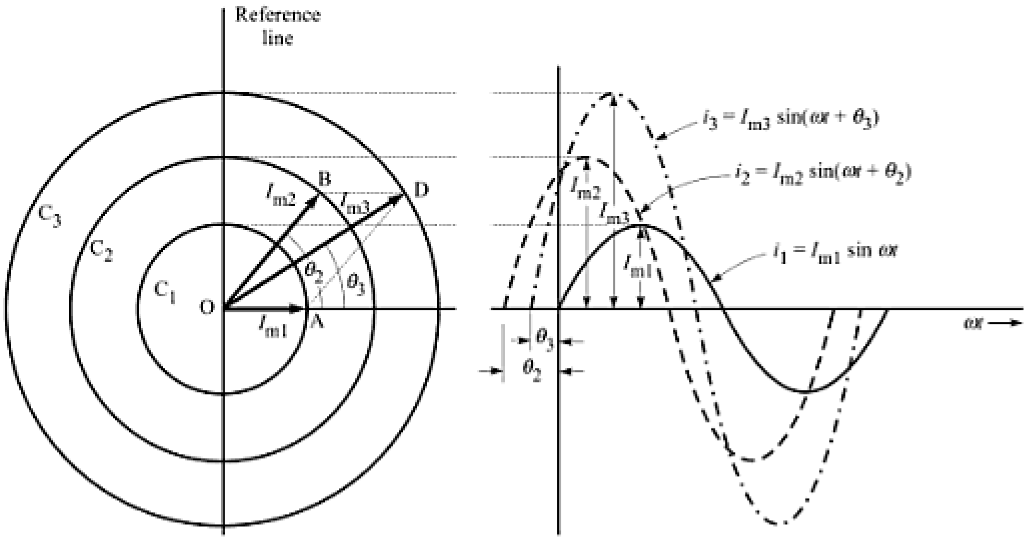
(1) *Addition Using Plots of Waveforms* This is the obvious way of finding the sum. Plot the two sinusoids and then make point by point summation of the two sine waves, as shown in Fig. 9.9*b*. You will find that the addition of two sinusoids is also a sinusoid of the same frequency. The amplitude I_{m3} and the phase θ_3 can be read from the plot of the resulting sinusoid. The current i_3 can then be expressed as

$$i_3 = I_{m3}\sin(\omega t + \theta_3) \tag{9.15}$$

This method, though straight forward, is quite laborious and time consuming. More so, when you add more than two sinusoids.

(2) *Addition Using Trigonometrical Identities* This proves to be a better way than the graphical procedure depicted in Fig. 9.9*b*. We proceed as follows.

$$\begin{aligned} i_1 = i_1 + i_2 &= I_{m1}\sin\omega t + I_{m2}\sin(\omega t + \theta_2) \\ &= I_{m1}\sin\omega t + I_{m2}[\sin\omega t\cos\theta_2 + \cos\omega t\sin\theta_2] \\ &= I_{m1}\sin\omega t + (I_{m2}\cos\theta_2)\sin\omega t + (I_{m2}\sin\theta_2)\cos\omega t \\ &= [I_{m1} + (I_{m2}\cos\theta_2)]\sin\omega t + (I_{m2}\sin\theta_2)\cos\omega t \end{aligned} \tag{9.16}$$



(*a*) Addition using phasors. (*b*) Addition using plots of waveforms.

Fig. 9.9 *Two methods of finding the sum of two sinusoids.*

For the convenience of writing the expression, let us put

$$a = I_{m1} + (I_{m2} \cos \theta_2) \qquad \text{and} \qquad b = I_{m2} \sin \theta_2 \tag{9.17}$$

Therefore, Eq. 9.16 can be written as

$$i_3 = a \sin \omega t + b \cos \omega t \tag{9.18}$$

We now use a standard technique to express i_3 as a single sinusoid of the form of Eq. 9.15,

$$i_3 = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2+b^2}} \sin \omega t + \frac{b}{\sqrt{a^2+b^2}} \cos \omega t \right) \tag{9.19}$$

Putting

$$\frac{a}{\sqrt{a^2+b^2}} = \cos \theta_3 \qquad \text{and} \qquad \frac{b}{\sqrt{a^2+b^2}} = \sin \theta_3 \tag{9.20}$$

Equation 9.19 can now be written as

$$i_3 = \sqrt{a^2 + b^2}\,(\sin \omega t \cos \theta_3 + \cos \omega t \sin \theta_3) = \sqrt{a^2 + b^2}\,\sin(\omega t + \theta_3)$$

Comparing this equation with Eq. 9.15, and from Eqs. 9.20, we get

$$I_{m3} = \sqrt{a^2 + b^2} \qquad \text{and} \qquad \theta_3 = \tan^{-1} \frac{b}{a} \tag{9.21}$$

The values of a and b can be computed from Eqs. 9.17 using the known quantities, I_{m1}, I_{m2} and θ_2.

This procedure, though does not require us to plot the sinusoid curves, is too cumbersome to be used in routine calculations.

(3) *Addition Using Concept of Phasors* As we shall see, this is much simpler compared to the first two methods. The sinusoidal currents i_1 and i_2 can be considered to be produced by the phasors OA and OB of amplitude I_{m1} and I_{m2} rotating counterclockwise in circles C_1 and C_2, as shown in Fig. 9.9*a*. Let us freeze these rotating phasors at $t = 0$, and treat them as ***vectors***. By applying the *law of parallelogram* or *law of triangles of vectors*, we can determine the sum of these two vectors as OD at an angle θ_3 with vector OA. We again treat this resultant vector OD as a phasor. Surprisingly, we find that this phasor OD when rotated in its circle C_3 produces the resulting sinusoidal current i_3 of amplitude I_{m3} and phase θ_3. Thus, we see **that *the laws of addition (or subtraction) of phasors are exactly the same as those for vectors***[2].

Important Points about Addition of Phasors Whenever we talk of ac sinusoidal quantities, we usually refer to their effective values and not their peak values. Therefore, there is no point in first finding the peak values I_{m1} and I_{m2} of the ac currents i_1 and i_2, and adding them in a phasor diagram to get peak value I_{m3} and then getting the rms value of i_3. Instead, the phasor diagram can directly be drawn in terms of the rms values I_1 and I_2, so as to directly get the rms value I_3 of the resultant current.

Furthermore, it is possible to find the addition of two phasors without even drawing the phasor diagram. This is done by representing the phasors by *complex numbers*. Many of us are not very familiar with complex numbers. So, let us refresh our knowledge of these complex numbers, since we are going to use them extensively in ac circuit.

9.5 MATHEMATICS OF COMPLEX NUMBERS

Most of us were introduced to complex numbers through the study of quadratic equations. We discovered that the solution to certain equations, like $x^2 + 4 = 0$ required the introduction of a new type of number:

$$x = \pm\sqrt{-4} = \pm i2$$

These new numbers are called ***imaginary*** and the symbol i is chosen (by mathematicians) to identify these numbers. The use of the word '*imaginary*' to identify such numbers is rather unfortunate. The terms '*real*' and '*imaginary*' are merely technical terms, just like '*positive*' and '*negative*'. The imaginary numbers are in fact as real as the real numbers, as would be clear from Example 9.6, given below.

The 'imaginary numbers' need to be distinguished from the 'real numbers' because different rules must be applied in the mathematical operations involving them.

The solutions of other quadratic equations are combination of the real and imaginary numbers, such as $2 \pm i\sqrt{2}$. These combinations are called ***complex numbers***. Complex numbers are appropriately named because the rules for manipulating them are more complicated than for real numbers.

Complex Plane All numbers can be represented as points in a complex plane, such as shown in Fig. 9.10. The horizontal axis represents *real numbers*, and the vertical axis represents *imaginary numbers*. Note that we are writing j instead of i to indicate imaginary numbers. This is customary among electrical engineers. It avoids confusion between imaginary numbers and currents, which have traditionally been symbolized by i. In Fig. 9.10, we have shown three complex numbers, $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{z}_3$, each having real and imaginary components. In general, a complex number[3] $\mathbf{z}$ (represented by point A in Fig. 9.10) can be expressed as

$$\mathbf{z} = a + jb \tag{9.22}$$

[2] This is the reason why some people refer to a *phasor diagram* as a *vector diagram*. In fact, *phasors are rotating vectors*. The calculations with phasors are done at a particular instant. One of the phasors is taken as reference and all others are expressed with respect to this reference.

[3] We have used **bold letters** for the symbols of complex numbers (also of vectors and phasors). While writing with a pen, it is not possible to make bold letters. So, we put a bar or an arrow over the symbol.

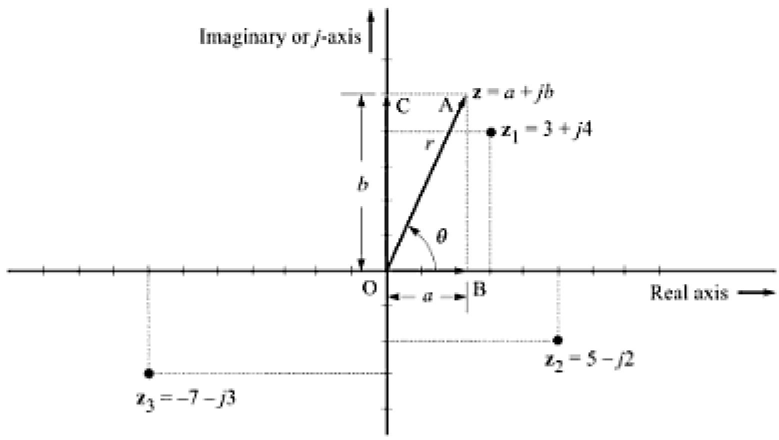


Fig. 9.10 *Representation of a phasor in a complex plane.*

This complex number represents phasor **OA**. Given the real part a and imaginary part b, we can determine the amplitude r and phase angle θ from the right angle triangle OAB as follows:

$$r = \sqrt{a^2 + b^2} \qquad \text{and} \qquad \theta = \tan^{-1}\frac{b}{a} \tag{9.23}$$

The phasor **OA** can then be expressed as **OA** = $r\angle\theta$. This representation is called ***polar form***. Conversely, given r and θ, we can express the phasor **OA** in ***rectangular*** or ***Cartesian form***, by finding a and b, as follows:

$$a = r\cos\theta \qquad \text{and} \qquad b = r\sin\theta \tag{9.24}$$

Since $e^{j\theta} = \cos\theta + j\sin\theta$, we can express the phasor **OA** (or the complex number) in the following forms:

$\mathbf{z} = a + jb$	(rectangular or Cartesian notation)
$= r(\cos\theta + j\sin\theta)$	(trigonometric notation)
$= re^{j\theta}$	(exponential notation)
$= r\angle\theta$	(polar notation)

Addition, Subtraction and Multiplication For these operations, just use ordinary algebra plus two more rules: (1) keep real and imaginary parts separate, and (2) treat j^2 as –1. For example, whenever we add complex numbers, we add the real parts and the imaginary parts separately:

$$\mathbf{z}_1 + \mathbf{z}_2 = (3 + j4) + (-7 - j3) = (3 - 7) + j(4 - 3) = -4 + j1$$

and similarly for subtraction. Thus, complex numbers are added and subtracted like vectors in a plane. This is one of the few properties common between complex numbers and vectors.

Similarly, multiplication of $\mathbf{z}_1$ and $\mathbf{z}_2$ is

$$\begin{aligned}\mathbf{z}_1\mathbf{z}_2 &= (3 + j4)(-7 - j3) = 3(-7) + j4(-j3) + 3(-j3) + j4(-7)\\ &= -21 - j^2 12 - j9 - j28 = -21 + 12 - j9 - j28 = (-21 + 12) - j(9 + 28)\\ &= -9 - j37\end{aligned}$$

Division and Conjugation Division requires a trick to get the results in standard form:

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{3+j4}{-7-j3} = \frac{3+j4}{-7-j3} \times \frac{-7+j3}{-7+j3}$$

$$= \frac{(3)(-7)+(j4)(j3)+(3)(j3)+(j4)(-7)}{(-7)^2-(j3)^2} = \frac{-21-12+j9-j28}{49+9}$$

$$= \frac{-33-j19}{58} = -\frac{33}{58} - j\frac{19}{58}$$

The first form we wrote to the right of the first equal sign is not a standard form. It contains a complex number in the denominator. To force the denominator to be real, we multiply numerator and denominator by the ***complex conjugate*** of $\mathbf{z}_2$. The complex conjugate of a complex number $\mathbf{z}$ is the same number except that the sign of the imaginary part is changed, and it is represented by the symbol $\mathbf{z}^*$. This forces the denominator to be real and positive. The final form is considered standard, because we can identify real and imaginary part at a glance. The process of forcing the denominator to be real and positive is called *rationalization* of the quotient.

Note that the polar form is merely a compact notation. When we write $r\angle\theta$, we actually mean $re^{j\theta}$. No mathematical operation can be performed on the notation $r\angle\theta$. The equivalent exponential form $re^{j\theta}$, being a legitimate mathematical function, can be subjected to any mathematical operation. Thus, the ***multiplication*** of two complex numbers is performed as follows:

$$(\mathbf{z}_1)(\mathbf{z}_2) = (r_1e^{j\theta_1})(r_2e^{j\theta_2}) = r_1r_2e^{j(\theta_1+\theta_2)}$$

Although the above proof rests on exponential form, the results are more easily expressed in polar form:

$$(\mathbf{z}_1)(\mathbf{z}_2) = (r_1\angle\theta_1)(r_2\angle\theta_2) = r_1r_2\angle(\theta_1+\theta_2) \tag{9.25}$$

This means that to multiply two complex numbers, just multiply their magnitudes and add their angles. In a similar way, we can show that dividing two complex numbers requires dividing the magnitudes and subtracting the angles:

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2) \tag{9.26}$$

Rotation of Phasor by 90° Consider a phasor represented by complex number $\mathbf{A} = 3 + j4 = 5\angle 53.1°$. If this phasor is multiplied by j, we get another phasor

$$\mathbf{B} = j\mathbf{A} = j3 + j^2 4 = -4 + j3 = 5\angle 143.1° = 5\angle(90° + 53.1°)$$

Thus, a phasor, when multiplied by j, rotates counterclockwise (in positive direction) by 90°. If multiplied by $-j$, it rotates clockwise (in negative direction) by 90°.

EXAMPLE 9.6

An ac current, denoted by a phasor in complex plane as $\mathbf{I} = 4 + j3$ amperes, is flowing through a resistor of 10 Ω. Determine the power consumed by the resistor.

Solution Let us first express the current $\mathbf{I}$ in the polar form,

$$\mathbf{I} = I_r + jI_i = 4 + j3 = \sqrt{4^2+3^2}\angle\tan^{-1}(3/4) = 5\angle 36.87° \text{ A}$$

Thus, we find that the magnitude (the rms value) of the given current is 5 A. Therefore, the power consumed is

$$P = I^2R = 5^2 \times 10 = \mathbf{250\ W}$$

It is interesting to determine the power due to the two components of the current. The power due to the 'real' component,

$$P_r = I_r^2 R = 4^2 \times 10 = 160 \text{ W}$$

The power due to the 'imaginary' component,

$$P_i = I_i^2 R = 3^2 \times 10 = 90 \text{ W}$$

The sum of these two powers,

$$P_t = P_r + P_i = 160 + 90 = 250 \text{ W}$$

This is seen to be the same as the actual power. Thus, we conclude that the imaginary component of current gives as much actual power as does the real component.

Additions of Phasors Using Complex Numbers

Let us now see how we use the concept of complex numbers to add two phasors $\mathbf{I}_1$ and $\mathbf{I}_2$, given as

$$\mathbf{I}_1 = I_1\angle 0° \qquad \text{and} \qquad \mathbf{I}_2 = I_2\angle \theta° \tag{9.27}$$

Here, I_1 and I_2 are the rms values of the two sinusoidal currents. If I_{m1} and I_{m2} are the peak values of these sinusoids, respectively, we should have $I_{m1} = \sqrt{2}I_1$ and $I_{m2} = \sqrt{2}I_2$. The instantaneous values of the two ac currents, if needed, can be written as

$$i_1 = I_{m1}\sin \omega t \qquad \text{and} \qquad i_2 = I_{m2}\sin(\omega t + \theta°) \tag{9.28}$$

We draw the phasors $\mathbf{I}_1$ and $\mathbf{I}_2$ on the complex plane, as shown Fig. 9.11. Phasor $\mathbf{I}_1$ is drawn as an arrow OA (of length I_1) along the real axis, as its phase angle is 0°. Hence, we can write

$$\mathbf{I}_1 = I_1 + j0 \tag{9.29}$$

Phasor $\mathbf{I}_2$ is drawn as an arrow OB (of length I_2), at an angle θ_2 with the real axis, as its phase angle is θ_2. Its components OM and ON along the two axes, are given as

$$\text{OM} = \text{OB}\cos\theta_2 = I_2\cos\theta_2 \qquad \text{and} \qquad \text{ON} = \text{OB}\sin\theta_2 = I_2\sin\theta_2$$

Hence, phasor $\mathbf{I}_2$ expressed as a complex number is

$$\mathbf{I}_2 = I_2\cos\theta_2 + jI_2\sin\theta_2 \tag{9.30}$$

The addition of phasors $\mathbf{I}_1$ and $\mathbf{I}_2$ is now obtained by adding the complex numbers of Eqs. 9.29 and 9.30,

$$\mathbf{I}_3 = \mathbf{I}_1 + \mathbf{I}_2 = (I_1 + j0) + (I_2\cos\theta_2 + jI_2\sin\theta_2) = (I_1 + I_2\cos\theta_2) = jI_2\sin\theta_2$$

In Fig. 9.11, the real component of $\mathbf{I}_3$ is simply the sum of the real components of the two phasors, i.e., OC = OA + OM. Similarly, for the imaginary component. The magnitude OD of phasor $\mathbf{I}_3$ is given as

$$I_3 = \sqrt{(\text{Real part})^2 + (\text{Imaginary part})^2} = \sqrt{(I_1 + I_2\cos\theta_2)^2 + (I_2\sin\theta_2)^2} \tag{9.31}$$

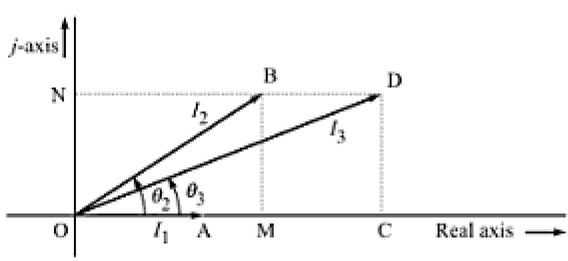


Fig. 9.11 *Use of complex numbers to add two phasors.*

And the phase angle θ_3 is given as

$$\theta_3 = \tan^{-1}\frac{\text{Imaginary part}}{\text{Real part}} = \tan^{-1}\frac{I_2 \sin\theta_2}{I_1 + I_2\cos\theta_2} \qquad (9.32)$$

EXAMPLE 9.7

A sinusoidal current of $10\angle 0°$ A is added to another sinusoidal current $20\angle 60°$ A. Find the resultant current.

Solution For adding two phasors, we first express them as complex numbers in Cartesian form. Thus,

$$\mathbf{I}_1 = 10\angle 0° \text{ A} = (10 + j0) \text{ A}$$

and

$$\mathbf{I}_2 = 20\angle 60° \text{ A} = (20\cos 60° + j20\sin 60°) \text{ A} = (10 + j10\sqrt{3}) \text{ A}$$

We can now get the summation of the two currents as

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = (10 + j0) + (10 + j10\sqrt{3}) = 20 + j10\sqrt{3} = \mathbf{26.46\angle 40.9° \text{ A}}$$

EXAMPLE 9.8

Find the expression for the sum of the currents

$$i_1 = 10\sqrt{2}\sin\omega t \text{ A} \qquad \text{and} \qquad i_2 = 20\sqrt{2}\sin(\omega t + 60°) \text{ A}$$

Also, determine the rms value of the sum of these two currents.

Solution Since we need to find the expression for the sum of the currents, there is no need of expressing the two phasors as complex number in terms of their rms values. We can directly find the peak value of the sum. Writing the peak values of the two currents as complex numbers, we get

$$\mathbf{I}_{m1} = 10\sqrt{2}\angle 0° \text{ A} \qquad \text{and} \qquad \mathbf{I}_{m2} = 20\sqrt{2}\angle 60° \text{ A}$$

Using *fx*-991-ES, the summation of these two currents is given as

$$\mathbf{I}_m = \mathbf{I}_{m1} + \mathbf{I}_{m2} = (10\sqrt{2}\angle 0° + 20\sqrt{2}\angle 60°) \text{ A} = 37.42\angle 40.9° \text{A}$$

Therefore, the expression for the sum of the two currents can be written as

$$\mathbf{i = I_m \sin(\omega t + 40.9°) \text{ A}}$$

The rms value of the sum is given as

$$I = \frac{I_m}{\sqrt{2}} = \frac{37.42}{\sqrt{2}} = \mathbf{26.46 \text{ A}}$$

EXAMPLE 9.9

Following three sinusoidal currents flow into a junction:

$$i_1 = 5\sin\omega t,\ i_2 = 5\sin(\omega t + 30°) \qquad \text{and} \qquad i_3 = 5\sin(\omega t - 120°)$$

Find the rms value of the resultant current (expressed in polar form) that leaves the junction.

Solution Since we are to determine the rms value of the resultant current in polar form, we work with rms values. First, we express the currents as complex numbers in polar form,

$$\mathbf{I}_1 = \frac{5}{\sqrt{2}}\angle 0° \text{ A}; \quad \mathbf{I}_2 = \frac{5}{\sqrt{2}}\angle 30° \text{ A}; \qquad \text{and} \qquad \mathbf{I}_3 = \frac{5}{\sqrt{2}}\angle -120° \text{ A}$$

Using *fx*-991ES calculator, the addition of the three currents is given as

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = \frac{5}{\sqrt{2}}\angle 0° + \frac{5}{\sqrt{2}}\angle 30° + \frac{5}{\sqrt{2}}\angle -120° = \mathbf{5\angle -15° \text{ A}}$$

Alternatively, we could work with peak values, expressed as complex numbers, add them up and then get the rms value of the resultant. Thus,

$$\mathbf{I}_{m1} = 5\angle 0° \text{ A}; \quad \mathbf{I}_{m2} = 5\angle 30° \text{ A}; \quad \text{and} \quad \mathbf{I}_{m3} = 5\angle -120° \text{ A}$$

Using *fx*-991ES calculator, the addition of these peak values is obtained as

$$\mathbf{I}_m = \mathbf{I}_{m1} + \mathbf{I}_{m2} + \mathbf{I}_{m3} = 5\angle 0° + 5\angle 30° + 5\angle -120° = 7.07\angle -15° \text{ A}$$

Therefore, the rms value of the resultant current is given as

$$\mathbf{I} = \frac{\mathbf{I}_m}{\sqrt{2}} = \frac{7.07}{\sqrt{2}} = \mathbf{5\angle -15° A}$$

Nonsinusoidal Waveforms

The ac power all over the world is supplied in the form of sinusoidal voltage. However, often we come across waveforms which are not sinusoidal. Basic definitions of average value and effective value of a nonsinusoidal waveform and of sinusoidal waveform are the same. Following example will make the process clear.

EXAMPLE 9.10

Determine the average and rms value of the resultant current in a wire carrying simultaneously a dc current of 10 A and a sinusoidal current of peak value 10 A.

Solution The resultant current at any instant is given as

$$i = 10 + 10 \sin \theta \text{ A}$$

The waveform of this resultant current is shown in Fig. 9.12. For this type of unsymmetric waveform, the average is found over one cycle (and not over half-cycle as in a sinusoidal waveform). As can be seen from Fig. 9.12, the current goes as much positive as negative around the value of 10 A. Hence, its *average value* is **10 A.**

There are two methods of finding the rms value of the resultant current.

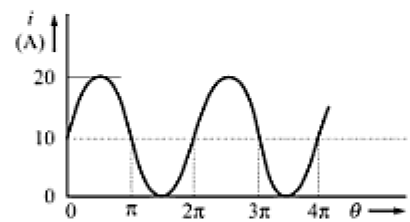


Fig. 9.12

First Method It is based on using the formula for determining the rms value,

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\theta)^2\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1 + 1\sin\theta)^2\, d\theta} = \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1 + \sin^2\theta + 2\sin\theta)\, d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}} = \sqrt{\frac{50}{\pi}(2\pi + \pi)} = \sqrt{150} = \mathbf{12.247\ A}$$

Second Method It is based on using the very basis of the meaning of *effective* (or rms) value. If two (or more than two) currents are simultaneously flowing through a resistor R, the net heating power is the sum of heating powers of the individual currents,

$$I^2R = I_1^2R + I_2^2R + I_3^2R + I_4^2R + \ldots; \quad \text{or} \quad I^2 = I_1^2 + I_2^2 + I_3^2 + I_4^2 + \ldots$$

Thus, we find that ***the rms values are added on square basis***. Since the rms value of the dc current is 10 A and of the sinusoidal ac current is $10/\sqrt{2}$ A, the rms value of the resultant current i is easily determined as

$$I = \sqrt{I_1^2 + I_2^2} = \sqrt{(10)^2 + \left(\frac{10}{\sqrt{2}}\right)^2} = \sqrt{100 + 50} = \sqrt{150} = \mathbf{12.247\ A}$$

NOTE

The second method of finding the rms value of a complex wave is much simpler than the first method using the standard formula.

EXAMPLE 9.11

One full cycle of an alternating voltage waveform is given in Fig. 9.13. Determine its average value.

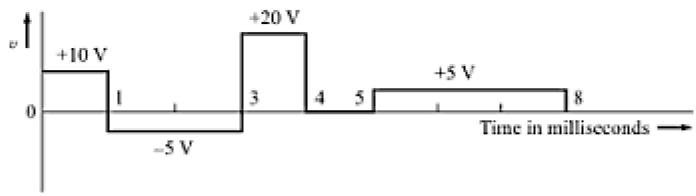


Fig. 9.13 *Voltage waveform for Example* 9.11.

Solution To get the average value of the voltage, we first find the total area under the curve and then divide it by the total period.

The area between 0 and 1 ms = $10 \times (1 \times 10^{-3}) = 10 \times 10^{-3}$

The area between 1 and 3 ms = $-5 \times (2 \times 10^{-3}) = -10 \times 10^{-3}$

The area between 3 and 4 ms = $20 \times (1 \times 10^{-3}) = 20 \times 10^{-3}$

The area between 4 and 5 ms = $0 \times (1 \times 10^{-3}) = 0$

The area between 5 and 8 ms = $5 \times (3 \times 10^{-3}) = 15 \times 10^{-3}$

$\therefore$ The total area between 0 and 8 ms

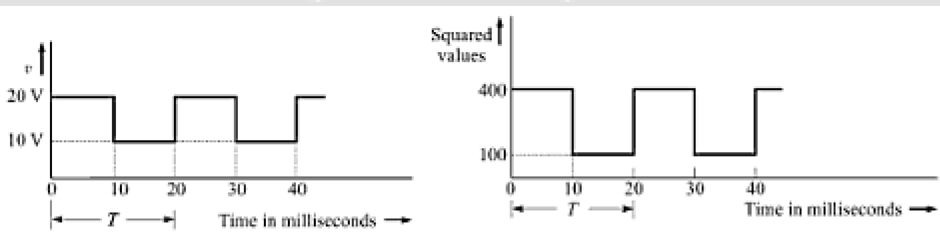
$$= (10 - 10 + 20 + 0 + 15) \times 10^{-3} = 35 \times 10^{-3}$$

Therefore, the average value is

$$V_{av} = \frac{\text{Total area}}{\text{Total period}} = \frac{35 \times 10^{-3}}{8 \times 10^{-3}} = \mathbf{4.375\ V}$$

EXAMPLE 9.12

Determine the effective value of the voltage wave form shown in Fig. 9.14*a*.



(*a*) Voltage waveform.

(*b*) Plot of squared values.

Fig. 9.14

Solution The given waveform is periodic with a period of 20 ms. The squared values of the voltages are plotted for two cycles in Fig. 9.14*b*.

The area under the squared curve from 0 and 10 ms

$$= 400 \times (10 \times 10^{-3}) = 4000 \times 10^{-3}$$

The area under the squared curve from 10 and 20 ms

$$= 100 \times (10 \times 10^{-3}) = 1000 \times 10^{-3}$$

The area under the squared curve from 0 and 20 ms

$$= (4000 + 1000) \times 10^{-3} = 5000 \times 10^{-3}$$

Since the length of the full cycle is 20×10^{-3} s, the average value of the squared waveform is

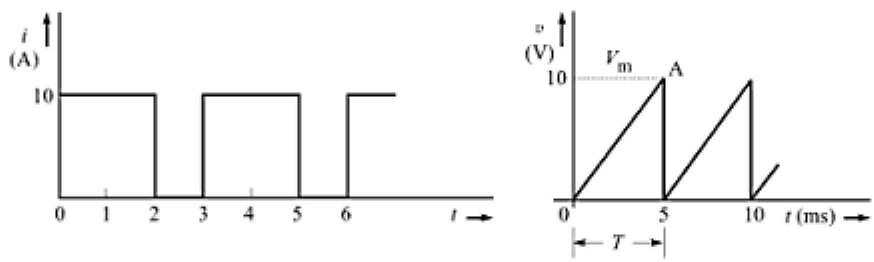
$$\frac{5000 \times 10^{-3}}{20 \times 10^{-3}} = 250$$

The square root of this quantity is the rms value of the waveform of Fig. 9.14*a*,

$$V_{\text{rms}} = \sqrt{250} = \mathbf{15.82\ V}$$

EXAMPLE 9.13

Determine the rms value, the average value and the form factor for the current waveform shown in Fig. 9.15*a*.



(*a*) (*b*)

Fig. 9.15

Solution The given waveform is periodic with a period of 3 seconds. Hence,

$$I_{\text{rms}} = \sqrt{\frac{10^2 \times 2 + 0^2 \times 1}{3}} = \mathbf{8.16\ A}$$

$$I_{\text{av}} = \frac{10 \times 2 + 0 \times 1}{3} = \mathbf{6.67\ A}$$

$$\therefore \qquad \text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{8.16}{6.67} = \mathbf{1.22}$$

EXAMPLE 9.14

Determine the average value, the rms value, the form factor and the peak factor for the saw-tooth voltage waveform of peak value $V_m = 10$ V and time period $T = 5$ ms, shown in Fig. 9.15*b*.

Solution The average value can be determined from its basic definition,

$$V_{\text{av}} = \frac{\text{Area under the curve in one cycle}}{\text{Duration of one cycle}} = \frac{(1/2) \times V_m \times T}{T} = \frac{V_m}{2} \tag{9.33}$$

$$= \frac{10}{2} = \mathbf{5\ V}$$

For determining the rms value, we first find the area under squared-value curve. For this, we have to resort to integration. First, we find the equation of straight line OA, as

$$y = mx + c \quad \text{or} \quad v = \left(\frac{V_m}{T}\right)t + 0 \quad \Rightarrow \quad v^2 = \left(\frac{V_m}{T}\right)^2 t^2$$

Thus, the rms value is given as

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}\right)^2 t^2\,dt} = \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T} = \frac{V_m}{\sqrt{3}} \tag{9.34}$$

$$= \frac{10}{\sqrt{3}} = \mathbf{5.77\ V}$$

$\therefore$ Form factor $= \dfrac{V_{rms}}{V_{av}} = \dfrac{5.77}{5} = \mathbf{1.154}$

and Peak factor $= \dfrac{V_m}{V_{rms}} = \dfrac{10}{5.77} = \mathbf{1.733}$

9.6 POWER AND POWER FACTOR

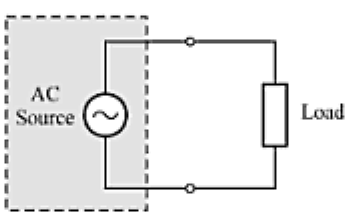
Consider a general ac circuit of Fig. 9.16*a*. An ac voltage source is delivering power to a load. Let us assume that the current i lags the voltage v by an angle θ. We can then write

$$v = V_m \sin \omega t \qquad \text{and} \qquad i = I_m \sin(\omega t - \theta)$$

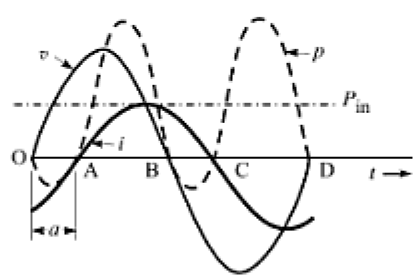
For one complete cycle, the waveforms for v and i are drawn in Fig. 9.16*b* by thin and thick lines, respectively.

Let $V = V_m/\sqrt{2}$ be the effective value of the voltage across the load, and $I = I_m/\sqrt{2}$ be the effective value of the current through the load. ***Apparently***, it seems that the power going to the load should be equal to VI. In fact, the product VI is given the name '***apparent power***'. But, is the *actual power* consumed by the load same as VI? As we shall see, this need not be true always.

At any instant, the power p consumed by the load is given as the product vi. If you multiply the value of v and the value of i from instant to instant, you get the waveform of the instantaneous power p, as shown in Fig. 9.16*b* by a dashed line. At points O, A, B, C and D, either the voltage or the current has zero value.



(a) AC source delivering power to a load.



(*b*) Waveforms of voltage, current and power.

Fig. 9.16 *AC circuit.*

Hence, a these points the power is zero. During the period form O to A, the voltage v has positive values but the current i has negative values. Therefore, the power ($p = vi$) has negative values. However, during the period from A to B, both the voltage v and current i have positive values. Hence the power p has positive values. From B to C, voltage is negative and current is positive; hence the power is again negative. From C to D, both the voltage and current are negative; hence their product ($= p$) is again positive.

It is seen that for some duration, the power p has positive values and for some it has negative values. A positive power means that power p flows from the ac source to the load. By the same token, a negative power means that the direction of power-flow reverses; it flows from the load to the source. Also, note that in one cycle of voltage (or current), there are two cycles of variations of power.

Average Power

The ***average power*** going into the load is the average of the waveform of instantaneous power p, and is shown as P_{in} in Fig. 9.16*b*. To find this value, let us write the expression of instantaneous power,

$$\begin{aligned} p &= vi = [V_m \sin \omega t]\,[I_m \sin(\omega t - \theta)] = V_m I_m \sin \omega t \sin(\omega t - \theta) \\ &= \frac{V_m I_m}{2}[\cos\{\omega t - (\omega t - \theta)\} - \cos\{\omega t + (\omega t - \theta)\}] \\ &= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}[\cos\theta - \cos(2\omega t - \theta)] = VI[\cos\theta - \cos(2\omega t - \theta)] \\ &= VI\cos\theta - VI\cos(2\omega t - \theta) \end{aligned} \tag{9.35}$$

The second term in the above expression represents a sinusoidal waveform of angular frequency 2ω; its average value is zero. However, the first term $VI\cos\theta$ is constant with time t. It therefore represents the average value of power P_{av} ($= P_{in}$) delivered to the load. Thus, the ***average power*** or ***actual power*** or ***real power*** consumed by the load in an ac circuit is given as

$$\boxed{P = VI\cos\theta} \tag{9.36}$$

Power Factor

The product VI is known as *apparent power*, and the *real power* is $VI\cos\theta$. The ***power factor*** is defined as the factor by which the apparent power is to be multiplied so as to get the real power. Thus,

$$\text{power factor } (pf) = \cos\theta \tag{9.37}$$

where, angle θ is the *phase angle*. If the current i lags the voltage v, as is usually the case, the *pf* is called *lagging pf* and is assigned a *positive sign*. On the other hand, if the current i lead the voltage v, the *pf* is called *leading pf* and is assigned a *negative sign*. The magnitude of power factor varies from 0 to 1. It can also be expressed in percentage. Thus, a *pf* of 0.8 can be expressed as *pf* of 80 %.

In case the phase angle is zero, the circuit is said to have ***unity pf***. *The real power is then same as the apparent power.*

EXAMPLE 9.15

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55\sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1\sin(\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Solution Here, the phase angle, $\theta = \pi/5$

The rms value of the voltage, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{55}{\sqrt{2}} = 38.89$ V

The rms value of the current, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{6.1}{\sqrt{2}} = 4.31$ A

Therefore, using Eq. 9.36, the average power is given as

$$P_{av} = VI\cos\theta = 38.89 \times 4.31 \times \cos\pi/5$$
$$= 167.62 \times 0.809 = \mathbf{135.6\ W}$$

The apparent power is

$$P_a = VI = 38.89 \times 4.31 = \mathbf{167.62\ VA}$$

Note that the apparent power is expressed in volt amperes (VA) and not in watts (W), since it is not a power in reality.

The instantaneous power at $\omega t = 0.3$ is given by Eq. 9.35, as

$$p = VI\cos\theta - VI\cos(2\omega t - \theta)$$
$$= 135.6 - 167.62 \times \cos(2 \times 0.3 - \pi/5) = \mathbf{-31.95\ VA}$$

The power factor is given as

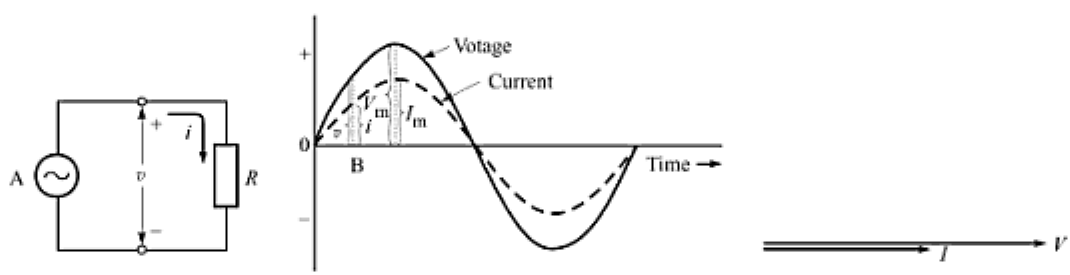
$$pf = \cos\theta = \cos\pi/5 = 0.809 = \mathbf{80.9\ \%}$$

9.7 BEHAVIOUR OF *R, L* AND *C* IN AC CIRCUITS

We shall now study the steady-state response of the basic circuit parameters (R, L and C) to a sustained sinusoidal function. The 'steady state' condition implies that the sinusoidal function had been recurring for a long time in the past, is recurring now, and will continue to recur in future too for a reasonably long time. We shall establish the phase angle relationships between the current through and the voltage across each of the circuit parameter. *These relationships remain fixed irrespective of the manner in which a given parameter is connected in a circuit.*

Purely Resistive Circuit

Consider the circuit of Fig. 9.17*a*. A resistance R is connected across the terminals of an ac voltage source A. Suppose that the ac voltage is a sine wave, $v = V_m \sin\omega t$, as shown in Fig. 9.17*b*. At any instant B, if the value of the voltage is v, the current in R is given as $i = v/R$. Since the current remains proportional to the voltage all the time, the waveform of the current is also a sine wave. The two waveforms are ***in phase*** with each other. This fact is illustrated in the phasor diagram of Fig. 9.17*c*.



(*a*) Resistive circuit. (*b*) Voltage and current waveforms. (*c*) Phasor diagram.

Fig. 9.17 *Response of a resistive circuit.*

For a source voltage $v = V_m \sin \omega t$, the current in the resistive circuit is given as

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \tag{9.38}$$

Power Waveform In a purely resistive circuit, the current and the voltage are in phase. The instantaneous power p, given by the product vi, fluctuates (sinusoidally) between zero and maximum value $V_m I_m$, as shown in Fig. 9.18. The average power flowing from the source to the load (a resistance) is seen to be

$$P = P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

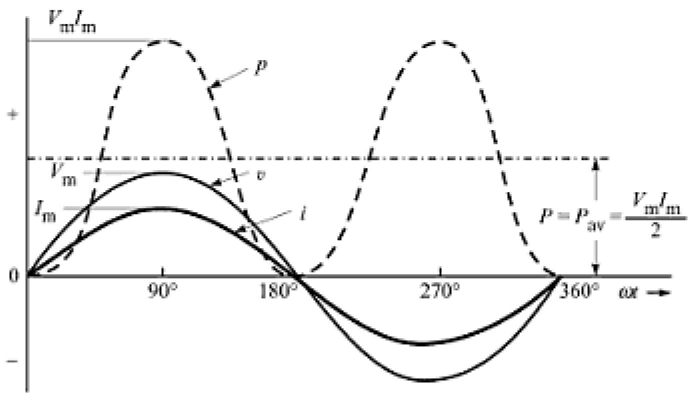


Fig. 9.18 *Power waveform for a resistive circuit.*

Power and Power Factor For a purely resistive circuit, the current is in phase with voltage; the phase angle $\theta = 0°$ (see the phasor diagram of Fig. 9.17*c*). Therefore, the real power is

$$P_r = VI \cos \theta = VI \cos 0° = VI = \text{Apparent power}$$

and $$pf = \cos \theta = \cos 0° = 1$$

Purely Inductive Circuit

Consider the circuit of Fig. 9.19*a*. A sinusoidal ac voltage source A is connected across a coil of inductance L having negligible resistance. We wish to find the relationship between the applied voltage v and the resulting current i. For our convenience, we start with the assumption that the current is given as

$$i = I_m \sin \omega t \tag{9.39}$$

and then we determine what should be the applied voltage v. When a varying current flows through an inductance, the induced emf is given as

$$e = -L\frac{di}{dt} = -L\frac{d}{dt}(I_m \sin \omega t) = -\omega L I_m \cos \omega t = \omega L I_m \sin(\omega t - \pi/2) \tag{9.40}$$

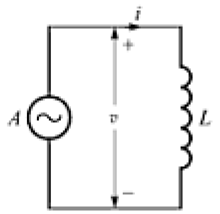
This shows that the induced emf e lags the current i by $\pi/2$, as we have shown in the waveform plot of Fig. 9.19*b* as well as in phasor diagram of Fig. 9.19*c*.

The induced emf e opposes the applied voltage v. Hence,

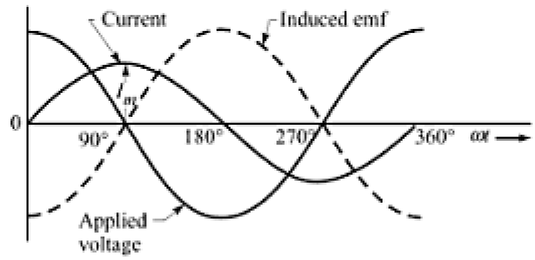
$$v = -e = \omega L I_m \cos \omega t = \omega L I_m \sin(\omega t + \pi/2) \tag{9.41}$$

On comparing this with Eq. 9.39, we conclude that the applied voltage v leads the current i by $\pi/2$, as shown in Fig. 9.19*b*. This fact is also shown in phasor diagram of Fig. 9.19*c*. In terms of phasors, if $\mathbf{I} = I\angle 0° = I + j0$, then the voltage phasor can be written as

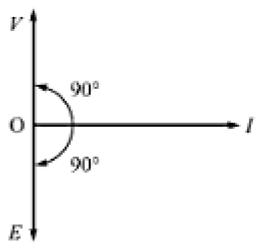
$$\mathbf{V} = V\angle 90° = 0 + jV$$



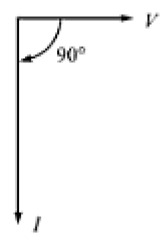
(*a*) Inductive circuit.



(*b*) Voltage and current waveforms.



(*c*) Phasor diagram (with *I* reference).



(d) Phasor diagram (with *V* reference).

Fig. 9.19 *Response of a purely inductive circuit.*

NOTE

We are at full liberty to take any phasor as reference, while drawing a phasor diagram. However, in electrical engineering, it is a normal practice to take the voltage as reference. Hence, the phasor diagram of Fig. 9.19*c* for a purely inductive circuit is redrawn as shown in Fig. 9.19*d*. It can now be seen that *the current lags applied voltage by* $\pi/2$.

Inductive Reactance A look at Eq. 9.41 shows that the peak or maximum value V_m of v is given as

$$V_m = \omega L I_m \quad \Rightarrow \quad \frac{V_m}{I_m} = \omega L$$

Since, the rms values of v and i are $V = V_m/\sqrt{2}$ and $I = I_m/\sqrt{2}$, respectively, the ratio of rms voltage to rms current for a purely inductive circuit is given as

$$\frac{V}{I} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_m}{I_m} = \omega L \tag{9.42}$$

This ratio V/I for purely inductive circuit is called ***inductive reactance***, and is represented by X_L. Like resistance, the inductive reactance is expressed in ohms (Ω). Hence,

$$X_L = \omega L = 2\pi f L \tag{9.43}$$

For a given ac voltage V, the current I in a purely inductive circuit can be determined, using Ohm's law,

$$I = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi f L} \tag{9.44}$$

As can be seen from Eqs. 9.43 and 9.44, the inductive reactance is proportional to the frequency and the current produced by a given voltage is inversely proportional to the frequency. This variation is shown in Fig. 9.22*a*.

Taking the current phasor as reference (as in phasor diagram of Fig. 9.19*c*), the voltage phasor across an inductor is given as $\mathbf{V} = jX_L I$, which is along $+j$ axis. Hence, we associate $+j$ with an inductive reactance.

Power and Power Factor In a purely inductive circuit, the current lags the applied voltage by $\pi/2$, as shown by the voltage and current waveforms (drawn for one complete cycle) in Fig. 9.20. The power waveform can be plotted by finding the product vi from instant to instant. At ωt = 0°, 90°, 180°, 270°, 360°, either the voltage or the current has zero value. Hence, at these instants, the power too is zero. Between 0° and 90°, the voltage is positive but the current is negative; and between 180° and 270°, the voltage is negative but the current is positive. Hence, during these intervals the power is *negative*. Between the 90°–180° and 270°–360°, both are either positive or negative. Hence, during these intervals the power is *positive*.

From the power-waveform in Fig. 9.20, we find that the power varies sinusoidally whose average value over a complete cycle is zero. That is, *the power consumed by the circuit is zero*. In fact, in a purely inductive load, same amount of energy keeps on flowing alternately from source to the load and then from load to the source.

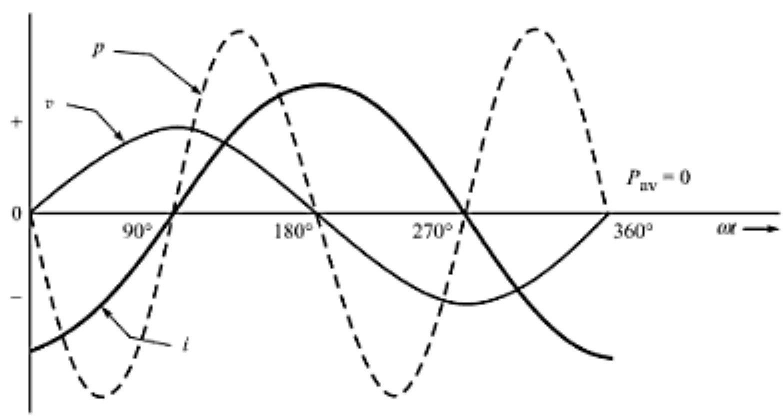


Fig. 9.20 *Power waveform for a purely inductive circuit.*

From Eq. 9.36, we can confirm the above result. The real power in a purely inductive circuit is given as

$$P = VI\cos\theta = VI\cos 90° = 0$$

and

$$pf = \cos\theta = \cos 90° = 0 \text{ (lagging)}$$

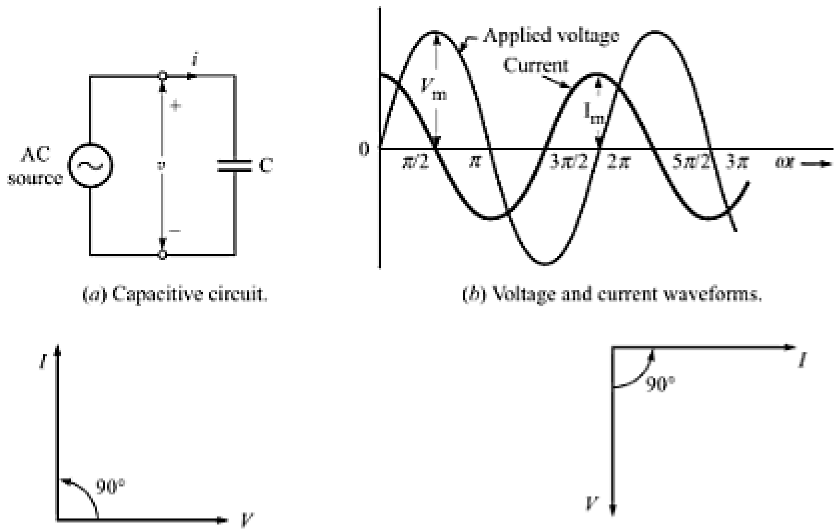
Purely Capacitive Circuit

Consider the circuit of Fig. 9.21*a*. A sinusoidal ac voltage source having $v = V_m \sin \omega t$ is connected across a capacitance C. Using the basic relationship between the applied voltage v and the resulting current i for a capacitor, we get

$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\, \omega t) = CV_m\, \omega \cos\, \omega t = \omega C V_m \cos\, \omega t$$

$$= \omega C V_m \sin\,(\omega t + \pi/2) \tag{9.45}$$

This shows that the resulting current i leads the applied voltage v, as illustrated in Fig. 9.19*b*.



(*a*) Capacitive circuit. (*b*) Voltage and current waveforms.

(*c*) Phasor diagram (with *V* reference). (*d*) Phasor diagram (with *I* reference).

Fig. 9.21 *Response of a purely capacitive circuit.*

The phasor diagram for the circuit is shown in Fig. 9.21*c*. Here, we have taken the voltage phasor as reference; that is $\mathbf{V} = V\angle 0° = V + j0$. The resulting current phasor will then be $\mathbf{I} = I\angle 90° = 0 + jI$.

From Eq. 9.45, it follows that the maximum value I_m of the current is $\omega C V_m$. Hence, if V and I are the rms values, the ratio

$$\frac{V}{I} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_m}{I_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

This ratio V/I for purely capacitive circuit is called ***capacitive reactance***, and is represented by X_C. Like inductive reactance, the capacitive reactance is expressed in ohms (Ω). Hence,

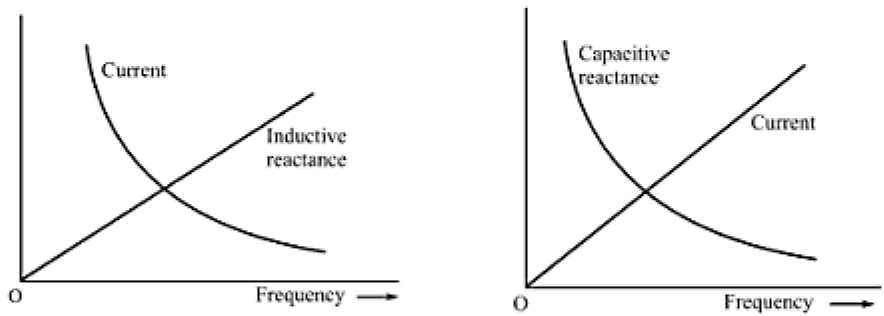
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \tag{9.46}$$

For a given voltage V, the resulting current in a purely capacitive circuit is given as

$$I = \frac{V}{X_C} = 2\pi f C V \tag{9.47}$$

Equations 9.46 and 9.47 show that for purely capacitive circuit, the reactance varies inversely as the frequency and the current varies directly as the frequency, as shown in Fig. 9.22*b*.

Taking the current phasor as reference, we have $\mathbf{I} = I\angle 0°$ (as in phasor diagram of Fig. 9.21*d*). The voltage phasor across the capacitor is then given as $\mathbf{V} = V\angle -90° = -jV = -jX_C I$, which is along $-j$ axis. It is for this reason that we associate $-j$ with a capacitive reactance.



(*a*) Purely inductive circuit. (*b*) Purely capacitive circuit.

Fig. 9.22 *Variation of reactance and current with frequency.*

Power and Power Factor Like an inductive circuit, the current and voltage are in quadrature[4] in a purely capacitive circuit. However, in a capacitive circuit, the current *leads* the applied voltage by $\pi/2$, as shown by the voltage and current waveforms (drawn for one complete cycle) in Fig. 9.23. The power waveform can be plotted by finding the product vi from instant to instant. Between 0°–90° and 180°–270°, same sign. Hence, during these intervals the power is *positive*. Between 90°–180° and 270°–360°, the voltage and the current are of opposite sign. Hence, during these intervals the power is *negative*.

From the power-waveform in Fig. 9.23, we find that the power varies sinusoidally whose average value over a complete cycle is zero. Like an inductive circuit, here too the same amount of energy keeps on flowing alternately from source to the load and then from load to the source. *The average power consumed by the circuit is zero.*

From Eq. 9.36, we can confirm the above result. The real power in a purely capacitive circuit is given as

$$P = VI\cos\theta = VI\cos 90° = 0$$

and

$$pf = \cos\theta = \cos 90° = 0 \text{ (leading)}$$

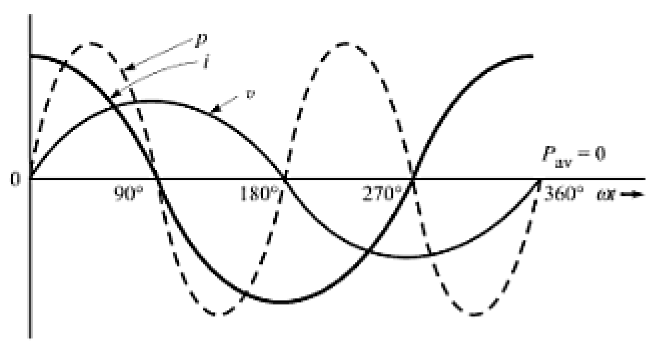


Fig. 9.23 *Power waveform for a purely capacitive circuit.*

[4] Two phasors are said to be in *quadrature* when the phase angle between them is 90°.

Comparison of *R*, *L* and *C*

All the three parameters are passive in nature. That is, none of these can act as a source of energy in a circuit. However, there exist marked differences in their properties and their behaviour in a circuit. The *resistance* is a dissipative parameter. It consumes electrical power and dissipates it as heat. But, *inductance* and *capacitance* do not consume any power. They temporarily store energy in one half-cycle and give away the same in next half cycle. The comparison among the three parameters is listed in Table 9.1.

Table 9.1 *Comparison of R, L and C*

Property	*Resistance*	*Inductance*	*Capacitance*
Current	V/R	V/X_L	V/X_C
Frequency dependency	Independent	$X_L \propto f$	$X_C \propto (1/f)$
Power	$VI = I^2R = V^2/R$	Zero	Zero
Phase difference	0°	90° lagging	90° leading
Reactance	R	$jX_L = j\omega L = j2\pi fL$	$-jX_C = 1/j\omega C = 1/j2\pi fC$

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 9.16

Find the average and rms values of the two waveforms given in Fig. 9.24.

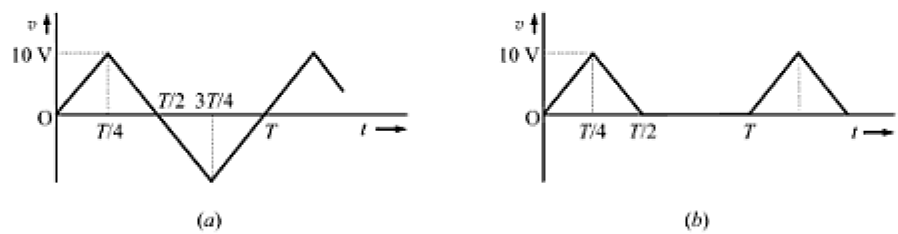


Fig. 9.24 *Triangular waveforms.*

Solution

(*a*) This is a symmetric wave, for which the average is found over half-cycle only. Therefore, we find the area from 0 to $T/2$ and then divide this area by $T/2$. This would be the same as finding the area from 0 to $T/4$ and dividing it by $T/4$, since the two right-angle triangles in the positive half-cycle are mirror-image of each other. Thus, the problem reduces to that of Example 9.14. Hence, the average value is given by Eq. 9.33 as

$$V_{av} = \frac{V_m}{2} = \frac{10}{2} = \mathbf{5\ V}$$

For finding the rms value, we have to first determine the average of the area under squared curve over one cycle. Since the square of a negative value is a positive quantity, the area under the squared curves would be same for the four time durations: (*i*) from 0 to $T/4$, (*ii*) from $T/4$ to $T/2$, (*iii*) from $T/2$ to $3T/4$, and (*iv*) from $3T/4$ to T. Therefore, instead of considering whole of one cycle, we would get the same result if we consider only the first quarter cycle (from 0 to $T/4$). Thus, the problem reduces to that of Example 9.14, and the rms value is given by

Eq. 9.34 as

$$V_{rms} = \frac{V_m}{\sqrt{3}} = \frac{10}{\sqrt{3}} = \mathbf{5.77\ V}$$

(*b*) This is an unsymmetrical waveform. Hence, its average has to be found over one cycle. Thus,

$$V_{av} = \frac{\text{Area under the curve in one cycle}}{\text{Duration of one cycle}} = \frac{(1/2)\times V_m \times (T/2) + 0}{T} = \frac{V_m}{4}$$

$$= \frac{10}{4} = \mathbf{2.5\ V}$$

The rms value is found as follows.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} v^2\,dt + \int_{T/2}^{T} 0\,dt\right]} = \sqrt{\frac{1}{T}\left[2\int_0^{T/4} v^2\,dt + 0\right]} = \sqrt{\frac{1}{T}\left[2\int_0^{T/4}\left(\frac{V_m}{T/4}\right)^2 t^2\,dt\right]}$$

$$= \sqrt{\frac{1}{T}\left[\frac{32V_m^2}{T^2}\left\{\frac{t^3}{3}\right\}_0^{T/4}\right]} = \sqrt{\frac{32V_m^2}{T^3}\cdot\frac{T^3/64}{3}} = \sqrt{\frac{V_m^2}{6}} = \frac{V_m}{\sqrt{6}} = \mathbf{4.08\ V}$$

EXAMPLE 9.17

An alternating current of frequency of 60 Hz has a maximum value of 12 A. (*a*) Write down the equation for its instantaneous value. (*b*) Find the value of the current after 1/360 second. (*c*) Find the time taken to reach 9.6 A for the first time.

Solution

(*a*) The angular frequency, $\omega = 2\pi f = 2 \times 3.141 \times 60 = 377$ rad/s. Therefore, the instantaneous value is given as

$$i = 12\sin 377t\ \mathbf{A}$$

(*b*) $i = 12\sin\dfrac{377\times 1}{360} = 12\sin\dfrac{377\times 1\times 180°}{360\times\pi} = \mathbf{10.34\ A}$

(*c*) $9.6 = 12\sin 377t \quad \Rightarrow \quad 377t = \sin^{-1}\dfrac{9.6}{12} = 0.927$ rad

$$\therefore \qquad t = \frac{0.927}{377} = 0.00246\text{ s} = \mathbf{2.46\ ms}$$

EXAMPLE 9.18

An alternating current has a maximum value of 10 units. At time $t = 0$, the current has a value of 5 units, and is increasing in the positive direction. Find the expression of the current in cosine form.

Solution The current is 5 units at $t = 0$ and is increasing in positive direction, hence the waveform would be as shown in Fig. 9.25. This is just a sine wave with a phase lead of ϕ, having equation, $i = 10\sin(\omega t + \phi)$.

The phase angle ϕ can be determined from the given condition,

$$5 = 10\sin(\omega \times 0 + \phi) \quad \Rightarrow \quad \phi = \sin^{-1}(5/10) = \pi/6$$

$$\therefore \qquad i = 10\sin(\omega t + \pi/6) = 10\cos\{(\pi/2) - (\omega t + \pi/6)\} = 10\cos\{-(\omega t - \pi/3)\}$$

$$= 10\cos(\omega t - \pi/3)$$

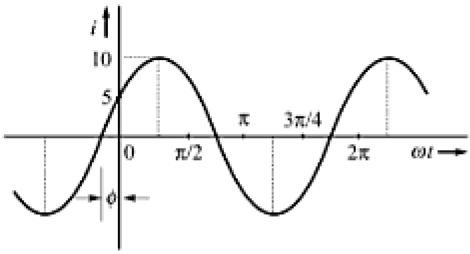


Fig. 9.25

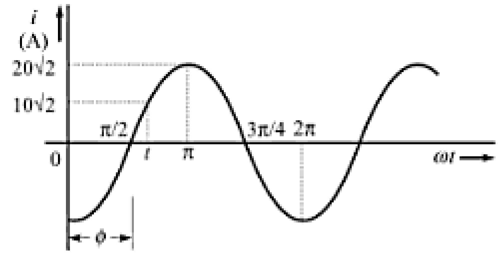


Fig. 9.26

EXAMPLE 9.19

An alternating current varying sinusoidally with a frequency of 50 Hz has a rms value of 20 A. At what time, measured from negative maximum value, instantaneous current will be $10\sqrt{2}$ A?

Solution The peak value of the current is $I_m = 20\sqrt{2}$ A. At negative maximum value, we put $t = 0$. Hence, the given current waveform should be as shown in Fig. 9.26. This is just a sine wave with a phase lag of $\phi = \pi/2$. Hence, the equation of the current should be $i = 20\sqrt{2}\sin(100\pi t - \pi/2)$ A. Therefore, the required time is given by

$$10\sqrt{2} = 20\sqrt{2}\sin(100\pi t - \pi/2) \quad \Rightarrow \quad 100\pi t - \pi/2 = \sin^{-1}(1/2) = \pi/6$$

$$\therefore \qquad t = \frac{\pi/6 + \pi/2}{100\pi} = 0.00667 \text{ s} = \mathbf{6.67\ ms}$$

EXAMPLE 9.20

Determine the average and rms values of a current given by $i = 10 + 5\cos 314t$ A.

Solution The given current is seen to be the combination of a dc current of 10 A and a sinusoidal current of peak value 5 A. The average value of the sinusoidal current being zero, the average of the overall current would be the same as the dc current. That is, I_{av} = **10 A**.

The rms value can easily be found by adding the rms values of the two components on square basis,

$$I_{rms} = \sqrt{(10)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{112.5} = \mathbf{10.6\ A}$$

EXAMPLE 9.21

Obtain the sum of the three voltages,

$$v_1 = 147.3\cos(\omega t + 98.1°)\text{ V}, \quad v_2 = 294.6\cos(\omega t - 45°)\text{ V} \quad \text{and} \quad v_3 = 88.4\sin(\omega t + 135°)\text{ V}$$

Solution We plot the above phasors in complex plane, in terms of their peak values. First, we write the voltages in terms of sine functions. Since, $\sin(90° + \theta) = \cos\theta$, we can write

$$v_1 = 147.3\sin(90° + \omega t + 98.1°)\text{ V} = 147.3\sin(\omega t + 188.1°)\text{ V}$$

$$v_2 = 294.6\sin(90° + \omega t - 45°)\text{ V} = 294.6\sin(\omega t + 45°)\text{ V}$$

and $$v_3 = 88.4\sin(\omega t + 135°)\text{ V}$$

The plot of these three phasors (in terms of peak values) is shown in Fig. 9.27.

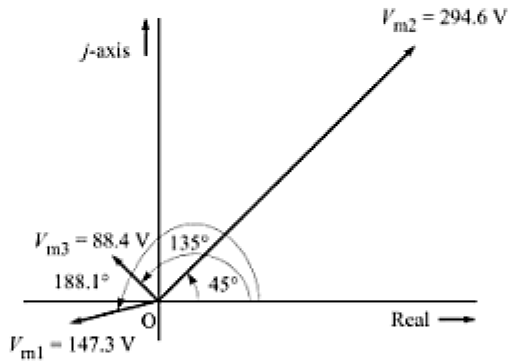


Fig. 9.27 *Addition of phasors in complex plane.*

By finding the components along the two axes, we now express the phasors in Cartesian form. By adding real and imaginary parts separately, the addition is found as

$$\mathbf{V}_{m1} = (-145.8 - j20.8)\ \text{V}$$
$$\mathbf{V}_{m2} = (208.3 + j208.3)\ \text{V}$$
$$\mathbf{V}_{m3} = (-62.5 + j62.5)\ \text{V}$$
$$\mathbf{V}_{m} = (0 + j250.0)\quad \text{V} = 250\angle 90^\circ$$

Therefore, the expression for the sum of the three voltages can be written as

$$v = \mathbf{250\ sin\ (\omega t + 90^\circ)\ V}$$

EXAMPLE 9.22

(*a*) What reactance will be offered (*i*) by an inductor of 0.2 H, (*ii*) by a capacitance of 10 μF, to an ac voltage source of 10 V, 100 Hz? (*b*) What, if the frequency of the source is changed to 140 Hz?

Solution

(*a*) (*i*) $X_L = 2\pi f L = 2\pi \times 100 \times 0.2 = \mathbf{125.66\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 100 \times 10 \times 10^{-6}} = \mathbf{159.15\ \Omega}$

(*b*) (*i*) $X_L = 2\pi f L = 2\pi \times 140 \times 0.2 = \mathbf{175.9\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 140 \times 10 \times 10^{-6}} = \mathbf{113.7\ \Omega}$

EXAMPLE 9.23

Study the ac circuits given in Fig. 9.28. Draw the phasor diagram and determine the values of the unknown quantity in each case.

Solution

(*a*) The two currents I_1 and I_2 are in phase with the applied voltage (Fig. 9.29*a*). Hence, the unknown current is

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_2 = 10\angle 0^\circ + 10\angle 0^\circ = \mathbf{20\angle 0^\circ\ A}$$

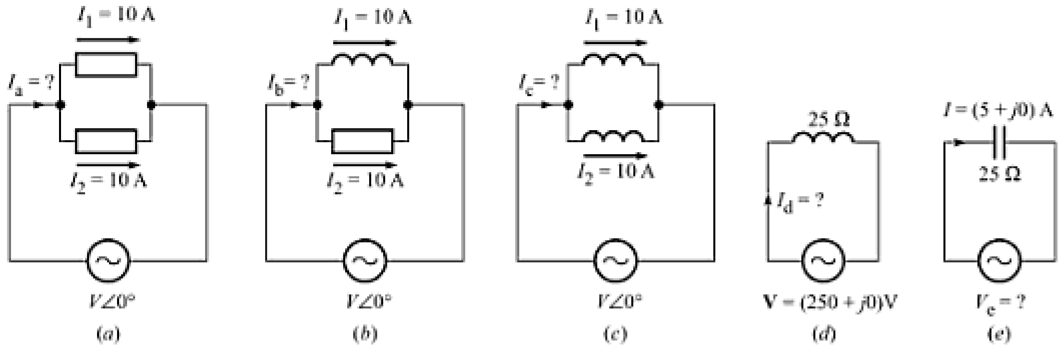


Fig. 9.28 *AC circuits.*

(*b*) The current I_2 is in phase with applied voltage, but the current I_1 lags the voltage by 90° (Fig. 9.29*b*). Hence, the unknown current is

$$I_b = \sqrt{I_1^2 + I_2^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}A \quad \Rightarrow \quad \mathbf{I}_b = \mathbf{10\sqrt{2}\angle{-45°}\ A}$$

(*c*) Both currents I_1 and I_2 lag the applied voltage by 90° (Fig. 9.29*c*). Hence, the unknown current is

$$I_c = I_1 + I_2 = 10 + 10 = 20 \text{ A} \quad \Rightarrow \quad \mathbf{I}_c = \mathbf{20\angle{-90°}\ A}$$

(*d*) The inductive reactance is taken as $j25\ \Omega$. Hence, the unknown current is given as

$$\mathbf{I}_d = \frac{250 + j0}{j25} = \frac{250\angle 0°}{25\angle 90°} = \mathbf{10\angle{-90°}\ A}$$

Hence, the current I_d is 10 A, and lags the applied voltage by 90° (Fig. 9.29*d*).

(*e*) The capacitive reactance is taken as $-j25\ \Omega$. Hence, the unknown voltage is given as

$$\mathbf{V}_e = (5 + j0)(-j25) = (5\angle 0°)(25\angle{-90°}) = \mathbf{125\angle{-90°}\ V}$$

Hence, the voltage V_e is 125 V, and lags the current by 90° (Fig. 9.29*e*).

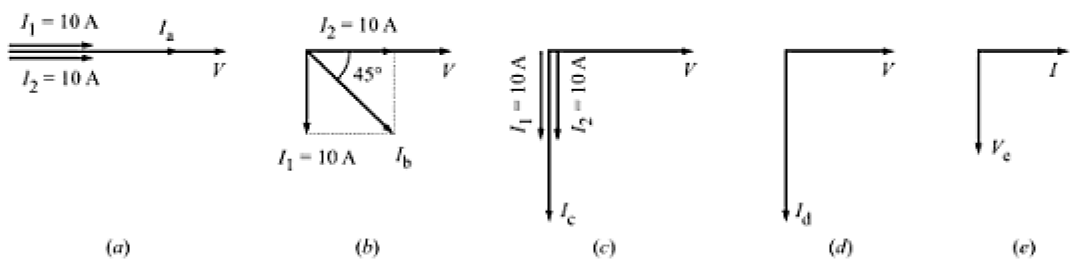


Fig. 9.29 *Phasor diagrams.*

SUMMARY

TERMS AND CONCEPTS

- An ***alternating*** system, the voltages and currents vary in a repetitive manner. A cycle of variation is the sequence of change before the repetition commences.
- The most basic form of alternating system is based on a ***sinusoidal*** variation.
- A sinusoid, even after repeated differentiation or integration, remains a sinusoid of the same frequency. Also, the sum or difference of a number of sinusoids is a sinusoid.

- The time taken to complete a cycle is the ***period***.
- The ***frequency*** is the number of cycles completed in one second.
- The ac supply in India is 220 V, 50 Hz (V_m = 311 V, T = 20 ms).
- A ***phasor*** is a line drawn to represent a sinusoidal alternating quantity. It is drawn to scale and its angle relative to the horizontal represents its phase shift in time.
- Addition (or subtraction) of two sinusoids of same frequency representing like quantities is done in exactly the same way as for vectors.
- All mathematical operations on phasors can be performed easily, if the phasors are represented by **complex numbers**.
- ***Phasor diagrams*** can be used to represent rms quantities in which case they are frozen in time.
- A phasor, when multiplied by j, rotates counterclockwise (in positive direction) by 90°; when multiplied by $-j$, it rotates clockwise (in negative direction) by 90°.
- The ***average value*** *of a symmetric periodic* waveform is determined for half-cycle only.
- The ***effective value*** of an ac current is the equivalent dc current that produces the same amount of heat. It is given by **r**oot of **m**ean of **s**quares of the values over one cycle. Hence, it is also called ***rms value***.
- In ac circuits, the product VI is called ***apparent power*** (measured in volt amperes); the ***actual*** or ***real power*** is given by $P = VI\cos\theta$, where $\cos\theta$ is called ***power factor***. Thus, power factor is the factor by which the apparent power is to be multiplied to get the real power.
- In a purely resistive circuit, the current and voltage are in phase.
- In a purely inductive circuit, the current lags voltage by 90°.
- In a purely capacitive circuit, the current leads voltage by 90°.
- The ***reactance*** of an inductor linearly increases with frequency.
- The ***reactance*** of a capacitor inversely decreases with frequency.

IMPORTANT FORMULAE

- $f = \dfrac{1}{T}$
- Sinusoidal current, $i = I_m \sin \omega t$, where $\omega = 2\pi f$ or $\omega = \dfrac{2\pi}{T}$
- For a sinusoidal ac and for a full-wave-rectified wave,
$$I_{av} = \frac{2I_m}{\pi}, \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}.$$
- For a sinusoidal ac and for a full-wave-rectified wave,
$$I_{av} = \frac{2I_m}{\pi}, \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}.$$
- For a half-wave-rectified wave,
$$I_{av} = \frac{I_m}{\pi}, \quad \text{and} \quad I_{rms} = \frac{I_m}{2}.$$
- Form factor, $K_f = \dfrac{V_{rms}}{V_{av}}$ = 1.11 (for sinusoidal waveform)
- Peak factor, $K_p = \dfrac{V_m}{V_{rms}}$ = 1.414 (for sinusoidal waveform)

Property	*Resistance*	*Inductance*	*Capacitance*
Current	V/R	V/X_L	V/X_C
Frequency dependency	Independent	$X_L \propto f$	$X_C \propto (1/f)$
Power	$VI = I^2R = V^2/R$	Zero	Zero
Phase difference	0°	90° lagging	90° leading
Reactance	R	$jX_L = j\omega L = j2\pi fL$	$-jX_C = 1/j\omega C = 1/j2\pi fC$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	A sinusoidal current $i = 2\sin \omega t$ flows through a resistance of 10 Ω. The power consumed by the resistor is 40 W.	☐	☐	
2.	In an ac circuit, the current and voltage are given as $i = 10\sin \omega t$ and $v = 20\sin(\omega t + 60°)$. It means the current lags voltage by 60°.	☐	☐	
3.	For a full-wave rectified wave, the rms value of current is given by $I_{rms} = I_m/\sqrt{2}$.	☐	☐	
4.	When a phasor is multiplied by $-j$, it rotates counterclockwise by 90°.	☐	☐	
5.	For an ac circuit, $I = 2$ A, $V = 10$ V and $P = 10$ W. It shows that the power factor is 0.5.	☐	☐	
6.	A complex number does not always represent a sinusoidal function.	☐	☐	
7.	The imaginary component of a current flowing through a resistor does not contribute to the real power.	☐	☐	
8.	If a constant-voltage ac source is connected across an inductor, the resulting current goes on increasing as the frequency of the source increases.	☐	☐	
9.	In a purely capacitive circuit, the voltage lags the current by $\pi/2$.	☐	☐	
10.	The capacitive reactance is directly proportional to the frequency of the ac source.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** True
6. True	**7.** False	**8.** False	**9.** True	**10.** False

REVIEW QUESTIONS

1. What are the requirements for a waveform to be classified as periodic?
2. How is the frequency related to the period of an ac waveform?
3. What is a phasor and how is the concept of phasor helpful in solving ac circuit?
4. How is the phase angle related to time in a sinusoidal voltage waveform?
5. What is meant by the average value of an ac voltage waveform?
6. How do you define the effective value of an ac waveform? Why is it also called rms value?
7. A square wave has equal positive and negative peak values. What are its average and effective values?
8. What advantages do you get by using complex algebra in solving ac problems?
9. Why are the multiplication and division of complex numbers preferably done in polar form?
10. What is the difference among a vector, a complex number and a phasor?
11. What is the difference between apparent power and real power in an ac circuit?
12. Describe the phasor relationship between current and voltage for (*a*) an inductor, and (*b*) a capacitor.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. A sinusoidal voltage is expressed as $v = 20\sin(314.16t + \pi/3)$ V. Its frequency and phase angle, respectively, are
   (*a*) 314.16 Hz, 60° (*b*) 60 Hz, 60°
   (*c*) 50 Hz, 60° (*d*) 50 Hz, –60°
2. A sinusoidal voltage v_1 leads another sinusoidal voltage v_2 by 180°. Then,
   (*a*) voltage v_2 leads voltage v_1 by 180°
   (*b*) both voltage have their zero values at the same time
   (*c*) both voltages have their peak values at the same time
   (*d*) all of the above
3. The rms value of an ac sinusoidal current is 10 A. Its peak value is
   (*a*) 7.07 A (*b*) 14.14 A
   (*c*) 10 A (*d*) 28.28 A
4. If **A** = $10\angle 45°$ and **B** = $5\angle 15°$, then the value of **A/B** will be
   (*a*) $50\angle 60°$ (*b*) $2\angle 60°$
   (*c*) $2\angle -30°$ (*d*) $2\angle 30°$
5. In an ac circuit, the active power and apparent power are equal in magnitude. Then the power factor of the circuit is
   (*a*) 1 (*b*) 0.8
   (*c*) 0.6 (*d*) 0
6. When a phasor is multiplied by $-j$, it gets rotated through ___ in the counterclockwise direction
   (*a*) 90° (*b*) 180°
   (*c*) 270° (*d*) none of the above
7. For a purely inductive circuit, the current leads the voltage by
   (*a*) 90° (*b*) 180°
   (*c*) 270° (*d*) 360°
8. In a purely capacitive circuit, the voltage lags the current by
   (*a*) 90° (*b*) 180°
   (*c*) 270° (*d*) 360°
9. The power consumed by a pure inductance connected to an ac source is
   (*a*) zero (*b*) very low
   (*c*) very high (*d*) infinite
10. If a 10-Ω resistance is connected to an ac supply $v = 100\sin(314t + 37°)$ V, the power dissipated by the resistance is
    (*a*) 10 000 W (*b*) 1000 W
    (*c*) 500 W (*d*) 250 W

ANSWERS

1. *c*	**2.** *d*	**3.** *b*	**4.** *d*	**5.** *a*	**6.** *c*	**7.** *c*	**8.** *a*	**9.** *a*	**10.** *c*

PROBLEMS

(A) SIMPLE PROBLEMS

1. The maximum value of a sinusoidal alternating current of frequency 50 Hz is 25 A. Write the equation for instantaneous value of the alternating current. Determine its value at 3 ms and 14 ms.
[**Ans.** $i = 25\sin 100\pi t$ A, 20.225 A, –23.78 A]
2. Calculate the rms value of a triangular wave in which voltage rises uniformly from zero to V_m, and completes the cycle by falling instantly back to zero. Find also its form factor and peak factor.
[**Ans.** $0.577V_m$, 1.1554, 1.733]
3. An ac voltage is mathematically expressed as $v = 141.42\sin(157.08t + \pi/2)$ volts.
Find its (*a*) effective value, (*b*) frequency, and (*c*) periodic time.
[**Ans.** (*a*) 100 V; (*b*) 25 Hz; (*c*) 40 ms]
4. The equation of an alternating current is given as $i = 62.35\sin 323t$ amperes. Determine its maximum value, frequency, rms value, average value and form factor.
[**Ans.** 62.35 A, 51.4 Hz, 44.1 A, 39.7 A, 1.11]
5. Find the rms value of the current given by $i = 10 + 5\cos(628t + 30°)$. [**Ans.** 10.6 A]
6. Determine the effective value of the voltage waveform represented by
$v = 200\sin \omega t + 100\sin 2\omega t + 50\sin 3\omega t$ V
[**Ans.** 162.02 V]
7. Find the rms value of a resultant current in a wire, which carries simultaneously a direct current of 5 A and a sinusoidal alternating current with a peak value of 5 A.
[**Ans.** 6.1237 A]
8. The waveform of a nonsinusoidal voltage has a form factor of 1.15 and a peak factor of 1.4. The peak value of the voltage is 322 V. Find (*a*) the rms value, and (*b*) the average value of this voltage.
[**Ans.** (*a*) 230 V, (*b*) 200 V]
9. A 10-Ω resistor is connected across 200-V, 50-Hz ac supply. Find the peak value, average value and the rms value of the current and the power dissipated by the resistor.
[**Ans.** 28.28 A, 18 A, 20 A, 4 kW]
10. Two phasors **A** and **B** are given as **A** = 3 + *j*1, and **B** = 4 + *j*3. Calculate the values of (*a*) **A** + **B**; (*b*) **A** – **B**; (*c*) **AB**; (*d*) **A/B**. Express the results in both polar and rectangular coordinates.
[**Ans.** (*a*) $7 + j4 = 8.06\angle 29.7°$;
(*b*) $-1 - j2 = 2.24\angle -116.57°$;
(*c*) $15.8\angle 55.3° = 8.99 + j12.99$;
(*d*) $0.632\angle -18.44° = 0.6 - j0.02$]

(B) TRICKY PROBLEMS

11. A 60-Hz sinusoidal current has an instantaneous value of 7.07 A at $t = 0$, and rms value of $10\sqrt{2}$ A. Assuming the current wave to enter positive half at $t = 0$, determine (*a*) the expression for current, (*b*) the magnitude of current at $t = 0.0125$ s and at $t = 0.025$ s after $t = 0$.
[**Ans.** (*a*) $i = 20\sin(120\pi t + 20.7°)$ A;
(*b*) –18.71 A, –7.07 A]
12. Determine the average value and effective value of the three voltage-waveforms shown in Fig. 9.30.
[**Ans.** (*a*) $0.7V_m$, $0.7746\,V_m$; (*b*) $0.54V_m$, $0.584V_m$; (*c*) $0.5434V_m$, $0.674V_m$]
13. A 200-mH inductance is connected across 230-V, 50-Hz ac supply. Find the inductive reactance and the current in the circuit.
[**Ans.** 62.832 Ω, $3.66\angle -90°$ A]
14. A 40-μF capacitor is connected to a 230-V, 50-Hz ac supply. Find the capacitive reactance and the current in the circuit. [**Ans.** 79.57 Ω, $2.89\angle 90°$ A]
15. A voltage $v = 141\sin(314t + 60°)$ V is applied separately to (*a*) a 20-Ω resistor, (*b*) a 0.1-H inductor, and (*c*) a 100-μF capacitor. Write, in each case, the expression for instantaneous current and find the rms value of current, power dissipated and power factor.
[**Ans.** (*a*) $i = 7.05\sin(314t + 60°)$ A, 4.985 A, 497 W, 1; (*b*) $i = 4.49\sin(314t - 30°)$ A, 3.176 A, 0 W, 0; (*c*) $i = 4.43\sin(314t + 150°)$ A, 3.13 A, 0 W, 0]
16. The instantaneous values of two emfs are $e_1 = 30\sin \omega t$ V and $e_2 = 20\sin(\omega t - \pi/4)$ V. Derive the expression for the instantaneous value of (*a*) $e_1 + e_2$, and (*b*) $e_1 - e_2$.
[**Ans.** (*a*) $e = 46.35\sin(\omega t - 17.76°)$ V;
(*b*) $e = 21.24\sin(\omega t + 41.72°)$ V]

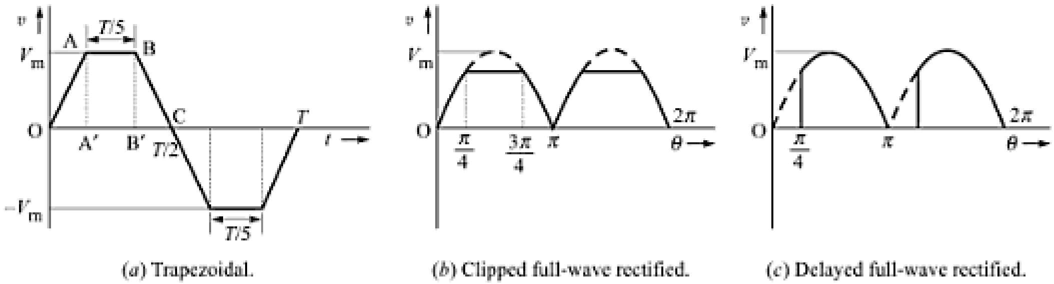


Fig. 9.30 *Different waveforms.*

(C) CHALLENGING PROBLEMS

17. There are three conducting wires connected to a junction. The currents flowing into the junction in two wires are $i_1 = 10\sin 314t$ A and $i_2 = 15\sin(314t - 45°)$ A. What is the current leaving the junction in the third wire? What is its value at $t = 0$?

[**Ans.** $i_3 = 23.17\sin(314t - 27.23°)$ A, -10.6 A]

18. Following impedances are connected in parallel. Find the equivalent impedance of this combination.

$\mathbf{Z}_1 = (8 + j6)\ \Omega$; $\mathbf{Z}_2 = (8 - j6)\ \Omega$; and $\mathbf{Z}_3 = (8.66 + j5)\ \Omega$.

[**Ans.** $(3.89 + j0.79)\ \Omega$]

AC CIRCUITS

10

OBJECTIVES

After completing this Chapter, you will be able to:

- *Explain what the impedance triangle for an ac circuit is.*
- *Define 'resistance', 'reactance' and 'complex impedance' with reference to an ac circuit.*
- *Draw phasor diagrams for a series and a parallel ac circuit containing R, L and C.*
- *Define 'complex power', 'apparent power', 'average power' and 'reactive power', in an ac circuit, and state their units.*

10.1 SERIES *RL* CIRCUIT

Figure 10.1*a* shows a series *RL* circuit. We know that a practical inductor possesses inductance and resistance effectively in series. Therefore, the following analysis of R and L in series is equivalent to the analysis of a circuit containing a practical inductor.

Our aim is to find its steady-state response (i.e., the current I) for the given applied ac voltage V. Note that the reference polarity of V is shown by means of an arrow. Let V_R and V_L be the voltage drops across resistance R and inductance L, respectively. Unlike dc circuits, here we cannot obtain the total voltage V just by adding the magnitudes of the voltages across the resistor and inductor. That is, to write

$$V = V_R + V_L \qquad \text{is } \textbf{\textit{wrong}}.$$

This does not mean that KVL is not applicable here. KVL is a basic law and is applicable to all circuits whether dc or ac. In an ac circuit, the voltages and currents are phasors. So, the two voltages must be added by treating them as ***phasors***,

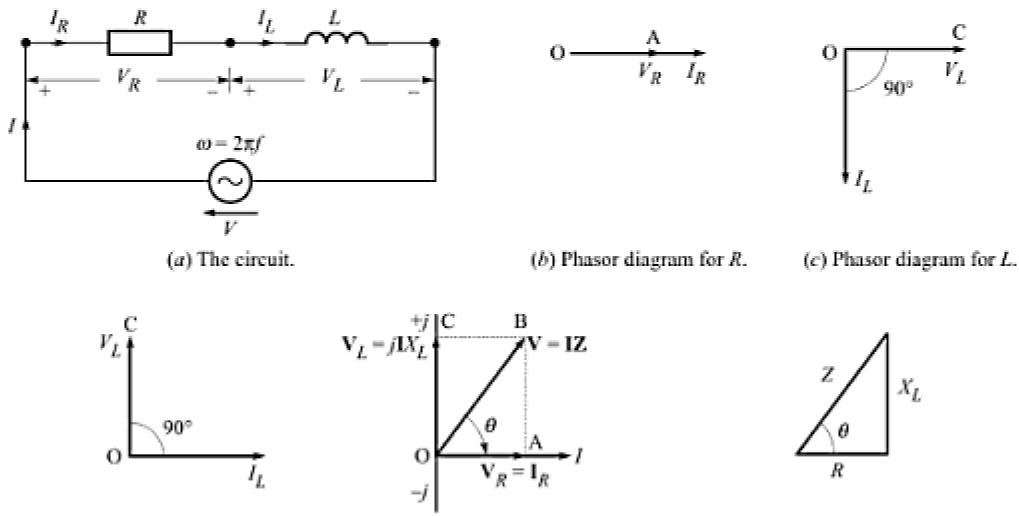
$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L \tag{10.1}$$

How to Draw a Phasor Diagram

We shall now explain the procedure of drawing the phasor diagram for an ac circuit. As an example, we take the series *RL* circuit of Fig. 10.1*a*.

Step 1 Mark the source voltage V, showing its polarity, either by an arrow or by using + and − signs. Mark the source current I showing its direction by an arrow. As a convention, the current I must leave the positive terminal of the source.

Step 2 Mark 'the voltage across' and 'the current through' each individual component of the circuit, following



(*a*) The circuit.
(*b*) Phasor diagram for *R*.
(*c*) Phasor diagram for *L*.
(*d*) Phasor diagram for *L*, rotated by +90°.
(*e*) The complete phasor diagram.
(*f*) Impedance triangle.

Fig. 10.1 *Series RL circuit.*

the passive sign convention (i.e., the current must enter the plus-marked terminal of the component). We have marked V_R and I_R for the resistance R, and V_L and I_L for the inductance L.

Step 3 Draw the phasor diagrams for individual components.

(*i*) ***For resistance*** R: The current is in phase with the voltage. Draw the voltage phasor V_R along the reference direction (i.e., along $+x$ direction). Draw the current phasor I_R also along the reference direction (see Fig. 10.1*b*).

(*ii*) ***For inductance*** L: The current lags the voltage by 90°. Draw the voltage phasor V_L along the reference direction. Draw the current phasor I_L 90° lagging, as shown in Fig. 10.1*c*.

Step 4 To get complete phasor diagram, superimpose all the individual phasor diagrams, by recognizing the ***common phasor*** among them. For this, you may have to rotate some phasor diagrams.

Here, we find that the common phasor is the current $I = I_R = I_L$. In Fig. 10.1*b*, the current phasor $I_R = I$ is already along the reference direction. But, in Fig. 10.1*c*, the current phasor $I_L = I$ is not along the reference direction. To align the two current phasors, we rotate the phasor diagram of Fig. 10.1*c* by 90° counter-clockwise (Fig. 10.1*d*). We can now superimpose Fig. 10.1*d* and Fig. 10.1*b*, to get the complete phasor diagram of the circuit, as shown in Fig. 10.1*e*.

Step 5 Find the phasor addition (same as vector additon) of V_R and V_L, by drawing the parallelogram OABC (here, it is a rectangle).

The resultant of this addition is given by the diagonal OB. The phasor OB must be equal to supply voltage V, as per KVL. From Fig. 10.1*e*, it becomes clear the current phasor I lags the supply voltage V by an angle BOA = θ.

Step 6 Once the phasor diagram is drawn, we can take the help of complex algebra to make calculations. Imagine that the phasor diagram is drawn in the complex plane. That is, mark the reference direction (+x-axis) as the positive real axis and the y-axis as the imaginary axis. We can then write

$$\mathbf{I} = I\angle 0°; \quad \mathbf{V}_R = \mathbf{I}R \qquad \text{and} \qquad \mathbf{V}_L = j\mathbf{I}X_L = \mathbf{I}jX_L = \mathbf{I}j\omega L$$

Therefore, Eq. 10.1 can be rewritten as

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L = \mathbf{I}R + \mathbf{I}j\omega L = \mathbf{I}(R + j\omega L) \tag{10.2}$$

Complex Impedance

In general, for an ac circuit, the ratio of the voltage phasor to the current phasor is a complex quantity, called ***complex impedance*** (represented by symbol **Z**). Its real part is called ***resistance*** and its imaginary part is called ***reactance***. Thus,

$$\textbf{\textit{Complex impedance}} = (\text{resistance}) + j(\text{reactance}) \quad \text{or} \quad \mathbf{Z} = R + jX \tag{10.3}$$

For the series RL circuit, from Eq. 10.2, we have

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R + j\omega L = Z\angle\theta \tag{10.4}$$

where,

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\frac{\omega L}{R} \tag{10.5}$$

From the phasor diagram of Fig. 10.1*e*, we can separate the voltage triangle OAB. If each side of this triangle is divided by I, the result is the ***impedance triangle***, shown in Fig. 10.1*f*. **Note** that an inductive circuit has an impedance triangle in the first quadrant of complex plane.

Voltage Phasor as Reference Often, we are given the source voltage and then we are required to find resulting current in a circuit. This can be done by taking the voltage as reference, and then writing the current as

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle 0°}{Z\angle\theta} = \frac{V}{Z}\angle -\theta \tag{10.6}$$

This shows that the current lags the applied voltage by an angle θ, given by Eq. 10.5. For a given ac voltage $v = V_m \sin \omega t$ volts, the equation of the resulting current is

$$i = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\{\omega t - \tan^{-1}(\omega L)/R\} \text{ amperes} \tag{10.7}$$

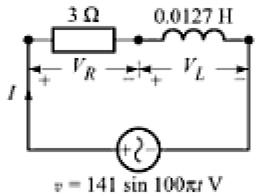
EXAMPLE 10.1

For the series RL circuit shown in Fig. 10.2*a*,

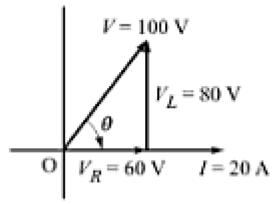
(*a*) Calculate the rms value of the steady state current and the relative phase angle.
(*b*) Write the expression for the instantaneous current.
(*c*) Find the average power dissipated in the circuit.
(*d*) Determine the power factor.
(*e*) Draw the phasor diagram.

Solution

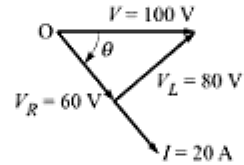
(*a*) Since the phase angle of the applied voltage is given as 0°. In the complex form, the applied voltage can be written as



(*a*) The circuit.



(*b*) Phasor diagram.



(*c*) Phasor diagram redrawn.

Fig. 10.2 *A series RL circuit.*

$$\mathbf{V} = V\angle 0° = \frac{V_m}{\sqrt{2}}\angle 0° = \frac{141}{\sqrt{2}}\angle 0° = 100\angle 0° = 100 + j0 \text{ volts}$$

The impedance, $\mathbf{Z} = R + j\omega L = 3 + j100\pi \times 0.0127 = 3 + j4 = 5\angle 53.1°$ ohms

$\therefore$ Current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{100\angle 0°}{5\angle 53.1°} = \mathbf{20\angle{-53.1°}\ A}$

Thus, the rms value of the steady state current is 20 A, and the phase angle is 53.1° lagging.

(*b*) The expression for the instantaneous current can be written as

$$i = 20\sqrt{2}\sin(100\pi t - 53.1°) = \mathbf{28.28\sin(100\pi t - 53.1°)\ A}$$

(*c*) Average power, $P = VI\cos\theta = 100 \times 20 \times \cos 53.1° = \mathbf{1200\ W}$

Or, $P = I^2R = (20)^2 3 = \mathbf{1200\ W}$

(*d*) $pf = \cos\theta = \cos 53.1° =$ **0.6 lagging**. Alternatively,

$$pf = \frac{\text{Average power}}{\text{Apparent power}} = \frac{P}{VI} = \frac{1200}{100 \times 20} = \mathbf{0.6\ lagging}$$

(*e*) Taking the current as reference, the phasor diagram is drawn in Fig. 10.2*b*, where

$$I = 20\text{ A};\quad V_R = IR = 20 \times 3 = 60\text{ V};\quad V_L = IX_L = 20 \times 4 = 80\text{ V} \quad \text{and} \quad V = 100\text{ V}$$

The same phasor diagram is redrawn in Fig. 10.2*c*, by rotating it clockwise by an angle 53.1°, so that the applied voltage becomes the reference phasor.

10.2 SERIES *RC* CIRCUIT

For the series *RC* circuit of Fig. 10.3*a*, we can apply KVL and get

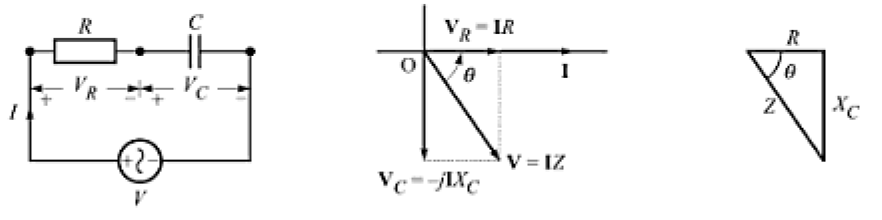
$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C = \mathbf{I}R - j\mathbf{I}X_C = \mathbf{I}(R - jX_C) = \mathbf{I}\left(R - j\frac{1}{\omega C}\right) = \mathbf{IZ} \tag{10.8}$$

where **Z** is the ***complex impedance*** for the series *RC* circuit. Since the voltage across a capacitor lags the current by 90°, we have put in the above equation,

$$\mathbf{V}_C = -j\mathbf{I}X_C = \mathbf{I}(-jX_C)$$

Also, note that $-j$ is associated with X_C (whereas $+j$ is associated with X_L).

Phasor Diagram Since the current is common to both the resistor and the capacitor, we take current phasor **I** as reference. For the resistance, the voltage $\mathbf{V}_R$ is drawn in phase with the current **I**; but for the capacitance, the voltage $\mathbf{V}_C$ is drawn *lagging* the current **I** by 90°. The complete phasor diagram for series *RC* circuit is drawn in Fig. 10.3*b*. Here, the supply voltage **V** is the phasor sum of $\mathbf{V}_R$ and $\mathbf{V}_C$. From this phasor



(*a*) The circuit. (*b*) Phasor diagram. (*c*) Impedance triangle.

Fig. 10.3 *Series RC circuit.*

diagram, we can obtain the impedance triangle of Fig. 10.3*c*. **Note** that a capacitive circuit has an impedance triangle in fourth quadrant.

For an impedance triangle to be in either the second or third quadrant, a circuit must have a negative resistance. This may occur if a circuit contains dependent sources.

10.3 COMPLEX POWER

We have seen that, if the terminal voltage is $\mathbf{V} = V\angle\theta$ and the current is $\mathbf{I} = I\angle\phi$ in an ac circuit, the average power absorbed by it is given as

$$P = VI\cos(\theta - \phi) \tag{10.9}$$

By using Euler's formula, the above equation can be written as

$$P = VI\,\mathrm{Re}[e^{j(\theta-\phi)}] = \mathrm{Re}[(Ve^{j\theta})\,(Ie^{-j\phi})] \tag{10.10}$$

The first factor $Ve^{j\theta}$ is simply the voltage phasor. The second term $Ie^{-j\phi}$ is seen to be the complex conjugate $\mathbf{I}^*$ of the current phasor $\mathbf{I} = Ie^{j\phi}$. Hence, Eq. 10.10 can be written as

$$P = \mathrm{Re}[\mathbf{VI}^*] \tag{10.11}$$

We now define the ***complex power*** (represented by symbol **S**) as

$$\mathbf{S} = P + jQ = \mathbf{VI}^* = VIe^{j(\theta-\phi)} \tag{10.12}$$

It is seen that the magnitude VI of **S** is the ***apparent power*** and the angle $(\theta - \phi)$ of **S** is the *pf* angle. The real part P of **S** is the ***average*** (***real*** or ***actual***) ***power*** absorbed by the circuit. The imaginary part Q of **S** is given the name ***reactive power***. From Eq. 10.12, the reactive power is given as

$$Q = VI\sin(\theta - \phi) \tag{10.13}$$

It is obvious that the dimensions of Q should be the same as that of P. However, to avoid confusion, the units of Q are taken as 'volt ampere reactive' (abbreviated as VAR). To summarize,

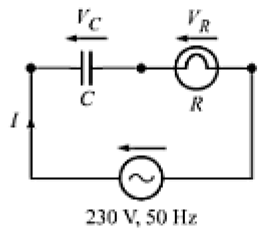
Name	***Symbol***	***Value***	***Units***
1. Apparent power	S	VI	volt amperes (VA)
2. Average power	P	$VI\cos(\theta - \phi)$	watts (W)
3. Reactive power	Q	$VI\sin(\theta - \phi)$	volt ampere reactive (VAR)

EXAMPLE 10.2

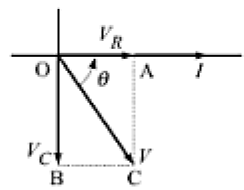
A metal-filament lamp, rated at 750 W, 100 V, is to be used on a 230-V, 50-Hz supply, by connecting a capacitor of suitable value in series. Determine (*a*) the capacitance required, (*b*) the phase angle, (*c*) the power factor, (*d*) the apparent power, and (*e*) the reactive power.

Solution The circuit is given in Fig. 10.4*a*, and the phasor diagram is shown in Fig. 10.4*b*. The voltage phasor V_R is in phase with the current I. The voltage phasor V_C lags the current I by 90°. The rated current of the lamp is

$$I = \frac{P}{V_R} = \frac{750}{100} = 7.5 \text{ A}$$



(*a*) The circuit.



(*b*) Phasor diagram.

Fig. 10.4 *Using a capacitor to light a lower-voltage-rating lamp on ac mains.*

(*a*) From ΔOAC in Fig. 10.4*b*, we can determine the voltage across the capacitor,

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(230)^2 - (100)^2} = 207 \text{ V}$$

Now, $$X_C = \frac{V_C}{I} = \frac{207}{7.5} = 27.6\ \Omega \Rightarrow \frac{1}{2\pi f C} = 27.6.$$

∴ $$C = \frac{1}{2\pi \times 50 \times 27.6} = 115 \times 10^{-6} \text{ F} = \mathbf{115\ \mu F}$$

(*b*) Phase angle, $\phi = \cos^{-1}\frac{V_R}{V} = \cos^{-1}\frac{100}{230} = \mathbf{64°12'}$

(*c*) Power factor, $pf = \cos\phi = (100/230) =$ **0.435 leading**

(*d*) Apparent power $= VI = 230 \times 7.5 =$ **1725 VA**

(*e*) Reactive power $= VI \sin\phi = 230 \times 7.5 \times \sin 64°12' =$ **1553 VAR**

EXAMPLE 10.3

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Solution Taking current as the reference phasor, $\mathbf{I} = 0.9\angle 0°$ A.

Impedance,	$\mathbf{Z} = 120 - j250 = \mathbf{277.3\angle{-64.4°}\ \Omega}$
Power factor,	$pf = \cos\theta = \cos(-64.4°) =$ **0.432 leading**
Supply voltage,	$\mathbf{V} = \mathbf{IZ} = (0.9\angle 0°)(277.3\angle{-64.4°}) = \mathbf{249.6\angle{-64.4°}\ V}$
Voltage across resistor,	$V_R = IR = (0.9\angle 0°) \times 120 = \mathbf{108\angle 0°\ V}$
Voltage across capacitor,	$V_C = IX_C = (0.9\angle 0°)(250\angle{-90°}) = \mathbf{225\angle{-90°}\ V}$
Apparent power,	$P_{app} = VI = 249.6 \times 0.9 = \mathbf{224.6\ VA}$
Actual power,	$P_a = VI\cos\theta = 249.6 \times 0.9 \times \cos 64.4° = \mathbf{97.06\ W}$
Reactive power,	$P_r = VI\sin\theta = 249.6 \times 0.9 \times \sin 64.4° = \mathbf{202.58\ VAR}$

10.4 PARALLEL *RL* CIRCUIT

Consider the circuit of Fig. 10.5*a*. Through this circuit, we shall introduce the concept of ***complex admittance*** represented by **Y**. The current $\mathbf{I}_R$ through resistor R and the current $\mathbf{I}_L$ through inductor L are given as

$$\mathbf{I}_R = \frac{\mathbf{V}}{R} \qquad \text{and} \qquad \mathbf{I}_L = \frac{\mathbf{V}}{jX_L} = \frac{\mathbf{V}}{j\omega L}$$

Using KCL, the total current **I** is given as

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L = \frac{\mathbf{V}}{R} + \frac{\mathbf{V}}{j\omega L} = \mathbf{V}\left(\frac{1}{R} + \frac{1}{j\omega L}\right) = \mathbf{VY} \tag{10.14}$$

where **Y** is the ***complex admittance*** of the parallel *RL* circuit. After rationalization, we can write

$$\mathbf{Y} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - j\frac{1}{\omega L} \tag{10.15}$$

In general, the real part of complex admittance **Y** is called ***conductance*** (represented by G) and the imaginary part is called ***susceptance*** (represented by B). Thus, we can write

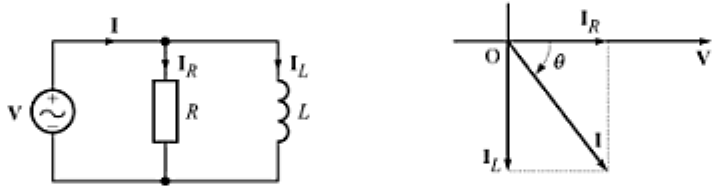
$$\mathbf{Y} = G + jB = (\text{conductance}) + j(\text{susceptance}) \tag{10.16}$$

For parallel *RL* circuit, $G = 1/R$ and $B = -1/\omega L$. Equation 10.14 can now be written as

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L = \mathbf{VY} = \mathbf{V}(G + jB) = \mathbf{V}\left(\frac{1}{R} - j\frac{1}{\omega L}\right) \tag{10.17}$$

This shows that in a parallel *RL* circuit, the current **I** lags the voltage **V** by the admittance angle, given by

$$\theta = \tan^{-1}\frac{-1/\omega L}{1/R} = \tan^{-1}\frac{-R}{\omega L} \tag{10.18}$$



(*a*) The circuit. (*b*) Phasor diagram.

Fig. 10.5 *Parallel RL circuit.*

Phasor Diagram Sometimes it is hard to know where to start drawing the phasor diagram. But, the rule is simple—start with the quantity that is common to the components of the circuit. Here, we are dealing with a parallel circuit, therefore the voltage **V** is the common quantity. Hence, we start the phasor diagram by taking the voltage phasor as reference. The current $\mathbf{I}_R$ is in phase with **V** and current $\mathbf{I}_L$ lags voltage **V** by 90° (see Fig. 10.5*b*). The resultant current phasor **I** is then found by adding phasors $\mathbf{I}_R$ and $\mathbf{I}_L$.

10.5 PARALLEL *RC* CIRCUIT

For the parallel *RC* circuit of Fig. 10.6*a*, the current $\mathbf{I}_R$ through resistor R and the current $\mathbf{I}_C$ through capacitor C are given as

$$\mathbf{I}_R = \frac{\mathbf{V}}{R} \qquad \text{and} \qquad \mathbf{I}_C = \frac{\mathbf{V}}{-jX_C} = \frac{\mathbf{V}}{-j(1/\omega C)} = \mathbf{V}(j\omega C)$$

Using KCL, the total current **I** is given as

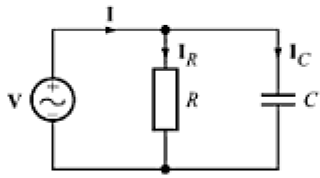
$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C = \frac{\mathbf{V}}{R} + \mathbf{V}(j\omega C) = \mathbf{V}\left(\frac{1}{R} + j\omega C\right) = \mathbf{VY} \tag{10.19}$$

Therefore, for this circuit, the complex admittance is given as

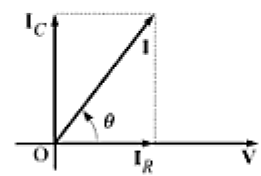
$$\mathbf{Y} = (G + jB) = \left(\frac{1}{R} + j\omega C\right) \tag{10.20}$$

Thus, the conductance $G = 1/R$ and susceptance, $B = \omega C$. Equation 10.19 shows that the current phasor **I** leads the voltage phasor **V** by the admittance angle, given by

$$\theta = \tan^{-1}\frac{B}{G} = \tan^{-1}\frac{\omega C}{1/R} = \tan^{-1}\omega CR \tag{10.21}$$



(*a*) The circuit.



(*b*) Phasor diagram.

Fig. 10.6 *Parallel RC circuit.*

Phasor Diagram Taking voltage **V** as reference, the current $\mathbf{I}_R$ is in phase with voltage **V**, but the current $\mathbf{I}_C$ leads voltage **V** by 90° (see Fig. 10.6*b*). The resultant current phasor **I** is then found by adding phasors $\mathbf{I}_R$ and $\mathbf{I}_C$.

EXAMPLE 10.4

An ac sinusoidal voltage, $\mathbf{V} = (160 + j120)$ V is applied to a circuit. The resulting current is $\mathbf{I} = (-4 + j10)$ A. Find the impedance of the circuit and state whether it is inductive or capacitive. Also, find the power factor, active power and reactive power.

Solution Given: $\mathbf{V} = 160 + j120 = 200\angle 36.87°$ V and $\mathbf{I} = -4 + j10 = 10.77\angle 111.8°$ A.

IMPORTANT NOTE

While converting the rectangular form of a phasor into its polar form, you should be careful. For example, for the current phasor **I**, the angle is calculated as

$$\theta = \tan^{-1}\frac{10}{-4} = -68.2° \qquad \text{(a \textbf{wrong} value !)}$$

(Had the current phasor been $\mathbf{I} = 4 - j10$, then also you would have got the same result for the angle.) The *correct way* is to first ignore the negative signs of the real part and imaginary part and to determine the acute angle. Then, decide the actual angle depending upon the quadrant in which the phasor lies. The given current phasor $\mathbf{I} = -4 + j10$ is seen to lie in the second quadrant. The acute angle, as determined by using a calculator, is 68.2°. Hence, the actual angle should be $\theta = 180° - 68.2° = 111.8°$.

Impedance, $$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{200\angle 36.87°}{10.77\angle 111.8°} = 18.57\angle -74.93° = \mathbf{(4.83 - \mathit{j}17.93)}\ \Omega$$

Since the imaginary part of the impedance is negative, the circuit is **capacitive**.

Power factor, $pf = \cos(-74.93°) = \mathbf{0.26\ leading}$

Active power, $P_a = VI\cos\theta = 200 \times 10.77 \times 0.26 = \mathbf{560\ W}$

Reactive power, $P_r = VI\sin\theta = 200 \times 10.77 \times 0.966 = \mathbf{2079.9\ VAR}$

EXAMPLE 10.5

When a two-element series circuit is connected across an ac source of frequency 50 Hz, it offers an impedance $\mathbf{Z} = (10 + j10)\ \Omega$. Determine the values of the two elements.

Solution The nature of the impedance indicates that the circuit is inductive.

$$\mathbf{Z} = R + jX_L = (10 + j10)\ \Omega$$

$$\therefore \quad R = \mathbf{10\ \Omega} \quad \text{and} \quad X_L = 10\ \Omega \ \Rightarrow\ L = \frac{X_L}{2\pi f} = \frac{10}{2\pi \times 50} = 0.0318\ \text{H} = \mathbf{31.8\ mH}$$

EXAMPLE 10.6

When a two-element parallel circuit is connected across an ac source of frequency 50 Hz, it offers an impedance $\mathbf{Z} = (10 - j10)\ \Omega$. Determine the values of the two elements.

Solution The nature of the impedance indicates that the circuit is capacitive. Since the circuit has two elements connected in parallel, we find the admittance of the circuit.

$$\mathbf{Y} = (G + jB) = \frac{1}{\mathbf{Z}} = \frac{1}{10 - j10} = \frac{1}{14.14\angle -45°} = 0.0707\angle 45°\ \text{S} = (0.05 + j0.05)\ \text{S}$$

$$\therefore \quad G = 0.05\ \text{S} \ \Rightarrow\ R = \frac{1}{G} = \frac{1}{0.05} = \mathbf{20\ \Omega} \quad \text{and} \quad B = 0.05\ \text{S} \ \Rightarrow\ C = \frac{B}{\omega} = \frac{0.05}{2\pi f} = \mathbf{159\ \mu F}$$

10.6 SERIES *RLC* CIRCUIT

Figure 10.7*a* shows a series *RLC* circuit. Writing KVL equation, we get applied voltage as

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C = \mathbf{I}R + \mathbf{I}(jX_L) + \mathbf{I}(-jX_C) = \mathbf{I}[R + j(X_L - X_C)] = \mathbf{IZ} \qquad (10.22)$$

where, **Z** is the complex impedance of the given circuit. We can write

$$\mathbf{Z} = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

or

$$\mathbf{Z} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1}\frac{\omega L - (1/\omega C)}{R} \qquad (10.23)$$

or

$$Z = |\mathbf{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{\omega L - (1/\omega C)}{R} \qquad (10.24)$$

Usually, we take the applied voltage as the reference phasor, i.e., $\mathbf{V} = V\angle 0°$. In such case the resulting current **I** is given as

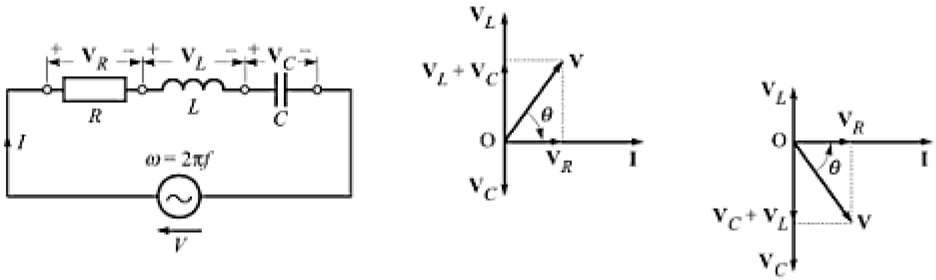
$$\mathbf{I} = I\angle\phi = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle 0°}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \angle -\tan^{-1}\frac{\omega L - (1/\omega C)}{R} \qquad (10.25)$$

In a series *RLC* circuit, one cannot definitely say whether the current lags or leads the applied voltage. It depends upon the relative values of the terms ωL and $1/\omega C$. There can be following three possibilities:

(1) *When* $\omega L > 1/\omega C$ The phase angle ϕ of the current phasor is negative (see Eq. 10.25). The current lags the voltage. The circuit behaves as an ***inductive circuit***.

(2) *When* $\omega L < 1/\omega C$ The phase angle ϕ of the current phasor is positive (see Eq. 10.25). The current leads the voltage. The circuit behaves as a ***capacitive circuit***.

(3) *When* $\omega L = 1/\omega C$ The phase angle $\phi = 0$. The current is in phase with voltage. The circuit behaves as a ***purely resistive circuit***. This is a special case, and is called ***resonance***. We shall talk about this phenomenon a little later.



(*a*) The circuit.

(*b*) Phasor diagram (when $\omega L > 1/\omega C$).

(*c*) Phasor diagram (when $\omega L < 1/\omega C$).

Fig. 10.7 *Series RLC circuit.*

Phasor Diagram Since it is a series circuit, we take current **I** as the reference phasor. The voltage $\mathbf{V}_R$ across resistor *R* will be in phase with the current **I**. The voltage $\mathbf{V}_L$ across inductor *L* leads the current **I** by 90°. The voltage $\mathbf{V}_C$ across capacitor *C* lags the current **I** by 90°. For $\omega L > 1/\omega C$, the phasor diagram is as shown in Fig. 10.7*b*. For the case, $\omega L < 1/\omega C$, the phasor diagram is as shown in Fig. 10.7*c*. Since the phasors $\mathbf{V}_L$ and $\mathbf{V}_C$ are along in opposite directions, we first find the phasor sum $\mathbf{V}_L + \mathbf{V}_C$. We then add to it the phasor $\mathbf{V}_R$, as shown in the figure.

EXAMPLE 10.7

For the circuit shown in Fig. 10.8*a* , calculate (*a*) the impedance, (*b*) the current, (*c*) the phase angle, (*d*) the voltage across each element, (*e*) the power factor, (*f*) the apparent power, and (*g*) the average power. Also, draw the phasor diagram for the circuit.

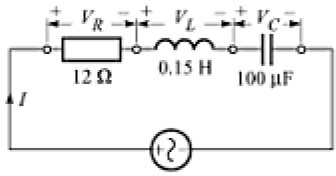
Solution $X_L = \omega L = 2\pi f L = 100\pi \times 0.15 = 47.1\ \Omega$; $X_C = 1/\omega C = 1/(100\pi \times 100 \times 10^{-6}) = 31.8\ \Omega$

(*a*) The impedance, $\mathbf{Z} = R + j(X_L - X_C) = 12 + j(47.1 - 31.8) = \mathbf{(12 + j15.3)\ \Omega}$

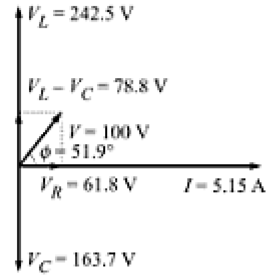
$$= \sqrt{12^2 + 15.3^2}\ \angle \tan^{-1}(15.3/12) = \mathbf{19.4\angle 51.9°\ \Omega}$$

(*b*) The current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{100\angle 0°}{19.4\angle 51.9°} = \mathbf{5.15\angle{-51.9°}\ A}$

(*c*) The phase angle, $\phi = \mathbf{-51.9°}$



(a) The circuit.



(b) Phasor diagram.

Fig. 10.8 *Series RLC circuit.*

(d) The voltage, $V_R = IR = 5.15 \times 12 =$ **61.8 V**; $V_L = IX_L = 5.15 \times 47.1 =$ **242.5 V**;
$V_C = IX_C = 5.15 \times 31.8 =$ **163.7 V**

(e) The power factor, $pf = \cos 51.9° =$ **0.617 lagging**

(f) The apparent power $P_{app} = VI = 100 \times 5.15 =$ **515 VA**

(g) The average power $P_{avg} = VI \cos 51.9° =$ **317.75 W**

The phasor diagram is given in Fig. 10.8*b*.

10.7 PARALLEL *RLC* CIRCUIT

Figure 10.9*a* shows a parallel *RLC* circuit. Writing KCL equation, we get total current **I** supplied by the applied voltage as

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \frac{\mathbf{V}}{R} + \frac{\mathbf{V}}{jX_L} + \frac{\mathbf{V}}{-jX_C} = \mathbf{V}G + \mathbf{V}(-jY_L) + \mathbf{V}(jY_C)$$

or

$$\mathbf{I} = \mathbf{V}[G + j(Y_C - Y_L)] = \mathbf{VY} \tag{10.26}$$

where, **Y** is the complex admittance of the given circuit. Obviously,

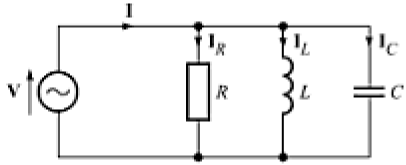
$$G = \frac{1}{R};\ Y_L = \frac{1}{X_L} = \frac{1}{\omega L} \quad \text{and} \quad Y_C = \frac{1}{X_C} = \frac{\omega C}{1} = \omega C$$

Note that $+j$ is associated with Y_C (and not with X_L) and $-j$ with Y_L. The complex admittance of the circuit can be written as

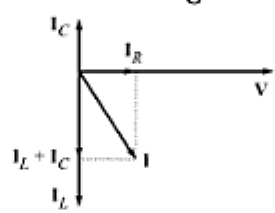
$$\mathbf{Y} = G + j(Y_C - Y_L) = \sqrt{G^2 + (Y_C - Y_L)^2}\ \angle \tan^{-1}\frac{Y_C - Y_L}{G}$$

Take the applied voltage as the reference phasor, i.e., $\mathbf{V} = V\angle 0°$, the resulting current **I** can be written as

$$\mathbf{I} = I\angle\phi = \mathbf{VY} = (V\angle 0°)\left(\sqrt{G^2 + (Y_C - Y_L)^2}\right)\angle \tan^{-1}\frac{Y_C - Y_L}{G}$$



(a) The circuit.



(b) Phasor diagram.

Fig. 10.9 *Parallel RLC circuit.*

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 10.8

A circuit consists of a resistance R in series with a capacitive reactance of 60 Ω. Determine the value of R for which the power factor of the circuit is 0.8.

Solution The power factor, $\cos\theta = 0.8$. Therefore, we get

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(0.8)^2} = 0.6 \quad \Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = \frac{3}{4}$$

From the impedance triangle, we know that

$$\tan\theta = \frac{X_C}{R} \quad \Rightarrow \quad \frac{3}{4} = \frac{X_C}{R} \quad \text{or} \quad \frac{3}{4} = \frac{60}{R} \quad \Rightarrow \quad R = \mathbf{80\ \Omega}$$

EXAMPLE 10.9

An iron choke takes 4 A current when connected to a 20-V dc supply. When connected to a 65-V, 50-Hz ac supply, it takes 5 A current. Determine (*a*) the resistance and inductance of the coil, (*b*) the power factor, and (*c*) the power drawn by the coil.

Solution

(*a*) The iron choke has both resistance R and inductance L. When connected to a dc supply, only its resistance is effective in limiting the current; when connected to an ac supply, its impedance becomes effective in limiting the current. Hence,

$$R = \frac{20\text{ V}}{4\text{ A}} = \mathbf{5\ \Omega} \quad \text{and} \quad Z = \frac{65\text{ V}}{5\text{ A}} = 13\ \Omega \quad \Rightarrow \quad X_L = \sqrt{Z^2 - R^2} = 12\ \Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{12}{2\pi \times 50} = 0.0382\text{ H} = \mathbf{38.2\ mH}$$

(*b*) The power factor, $pf = \cos\theta = \frac{R}{Z} = \frac{5}{13} = \mathbf{0.38\ (lagging)}$

(*c*) The power, $P = VI\cos\theta = 65 \times 5 \times 0.38 = \mathbf{123.5\ W}$

EXAMPLE 10.10

A choke coil is connected to a 240-V ac supply. When the frequency of the supply is 50 Hz, an ammeter connected in series with the choke reads 60 A. On increasing the frequency of the ac supply to 100 Hz, the same ammeter reads 40 A. Calculate the resistance and inductance of the coil.

Solution On changing the frequency of the ac supply, only the reactance part of the choke changes. Thus,

$$\text{At } f = 50\text{ Hz}: Z_1 = \frac{V}{I_1} = \frac{240\text{ V}}{60\text{ A}} = 4\ \Omega \quad \Rightarrow \quad R^2 + (2\pi \times 50L)^2 = 4^2 \qquad (i)$$

$$\text{At } f = 100\text{ Hz}: Z_2 = \frac{V}{I_2} = \frac{240\text{ V}}{40\text{ A}} = 6\ \Omega \quad \Rightarrow \quad R^2 + (2\pi \times 100L)^2 = 6^2 \qquad (ii)$$

Subtracting Eq. (*i*) from Eq. (*ii*), we get

$$(200\pi)^2 L^2 - (100\pi)^2 L^2 = 20 \quad \Rightarrow \quad L = \sqrt{\frac{20}{(200\pi)^2 - (100\pi)^2}} = \mathbf{8.22\ mH}$$

$$X_1 = 2\pi f_1 L = 2\pi \times 50 \times 8.22 \times 10^{-3} = 2.58\ \Omega;$$

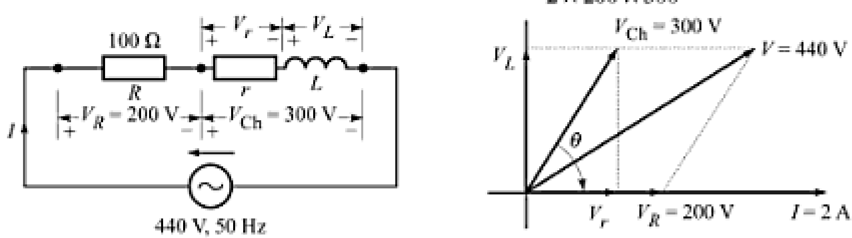
$$\Rightarrow \quad R = \sqrt{Z_1^2 - X_1^2} = \sqrt{4^2 - 2.58^2} = \mathbf{3.05\ \Omega}$$

EXAMPLE 10.11

A 100-Ω resistor is connected in series with a choke coil. When a 440-V, 50-Hz ac voltage is applied to this combination, the voltage across the resistor and the choke coil are 200 V and 300 V, respectively. Find the power consumed by the choke coil. Sketch a neat phasor diagram, indicating the current and all voltages.

Solution A choke coil has some resistance, say r, and some inductance, say L. Though the resistance is distributed throughout the coil winding, we can show it as lumped parameter in its equivalent circuit, as given in Fig. 10.10*a*. Because of the presence of this resistance, the voltage across the choke coil, V_{Ch}, leads the current I by an angle less than 90° (say, angle θ), as shown in the phasor diagram of Fig. 10.10*b*. As per KVL, the phasor voltages $\mathbf{V}_R$ and $\mathbf{V}_{Ch}$ must add up to the supply voltage **V**. That is, $\mathbf{V} = \mathbf{V}_R + \mathbf{V}_{Ch}$. Hence,

$$V^2 = V_R^2 + V_{Ch}^2 + 2V_R V_{Ch} \cos\theta \quad \Rightarrow \quad \cos\theta = \frac{(440)^2 - (200)^2 - (300)^2}{2 \times 200 \times 300} = 0.53$$



(*a*) The circuit. (*b*) Phasor diagram.

Fig. 10.10 *A series ac circuit.*

Now, $I = \dfrac{V_R}{R} = \dfrac{200}{100} = 2$ A; P (in the choke coil) $= V_{Ch} \times I \times \cos\theta = 300 \times 2 \times 0.53 = \mathbf{318\ W}$

EXAMPLE 10.12

A coil having a resistance of 15 Ω and inductance of 0.2 H is connected in series with another coil having a resistance of 25 Ω and inductance of 0.04 H to a 230-V, 50-Hz supply. Determine (*a*) the voltage across each coil, (*b*) the power dissipated in each coil, and (*c*) the power factor of the whole circuit

Solution To find the total complex impedance, **Z**, we first find the complex impedances of the individual coils:

$$X_1 = 2\pi f L_1 = 100\pi \times 0.2 = 62.8\ \Omega; \qquad \text{and} \qquad X_2 = 2\pi f L_2 = 100\pi \times 0.04 = 12.6\ \Omega$$

$$\therefore \quad \mathbf{Z}_1 = (15 + j62.8)\ \Omega = 64.6\angle 76.6°\ \Omega; \qquad \text{and} \qquad \mathbf{Z}_2 = (25 + j12.6)\ \Omega = 28\angle 26.7°\ \Omega$$

$$\therefore \quad \mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = (40 + j75.4)\ \Omega = 85.4\angle 62°\ \Omega \quad \Rightarrow \quad I = \frac{V}{Z} = \frac{230}{85.4} = 2.7\ \text{A}$$

(*a*) $V_1 = IZ_1 = 2.7 \times 64.6 = 174.42 \approx \mathbf{174\ V}$; and $V_2 = IZ_2 = 2.7 \times 28 = 75.6 \approx \mathbf{76\ V}$

(*b*) $P_1 = I^2 R_1 = (2.7)^2 \times 15 = 109.35 \approx \mathbf{109\ W}$; and $P_2 = I^2 R_1 = (2.7)^2 \times 25 = 182.25 \approx \mathbf{182\ W}$

(*c*) $pf = \cos\theta = \cos 62° = \mathbf{0.469}$ **(lagging)**

EXAMPLE 10.13

When a 100-V, 50-Hz ac source is connected to a coil A, the resulting current is 8 A and the power delivered is 120 W. When the same source is connected to coil B, the resulting current is 10 A and the power delivered is 500 W. What current and power will be taken from the same source, if the two coils joined in series are connected to it?

Solution

For coil A: $Z_1 = \dfrac{100}{8} = 12.5\ \Omega$; $R_1 = \dfrac{P}{I^2} = \dfrac{120}{8^2} = 1.875\ \Omega \quad \Rightarrow \quad X_1 = \sqrt{Z_1^2 - R_1^2} = 12.36\ \Omega$

For coil B: $Z_2 = \dfrac{100}{10} = 10\ \Omega$; $R_1 = \dfrac{P}{I^2} = \dfrac{500}{10^2} = 5\ \Omega \quad \Rightarrow \quad X_2 = \sqrt{Z_2^2 - R_2^2} = 8.66\ \Omega$

For the two coils joined in series, we have

$$R = R_1 + R_2 = 1.875 + 5 = 6.875\ \Omega \qquad \text{and} \qquad X = X_1 + X_2 = 12.36 + 8.66 = 21.02\ \Omega$$

$$\therefore \qquad Z = \sqrt{R^2 + X^2} = 22.115\ \Omega;\ I = \frac{V}{Z} = \frac{100}{22.115} = \mathbf{4.52\ A};$$

$$P = I^2R = (4.52)^2 \times 6.875 = \mathbf{140.46\ W}$$

EXAMPLE 10.14

Two coils A and B are connected in series across a 240-V, 50-Hz supply. The resistance R_A of A is 5 Ω and inductance L_B of B is 0.015 H. The active and reactive powers are 3 kW and 2 kVAR, respectively. Find (*a*) the resistance R_B of coil B, (*b*) the inductance L_A of coil A, (*c*) the voltage drop across each coil.

Solution

(*a*) The apparent power (magnitude of complex power) is given as

$$S = \sqrt{P^2 + Q^2} = \sqrt{3^2 + 2^2} = 3.6055\ \text{kVA}$$

$$\Rightarrow \qquad I = \frac{S}{V} = \frac{3.6055 \times 10^3}{240} = 15.02\ \text{A}$$

The real power is given as

$$P = I^2(R_A + R_B) \quad \text{or} \quad 3 \times 10^3 = (15.02)^2(R_A + R_B) \quad \Rightarrow \quad (R_A + R_B) = 13.3\ \Omega$$

$$\Rightarrow \qquad R_B = 13.3 - R_A = 13.3 - 5 = \mathbf{8.3\ \Omega}$$

(*b*) The reactive power is given as

$$Q = I^2(X_A + X_B) \quad \text{or} \quad 2 \times 10^3 = (15.02)^2(X_A + X_B) \quad \Rightarrow \quad (X_A + X_B) = 8.865\ \Omega$$

But, $\quad X_B = 2\pi f L_B = 2\pi \times 50 \times 0.015 = 4.71\ \Omega; \qquad \therefore \quad X_A = 8.865 - 4.71 = 4.155\ \Omega$

$$\therefore \qquad L_A = \frac{X_A}{2\pi f} = \frac{4.155}{2\pi \times 50} = \mathbf{0.0132\ H}$$

(*c*) $Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.155^2} = 6.50\ \Omega$; $Z_B = \sqrt{R_B^2 + X_B^2} = \sqrt{8.3^2 + 4.71^2} = 9.543\ \Omega$

$\therefore \quad V_A = IZ_A = 15.02 \times 6.50 = \mathbf{97.63\ V}; \qquad \text{and} \qquad V_B = IZ_B = 15.02 \times 9.543 = \mathbf{143.336\ V}$

EXAMPLE 10.15

It is desired to operate a 100-W, 120-V electric bulb at its rated current on a 240-V, 50-Hz supply. The simplest arrangement is to use either (*a*) a resistor, or (*b*) a capacitor, or (*c*) an inductor having 10-Ω resistance, in series with the electric bulb so as to drop the excess voltage. Calculate the value of the component used, the total power consumed and the power factor in each case. Giving reasons, state which alternative is the best.

Solution Rated current of the electric bulb, $I = \dfrac{P}{V} = \dfrac{100}{120} = 0.833$ A.

(*a*) $V_R = V - V_b = 240 - 120 = 120$ V; $\quad \Rightarrow \quad R = \dfrac{V_R}{I} = \dfrac{120}{0.833} = \mathbf{144\ \Omega}$; $pf = \mathbf{1}$

Total power consumed, $P_t = VI = 240 \times 0.833 = \mathbf{200\ W}$

(*b*) $V_C = \sqrt{240^2 - 120^2} = 207.84$ V; $\Rightarrow$ $X_C = \dfrac{207.84}{0.833} = 249.5\ \Omega$; $C = \dfrac{1}{100\pi X_C} = \mathbf{12.76\ \mu F}$

$pf = \cos\phi = \dfrac{120}{240} = \mathbf{0.5\ (leading)}$; $P = VI\cos\phi = 240 \times 0.833 \times 0.5 = \mathbf{100\ W}$

(*c*) $V_r = Ir = 0.833 \times 10 = 8.33$ V; $V_L = \sqrt{240^2 - (120 + 8.33)^2} = 202.8$ V $\Rightarrow$ $X_L = \dfrac{202.8}{0.833} = 243.46\ \Omega$

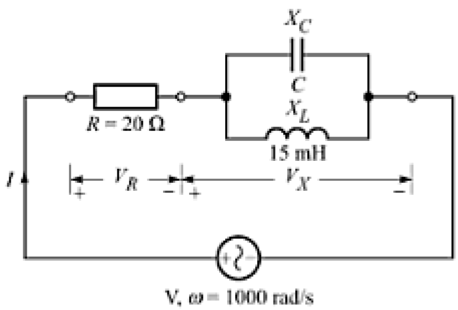
$\therefore\quad L = \dfrac{X_L}{100\pi} = \dfrac{243.46}{100\pi} = \mathbf{0.775\ H}$; Total resistive voltage drop = 120 + 8.33 = 128.33 V

$pf = \cos\phi = \dfrac{128.33}{240} = \mathbf{0.535\ (lagging)}$; $P = 240 \times 0.833 \times 0.535 = \mathbf{107\ W}$

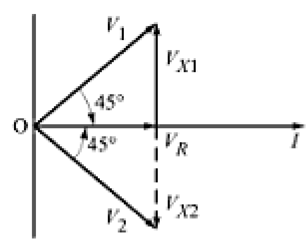
It is obvious that the method (*b*) is most economical, as it involves least wastage of power. Method (*a*) is the worst, as it involves wastage of as much as 100 W of power for utilizing 100 W for lighting the bulb. Method (*b*) is better than the method (*c*), as it involves less wastage of power and, furthermore, a capacitor is much cheaper than a choke coil.

EXAMPLE 10.16

A 20-Ω resistance is connected in series with a parallel combination of a capacitance *C* and a 15-mH pure inductance, as shown in Fig. 10.11*a*. This circuit is connected to a voltage source of angular frequency ω = 1000 rad/s. Find the value of capacitance *C* such that the line current is 45° out of phase with the source voltage.



(*a*) The ac circuit.



(*b*) The desired phasor diagram.

Fig. 10.11

Solution The inductive reactance, $X_L = \omega L = 1000 \times 0.015 = 15\ \Omega$. The impedance of the circuit is given as $Z = R \pm jX$, because the net reactance *X* of the parallel combination of *L* and *C* can be either inductive or capacitive.

For these two possibilities, the desired phasor diagram is shown in Fig. 10.11*b*. The voltage V_R is in phase with current *I*. If *X* is *inductive*, the voltage phasor V_{X1} leads the current phasor *I* by 90°, and the source voltage V_1 leads the current *I* by 45°. In case *X* is *capacitive*, the voltage phasor V_{X2} lags the current phasor *I* by 90°, and the source voltage V_2 lags the current *I* by 45°. For both cases, the phase angle is 45°. Hence,

$$\tan 45° = \frac{X}{R} \quad \Rightarrow \quad X = R = 20\ \Omega$$

Case I (*X* inductive): The net reactance *X* is given as

$$\frac{1}{jX} = \frac{1}{jX_L} + \frac{1}{-jX_C} \quad \Rightarrow \quad \frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} \quad \text{or} \quad \frac{1}{X_C} = \frac{1}{X_L} - \frac{1}{X} = \frac{1}{15} - \frac{1}{20} = \frac{1}{60}$$

$$\Rightarrow \qquad X_C = 60\ \Omega \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{1000 \times 60} = \mathbf{16.67\ \mu F}$$

Case II (X capacitive) : The net reactance X is given as

$$\frac{1}{-jX} = \frac{1}{jX_L} + \frac{1}{-jX_C} \quad \Rightarrow \quad -\frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} \quad \text{or} \quad \frac{1}{X_C} = \frac{1}{X_L} + \frac{1}{X} = \frac{1}{15} + \frac{1}{20} = \frac{7}{60}$$

$$\Rightarrow \qquad X_C = \frac{60}{7}\,\Omega \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{7}{1000 \times 60} = \mathbf{116.67\ \mu F}$$

EXAMPLE 10.17

Two impedances, $Z_1 = (12 + j15)\ \Omega$ and $Z_2 = (8 - j4)\ \Omega$, are connected in parallel. If the potential difference across this combination is $(230 + j0)$ V, calculate (*a*) the current supplied to each branch and the total current, (*b*) the power consumed by each branch and the total power, and (*c*) the *pf* of each branch and the overall *pf*.

Solution $Z_1 = (12 + j15)\ \Omega = 19.2\angle 51.3°\ \Omega$; $Z_2 = (8 - j4)\ \Omega = 8.94\angle -26.56°\ \Omega$;
$V = (230 + j0)\text{ V} = 230\angle 0°\text{ V}$

(*a*) The currents are given as

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z_1}} = \frac{230\angle 0°\text{ V}}{19.2\angle 51.3°\,\Omega} = \mathbf{11.98\angle{-51.3°}\ A} = (7.49 - j9.35)\text{ A}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z_2}} = \frac{230\angle 0°\text{ V}}{8.94\angle{-26.56°}\,\Omega} = \mathbf{25.73\angle 26.56°\ A} = (23 + j11.5)\text{ A}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = (7.49 - j9.35) + (23 + j11.5) = (30.49 + j2.15) = \mathbf{30.56\angle 4.03°\ A}$$

(*b*) $P = VI\cos\theta = 230 \times 30.56 \times \cos 4.03° = \mathbf{7011.42\ W}$

$P_1 = I_1^2 R_1 = (11.98)^2 \times 12 = \mathbf{1722.24\ W}$; $P_2 = I_2^2 R_2 = (25.73)^2 \times 8 = \mathbf{5296.26\ W}$

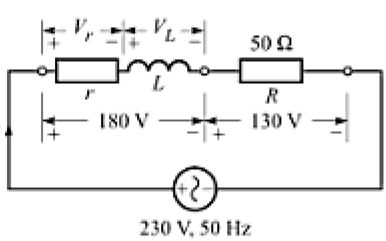
(*c*) $pf_1 = \dfrac{R_1}{Z_1} = \dfrac{12}{19.2} = \mathbf{0.625\ (lag)}$; $pf_2 = \dfrac{R_2}{Z_2} = \dfrac{8}{8.94} = \mathbf{0.894\ (lead)}$; $pf = \cos 4.03° = \mathbf{0.998\ (lead)}$

EXAMPLE 10.18

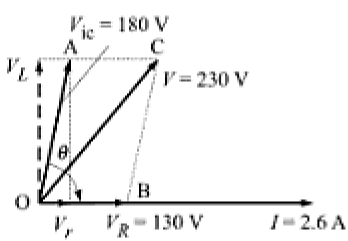
An inductive coil is connected in series with a resistance of 50 Ω across a 230-V, 50-Hz ac supply. The voltage across the coil is 180 V; and across the resistance is 130 V. Calculate (*a*) the resistance and inductance of the coil, (*b*) the power dissipated in the coil. Also, draw the phasor diagram.

Solution The circuit is shown in Fig. 10.12*a*, and its phasor diagram is given in Fig. 10.12*b*. The magnitude of the current I is given as

$$I = \frac{V_R}{R} = \frac{130}{50} = 2.6\text{ A}$$



(*a*) The circuit.



(*b*) The phasor diagram.

Fig. 10.12

(*a*) From the parallelogram OACB, the angle θ is given by

$$230^2 = 130^2 + 180^2 + 2 \times 180 \times 130 \times \cos\theta \quad \Rightarrow \quad \cos\theta = 0.0769 \quad \text{or} \quad \theta = 85.6°$$

$\therefore$ $$V_L = V_C \sin\theta = 180 \times 0.997 = 179.46 \text{ V}$$

and $$V_r = V_C \cos\theta = 180 \times 0.0769 = 13.84 \text{ V}$$

$\therefore$ $$L = \frac{X_L}{\omega} = \frac{V_L / I}{\omega} = \frac{V_L}{I\omega} = \frac{179.46}{2.6 \times 2\pi \times 50} = \mathbf{0.22\ H}$$

and $$r = \frac{V_r}{I} = \frac{13.84}{2.6} = \mathbf{5.32\ \Omega}$$

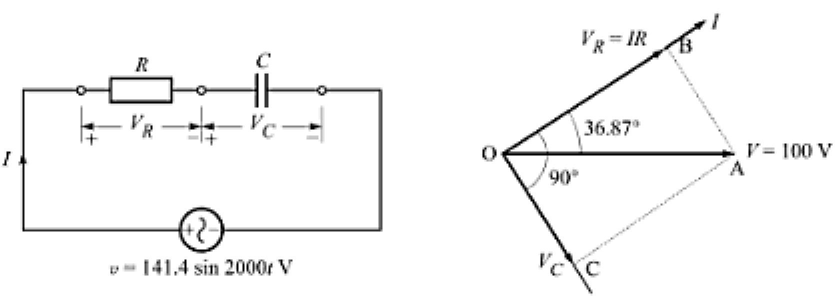
(*b*) $P = IV_r = 2.6 \times 13.84 =$ **36 W**. The phasor diagram is drawn in Fig. 10.12*b*.

EXAMPLE 10.19

The terminal voltage and current for a series circuit are $v = 141.4 \sin 2000t$ V and $i = 7.07 \sin(2000t + 36.87°)$ A. Obtain the simplest two-element series circuit which would have the above v-i relationship.

Solution The simplest *RC* series circuit is shown in Fig. 10.13*a*, and the phasor diagram is given in Fig. 10.13*b*. From the given expressions of v and i, we have

$$V = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100 \text{ V} \qquad \text{and} \qquad I = \frac{I_m}{\sqrt{2}} = \frac{7.07}{\sqrt{2}} = 5 \text{ A}$$



(*a*) The circuit. (*b*) The phasor diagram.

Fig. 10.13

From the circuit, we have

$$V = IZ \quad \text{or} \quad 100 = 5 \times \sqrt{R^2 + X_C^2} \quad \Rightarrow \quad R^2 + X_C^2 = \frac{100^2}{5^2} = 400 \qquad (i)$$

From ΔOAB in Fig. 10.13*b*,

$$\cos(36.87°) = \frac{V_R}{V} = \frac{IR}{V} = \frac{5R}{100} \quad \Rightarrow \quad R = \mathbf{16\ \Omega} \qquad (ii)$$

Putting this value of R in Eq. (*i*), we get

$$X_C^2 = 400 - 16^2 = 144 \quad \text{or} \quad X_C = 12\ \Omega \quad \Rightarrow \quad C = \frac{1}{\omega X_C} = \frac{1}{2000 \times 12} = \mathbf{41.67\ \mu F}$$

SUMMARY

TERMS AND CONCEPTS

- If a circuit contains both resistance and inductance, the current lags the voltage by an angle less than 90° but greater than 0°.
- If a circuit contains both resistance and capacitance, the current leads the voltage by an angle less than 90° but greater than 0°.
- If a circuit contains resistance, inductance and capacitance, the current may lead, lag or be in phase with the voltage depending on the relative values of inductive and capacitive reactances.
- The quantity **Z** is called ***complex impedance***, whose real part is called ***resistance*** (R) and imaginary part is called ***reactance*** (X), either *inductive* or *capacitive*.
- The inverse of impedance is called ***admittance* (Y)**, whose real part is called ***conductance*** (G) and the imaginary part is called ***susceptance*** (B), either *inductive* or *capacitive*.
- We associate $+j$ with X_L and $-j$ with X_C.

IMPORTANT FORMULAE

- In a series RL circuit, $\mathbf{Z} = R + j\omega L$ and $\mathbf{I} = \dfrac{(V\angle 0°)}{\mathbf{Z}} = \dfrac{V}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}(\omega L / R)$.
- In a series RC circuit, $\mathbf{Z} = R - j(1/\omega C)$ and $\mathbf{I} = \dfrac{(V\angle 0°)}{\mathbf{Z}} = \dfrac{V}{\sqrt{R^2 + (1/\omega C)^2}} \angle \tan^{-1}(1/\omega C R)$.
- The ***complex power***, $\mathbf{S} = P + jQ = \mathbf{VI}^* = VIe^{j(\theta - \phi)}$.
- The ***apparent power***, $S = VI$ (measured in VA).
- The ***active*** or ***real power***, $P = VI\cos(\theta - \phi)$ (measured in W).
- The ***reactive power***, $Q = VI\sin(\theta - \phi)$ (measured in VAR).
- For a series RLC circuit,

$$\mathbf{Z} = R + j(X_L - X_C) = R + j(\omega L - 1/\omega C) = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \angle \tan^{-1} \frac{\omega L - (1/\omega C)}{R}$$

$$\mathbf{I} = I\angle\phi = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle 0°}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \angle -\tan^{-1} \frac{\omega L - (1/\omega C)}{R}$$

- For a parallel RLC circuit,

$$\mathbf{I} = I\angle\phi = \mathbf{VY} = (V\angle 0°)\left(\sqrt{G^2 + (Y_C - Y_L)^2}\right)\angle \tan^{-1}\frac{Y_C - Y_L}{G}$$

where, $$G = \frac{1}{R};\ Y_L = \frac{1}{X_L} = \frac{1}{\omega L} \quad \text{and} \quad Y_C = \frac{1}{X_C} = \frac{\omega C}{1} = \omega C$$

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	All phasors can be represented by complex numbers. Hence, all complex numbers, such as impedance and admittance, represent phasors.	☐	☐	
2.	In a series *RL* circuit, the current always lags the voltage.	☐	☐	
3.	The real part of the complex admittance of a series *RL* circuit is called *conductance*, and is given as $G = 1/R$.	☐	☐	
4.	The complex admittance of a parallel *RC* circuit can be written as $\mathbf{Y} = (1/R) - j\omega C$.	☐	☐	
5.	For a parallel *RC* circuit, if the current phasor is represented as $I\angle 0°$, then the voltage can be represented as $V\angle -\theta$, where angle θ has positive values.	☐	☐	
6.	When an iron-core choke coil is connected in an ac circuit, the voltage across it leads the current through it by an angle less than 90°.	☐	☐	
7.	For a parallel ac circuit, we usually take the voltage as reference phasor, because the voltage across each of the components is the same.	☐	☐	
8.	If the admittance of a two-element series circuit is given as $\mathbf{Y} = (3 + j4)$ S, we can definitely say that the circuit consists of a resistance in series with an inductance.	☐	☐	
9.	If the impedance of a two-element parallel circuit is given as $\mathbf{Z} = (3 + j4)\ \Omega$, we can definitely say that the circuit consists of a resistance in parallel with a capacitance.	☐	☐	
10.	In a series *RC* circuit, the current lags the voltage by an angle greater than 270° but less than 360°.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** False	**4.** False	**5.** True
6. True	**7.** True	**8.** False	**9.** False	**10.** True

REVIEW QUESTIONS

1. In a series *RL* circuit, when can you neglect the inductive reactance as compared to the resistance?
2. What is the impedance of a parallel *RL* circuit? How does the series resistance in inductive branch affect this impedance?
3. How is that in a purely inductive circuit, although the current flows but the active power *P* is zero?
4. How and why does the frequency of the ac supply affect the magnitude of current in an ac circuit?
5. Show the wave shape of the current and voltage for a series *RL* circuit.
6. Draw the phasor diagram for (*a*) a parallel *RC* circuit, and (*b*) a series *RC* circuit.
7. Draw the phasor diagram for (*a*) a parallel *RL* circuit, and (*b*) a series *RL* circuit.
8. For a series ac circuit having resistive and reactive components,
   (*a*) How do you determine the active power consumed? Give two equations for calculating the active power.
   (*b*) What does volt-ampere (VA) mean?
   (*c*) What does volt-ampere-reactive (VAR) mean?
   (*d*) How is the VAR calculated?

9. What do you understand by the term 'power factor'? What is its significance? When is the power factor leading? When is it lagging?
10. What is an impedance triangle? Draw the impedance triangle for a series *RL* circuit and series *RC* circuit.
11. Derive expressions for calculating the impedance, admittance, conductance and susceptance of a series *RL* circuit.
12. In a series *RLC* circuit, the current is seen to be lagging the voltage. What does this mean? What is the nature of the circuit?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. In a series *RL* circuit, the phase difference between the applied voltage and the current increases if
   (*a*) X_L is increased
   (*b*) X_L is decreased
   (*c*) R is increased
   (*d*) supply frequency is decreased
2. The power factor of a series *RL* ac circuit is given by
   (*a*) X_L/R (*b*) R/X_L
   (*c*) R/Z (*d*) Z/R
3. For greater accuracy, the value of phase angle θ in an ac circuit should be determined from
   (*a*) $\cos\theta$ (*b*) $\sin\theta$
   (*c*) $\tan\theta$ (*d*) $\sec\theta$
4. A low power factor of an ac circuit means that
   (*a*) it causes less voltage drop in the line
   (*b*) it draws more active power
   (*c*) it draws less line current
   (*d*) it draws more reactive power
5. The impedance of an circuit is given as $15.5 \angle -30°\ \Omega$. It means that the circuit is
   (*a*) capacitive (*b*) inductive
   (*c*) purely resistive (*d*) none of the above
6. If the active and apparent power of an ac circuit are equal in magnitude, its power factor is
   (*a*) 0.5 (*b*) 0.707
   (*c*) 0.8 (*d*) 1
7. For the ac circuit given in Fig. 10.14, the power factor is
   (*a*) 0.4 lagging (*b*) 0.6 lagging
   (*c*) 0.75 lagging (*d*) 0.8 lagging
8. The power consumed in the circuit of Fig. 10.14 is
   (*a*) 4000 W (*b*) 2400 W
   (*c*) 2000 W (*d*) 1200 W

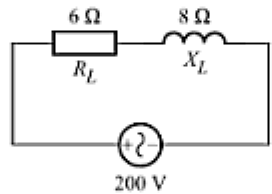


Fig. 10.14

9. The reactive power drawn in the circuit of Fig. 10.14 is
   (*a*) 4000 VAR (*b*) 3200 VAR
   (*c*) 2000 VAR (*d*) 1200 VAR
10. The apparent power drawn in the circuit of Fig. 10.14 is
   (*a*) 4000 VA (*b*) 3200 VA
   (*c*) 2000 VA (*d*) 1200 VA
11. In a series *RLC* circuit, the inductive reactance is 10 Ω and the capacitive reactance is 15 Ω. The total reactance in the circuit is
   (*a*) 25 Ω (*b*) 18.03 Ω
   (*c*) 5 Ω (*d*) 1.5 Ω
12. A series *RL* circuit has a resistance of 6 Ω and a reactance of 8 Ω. The impedance of the circuit is
   (*a*) 10 Ω (*b*) 14 Ω
   (*c*) 2 Ω (*d*) 13.43 Ω
13. The resistance and the reactance in a series *RC* circuit are 7.5 Ω each. In this circuit,
   (*a*) the voltage leads the current by 45°
   (*b*) the current leads the voltage by 45°
   (*c*) the voltage leads the current by 60°
   (*d*) the current leads the voltage by 15°
14. In an *RC* circuit, the resistance and reactance are 1 Ω and 10 Ω, respectively. In this circuit,
   (*a*) the voltage leads the current by 84.3°
   (*b*) the current leads the voltage by 5.7°
   (*c*) the voltage leads the current by 5.7°
   (*d*) the current leads the voltage by 84.3°

15. In a series *RLC* circuit, $R = 5\ \Omega$, $X_L = 10\ \Omega$ and $X_C = 15\ \Omega$. If this circuit is connected to a voltage source $v = 100 \sin(314t + 30°)$ V, the rms value of the current will be
(*a*) 14.14 A (*b*) 10 A
(*c*) 5 A (*d*) 3.33 A

16. The impedance of an *RL* circuit is 25 Ω at a frequency of 50 Hz. At a frequency of 60 Hz, its impedance will be
(*a*) greater than 25 Ω (*b*) exactly 25 Ω
(*c*) less than 25 Ω (*d*) 0 Ω

17. The impedance of an *RC* circuit is 20 Ω at a frequency of 50 Hz. At a frequency of 60 Hz, its impedance will be
(*a*) greater than 20 Ω (*b*) exactly 20 Ω
(*c*) less than 20 Ω (*d*) 0 Ω

18. The maximum and minimum values of power factor in an ac circuit can be
(*a*) 1 and 0 (*b*) +1 and –1
(*c*) +1 and –5 (*d*) +10 and –10

ANSWERS

1. *a*	**2.** *c*	**3.** *c*	**4.** *d*	**5.** *a*	**6.** *d*	**7.** *b*	**8.** *b*	**9.** *b*	**10.** *a*
11. *c*	**12.** *a*	**13.** *b*	**14.** *d*	**15.** *b*	**16.** *a*	**17.** *c*	**18.** *b*		

PROBLEMS

(A) SIMPLE PROBLEMS

1. A series circuit having a resistance of 10 Ω and an inductance of 20 mH is carrying a current given as $i = 2 \sin 500t$ A. Obtain the total voltage across the series circuit, and the angle by which the current lags the voltage. [**Ans.** 20 V, 45° (lag)]

2. A circuit is composed of a resistance of 8 Ω and a capacitive reactance of 6 Ω in series. A voltage, $v = 141.4 \sin(314t)$ V is applied to the circuit.
(*a*) Find the complex impedance and draw the impedance triangle.
(*b*) Determine the rms and instantaneous values of the current.
(*c*) Calculate the power delivered to the circuit.
(*d*) Find the equation for the voltage appearing across the capacitor.
(*e*) Find the value of the capacitance.
[**Ans.** (*a*) $10\angle{-36.8°}\ \Omega$; (*b*) 10 A, $i = 14.14 \sin(314t + 36.8°)$ A; (*c*) 800 W; (*d*) $v = 84.8 \sin(314t - 53.2°)$ V; (*e*) 531 μF]

3. A series *RC* circuit, having $R = 4\ \Omega$ and $C = 120\ \mu F$, is connected across 230-V, 50-Hz supply. Calculate (*a*) the reactance, (*b*) the impedance, (*c*) the current drawn by the circuit, and (*d*) the power factor of the circuit.
[**Ans.** (*a*) 26.54 Ω; (*b*) 26.84 Ω; (*c*) 8.57 A; (*d*) 0.149 (leading)]

4. A series *RL* circuit, having $R = 4\ \Omega$ and $L = 0.2$ H, is connected across $230\angle 30°$ V, 50-Hz supply. Calculate (*a*) the current drawn by the circuit, and (*b*) the power factor of the circuit.
[**Ans.** (*a*) $3.656\angle{-56.36°}$ A; (*b*) 0.063 (lagging)]

5. Two impedances $\mathbf{Z}_1 = (5 + j7)\ \Omega$ and $\mathbf{Z}_2 = (10 - j5)\ \Omega$ are connected in series across a 200-V, 50-Hz supply. Determine the current, active power, apparent power and power factor of the circuit.
[**Ans.** 13.2 A, 2614 W, 2640 VA, 0.991 (lagging)]

6. Given $v = 200 \sin 377t$ V and $i = 8 \sin(377t - 30°)$ A for an ac circuit. Determine (*a*) power factor, (*b*) true power, (*c*) apparent power, and (*d*) reactive power.
[**Ans.** (*a*) 0.866 (lagging); (*b*) 692.8 W; (*c*) 800 VA; (*d*) 400 VAR]

7. A coil has a resistance of 10 Ω and draws a current of 5 A when connected across a 100-V, 60-Hz source. Determine (*a*) the inductance of the coil, (*b*) the power factor of the circuit, and (*c*) the reactive power.
[**Ans.** (*a*) 45.94 mH; (*b*) 0.5 lagging; (*c*) 433 VAR]

8. A 20-Ω resistor is connected in parallel to a 26.52-mH inductor. The circuit operates at 60 Hz. What is (*a*) the input admittance, and (*b*) the input impedance of the circuit?
[**Ans.** (*a*) $0.1118\angle{-63.43°}$ S; (*b*) $8.94\angle 63.43°\ \Omega$]

(B) TRICKY PROBLEMS

9. An adjustable resistance R in series with a capacitance of 25 μF draws a current of 0.8 A when connected across a 50-Hz supply. Calculate (*a*) the value of the resistance so that the voltage across the capacitor is half of the supply voltage, (*b*) the power, and (*c*) the pf.

[**Ans.** (*a*) 220.8 Ω; (*b*) 141 W; (*c*) 0.866 (leading)]

10. When a two-element series circuit is connected to an ac source, $v = 200\sqrt{2}\sin(314t + 20°)$ V, the resulting current is found to be $i = 10\sqrt{2}\cos(314t - 25°)$ A. Determine the values of the elements of the circuit.

[**Ans.** $R = 14.14\ \Omega$; $C = 225.2\ \mu F$]

11. A parallel RL circuit, having $R = 50\ \Omega$ and $X_L = 40\ \Omega$, is connected across 100-V ac source. Calculate (*a*) the current drawn from the source, (*b*) the apparent power, (*c*) the real power, and (*d*) the reactive power.

[**Ans.** (*a*) 3.2 A; (*b*) 320 VA; (*c*) 200 W; (*d*) 249.7 VAR]

12. A 46-mH inductive coil has a resistance of 10 Ω. (*a*) How much current will it draw if connected across a 100-V, 60-Hz source? (*b*) What is the power factor of the coil?

[**Ans.** (*a*) $5\angle -60°$ A; (*b*) 0.5 lagging]

13. A capacitor is connected across the coil of Prob. 12 to make the power factor of overall circuit unity. Determine the value of the capacitance.

[**Ans.** 153 μF]

14. A two-element series circuit draws 600 W of power and 10 A of current at 100 V and 500/π Hz. Specify the values of these elements.

[**Ans.** $R = 6\ \Omega$ and $L = 8$ mH or $C = 125\ \mu F$]

15. A parallel RL circuit has $R = 4\ \Omega$ and $X_L = 3\ \Omega$. Obtain its series equivalent such that the series circuit draws the same current and power at a given voltage.

[**Ans.** $R = 1.44\ \Omega$, $X_L = 1.92\ \Omega$]

(C) CHALLENGING PROBLEMS

16. A 100-V, 60-W lamp is to be operated from a 220-V, 50-Hz mains. What (*a*) pure resistance, and (*b*) pure inductance, placed in series with the lamp, will enable it to be used without being over-run? Which method would be more economical?

[**Ans.** (*a*) 200 Ω, (*b*) 1.039 H; Second method]

17. A choke coil, when connected to a 230-V, 50-Hz supply, takes a current of 15 A. On decreasing the frequency of the supply to 40 Hz, the current increases to 17.2 A. Determine the resistance and inductance of the coil.

[**Ans.** $R = 8.884\ \Omega$; $L = 39.8$ mH]

18. A capacitor is used in series with a tungsten-filament bulb rated at 500 W, 100 V, so that it give its rated illumination when connected to a 220-V, 50-Hz supply. Calculate (*a*) the value of the capacitor, and (*b*) the power factor of the current drawn from the supply.

[**Ans.** (*a*) 81.2 μF; (*b*) 0.455 (leading)]

19. It is desired to run a bank of ten 100-W, 100-V lamps in parallel from a 230-V, 50-Hz supply by inserting a choke coil in series with the bank of lamps. If the choke coil has a *pf* 0.2, find its resistance, reactance and inductance. [**Ans.** 3.76 Ω, 18.4 Ω, 0.058 H]

20. A noninductive resistor is connected in series with an iron-cored coil and a capacitor. The circuit is connected to a single-phase ac supply. When current flowing through the circuit is 0.345 A, the

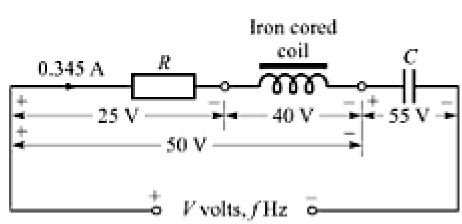


Fig. 10.15

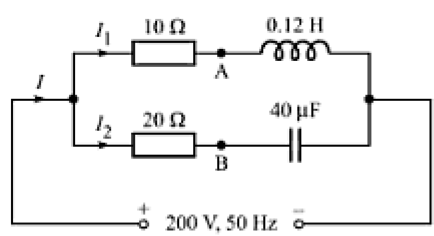


Fig. 10.16

voltages across the elements are as indicated in the Fig. 10.15. Find the applied voltage and the power loss in the coil. **[Ans.** 34.17 V, 1.9 W]

21. Two branches A and B are connected in parallel across a 200-V, 50-Hz supply, as shown in Fig. 10.16. Find (*a*) the currents in each branch, (*b*) the source current, and (*c*) the power factor.
[Ans. (*a*) $5.12\angle{-75°}$ A, $2.424\angle 76°$ A; (*b*) $3.22\angle{-53.6°}$ (*c*) 53.6° (lag)]

22. Three circuit elements $R = 2.5\ \Omega$, $X_L = 4\ \Omega$, and $X_C = 10\ \Omega$ are connected in parallel, the reactances being at 50 Hz. (*a*) Determine the admittance of each element and hence obtain the input admittance. (*b*) If this circuit is connected across a 10-V, 50-Hz ac source, determine the current in each branch and the total input current.
[Ans. (*a*) $\mathbf{Y}_R = (0.4 + j0)$ S, $\mathbf{Y}_L = (0 - j0.25)$ S, $\mathbf{Y}_C = (0 + j0.10)$ S, $\mathbf{Y} = (0.4 - j0.15)$ S; (*b*) $\mathbf{I}_R = 4\angle 0°$ A, $\mathbf{I}_L = 2.5\angle{-90°}$ A, $\mathbf{I}_C = 1.0\angle 90°$ A, $\mathbf{I} = 4.272\angle{-20.56°}$ A]

23. The resistance R and the inductance L of a coil are to be determined experimentally. The available equipments are a voltmeter and an 8-Ω resistor. The 8-Ω resistor is connected in series with the coil and the combination across a 120-V, 50-Hz source. If the voltmeter reads 32 V across the resistor and 104 V across the coil, determine R and L.
[Ans. $R = 10\ \Omega$, $L = 76.4$ mH]

24. A circuit draws 2-A current and consumes 220-W power when connected to a 220-V, 50-Hz supply. When a capacitor of unknown value is connected in series with the given circuit and the combination put across the same voltage source, the magnitude of the input current is found to increase. Determine the values of the original circuit elements.
[Ans. $R = 55\ \Omega$, $L = 303.22$ mH]

25. For the circuit shown in Fig. 10.17, calculate the value of C such that the input current is 45° out of phase with the input voltage at frequency $\omega = 2000$ rad/s. **[Ans.** 41.66 μF]

26. The circuit shown in Fig. 10.18 operates at a frequency of 50 Hz. Determine the value of C such that the input voltage **V** and the input current **I** are in the same phase. **[Ans.** 1325.75 μF]

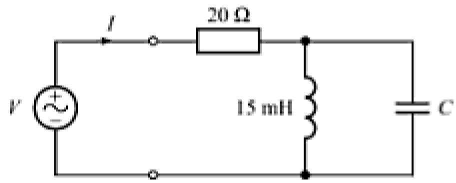


Fig. 10.17

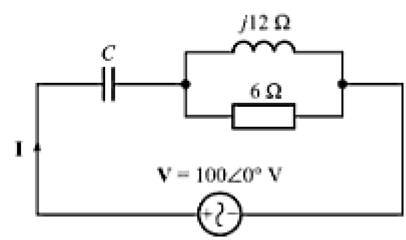


Fig. 10.18

EXPERIMENTAL EXERCISE 10.1

SERIES *RL* CIRCUIT

Objectives

1. To observe the variation of current I when the impedance of a series RL circuit is varied.
2. To draw the locus diagram for a series RL circuit by varying the resistance.

Apparatus Single-phase ac supply; One variac 250 V, 5 A; One choke coil with negligible resistance; One ammeter (MI type) 0-5 A; Three voltmeters (MI Type) 0-300 V; One rheostat 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 10.19.

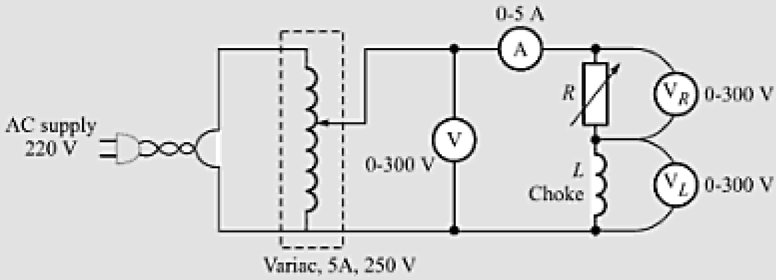
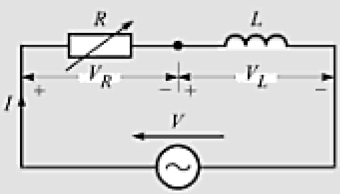


Fig. 10.19

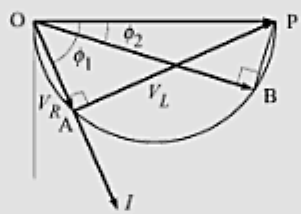
Brief Theory For the series RL circuit of Fig. 10.20*a*, the quantities V, I, V_R and V_L represent the rms values of the supply voltage, the current, the voltage across the resistance and the voltage across the inductor, respectively. According to the Kirchhoff's voltage law, the phasor sum of voltages V_R and V_L must be equal to the phasor V. That is,

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L \tag{i}$$

As shown in the phasor diagram of Fig. 10.20*b*, the current I lags the supply voltage V by an angle ϕ_1 for the inductive load. The voltage V_R (represented by phasor OA) is in phase with the current I, but the voltage V_L (represented by phasor AP) leads the current by 90°. The phasor OP is then the phasor sum of voltages V_R and V_L, and hence must represent the supply voltage V.



(*a*) A series RL circuit.



(*b*) Its phasor diagram.

Fig. 10.20

Keeping the supply voltage constant, if we increase the resistance R, the voltage V_R increases (as represented by phasor OB) and the voltage V_L adjusts itself so as to satisfy Eq. (*i*). Whatever be the value of resistance R, the phasor V_L is always perpendicular to phasor V_R. Hence we must have

$$V^2 = V_R^2 + V_L^2 \tag{ii}$$

This shows that the *locus* of the tip of the phasor OA is a semicircle of the diameter equal to supply voltage V.

For a given value of the resistance R, the phase angle can be calculated by

$$\cos\phi = \frac{V_R}{V} \quad \text{or} \quad \phi = \cos^{-1}\left(\frac{V_R}{V}\right) \tag{iii}$$

Procedure

1. Connect the circuit as shown in Fig. 10.19.
2. Switch on the power supply. Adjust the variac so that the voltmeter V reads 200 V. Throughout the experiment, keep this voltage constant at 200 V.

3. Adjust the rheostat R to its minimum value (say, 0 Ω).
4. Note the readings of the ammeter A and voltmeters V_R and V_L.
5. Go on increasing the value of resistance R in steps, and for each step note the above readings.
6. For each set of readings, calculate the phase angle using Eq. (*iii*).
7. Draw the phasor diagram taking the phasor V (= 200 V) as reference and phasor I lagging at an angle Φ.
8. Mark the phasor V_R along phasor I, and join point A to point P.
9. Draw a semicircle taking OA as diameter.
10. Switch OFF the supply.

Observations and Calculations

Supply voltage, V = …… V

S. No.	V_R	V_L	I	ϕ
1.				
2.				
3.				
4.				
5.				

Results

1. The current I goes on decreasing as the resistance R increases.
2. From the phasor diagram drawn, it is seen that the locus of point A for different set of readings falls on the semicircle.

Precautions

1. Before switching ON the ac supply, the zero readings of the ammeter and voltmeters should be checked.
2. The terminals of the rheostat should be connected properly.
3. While setting the rheostat, care should be taken that the current recorded by the ammeter does not exceed 5 A, the current rating of the rheostat.
4. The supply voltage V should remain constant throughout the experiment. If required, the variac may be adjusted in between.

Viva-Voce

1. What would be the power factor of the series *RL* circuit, if the resistance becomes zero?
Ans.: As power factor, $pf = \cos\phi = R/Z$, it becomes zero when $R = 0$.

2. What is the power factor of circuit having pure inductance?
Ans.: For such a circuit, $\phi = 90°$, hence pf is zero.

3. What is the frequency of your ac supply?
Ans.: 50 Hz.

4. Why are the ac quantities expressed in rms values and not in average values?
Ans.: The average value of an ac current or voltage is zero over a complete cycle. So, while specifying an ac quantity we cannot take average value.

5. What is meant by rms value?
Ans.: It means the **root** of **mean** of **squares** of the quantity (a voltage or a current). It is its effective value from energy transfer point of view.

6. When you say that the ac supply is 220 V, what does it mean?
Ans.: It means the rms value of the ac supply is 220 V. Its amplitude will be $\sqrt{2}$ times this value (that is, $V_m = \sqrt{2} \times 220\text{ V} \approx 311\text{ V}$).

7. Is the amplitude positive 311 V or negative 311 V?
   Ans.: We do not associate any algebraic sign with amplitude.
8. What does the form factor of an ac wave indicate?
   Ans.: It indicates how far the ac wave departs from a sinusoidal wave. The form factor of a sinusoidal wave is 1.11. If the form factor of the given ac wave is less than 1.11, it is said to be flat-topped. On the other hand, if it is more than 1.11, the wave is said to be peaky.
9. If a series *RL* circuit has $R = 4\ \Omega$ and $X_L = 3\ \Omega$, is the total impedance of the circuit 7 Ω?
   Ans.: No. As $\mathbf{Z} = R + jX_L$; $Z = \sqrt{R^2 + X_L^2} = \sqrt{4^2 + 3^2} = 5\ \Omega$.

EXPERIMENTAL EXERCISE 10.2

PARALLEL AC CIRCUIT

Objectives

1. To observe the variation of current *I* when the resistance in the parallel ac circuit is varied.
2. To draw the phasor diagram for the parallel ac circuit for *four* sets of impedances obtained by varying the resistance.
3. To calculate the circuit parameters (*R*, *L* and *C*) for the four sets of observations, assuming the ac supply frequency to be 50 Hz.
4. To compare the two values of power-factors—one obtained from the readings and the other obtained from the phasor diagram—for the four sets of observations.

Apparatus Single-phase ac supply; One Variac 0-250 V, 5 A; One choke coil with negligible resistance; One wattmeter, Three ammeters (MI type) 0-5 A; Three voltmeters (MI Type) 0-300 V; One rheostat 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 10.21.

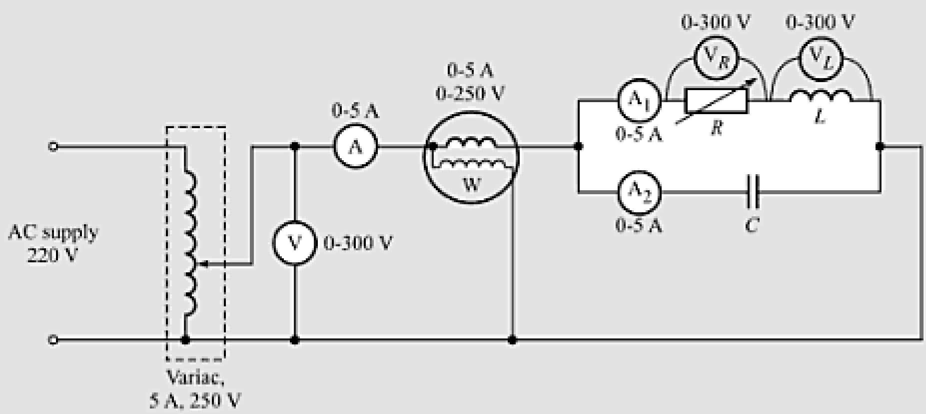
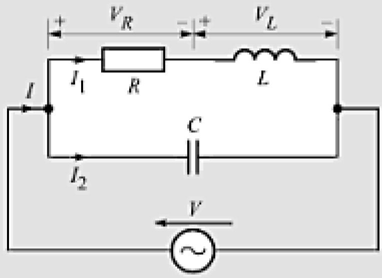


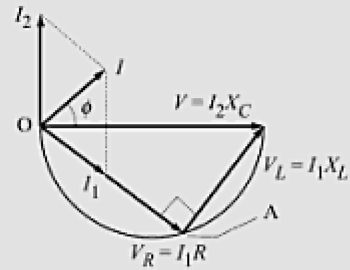
Fig. 10.21

Brief Theory Figure 10.22 shows an ac circuit having two branches in parallel. One branch has a resistance *R* in series with an inductance *L*, and the other branch has only a capacitance *C*. Various voltage and current relationships are

$$V_R = I_1 R, \quad V_L = I_1 X_L, \quad V_C = I_2 X_C, \quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 \quad \text{and} \quad \mathbf{V} = \mathbf{V}_C = \mathbf{V}_R + \mathbf{V}_L \qquad (i)$$



(a) A parallel ac circuit.



(b) Its phasor diagram.

Fig. 10.22

Keeping the supply voltage constant, if we increase the resistance R, the voltage V_R increases (as represented by phasor OA) and the voltage V_L adjusts itself so as to satisfy Eq. (*i*). Whatever be the value of resistance R, the phasor V_L is always perpendicular to phasor V_R. Hence we must have

$$V^2 = V_R^2 + V_L^2 \qquad (ii)$$

This shows that the *locus* of the tip of the phasor OA is a semicircle of diameter equal to supply voltage V.

The current I_2 leads the voltage V by 90°.

Procedure

1. Connect the circuit as shown in Fig. 10.21.
2. Check the zero reading of the three ammeters, the voltmeter and the wattmeter.
3. Set the variac to its zero position and switch ON the power supply.
4. With the help of the variac, increase the supply voltage so that some observable readings are obtained in all the meters.
5. Note the readings of the wattmeter and all the three ammeters.
6. Using a voltmeter and a pair of probes, note the readings of the voltages across each component (R, L and C).
7. Repeat the experiment with four more values of the applied voltage, keeping the setting of the rheostat different every time.
8. Bring the variac to zero position and switch OFF the supply.

Observations

S. No.	V	I	I_1	I_2	I_3	V_R	V_L	V_C	P
1									
2									
3									
4									

Calculations

1. For each set of observations, calculate the values of the circuit parameters, R, L and C as follows:

$$R = \frac{V_R}{I_1};\quad X_L = \frac{V_L}{I_1} \quad\Rightarrow\quad L = \frac{X_L}{2\pi \times 50};\quad X_C = \frac{V_C}{I_2} \quad\Rightarrow\quad C = \frac{1}{2\pi \times 50 \times X_C}$$

2. For each set of observations, calculate the value of the net impedance of the ac circuit as follows:

$$Z = \frac{V}{I}$$

3. For each set of observations, calculate the value of the power factor of the ac circuit as follows:

$$pf = \frac{P}{VI}$$

4. For each set of observations, draw the phasor diagram (preferably on a graph paper), choosing a suitable scale. Measure the phase angle ϕ and calculate the value of cos ϕ.
5. Tabulate the results of the calculations in the following table.
6. For each set of observations, compare the two values of the power factor (pf), one obtained from measurement and the other obtained form the phasor diagram.

S. No.	*R*	*L*	*C*	*Z*	*pf*	*pf (from phasor diagram)*
1						
2						
3						
4						

Results

1. For different sets of observations, the calculated values of R, Z and pf are different; but the values of L and C are found to be almost same.
2. The two values of the pf are almost same for each set of observations.

Precautions

1. Before switching on the ac supply, the zero readings of the ammeters and voltmeters should be checked.
2. The terminals of the rheostat should be connected properly.
3. While setting the rheostat, care should be taken that the current recorded by the ammeter A_1 does not exceed 5 A, the current rating of the rheostat.

Viva-Voce

1. While solving a parallel ac circuit, we normally work with admittances rather than impedances. Why?
Ans.: For a parallel circuit, the equivalent admittance is sum of admittances of individual branches.

2. What is the sign of inductive reactance?
Ans.: It is conventionally taken as positive.

3. Is the sign of inductive susceptance also positive?
Ans.: No. If the sign of inductive reactance is taken positive, the sign of its susceptance will be negative.

4. What is the sign of capacitive reactance?
Ans.: It is negative.

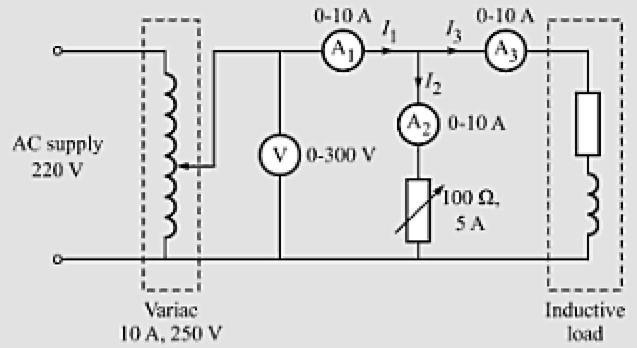
EXPERIMENTAL EXERCISE 10.3

THREE-AMMETER METHOD

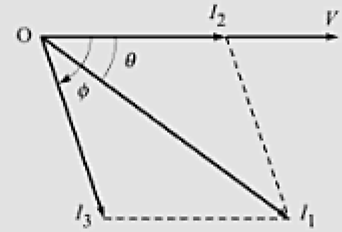
Objectives To measure power and power factor in an ac circuit using three-ammeter method.

Apparatus Single-phase ac supply; One variac 250 V, 10 A; One inductive load; Three ammeters (MI type) 0-10 A; One voltmeter (MI Type) 0-300 V; One rheostat 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 10.23*a*.



(*a*)



(*b*)

Fig. 10.23

Brief Theory The power consumed by the inductive load is given as

$$P = VI\cos\phi = VI_3\cos\phi \qquad (i)$$

Therefore, unlike for a dc circuit, it is not possible to find power in an ac circuit, simply from the readings of a voltmeter and an ammeter. The power is normally measured by a wattmeter. However, it is possible to measure power in an ac circuit by using three ammeters.

Figure 10.23*b* shows the phasor diagram for the ac circuit. The current I_2 through the rheostat is in phase with the applied voltage V. The current I_3 through the inductive load lags the voltage V by an angle ϕ. The total current I_1 is the phasor sum of currents I_2 and I_3. Therefore, we can write

$$I_1^2 = I_2^2 + I_3^2 + 2I_2I_3\cos\phi \qquad (ii)$$

$$\therefore \qquad \text{power factor, } pf = \cos\phi = \frac{I_1^2 - I_2^2 - I_3^2}{2I_2I_3} \qquad (iii)$$

Now, from Eq. (*i*),

$$I_3\cos\phi = \frac{P}{V}$$

Putting this value in Eq. (*ii*), we get

$$I_1^2 = I_2^2 + I_3^2 + 2I_2\frac{P}{V} \quad \Rightarrow \quad P = (I_1^2 - I_2^2 - I_3^2)\frac{V}{2I_2} \qquad (iv)$$

Thus, using Eqs. (*iii*) and (*iv*), we can calculate the power factor and the power consumed by the load by taking the readings of the three ammeters and the voltmeter.

Procedure

1. Connect the circuit as shown in Fig. 10.23*a*.
2. Check the zero reading of the three ammeters and the voltmeter.
3. Set the variac to its zero position and switch ON the power supply.
4. With the help of the variac, change the supply voltage so that some observable readings are obtained in all the meters.
5. Note the readings of the three ammeters and the voltmeter.
6. Change the position of the rheostat and repeat steps 4 and 5 four more times.
7. Switch OFF the supply.

Observations and Calculations

S. No.	V (*in* V)	I_1 (*in* A)	I_2 (*in* A)	I_3(*in* A)	P (*in* W)	*pf*
1						
2						
3						
4						
5						

Results

1. For all sets of observations, the value of the power factor remains the same.
2. On changing the power supply voltage, the power consumed by the inductive load also changes.

Precautions

1. Before switching ON the ac supply, the zero readings of the ammeters and voltmeter should be checked.
2. The terminals of the rheostat should be connected properly.
3. While setting the rheostat, care should be taken that the current recorded by the ammeter does not exceed 5 A, the current rating of the rheostat.

Viva-Voce

1. In an ac circuit, which power has higher value—the apparent power or the real power?
Ans.: The apparent power.

2. In an ac circuit, can these two powers ever have the same value?
Ans.: Yes. It is possible when the load is purely resistive.

3. Is the load in your experiment purely inductive or simply inductive?
Ans.: It is simply inductive load.

4. What is the basic difference between these two types of load?
Ans.: An inductive load includes a resistance in addition to an inductance. On the other hand, a purely inductive load has only inductance. In practice, it is not possible to have a purely inductive load, as an inductance has some resistance of its own.

5. For measuring power in an ac circuit, why do you use three-ammeter method, instead of using a single wattmeter?
Ans.: A wattmeter is an expensive instrument. In absence of a wattmeter, we can alternatively use three-ammeter method.

EXPERIMENTAL EXERCISE 10.4

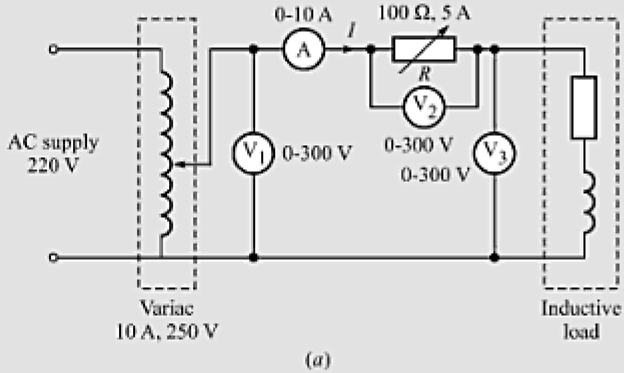
THREE-VOLTMETER METHOD

Objectives To measure power and power factor in an ac circuit using three-voltmeter method.

Apparatus Single-phase ac supply; One variac 250 V, 10 A; One inductive load; Three voltmeters (MI type) 0-300 V; One ammeter (MI Type) 0-10 A; One rheostat 100 Ω, 5 A.

Circuit Diagram The circuit diagram is shown in Fig. 10.24*a*.

Brief Theory Figure 10.24*b* shows the phasor diagram for the ac circuit. The current I is in phase with the voltage V_2 across the rheostat. The voltage V_3 across the inductive load leads the current I by an angle ϕ. The total voltage V_1 is the phasor sum of the voltages V_2 and V_3. Therefore, we can write



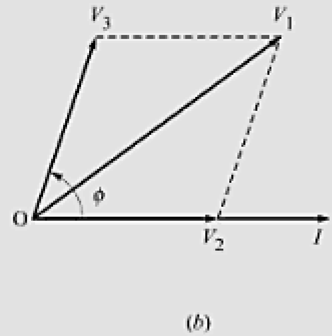


Fig. 10.24

$$V_1^2 = V_2^2 + V_3^2 + 2V_2V_3\cos\phi \qquad (i)$$

$$\therefore \qquad \text{power factor, } pf = \cos\phi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3} \qquad (ii)$$

As the power consumed by the inductive load is given as $P = IV_3\cos\phi$, we have

$$V_3\cos\phi = \frac{P}{I}$$

Putting this value in Eq. (*i*), we get

$$V_1^2 = V_2^2 + V_3^2 + 2V_2\frac{P}{I} \quad \Rightarrow \quad P = (V_1^2 - V_2^2 - V_3^2)\frac{I}{2V_2} \qquad (iii)$$

Thus, using Eqs. (*ii*) and (*iii*), we can calculate the power factor and power consumed by the load by taking the readings of the three voltmeters and the ammeter.

Procedure

1. Connect the circuit as shown in Fig. 10.24*a*.
2. Check the zero reading of the three voltmeters and the ammeter.
3. Set the variac to its zero position and switch ON the power supply.
4. With the help of the variac, change the supply voltage so that some observable readings are obtained in all the meters.
5. Note the readings of the three voltmeters and the ammeter.
6. Change the position of the rheostat and repeat steps 4 and 5 four more times.
7. Switch OFF the supply.

Observations and Calculations

S. No.	I (*in* A)	V_1 (*in* V)	V_2 (*in* V)	V_3 (*in* V)	P (*in* W)	*pf*
1						
2						
3						
4						
5						

Results

1. For all sets of observations, the value of the power factor remains the same.
2. On changing the power supply voltage, the power consumed by the inductive load also changes.

Precautions

1. Before switching ON the ac supply, the zero readings of the ammeter and voltmeters should be checked.
2. The terminals of the rheostat should be connected properly.
3. While setting the rheostat, care should be taken that the current recorded by the ammeter does not exceed 5 A, the current rating of the rheostat.

Viva-Voce

1. How can you minimize the error in this three-voltmeter method of measuring power?
Ans.: Instead of using three separate voltmeters, we can use a single voltmeter with probes to measure the three voltages.

2. Why have you taken the current as reference phasor in your phasor diagram?
Ans.: Because in the series circuit, the current remains the same for all the elements.

RESONANCE IN AC CIRCUITS

11

OBJECTIVES

After completing this Chapter, you will be able to:

- *Derive the expression for resonant frequency for a series RLC circuit.*
- *Explain the effect of variation of frequency on impedance and phase angle of a series RLC circuit.*
- *Define quality factor of a coil.*
- *Derive the expression of quality factor for a series resonant circuit.*
- *Classify coils according to their Q values.*
- *Explain the meaning of Q-gain in a series resonant circuit.*
- *Define 'half-power frequencies' and 'bandwidth' with reference to a resonance curve.*
- *Explain the meaning of 'selectivity'.*
- *Derive the relationship among f_1, f_2, f_0 and BW.*
- *Derive the expression of resonant frequency for a parallel resonant circuit.*
- *Compare series resonant circuit with parallel resonant circuit in respect of impedance and current at resonance.*

11.1 INTRODUCTION

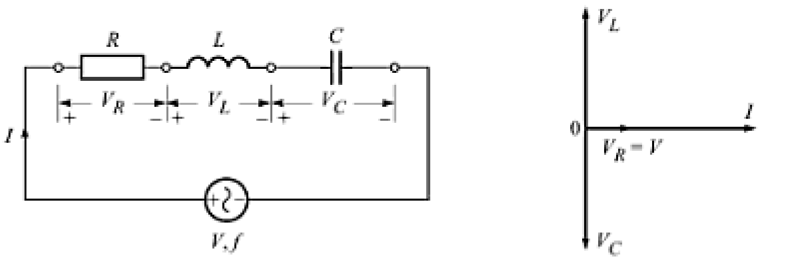
Resonance is the condition that exists in ac circuits under steady state when the input current is in phase with the input voltage. When in resonance, the ac circuit is *purely resistive* and draws power at unity power factor. The condition of resonance can be obtained by connecting an inductor and a capacitor in series (or in parallel) across an ac voltage source of variable frequency. At resonance, the impedance of the circuit is minimum (or maximum). Hence, for a given applied voltage the current becomes maximum (or minimum).

11.2 SERIES RESONANT CIRCUIT

Consider the circuit shown in Fig. 11.1*a*. An inductor and a capacitor are connected in series across an ac voltage source of voltage V and frequency f. An inductor (say, an iron-cored coil) has a small resistance R and an inductance L. A capacitor usually has no losses, and hence it is represented as a pure capacitance C with no resistance. The general expression for the total impedance of this series RLC circuit is

$$\mathbf{Z} = R + j(X_L - X_C) \tag{11.1}$$

If the reactance term $(X_L - X_C)$ is zero, the impedance Z of the circuit is R alone, and the condition of resonance is satisfied. If this condition is met, the voltage drops $V_L(= IX_L)$ and $V_C(= IX_C)$ must be equal, and hence the phasor diagram is as shown in Fig. 11.1*b*. The voltage V_L cancels the voltage V_C. The applied voltage V is then same as the voltage V_R. If the resistance is small compared to the inductive and capacitive reactances, the voltages V_L and V_C are many times the supply voltage V, as shown in the phasor diagram in Fig. 11.1*b*.



(*a*) The circuit. (*b*) Phasor diagram at resonance.

Fig. 11.1 *Series resonant circuit.*

We shall represent the condition of resonance by putting subscript "0", e.g., f_0, Z_0, X_{C0}, etc. Thus, the resonance in a series *RLC* circuit requires that

$$X_{L0} - X_{C0} = 0 \quad \text{or} \quad X_{L0} = X_{C0} \quad \text{or} \quad \omega_0 L = \frac{1}{\omega_0 C} \tag{11.2}$$

Solving the above for ω_0, we get the ***frequency of resonance*** as

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{11.3}$$

When resonance occurs, the impedance of the circuit assumes a minimum value given as $Z_0 = R + j0 = R$, and the current has a maximum value given by

$$I_0 = \frac{V}{Z_0} = \frac{V}{R} \tag{11.4}$$

Effect of Variation of Frequency

Let the values of R, L, C and V in the circuit of Fig. 11.1*a* be fixed. We now examine what happens when the frequency f of the ac source is varied from 0 to ∞. Since $+j$ is associated with X_L and $-j$ with X_C, we plot the values of X_L on the positive imaginary axis and of X_C on the negative imaginary axis.

Since $X_L = 2\pi fL$, X_L varies directly with the frequency f. The plot of X_L versus f is a straight line starting from the origin, as shown in Fig. 11.2*a*. Since $X_C = 1/2\pi fC$, X_C varies inversely as frequency f. The plot of X_C versus f is a rectangular hyperbola. At a frequency f_0, the values of X_L and X_C are seen to be equal. Hence the net reactance of the circuit, $X = (X_L - X_C)$ is zero at this frequency. At f_0, the circuit behaves as *purely resistive*. Below f_0, X has negative values (i.e., the circuit is capacitive). Above f_0, X has positive values (i.e., the circuit is inductive).

The impedance of a series *RLC* circuit is given as

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1} \frac{X_L - X_C}{R} \tag{11.5}$$

At $f = 0$, the inductance behaves as a short-circuit ($X_L = 0$) and the capacitance behaves as an open-circuit ($X_C = \infty$). The impedance Z is infinitely large and the current $I = 0$. As the frequency is increased, X_L increases but X_C decreases. The circuit remains capacitive in nature for $f < f_0$. At $f = f_0$, X_L equals X_C, the impedance has minimum value and the current has maximum value. As the frequency is further increased, X_L becomes

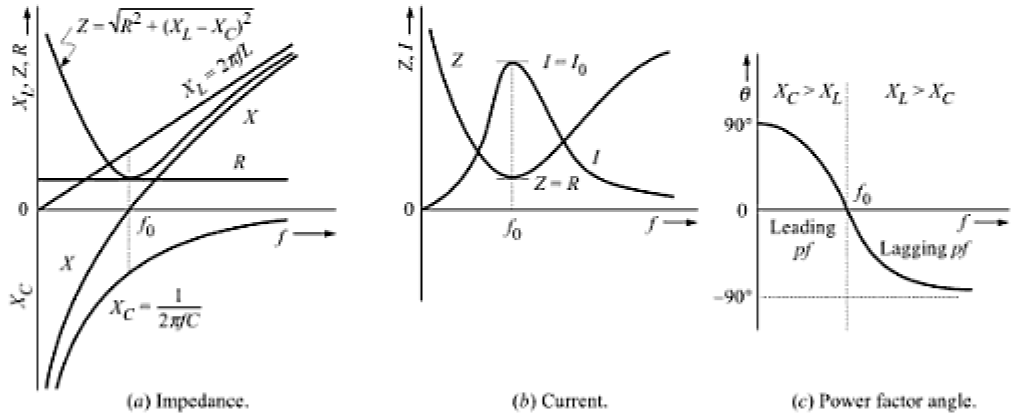


Fig. 11.2 *Effects of variation of frequency in a series RLC circuit.*

greater than X_C, the impedance increases and consequently the current goes on decreasing. It approaches zero as f becomes infinitely large. The variation of current with frequency is shown in Fig. 11.2*b*.

In general, the current is given as

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle 0^\circ}{\sqrt{R^2 + (X_L - X_C)^2}\,\angle \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}\,\angle -\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

This shows that the power factor angle for *RLC* circuit is given as

$$\theta = -\tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) \qquad (11.6)$$

The plot of θ versus frequency is shown in Fig. 11.2*c*.

Importance of Resonant Circuit

Since the current becomes maximum at resonant frequency, the series *RLC* resonant circuit is also called an ***acceptor circuit***. Such a circuit is extremely important in communications, e.g., radio and TV. It provides a simple method of increasing the sensitivity of a receiver. Also it gives ***selectivity***, i.e., it enables a signal of given frequency to be considerably magnified so that it can be separated from signals of other frequencies[1].

11.3 DIFFERENT ASPECTS OF RESONANCE

Variation of Voltage across *C* and *L* with Frequency

Consider a series *RLC* circuit connected to an ac voltage source of voltage V (constant magnitude) and frequency f (which can be varied). The magnitude of I is given as

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (\omega L - (1/\omega C))^2}}$$

[1] This process is called ***tuning***. A variable capacitor is used in a resonant circuit to tune a desired station in radio (or channel in TV).

Therefore, the voltage across capacitor,

$$V_C = IX_C = \frac{V}{\sqrt{R^2 + (\omega L - (1/\omega C))^2}} \cdot \frac{1}{\omega C}$$

$$\Rightarrow \qquad V_C^2 = \frac{V^2}{\omega^2 C^2 \{R^2 + (\omega L - (1/\omega C))^2\}} = \frac{V^2}{R^2 \omega^2 C^2 + (\omega^2 LC - 1)^2} \tag{11.7}$$

To find the frequency at which the voltage V_C is maximum, we should have

$$\frac{dV_C}{d\omega} = 0 \quad \text{or} \quad \frac{dV_C^2}{d\omega} = 0$$

or
$$V^2 \cdot \left[\frac{0 - \{2\omega R^2 C^2 + 2(\omega^2 LC - 1)\, 2\omega LC\}}{\{R^2 \omega^2 C^2 + (\omega^2 LC - 1)^2\}^2}\right] = 0$$

or
$$2\omega R^2 C^2 + 2(\omega^2 LC - 1)2\omega LC = 0$$

or
$$\omega^2 = \frac{1}{2L^2 C}(2L - CR^2) = \frac{1}{LC} - \frac{R^2}{2L^2}$$

Thus, the frequency at which the voltage V_C is maximum is given as

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \tag{11.8}$$

Now, the voltage across inductance is given as

$$V_L = IX_L = \frac{V}{\sqrt{R^2 + (\omega L - (1/\omega C))^2}} \cdot \omega L$$

$$\Rightarrow \qquad V_L^2 = \frac{V^2 \omega^2 L^2}{R^2 + (\omega L - (1/\omega C))^2} = \frac{V^2 \omega^4 L^2 C^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2} \tag{11.9}$$

To find the frequency at which the voltage V_L is maximum, we should have

$$\frac{dV_L}{d\omega} = 0 \quad \text{or} \quad \frac{dV_L^2}{d\omega} = 0$$

or
$$2\omega^2 LC - \omega^2 C^2 R - 2 = 0 \quad \text{or} \quad \omega^2(2LC - C^2 R) = 2$$

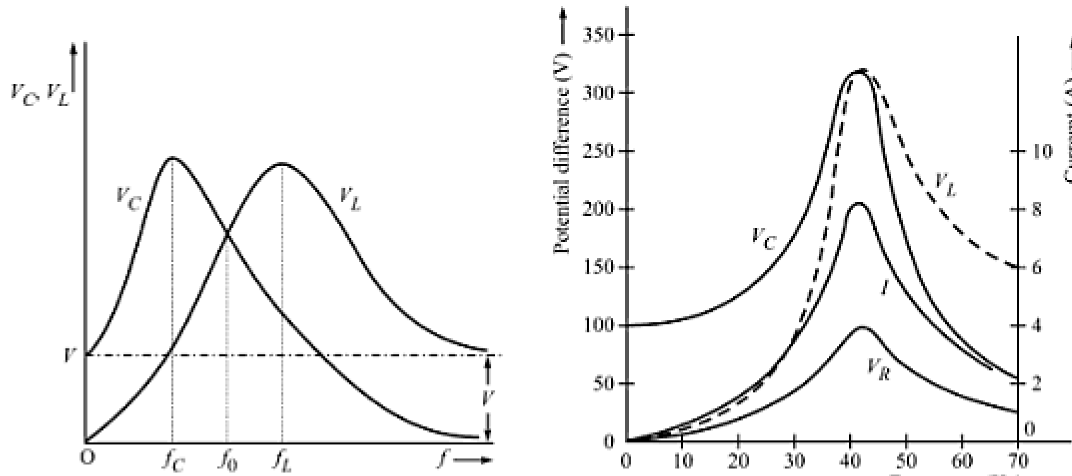
or
$$\omega^2 = \frac{1}{LC - (R^2 C^2/2)}$$

Thus, the frequency at which the voltage V_L is maximum is given as

$$f_L = \frac{1}{2\pi\sqrt{LC - (R^2 C^2/2)}} \tag{11.10}$$

The Plots of V_C and V_L versus Frequency The variations of V_C with frequency (Eq. 11.7) and of V_L with frequency (Eq. 11.9) are plotted in Fig. 11.3*a*. At $f = 0$, the capacitor behaves as an open circuit, and hence the current in the circuit is zero. The entire source voltage V appears across the capacitor. With increase in frequency, the voltage V_C also increases, reaches a maximum value at f_C (given by Eq. 11.8) and then falls again. As the frequency f approaches infinity, the voltage V_C approaches zero, as the capacitor tends to behave as a short circuit.

At $f = 0$, the inductor behaves as a short circuit, and hence the voltage V_L is zero. As the frequency increases, the voltage V_L rises, reaches a maximum value at f_L (given by Eq. 11.10) and then falls again. As



(*a*) When R has appreciable value compared to X_{L0} (or X_{C0}).

(*b*) When R is very small compared to X_{L0} (or X_{C0}).

Fig. 11.3 *Plots of V_C and V_L versus frequency.*

the frequency f approaches infinity, the voltage V_L approaches the source voltage V, as the inductor tends to behave as an open circuit.

The point of intersection of these two curves gives the resonant frequency f_0. At this frequency, $V_C = V_L$, and the circuit becomes purely resistive.

If the resistance R in a series RLC circuit is small compared to X_{L0} (or X_{C0}), the frequencies f_C and f_L come close together and coincide at f_0, as shown by the curves in Fig. 11.3*b*. Also, under this condition, at $f = f_0$ the voltages across C and L are equal and each is greater than the source voltage V.

Energy Considerations

At resonance, the circuit impedance is a minimum being equal to the resistance R. The resonant current I_0 is dependent on the resistance, as given by Eq. 11.4. This current is considerably higher than that at other frequencies. The power dissipation ($I_0^2 R$) in the circuit is maximum, and also the peak rates of energy storage in the two reactive components (L and C) become equal and maximum.

As the circuit operates at unity power factor at resonance, it has no reactive power. The energy stored by the reactive components is a constant and it oscillates between electric and magnetic modes of storage. The predominance of this energy oscillation over the energy associated with the resistance is important to electronics and communication engineers. The quality of a resonant circuit to accept current (and power) at the resonant frequency to the exclusion of other frequencies is measured by a factor termed ***quality factor*** (Q factor), described below.

Quality Factor

We can define the *quality factor* of a resonant circuit as follows :

$$Q = \frac{2\pi(\text{Maximum energy stored in } L \text{ or } C \text{ per cycle})}{\text{Energy dissipated per cycle}} \qquad (11.11)$$

Q Factor for Series Resonant Circuit Using this definition, we can obtain the expression for Q in various forms for a resonant series RLC circuit. Thus, if I_m is the peak value of the current at resonance and T_0 is the time period at the resonant frequency, we get

$$Q = \frac{2\pi[1/2\ L(I_m)^2]}{I_0^2\ RT_0}$$

Replacing I_m by its rms value $I_0(I_m = \sqrt{2}\,I_0)$ and $T_0 = 1/f_0 = 2\pi/\omega_0$, we get

$$Q = \frac{2\pi[1/2\ L(\sqrt{2}\ I_0)^2]}{I_0^2\ R(2\pi/\omega_0)} = \frac{\omega_0 L}{R} \tag{11.12}$$

Since at resonance, we have from Eq. 11.2, $\omega_0 L = 1/\omega_0 C$, the above equation can be modified to

$$Q = \frac{1}{\omega_0 CR} \tag{11.13}$$

Another form for Q can be obtained by putting $\omega_0 = 1/\sqrt{LC}$ in the above expression,

$$Q = \frac{1}{\omega_0 CR} = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{11.14}$$

A capacitor usually has no losses; hence it is represented as a pure capacitance C with no resistance. However, in practice an inductor (say, an iron-cored coil) always has a small resistance R in addition to an inductance L. The quality factor Q of a series inductor-capacitor circuit, therefore, is the same as the quality factor Q of the coil used. In fact, Q of the coil is used as a figure of merit for the coil. The greater the value of Q, the better is the coil. Coils having Q less than 10 are described as ***low-Q coils.*** The coils having Q equal to or greater than 10 are described as ***high-Q coils***. Coils having Q as high as 200–300 are used in electronic circuits.

Voltage Magnification

Let us determine the voltage drops across each element of the series RLC circuit at resonance.

Voltage drop across resistance, $\quad V_R = I_0 R = \dfrac{V}{R} R = V$

Voltage drop across inductance, $\quad V_L = I_0 X_{L0} = \dfrac{V}{R} X_{L0} = V\dfrac{X_{L0}}{R} = V\dfrac{\omega_0 L}{R} = VQ$

Voltage drop across capacitance, $\quad V_C = I_0 X_{C0} = \dfrac{V}{R} X_{C0} = V\dfrac{X_{C0}}{R} = V\dfrac{1}{\omega_0 CR} = VQ$

Note that we have made use of Eqs. 11.12 and 11.13. We find that at resonance the entire supply voltage V appears across R. More surprising is the fact that the voltage V_L (and voltage V_C) is Q times the supply voltage. For example, if the supply voltage is 230 V and the Q of the coil is 10, at resonance the voltage V_R will also be 230 V, but the voltages across the coil and the capacitor will each be 2300 V ! Therefore, extreme care must be taken while working on series ac circuits that may become resonant. The multiplication of voltage by the Q of the coil in a series resonant circuit is often called ***Q gain*** in electronics. Since, the voltage increases Q times in a series resonant circuit, it is also known as a ***voltage resonant circuit***.

EXAMPLE 11.1

A series RLC circuit has $R = 12\ \Omega$, $L = 0.15$ H and $C = 100\ \mu$F. It is connected to an ac source of voltage 100 V, whose frequency can be varied. Determine (*a*) the frequency of the source at which the current supplied by it is maximum, (*b*) the value of this current, (*c*) the frequency at which the voltage across the capacitor is maximum, (*d*) the frequency at which the voltage across the inductor is maximum, (*e*) the inductive reactance at resonant frequency, (*f*) the capacitive reactance at resonant frequency, (*g*) the quality factor of the circuit, and (*h*) the voltages across each element, under resonant condition.

Solution

(*a*) The current becomes maximum at the resonant frequency of the circuit,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.15 \times 100 \times 10^{-6}}} = \mathbf{41.09\ Hz}$$

(*b*) The maximum current supplied by the source is

$$I_0 = \frac{V}{R} = \frac{100}{12} = \mathbf{8.3\ A}$$

(*c*) The frequency at which the voltage across the capacitor is maximum,

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.15 \times 100\ \times 10^{-6}} - \frac{(12)^2}{2(0.15)^2}} = \mathbf{40.09\ Hz}$$

(*d*) The frequency at which the voltage across the inductor is maximum,

$$f_L = \frac{1}{2\pi\sqrt{LC - (R^2C^2/2)}} = \frac{1}{2\pi\sqrt{0.15 \times 100 \times 10^{-6} - \{12^2 \times (100 \times 10^{-6})^2/2\}}} = \mathbf{41.09\ Hz}$$

(*e*) The inductive reactance, $X_{L0} = 2\pi f_0 L = 2\pi \times 41.09 \times 0.15 = \mathbf{38.72\ Hz}$

(*f*) The capacitive reactance, $X_{C0} = 1/2\pi f_0 C = 1/2\pi \times 41.09 \times 100 \times 10^{-6} = \mathbf{38.73\ Hz}$

(*g*) The Q of the circuit, $Q = \dfrac{\omega_0 L}{R} = \dfrac{X_{L0}}{R} = \dfrac{38.72}{12} = \mathbf{3.22}$

(*h*) The voltage drops across the elements are

$$V_R = V = \mathbf{100\ V};\ V_L = V_C = QV = 3.22 \times 100 = \mathbf{322\ V}$$

NOTE

The curves drawn in Fig. 11.3*b* correspond to this series *RLC* circuit.

EXAMPLE 11.2

A series combination of a resistance of 4 Ω, an inductance of 0.5 H and a variable capacitance is connected across a 100-V, 50-Hz supply. Calculate (*a*) the capacitance to give resonance, (*b*) the voltage across the inductance and the capacitance, and (*c*) the Q factor of the circuit.

Solution

(*a*) For resonance, we should have

$$X_{L0} = X_{C0} \quad \text{or} \quad 2\pi f_0 L = 1/2\pi f_0 C$$

$$\Rightarrow \qquad C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 50)^2 \times 0.5} = \mathbf{20.3\ \mu F}$$

(*b*) At resonance, $I_0 = \dfrac{V}{R} = \dfrac{100}{4} = 25$ A

∴ the voltage across $L = I_0 X_L = 25 \times (2\pi \times 50 \times 0.5) = \mathbf{3925\ V}$

and the voltage across C = voltage across L = **3925 V** (*c*) From Eq. 11.12,

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 50 \times 0.5}{4} = \mathbf{39.25}$$

EXAMPLE 11.3

The voltage applied to a series resonant circuit is 0.85 V. The Q of the coil used is 50 and the value of the capacitor is 320 pF. The circuit is required to resonate at a frequency of 175 kHz. Find the value of the inductance, the circuit current and the voltage across the capacitor under resonance. Draw the phasor diagram.

Solution Since $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, the required inductance is given as

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \times (175 \times 10^3)^2 \times 320 \times 10^{-12}} = \mathbf{2.58\ mH}$$

The reactance of the coil (and also of the capacitor) at resonance is given as

$$X_{L0} = 2\pi f_0 L = 2\pi \times 175 \times 10^3 \times 2.58 \times 10^{-3} = 2836.85\ \Omega$$

The resistance of the coil is given as

$$R = \frac{X_{L0}}{Q} = \frac{2836.85}{50} = 56.74\ \Omega$$

$$\therefore \qquad I = \frac{V}{R} = \frac{0.85}{56.74} = \mathbf{14.98\ mA}$$

The voltage across the capacitor (and also across the inductance part of the coil) is

$$V_C = QV = 50 \times 0.85 = \mathbf{42.5\ V}$$

Using the above numerical values, we can draw the phasor diagram, as shown in Fig. 11.4.

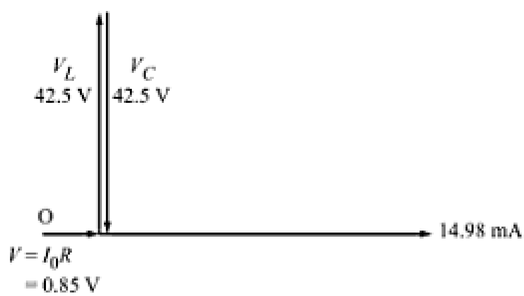


Fig. 11.4 *Phasor diagram at resonance.*

EXAMPLE 11.4

A coil with inductance 1.0 mH and resistance 2.0 Ω is connected in series with a capacitor and a 120-V, 5-kHz supply. Determine the value of capacitance that will cause the system to be in resonance. Also, find (*a*) the current at the resonance frequency, and (*b*) the maximum instantaneous energy stored in the magnetic field of the inductance at the resonance frequency.

Solution The required value of the capacitance is given as

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times (5 \times 10^3)^2 \times 1 \times 10^{-3}} = \mathbf{1.01\ \mu F}$$

(*a*) The current, $I_0 = \frac{V}{R} = \frac{120}{2} = \mathbf{60\ A}$

(*b*) The maximum instantaneous energy, $U_m = (1/2)LI_m^2 = LI_{rms}^2 = 1 \times 10^{-3} \times (60)^2 = \mathbf{3.6\ J}$

EXAMPLE 11.5

For the circuit shown in Fig. 11.5, R_1, R_2 and R_3 are 0.51 Ω, 1.3 Ω and 0.24 Ω respectively; C_1 and C_2 are 25 µF and 62 µF respectively; and L_1 and L_2 are 32 mH and 15 mH respectively. Determine (*a*) the resonance frequency f_0, (*b*) the quality factor of the overall circuit Q, (*c*) the quality factor of coil-1, Q_1, and (*d*) the quality factor of coil-2, Q_2.

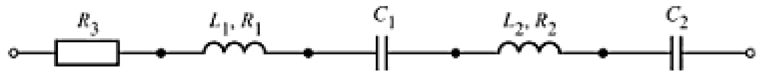


Fig. 11.5 *A series resonant circuit.*

Solution $R_{eq} = R_1 + R_2 + R_3 = 0.51 + 1.3 + 0.24 = 2.05\ \Omega$

$$L_{eq} = L_1 + L_2 = (32 + 15)\text{ mH} = 47\text{ mH};\ C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{62 \times 25}{62 + 25}\mu\text{F} = 17.8\ \mu\text{F}$$

(*a*) $\therefore\ f_0 = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{47 \times 10^{-3} \times 17.8 \times 10^{-6}}} = \mathbf{174\ Hz}$

(*b*) $Q = \frac{1}{R_{eq}}\sqrt{\frac{L_{eq}}{C_{eq}}} = \frac{1}{2.05}\sqrt{\frac{47 \times 10^{-3}}{17.8 \times 10^{-6}}} = \mathbf{25}$

(*c*) $Q_1 = \frac{\omega_0 L_1}{R_1} = \frac{2\pi \times 174 \times 32 \times 10^{-3}}{0.51} = \mathbf{68.6}$

(*d*) $Q_2 = \frac{\omega_0 L_2}{R_2} = \frac{2\pi \times 174 \times 15 \times 10^{-3}}{1.3} = \mathbf{12.6}$

11.4 RESONANCE CURVE

The current versus frequency curve for a resonant circuit is called its *resonance curve.* For a series resonant circuit, the resonance curve is shown in Fig. 11.2*b*. It has been redrawn in Fig. 11.6 for two values of resistances R_1 and R_2 ($R_1 < R_2$). The range of frequencies within which the current does not drop below 0.707(= $1/\sqrt{2}$) times the maximum value is called the *passband* or ***bandwidth***. Thus, for $R = R_1$, the bandwidth is

$$BW = f_2 - f_1 \tag{11.15}$$

The frequencies f_1 and f_2 are often called *lower* and *upper* ***cutoff frequencies***. At these frequencies, the current reduces to $I_0/\sqrt{2}$. Since the power dissipated at f_0 is given as $P_0 = I_0^2R$, the power dissipated at the cutoff frequencies is $(I_0/\sqrt{2})^2 R = P_0/2$. That is, the power at f_1 or f_2 is half the maximum power P_0. Hence, these frequencies are also called ***half-power frequencies***.

Selectivity of Tuned Circuit

As we can see from Fig. 11.6, the lower the value of R, the sharper is the resonance curve. The curve sharply falls to zero on the two sides of resonance frequency. The response of the circuit at the desired frequency

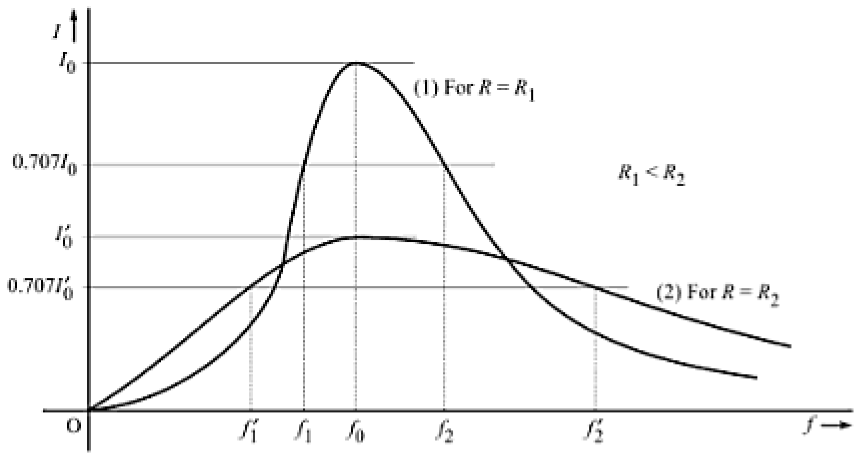


Fig. 11.6 *Resonance curves for two values of resistances.*

(i.e., f_0) is high, but it is quite low for the nearby other frequencies. In other words, it has a good capability of *selecting* the desired frequency and of *rejecting* the nearby undesired frequencies. We say that such a resonant circuit has high ***selectivity***.

For larger values of R, not only the peak value of current falls, but even the response curve becomes less sharp (see the curve for R_2 in Fig. 11.6). Its bandwidth increases. In other words, its selectivity reduces. We use such resonance circuits with high quality factor to tune in the desired broadcasting station in the radio receiver.

Relation between f_0, f_1 and f_2

At f_1 and f_2 the current drops to $1/\sqrt{2}$ of its resonant value I_0. It means that at these frequencies, the impedance must be equal to $\sqrt{2}$ times the resonant value R. That is,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}\,R \quad \Rightarrow \quad X_L - X_C = \pm R \quad \text{or} \quad \omega L - \frac{1}{\omega C} = \pm R \tag{11.16}$$

For $X_L < X_C$, which corresponds to f_1 or ω_1, Eq. 11.16 yields

$$\omega_1 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \tag{11.17}$$

Similarly, for $X_L > X_C$, which corresponds to f_2 or ω_2, Eq. 11.16 gives

$$\omega_2 = \frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \tag{11.18}$$

Multiplying Eqs. 11.17 and 11.18, we get

$$\omega_1\omega_2 = \frac{1}{4}\left[\left(\frac{R}{L}\right)^2 + \frac{4}{LC}\right] - \frac{1}{4}\left(\frac{R}{L}\right)^2 = \frac{1}{LC} = (\omega_0)^2 \quad \text{or} \quad \omega_0 = \sqrt{\omega_1\omega_2}$$

Hence,
$$f_0 = \sqrt{f_1 f_2} \tag{11.19}$$

Bandwidth (*BW*) in Terms of Circuit Parameters

From Eqs. 11.17 and 11.8, we get

$$BW = f_2 - f_1 = \left[\frac{R}{4\pi L} + \frac{1}{4\pi}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] - \left[-\frac{R}{4\pi L} + \frac{1}{4\pi}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$BW = \frac{R}{2\pi L} \tag{11.20}$$

Bandwidth (*BW*) in Terms of Q and f_0

From Eq. 11.12, we have $R/L = \omega_0/Q = (2\pi f_0)/Q$. Substituting this into Eq. 11.20, we get

$$BW = \frac{R}{2\pi L} = \frac{1}{2\pi}\left(\frac{R}{L}\right) = \frac{1}{2\pi}\left(\frac{2\pi f_0}{Q}\right) = \frac{f_0}{Q} \tag{11.21}$$

Comments on Resonance

As indicated in Fig. 11.6, the resonance frequency is *not* centrally located with respect to the two half-power frequencies, especially when the BW is large (i.e., when the Q is small). Actually, as can be seen from Eq. 11.19, f_0 is the *geometric mean* of f_1 and f_2; and the geometric mean is always less than the arithmetic mean. However, if $Q > 10$, the resonant frequency f_0 is sufficiently centered with respect to the two half-power frequencies f_1 and f_2 (as for the curve (1) in Fig. 11.6), and we can then write

$$f_1 \approx f_0 - \frac{BW}{2} \quad \text{and} \quad f_2 \approx f_0 + \frac{BW}{2} \tag{11.22}$$

For low-Q coils, we cannot use the above approximations. The exact relations are given in Eqs. 11.17 and 11.18.

EXAMPLE 11.6

A series ac circuit has a resonance frequency of 150 kHz and a bandwidth of 75 kHz. Determine its half-power frequencies.

Solution From Eq. 11.21, $Q = 150/75 = 2$ (which is less than 10). Hence, we cannot use the approximate relations given in Eq. 11.15. Using the exact relations given in Eqs. 11.17 and 11.18 and working in kHz, we have

$$75 = f_2 - f_1 \quad \text{and} \quad 150 = \sqrt{f_2 f_1}$$

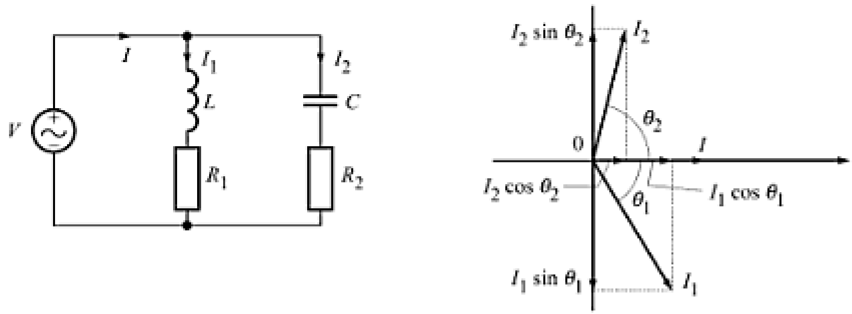
Eliminating f_2 between the two equations, we get

$$f_1^2 + 75 f_1 - 22\,500 = 0 \quad \Rightarrow \quad f_1 = 117.1 \text{ kHz} \quad \text{or} \quad -192.1 \text{ kHz}$$

Ignoring the negative value, we have f_1 = **117.1 kHz**. Hence, $f_2 = 75 + f_1$ = **192.1 kHz**.

11.5 PARALLEL RESONANT CIRCUIT

In a parallel resonant circuit, the ac source is connected across a parallel combination of an inductor and a capacitor, as shown in Fig. 11.7*a*. In general, both the inductor and the capacitor have some losses. In a circuit, these losses are accounted for by inserting equivalent series resistances R_1 and R_2, respectively. By varying the frequency of the source, resonant condition may reach when the reactive (or wattless) component of line current I reduces to zero.



(*a*) The circuit for a general case of parallel resonance.

(*b*) The phasor diagram.

Fig. 11.7 *Parallel resonant circuit.*

Resonance Frequency

Figure 11.7*b* shows the phasor diagram of the circuit under resonance condition. The branch current I_1 through the inductor lags the supply voltage V by an angle θ_1. The branch current I_2 through the capacitor leads the voltage V by an angle θ_2. Under resonance condition, the reactive components of these two currents are equal in magnitude (but opposite in phase). That is,

$$I_1 \sin\theta_1 = I_2 \sin\theta_2$$

or $$\frac{V}{\sqrt{R_1^2 + (\omega_0 L)^2}} \times \frac{(\omega_0 L)}{\sqrt{R_1^2 + (\omega_0 L)^2}} = \frac{V}{\sqrt{R_2^2 + (1/\omega_0 C)^2}} \times \frac{(1/\omega_0 C)}{\sqrt{R_2^2 + (1/\omega_0 C)^2}}$$

or $$\frac{(\omega_0 L)}{R_1^2 + (\omega_0 L)^2} = \frac{(1/\omega_0 C)}{R_2^2 + (1/\omega_0 C)^2} \quad \text{or} \quad \frac{\omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{\omega_0 C}{R_2^2\,\omega_0^2 C^2 + 1}$$

$\Rightarrow$ $$L(R_2^2\omega_0^2 C^2 + 1) = C(R_1^2 + \omega_0^2 L^2) \quad \text{or} \quad \omega_0^2 LC(R_2^2 C - L) = CR_1^2 - L$$

or $$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_1^2 - (L/C)}{R_2^2 - (L/C)}}$$

or $$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_1^2 - (L/C)}{R_2^2 - (L/C)}} \qquad (11.23)$$

Practical Parallel Resonance Circuit

It is possible to get a capacitor having negligible losses. It means that the resistance R_2 in series with capacitor C can be ignored. However, in practice an inductor does have some losses. The circuit of Fig. 11.8*a* therefore represents a practical parallel resonant circuit. By putting $R_2 = 0$ and $R_1 = R$ in Eq. 11.23, we get the resonance frequency of this circuit as

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R^2 - (L/C)}{-(L/C)}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2 C/L} \qquad (11.24)$$

Note that if $R^2C/L > 1 \Rightarrow R^2 > (L/C)$, f_0 as given by the above equation is imaginary, and therefore resonance cannot occur.

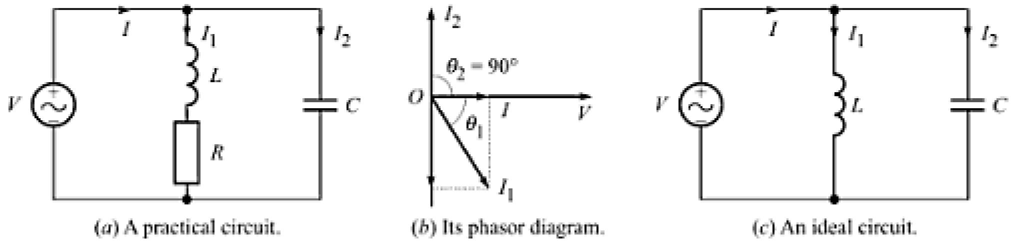


Fig. 11.8 *Parallel resonant circuit.*

Ideal Parallel Resonant Circuit It would have been ideal if the inductor too were lossless, though it is not physically possible. In such a case, we could ignore resistance R in Fig. 11.8*a*, so that the circuit reduces to that shown in Fig. 11.8*c*. The resonance frequency of this circuit can be obtained from Eq. 11.24, as

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{11.25}$$

Note that this equation is same as Eq. 11.3 for series resonance circuit.

Current at Resonance

In the practical circuit of Fig. 11.8*a*, at resonance we have

$$\textit{For branch } 1: Z_1 = \sqrt{R^2 + (\omega_0 L)^2}\ ; I_1 = \frac{V}{Z_1} \qquad \text{and} \qquad \theta_1 = \cos^{-1}\frac{R}{Z_1} = \sin^{-1}\frac{\omega_0 L}{Z_1}$$

$$\textit{For branch } 2: Z_2 = 1/\omega_0 C; I_2 = \frac{V}{Z_2} = \frac{V}{1/\omega_0 C} = V\omega_0 C \qquad \text{and} \qquad \theta_2 = 90°$$

Equating the reactive components of the two currents, we get

$$I_1 \sin\theta_1 = I_2 \sin 90° \quad \text{or} \quad \frac{V}{Z_1}\cdot\frac{\omega_0 L}{Z_1} = V\omega_0 C \;\Rightarrow\; Z_1 = \sqrt{\frac{L}{C}}$$

Since the reactive components of the two currents cancel each other, the line current is

$$I = I_1\cos\theta_1 + I_2\cos\theta_2 = \frac{V}{Z_1}\cdot\frac{R}{Z_1} + 0 = \frac{VR}{Z_1^2} = \frac{VR}{L/C} = \frac{V}{L/CR} = \frac{V}{Z_0}$$

Thus, the *effective* or *equivalent* or *dynamic impedance* of the parallel resonance circuit is given as

$$Z_0 = \frac{L}{CR} \tag{11.26}$$

Effect of Variation of Frequency

Let us first examine how the admittance of the parallel resonant circuit varies with frequency. For the practical parallel resonant circuit of Fig. 11.8*a*, the admittances of the two branches are given as

$$\mathbf{Y}_1 = \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega L)^2} - j\frac{\omega L}{R^2 + (\omega L)^2} = G - jB_L \qquad \text{and} \qquad \mathbf{Y}_2 = \frac{1}{(1/j\omega C)} = +j\omega C = +jB_C$$

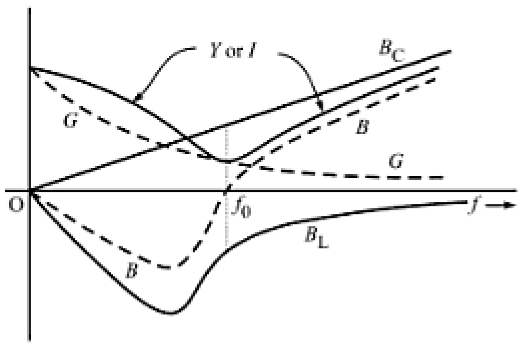
Thus, the conductance and susceptance of the inductive branch, respectively, are

$$G = \frac{R}{R^2 + (\omega L)^2} \qquad \text{and} \qquad B_L = \frac{\omega L}{R^2 + (\omega L)^2}$$

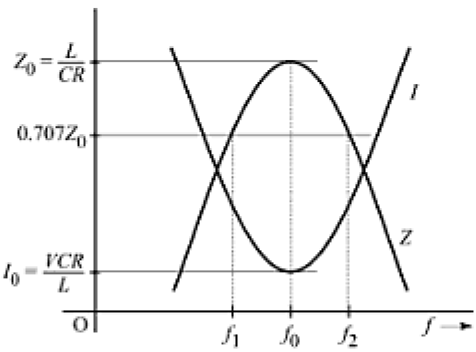
The conductance of the capacitive branch is zero and its susceptance is $B_C = \omega C$. Total admittance of the circuit is given as

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 = \frac{R}{R^2 + (\omega L)^2} - j\frac{\omega L}{R^2 + (\omega L)^2} + j\omega C = G + j(B_C - B_L)$$

From the above expressions, it can be seen that as frequency increases, the value of conductance G decreases. The plot of G versus frequency is shown in Fig. 11.9*a* by a dashed curve. The inductive susceptance B_L is considered negative, since $-j$ is associated with it. For low frequencies, $\omega L < R$ and $(\omega L)^2 << R^2$, hence the magnitude of B_L is directly proportional to the frequency. As a result, the plot of B_L versus f is a straight line in the low-frequency region. But, for high frequencies, $\omega L > R$ and $(\omega L)^2 >> R^2$, hence the magnitude of B_L is inversely proportional to the frequency. Consequently, the plot of B_L versus f is a rectangular hyperbola in the high-frequency region. The overall plot of B_L versus f takes a shape as shown in the figure.



(*a*) Admittance versus frequency.



(*b*) Current versus frequency.

Fig. 11.9 *Effect of variation of frequency in a parallel resonant circuit.*

The capacitive susceptance is considered positive. Its magnitude is seen to vary directly with frequency. Thus, as shown in Fig. 11.9*a*, the plot of B_C versus f is a straight line. By algebraic addition of B_L and B_C, the plot of the net susceptance B versus f can be determined (as shown by dotted curve). Since the net admittance of the circuit is given as $Y = \sqrt{G^2 + B^2}$, the plot of Y versus f is as shown by solid curve (V-shaped) in the figure.

Current versus Frequency Since the impedance is inverse of admittance ($Z = 1/Y$), the Z verses f plot is just the inverted figure of Y versus f plot, as shown in Fig. 11.9*b*. At resonance frequency f_0, the impedance has maximum value Z_0, given by Eq. 11.26. The plot of current I versus f is also shown in the figure. At resonance frequency, the line current is seen to have minimum value given by

$$I_0 = \frac{V}{Z_0} = \frac{V}{L/CR} = \frac{VCR}{L} \tag{11.27}$$

If the resistance R is zero (as in an *ideal tank circuit* of Fig. 11.8*c*), the line current I_0 at resonance would be zero.

Bandwidth As shown in Fig. 11.9*b*, the half-power frequencies f_1 and f_2 are defined as the frequencies where the impedance reduces to $1/\sqrt{2}$ (or 0.707) times the impedance Z_0 at resonance. The range from f_1 to f_2 is called bandwidth (BW). To examine how the nature of the circuit (inductive or capacitive) varies with frequency, let us write the expression for the impedance,

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{G + j(B_C - B_L)} = \frac{G}{G^2 + (B_C - B_L)^2} - j\frac{(B_C - B_L)}{G^2 + (B_C - B_L)^2}$$

At the resonance frequency f_0, $(B_C - B_L) = 0$, hence the circuit is purely resistive; the power factor is unity. For frequencies below f_0, $B_C < B_L$ (see Fig. 11.9*a*), the imaginary part of **Z** is positive, and hence the circuit is predominantly inductive. For frequencies above f_0, $B_C > B_L$, the imaginary part of **Z** is negative, and hence the circuit is predominantly capacitive.

At frequencies f_1 and f_2, the impedance $Z = Z_0/\sqrt{2}$, which is possible only when $R = X$, so that $\mathbf{Z} = R \pm jX = \sqrt{2}\,R\angle\pm 45°\ \Omega$. The impedance angle is 45°; the power factor is $1/\sqrt{2}$ (or 0.707).

Quality Factor

It can be seen from the phasor diagram of Fig. 11.8*b* that both the branch-currents I_1 and I_2 are much larger than the line current I. This shows that in a parallel circuit, the current taken from the ac source is greatly magnified. The amount by which the line current is magnified in a parallel resonant circuit is called its ***Q-factor***. At resonance, the reactive components of the two branch currents are equal and opposite in phase. That is,

$$I_2 \sin 90° = I_1 \sin\theta_1 \quad \text{or} \quad I_2 = I_1 \sin\theta_1$$

$$\therefore \qquad Q = \frac{\text{Current through the capacitor}}{\text{Line current}} = \frac{I_2}{I} = \frac{I_1\sin\theta_1}{I_1\cos\theta_1} = \tan\theta_1 = \frac{\omega_0 L}{R}$$

SOME IMPORTANT POINTS

Following are some of the important points about parallel circuit at resonance:

1. The reactive (or wattless) component of line current is zero; hence the circuit power factor is unity.
2. The circuit is purely *resistive*; the impedance has maximum value given as $Z_0 = R_0 = L/CR$.
3. The line current is minimum, given as $I_0 = V/(L/CR)$.
4. Since the circuit rejects the current at resonance (i.e., it has minimum value), the parallel resonant circuit is also called ***rejector circuit*** or ***anti-resonant circuit***.
5. Since the circulating current between the two branches is many times the line current, the parallel resonant circuit is also called ***current resonant circuit***.
6. The circuit is also called a ***tank circuit.***

11.6 COMPARISON BETWEEN SERIES AND PARALLEL RESONANCE

The resonance in a series and in a parallel circuit differs in its characteristics and properties. These differences are summarized in Table 11.1.

EXAMPLE 11.7

An inductor coil having a resistance of 20 Ω and inductance of 200 μH is connected in parallel with a variable capacitor. This combination is connected in series with a resistance of 8 kΩ. The circuit is then connected across a 230-V, 1-MHz ac source. Determine (*a*) the value of capacitance to cause resonance, (*b*) the *Q*-factor of the circuit, (*c*) the dynamic impedance of the parallel resonant circuit, and (*d*) the line current.

Table 11.1 *Comparison of resonance in series and parallel circuits.*

S.No.	Property	Series circuit	Parallel circuit
1.	Impedance at resonance	$Z_0 = R$, the minimum	$Z_0 = L/CR$, the maximum
2.	Current at resonance	$I_0 = V/R$, the maximum	$I_0 = V/(L/CR)$, the minimum
3.	Resonance frequency	$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$,	$f_0 = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2 C/L}$,
4.	Magnification of	Voltage (not affected by R)	Current (affected by R)
5.	Nature of the circuit:		
	(*i*) Below f_0	(*i*) Capacitive	(*i*) Inductive
	(*ii*) Above f_0	(*ii*) Inductive	(*ii*) Capacitive

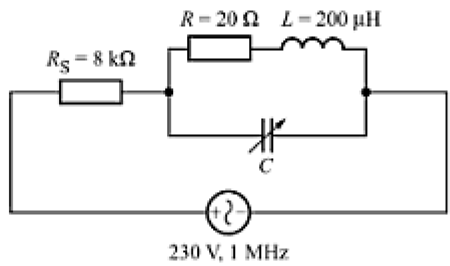


Fig. 11.10 *A series-parallel ac circuit.*

Solution The circuit diagram is shown in Fig. 11.10.

$$X_L = 2\pi f L = 2\pi \times 1 \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$$

(*a*) Since $R << X_L$, we can use the approximate relation of Eq. 11.25 to calculate the required value of the capacitor,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times (1 \times 10^6)^2 \times 200 \times 10^{-6}} = \mathbf{126.6\ pF}$$

(*b*) The Q-factor of the circuit $= \dfrac{X_L}{R} = \dfrac{1256.6}{20} = \mathbf{62.83}$

(*c*) The dynamic impedance, $Z_0 = \dfrac{L}{CR} = \dfrac{200 \times 10^{-6}}{126.6 \times 10^{-12} \times 20} = \mathbf{78\,989\ \Omega}$

(*d*) The total equivalent impedance of the circuit is

$$Z = Z_0 + R_S = 78\,989 + 8000 = 86\,989\ \Omega$$

Therefore, the total line current is

$$I = \frac{V}{Z} = \frac{230}{86\,989} = \mathbf{2.644\ mA}$$

EXAMPLE 11.8

A practical parallel resonant circuit consists of a coil, having a resistance of 150 Ω and an inductance of 0.24 H, in parallel with a lossless capacitor of capacitance 3 μF. Find its resonance frequency, Q-factor and bandwidth.

Solution Using Eq. 11.24, we have

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2 C/L} = \frac{1}{2\pi\sqrt{0.24 \times 3 \times 10^{-6}}}\sqrt{1 - \frac{(150)^2 \times 3 \times 10^{-6}}{0.24}} = \mathbf{159.05\ Hz}$$

The quality factor, $Q = \dfrac{X_L}{R} = \dfrac{2\pi \times 159.05 \times 0.24}{150} = \mathbf{1.6}$

The bandwidth, $BW = \dfrac{f_0}{Q} = \dfrac{159.05}{1.6} = \mathbf{99.4\ Hz}$

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 11.9

A 125-V ac source supplies a series circuit consisting of a 20.5-μF capacitor and a coil whose resistance and inductance are 1.06 Ω and 25.4 mH, respectively. The source frequency is adjusted so as to bring the circuit to resonance.

(*a*) Determine (*i*) the source frequency, and (*ii*) the current supplied by the source.

(*b*) Determine the voltage across (*i*) the capacitor and (*ii*) the coil.

(*c*) Determine the resistance that must be connected in series with the circuit to limit the capacitor voltage to 300 V.

Solution

(*a*) (*i*) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(25.4\times10^{-3})\times(20.5\times10^{-6})}} = \mathbf{220.6\ Hz}$

(*ii*) $I_0 = \dfrac{V}{Z_0} = \dfrac{V}{R} = \dfrac{125}{1.06} = \mathbf{117.9\ A}$

(*b*) (*i*) $V_C = V_L = IX_L = I(2\pi f_0 L) = (117.9)\,(2\pi\times220.6\times25.4\times10^{-3}) = \mathbf{4151\ V}$

(*ii*) $R = 1.06\ \Omega;\ X_L = 2\pi f_0 L = 2\pi\times220.6\times25.4\times10^{-3} = 35.21\ \Omega$

$$\mathbf{V}_{\text{coil}} = \mathbf{IZ}_{\text{coil}} = (117.9\angle0^\circ)\,(1.06 + j35.21) = \mathbf{4154\angle88.3^\circ\ V}$$

(*c*) $V_C = IX_C = IX_L$ or $300 = I(35.21) \Rightarrow I = 8.52$ A

Now, $I = \dfrac{V}{R}$ or $8.52 = \dfrac{125}{R} \Rightarrow R = 14.67\ \Omega;\quad \therefore\ R_x = 14.67 - 1.06 = \mathbf{13.61\ \Omega}$

EXAMPLE 11.10

A coil has a resistance of 3 Ω and inductance of 12 mH. What is the value of the capacitance that must be connected in series with this coil so that the circuit resonates at a frequency of 9 kHz? Assuming the supply voltage to be 240 V, calculate the maximum instantaneous energy stored in the inductor.

Solution Since $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, the required value of the capacitance is given as

$$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi\times9\times10^3)^2\times12\times10^{-3}} = \mathbf{26\ nF}$$

Now, the current, $I_0 = \dfrac{V}{R} = \dfrac{240}{3} = 80$ A

$\therefore$ Energy stored $= \dfrac{1}{2}LI_0^2 = \dfrac{1}{2}\times12\times10^{-3}\times(80)^2 = \mathbf{38.4\ J}$

EXAMPLE 11.11

Determine the parameters of a series *RLC* circuit that will resonate at 10 kHz, have a bandwidth of 1 kHz, and draw 15.3 W from a 200-V generator operating at the resonance frequency of the circuit.

Solution At resonance, $V_R = V = 200$ V. The power drawn is given as

$$P = \frac{V_R^2}{R} \quad \Rightarrow \quad R = \frac{V_R^2}{P} = \frac{(200)^2}{15.3} = \mathbf{2.61\ k\Omega}$$

Now, $Q = \dfrac{f_0}{BW}$, and as per definition $Q = \dfrac{2\pi f_0 L}{R}$. Therefore,

$$\frac{f_0}{BW} = \frac{2\pi f_0 L}{R} \quad \Rightarrow \quad L = \frac{R}{2\pi (BW)} = \frac{2610}{2\pi \times 1000} = \mathbf{415.4\ mH}$$

$$f_0 = \frac{1}{2\pi \sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 10 \times 10^3)^2 \times 415.4 \times 10^{-3}} = \mathbf{610\ pF}$$

EXAMPLE 11.12

A 400-V, 200-Hz ac source is connected in series with a capacitor and a coil whose resistance and inductance are 20 mΩ and 6 mH, respectively. The circuit is in resonance at 200 Hz. Determine (*a*) the capacitance of the capacitor, (*b*) the circuit current, (*c*) the voltage across the capacitor, (*d*) the maximum energy stored in the coil, and (*e*) the half-power frequencies.

Solution

(*a*) $f_0 = \dfrac{1}{2\pi \sqrt{LC}} \quad \Rightarrow \quad C = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{(2\pi \times 200)^2 \times 6 \times 10^{-3}} = \mathbf{105.54\ \mu F}$

(*b*) $I_0 = \dfrac{V}{R} = \dfrac{400}{0.020} = \mathbf{20\ kA}$

(*c*) $V_C = IX_C = (20 \times 10^3) \times \dfrac{1}{2\pi \times 200 \times 105.54 \times 10^{-6}} = \mathbf{151\ kV}$

(*d*) $U_{max} = 1/2 L I_m^2 = (1/2) \times (0.006)\,(20\,000\sqrt{2})^2 = \mathbf{2.4\ MJ}$

(*e*) $Q = \dfrac{\omega_0 L}{R} = \dfrac{2\pi \times 200 \times 0.006}{0.020} = 377$ and $BW = \dfrac{f_0}{Q} = \dfrac{200}{377} = 0.53$ Hz

For $Q > 10$, $f_2 \approx f_0 + \dfrac{BW}{2} = 200 + 0.265 = \mathbf{200.265\ Hz}$

and $f_1 \approx f_0 - \dfrac{BW}{2} = 200 - 0.265 = \mathbf{199.735\ Hz}$

EXAMPLE 11.13

A series *RLC* circuit has a resistance of 1 kΩ and half-power frequencies of 20 kHz and 100 kHz. Determine (*a*) the bandwidth, (*b*) the resonance frequency, (*c*) the inductance, and (*d*) the capacitance.

Solution

(*a*) The bandwidth, $BW = f_2 - f_1 = 100 - 20 = \mathbf{80\ kHz}$

(*b*) The resonance frequency, $f_0 = \sqrt{f_1 f_2} = \sqrt{20 \times 100} = \mathbf{44.72\ kHz}$

(*c*) $Q = \dfrac{f_0}{BW} = \dfrac{44.72}{80} = 0.56$; but $Q = \dfrac{2\pi f_0 L}{R} \quad \Rightarrow \quad L = \dfrac{QR}{2\pi f_0} = \dfrac{0.56 \times 10^3}{2\pi \times 44.72 \times 10^3} = \mathbf{2\ mH}$

(*d*) The capacitance, $C = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{(2\pi \times 44.72 \times 10^3)^2 \times 2 \times 10^{-3}} = \mathbf{6.33\ nF}$

EXAMPLE 11.14

A series *RLC* circuit having $R = 5\ \Omega$ operates from a 20-V source. Determine the power at half-power frequencies.

Solution $I_0 = \dfrac{V}{Z_0} = \dfrac{V}{R} = \dfrac{20}{5} = 4$ A; $P_0 = I_0^2 R = (4)^2 \times 5 = 80$ W

$$\therefore \quad P_{\text{half-power}} = (1/2)P_0 = (1/2) \times 80 = \mathbf{40\ W}$$

EXAMPLE 11.15

A 240-V, 100-Hz ac source is connected to a series *RLC* circuit consisting of a coil and a variable capacitor. The coil has a resistance of 55 mΩ and an inductance of 7 mH. The capacitor is varied so as to achieve resonance. Determine (*a*) the value of the capacitance, (*b*) the circuit quality factor, and (*c*) the half-power frequencies.

Solution

(*a*) Since, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, the value of the capacitance is given as

$$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 100)^2 \times 7 \times 10^{-3}} = \mathbf{361.86\ \mu F}$$

(*b*) The quality factor, $Q = \dfrac{2\pi f_0 L}{R} = \dfrac{2\pi \times 100 \times 7 \times 10^{-3}}{55 \times 10^{-3}} = \mathbf{79.93}$

(*c*) The bandwidth, $BW = \dfrac{f_0}{Q} = \dfrac{100}{79.93} = \mathbf{1.251\ Hz}$

Therefore, the half-power frequencies are

$$f_1 = f_0 - \frac{BW}{2} = 100 - \frac{1.251}{2} = \mathbf{99.3745\ Hz} \quad \text{and} \quad f_2 = f_0 + \frac{BW}{2} = \mathbf{100.6255\ Hz}$$

EXAMPLE 11.16

A coil of resistance and inductance 20 Ω and 0.2 H, respectively, is connected in parallel with a 100-μF capacitor. Determine the frequency at which the circuit behaves as a non-inductive resistance. Find the value of this resistance.

Solution The circuit behaves as a pure resistance at resonance frequency, given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2 C/L} = \frac{1}{2\pi\sqrt{0.2 \times 100 \times 10^{-6}}}\sqrt{1 - \frac{(20)^2 \times 100 \times 10^{-6}}{0.2}} = \mathbf{31.82\ Hz}$$

The effective resistance, $Z_0 = R_0 = \dfrac{L}{CR} = \dfrac{0.2}{100 \times 10^{-6} \times 20} = \mathbf{100\ \Omega}$

EXAMPLE 11.17

The medium-wave band in a radio receiver spreads from 570 kHz to 1560 kHz. The radio is tuned to the desired amplitude-modulated carrier frequency by using a series resonant circuit consisting of a coil and a variable capacitor.

(*a*) If the coil has an inductance of 20-μH, find the range of the capacitance needed to tune over the entire range.

(*b*) If the Q of the circuit is 50 at a frequency of 570 kHz, find the resistance of the coil and the bandwidth of the circuit.

(*c*) Find the Q of the circuit at the upper tuning frequency.

Solution

(*a*) Since the tuning frequency (or resonance frequency) is given as

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \text{ we have } C = \frac{1}{(2\pi f_0)^2 L}$$

Therefore, at the lower tuning frequency, the value of the capacitor should be

$$C_1 = \frac{1}{(2\pi f_{01})^2 L} = \frac{1}{(2\pi \times 570 \times 10^3)^2 \times 20 \times 10^{-6}} = \mathbf{3.9\ nF}$$

The value of the capacitor at the upper tuning frequency should be

$$C_2 = \frac{1}{(2\pi f_{02})^2 L} = \frac{1}{(2\pi \times 1560 \times 10^3)^2 \times 20 \times 10^{-6}} = \mathbf{0.52\ nF}$$

Thus, the range of the tuning capacitor is from **0.52 nF to 3.9 nF**.

(*b*) Since $Q_1 = \dfrac{\omega_{01} L}{R} = \dfrac{2\pi f_{01} L}{R}$, the resistance of the coil is given as

$$R = \frac{2\pi f_{01} L}{Q_1} = \frac{2\pi \times (570 \times 10^3) \times (20 \times 10^{-6})}{50} = \mathbf{1.433\ \Omega}$$

Bandwidth, $BW = \dfrac{f_{01}}{Q_1} = \dfrac{570 \times 10^3}{50} = \mathbf{11.4\ kHz}$

(*c*) At the upper tuning frequency,

$$Q_2 = \frac{\omega_{02} L}{R} = \frac{2\pi f_{02} L}{R} = \frac{2\pi \times (1560 \times 10^3) \times (20 \times 10^{-6})}{1.433} = \mathbf{136.8}$$

SUMMARY

TERMS AND CONCEPTS

- ***Resonance*** is the condition that exists in ac circuits under steady state when the input current is in phase with the input voltage.
- ***Resonant frequency*** (f_0) is the frequency at which resonance occurs.
- At resonance in a series RLC circuit, the impedance is minimum and purely resistive. The power factor is unity.
- In a series RLC circuit, when $f < f_0$, the circuit is capacitive and the power factor is leading; when $f > f_0$, the circuit is inductive and the power factor is lagging.
- ***Bandwidth*** (BW) or *passband* is the range of frequencies within which the current does not drop below 0.707 $(=1/\sqrt{2})$ times the maximum value (I_0).
- The frequencies f_1 and f_2 at which the current reduces to 0.707 times the maximum value (or, equivalently, at which the power reduce to half the maximum value) are called *lower* and *upper* ***cutoff frequencies*** or ***half-power frequencies***.

- The greater the Q factor, the narrower is the BW and the greater is the ***selectivity*** of a tuned circuit.
- In a series RLC circuit, at resonance, the voltages across capacitor and inductor are magnified Q times. Hence it is called ***voltage resonant circuit.***
- A parallel resonant circuit (also called ***anti-resonant*** or ***tank*** circuit) has a coil and a capacitor in parallel. It is said to resonate when the reactive (or wattless) component of line current I reduces to zero.
- A practical parallel resonance circuit is represented by an inductance and a resistance in one branch and a capacitance in other branch.
- At resonance in a practical parallel resonance circuit, the impedance is maximum and the line current is minimum. Hence, it is called a ***rejector circuit***.
- In a parallel resonant circuit, when $f < f_0$, the circuit is inductive and pf is lagging; when $f > f_0$, the circuit is capacitive and pf is leading.
- At resonance, the circulating current between the two parallel branches is many times the line current. Hence, the parallel resonant circuit is also called ***current resonant circuit***.
- The parallel resonant circuit is also called a ***tank circuit***.

IMPORTANT FORMULAE

For series resonant circuit:

- Resonant frequency, $\omega_0 = \dfrac{1}{\sqrt{LC}}$ or $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$.
- Impedance at resonance, $Z_0 = R$.
- Frequency at which V_C is maximum, $f_C = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{2L^2}}$.
- Frequency at which V_L is maximum, $f_L = \dfrac{1}{2\pi\sqrt{LC - (R^2C^2/2)}}$.
- $f_0 = \sqrt{f_1 f_2}$.
- The quality factor, $Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$.
- The bandwidth, $BW = f_2 - f_1 = \dfrac{f_0}{Q}$. Also, $BW = \dfrac{R}{2\pi L}$.
- For high-Q coils ($Q > 10$), $f_1 \approx f_0 - \dfrac{BW}{2}$ and $f_2 \approx f_0 + \dfrac{BW}{2}$.

For parallel resonant circuit:

- Resonant frequency:

  (a) General circuit: $f_0 = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{\dfrac{R_1^2 - (L/C)}{R_2^2 - (L/C)}}$.

  (b) Practical circuit: $f_0 = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{1 - R^2C/L}$.

  (c) Ideal circuit: $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$.

- Impedance at resonance, $Z_0 = \dfrac{L}{CR}$.
- The quality factor, $Q = \dfrac{\omega_0 L}{R}$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The ratio of bandwidth to the resonance frequency of a resonant circuit is called its quality factor.	☐	☐	
2.	In a *series resonant circuit*, the lower the resistance in the circuit, the steeper is its current response.	☐	☐	
3.	When a capacitor is connected in parallel to an inductive circuit, the phase angle increases and the power factor decreases.	☐	☐	
4.	In a practical resonant circuit, the value of the resistance affects the resonant frequency.	☐	☐	
5.	The impedance of both the series and parallel resonant circuit increase with increase in frequency.	☐	☐	
6.	In a series resonant circuit, the impedance for the frequencies above resonant frequency is inductive.	☐	☐	
7.	When the frequency is much greater than the resonant frequency of a series resonant circuit, the angle of impedance Z approaches 0°.	☐	☐	
8.	For a series resonant circuit, the resonance curve is a plot of frequency versus voltage.	☐	☐	
9.	At half-power points of a resonance curve of a series resonant circuit, the value of current is $1/\sqrt{2}$ times the current at resonance.	☐	☐	
10.	A parallel ac circuit draws maximum current when in resonance.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** False	**4.** True	**5.** False
6. True	**7.** False	**8.** False	**9.** True	**10.** False

REVIEW QUESTIONS

1. Discuss briefly the phenomenon of resonance in electrical circuits. What is the practical application of this phenomenon?
2. In a series *RLC* circuit, the resonance condition can be achieved in three ways. Explain them.
3. An ac voltage is connected to a circuit containing R, L and C in series. The supply frequency is gradually varied. Explain the observations.
4. For a series *RLC* circuit, discuss the variation of the following quantities with frequency of the ac source: X_L, X_C, $X_L - X_C$, Z and I.

5. Define half-power points and bandwidth for a series *RLC* circuit. How is the bandwidth affected by Q of the circuit?
6. At resonance in a series *RLC* circuit, the voltage across the capacitor may become many times the supply voltage. Explain how.
7. How is a practical resonant circuit different from a theoretical parallel resonant circuit? Derive the expressions for the resonance frequency, the quality factor and the bandwidth for a practical parallel resonant circuit.
8. Show that a practical parallel resonant circuit behaves as purely resistive with $R_0 = L/CR$, at resonance.
9. Show that a parallel resonant circuit is inductive when $f < f_0$, and capacitive when $f > f_0$.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. In a series *RLC* circuit, if the frequency of the source is below the resonance frequency, then
   (*a*) $X_C = X_L$ (*b*) $X_C > X_L$
   (*c*) $X_C < X_L$ (*d*) none of the above
2. A series resonant circuit magnifies
   (*a*) voltage (*b*) current
   (*c*) both voltage and current (*d*) none of the above
3. A parallel resonant circuit magnifies
   (*a*) voltage (*b*) current
   (*c*) both voltage and current (*d*) none of the above
4. In a parallel ac circuit, if the supply frequency is greater than the resonance frequency, the circuit is
   (*a*) resistive (*b*) inductive
   (*c*) capacitive (*d*) none of the above
5. A parallel resonant circuit draws a current of 2 mA at resonance. If the Q of the circuit is 100, the current drawn by the capacitor is
   (*a*) 0.02 mA (*b*) 1 mA
   (*c*) 2 mA (*d*) 200 mA
6. If a parallel resonant circuit is shunted by a resistance, then
   (*a*) the circuit impedance is decreased
   (*b*) the Q of the circuit is increased
   (*c*) the gain of the circuit is increased
   (*d*) none of the above
7. A series *RLC* circuit has unity power factor if operated at a frequency of
   (*a*) $1/LC$ Hz (*b*) $1/\pi LC$ Hz
   (*c*) $1/2\pi LC$ Hz (*d*) $1/2\pi\sqrt{LC}$ Hz
8. In a series *RLC* circuit at resonance,
   (*a*) its impedance is maximum
   (*b*) its admittance is maximum
   (*c*) its impedance is purely reactive
   (*d*) its current is minimum
9. A series *RLC* resonant circuit implies that it has
   (*a*) zero *pf* and maximum current
   (*b*) zero *pf* and minimum current
   (*c*) unity *pf* and minimum current
   (*d*) unity *pf* and maximum current
10. A series *RLC* circuit, consisting of $R = 10\ \Omega$, $X_L = X_C = 20\ \Omega$, is connected across 100-V, 50-Hz supply. The magnitude and the phase angle (with reference to the supply voltage) of the voltage across the inductance are, respectively,
    (*a*) 100 V, 90° (*b*) 100 V, –90°
    (*c*) 200 V, –90° (*d*) 200 V, 90°
11. A resonant circuit with a high-Q coil has
    (*a*) large bandwidth (*b*) high losses
    (*c*) low losses (*d*) flat response
12. If the Q factor of a resonant circuit is high, then
    (*a*) its power factor is high
    (*b*) its impedance is high
    (*c*) its bandwidth is large
    (*d*) none of these
13. In a series resonant circuit, if the value of inductance L is increased,
    (*a*) its resonance frequency decreases
    (*b*) its bandwidth decreases
    (*c*) its Q increases
    (*d*) all of the above
14. In a series resonant circuit, if the value of capacitance C is increased,
    (*a*) its bandwidth increases
    (*b*) its resonance frequency decreases

(*c*) its Q decreases
(*d*) both (*b*) and (*c*)

15. In case of a series resonant circuit, a change in supply voltage will change
(*a*) the resonance frequency
(*b*) the bandwidth of the circuit
(*c*) the current drawn
(*d*) the Q of the circuit

16. For a series *RLC* circuit, at half-power frequencies, we must have
(*a*) $X_C = X_L$ (*b*) $X_C = 2X_L$
(*c*) $R = 0$ (*d*) $X_C - X_L = \pm R$

17. For a series *RLC* circuit, the resonance frequency f_0 is related to the half-power frequencies f_1 and f_2 as
(*a*) $f_0 = \sqrt{f_1 f_2}$
(*b*) $f_0 = (f_1 + f_2)/2$
(*c*) $f_0 = f_2 - f_1$
(*d*) $f_0 = \sqrt{f_1} + \sqrt{f_2}$

18. The power factor of a series *RLC* circuit at its half-power frequencies is
(*a*) unity (*b*) leading
(*c*) lagging (*d*) leading or lagging

19. In a parallel resonant circuit, the current through the capacitor is
(*a*) equal to the line current
(*b*) greater than the line current
(*c*) less than the line current
(*d*) zero

20. A parallel resonant circuit can be employed
(*a*) to amplify voltage
(*b*) to reject a small band of frequencies
(*c*) as a high impedance
(*d*) both (*b*) and (*c*)

ANSWERS

1. *b*	**2.** *a*	**3.** *b*	**4.** *c*	**5.** *d*	**6.** *a*	**7.** *d*	**8.** *b*	**9.** *d*	**10.** *d*
11. *c*	**12.** *d*	**13.** *d*	**14.** *d*	**15.** *c*	**16.** *d*	**17.** *a*	**18.** *d*	**19.** *b*	**20.** *d*

PROBLEMS

(A) SIMPLE PROBLEMS

1. A 120-V, 20-Hz source is connected to a series circuit consisting of 5.0-Ω capacitive reactance, a 1.6-Ω resistor, and a coil with resistance and inductive reactance of 3.0-Ω and 1.2-Ω, respectively. Determine (*a*) the input impedance, and (*b*) the circuit current.
[**Ans.** (*a*) $5.97\angle{-39.56°}$ Ω; (*b*) $20.11\angle 39.56°$ A]

2. For Prob. 1, calculate (*a*) the voltage across the coil and (*b*) the resonance frequency.
[**Ans.** (*a*) $64.98\angle 61.36°$ V; (*b*) 40.93 Hz]

3. A series *RLC* circuit consists of $R = 1$ Ω, $L = 140$ mH and $C = 100$ μF. Determine the frequency at which resonance occurs. If the applied voltage is 220 V at 50 Hz, determine the current and voltage drops across R, L and C.
[**Ans.** 42.5 Hz, 18 A, 18 V, 791.64 V, 572.94 V]

4. A coil with resistance and inductance of 20 Ω and 3.0 mH, respectively, is connected in series with a capacitor and a 12-V, 5.0-kHz source. Determine (*a*) the value of capacitance that will cause the system to be in resonance, and (*b*) the circuit current at the resonance frequency.
[**Ans.** (*a*) 338 nF; (*b*) 0.6 A]

5. What is the maximum stored energy in the capacitor of Prob. 4? [**Ans.** 1.08 mJ]

6. A 24-μF capacitor is connected in series with a coil whose inductance is 5.0 mH. Determine (*a*) the resonance frequency, (*b*) the resistance of the coil if a 40-V source operating at the resonance frequency causes a current of 3.6 mA, and (*c*) the Q of the coil.
[**Ans.** (*a*) 459.5 Hz; (*b*) 11.1 kΩ; (*c*) 1.3×10^{-3}]

7. A coil having a resistance of 2 Ω is connected in series with a 50-μF capacitor. The circuit resonates at 100 Hz. (*a*) Calculate the inductance of the coil. (*b*) If the circuit is connected across a 100-V, 100-Hz ac source, determine the power dissipated in the coil. (*c*) Calculate the voltage across the capacitor and the coil.

[**Ans.** (*a*) 50.66 mH; (*b*) 5 kW; (*c*) 1591.5 V, 1594.6 V]

8. A coil having resistance and inductance of 5.0 Ω and 32 mH, respectively, is connected in series with a 796-pF capacitor. Determine (*a*) the resonance frequency of the circuit, (*b*) the quality factor, and (*c*) the bandwidth.

[**Ans.** (*a*) 31.53 kHz; (*b*) 1268; (*c*) 24.9 Hz]

9. The circuit of Prob. 8 is connected to a 120-V source and operates at resonant frequency. Calculate (*a*) the input current, and (*b*) the voltage across the capacitor.

[**Ans.** (*a*) 24 A; (*b*) 152 kV]

(B) TRICKY PROBLEMS

10. A generator supplies a variable frequency voltage of constant amplitude of 100 V (rms) to a series *RLC* circuit, having $R = 5\ \Omega$, $L = 4$ mH and $C = 0.1\ \mu$F. The frequency of the generator is varied until a maximum current is obtained. Determine the maximum current, the frequency at which it occurs, and the resulting voltage across the inductance and the capacitance. [**Ans.** 20 A, 7.958 kHz, 4 kV]

11. A series *RLC* circuit, having a variable inductor, is connected to a sinusoidal source $200\sqrt{2}\sin 100\pi t$ V. By varying the inductance, the maximum current obtained is 0.314 A, and under this condition the voltage across the capacitor is 300 V. Find the values of the circuit elements.

[**Ans.** $R = 637\ \Omega$, $C = 3.332\ \mu$F, $L = 3.04$ H]

12. A coil with resistance and inductance of 40 Ω and 50 mH, respectively, is connected in series with a 450-pF capacitor and a generator. Determine (*a*) the resonance frequency, and (*b*) the circuit impedance at the resonance frequency.

[**Ans.** (*a*) 33.553 kHz; (*b*) 40 Ω]

13. A 60-V source having an internal resistance of 10 Ω is connected to the circuit of Prob. 12. Determine (*a*) the circuit current, and (*b*) the voltage across the capacitor. [**Ans.** (*a*) 1.2 A; (*b*) 12.65 kV]

14. The bandwidth of a series *RLC* circuit is 100 Hz and the resonance frequency is 1000 Hz. If the circuit resistance is 10 Ω, determine *L* and *C*.

[**Ans.** 15.91 mH, 1.59 μF]

15. Find the half-power frequencies of a series *RLC* circuit having $Q = 60$ and $f_0 = 12$ kHz.

[**Ans.** 11 900 Hz, 12 100 Hz]

(C) CHALLENGING PROBLEMS

16. A series *RLC* circuit consists of a 100-Ω resistance, and inductance of 0.318 H and a capacitance of unknown value. When this circuit is energized by a $230\angle 0°$ V, 50-Hz sinusoidal ac supply, the current was found to be $2.3\angle 0°$ A. Find (*a*) the value of the capacitance, (*b*) the voltage across the inductance, and (*c*) the total power consumed.

[**Ans.** 31.86 μF, $230\angle 90°$ V, 529 W]

17. A series *RLC* circuit has a *Q* of 5.1 at its resonance frequency of 100 kHz. (*a*) Assuming that the power dissipation of the circuit is 100 W when drawing a current of 0.80 A, determine the circuit parameters. (*b*) What is the bandwidth of the circuit? (*c*) Determine the half-power frequencies of the circuit?

[**Ans.** (*a*) $R = 156\ \Omega$, $L = 1.26$ mH, $C = 2.01$ nF; (*b*) 19.6 kHz; (*c*) 90.7 kHz, 110.3 kHz]

18. A practical parallel resonant circuit consists of a 65-pF capacitor in parallel with a coil whose inductance and resistance are 56 μH and 60 Ω, respectively. Determine (*a*) its resonant frequency, and (*b*) the quality factor at resonance frequency.

[**Ans.** (*a*) 2.63 MHz; (*b*) 15.4]

THREE-PHASE CIRCUITS AND SYSTEMS 12

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the advantages of three-phase system over single-phase system.*
- *Use double-subscript notation for voltages and currents in an electrical circuit.*
- *Explain the concept of three-phase voltages.*
- *Explain the principle of generation of three-phase voltages.*
- *Connect three loads in star or delta.*
- *Derive the relations between phase voltage and line voltage in a balanced star- or delta-connection.*
- *Derive the relations between phase current and line current in a balanced star- or delta-connection.*
- *State the advantages of 4-wire, 3-phase system over 3-wire, 3-phase system.*
- *Explain the term 'neutral shift' or 'floating neutral'.*
- *Derive an expression to calculate total active power in a 3-phase system for a balanced 3-phase load (star or delta).*
- *Compare the star-connected systems and delta-connected systems.*
- *Explain two-wattmeter method of measuring power in a balanced or unbalanced three-phase load.*
- *Explain two-wattmeter method of measuring power factor in a balanced three-phase load.*

12.1 INTRODUCTION

The earliest application of ac current was for heating the filament of an electric lamp. For this purpose, the single-phase system was quite satisfactory. Some years later, ac motors were developed. It was found that single-phase ac supply was not very satisfactory for this application. For instance, the single-phase induction motor—the type most commonly used—was not self-starting unless it was fitted with an auxiliary winding. It was found that by using two separate windings with currents differing in phase by 90° or three windings with currents differing in phase by 120°, the induction motor became self-starting, had better efficiency and power factor.

The system utilising two windings is referred to as a *two-phase system* and that utilising three windings is referred to as a *three-phase system**.

Almost all the electrical power used in the country is generated and distributed in the form of three-phase ac supply. The single-phase ac supply used in homes, offices, factories, etc., originates as a part of 3-phase system.

* A 'three-phase system' is sometimes written in brief as '3-ϕ system'.

The use of *polyphase systems* having higher number of phases, such as 6- and 12-phase systems, is limited to the supply to large rectifiers. Here, the rectifiers convert ac into dc, which is required for certain processes such as electrolysis. As the number of phases increases, the ripple in the output of the rectifiers decreases.

Advantages of Three-Phase System

The advantages of using 3-ϕ systems over 1-ϕ systems are as follows:

1. Three-phase transmission lines require much less conductor material. Since the phasor sum of currents in all the phases is zero, there is substantial saving by eliminating the return conductor or replacing it by a single neutral conductor of comparatively small size.
2. For a given frame size, a three-phase machine gives a higher output than a single-phase machine.
3. The power in a single-phase system pulsates at twice the line frequency. However, the sum of powers in the three phases in a three-phase system remains constant. Therefore, a three-phase motor develops a uniform torque, whereas a single-phase motor develops pulsating torque.
4. Since the three-phase supply can generate a rotating field, the three-phase induction motors are self-starting.
5. The three-phase system can be used to supply domestic as well as industrial (or commercial) power.
6. The voltage regulation in three-phase system is better than that in single-phase supply.

12.2 DOUBLE-SUBSCRIPT NOTATION

Double-Subscript Notation for Voltages

It is convenient to describe three-phase voltages and currents using double-subscript notation. With this notation, a voltage, such as $\mathbf{V}_{ab}$, has more meaning than if it were denoted simply as $\mathbf{V}_1$ or $\mathbf{V}_x$. The notation $\mathbf{V}_{ab}$ represents the voltage of point a with respect to point b. Thus, a plus sign is located at a, as shown in Fig. 12.1*a*. We therefore consider the double-subscript notation to be *equivalent* to a plus-minus-sign pair. The use of both would be redundant.

With reference to Fig. 12.1*b*, it is obvious that $\mathbf{V}_{ad} = \mathbf{V}_{ab} + \mathbf{V}_{cd}$. The advantage of the double subscript lies in the fact that KVL requires the voltage between two points to be the same, regardless of the path chosen between the points. Thus,

$$\mathbf{V}_{ad} = \mathbf{V}_{ab} + \mathbf{V}_{bd} = \mathbf{V}_{ac} + \mathbf{V}_{cd} = \mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{cd}, \text{ etc}$$

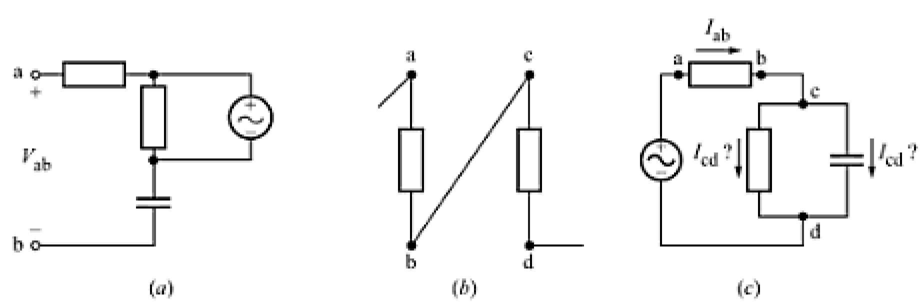


Fig. 12.1 *Double-subscript notation.*

It is apparent that KVL may be satisfied without reference to the circuit diagram. For example, we may write $\mathbf{V}_{ab} = \mathbf{V}_{ax} + \mathbf{V}_{xb}$, where x identifies the location of any interesting point of our choice, which is not even marked in the diagram.

Double-Subscript Notation for Currents

The double-subscript notation may also be applied to currents. We define the current $\mathbf{I}_{ab}$ as the current flowing from a to b *by the direct path*. In every complete circuit we consider, there must exist at least two possible paths between points a and b. We agree that we shall not use double-subscript notation unless it is obvious that one path is much shorter, or much more direct.

Usually, the path of the current is through a single element. Thus, the current $\mathbf{I}_{ab}$ is correctly indicated in Fig. 12.1*c*. In fact, we do not even need the direction arrow when talking about this current. The subscript ab tells us the direction. However, if we merely write the current from c to d as $\mathbf{I}_{cd}$ without marking the arrow, it may cause confusion (see Fig. 12.1*c*).

12.3 CONCEPT OF THREE-PHASE VOLTAGES

A three-phase voltage system requires three single-phase emfs having the same amplitude and frequency but phase-displaced by 120°. Figure 12.2*a* shows the three single-phase emf sources, e_a, e_b, and e_c. The phasor relationship of these three emfs is shown in phasor diagram of Fig. 12.2*b*. Length of each phasor represents maximum or peak value of each emf. Since the three emfs have equal amplitudes, we can write

$$E_{ma} = E_{mb} = E_{mc} = E_m$$

The phasors are assumed to be rotating in the counterclockwise (positive) direction at an angular frequency ω.

The time diagram of Fig. 12.2*c* is generated by projecting each of the three phasors of Fig. 12.2*b* onto the vertical line, when the phasors revolve. Note that the positive maximum value first occurs for phase a and then in succession for phases b and c. Thus, we see that phase b lags phase a by 120° (or one-third of a cycle), and phase c lags phase b by 120°. For this reason, the three-phase voltage system of Fig. 12.2 is said to have the ***phase order*** *abc*. The terms *phase sequence* and *phase rotation* are also used in place of phase order.

The phase order of a three-phase voltage system is important in many applications. For example, in a three-phase induction motor it is the phase order that determines the direction of its rotation.

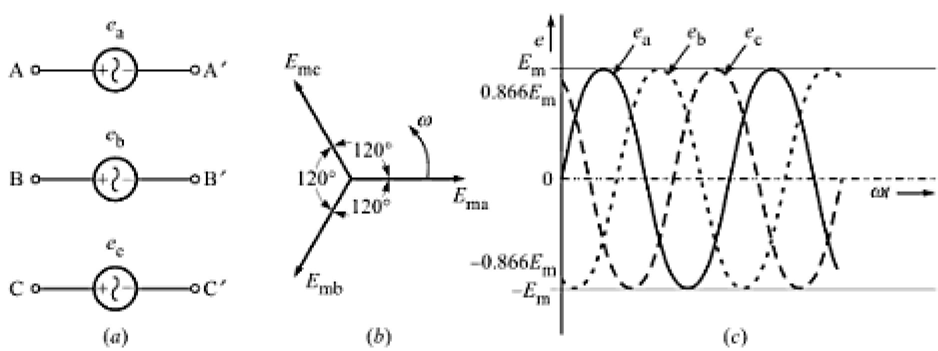


Fig. 12.2 *Three-phase emfs.*

It can be seen from the phasor diagram of Fig. 12.2*b* that the phasor sum of the three emfs, e_a, e_b, and e_c is zero. That is, in terms of the rms values, we have

$$\mathbf{E}_a + \mathbf{E}_b + \mathbf{E}_c = 0 \tag{12.1}$$

Since, $\mathbf{E}_a$ has been taken as the reference phasor, the instantaneous values of the three emfs are given as

$$e_a = E_m \sin \omega t \tag{12.2}$$

$$e_b = E_m \sin(\omega t - 120°) \tag{12.3}$$

and

$$e_c = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°) \tag{12.4}$$

Let us check whether the sum of the instantaneous values of these three emfs at $t = 0$ is zero. Putting $t = 0$ in Eqs. 12.2, 12.3 and 12.4, we get

$$e_a = 0,\ e_b = E_m \sin(-120°) = -0.866E_m \qquad \text{and} \qquad e_b = E_m \sin 120° = +0.866E_m$$

Thus, at $t = 0$, we have

$$e_a + e_b + e_c = 0 + (-0.866E_m) + 0.866E_m = 0$$

12.4 GENERATION OF THREE-PHASE VOLTAGES

Figure 12.3*a* illustrates the principle of generation of three-phase voltage system. In practice, instead of naming the phase order as abc, we name it as **RYB**, which stands for ***R***ed, ***Y***ellow and ***B***lue coloured wires used in the three-phase power supply. The three coils RR′, YY′, and BB′ wound on a rotor have a space-displacement of 120° from one another. The rotor is rotated counterclockwise in a uniform magnetic field with a uniform angular speed ω. A sinusoidal emf is generated in each coil. Because of the space-displacement of the coils, after one-third cycle of rotation, coil YY′ occupies the same position as the coil RR′ did. Therefore, the emf in coil YY′ lags behind the emf in coil RR′ by 120°. Similarly, the emf in coil BB′ lags behind the emf in coil YY′ by 120°. Thus, if the three coils are identical, we get three emfs having the same amplitude and frequency but time-displaced by 120°, as shown in Fig. 12.2*b* or *c*.

For generating an emf in a coil what is needed is the relative motion between the coil and magnetic field. Thus, instead of rotating the coils in a fixed magnetic field, we can do the reverse and still get the same result. Figure 12.3*b* shows such an arrangement. Here, three coils RR′, YY′ and BB′ are wound on a stator,

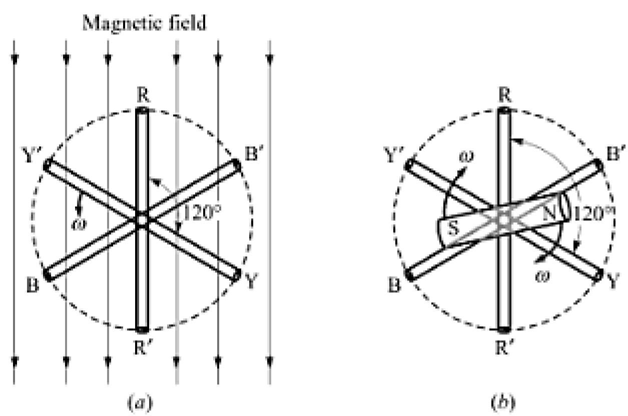


Fig. 12.3 *Principle of generation of three-phase voltage system.*

space-displaced by 120°. The magnetic field is provided by the rotor. The rotor (and hence the magnetic field) is made to rotate *clockwise* at a uniform angular speed ω. The effect would be the same as if the three coils were rotated counterclockwise at a uniform angular speed ω in a stationary magnetic field.

In practice, the scheme of Fig. 12.3*b* is preferred over that of Fig. 12.3*a*. It is much safer and easier to make external connections to stationary coils rather than to rotating ones.

Note that each coil has two terminals—start and finish. The finish (or return) terminal of a coil is diametrically opposite to its start. Thus, the terminals on the periphery (Fig. 12.3*b*) appear in the order: R, B′, Y, R′, B, Y′. We maintain same sequence of coil terminals in three-phase generators and motors.

For convenience, the three phases of Fig. 12.3 can be represented as in Fig. 12.4. Here, the phases are shown isolated from one another. We shall assume an emf to be positive when acting from 'finish' to 'start' of the phase winding. Thus, the three emfs can be represented by arrows e_R, e_Y and e_B in Fig. 12.4. These three emfs are shown connected to three respective loads L_1, L_2 and L_3. This arrangement necessitates the use of six line conductors. Obviously, it is cumbersome and expensive. Let us now consider how it may be simplified.

Depending on the interconnection of windings, there are *two* kinds of three-phase systems: (*i*) Star or wye (Y) connection, and (*ii*) Delta (Δ) or mesh connection.

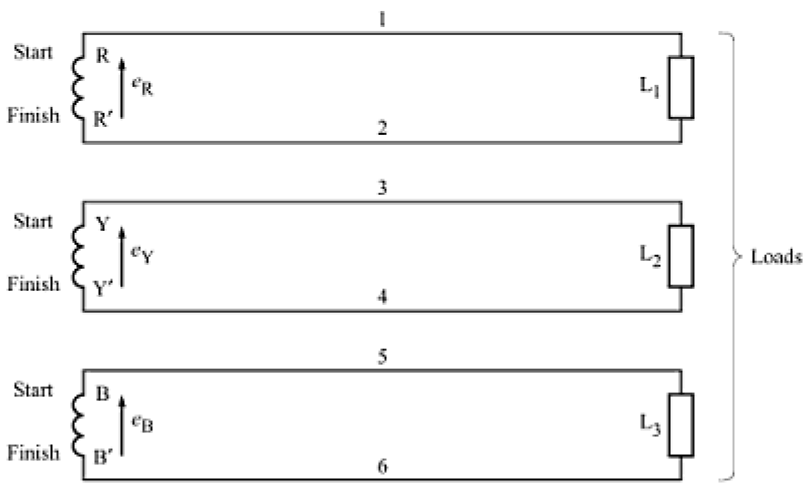


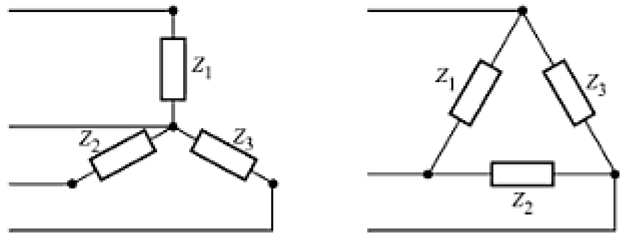
Fig. 12.4 *Three windings connected to three loads using six line conductors.*

12.5 THREE-PHASE LOADS

The three-phase voltage system can be used in the following manner at the consumer end:

(*a*) Three phases can be used independent of each other supplying power to different loads. Each load gets power from a single phase. This system is adopted for domestic power supply.

(*b*) The loads are connected to form a three-phase system. The loads (i.e., impedances Z_1, Z_2 and Z_3) can then be either star-connected or delta-connected, as shown in Fig. 12.5. Such a load is called a three-phase load.



(*a*) Star (Y) connected. (*b*) Delta (Δ) connected.

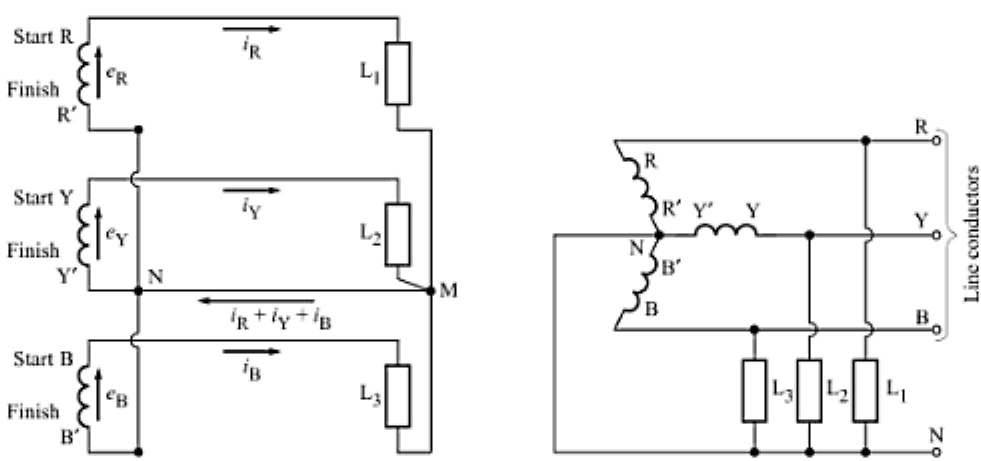
Fig. 12.5 *Three-phase loads.*

In a three-phase load, if all the three impedances are equal (both in resistive as well as in reactive parts), the load is said to be a *balanced load.* An example of such a balanced three-phase load is a three-phase motor, which has three identical windings. To such a balanced load, if a balanced three-phase supply is applied, the currents will also be balanced. Conversely, if it is carrying balanced currents, the voltages across the circuit will also be balanced.

12.6 STAR (Y) CONNECTED THREE-PHASE SYSTEM

Let us go back to Fig. 12.4 and join together the three 'finishes', R′, Y′ and B′ at N, as shown in Fig. 12.6*a*. This way, the three conductors 2, 4 and 6 of Fig. 12.4 are replaced by a single conductor NM of Fig. 12.6*a*.

Since the emf has been assumed positive when acting from 'finish' to 'start', the current in each phase must also be taken as positive flowing in that direction, as shown in Fig. 12.6*a*. Let i_R, i_Y and i_B be the instantaneous values of the three phase currents. Then the current in the common wire MN is ($i_R + i_Y + i_B$), having positive direction from M to N.



(*a*) The three-phase windings connected in star.

(*b*) Conventional representation of star-connected system.

Fig. 12.6 *Star-connected four-wire three-phase voltage system.*

This arrangement is referred to as a *four-wire star-connected* voltage system. It is more conveniently represented as in Fig. 12.6*b*. In appearance, the three windings look like a star or 'Y'. Hence, it gets its name. The junction N is referred to as the ***star*** or ***neutral point***. Three-phase motors are connected to the line conductors R, Y and B. However, simple gadgets such as lamps, heaters, microwave ovens, etc. are connected between the line and neutral conductors, as indicated by L_1, L_2 and L_3. In such cases, the total load is distributed as equally as possible between the three lines. If the three loads are exactly equal, the phase currents have the same peak value, I_m, and differ in phase by 120°. Hence, if the instantaneous current in L_1 is represented by

$$i_R = I_m \sin \omega t$$

The instantaneous currents in L_2 and L_3, respectively, are

$$i_Y = I_m \sin(\omega t - 120°) \qquad \text{and} \qquad i_B = I_m \sin(\omega t - 240°)$$

Hence, the instantaneous value of current in neutral conductor MN is

$$i_R + i_Y + i_B = I_m\{\sin \omega t + \sin(\omega t - 120°) + \sin(\omega t - 240°)\} = I_m \times 0 = 0$$

Thus, with a *balanced load* (such as a three-phase motor), the resultant current in the neutral* conductor is zero at *every* instant. Hence, this conductor can be removed altogether from the system, thereby giving us the *three-wire star-connected* system as shown in Fig. 12.7.

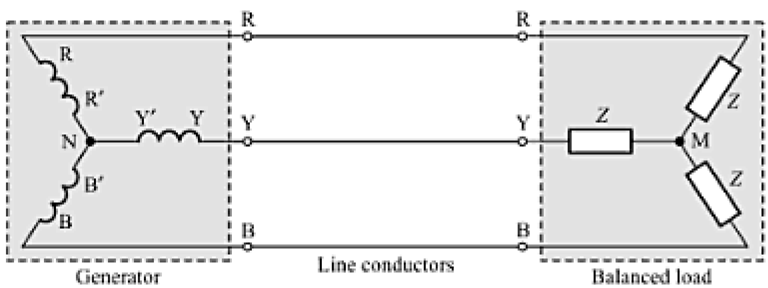


Fig. 12.7 *Three-wire star-connected voltage system with balance load.*

Note that in the three-wire star-connected system with a balanced load, the distribution of currents in the three lines continually changes, but at every instant the algebraic sum of the three currents remains zero.

Unbalanced Three-Phase System

We prefer using a three-phase system to supply power both to the domestic consumers as well as to the industrial consumers, as shown in Fig. 12.8.

The industrial load (such as a 3-ϕ motor) may be connected in star or in delta. It normally constitutes a balanced load.

In a residential colony, one set of houses, constituting load L_1, is provided electric supply from phase *R* and common point (or *local neutral point*) M; the second set of houses, constituting load L_2, from phase Y and common point M; and the third set of houses, constituting load L_3, from phase B and common point M. Although an effort is made to make this star-connected domestic load balanced, but practically it may not be possible to do so all the time.

* The term '*neutral*' is used for this conductor as it carries no current.

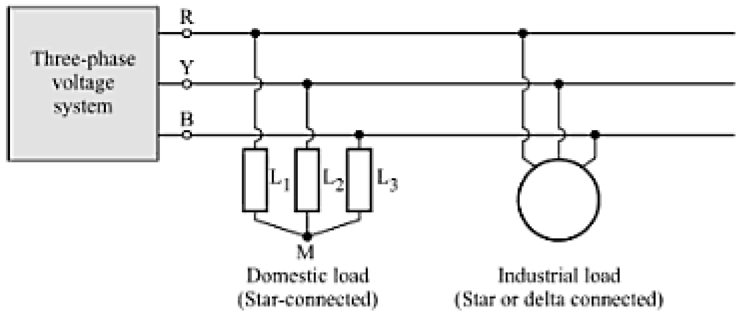


Fig. 12.8 *Three-phase system supplying power to domestic as well as industrial loads.*

If an unbalanced star-connected load is connected to a 3-wire 3-ϕ system (shown in Fig. 12.8), the currents in the three phases of the load would be different. However, as per KCL, the sum of the three line currents must still be zero. This can happen only if the voltage drops across different load branches are different. In fact, the voltages across the three loads get adjusted such that the sum of the line currents becomes zero. This results in a situation known as ***neutral shift*** or ***floating neutral***.

The potential of the neutral point does not remain zero but depends on the load impedances. Under such a situation, it is quite possible that one phase voltage becomes much larger than the other phase voltages. The equipment connected across this phase may get damaged due to overvoltage. On the other hand, the equipment connected to a phase whose voltage is too low may not operate properly. This situation is definitely undesirable. In view of this, unbalanced star connected loads are **not** normally used on a 3-wire 3-phase system.

***Four-Wire Three-Phase Voltage System*:** This system (Fig. 12.6) is very flexible. It is widely used in distribution of electric power to domestic, industrial and other consumers. The domestic and other single-phase loads are connected between a phase and neutral. Three-phase loads and other loads needing higher voltages are connected between the lines. In such cases, the currents in the three lines may not be equal; their phasor sum would return through the neutral conductor.

12.7 DELTA (Δ) CONNECTED THREE-PHASE SYSTEM

Let us again go back to Fig. 12.4. This time, we join R′ and Y thereby replacing conductors 2 and 3 by a single conductor. Similarly, we join Y′ and B together replacing conductors 4 and 5 by another single conductor. The result is shown in Fig. 12.9*a*.

Before we can proceed to join 'finish' B′ to 'start' R, we have to prove that the resultant emf between these two points is zero at every instant. It will then be ensured that no circulating current is set up when these two conductors are connected together. The instantaneous value of the total emf from B′ to R is

$$
\begin{aligned}
e_R + e_Y + e_B &= E_m[\sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°)] \\
&= E_m[\sin\theta + \{\sin\theta \cdot \cos 120° - \cos\theta \cdot \sin 120°\} + \{\sin\theta \cdot \cos 240° - \cos\theta \cdot \sin 240°\}] \\
&= E_m[\sin\theta - 0.5\sin\theta - 0.866\cos\theta - 0.5\sin\theta + 0.866\cos\theta] \\
&= 0
\end{aligned}
$$

Since this condition holds for every instant, we can join R and B′ together, as shown in Fig. 12.9*b*, without creating any circulating current in the circuit. The three line conductors are joined to the junctions thus formed.

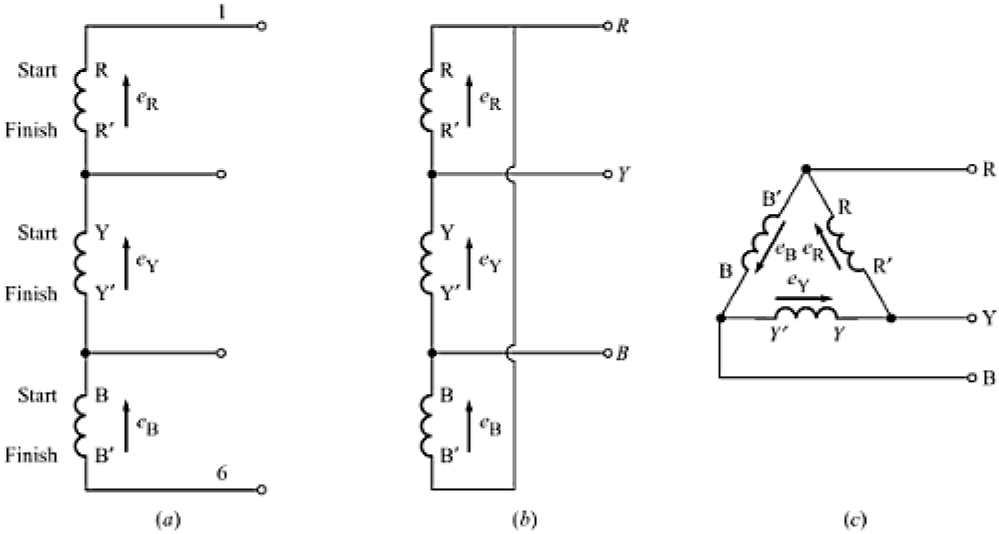


Fig. 12.9 *Connecting the three phase windings of Fig.* 12.4 *to make delta* (Δ) *connected three-phase system.*

The circuit derived in Fig. 12.9*b* is conventionally drawn as in Fig. 12.9*c*. In appearance, it looks like Greek letter delta (Δ). Hence, this connection is given this name. Note that in Fig. 12.9*c*, the 'finish' of one phase is connected to the 'start' of another phase. This ensures that the arrows representing the positive directions of the emfs point in the same direction round the mesh formed by the windings.

12.8 VOLTAGES AND CURRENTS RELATIONS IN 3-ϕ SYSTEMS

In a three-phase system, there are two sets of voltages: (*i*) line voltages, and (*ii*) phase voltages. Similarly, there are two sets of currents: (*i*) line currents, and (*ii*) phase currents. We shall now determine the relations between these two sets of voltage and two sets of currents in both the star-connected system as well as delta-connected system.

(1) Star-Connected System

Let us again assume the emf in each phase to be positive when acting from the neutral point outwards, as shown in Fig. 12.10. The rms values of the emfs generated in the three phases are E_{RN}, E_{YN} and E_{BN}. In practice, it is the voltage between two lines or between a line conductor and the neutral point that is measured. Due to the impedance voltage-drop in the windings, this potential difference (pd) is different from the corresponding emf generated in the winding, except when the generator is on open circuit. Hence, in general it is preferable to work with the potential difference, V, rather than the emf, E.

In a three-phase system, there are two sets of voltages we are interested in. One is the set of *phase voltages*, and the other is the set of *line voltages*. In Fig. 12.10, $\mathbf{V}_{RN}$ is the rms value of the voltage drop from R to N. That is, this is the phase voltage of *phase R*. Thus, $\mathbf{V}_{RN}$, $\mathbf{V}_{YN}$ and $\mathbf{V}_{BN}$ denote the set of three phase voltages.

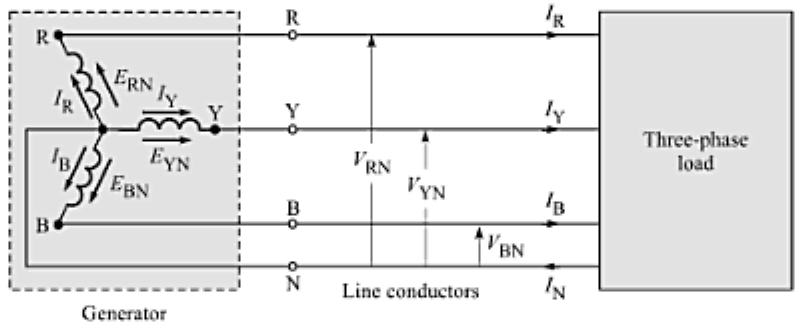


Fig. 12.10 *A star-connected generator supplying power to a three-phase load.*

The term 'line voltage' is used to denote the voltage between two lines. Thus, $\mathbf{V}_{RY}$ represents line voltage between the lines R and Y. The other line voltages are $\mathbf{V}_{YB}$ and $\mathbf{V}_{BR}$.

To determine the relation between phase voltages and line voltages, we analyse the phasor diagram, shown in Fig. 12.11*a*. Note that a phasor diagram by itself is meaningless. It is essential to relate the quantities in the phasor diagram to a circuit diagram and to indicate the directions in which the voltage and current is assumed to be positive. The phasor diagram of Fig. 12.11*a* is same as that drawn in Fig. 12.2*b*, except two differences. First, it is now drawn in terms of effective (or rms) values rather than peak values. Secondly, it shows voltages (which can be measured) rather than the emfs generated in the windings of the generator. In Fig. 12.11*a*, $\mathbf{V}_{RN}$ represents rms value of the voltage of phase R line with respect to the neutral line N.

By applying Kirchhoff's voltage law, we can get the magnitude and phase angle of the line voltage $\mathbf{V}_{RY}$ (which is the voltage drop from R via N to Y, and can be represented by an unambiguous symbol $\mathbf{V}_{RNY}$):

$$\mathbf{V}_{RY} = \mathbf{V}_{RNY} = \mathbf{V}_{RN} + \mathbf{V}_{NY}$$

This equation simply states that the voltage drop existing from *R* to *Y* is equal to the voltage drop from *R* to *N* plus the voltage drop from *N* to *Y*. The above equation can be written as

$$\mathbf{V}_{RY} = \mathbf{V}_{RNY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = \mathbf{V}_{RN} + (-\mathbf{V}_{YN}) \tag{12.5}$$

This shows that to determine $\mathbf{V}_{RY}$, first we reverse the phasor $\mathbf{V}_{YN}$ to get $-\mathbf{V}_{YN}$ and then add the phasors $\mathbf{V}_{RN}$ and $-\mathbf{V}_{YN}$, as shown in Fig. 12.11*a*.

Analytical Analysis In a balanced system, each phase voltage has the same magnitude. So, we can write

$$|\mathbf{V}_{RN}| = |\mathbf{V}_{YN}| = |\mathbf{V}_{BN}| = V_{ph} \text{ (say)}$$

The three phasors representing the set of phase voltages can be written as

$$\mathbf{V}_{RN} = V_{ph}\angle 0°; \quad \mathbf{V}_{YN} = V_{ph}\angle -120°; \quad \mathbf{V}_{BN} = V_{ph}\angle -240° = V_{ph}\angle 120°$$

Equation 12.5 can then be written as

$$\begin{aligned}\mathbf{V}_{RY} &= \mathbf{V}_{RN} - \mathbf{V}_{YN} = V_{ph}\angle 0° - V_{ph}\angle -120° = V_{ph} - V_{ph}(\cos 120° - j\sin 120°)\\ &= V_{ph} - V_{ph}(-1/2 - j\sqrt{3}/2) = V_{ph}(3/2 + j\sqrt{3}/2)\end{aligned}$$

Thus, the magnitude of $\mathbf{V}_{RY}$ is given as

$$V_{RY} = V_{ph}\sqrt{(9/4 + 3/4)} = \sqrt{3}V_{ph} \tag{12.6}$$

And the phase angle of $\mathbf{V}_{RY}$ with respect to the reference phasor $\mathbf{V}_{RN}$ is given as

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{3/2}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30° \tag{12.7}$$

Hence the phasor $\mathbf{V}_{RY}$ can be written as

$$\mathbf{V}_{RY} = \sqrt{3}V_{ph} \angle 30° \tag{12.8}$$

Similarly, we can get

$$\mathbf{V}_{YB} = \sqrt{3}V_{ph} \angle -90° \qquad \text{and} \qquad \mathbf{V}_{BR} = \sqrt{3}V_{ph} \angle 150° \tag{12.9}$$

Thus, we can say that the magnitude of the line voltage V_L for star connection is given as

$$\boxed{V_L = \sqrt{3}V_{ph}} \tag{12.10}$$

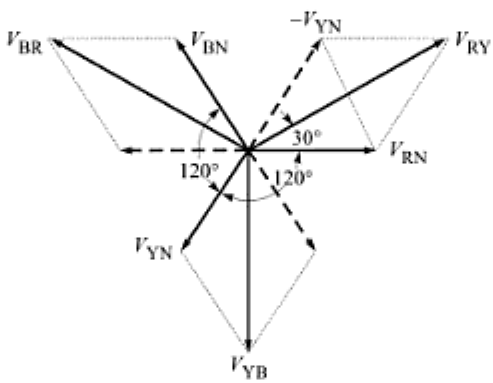
Geometrical Analysis Because of the symmetry in Fig. 12.11*a*, it is evident that the line voltages are equal and are spaced 120° apart. Further, since sides of all the parallelograms are of equal length, the diagonals bisect one another at right angles. Also, they bisect the angle of their respective parallelograms. Since the angle between $\mathbf{V}_{RN}$ and $-\mathbf{V}_{YN}$ is 60°, we have

$$V_{RY} = 2(V_{RN} \cos 30°) \quad \text{or} \quad V_L = 2V_{ph}(\sqrt{3}/2) = \sqrt{3}V_{ph}$$

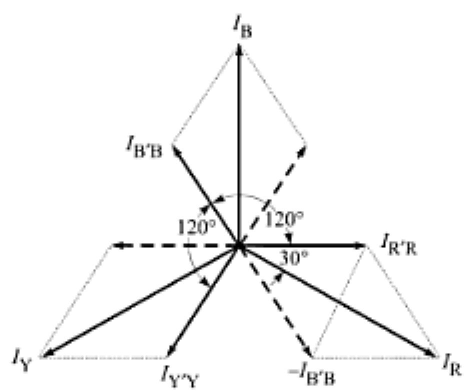
which is same as Eq. 12.10.

It is obvious that any current that flows out of the line terminal *R* must be the same as that which flows due to the phase source voltage appearing between terminals *R* and *N*. Therefore, for star-connection, we have

$$\boxed{I_L = I_{ph}} \tag{12.11}$$



(*a*) Star-connected system.



(*b*) Delta-connected system.

Fig. 12.11 *Phasor diagrams.*

(2) Delta-Connected System

Let $I_{R'R}$, $I_{Y'Y}$, $I_{B'B}$, be the rms values of the phase currents in the three windings of the generator. Their assumed positive directions are indicated by arrows in Fig. 12.12. Since the load is assumed balanced, these

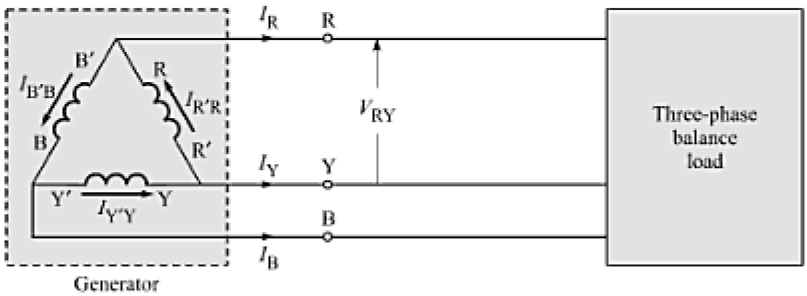


Fig. 12.12 *A star-connected generator supplying power to a 3-Φ load.*

currents are equal in magnitude and differ in phase by 120°, as shown in the phasor diagram of Fig. 12.11*b*. Therefore, we can write

$$|\mathbf{I}_{R'R}| = |\mathbf{I}_{Y'Y}| = |\mathbf{I}_{B'B}| = I_{ph} \text{ (say)}$$

The three phasors representing the set of phase currents can be written as

$$\mathbf{I}_{R'R} = I_{ph}\angle 0°; \quad \mathbf{I}_{Y'Y} = I_{ph}\angle -120°; \quad \mathbf{I}_{B'B} = I_{ph}\angle -240° = I_{ph}\angle 120°$$

From Fig. 12.12, it can be seen that the phase current $I_{R'R}$ flows towards the line conductor R, whereas the phase current $I_{B'B}$ flows away from it. Applying KCL at the terminal R, we can write

$$\mathbf{I}_R = \mathbf{I}_{R'R} - \mathbf{I}_{B'B}$$

Above vector addition of $\mathbf{I}_{R'R}$ and $-\mathbf{I}_{B'B}$ is shown in the phasor diagram of Fig. 12.11*b*. From the symmetrical geometry of the diagram, it is evident that the line currents are equal in magnitude and differ in phase by 120°. Also

$$I_R = 2(I_{ph}\cos 30°) = \sqrt{3}\, I_{ph}$$

Hence, for a delta-connected system with balanced load, the magnitudes of line current and of phase current are related as

$$\boxed{I_L = \sqrt{3}\, I_{ph}} \tag{12.12}$$

From Fig. 12.12, it is obvious that the line voltage V_{RY} is same as the phase voltage $V_{RR'}$. Hence, for a delta-connected system, we have

$$\boxed{V_L = V_{ph}} \tag{12.13}$$

Important Points about Three-Phase Systems

Following important points should be noted while dealing with three-phase systems:

(*i*) For a three-phase system, unless otherwise mentioned, it is normal practice to specify the values of the line voltages and line currents. Thus, when we say that a 3-phase, 11-kV circuit is carrying a current of 500 A, it implies that V_L = 11 kV and I_L = 500 A. One can calculate the phase voltage and phase current using Eqs. 12.10 and 12.11 for star-connection, and Eqs. 12.12 and 12.13 for Δ-connection.

(*ii*) The current in any phase can be determined by dividing the phase voltage by its impedance. That is, $I_{ph} = V_{ph}/Z$. The power factor of Z is the same as the cosine of the phase difference between V_{ph} and I_{ph}.

EXAMPLE 12.1

A 400-V, 3-ϕ supply is connected across a balanced load of three impedances each consisting of a 32-Ω resistance and 24-Ω inductive reactance. Determine the current drawn from the power mains, if the three impedances are (*a*) Y-connected, and (*b*) Δ-connected.

Solution Each impedance, $\mathbf{Z} = R + jX = (32 + j24)\ \Omega$.

$$\therefore \qquad Z = \sqrt{R^2 + X^2} = \sqrt{32^2 + 24^2} = 40\ \Omega$$

(*a*) Y-*connection*: $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}\ \text{V} \Rightarrow I_{ph} = \dfrac{V_{ph}}{Z} = \dfrac{400/\sqrt{3}}{40} = \dfrac{10}{\sqrt{3}}\ \text{A}$

$$\therefore \qquad I_L = I_{ph} = \frac{10}{\sqrt{3}} = \mathbf{5.78\ A}$$

(*b*) For Δ-*connection*: $V_{ph} = V_L = 400\ \text{V} \Rightarrow I_{ph} = \dfrac{V_{ph}}{Z} = \dfrac{400}{40} = 10\ \text{A}$

$$\therefore \qquad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 10 = \mathbf{17.32\ A}$$

EXAMPLE 12.2

In a four-wire, three-phase system, the line voltage is 415 V. Non-inductive loads of 10 kW, 8 kW and 5 kW are connected between the three line conductors and the neutral as shown in Fig. 12.13. Calculate (*a*) the current in each line, and (*b*) the current in the neutral conductor.

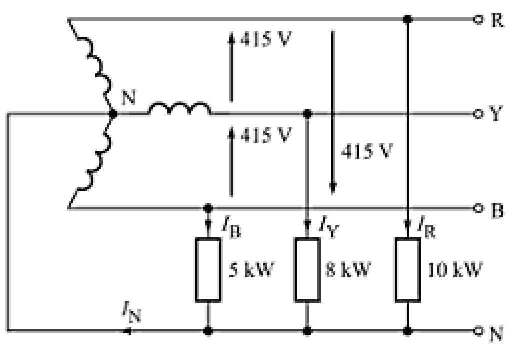


Fig. 12.13 *Circuit diagram for Example* 12.2.

Solution

(*a*) Here, the load is connected in star, for which the phase voltage is given as

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240\ \text{V}$$

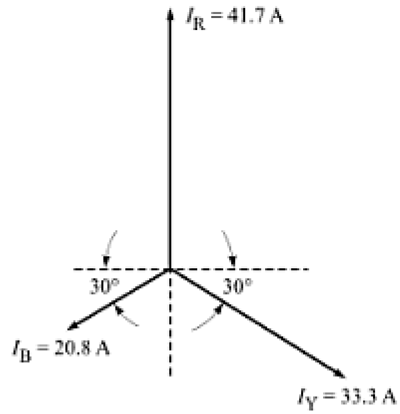
The currents taken by the 10-kW, 8-kW and 5-kW loads, respectively, are

$$I_R = 10 \times 1000/240 = \mathbf{41.67\ A}$$

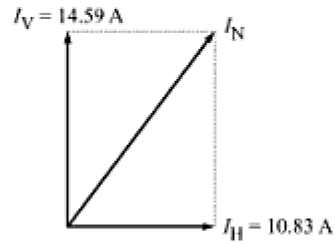
$$I_Y = 8 \times 1000/240 = \mathbf{33.33\ A}$$

and $$I_B = 5 \times 1000/240 = \mathbf{20.83\ A}$$

These currents are represented by respective phasors in Fig. 12.14*a*.



(*a*) Phasor diagram of line currents.



(*b*) Horizontal and vertical components of current I_N.

Fig. 12.14 *Calculation of current in the neutral conductor.*

(*b*) The current in the neutral conductor is the phasor sum of the three line currents. A simple method of adding such quantities is to calculate the resultant horizontal and vertical components. These components are

$$I_H = I_Y \cos 30° - I_B \cos 30° = 0.866(33.33 - 20.83) = 10.83 \text{ A}$$

and
$$I_V = I_R - I_Y \sin 30° - I_B \sin 30° = 41.67 - 0.5(33.33 + 20.83) = 14.59 \text{ A}$$

These components are represented in Fig. 12.14*b*. The current in the neutral conductor is given as

$$I_N = \sqrt{(10.83)^2 + (14.59)^2} = \mathbf{18.2 \text{ A}}$$

EXAMPLE 12.3

A three-phase, 50-Hz, 415-V supply is connected to a delta-connected load comprising a resistance of 100 Ω, a resistance of 20 Ω in series with an inductance of 191 mH, and a capacitance of 30 μF, as shown in Fig. 12.15. Calculate (*a*) the phase currents and (*b*) line currents.

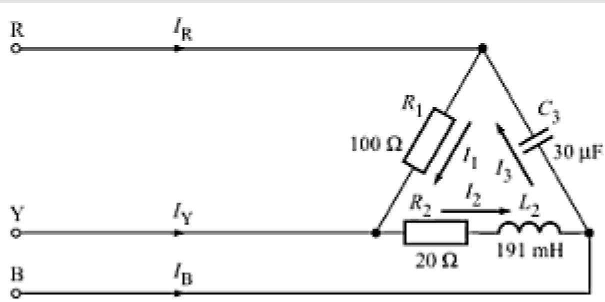


Fig. 12.15 *Circuit diagram for Example* 12.3.

Solution

(*a*) The three impedances in the delta-connected load are

$$Z_1 = R_1 = 100 \text{ Ω}; \quad \phi_1 = 0°$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(20)^2 + (2\pi f L_2)^2} = \sqrt{(20)^2 + (2\pi \times 50 \times 0.191)^2} = 63.25\ \Omega$$

$$\phi_2 = \tan^{-1}\frac{60}{20} = 71°56' \text{ (lagging)}$$

$$Z_3 = 1/(2\pi f C_3) = 1/(2\pi \times 50 \times 30 \times 10^{-6}) = 106.1\ \Omega$$

$$\phi_3 = 90° \text{ (leading)}$$

Since the phase sequence is R Y B, V_{RY} leads V_{YB} by 120° and V_{YB} leads V_{BR} by 120°, as shown in Fig. 12.16*a*. The phase current I_1, I_2 and I_3 in loads RY, YB and BR, respectively are

$$I_1 = \frac{415}{Z_1} = \frac{415}{100} = \mathbf{4.15\ A}, \text{ in phase with } V_{RY}.$$

$$I_2 = \frac{415}{Z_2} = \frac{415}{63.25} = \mathbf{6.56\ A}, \text{ lagging } V_{YB} \text{ by } \phi_2 (= 71°34')$$

$$I_3 = \frac{415}{Z_3} = \frac{415}{106.1} = \mathbf{3.91\ A}, \text{ leading } V_{BR} \text{ by } 90°.$$

(*b*) Figure 12.15 shows assumed positive directions of currents. Using KCL, the current in the line conductor *R* is given as

$$\mathbf{I}_R = \mathbf{I}_1 - \mathbf{I}_3$$

The process of finding the above phasor difference is shown in Fig. 12.16*a*. It is obvious that the angle between I_1 and I_3 (reversed) is 30°. Hence, the line current I_R is given as

$$I_R = \sqrt{(4.15)^2 + (3.91)^2 + 2 \times 4.15 \times 3.91 \times \cos 30°} = \sqrt{60.53} = \mathbf{7.78\ A}$$

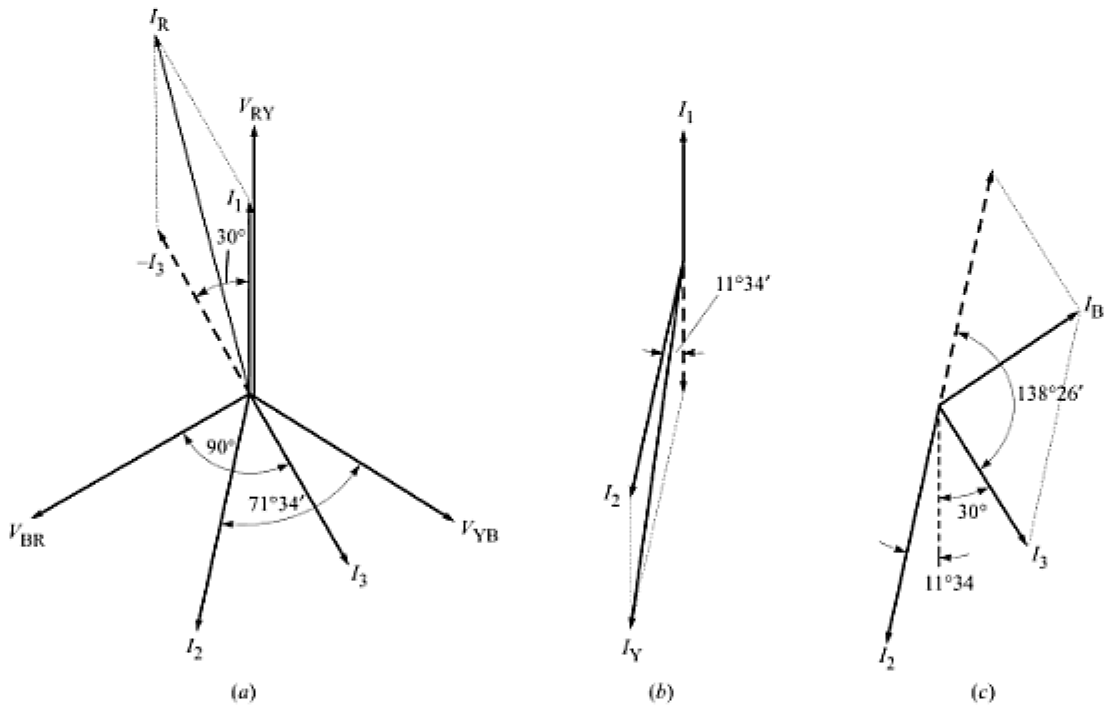


Fig. 12.16 *Phasor diagrams.*

The line current I_Y is found by determining the phasor difference $\mathbf{I}_2 - \mathbf{I}_1$ (see Fig. 12.16*b*). The angle between I_2 and I_1 (reversed) is

$$\theta_Y = \phi_2 - 60° = 71°34' - 60° = 11°34'$$

$$\therefore \quad I_Y = \sqrt{(4.15)^2 + (6.56)^2 + 2 \times 4.15 \times 6.56 \times \cos 11°34'} = \mathbf{10.66\ A}$$

Similarly, the line current I_B is obtained by subtracting I_2 from I_3, as shown in Fig. 12.16*c*. The angle between I_3 and I_2 (reversed) is given as

$$\theta_B = 180° - 11°34' - 30° = 138°26'$$

$$\therefore \quad I_B = \sqrt{(6.56)^2 + (3.91)^2 + 2 \times 6.56 \times 3.91 \times \cos 138°26'} = \mathbf{4.47\ A}$$

12.9 POWER IN THREE-PHASE SYSTEM WITH A BALANCED LOAD

Consider one phase only. For this load, the voltage is V_{ph} and the current is I_{ph}. The average active power consumed by this load is given by

$$P_1 = V_{ph} I_{ph} \cos \phi$$

where ϕ is the phase angle of the load.

As the load is balanced, the power in other two phase circuits will also be the same. Hence, the total power consumed is

$$P = 3P_1 = 3V_{ph} I_{ph} \cos \phi \tag{12.14}$$

Above expression for the total power is in terms of phase voltage and phase current. However, it is a normal practice to mention line voltage and line current in a three-phase system. Hence, let us determine the expression for the total power in terms of V_L and I_L.

For a *star-connected system*, we have $V_L = \sqrt{3} V_{ph}$ and $I_L = I_{ph}$. Hence,

$$P = 3(V_L / \sqrt{3}) I_L \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

For a *delta-connected system*, we have $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$. Hence,

$$P = 3V_L (I_L / \sqrt{3}) \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

Thus, it follows that, for *any balanced load* (connected in either Y or Δ), the total power is given as

$$\boxed{P = \sqrt{3} V_L I_L \cos \phi} \tag{12.15}$$

While using above equation, it is important to note that ϕ is the angle of the load impedance per phase and not the angle between V_L and I_L.

Figure 12.17 shows the variation of the average power per phase as well as the total instantaneous power in the three-phase load with time. The total power is three times the average power per phase. It can be seen that in a single-phase system, the power pulsates at twice the line frequency. But, in a three-phase system with balanced load (such as a three-phase motor), there is no variation of power at all. This is the reason why for driving heavy mechanical loads we prefer a three-phase motor rather than a single-phase motor.

EXAMPLE 12.4

A 400-V, 3-ϕ supply is connected to a balanced network of three impedances each consisting of a 20-Ω resistance and a 15-Ω inductive reactance. Determine (*i*) the line current, (*ii*) the power factor, and (*iii*) the total power in kW, when the three impedances are (*a*) star-connected, and (*b*) delta-connected.

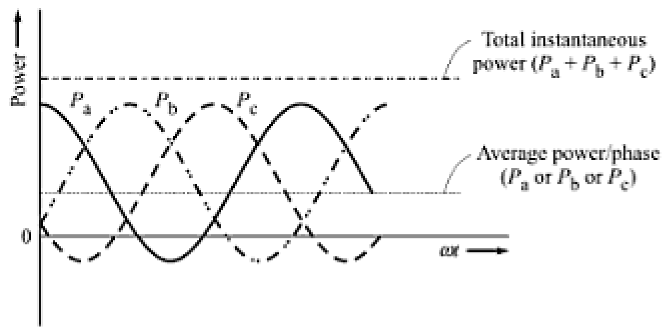


Fig. 12.17 *Total instantaneous power in a three-phase system.*

Solution

(*a*) *For star-connected load*: $V_L = \sqrt{3}V_{ph}$ and $I_L = I_{ph}$. Therefore,

$$\therefore \quad V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

The impedance per phase, $Z_{ph} = \sqrt{20^2 + 15^2} = 25\ \Omega$

(*i*) The line current is given as

$$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = \mathbf{9.24\ A}$$

(*ii*) The power factor is given as

$$\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{20}{25} = \mathbf{0.8\ (lagging)}$$

(*iii*) The total active power is given as

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 = \mathbf{5.12\ kW}$$

(*b*) *For delta-connected load*: $V_L = V_{ph}$ and $I_L = \sqrt{3}\,I_{ph}$

$$\therefore \quad V_L = V_{ph} = 400 \text{ V}$$

(*i*) The phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{25} = 16 \text{ A}$

$$\therefore \quad I_L = \sqrt{3}\,I_{ph} = \sqrt{3} \times 16 = \mathbf{27.71\ A}$$

(*ii*) The power factor is same as above, *pf* = **0.8 (lagging)**

(*iii*) The total active power is given as

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 400 \times 27.71 \times 0.8 = \mathbf{15.36\ kW}$$

12.10 COMPARISON BETWEEN TWO THREE-PHASE SYSTEMS

Though 2-ϕ, 6-ϕ, or 12-ϕ systems are sometimes used, but the 3-ϕ systems are most popular for distribution of electrical power. Table 12.1 provides a comparison between the two 3-ϕ systems.

Table 12.1 *Comparison between star- and delta-connected systems.*

S. No.	*Star-Connected System*	*Delta-Connected System*
1.	Similar ends are joined together.	Dissimilar ends are joined.
2.	$V_L = \sqrt{3}\,V_{ph}$ and $I_L = I_{ph}$	$V_L = V_{ph}$ and $I_L = \sqrt{3}\,I_{ph}$
3.	Neutral wire is available.	Neutral wire is not available.
4.	4-wire, 3-ϕ system is possible.	4-wire, 3-ϕ system is not possible.
5.	Both domestic and industrial loads can be handled.	Only industrial loads can be handled.
6.	By earthing the neutral wire, relays and protective devices can be provided in alternators for safety.	Due to absence of neutral wire, it is not possible.

12.11 MEASUREMENT OF POWER

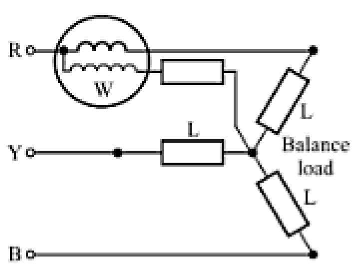
The method of measurement of total power in three-phase depends upon the type of system and that of the load. There exists the following methods.

(*i*) *Three-Wattmeter Method* This is the simplest and straight forward method. One wattmeter is inserted in each of the phases. The power consumed by the load is the algebraic sum of the three wattmeter-readings.

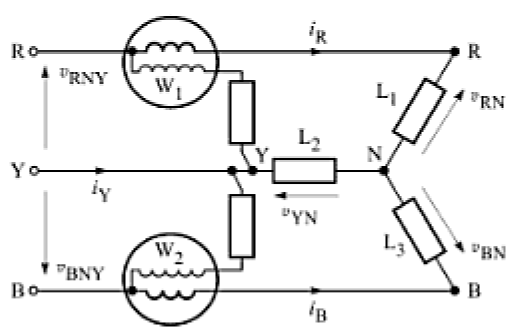
(*ii*) *One-Wattmeter Method* This can be used to determine the total power consumed by a star-connected balanced load, with neutral point accessible. The current coil of the wattmeter is connected in one line and the potential coil is connected between that line and the neutral point, as shown in Fig. 12.18*a*. The reading of the wattmeter gives the power per phase. Therefore,

$$\text{Total power} = 3 \times \text{wattmeter reading}$$

(*iii*) *Two-Wattmeter Method* This can be used for any balanced or unbalanced load, star- or delta-connected. Details of this method are explained below.



(*a*) Star-connected balanced load.



(*b*) Star-connected unbalanced load.

Fig. 12.18 *Measurement of power in 3-ϕ load.*

Power Measurement by Two-Wattmeter Method

Suppose that the three loads L_1, L_2 and L_3 are connected in star, as shown in Fig. 12.18*b*. The current coils (CC) of the two wattmeters W_1 and W_2 are connected in any two lines, say, the 'red' and 'blue' lines. The potential coils (PC) of the wattmeters are connected between these lines and the third line. The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases.

Proof Let v_{RN}, v_{YN} and v_{BN} be the instantaneous values of the voltages across the loads, with the positive direction marked by arrows in the diagram. Let i_R, i_Y and i_B be the corresponding instantaneous values of the line (and phase) currents.

$\therefore$ $$\text{Total instantaneous power} = i_R v_{RN} + i_Y v_{YN} + i_B v_{BN}.$$

Since the current through the current coil of W_1 is i_R, and the potential across its potential coil is $v_{RN} - v_{YN}$, we have

$$\text{the instantaneous power measured by } W_1,\ p_1 = i_R(v_{RN} - v_{YN})$$

Similarly, $$\text{the instantaneous power measured by } W_2,\ p_2 = i_B(v_{BN} - v_{YN})$$

Hence, the sum of the instantaneous powers of W_1 and W_2 is

$$p_1 + p_2 = i_R(v_{RN} - v_{YN}) + i_B(v_{BN} - v_{YN}) = i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN}$$

From KCL, the algebraic sum of the instantaneous currents at *N* is zero, i.e.,

$$i_R + i_Y + i_B = 0 \quad \Rightarrow \quad (i_R + i_B) = -i_Y$$

$\therefore$ $$p_1 + p_2 = i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN} = i_R v_{RN} + i_B v_{BN} + i_Y v_{YN}$$
$$= \text{total instantaneous power}$$

Actually, the power measured by each wattmeter varies from instant to instant, but due to the inertia of the moving system the pointer stays at the average value of the power.

Since the above proof does not assume a balanced load or a sinusoidal waveform, it follows that *the sum of the two wattmeter readings gives the total power under all conditions.* The above proof was derived for a star-connected load. One could derive the same conclusion for a delta-connected load.

Power Factor Measurement by Two-Wattmeter Method

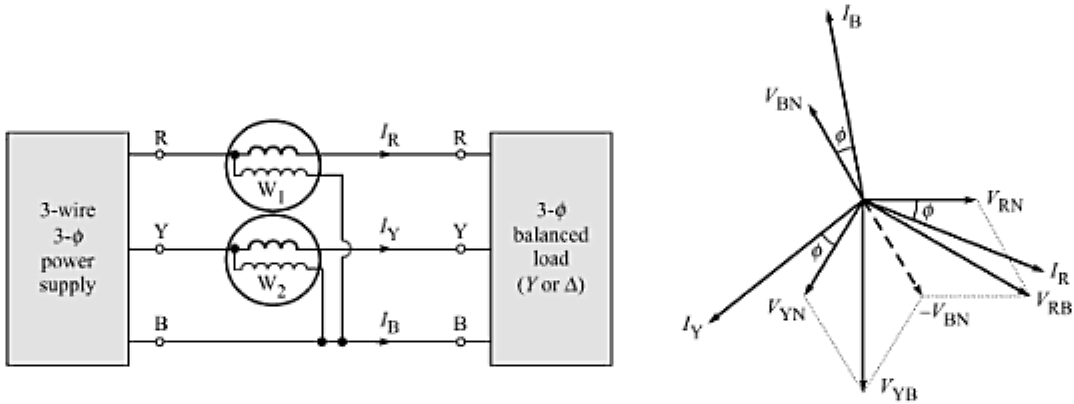
Consider a balanced three-phase inductive load at a power factor $\cos\phi$ (lagging), connected to a 3-wire, 3-ϕ system, as shown in Fig. 12.19*a*. The phase sequence is R Y B. The current coils of the two wattmeters W_1 and W_2 are connected in the line conductors R and Y, respectively. Their potential coils are connected between the corresponding line conductor and the third line conductor B.

Let I_R, I_Y and I_B be the three line currents, and V_{RN}, V_{YN} and V_{BN} be the three phase-voltages*. Since the load is balanced, the three line currents and the three line voltages will have same magnitude, i.e.,

$$I_R = I_Y = I_B = I_L \text{ (say)} \qquad \text{and} \qquad V_{RN} = V_{YN} = V_{BN} = V_{ph} \text{ (say)}$$

Each line current lags by angle ϕ its corresponding voltage as shown in the phasor diagram of Fig. 12.19*b*. Since $\mathbf{V}_{RB} = \mathbf{V}_{RN} - \mathbf{V}_{BN}$, and $\mathbf{V}_{YB} = \mathbf{V}_{YN} - \mathbf{V}_{BN}$, we can determine the line voltages V_{RB} and V_{YB} by phasor method. It is seen from Fig. 12.19*b* that the line voltage V_{RB} lags the phase voltage V_{RN} by 30° and V_{YB} leads V_{YN} by 30°. Thus, the phase angle between the line voltage V_{RB} and the line current I_R is $(30° - \phi)$. Similarly,

* Here, point *N* may be the common point of the loads connected in star, or it may be just an imaginary reference point at ground potential in case of delta connected loads.



(a) Connection of two wattmeters.

(b) Phasor diagram.

Fig. 12.19 *Measurement of power factor by two-wattmeter method.*

the phase angle between the line voltage V_{YB} and the line current I_Y is $(30° + \phi)$. Therefore, the readings of the two wattmeters are

$$P_1 = V_{RB} I_R \cos(30° - \phi) = V_L I_L \cos(30° - \phi) \tag{12.16}$$

and

$$P_2 = V_{YB} I_Y \cos(30° + \phi) = V_L I_L \cos(30° + \phi) \tag{12.17}$$

Dividing Eq. 12.16 by Eq. 12.17, we get

$$\frac{P_1}{P_2} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

By applying componendo and dividendo to the above, we get

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\cos(30° - \phi) - \cos(30° + \phi)}{\cos(30° - \phi) + \cos(30° + \phi)} = \frac{2\sin 30° \sin\phi}{2\cos 30° \cos\phi} = \tan 30° \tan\phi$$

or

$$\boxed{\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right]} \tag{12.18}$$

We can calculate the phase angle ϕ from the above relation, and then determine the power factor $\cos\phi$.

Let us check whether we get total power consumed by the load by adding P_1 and P_2. Using Eqs. 12.16 and 12.17, we have

$$\begin{aligned} P_1 + P_2 &= V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi) = V_L I_L [\cos(30° - \phi) + \cos(30° + \phi)] \\ &= V_L I_L [2\cos 30° \cos\phi] = V_L I_E [2(\sqrt{3}/2)\cos\phi] = \sqrt{3}\, V_L I_L \cos\phi \end{aligned}$$

The right hand side of the above equation gives the total power consumed by a balanced three-phase load, as per Eq. 12.15. Hence, we have

$$\text{Total power consumed} = P_1 + P_2 \tag{12.19}$$

This is an ***alternative method*** of proving that the sum of the two wattmeter-readings gives the total power. But, note that this proof assumed a balanced load.

The line currents are

$$\mathbf{I}_R = \mathbf{I}_{RY} - \mathbf{I}_{BR} = (13.2 - j17.6) - (8.65 + j20.2) = \mathbf{38.1\angle{-83.13°}\ A}$$
$$\mathbf{I}_Y = \mathbf{I}_{YB} - \mathbf{I}_{RY} = (-21.85 - j2.63) - (13.2 - j17.6) = \mathbf{38.1\angle 156.87°\ A}$$
$$\mathbf{I}_B = \mathbf{I}_{BR} - \mathbf{I}_{YB} = (8.65 + j20.2) - (-21.85 - j2.63) = \mathbf{38.1\angle 36.81°\ A}$$

Power consumed, $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 220 \times 38.1 \times \cos 53.13° = \mathbf{8710.85\ W}$

EXAMPLE 12.9

Three identical impedances of $12\angle 30°\ \Omega$ are connected in delta, and another set of three identical impedances of $5\angle 45°\ \Omega$ are connected in star. Both these sets are connected across a 3-ϕ, 3-wire, 400-V supply. Find the line current and the total power supplied.

Solution The delta-connected impedances can be converted into its equivalent start-connection. Each equivalent impedance is given as

$$Z_A = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \quad \text{or} \quad Z_Y = \frac{Z_\Delta}{3} = \frac{12\angle 30°}{3} = 4\angle 30°\ \Omega$$

Thus, each phase has two impedances in parallel. The equivalent load impedance in each phase is

$$\mathbf{Z}_{eq} = \frac{(5\angle 45°)(4\angle 30°)}{(5\angle 45°) + (4\angle 30°)} = 2.24\angle 36.65°\ \Omega$$

Thus, we have a single star-connected balanced three-phase load each of impedance Z_{eq}.

The line to neutral voltage, $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231\ \text{V}$

The line current, $I_L = I_{ph} = \dfrac{V_{ph}}{Z_{eq}} = \dfrac{231}{2.24} = \mathbf{103.1\ A}$

Power, $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 103.1 \times \cos 36.65° = 57308\ \text{W} \approx \mathbf{57.3\ kW}$

EXAMPLE 12.10

A balanced 3-phase, star-connected load of 150 kW takes a leading line current of 100 A from a 1100-V, 50-Hz, three-phase supply. Determine the constants of the load per phase.

Solution For the star-connected load, the total power is $P = 3I_L^2 R$. Hence the resistance of the load is given as

$$R = \frac{P}{3I_L^2} = \frac{150 \times 1000}{3 \times (100)^2} = \mathbf{5\ \Omega}$$

Since the line current is given as $I_L = I_{ph} = \dfrac{V_{ph}}{Z} = \dfrac{V_L/\sqrt{3}}{Z}$, the load impedance is

$$Z = \frac{V_L/\sqrt{3}}{I_L} = \frac{1100/\sqrt{3}}{100} = 6.35\ \Omega$$

Since the load takes a leading current, the load must be capacitive. Hence, we have

$$Z = \sqrt{R^2 + X_C^2} \quad \text{or} \quad X_C = \sqrt{Z^2 - R^2} = \sqrt{6.35^2 - 5^2} = 3.914\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 3.914} = \mathbf{813.26\ \mu F}$$

EXAMPLE 12.11

A balanced delta-connected 3-ϕ load is fed from a 3-ϕ, 400-V supply. The line current is 20 A and the total power absorbed by the load is 10 kW. Calculate (*a*) the impedance in each branch, (*b*) the power factor, and (*c*) total power consumed if the same impedances are star-connected.

Solution For delta-connection: $V_{ph} = V_L = 400$ V; $I_{ph} = I_L/\sqrt{3} = 20/\sqrt{3} = 11.55$ A

(*a*) Impedance in each branch, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{400}{11.55} = \mathbf{34.64\ \Omega}$

(*b*) Power factor $= \cos\phi = \dfrac{P}{\sqrt{3}V_L I_L} = \dfrac{10\,000}{\sqrt{3}\times 400\times 20} = \mathbf{0.7216}$

(*c*) For star-connection: $V_{ph} = V_L/\sqrt{3} = 400/\sqrt{3} = 231$ V; $I_{ph} = I_L$

$$\therefore \qquad I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{34.64} = 6.67 \text{ A}$$

Total power consumed, $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}\times 400\times 6.67\times 0.7216 = \mathbf{3.33\ kW}$

EXAMPLE 12.12

An unbalanced delta-connected load consists of $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, and a coil of $L_3 = 0.3$ H with negligible resistance. This load is fed from a three-phase, 400-V, 50-Hz source, as shown in Fig. 12.20. Calculate (*a*) the total power in the system, and (*b*) the total volt ampere reactive.

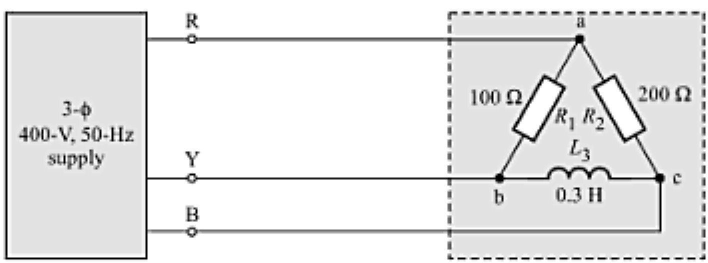


Fig. 12.20 *An unbalanced delta-connected load fed from a three-phase supply.*

Solution $X_3 = 2\pi fL = 2\pi\times 50\times 0.3 = 94.2\ \Omega$. Let the phase voltages for the load be represented as

$$\mathbf{V}_{ab} = 100\angle 0°\text{ V};\quad \mathbf{V}_{bc} = 100\angle{-120°}\text{ V};\quad \text{and}\quad \mathbf{V}_{ca} = 100\angle{-240°}\text{ V}$$

The phase impedances are

$$\mathbf{Z}_{ab} = (100 + j0)\ \Omega = 100\angle 0°\ \Omega;\ \mathbf{Z}_{bc} = (0 + j94.2)\ \Omega = 94.2\angle 90°\ \Omega;\text{ and}$$
$$\mathbf{Z}_{ca} = (200 + j0)\ \Omega = 200\angle 0°\ \Omega$$

Therefore, the phase currents are

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{100\angle 0°}{100\angle 0°} = \mathbf{1\angle 0°\ A};\ \mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{100\angle{-120°}}{94.2\angle 90°} = \mathbf{1.06\angle{-210°}\ A};\text{ and}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{100\angle{-240°}}{200\angle 0°} = \mathbf{0.5\angle{-240°}\ A}$$

(*a*) *Active power*: $P_{ab} = \dfrac{V_{ab}^2}{R_1} = \dfrac{100^2}{100} = 100$ W; $P_{bc} = 0$; and $P_{ca} = \dfrac{V_{ca}^2}{R_2} = \dfrac{100^2}{200} = 50$ W

$\therefore$ Total active power, $P = P_{ab} + P_{bc} + P_{ca} = 100 + 0 + 50 =$ **150 W**

(*b*) *Volt ampere reactive*: Since branches ab and ca have no reactance, their reactive power is zero.

$$\text{Reactive power in branch bc } = V_{bc} I_{bc} \sin\phi = 100 \times 1.06 \times \sin 90° = 106 \text{ VAR}$$

$$\therefore \quad \text{Total volt ampere reactive } = 106 + 0 + 0 = \mathbf{106\ VAR}$$

EXAMPLE 12.13

A balanced three-phase, star-connected, 210 kW load takes a leading current when connected across a balanced three-phase, 1.1-kV, 50-Hz supply. Find the load circuit parameters per phase.

Solution From Eq. 12.15, the power factor is given as

$$pf = \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{210 \times 10^3}{\sqrt{3} \times 1.1 \times 10^3 \times 160} = 0.6888 \text{ (leading)}$$

The load impedance per phase, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{V_L/\sqrt{3}}{I_L} = \dfrac{1.1 \times 10^3/\sqrt{3}}{160} = 3.97\ \Omega$

Since the power factor is leading, the equivalent load must consist of a resistance R and a capacitance C in series. Thus,

$$R = Z_{ph}\cos\phi = 3.97 \times 0.6888 = \mathbf{2.734\ \Omega}; \quad X_C = Z_{ph}\sin\phi = 3.97 \times 0.7249 = 2.877\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 2.877} = \mathbf{1106\ \mu F}$$

EXAMPLE 12.14

A balanced, 3-phase, star-connected load is fed from a 400-V, 3-phase, 50-Hz supply. The current per phase is 25 A (lagging), and the total active power absorbed by the load is 13.856 kW. Determine (*a*) the resistance and inductance of the load per phase, (*b*) the total reactive power, and (*c*) the total apparent power.

Solution

(*a*) From Eq. 12.15, the power factor is given as

$$pf = \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{13.856 \times 10^3}{\sqrt{3} \times 400 \times 25} = 0.8 \text{ (lagging)}$$

The load impedance per phase, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{V_L/\sqrt{3}}{I_L} = \dfrac{400/\sqrt{3}}{25} = 9.24\ \Omega$

$$\therefore \quad R_{ph} = Z_{ph}\cos\phi = 9.24 \times 0.8 = \mathbf{7.392\ \Omega}; \quad \text{and} \quad X_{ph} = Z_{ph}\sin\phi = 9.24 \times 0.6 = 5.544\ \Omega$$

$$\therefore \quad L = \frac{X_{ph}}{2\pi f} = \frac{5.544}{2\pi \times 50} = \mathbf{17.6\ mH}$$

(*b*) The total reactive power consumed by the load is

$$Q = 3 V_{ph} I_{ph} \sin\phi = 3 \times (400/\sqrt{3}) \times 25 \times 0.6 = \mathbf{10.392\ kVAR}$$

(*c*) The total apparent power is

$$S = 3 V_{ph} I_{ph} = 3 \times (400/\sqrt{3}) \times 25 = \mathbf{17.321\ kVA}$$

EXAMPLE 12.15

Each phase of a star-connected load consists of a resistance of 100 Ω in parallel with a capacitance of 32 μF. If this load is connected to a 415-V, 3-phase, 50-Hz supply, calculate the line current, the power factor, the power absorbed and the total kVA.

Solution $\mathbf{Z}_1 = (100 + j0)\ \Omega = 100\angle 0°\ \Omega$;

and $\mathbf{Z}_2 = -jX_C = 0 - j(1/2\pi fC) = (0 - j99.5)\ \Omega = 99.5\angle -90°\ \Omega$

$$\therefore \quad \mathbf{Z}_{ph} = \mathbf{Z}_1 \| \mathbf{Z}_2 = \frac{(100\angle 0°)\times(99.5\angle -90°)}{(100\angle 0°)+(99.5\angle -90°)} = 70.5\angle -45.2°\ \Omega$$

Line current, $I_L = I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{V_L/\sqrt{3}}{Z_{ph}} = \dfrac{415/\sqrt{3}}{70.5} = \mathbf{3.39\ \Omega}$

Power factor, $pf = \cos\phi = \cos 45.2° = \mathbf{0.7046\ (leading)}$

Power absorbed, $P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 415 \times 3.39 \times 0.7046 = \mathbf{1.717\ kW}$

Total kVA $= \sqrt{3}\,V_L I_L = \sqrt{3} \times 415 \times 3.39 = \mathbf{2.437\ kVA}$

EXAMPLE 12.16

The input power to a 1.6-kV, 50-Hz, 3-phase motor is measured by using two-wattmeter method. The motor is running on full-load with an efficiency of 86 %. The readings of the two wattmeters are 255 kW and 85 kW. Determine (*a*) the input power, (*b*) the power factor, (*c*) the line current, and (*d*) the output power.

Solution

(*a*) The input power, $P = P_1 + P_2 = 255 + 85 = \mathbf{340\ kW}$

(*b*) The phase angle, $\phi = \tan^{-1}\sqrt{3}\left[\dfrac{P_1 - P_2}{P_1 + P_2}\right] = \tan^{-1}\sqrt{3}\left[\dfrac{255-85}{255+85}\right] = \tan^{-1}0.866 = 40.9°$

$\therefore$ Power factor, $pf = \cos\phi = \cos 40.9° = \mathbf{0.756\ (lagging)}$

(*c*) The line current, $I_L = \dfrac{P}{\sqrt{3}V_L\cos\phi} = \dfrac{340\times 10^3}{\sqrt{3}\times 1.6\times 10^3 \times 0.756} = \mathbf{162.28\ A}$

(*d*) The output power, P_o = Input power × Efficiency = 340 × 0.86 = **292.4 kW**

EXAMPLE 12.17

During the measurement of power by two-wattmeter method, the total input power to a 3-phase, 440-V motor running at a power factor of 0.8, was found to be 25 kW. Find the readings of the two wattmeters.

Solution Given: $P = P_1 + P_2 = 25$ kW and $pf = \cos\phi = 0.8$.

$\therefore \quad \phi = 36.87°$ and $\tan\phi = 0.75$.

Since $\tan\phi = \sqrt{3}\left[\dfrac{P_1 - P_2}{P_1 + P_2}\right] \Rightarrow P_1 - P_2 = \dfrac{1}{\sqrt{3}}(P_1 + P_2)\tan\phi = \dfrac{1}{\sqrt{3}} \times 25 \times 0.75 = 10.825$ kW

On solving, we get P_1 = **17.9125 kW** and P_2 = **7.0875 kW**

EXAMPLE 12.21

A 3-ϕ, 75-kW, delta-connected induction motor operates at rated load in parallel with a star-connected load having impedance of (3 + *j*4) Ω per phase, on a 3-ϕ, 440-V, 50-Hz supply, as shown in Fig. 12.23. The motor is running at a power factor of 0.8 lagging, and can be considered as three impedances. Determine (*a*) the line and phase current in motor, (*b*) the line and phase current in load, (*c*) the total line current, and (*d*) the total power consumed.

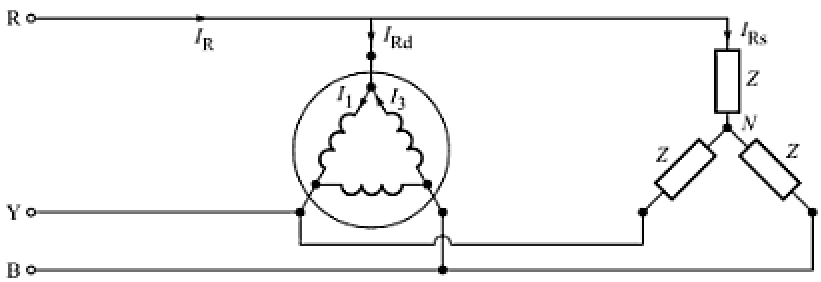


Fig. 12.23 *A motor and a balanced load connected to 3-ϕ supply.*

Solution Let us take the line voltage V_{RY} as the reference for the phasor diagram. That is, let $\mathbf{V}_{RY} = 440\angle 0°$ V.

(*a*) *For Motor* (*Delta-Connected*) : $pf = \cos\phi_d = 0.8 \;\Rightarrow\; \phi_d = 36.87°$

$$I_L = I_R = P/(\sqrt{3}\,V_L \cos\phi_d) = 75\,000/(\sqrt{3}\times 440\times 0.8) = 123 \text{ A}$$

$\therefore$ $\mathbf{I}_L = \mathbf{I}_{Rd}(= \mathbf{I}_1 - \mathbf{I}_3) = 123\angle(-30° - \phi_d) = \mathbf{123\angle{-66.87°}\ A}$ (see the phasor diagram in Fig. 12.24)

$$\therefore \quad \mathbf{I}_{ph} = \mathbf{I}_1 = \frac{I_L}{\sqrt{3}}\angle{-\phi_d} = \frac{123}{\sqrt{3}}\angle{-36.87°} = \mathbf{71\angle{-36.87°}\ A}$$

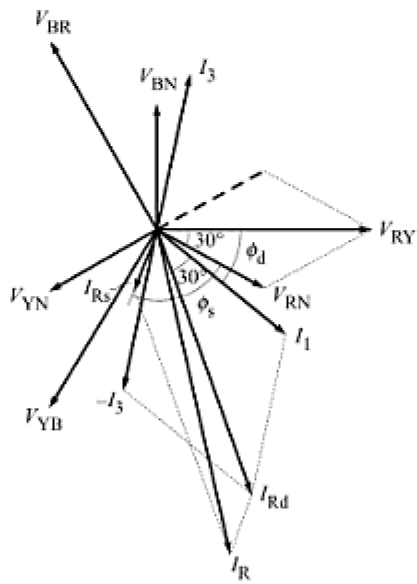


Fig. 12.24 *Phasor diagram.*

(*b*) *For Load* (*Star-Connected*): $\mathbf{Z} = (3 + j4)\ \Omega = 5\angle 53.13°\ \Omega$

$$\mathbf{V}_{ph} = \mathbf{V}_{RN} = \frac{440}{\sqrt{3}}\angle -30° = 254\angle -30°\ \text{V}$$

$$\mathbf{I}_L = \mathbf{I}_{Rs} = \mathbf{I}_{ph} = \frac{\mathbf{V}_{ph}}{\mathbf{Z}_{ph}} = \frac{254\angle -30°}{5\angle 53.13°} = \mathbf{50.8\angle -83.13°\ A}$$

(*c*) *Total Line Current*:

$$\mathbf{I}_L = \mathbf{I}_R = \mathbf{I}_{Rd} + \mathbf{I}_{Rs} = (123\angle -66.87°) + (50.8\angle -83.13°) = \mathbf{172.3\angle -71.6°\ A}$$

(*d*) *Total Power Consumed*:

Power consumed by the motor, $P_1 = \sqrt{3}\, V_L I_{Rd} \cos\phi_d = \sqrt{3} \times 440 \times 123 \times 0.8 = 74\,991$ W

Power consumed by the load, $P_2 = \sqrt{3}\, V_L I_{Rs} \cos\phi_s = \sqrt{3} \times 440 \times 50.8 \times \cos 53.13°$
$= 23\,229$ W

Total power consumed, $P = P_1 + P_2 = 74\,991 + 23\,229 = 98\,220$ W = **98.22 kW**

SUMMARY

TERMS AND CONCEPTS

- **Three-phase systems** are best suited for transmitting high powers efficiently and also for providing powerful motor drives.
- Three-phase systems often identify the phases by the colours ***R***ed, ***Y***ellow and ***B***lue, although some systems use letters *a*, *b* and *c*, and some others use numbers 1, 2 and 3.
- Three phases can be connected either in *star* (Y) or *delta* (Δ)
- The voltages across and the currents through the components of the load or source are termed the *phase values* (represented as V_{ph} and I_{ph}). The voltage between the supply conductors and the currents in these conductors are termed the *line values* (represented as V_L and I_L).
- In both the star-connected and delta-connected systems, the line voltages are mutually displaced by 120°.
- It is normal practice to state the value of the line voltage (and not the phase voltage) for a three-phase system.
- In a three-wire system, the sum of line currents is always zero.
- In a three-phase system, if the load is balanced, the power can be given by measuring the power in one phase and then multiplying it by 3.
- Two wattmeters can be used to measure the total power in a three-phase load, whether the load is balanced or not.

IMPORTANT FORMULAE

- In a star-connection, $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$.
- In a delta-connection, $V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$.
- For *any balanced load*, $P = \sqrt{3}\, V_L I_L \cos\phi$, where ϕ is the phase angle in one phase.
- The phase angle ϕ of a balanced three-phase load, $\tan\phi = \sqrt{3}\left[\dfrac{P_1 - P_2}{P_1 + P_2}\right]$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The power transmission and distribution by three-phase system is costlier than that by single phase.	☐	☐	
2.	In a three-phase generator, the windings of the 3 phases are displaced by 120°.	☐	☐	
3.	For a three-phase, star-connected load, the phase voltage is always equal to the line voltage.	☐	☐	
4.	If three unequal impedances are connected in star on a 3-wire, 3-ϕ system, each phase current is equal to the corresponding line current.	☐	☐	
5.	If an unbalanced delta-connected load is connected to a 3-wire, 3-ϕ system, the phase voltages become unbalanced.	☐	☐	
6.	For a three-phase load, the power factor is the cosine of the phase angle between the phase voltage and phase current.	☐	☐	
7.	A three-phase system feeds a balanced star-connected load and supplies a total power P. If the same impedances are connected in delta and fed from the same three-phase system, the total power will be $9P$.	☐	☐	
8.	If a balanced three-phase system feeds an unbalanced delta-connected load, the line currents will be balanced.	☐	☐	
9.	The phenomenon of neutral shift does not occur in 4-wire, 3-phase system.	☐	☐	
10.	In two-wattmeter method of power measurement, if the reading of one wattmeter is zero, the power factor of the load must be 0.5.	☐	☐	

Your Score

ANSWERS

1. False **2.** True **3.** False **4.** True **5.** False
6. True **7.** False **8.** False **9.** True **10.** True

REVIEW QUESTIONS

1. Explain what is meant by a balanced three-phase voltage system.

2. Explain briefly the principle of generation of three-phase voltages.

3. To generate three-phase voltages, why is it considered better to rotate the magnetic field instead of rotating the coils?

4. What are the advantages of three-phase voltage system over single-phase voltage system?

5. Explain the term 'phase sequence'. What is its significance? Draw waveforms of 3 balanced voltages A, B and C with phase sequence (*a*) ABC, and (*b*) ACB.

6. Derive the relations between phase and line voltages, and phase and line currents for a balanced three-phase load when (*a*) star-connected, and (*b*) delta-connected.

7. Three single phase loads can be connected in either star or in delta to form a three-phase load. Which of these connections results in higher current, when connected to a three-phase supply?
8. Explain why an unbalanced star-connected load is not normally used on a 3-wire, 3-phase system.
9. Derive, for both star- and delta-connected systems, an expression for the total power input for a balanced three-phase load in terms of line voltage, line current and power factor.
10. Show that the total power consumed by a three-phase load, whether balanced or unbalanced, can be measured by using only two wattmeters.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. In double subscript notation, the symbol V_{ab} represents
   (*a*) the voltage of terminal b with respect to that of terminal a
   (*b*) the voltage of terminal a with respect to that of terminal b
   (*c*) the voltage difference between terminals a and b
   (*d*) the voltage difference between terminals b and a
2. In a three-phase, star-connected system, the three phase voltages are $100\angle 0°$ V, $100\angle -120°$ V and $100\angle -240°$ V. Then the three line voltages must be
   (*a*) $173.2\angle 0°$ V, $173.2\angle -120°$ V and $173.2\angle -240°$ V
   (*b*) $173.2\angle -30°$ V, $173.2\angle -150°$ V and $173.2\angle -270°$ V
   (*c*) $173.2\angle 30°$ V, $173.2\angle -90°$ V and $173.2\angle 150°$ V
   (*d*) $57.7\angle 0°$ V, $57.7\angle -120°$ V and $57.7\angle -240°$ V
3. In a balanced 3-ϕ, star-connected load, the three phase currents are $10\angle -30°$ A, $10\angle -150°$ A, and $10\angle 90°$ A. Then the three line currents must be
   (*a*) $17.32\angle -30°$ A, $17.32\angle -150°$ A and $17.32\angle 90°$ A
   (*b*) $17.32\angle 0°$ A, $17.32\angle -120°$ A and $17.32\angle 120°$ A
   (*c*) $10\angle 0°$ A, $10\angle -120°$ A and $10\angle 120°$ A
   (*d*) $10\angle -30°$ A, $10\angle -150°$ A and $10\angle -270°$ A
4. A 3-phase, 4-wire system supplies power to a balanced star-connected load. The current in each phase is 15 A. The current in the neutral wire will be
   (*a*) 15 A (*b*) 30 A
   (*c*) 45 A (*d*) 0
5. Three unequal impedances are connected in star on a 3-wire, 3-ϕ system. The sum of the three line currents must be equal to
   (*a*) three times the average value of the three phase currents
   (*b*) the average value of the three phase currents
   (*c*) $\sqrt{3}$ times the average value of the three phase currents
   (*d*) zero
6. Three unequal impedances are connected in delta on a 3-wire, 3-ϕ system. Then
   (*a*) the voltages across the three phases will be unequal
   (*b*) the phase currents will be unbalanced but the line currents will be balanced
   (*c*) the three line-currents will be equal
   (*d*) the sum of the three line-currents will be zero
7. The phase sequence R B Y denotes that
   (*a*) the emf of phase R lags that of phase B by 120°
   (*b*) the emf of phase R lags that of phase Y by 120°
   (*c*) the emf of phase R leads that of phase Y by 120°
   (*d*) the emf of phase B leads that of phase R by 120°
8. In a three-phase power measurement by two-wattmeter method, both wattmeters read the same value. The power factor of the load must be
   (*a*) unity (*b*) 0.707 lagging
   (*c*) 0.707 leading (*d*) zero
9. In a three-phase power measurement by two-wattmeter method, the reading of one of the wattmeters was zero. The power factor of the load must be
   (*a*) unity (*b*) 0.8
   (*c*) 0.5 (*d*) zero

10. In a three-phase power measurement by two-wattmeter method, the reading of one of the wattmeters was negative. The power factor of the load must be

(*a*) greater than 0.5
(*b*) less than 0.5
(*c*) equal to 0.5
(*d*) negative

ANSWERS

1. *b*	**2.** *c*	**3.** *d*	**4.** *d*	**5.** *d*	**6.** *d*	**7.** *b*	**8.** *a*	**9.** *c*	**10.** *b*

PROBLEMS

(A) SIMPLE PROBLEMS

1. A balanced three-phase load consists of three coils, each of 4-Ω resistance and 0.02-H inductance. Determine the total active power when the coils are (*a*) star-connected, (*b*) delta-connected to a 415-V, 50-Hz, three-phase supply.

[**Ans.** (*a*) 12.41 kW; (*b*) 37.23 kW]

2. A balanced three-phase load consists of three coils, each of resistance 20 Ω and inductance 0.5 H. Determine the line current and the total active power when the coils are (*a*) star-connected, (*b*) delta-connected to a 400-V, 50-Hz, three-phase supply.

[**Ans.** (*a*) 1.459 A, 127.66 W; (*b*) 4.38 A, 383.3 W]

3. Three similar resistors, each of 10 Ω, are connected in star to a 3-phase, 400-V, 50-Hz supply. Calculate (*a*) the phase voltage, (*b*) the line current, and (*c*) the power absorbed.

[**Ans.** (*a*) 231 V; (*b*) 23.1 A; (*c*) 16 kW]

4. A balanced star-connected load with impedance $(10 + j5)\ \Omega$ per phase is fed from a balanced 3-phase, 400-V supply. Find (*a*) the line current, and (*b*) the power factor.

[**Ans.** (*a*) $20.65\angle{-26.57°}$ A; (*b*) 0.894 (lagging)]

5. Three identical coils connected in delta are fed from a 440-V, 3-phase supply. This load takes a total power of 50 kW at a line current of 90 A. Determine (*a*) the phase current, (*b*) the power factor, and (*c*) the total apparent power.

[**Ans.** (*a*) 51.96 A; (*b*) 0.8; (*c*) 62.354 kVA]

6. Three equal impedances, each having a resistance of 8 Ω and inductive reactance of 6 Ω, are connected in (*a*) star, (*b*) delta across a 3-phase, 400-V system. Find (*i*) the phase current, (*ii*) the line current, and (*iii*) the power consumed in the two cases.

[**Ans.** (*a*) (*i*) $23.1\angle{-36.87°}$ A (*ii*) 23.1 A (*iii*) 12.8 kW; (*b*) (*i*) $40\angle{-36.87°}$ A (*ii*) 69.28 A (*iii*) 38.4 kW]

7. Calculate the current flowing into each terminal and in each phase of the winding of a 3-phase delta-connected motor developing an output of 250 hp at 2300 V between the terminals at a power factor of 0.75 and efficiency of 85 %.

[**Ans.** 73.44 A, 42.4 A]

8. A 3-ϕ, delta-connected load consumes a power of 120 kW, drawing a lagging line current of 200 A from a 3-ϕ, 400-V, 50-Hz supply. (*a*) Find the parameters of each phase. (*b*) What would be the power consumed if the loads were connected in star? [**Ans.** (*a*) 3 Ω, 5.51 mH; (*b*) 40 kW]

9. Two wattmeters connected to measure input power to a balanced three-phase circuit indicate 2500 W and 500 W respectively. Find the power factor of the circuit (*a*) when both readings are positive, and (*b*) when the latter reading is obtained after reversing the connections to the current coil.

[**Ans.** (*a*) 0.6547; (*b*) 0.3592]

10. Two-wattmeter method is used to measure input power to a three-phase load. The load is found to consume total power of 30 kW at 0.7 *pf* (lagging). What are the readings of the two wattmeters?

[**Ans.** 23.84 kW and 6.16 kW]

11. In a balanced 3-ϕ system, power is measured by two wattmeters. The ratio of two readings is found to be 2 : 1. Determine the power factor of the system. [**Ans.** 0.866]

12. The readings of the two wattmeters connected to measure the total power in a three-phase, star-connected, 400-V system, are 3000 W and 5000 W. Find the power factor, the total power and the line current. [**Ans.** 0.9177, 8000 W, 12.67 A]

(B) TRICKY PROBLEMS

13. If the phase voltage of a balanced three-phase star-connected generator is 100 V, what will be the line voltages: (*a*) when the phase voltages are correctly connected, and (*b*) when the connections to one of the phases are reversed?

[**Ans.** (*a*) 173 V, 173 V, 173 V;
(*b*) 173 V, 100 V, 100 V]

14. A three-phase motor operating on a 400-V supply is developing a power of 25 hp at an efficiency of 87 % and a power factor of 0.82. Calculate (*a*) the line current, and (*b*) phase current, if the windings are delta-connected. [**Ans.** (*a*) 37.2 A; (*b*) 21.5 A]

15. A balanced star-connected load is supplied from a symmetrical 3-phase, 400-V system. The current in each phase is 30 A, and lags by 30° behind the voltage. Find (*a*) the impedance in each phase, and (*b*) the total power drawn. Draw the phasor diagram. [**Ans.** (*a*) $7.7\angle 30°\ \Omega$; (*b*) 18 kW]

16. A balanced 3-phase, star-connected load of 100 kW takes a leading current of 80 A when connected to a 3-phase, 1.1-kV, 50-Hz supply. Find the resistance, the impedance and the capacitance of the load per phase. Also calculate the power factor of the load.

[**Ans.** 5.21 Ω, $7.939\angle -49°\ \Omega$,
530 μF, 0.656 (leading)]

17. When a balanced delta-connected load is fed from a 3-phase, 400-V, 50-Hz supply, the line current drawn is 20 A at a power factor of 0.3 lagging. Calculate the resistance and inductance per phase.

[**Ans.** 10.39 Ω, 0.105 H]

18. A balanced star-connected load of $(8 + j6)\ \Omega$ per phase is connected across a three-phase, 230-V supply. Find the line current, power factor, active power, reactive power and total volt amperes.

[**Ans.** $13.28\angle -36.87°$ A, 0.8 (lagging),
4.232 kW, 3.174 kVAR, 5.29 kVA]

19. A balanced star-connected load consumes a power of 3 kW at a power factor of 0.8 (lagging) when connected to a 3-ϕ supply. If the line current is 12.5 A, calculate the resistance and the reactance in each branch of the load. What will be the line current, reactive power and power consumed, if the same load is connected in delta?

[**Ans.** 6.4 Ω, 4.8 Ω; 6750 VAR, 9000 W]

20. A delta-connected load has $Z_{AB} = 52\angle 45°\ \Omega$, $Z_{BC} = 52\angle -30°\ \Omega$ and $Z_{CA} = 10\angle 0°\ \Omega$. This load is connected to a 3-ϕ, 230-V supply having phase sequence *ABC*. Determine the magnitude of line currents.

[**Ans.** $27.3\angle -57.6°$ A, $3.385\angle -157.55°$ A,
$26.92\angle -115.29°$ A]

21. Two wattmeters are used to measure power supplied to a balanced 3-ϕ load. Each wattmeter indicates 60 kW. If the power factor of the load is changed to 0.866 leading, the total power still remaining the same, determine the readings of the two wattmeters. [**Ans.** 80 kW and 40 kW]

(C) CHALLENGING PROBLEMS

22. Three identical impedances $\mathbf{Z} = (12 + j16)\ \Omega$ are connected in delta, and another set of three identical impedances $\mathbf{Z} = (12 + j16)\ \Omega$ are connected in star. Both these sets are connected across a 3-ϕ, 3-wire, 200-V supply. Find the line current and the total power supplied. [**Ans.** 23.1 A, 4802.5 W]

23. A 3-ϕ, 75-kW, delta-connected induction motor operates at rated load in parallel with a star-connected load having impedance of $(20 + j15)\ \Omega$ per phase, on a 3-ϕ, 440-V, 50-Hz supply. The motor is running at a power factor of 0.8 lagging, and can be considered as three impedances. Determine (*a*) the line and phase current in motor, (*b*) the line and phase current in load, (*c*) the total line current, and (*d*) the total power consumed.

[**Ans.** (*a*) $123\angle-66.87°$ A, $71\angle-36.87°$ A; (*b*) $10.16\angle-66.87°$ A; (*c*) $133.14\angle-66.87°$ A; (*d*) 81.173 kW]

24. (*a*) The load connected to a three-phase supply comprises three similar coils connected in star. The line current is 25 A, and the real and apparent powers are 11 kW and 20 kVA respectively. Find the line voltage, phase voltage, and the resistance and reactance of each coil. (*b*) If the three coils are now connected in delta, and then to the same three-phase supply, calculate the line current and the power taken.

[**Ans.** (*a*) 462 V, 267 V, 5.87 Ω, 8.92 Ω; (*b*) 75 A, 33 kW]

25. A balanced delta-connected load consumes 2 kW of power when connected to a 3-ϕ, 400-V, 50-Hz supply. The same load, when connected to a 3-ϕ, 230-V, 50-Hz supply, draws a current of 2A at a lagging power factor. Determine (*a*) the load power factor, (*b*) the resistance and inductance per phase, (*c*) the power consumed when the load connection is changed to star and the supply voltage is 400 V.

[**Ans.** (*a*) 0.8333 (lag); (*b*) 166 Ω, 0.355 H; (*c*) 667 W]

26. A 400-V, 3-ϕ motor has a full-load output of 20 hp, the efficiency being 88 % and the power factor being 0.8 lagging. Find the reading on each of two wattmeters connected to measure the input. What is the full-load line current?

[**Ans.** 12.148 kW, 4.807 kW, 30.59 A]

EXPERIMENTAL EXERCISE 12.1

TWO-WATTMETER METHOD

Objectives To measure power and power factor in a balanced three-phase circuit using two single-phase wattmeters.

Apparatus A 400-V, Three-phase supply; One three-phase variac; One three-phase variable reactive load; Two single-phase wattmeters, (0 – 10 A), 300 V; One voltmeter, MI, (0 – 300 V); One ammeter, MI, (0 – 10 A).

Circuit Diagram The circuit diagram is shown in Fig. 12.25.

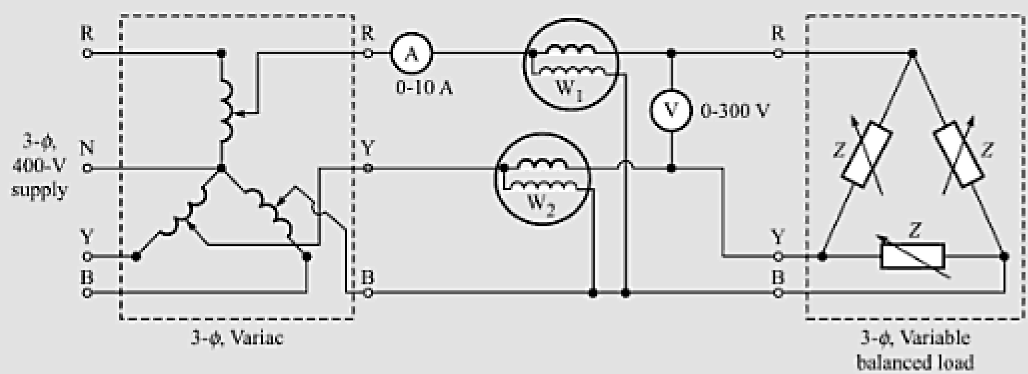


Fig. 12.25 *Circuit arrangement to measure power in a three-phase load using two wattmeters.*

Brief Theory Surprisingly, only two single-phase wattmeters are sufficient to measure the power factor and the total power in a balanced three-phase load. The current coils of the wattmeters are connected in series with any two lines, say, R and Y. The voltage (or pressure) coils of the two wattmeters are connected between that line and the third line. If P_1 and P_2 are the readings of the two wattmeters, the total power consumed by the three-phase load is simply given as

$$P = P_1 + P_2 \qquad (i)$$

The phase angle of the load can be calculated from the expression,

$$\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right] \quad \text{or} \quad \phi = \tan^{-1}\sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right] \qquad (ii)$$

The power factor is then given as

$$pf = \cos\phi \qquad (iii)$$

If V_L and I_L are the line voltage and line current for the balanced load, the total power is given as

$$P = \sqrt{3}\, V_L I_L \cos\phi \qquad (iv)$$

Procedure

1. Make the connections as shown in Fig. 12.25.
2. Keep the three-phase variac at its zero position.
3. Adjust the three-phase load to have maximum impedance.
4. Switch ON the three-phase supply.
5. Gradually increase the position of the variac towards higher voltage till all the meters give enough deflection.
6. Note the readings of all the meters.
7. Increase further the voltage applied to the circuit by adjusting the variac and repeat steps 5 and 6 four times. In case, changing the applied voltage does not give sufficient readings, the three-phase load can also be changed.
8. Decrease the voltage applied to the circuit using the variac.
9. Switch off the three-phase supply.

Observations

S. No.	I_L (*in* A)	V_L (*in* V)	P_1 (*in* W)	P_2 (*in* W)
1				
2				
3				
4				
5				

Calculations

S. No.	$P = P_1 + P_2$ (*in* W) [*Eq.* (*i*)]	*Phase angle* [*Eq.* (*ii*)]	*Power factor* [*Eq.* (*iii*)]	*P* (*in* W) [*Eq.* (*iv*)]
1				
2				
3				
4				
5				

Results

1. For five different observations, the total power has been measured using Eq. (*i*).
2. For the same observations, the power factor has been calculated using Eqs. (*ii*) and (*iii*).
3. For the same observations, the total power has also been calculated using Eq. (*iv*). The results obtained in Column 4 quite tally with the results obtained in Column 1.

Precautions

1. Before switching on the supply, the zeroes of the ammeter, voltmeter, and wattmeters should be checked.
2. The readings in the ammeter should not exceed the current ratings of the wattmeter.
3. During the experiment, one of the wattmeters may give a negative reading. Since the meter does not have any markings for negative readings, the connections of either the current coil or the pressure coil should be reversed. The reading of this meter should then be recorded with negative sign.

Viva-Voce

1. The power in a three-phase circuit can easily be measured by a *single* three-phase wattmeter. Then, why do you measure power by using *two* single-phase wattmeter?
   Ans.: A three-phase wattmeter is not readily available usually. In this experiment, we learn to measure power in a three-phase circuit by using only *two* (and not *three*) single-phase wattmeters.
2. Can this method also be used to measure power, even if the three-phase load is unbalanced?
   Ans.: Yes, the Eqs. (*i*) and (*ii*) are valid even for an unbalanced load.
3. Is it possible to measure reactive power in a three-phase circuit using this method?
   Ans.: Yes, the reactive power is given as
   $$Q = \sqrt{3}\,(P_1 - P_2)$$
4. How are the readings of the two wattmeters affected, when the load is purely resistive?
   Ans.: For purely resistive load, the phase angle is zero. Then, according to Eq. (*ii*), we must have $P_1 = P_2$. That is, both the wattmeters should give the same readings.
5. What happens to the readings of the two wattmeters, if the load is purely reactive?
   Ans.: For purely reactive load, the power factor is zero and the phase angle is 90°. Then, according to Eq. (*ii*), we must have $P_1 = -P_2$. Thus, the two wattmeters should give the same readings, but only after reversing the connections of one of them.
6. If one of the wattmeters reads zero, what can you say about the power factor?
   Ans.: Putting $P_2 = 0$ in Eq. (*ii*), we get $\tan\phi = \sqrt{3} \Rightarrow \phi = 60°$. Hence, the power factor is $\cos 60° = 0.5$.
7. What is the phase sequence of a 3-phase system in general?
   Ans.: The phase sequence is generally *R*, *Y*, *B*.
8. What do you mean by 'phase-sequence'?
   Ans.: It means the order in which the individual phase voltages attain their respective maximum values.
9. How will you determine the phase sequence of a 3-phase system?
   Ans.: We connect two 100-W lamps, L_1 and L_2, and a capacitor *C* to the three lines (*A*, *B* and *C*) of the 3-phase system, as shown in Fig. 12.26. When the supply is switched on, we shall always find one of the lamps brighter than the other. Then the phase sequence is in the order: bright-lamp, dim-lamp, capacitor. For instance, if lamp L_2 is brighter than the lamp L_1 in Fig. 12.26, the phase sequence is B A C.

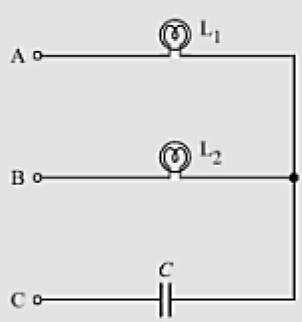


Fig. 12.26 *Circuit for determining the phase sequence.*

SUPPLEMENTARY EXERCISES C

C.1 Solved Problems
C.2 Practice Problems

PART C: AC CIRCUITS

Assemblage of

- Chapter 9: Alternating Voltage and Current
- Chapter 10: AC Circuits
- Chapter 11: Resonance in AC Circuits
- Chapter 12: Three-Phase Circuits and Systems

C.1. SOLVED PROBLEMS

PROBLEM C-1

Find the angle by which the current i_1 lags the voltage v_1, if $v_1 = 80\cos(100\pi t - 40°)$ V and i_1 is equal to

(*a*) $i_1 = 2.5\cos(100\pi t + 20°)$ A; (*b*) $i_1 = 21.4\sin(100\pi t - 70°)$ A;

(*c*) $i_1 = -0.8\cos(100\pi t - 110°)$ A; (*d*) $i_1 = -5.5\sin(100\pi t + 50°)$ A

Solution

(*a*) Since both v_1 and i_1 have positive amplitude, and are expressed as cosine function, the angle by which i_1 lags the voltage v_1 is given as

$$\phi = (-40°) - (20°) = \mathbf{-60°}$$

(*b*) First, we express i_1 as a cosine function,

$$i_1 = 21.4\sin(100\pi t - 70°) = 21.4\cos(100\pi t - 70° - 90°) = 21.4\cos(100\pi t - 160°)$$

We can now say that i_1 lags v_1 by $\phi = (-40°) - (-160°) = \mathbf{120°}$

(*c*) First, we express i_1 with a positive amplitude,

$$i_1 = -0.8\cos(100\pi t - 110°) = 0.8\cos(100\pi t - 110° + 180°) = 0.8\cos(100\pi t + 70°)$$

Therefore, i_1 lags v_1 by $\phi = (-40°) - (70°) = \mathbf{-110°}$

(*d*) First, we express i_1 as a cosine function with a positive amplitude,

$$i_1 = -5.5\sin(100\pi t + 50°) = 5.5\cos(100\pi t + 50° + 90°) = 5.5\cos(100\pi t + 140°)$$

Thus, i_1 lags v_1 by $\phi = (-40°) - (140°) = \mathbf{-180°}$. That is, i_1 and v_1 are out of phase.

PROBLEM C-2

A series combination of two impedances Z_1 and Z_2 is connected across an ac voltage source v, so that the voltages v_1 and v_2 across them, as shown in Fig. C-1*a*, are given as

$$v_1 = 40\cos(100\pi t - 40°)\text{ V} \qquad \text{and} \qquad v_2 = 20\sin(100\pi t + 170°)\text{ V}$$

The supply voltage v can be written as

$$v = (A\cos 100\pi t + B\sin 100\pi t)\text{ V} = C\cos(100\pi t + \phi)\text{ V}$$

Find A, B, C, and ϕ.

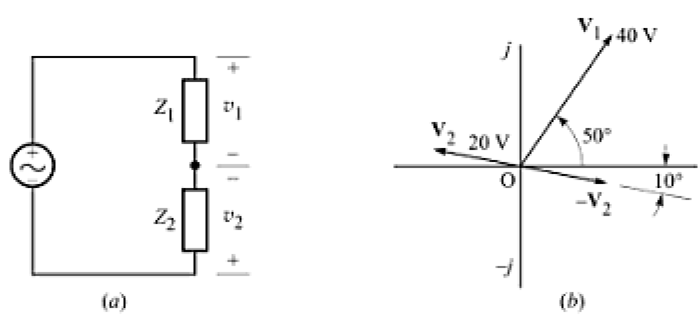


Fig. C-1

Solution Applying KVL, we get

$$v = v_1 + (-v_2) = \{40\cos(100\pi t - 40°)\} + \{-20\sin(100\pi t + 170°)\}\text{ V}$$

The voltage v can more conveniently be determined by using the phasor diagram drawn on a complex plane. First, all sinusoids should be expressed in sine-wave form. Hence, we express v_1 as

$$v_1 = 40\cos(100\pi t - 40°) = 40\sin(100\pi t - 40° + 90°)$$
$$= 40\sin(100\pi t + 50°)\text{ V}$$

As shown in Fig. C-1*b*, the phasor $\mathbf{V}_1$ has a magnitude of 40 V (peak or maximum value) and makes an angle of 50° with the reference axis. Similarly, phasor $\mathbf{V}_2$ has a magnitude of 20 V and makes an angle of 170° with the reference axis. Reversing its direction gives us phasor $-\mathbf{V}_2 = 20\sin(100\pi t - 10°)$ V, which also has a magnitude of 20 V but makes an angle of –10° with the reference axis. Thus,

$$\mathbf{V} = \mathbf{V}_1 + (-\mathbf{V}_2) = 40\angle 50° + 20\angle -10°$$
$$= (40\cos 50° + j40\sin 50°) + \{20\cos(-10°) + j20\sin(-10°)\}$$
$$= (25.71 + j30.64) + (19.70 - j3.47) = (45.41 + j27.17)$$

or $$\mathbf{V} = (45.41 + j27.17) \qquad (i)$$

Writing Eq. (*i*) as summation of two phasors, we get

$$\mathbf{V} = (45.41\angle 0° + 27.17\angle 90°)\text{ V}$$

which can be written as

$$v = 27.17\sin(100\pi t + 90°) + 45.41\sin 100\pi t$$
$$= (27.17\cos 100\pi t + 45.41\sin 100\pi t)\text{ V}$$

Hence, we have A = **27.17 V** and B = **45.41 V**.

Now, we can re-write Eq. (*i*) in the polar form as

$$\mathbf{V} = (45.41 + j27.17) = 52.92\angle 30.9°\text{ V}$$

which can be written as

$$v = 52.92\sin(100\pi t + 30.9°) = 52.92\cos(100\pi t + 30.9° - 90°)$$
$$= 52.92\cos(100\pi t - 59.1°)\text{ V}$$

Therefore, we have C = **52.92 V** and ϕ = **59.1°**

PROBLEM C-3

Find the average values of the periodic waves shown in Fig. C-2.

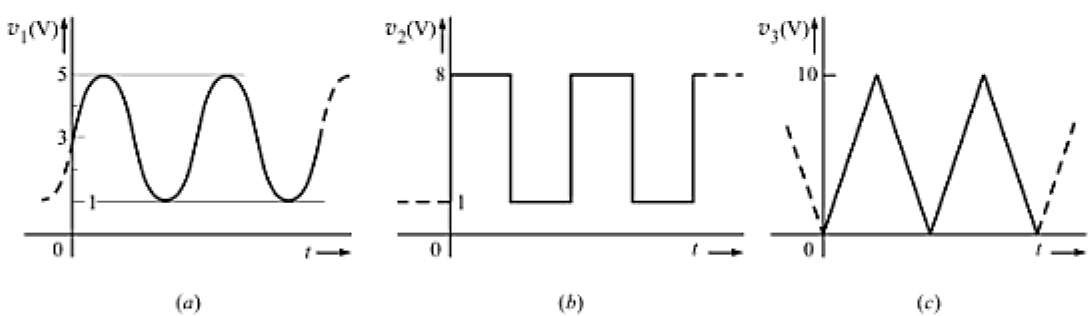


Fig. C-2

Solution

(*a*) It is a voltage sinusoid of peak value 5 – 3 = 2 V, riding over a constant voltage of 3 V. Since the average value of the sinusoid is zero, the average of the given waveform is **3 V**.

(*b*) The given wave is at 8 V for first-half period and is at 1 V for the second-half period. The area underneath the curve for one cycle is

$$A = 8 \times (T/2) + 1 \times (T/2) = 4.5T$$

Therefore, the average value of the wave is

$$V_{2(\text{avg})} = \frac{\text{Area}}{\text{Period}} = \frac{4.5T}{T} = \mathbf{4.5\ V}$$

(*c*) It is a triangular wave of height 10 V. The area underneath the curve for one cycle is

$$A = \frac{1}{2} \times 10 \times T = 5\,T$$

Therefore, the average value of the wave is

$$V_{3(\text{avg})} = \frac{\text{Area}}{\text{Period}} = \frac{5T}{T} = \mathbf{5\ V}$$

PROBLEM C-4

Determine the period of the following waves:

(*a*) $9 - 3.2\cos(400t + 30°)$, (*b*) $5\sin^2 4t$, and (*c*) $10\sin 3t \cos 3t$

Solution

(*a*) The expression is a sinusoid riding over a constant 9. Since, the time period is contributed by sinusoid only, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{400} = 15.7 \times 10^{-3}\text{s} = \mathbf{15.7\ ms}$$

(*b*) Because of the square term, the time period is not very obvious. We should first eliminate the square by writing,

$$5\sin^2 4t = 5\left[\frac{1-\cos(2\times 4t)}{2}\right] = 2.5\,(1 - \cos 8t)$$

For the cosine-wave portion, the period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8} = \mathbf{0.785\ s}$$

(*c*) Before the period can be determined, we must simplify the product of two sinusoids, by using the identity $\sin 2\theta = 2\sin\theta\cos\theta$. Thus, the given expression can be written as

$$10\sin 3t\cos 3t = 10\left[\frac{\sin(2\times 3t)}{2}\right] = 5\sin 6t$$

From this, the time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \mathbf{1.05\ s}$$

PROBLEM C-5

An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 10 A. Write the equation of instantaneous value, and find its value (*a*) 0.0025 s after passing through positive maximum value, (*b*) 0.0075 s after passing through zero and increasing negatively.

Solution $I_m = \sqrt{2} \times 10 = 14.14$ A; $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ rad/s

Hence, the equation for instantaneous current is

$$i = 14.14 \sin 100\pi t = \mathbf{14.14 \sin 314}t\ \mathbf{A} \qquad (i)$$

(*a*) If we are to take $t = 0$ at the positive maximum, we should shift the wave represented by Eq. (*i*) to the left by $\pi/2$ to get

$$i = 14.14 \sin(314t + \pi/2)\ \text{A} = 14.14 \cos 314t\ \text{A}$$

$$\therefore \quad i(0.0025\,\text{s}) = 14.14 \cos(314 \times 0.0025)\ \text{A} = \mathbf{10\ A}$$

(*b*) If we are to take $t = 0$ at the instant when the wave is passing through zero and increasing negatively, we should shift the wave represented by Eq. (*i*) to the left by π to get

$$i = 14.14 \sin(314t + \pi) = -14.14 \sin 314t\ \text{A}$$

$$\therefore \quad i(0.0075\,\text{s}) = -14.14 \sin(314 \times 0.0075) = \mathbf{-10\ A}$$

PROBLEM C-6

A current wave consists of two components: (*i*) a dc current of 10 A, and (*ii*) a sinusoidal ac current of peak value 5 A and frequency 50 Hz. (*a*) Write the expression of the current wave, taking $t = 0$ at an instant where the ac component is zero and increasing in positive direction. (*b*) Find the average value of the current wave. (*c*) Find the rms value of the current wave.

Solution

(*a*) The resultant current, $i = i_{1(dc)} + i_{2(ac)} = 10 + 5 \sin 2\pi \times 50t = 10 + 5 \sin 314t$

or $\quad i = 10 + 5 \sin \theta, \quad$ where $\quad \theta = 314t$

(*b*) The dc component remains constant at 10 A, and hence its average value is 10 A. The average value of the ac component over full cycle is zero. Hence, the average of the given current wave,

$$I_{avg} = 10 + 0 = \mathbf{10\ A}$$

Alternatively, the average value can be determined using standard procedure, as follows.

$$I_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} (10 + 5\sin\theta)\,d\theta = \frac{1}{2\pi}[10\theta - 5\cos\theta]_0^{2\pi}$$

$$= \frac{1}{2\pi}[10 \times 2\pi - 5 + 5]_0^{2\pi} = \mathbf{10\ A}$$

(*c*) Using the standard procedure, the rms value of the current wave is given as

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\,d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 5\sin\theta)^2\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100 + 2 \times 10 \times 5 \sin\theta + 5^2 \sin^2\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left[100 + 100\sin\theta + 25\left(\frac{1 - \cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}[112.5 + 100\sin\theta - 12.5\cos 2\theta]d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[112.5\theta - 100\cos\theta - \frac{12.5\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{1}{2\pi}[112.5\times 2\pi - 100 + 100 + 0]} = \sqrt{112.5} = \mathbf{10.61\ A}$$

Alternatively, the rms value of the given current wave can *easily* be determined if we apply the basic definition of rms or effective value. The effective value is defined on the basis of effective power. Suppose that the effective or rms value of the given current wave is I_{rms}. If this current flows through a resistance R, the power delivered is $I_{\text{rms}}^2 R$. The same amount of power is delivered jointly by the two components of the current wave. That is,

$$I_{\text{rms}}^2 R = I_{1(\text{rms})}^2 R + I_{2(\text{rms})}^2 R \quad \Rightarrow \quad I_{\text{rms}}^2 = I_{1(\text{rms})}^2 + I_{2(\text{rms})}^2$$

Hence, the rms value of the current wave is simply given as

$$I_{\text{rms}} = \sqrt{I_{1(\text{rms})}^2 + I_{2(\text{rms})}^2} = \sqrt{10^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{100 + 12.5} = \mathbf{10.61\ A}$$

PROBLEM C-7

Find the rms value of an ac voltage wave of peak value V_{m}, which can be represented by alternate positive and negative semicircles of radius V_{m}.

Solution Figure C-3 shows the given ac voltage wave, taking voltage v along y-axis and angle ωt along x-axis. For convenience, the centre of the first (positive) semicircle is taken at the origin O. Obviously, V_{m} is same as the radius R of the circle. Any point (x, y) on the semicircle is then given by

$$x^2 + y^2 = R^2 \quad \text{or} \quad v^2 = V_{\text{m}}^2 - \theta^2$$

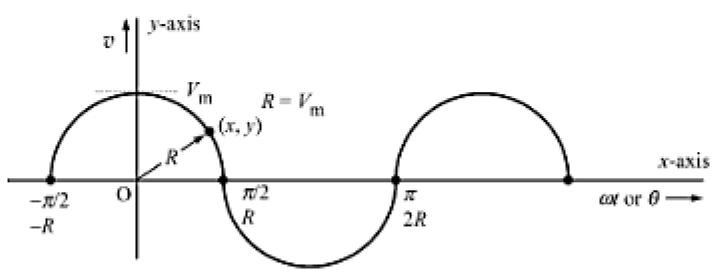


Fig. C-3 *The voltage wave shape.*

While determining the rms value, we take the mean of squares of voltage. Since, the square of negative quantities is positive, we need to consider only half of the wave. Hence, we consider only the positive semicircle, for which θ varies from $-V_{\text{m}}$ to $+V_{\text{m}}$, or equivalently from $-R$ to $+R$. Thus, the rms value of the voltage can now be determined as

$$V_{\text{rms}} = \sqrt{\frac{1}{2V_{\text{m}}}\int_{-V_{\text{m}}}^{+V_{\text{m}}} v^2 \cdot d\theta} = \sqrt{\frac{1}{2V_{\text{m}}}\int_{-V_{\text{m}}}^{+V_{\text{m}}} (V_{\text{m}}^2 - \theta^2)\cdot d\theta} = \sqrt{\frac{1}{2V_{\text{m}}}\left[V_{\text{m}}^2\theta - \frac{\theta^3}{3}\right]_{-V_{\text{m}}}^{+V_{\text{m}}}}$$

$$= \sqrt{\frac{1}{2V_m}\left[\left(V_m^2 V_m - \frac{(V_m)^3}{3}\right) - \left(-V_m^2 V_m - \frac{(-V_m)^3}{3}\right)\right]} = \sqrt{\frac{1}{2V_m}\left(2V_m^3 - \frac{2(V_m)^3}{3}\right)}$$

$$= \sqrt{\frac{2}{3}V_m^2} = \mathbf{0.818}V_m$$

PROBLEM C-8

Find i_S for the circuit shown in Fig. C-4.

Solution The current i_S can be determined from $i_S = i_R + i_L + i_C$. From Ohm's law,

$$i_R = \frac{150 \sin(2500t - 34°)}{10} = 15 \sin(2500t - 34°)\text{ V}$$

For the *inductor branch*, $X_L = \omega L = 2500 \times 6 \times 10^{-3} = 15\ \Omega$. Thus, peak value of the current is given as

$$I_m = \frac{V_m}{X_L} = \frac{150}{15} = 10\text{ A}$$

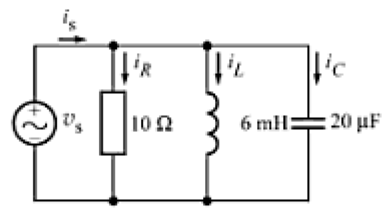


Fig. C-4

The current i_L lags the applied voltage by 90°. Hence,

$$i_L = 10 \sin(2500t - 34° - 90°) = 10 \sin(2500t - 124°)\text{ A}$$

For the *capacitor branch*, $X_C = 1/\omega L = 1/(2500 \times 20 \times 10^{-6}) = 20\ \Omega$. The peak value of the current is given as

$$I_m = \frac{V_m}{X_C} = \frac{150}{20} = 7.5\text{ A}$$

The current i_C leads the applied voltage by 90°. Hence,

$$i_C = 7.5 \sin(2500t - 34° + 90°) = 7.5 \sin(2500t + 56°)\text{ A}$$

To add the sinusoids directly and then to simplify it by using trigonometrical identities is a very tedious job. Instead, phasors based on peak values can be used to add sinusoids. Thus,

$$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = 15\angle{-34°} + 10\angle{-124°} + 7.5\angle 56° = 15.2\angle{-43.46°}\text{ A}$$

The above result can be expressed as

$$i_S = \mathbf{15.2 \sin(2500}t\mathbf{ - 43.46°)\ A}$$

PROBLEM C-9

Transform the series ac circuit given in Fig. C-5*a* to its equivalent in phasor-domain, and then determine the current *i*.

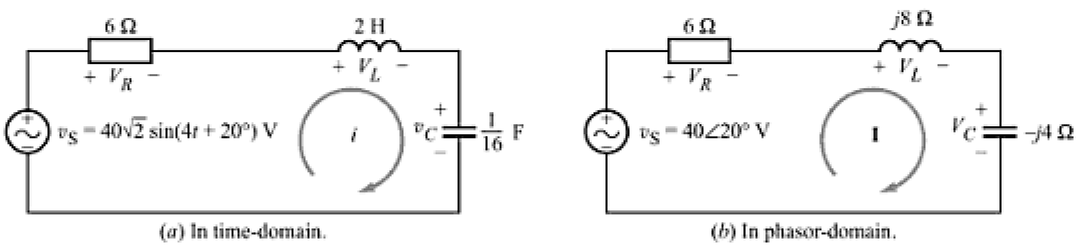


Fig. C-5 *An ac series circuit.*

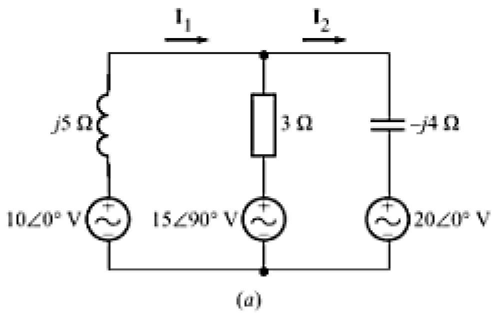


Fig. C-8

PROBLEM C-13

Determine the time-domain node voltages $v_1(t)$ and $v_2(t)$ in the circuit shown in Fig. C-9.

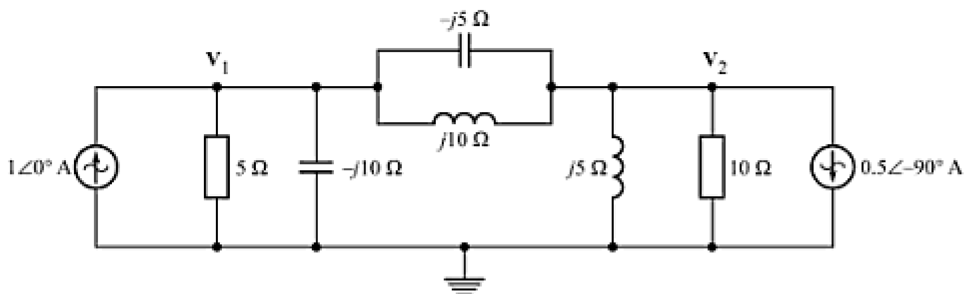


Fig. C-9

Solution We first solve the phasor-domain node voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ and then transform the results into time-domain. Applying KCL at the left node, gives

$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 1\angle 0°$$

or
$$\mathbf{V}_1\left(\frac{1}{5} - \frac{1}{j10} - \frac{1}{j5} + \frac{1}{j10}\right) - \mathbf{V}_2\left(-\frac{1}{j5} + \frac{1}{j10}\right) = 1$$

or
$$(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 = 1 \qquad (i)$$

Applying KCL at the right node,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{10} = -0.5\angle -90°$$

or
$$-\mathbf{V}_1\left(\frac{1}{-j5} + \frac{1}{j10}\right) + \mathbf{V}_2\left(-\frac{1}{j5} + \frac{1}{j10} + \frac{1}{j5} + \frac{1}{10}\right) = j0.5$$

or
$$-j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 = j0.5 \qquad (ii)$$

Solving Eqs. (*i*) and (*ii*) for $\mathbf{V}_1$ and $\mathbf{V}_2$ we get

$$\mathbf{V}_1 = 1 - j2 = 2.24\angle -63.4°\text{ V} \qquad \text{and} \qquad \mathbf{V}_2 = -2 + j4 = 4.47\angle 116.6°\text{ V}$$

Thus, the time-domain solutions are

$$v_1(t) = 2.24\sqrt{2}\sin(\omega t - 63.4°)\text{ V} \quad \text{and} \quad v_2(t) = 4.47\sqrt{2}\sin(\omega t + 116.6°)\text{ V}$$

or

$$v_1(t) = \mathbf{3.168\sin(\omega t - 63.4°)\ V} \quad \text{and} \quad v_2(t) = \mathbf{6.322\sin(\omega t + 116.6°)\ V}$$

PROBLEM C-14

Use superposition theorem to find $\mathbf{V}_1$ for the circuit of Fig. C-9.

Solution First we reduce each pair of parallel impedances by a single equivalent impedance. Thus,

$$(5\ \Omega)\|(-j10\ \Omega) = \frac{1}{0.2 + j0.1} = (4 - j2)\ \Omega; \quad (j10\ \Omega)\|(-j5\ \Omega) = \frac{1}{-j0.1 + j0.2} = -j10\ \Omega$$

$$(j5\ \Omega)\|(10\ \Omega) = \frac{1}{-j0.2 + 0.1} = (2 + j4)\ \Omega$$

Therefore, the circuit of Fig. C-9 simplifies to that of Fig. C-10.

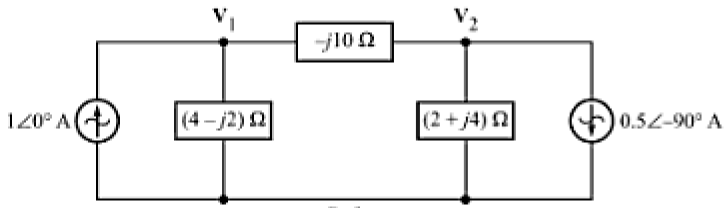


Fig. C-10

To use the superposition, we consider only one source at a time. First, activating only the *left* source and deactivating the right source (i.e., open-circuiting it), the equivalent admittance across the current source is given as

$$\mathbf{Y}_1 = \frac{1}{4 - j2} + \frac{1}{(-j10) + (2 + j4)} = (0.2 + j0.1) + (0.05 + j0.15) = (0.25 + j0.25)\text{ S}$$

Hence, the partial response $\mathbf{V}_{1L}$ due to only the left source is given as

$$\mathbf{V}_{1L} = \mathbf{I}/\mathbf{Y}_1 = (1\angle 0°)/(0.25 + j0.25) = (2 - j2)\text{ V}$$

Now, we activate only the *right* source and deactivate the left source. Using current division concept, the current through $(4 - j2)\ \Omega$ impedance is given as

$$\mathbf{I}_1 = (-0.5\angle -90°) \times \frac{(2 + j4)}{(2 + j4) + (-j10 + 4 - j2)} = (-0.2 - j0.1)\text{ A}$$

Using Ohm's law, we get the partial response $\mathbf{V}_{1R}$ due to only the right source as

$$\mathbf{V}_{1R} = \mathbf{I}_1\mathbf{Z} = (-0.2 - j0.1)(4 - j2) = -1.0\text{ V}$$

Finally, adding the two partial responses, we get

$$\mathbf{V}_1 = \mathbf{V}_{1L} + \mathbf{V}_{1R} = (2 - j2) - 1 = \mathbf{(1 - j2)\ V}$$

PROBLEM C-15

A noninductive load takes 20 A at 200 volts. Calculate the inductance of a reactor to be connected in series in order that the same current be supplied from a 230-V, 50-Hz supply. Also, determine the power factor of this circuit.

PROBLEM C-17

An inductive coil, when connected across a 200-V, 50-Hz supply, draws a current of 6.25 A and a power of 1000 W. Another coil, when connected across the same supply, draws a current of 10.75 A and a power of 1155 W. Find the current drawn and the power consumed when the two coils are connected in series across the same supply.

Solution For coil 1, we have

$$R_1 = \frac{P_1}{I_1^2} = \frac{1000}{(6.25)^2} = 25.6\ \Omega;\ Z_1 = \frac{V}{I_1} = \frac{200}{6.25} = 32\ \Omega,\ X_{L1} = \sqrt{Z_1^2 - R_1^2} = \sqrt{32^2 - 25.6^2} = 19.2\ \Omega$$

Similarly, for coil 2, we have

$$R_2 = \frac{P_2}{I_2^2} = \frac{1155}{(10.75)^2} = 10\ \Omega;\ Z_2 = \frac{V}{I_2} = \frac{200}{10.75} = 18.6\ \Omega,\ X_{L2} = \sqrt{Z_2^2 - R_2^2} = \sqrt{18.6^2 - 10^2} = 15.7\ \Omega$$

When the two coils are connected in series, the net impedance is given as

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = (25.6 + j19.2) + (10 + j15.7) = (35.6 + j34.9)\ \Omega = 49.85\angle 44.43^\circ\ \Omega$$

Therefore, the current drawn, $I = \dfrac{V}{Z} = \dfrac{200}{49.85} = \mathbf{4.01\ A}$

The power drawn, $P = I^2R = (4.01)^2 \times 35.6 = \mathbf{572.5\ W}$

PROBLEM C-18

A 50-Hz sinusoidal voltage $v = 141.4 \sin \omega t$ V is applied to a series *RL* circuit consisting of $R = 3\ \Omega$ and $L = 0.01272$ H. Compute (*a*) the effective value of the steady-state current and its phase angle, (*b*) the expression for the instantaneous current, (*c*) the effective value and phase angle of the voltage drops across each element, (*d*) the average power and the power factor of the circuit, and (*e*) the reactive power.

Solution $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{141.4}{\sqrt{2}} = 100$ V; $X_L = \omega L = 2\pi \times 50 \times 0.01272 = 4\ \Omega$;

(*a*) The impedance of the circuit, $\mathbf{Z} = 3 + j4 = 5\angle 53.13^\circ\ \Omega$

Hence, the current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{100\angle 0^\circ}{5\angle 53.13^\circ} = \mathbf{20\angle{-53.13^\circ}\ A}$

The effective (rms) current, $I = \mathbf{20\ A}$

The phase angle, $\phi = \mathbf{53.13^\circ}$ **(lagging)**

(*b*) Peak value of current, $I_m = I\sqrt{2} = 20\sqrt{2} = 28.28$ A

The expression for the instantaneous current can be written as

$$i(t) = \mathbf{28.28\ sin\ (\omega t - 53.13^\circ)\ A}$$

(*c*) The voltage drop across the resistor, $\mathbf{V}_R = \mathbf{I}R = (20\angle{-53.13^\circ}) \times 3 = \mathbf{60\angle{-53.13^\circ}\ V}$

The voltage drop across the inductor,

$$\mathbf{V}_L = \mathbf{I}(jX_L) = (20\angle{-53.13^\circ}) \times (4\angle 90^\circ) = 80\angle(90^\circ - 53.13^\circ) = \mathbf{80\angle 36.87^\circ\ V}$$

(*d*) The average power, $P = VI\cos\phi = 100 \times 20 \times \cos 53.13^\circ = \mathbf{1200\ W}$

The power factor $= \cos 53.13^\circ = \mathbf{0.6}$ **(lagging)**

(*e*) The reactive power, $Q = VI\sin\phi = 100 \times 20 \times \sin 53.13^\circ = \mathbf{1600\ VAR}$

Solution

(*a*) *Case* 1 (*Coil with iron core*): Impedance, $Z_1 = \dfrac{V_1}{I_1} = \dfrac{110}{3} = 36.67\ \Omega.$

The phase angle, $\phi_1 = \cos^{-1} 0.7 = 45.57° \quad \Rightarrow \quad \sin \phi_1 = 0.7141$

$\therefore \quad X_{L1} = Z_1 \sin \phi_1 = 36.67 \times 0.7141 = 26.19\ \Omega$

Hence, the inductance, $L_1 = \dfrac{X_{L1}}{2\pi f} = \dfrac{26.19}{2\pi \times 50} = 0.08336$ H = **83.36 mH**

Case 2 (*Coil without iron core*): Impedance, $Z_2 = \dfrac{V_2}{I_2} = \dfrac{30}{4.5} = 6.667\ \Omega.$

The phase angle, $\phi_2 = \cos^{-1} 0.9 = 25.84° \quad \Rightarrow \quad \sin \phi = 0.4359$

$\therefore \quad X_{L2} = Z_2 \sin \phi_2 = 6.667 \times 0.4359 = 2.906\ \Omega$

Hence, the inductance, $L_2 = \dfrac{X_{L2}}{2\pi f} = \dfrac{2.906}{2\pi \times 50} = 0.00925$ H = **9.25 mH**

(*b*) In *Case* 1 (*Coil with iron core*): the equivalent resistance responsible for losses,

$$R_1 = Z_1 \cos \phi_1 = 36.67 \times 0.7 = 25.67\ \Omega$$

In *Case* 2 (*Coil without iron core*): the equivalent resistance responsible for losses,

$$R_2 = Z_2 \cos \phi_2 = 6.667 \times 0.9 = 6.0\ \Omega$$

Hence, the equivalent resistance responsible for iron losses,

$$R_i = R_1 - R_2 = 25.67 - 6.0 = 19.67\ \Omega$$

Thus, the iron loss at 3 A current, $P_i = I^2 R_i = 3^2 \times 19.67 =$ **177 W**

PROBLEM C-21

A coil having a resistance of 15 Ω and an inductance of 0.2 H is connected in series with another coil having a resistance of 25 Ω and an inductance of 0.04 H to a 230-V, 50-Hz supply, as shown in Fig. C-12. Determine (*a*) the voltage across each coil, (*b*) the power dissipated in each coil, and (*c*) the power factor of the whole circuit.

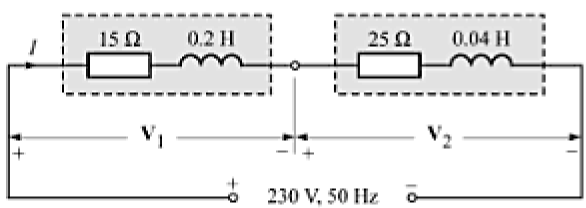


Fig. C-12

Solution $R = R_1 + R_2 = 15 + 25 = 40\ \Omega;\ L = L_1 + L_2 = 0.2 + 0.04 = 0.24$ H

The impedances of each coil and of entire circuit are given as

$$\mathbf{Z_1} = R_1 + jX_{L1} = 15 + j(2\pi \times 50 \times 0.2) = (15 + j62.83) = 64.60\angle 76.57°\ \Omega$$
$$\mathbf{Z_2} = R_2 + jX_{L2} = 25 + j(2\pi \times 50 \times 0.04) = (25 + j12.57) = 27.98\angle 26.69°\ \Omega$$
$$\mathbf{Z} = R + jX_L = 40 + j(2\pi \times 50 \times 0.24) = (40 + j75.4) = 85.35\angle 62.05°\ \Omega$$

Therefore, the magnitude of the current through the circuit is given as,

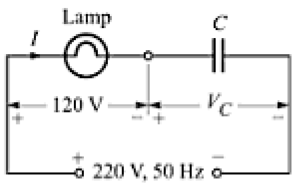
$$I = \frac{V}{Z} = \frac{230}{85.35} = 2.695 \text{ A}$$

The voltage across the inductance, $V_L = \sqrt{V^2 - V_B^2} = \sqrt{220^2 - 120^2} = 184.4 \text{ V}$

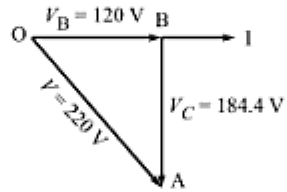
$\therefore$ The required inductive reactance, $X_L = \dfrac{V_L}{I} = \dfrac{184.4}{0.5} = 368.8\ \Omega$

Hence, the inductance, $L = \dfrac{X_L}{2\pi f} = \dfrac{368.8}{2\pi \times 50} = \mathbf{1.174\ H}$

(*iii*) Let C be the required capacitance to be connected in series with the lamp, as shown in Fig. C-15*a*. The voltage V_C lags the current I by 90°, as shown in phasor diagram of Fig. C-15*b*.



(*a*) The circuit.



(*b*) The phasor diagram.

Fig. C-15

The voltage across the capacitance, $V_C = \sqrt{V^2 - V_B^2} = \sqrt{220^2 - 120^2} = 184.4 \text{ V}$

$\therefore$ The required capacitive reactance, $X_C = \dfrac{V_C}{I} = \dfrac{184.4}{0.5} = 368.8\ \Omega$

Hence, the capacitance, $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi \times 50 \times 368.8} = \mathbf{8.63\ \mu F}$

COMMENTS

In the first alternative of using a resistance, there is unnecessary power loss ($P_{loss} = I^2R = (0.5)^2 \times 200 = 50$ W). Using L or C is a better alternative, as no power loss occurs. However, using a capacitor is preferred, as a capacitor costs much less than an inductor.

PROBLEM C-23

Find the simplest parallel circuit that has the same impedance at 400 Hz as the series combination of a 300-Ω resistor, a 0.25-H inductor and a 1-μF capacitor.

Solution Let us first determine the impedance of the given series combination,

$$\mathbf{Z} = 300 + j2\pi(400)\,(0.25) - j1/[2\pi(400)\,(1 \times 10^{-6})] = 300 + j230 = 378\angle 37.5^\circ\ \Omega$$

Thus, the admittance of the parallel circuit should be

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{378\angle 37.5^\circ} = 2.64 \times 10^{-3}\angle{-37.5^\circ} \text{ S} = (2.096 \times 10^{-3} - j1.61 \times 10^{-3}) \text{ S}$$

The simplest parallel circuit which has this admittance is a resistor and an inductor. From the above expression for admittance, we have

$$R = \frac{1}{G} = \frac{1}{2.096 \times 10^{-3}} = \mathbf{477\ \Omega}$$

and $$B_L = -1.61 \times 10^{-3}\text{ S} \quad \Rightarrow \quad L = \frac{-1}{\omega B_L} = \frac{-1}{2\pi(400)(-1.61\times 10^{-3})}\text{ H} = \mathbf{247\ mH}$$

PROBLEM C-24

Find $\mathbf{I}_{\text{in}}$ and $\mathbf{I}_L$ for the circuit shown in Fig. C-16.

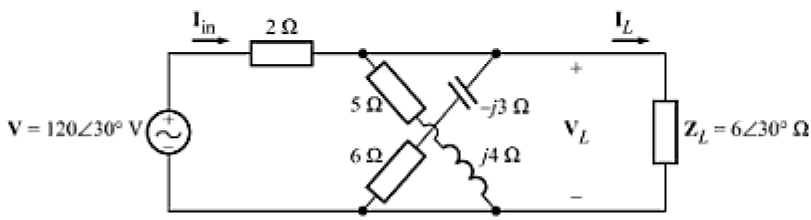


Fig. C-16

Solution The three branches on the right are in parallel and have a total admittance of

$$\mathbf{Y} = \frac{1}{5+j4} + \frac{1}{6-j3} + \frac{1}{6\angle 30^\circ} = 0.416\angle{-16^\circ}\text{ S}$$

Therefore, the total input impedance of the circuit is

$$\mathbf{Z}_{\text{in}} = 2 + \frac{1}{\mathbf{Y}} = 2 + \frac{1}{0.416\angle{-16^\circ}} = 4.36\angle 8.72^\circ\ \Omega$$

Hence, the current, $\mathbf{I}_{\text{in}} = \dfrac{\mathbf{V}}{\mathbf{Z}_{\text{in}}} = \dfrac{120\angle 30^\circ}{4.36\angle 8.72^\circ} = \mathbf{27.5\angle 21.3^\circ\ A}$

The current $\mathbf{I}_L$ can be found from the load voltage $\mathbf{V}_L$ and its impedance $\mathbf{Z}_L$. The load voltage $\mathbf{V}_L$ is equal to the current $\mathbf{I}_{\text{in}}$ divided by the total admittance of the three parallel branches,

$$\mathbf{V}_L = \frac{\mathbf{I}_{\text{in}}}{\mathbf{Y}} = \frac{27.5\angle 21.3^\circ}{0.416\angle{-16^\circ}} = 66.2\angle 37.3^\circ\text{ V}$$

and $$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{66.2\angle 37.3^\circ}{6\angle 30^\circ} = \mathbf{11\angle 7.3^\circ\ A}$$

PROBLEM C-25

Determine the Thevenin's equivalent as seen by the $-j10\ \Omega$ impedance in the circuit shown in Fig. C-17, and hence find the current directed from node 1 to node 2.

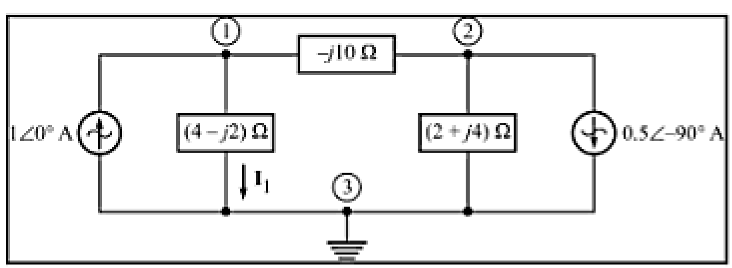


Fig. C-17

Solution On first glance at the circuit, we might be tempted to quickly write two nodal equations to find the voltage across the 10-Ω resistor. However, we just cannot do this, since the *two* sources are operating at *different* frequencies. In such a situation, there is no way to compute the impedance of any capacitor or inductor in a circuit, as we are in a dilemma as to which frequency to use.

The only way out of this dilemma is to use principle of ***superposition.*** We can consider one source at a time and calculate the resulting current after finding the impedances at the frequency of the source.

Let us first find the current **I′** due to the current source $2\sin 5t$ A acting alone. The other current source is deactivated (open-circuited), and all impedances calculated at $\omega = 5$ rad/s. The resulting subcircuit in phasor domain is shown in Fig. C-20*a*.

$$\mathbf{Z}_{C1} = \frac{-j1}{\omega C_1} = \frac{-j}{5\times 0.2} = -j\ \Omega;\quad \mathbf{Z}_{C2} = \frac{-j1}{\omega C_2} = \frac{-j}{5\times 0.5} = -j0.4\ \Omega;\quad I = \frac{I_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.414\ \text{A}$$

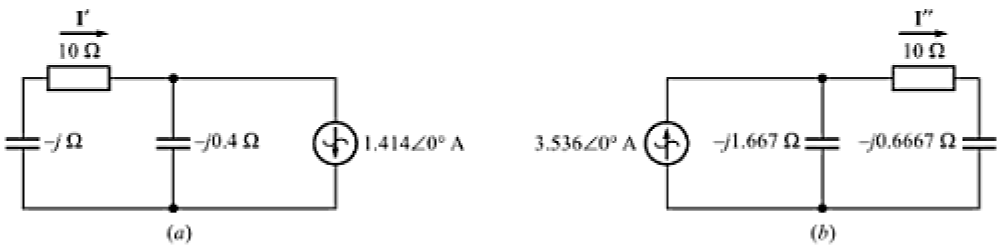


Fig. C-20

Using current division, we find the current **I′** from the circuit of Fig. C-20*a*,

$$\mathbf{I}' = (1.414\angle 0°)\left(\frac{-j0.4}{10 - j - j0.4}\right) = 56\angle{-82.03°}\ \text{mA}$$

Similarly, considering the current source $5\sin 3t$ A acting alone, we find the current **I″** through 10-Ω resistor. We convert the subcircuit in phasor domain at $\omega = 3$ rad/s, as shown in Fig. C-20*b*.

$$\mathbf{Z}_{C1} = \frac{-j1}{\omega C_1} = \frac{-j}{3\times 0.2} = -j1.667\ \Omega;\quad \mathbf{Z}_{C2} = \frac{-j1}{\omega C_2} = \frac{-j}{3\times 0.5} = -j0.6667\ \Omega;\quad I = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.536\ \text{A}$$

Using current division, we find the current **I″** from the circuit of Fig. C-20*b*,

$$\mathbf{I}'' = (3.536\angle 0°)\left(\frac{-j1.667}{10 - j0.6667 - j166.7}\right) = 574\angle{-76.86°}\ \text{mA}$$

NOTE

At this point, no matter how tempted we might be to add the two phasor currents **I′** and **I″**, this would be ***incorrect***, as the two currents are at different frequencies. However, it is perfectly justified to add the heating power by these two currents to get the net heating power of the resistor.

Hence the power dissipated by the 10-Ω resistor is given as

$$P = (I')^2R + (I'')^2R = [(0.056)^2 + (0.574)^2] \times 10 = \mathbf{3.326\ W}$$

PROBLEM C-27

Transform the practical ac voltage source given in Fig. C-21*a* into its equivalent current source.

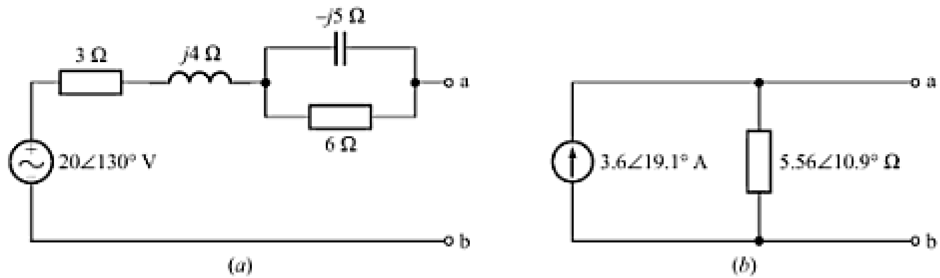


Fig. C-21

Solution Total series impedance,

$$\mathbf{Z} = 3 + j4 + \{6 \parallel (-j5)\} = 3 + j4 + \frac{6 \times (-j5)}{(6 - j5)} = 3 + j4 + (2.46 - j2.95)$$

$$= (5.46 + j1.05) = 5.56\angle 10.9°\ \Omega$$

The current source of the equivalent circuit is given as

$$\mathbf{I}_s = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{20\angle 30°}{5.56\angle 10.9°} = 3.6\angle 19.1°\text{ A}$$

Thus, the equivalent practical current source is as shown in Fig. C-21*b*.

PROBLEM C-28

An inductive choke coil having a resistance of 5 Ω and a self inductance of 0.06 H is connected to a 200-V, 50-Hz supply mains. Estimate the current taken and its power factor. Find also what value of capacitance must be arranged in series with it to give 2000 V across the capacitor with minimum applied potential difference. Estimate the necessary pd and also the pd across the inductive coil.

Solution Refer to Fig. C-22*a*.

$$\mathbf{X}_L = j(2\pi fL) = j(100\pi \times 0.06) = j18.85\ \Omega$$

∴ Choke impedance, $\mathbf{Z}_{Ch} = (5 + j18.85) = 19.5\angle 75.14°\ \Omega$

Therefore, the current taken from the supply mains,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_{Ch}} = \frac{200\angle 0°}{19.5\angle 75.14°} = \mathbf{10.26\angle -75.14°\ A}$$

Power factor, $pf = \cos 75.14° = $ **0.256 (lagging)**

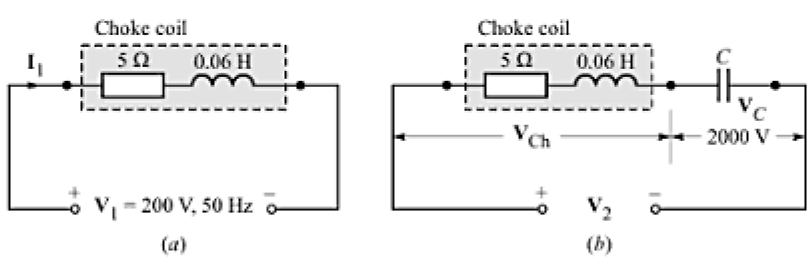


Fig. C-22

PROBLEM C-30

Determine the mesh currents in the circuit shown in Fig. C-24.

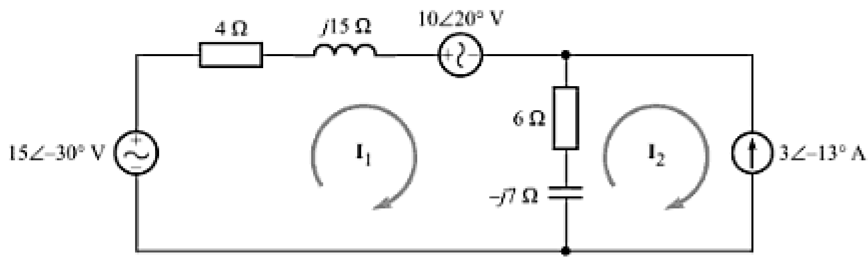


Fig. C-24

Solution For writing the mesh equations, we can apply the self-impedance and mutual impedance approach. For mesh 1,

the self-impedance, $\mathbf{Z}_{11} = 4 + j15 + 6 - j7 = (10 + j8)\ \Omega$

and the mutual impedance, $\mathbf{Z}_{12} = (6 - j7)\ \Omega$

The source voltage $15\angle{-30°}$ V is aiding the current $\mathbf{I}_1$, but the source voltage $10\angle 20°$ V is opposing it. Hence, the net source voltage for mesh 1 is

$$15\angle{-30°}\text{ V} - 10\angle 20°\text{ V} = 11.5\angle{-71.8°}\text{ V}$$

Hence, the KVL equation for mesh 1 is

$$(10 + j8)\mathbf{I}_1 - (6 - j7)\mathbf{I}_2 = 11.5\angle{-71.8°}\text{ V}$$

No KVL equation is needed for mesh 2, since $\mathbf{I}_2$ is the only mesh current through the current source of $3\angle{-130°}$ A. Since the direction of this source current is opposite to the mesh current $\mathbf{I}_2$, we have

$$\mathbf{I}_2 = \mathbf{-3\angle{-13°}\ A}$$

Substituting this value of $\mathbf{I}_2$ into the mesh 1 equation gives

$$(10 + j8)\mathbf{I}_1 - (6 - j7)(-3\angle{-13°}) = 11.5\angle{-71.8°}$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{11.5\angle{-71.8°} + (6 - j7)(-3\angle{-13°})}{(10 + j8)} = \mathbf{1.28\angle 85.5°\ A}$$

PROBLEM C-31

In the circuit shown in Fig. C-25, the load $\mathbf{Z}_L$ is a resistor with resistance R. Determine the value of R that will cause a current of 0.1 A through the load.

Solution We first find the Thevenin's equivalent of the circuit to the left of terminals a-b. The open-circuit or Thevenin's voltage is simply the voltage drop across the $j8$-Ω impedance,

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{oc}} = (1\angle 30°) \times \frac{j8}{6 + j8} = 0.8\angle 66.87°\text{ V}$$

The Thevenin's impedance is the impedance across the terminals a and b with the load impedance removed and the voltage source replaced by a short-circuit. Thus,

$$\mathbf{Z}_{\text{Th}} = -j4 + \frac{6(j8)}{6 + j8} = (3.84 - j1.12) = 4\angle{-16.26°}\ \Omega$$

Now, for a given load $\mathbf{Z}_L$, the load current is given as

$$\mathbf{I}_L = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \qquad \text{from which} \qquad \mathbf{Z}_{Th} + \mathbf{Z}_L = \frac{\mathbf{V}_{Th}}{\mathbf{I}_L}$$

Since only the rms load current is specified (angles are not known), only magnitudes must be used. Therefore,

$$|\mathbf{Z}_{Th} + \mathbf{Z}_L| = \frac{V_{Th}}{I_L} = \frac{0.8}{0.1} = 8\ \Omega \quad \text{or} \quad |3.84 - j1.12 + R| = 8$$

$$\Rightarrow \qquad \sqrt{(3.84 + R)^2 + (1.12)^2} = 8 \quad \text{or} \quad (3.84 + R)^2 + (1.12)^2 = 64$$

$$\text{or} \qquad R^2 + 7.86R - 48 = 0 \quad \Rightarrow \quad R = 4.08\ \Omega \quad \text{or} \quad -11.85\ \Omega$$

Since a resistance cannot have a negative value, R = **4.08 Ω**

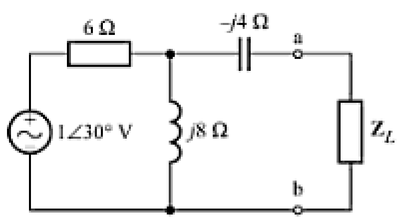


Fig. C-25

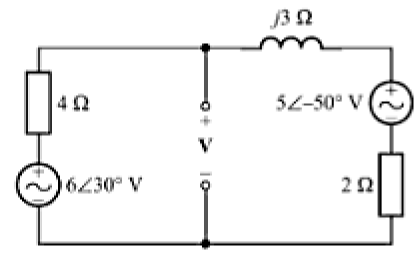


Fig. C-26

PROBLEM C-32

Use superposition to find **V** in the circuit of Fig. C-26.

Solution The voltage **V** can be considered to have two components, $\mathbf{V}_1$ and $\mathbf{V}_2$. The component $\mathbf{V}_1$ is due the source 6∠30° V acting alone, and the component $\mathbf{V}_2$ is due the source 5∠–50° V acting alone. Thus,

$$\mathbf{V}_1 = (6\angle 30°) \times \frac{2 + j3}{4 + 2 + j3} = 3.22\angle 59.7°\ \text{V}$$

and

$$\mathbf{V}_2 = (5\angle -50°) \times \frac{4}{4 + 2 + j3} = 2.98\angle -76.6°\ \text{V}$$

Therefore,

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = 3.22\angle 59.7° + 2.98\angle -76.6° = \mathbf{2.32\angle -2.96°\ V}$$

PROBLEM C-33

Find $\mathbf{Z}_{Th}$, $\mathbf{V}_{Th}$, and $\mathbf{I}_N$ for Thevenin's and Norton's equivalents across terminals a and b, of the circuit shown in Fig. C-27.

Solution To find $\mathbf{Z}_{Th}$, the current source is replaced by an open circuit and the voltage source is replaced by a short circuit. Thus,

$$\mathbf{Z}_{Th} = 4 + j3 = \mathbf{5\angle 36.87°\ \Omega}$$

We shall first find the short-circuit current $\mathbf{I}_{sc} = \mathbf{I}_N$, and then we shall use this to find $\mathbf{V}_{Th}$. When we short circuit the terminals a and b, the voltage source of 40∠60° V gets applied across the series combination of 4-Ω and j3-Ω impedances. Hence, the current (from left to the right) through the j3-Ω impedance is

$$\mathbf{I}_1 = \frac{40\angle 60°}{4 + j3} = 8\angle 23.1°\ \text{A}$$

Of course, the current (from left to the right) through the 100-Ω impedance is the same as the current source,

$$\mathbf{I}_2 = 6\angle 20° \text{ A}$$

Applying KCL at the terminal a, we get

$$\mathbf{I}_{sc} = \mathbf{I}_N = 6\angle 20° - 8\angle 23.1° = 2.04\angle -147.6° \text{ A} = \mathbf{-2.04\angle 32.4° \text{ A}}$$

Finally, we can determine Thevenin's voltage as

$$\mathbf{V}_{Th} = \mathbf{I}_N \mathbf{Z}_{Th} = (-2.04\angle 32.4°)\,(5\angle 36.87°) = \mathbf{-10.2\angle 69.27° \text{ V}}$$

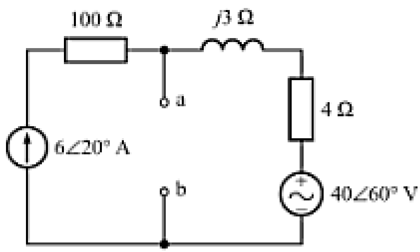


Fig. C-27

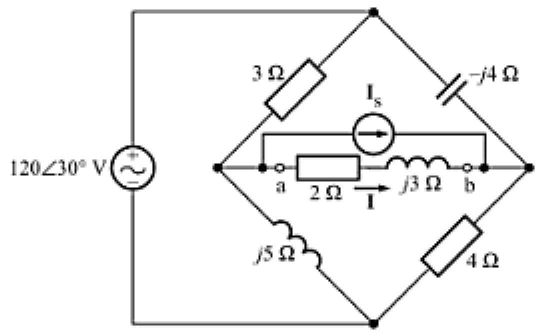


Fig. C-28

PROBLEM C-34

For the circuit shown in Fig. C-28, find (*a*) the current **I** if $\mathbf{I}_s = 0$ A, (*b*) the current **I** if $\mathbf{I}_s = 10\angle -50°$ A.

Solution

(*a*) Since the current source produces 0 A, it is equivalent to an open circuit and can be removed form the circuit. We shall make use of Thevenin's equivalent to find the current **I**. Hence, we treat the impedance $(2 + j3)$ Ω as load, and temporarily remove this from the circuit. For finding $\mathbf{Z}_{Th}$, we short circuit the voltage source so that the 3-Ω and $j5$-Ω impedances, and as well as the $-j4$-Ω and 4-Ω impedances, come in parallel. These two parallel combinations are in series between *a* and *b*. Hence,

$$\mathbf{Z}_{Th} = 3 \| (j5) + 4 \| (-j4) = \frac{3(j5)}{3 + j5} + \frac{4(-j4)}{4 - j4} = 4.26\angle -9.14° \ \Omega$$

The open-circuit voltage or Thevenin's voltage is equal to the difference in voltage drops across the $j5$-Ω and 4-Ω impedances. These voltage drops can be found by applying voltage division. Thus,

$$\mathbf{V}_{Th} = (120\angle 30°) \times \frac{j5}{3 + j5} - (120\angle 30°) \times \frac{4}{4 - j4} = 29.1\angle 16° \text{ V}$$

We can now find the current **I** through the load, as

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{29.1\angle 16°}{4.26\angle -9.14° + (2 + j3)} = \mathbf{4.39\angle -4.5° \text{ A}}$$

(*b*) The current source does not affect $\mathbf{Z}_{Th}$, and its value remains the same as found above: $\mathbf{Z}_{Th} = 4.26\angle -9.14°$ Ω. However, the current source does affect Thevenin's voltage. By superposition, it contributes a voltage given by the source-current multiplied with the impedance between terminals *a* and *b* (with the load replaced by an open circuit). This impedance is $\mathbf{Z}_{Th}$. Hence, the voltage contribution of the current source $\mathbf{I}_s$ is

$$\mathbf{V}_s = \mathbf{I}_s \mathbf{Z}_{Th} = (10\angle -50°)\,(4.26\angle -9.14°) = 42.6\angle -59.14° \text{ V}$$

PROBLEM C-36

For the circuit shown in Fig. C-31, what is the power factor at 60 Hz? What capacitor connected across the input terminals causes the power factor to be unity? What capacitor causes the overall power factor to be 0.85 lagging?

Solution $\mathbf{X}_L = j\omega L = j(120\pi)(0.03) = j11.3\ \Omega$. Therefore, the impedance of the circuit is

$$\mathbf{Z} = 4 + \frac{15(j11.3)}{15 + j11.3} = 11.9\angle 37.38°\ \Omega$$

Hence, the power factor of the circuit is given as

$$pf = \cos 37.38° = \mathbf{0.795\ (lagging)}$$

Since the capacitor is to be connected in parallel, let us determine the admittance of the circuit. Before the capacitor is connected, the admittance is

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{11.9\angle 37.38°} = 0.0842\angle -37.38°\ \text{S} = (66.9 - j51.1)\ \text{mS}$$

For unity power factor, the imaginary part of the total admittance should be zero. It means that the capacitor to be connected must have a susceptance of 51.1 mS. Consequently, its capacitance must be

$$C = \frac{B}{\omega} = \frac{51.1 \times 10^{-3}}{120\pi} = 136 \times 10^{-6} = \mathbf{136\ \mu F}$$

We need a different capacitor to make the overall power factor to be 0.85 lagging. New power factor angle is given as

$$\theta = \cos^{-1} 0.85 = 31.79°$$

By adding a parallel capacitor to a circuit, the conductance G of the circuit does not change. That is, the new conductance, $G = 66.9$ mS. The new susceptance B can be determined from the admittance triangle as

$$B = G\tan(-\theta) = (66.9 \times 10^{-3})\tan(-31.79°) = -41.5 \times 10^{-3}\ \text{S}$$

Because, it is the added capacitor that brings the change in susceptance of the circuit, its capacitance is given as

$$C = \frac{\Delta B}{\omega} = \frac{51.1 \times 10^{-3} - 41.5 \times 10^{-3}}{120\pi} = 25.6 \times 10^{-6}\ \text{F} = \mathbf{25.6\ \mu F}$$

PROBLEM C-37

Operating at a maximum capacity, a 12.47-kV alternator supplies 35 MW at a 0.7 lagging power factor. What is the maximum real power that the alternator can deliver?

Solution The maximum volt-amperes or apparent power is the limitation on the capacity of an alternator. Since the real power is the product of the apparent power and the power factor, the maximum apparent power of the given alternator is

$$S_{\max} = \frac{P}{pf} = \frac{35}{0.7} = 50\ \text{MVA}$$

At unity power factor, all of the apparent power would be the real power. This means that the maximum real power that the alternator can deliver is

$$P_{\max} = S_{\max} \times 1 = \mathbf{50\ MW}$$

PROBLEM C-39

A single-phase induction motor takes a current of 40 A at a power factor of 0.7 lagging from a 440-V, 50-Hz supply. What capacitor is to be connected across the input terminals to raise the power factor to 0.9 lagging?

Solution The power factor angle of the motor is given as

$$\theta_1 = \cos^{-1} 0.7 = 45.573°$$

After connecting the capacitor in parallel, the new power factor angle is given as

$$\theta_2 = \cos^{-1} 0.9 = 25.842°$$

The parallel connected capacitor takes a current I_C leading the applied voltage by 90°, and changes the input current I_1(= 40 A) to I_2, as shown in the phasor diagram of Fig. C-34. Since the power drawn by the circuit remains the same, the active component of the current does not change by connecting a capacitor. That is,

$$I_1 \cos \theta_1 = I_2 \cos \theta_2 \quad \text{or} \quad 40 \times 0.7 = I_2 \times 0.9$$

or

$$I_2 = \frac{40 \times 0.7}{0.9} = 31.11 \text{ A}$$

From the phasor diagram, we have

$$I_C = \text{AB} = \text{AC} - \text{BC} = I_1 \sin \theta_1 - \sin \theta_2$$
$$= 40 \sin 45.573° - 31.11 \sin 25.842° = 15 \text{ A}$$

The reactance of the capacitor, $X_C = \dfrac{V}{I_C} = \dfrac{440}{15} = 29.33\ \Omega$

Thus, the capacitance, $C = \dfrac{1}{\omega X_C} = \dfrac{1}{100\pi \times 29.33} = \mathbf{108.5\ \mu F}$

PROBLEM C-40

A series *RLC* circuit has L = 50 μH, C = 2 μF, and R = 10 Ω. (*a*) Calculate the *Q*-factor of the circuit. (*b*) If the inductance is doubled, determine the new value of C required for resonance at the same frequency, and the new *Q*-factor.

Solution

(*a*) The *Q*-factor $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{50 \times 10^{-6}}{2 \times 10^{-6}}}\ \mathbf{0.5}$

(*b*) The resonant frequency, $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{50 \times 10^{-6} \times 2 \times 10^{-6}}} = \dfrac{10^5}{2\pi}$ Hz

The changed inductance, $L_1 = 2L = 2 \times 50 \times 10^{-6} = 10^{-4}$ H.

If C_1 is the required capacitance for keeping the resonant frequency same, we must have

$$\frac{10^5}{2\pi} = \frac{1}{2\pi\sqrt{L_1 C_1}} \quad \text{or} \quad C_1 = \frac{1}{L_1 \times 10^{10}} = \frac{1}{10^{-4} \times 10^{10}} = 10^{-6} \text{ F} = \mathbf{1\ \mu F}$$

The new *Q*-factor $= \dfrac{1}{R}\sqrt{\dfrac{L_1}{C_1}} = \dfrac{1}{10}\sqrt{\dfrac{100 \times 10^{-6}}{1 \times 10^{-6}}} = \mathbf{1.0}$

PROBLEM C-41

A series *RLC* circuit with $R = 25\ \Omega$ and $L = 0.6$ H results in a leading phase angle of 60° at a frequency of 40 Hz. At what frequency will the circuit resonate?

Solution The reactance of the inductance at 40 Hz is

$$X_L = 2\pi fL = 2\pi \times 40 \times 0.6 = 150.8\ \Omega$$

If X_C is the reactance of the capacitor in the circuit, its total impedance is given as

$$\mathbf{Z} = R + j(X_L - X_C) = 25 + j(150.8 - X_C)\ \Omega$$

Since, the phase angle is 60° *leading*, we should have

$$\frac{150.8 - X_C}{25} = -\tan 60° \quad \Rightarrow \quad X_C = 193.3\ \Omega$$

Therefore, the value of the capacitance must be

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 40 \times 19.3} = 20.58\ \mu\text{F}$$

We can now calculate the frequency of resonance of the circuit as

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.6 \times 20.58 \times 10^{-6}}} = \mathbf{45.29\ Hz}$$

PROBLEM C-42

A coil of resistance 40 Ω and inductance 0.75 H forms a part of a series circuit for which the resonant frequency is 55 Hz. If the supply is 250 V, 50 Hz, find (*a*) the line current, (*b*) the power factor, (*c*) the power consumed, and (*d*) the voltage across the coil.

Solution

(*a*) For a series resonant circuit, the resonant frequency is given as

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 55)^2 \times 0.75} = 11.16\ \mu\text{F}$$

The reactances of the inductance and capacitance at 50 Hz are given as

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.75 = 235.6\ \Omega$$

and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 11.16 \times 10^{-6}} = 285.2\ \Omega$$

Therefore, the impedance of the circuit is

$$\mathbf{Z} = R + j(X_L - X_C) = 40 + j(235.6 - 285.2) = (40 - j49.6)\ \Omega = 63.72\angle{-51.12°}\ \Omega$$

We can now calculate the line current as

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{250\angle 0°}{63.73\angle{-51.13°}} = \mathbf{3.92\angle 51.13°\ A}$$

(*b*) The power factor, $pf = \cos 51.13° =$ **0.6276 (leading)**

(*c*) The power consumed, $P = VI\cos\phi = 250 \times 3.92 \times \cos 51.13° =$ **615 W**

(*d*) The voltage across the coil,

$$\mathbf{V}_{\text{coil}} = \mathbf{I}\mathbf{Z}_{\text{coil}} = \mathbf{I}(R + jX_L) = (3.92\angle 51.13°)\,(40 + j235.6) = \mathbf{936.8\angle 131.5°\ V}$$

PROBLEM C-43

An inductive circuit of resistance 2 Ω and inductance 0.01 H is connected to a 250-V, 50-Hz. What value of capacitance to be placed in parallel with the inductive circuit will produce resonance? Also, find the current taken from the supply at resonance.

Solution The circuit diagram is shown in Fig. C-35*a*. The impedance of the inductive branch is given as

$$\mathbf{Z}_1 = 2 + j(2\pi \times 50 \times 0.01) = (2 + j3.142)\ \Omega = 3.725\angle 57.52°\ \Omega$$

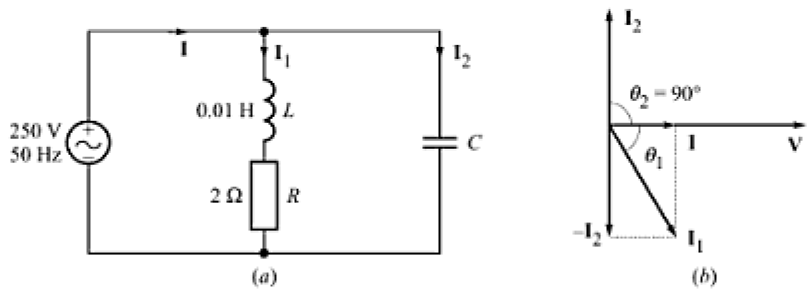


Fig. C-35

At resonance, the net current **I** should be in phase with the applied voltage **V**, as shown in Fig. C-35*b* This can happen only if the reactive component of the inductive-circuit current I_1 is equal and opposite in phase to the current I_2 through the capacitor. Hence,

$$I_2 = I_1 \sin\theta_1.$$

or
$$\frac{V}{X_C} = \frac{V}{Z_1}\cdot\frac{X_L}{Z_1} \quad\Rightarrow\quad Z_1^2 = X_L X_C = (\omega_0 L)\frac{1}{(\omega_0 C)} = \frac{L}{C}$$

Hence,
$$C = \frac{L}{Z_1^2} = \frac{0.01}{(3.725)^2} = \mathbf{720.7\ \mu F}$$

The current at resonance, $I = I_1\cos\theta_1 = \frac{V}{Z_1}\cdot\frac{R}{Z_1} = \frac{VR}{Z_1^2} = \frac{250\times 2}{(3.725)^2} = \mathbf{36.03\ A}$

PROBLEM C-44

A coil having a resistance of 50 Ω and an inductance of 0.2 H is connected in series with a capacitor C_1 across an ac voltage source of variable frequency, as shown in Fig. C-36*a*. The current through the circuit is found to be maximum when the frequency is 50 Hz. Calculate the capacitance C_2 of the condenser to be connected in parallel (Fig. C-36*b*) with the above series combination so that the total current taken from the ac supply is a minimum when the frequency is 100 Hz. Taking the supply voltage as reference, draw the phasor diagram representing all the currents at 100 Hz.

Solution When the current is maximum at 50 Hz, we must have

$$X_{L1} = X_{C1} \quad\Rightarrow\quad C_1 = \frac{1}{(2\pi f)^2 L_1} = \frac{1}{(2\pi\times 50)^2\times 0.2} = 50.66\ \mu\text{F}$$

At a frequency of 100 Hz, we have

$$X'_{L1} = 2\pi f' L_1 = 2\pi \times 100 \times 0.2 = 125.7\ \Omega$$

and
$$X'_{C1} = \frac{1}{2\pi f' L_1} = \frac{1}{2\pi\times 100\times 50.66\times 10^{-6}} = 31.42\ \Omega$$

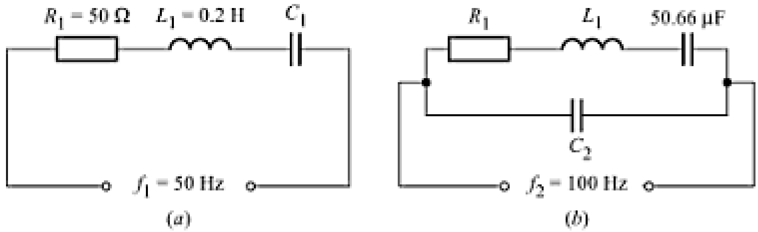


Fig. C-36

Hence, $\mathbf{Z}_1 = 50 + j(125.7 - 31.42) = (50 + j94.28)\ \Omega = 106.7\angle 62°\ \Omega$

and $\mathbf{Z}_2 = 0 - jX_{C2} = X_{C2}\angle -90°\ \Omega$

The corresponding admittances of the parallel branches are

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{106.7\angle 62°} = 9.372\angle -62°\ \text{mS} \qquad \text{and} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{X_{C2}\angle -90°} = \omega C_2\angle 90°\ \text{S}$$

Therefore, the equivalent admittance of the parallel combination,

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 = 9.372 \times 10^{-3}\angle -62° + \omega C_2\angle 90° = 4.4 \times 10^{-3} - j(8.275 \times 10^{-3} - \omega C_2)$$

Since, $\mathbf{I} = \mathbf{VY}$, for minimum supply current (that is, for resonant condition), we must have minimum **Y.** This can happen only when the imaginary component of the **Y** is made zero,

$$8.275 \times 10^{-3} - j\omega C_2 = 0 \quad \Rightarrow \quad C_2 = \frac{8.275 \times 10^{-3}}{2\pi \times 100} = \mathbf{13.17\ \mu F}$$

At resonant condition, the overall admittance is $\mathbf{Y} = 4.4 \times 10^{-3}$ S, the admittance of the second branch is

$$\mathbf{Y}_2 = \omega C_2\angle 90° = (2\pi \times 100 \times 13.17 \times 10^{-6})\angle 90° = 0.008275\angle 90°\ \text{S}$$

Let the supply voltage be $V\angle 0°$ volts. Then, the supply current and the branch currents are given as

$$\mathbf{I} = \mathbf{YV} = (4.4 \times 10^{-3})\,(V\angle 0°) = 0.0044\,V\ \text{A}$$

$$\mathbf{I}_1 = \mathbf{Y}_1\mathbf{V} = (9.372 \times 10^{-3}\angle -62°)\,(V\angle 0°) = 0.009372\,V\angle -62°\ \text{A}$$

and $\mathbf{I}_2 = \mathbf{Y}_2\mathbf{V} = (0.008275\angle 90°)\,(V\angle 0°) = 0.008275\,V\angle 90°\ \text{A}$

Thus, the phasor diagram of the circuit of Fig. C-36*b* at resonance can be drawn as shown in Fig. C-37.

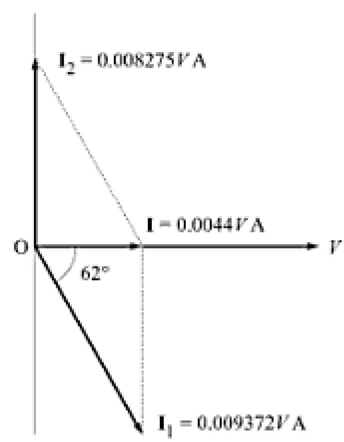


Fig. C-37

PROBLEM C-45

Each phase of a star-connected load consists of a resistance of 100 Ω in parallel with a capacitance of 32 μF. Calculate the line current, the power absorbed, the total kVA, and the power factor, when connected to a 415-V, 50-Hz, 3-phase supply.

Solution For each phase: $\mathbf{Z}_1 = 100\ \Omega$; $\mathbf{Z}_2 = -j/(2\pi \times 50 \times 32 \times 10^{-6}) = -j99.5\ \Omega$

Therefore, the load impedance per phase,

$$\mathbf{Z}_{\text{ph}} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{100(-j99.5)}{100 - j99.5} = 70.53\angle -45.14^\circ\ \Omega$$

For star-connected load, $V_{\text{ph}} = V_{\text{L}}/\sqrt{3} = 415/\sqrt{3} = 239.6$ V

We can now calculate the phase current,

$$I_{\text{ph}} = \frac{V_{\text{ph}}}{Z_{\text{ph}}} = \frac{239.6}{70.53} = 3.397 \text{ A}$$

∴ Line current, $I_L I_{\text{ph}}$ = **3.397 A**

The power factor, $pf = \cos 45.14^\circ$ = **0.7054 (leading)**

The power absorbed, $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 415 \times 3.397 \times \cos 45.14^\circ$ = **1722 W**

The total kVA $= \sqrt{3}\, V_L I_L = \sqrt{3} \times 415 \times 3.397$ = **2442 kVA**

PROBLEM C-46

A 440-V, 50-Hz, 3-phase supply drives a 3-phase, delta-connected induction motor operating at rated load in parallel with another star-connected load having impedance of (20 + j15.65) Ω per phase, as shown in Fig. C-38. The motor draws a power of 75 kW at a power factor of 0.8 lagging and it can be considered as three impedances. Determine (*a*) the line and phase currents in the motor, (*b*) the line and phase currents in the impedance load, (*c*) total line current, and (*d*) the total power consumed.

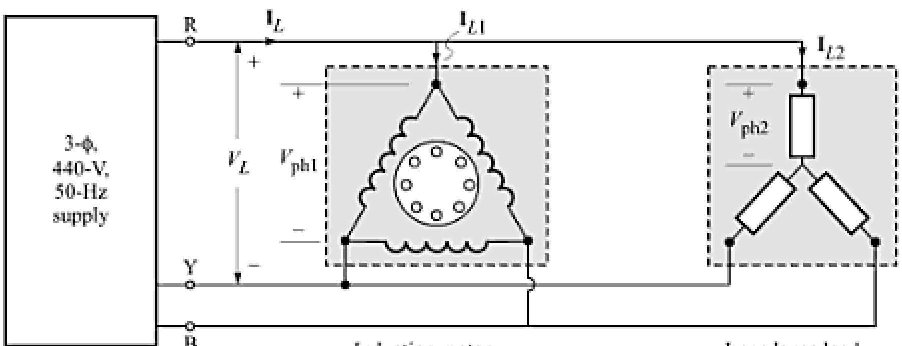


Fig. C-38 *An induction motor and an impedance load connected to a 3-phase supply.*

Solution

(*a*) For the *delta-connected motor*, $\mathbf{V}_{\text{ph1}} = \mathbf{V}_L = 440\angle 0^\circ$ V.

The magnitude of the line current for the motor can be found from the power draw by it,

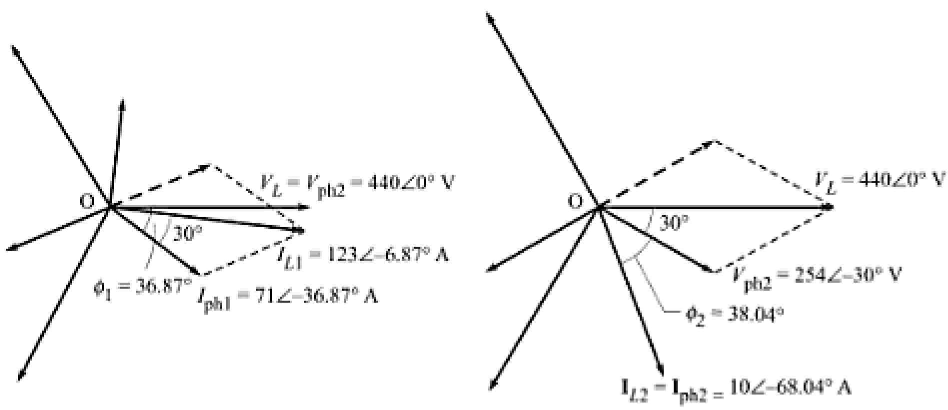
$$I_{L1} = \frac{P}{\sqrt{3}V_L \cos\phi} = \frac{75000}{\sqrt{3} \times 440 \times 0.8} = 123 \text{ A}$$

The phase angle of the motor, $\phi_1 = \cos^{-1}0.8 = 36.87°$. As can be seen from the phasor diagram given in Fig. C-39*a*, the current I_{ph1} lags the voltage V_{ph1} by 36.87°, and the line current I_{L1} leads the phase current I_{ph1} by 30°. Hence, the phase angle of line current I_{L1},

$$\phi_{L1} = 30° - 36.87° = -6.87°$$

Thus, the line current and the phase current for the motor are given as

$$\mathbf{I}_{L1} = \mathbf{123\angle{-6.87°}\ A} \qquad \text{and} \qquad \mathbf{I}_{ph1} = \frac{123}{\sqrt{3}}\angle{-36.87°} = \mathbf{71\angle{-36.87°}\ A}$$



(*a*) For the induction motor. (*b*) For the impedance load.

Fig. C-39 *Phasor diagrams.*

(*b*) For the *star-connected impedance load*, as can be seen from the phasor diagram shown in Fig. C-39*b*, the phase voltage is given as

$$\mathbf{V}_{ph2} = \frac{V_L}{\sqrt{3}}\angle{-30°} = \frac{440}{\sqrt{3}}\angle{-30°} = 254\angle{-30°}\ \text{V}.$$

The impedance of the load per phase, $\mathbf{Z}_{ph2} = (20 + j15.65)\ \Omega = 25.4\angle 38.04°\ \Omega$.

Hence, the line current and phase current for the impedance load are given as

$$\mathbf{I}_{L2} = \mathbf{I}_{ph2} = \frac{\mathbf{V}_{ph2}}{\mathbf{Z}_{ph2}} = \frac{254\angle{-30°}}{25.4\angle 38.04°} = \mathbf{10\angle{-68.04°}\ A}$$

(*c*) The total line current, $\mathbf{I}_L = \mathbf{I}_{L1} + \mathbf{I}_{L2} = (123\angle{-6.87°}) + (10\angle{-68.04°}) = \mathbf{128.1\angle{-10.79°}\ A}$

(*d*) The total power consumed,

$$\begin{aligned} P &= P_1 + P_2 = 3V_{ph1}I_{ph1}\cos\phi_1 + 3V_{ph2}I_{ph2}\cos\phi_2 \\ &= 3 \times 440 \times 71 \times \cos 36.87° + 3 \times 254 \times 10 \times \cos 38.04° \\ &= 75000 + 6001 \approx \mathbf{81\ kW} \end{aligned}$$

PROBLEM C-47

A three-phase balanced supply system with phase voltage of 120 V and phase-sequence of abc is connected to an unbalanced delta-connected load, as shown in Fig. C-40. Find (*a*) $\mathbf{I}_{aA}$, (*b*) $\mathbf{I}_{bB}$, (*c*) $\mathbf{I}_{cC}$, and (*d*) the total complex power supplied by the source.

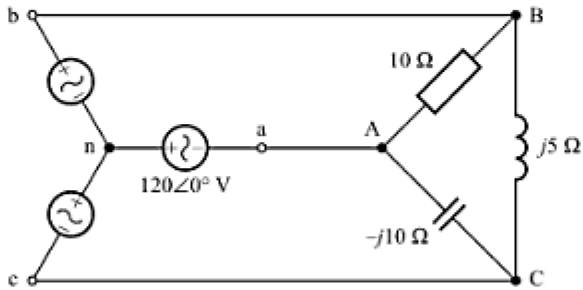


Fig. C-40 *An unbalanced 3-phase load connected to a balanced 3-phase supply.*

Solution The phase voltages of the 3-phase supply are given as

$$\mathbf{V}_{an} = 120\angle 0° \text{ V}; \quad \mathbf{V}_{bn} = 120\angle{-120°} \text{ V}; \quad \text{and} \quad \mathbf{V}_{cn} = 120\angle 120° \text{ V}$$

Therefore, the line voltages are

$$\mathbf{V}_{AB} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 0° - 120\angle{-120°} = 207.8\angle 30° \text{ V}$$

$$\mathbf{V}_{BC} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = 120\angle{-120°} - 120\angle 120° = 207.8\angle{-90°} \text{ V}$$

and

$$\mathbf{V}_{CA} = \mathbf{V}_{cn} - \mathbf{V}_{an} = 120\angle 120° - 120\angle 0° = 207.8\angle 150° \text{ V}$$

We first calculate the currents in each phase of the load,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{207.8\angle 30°}{10} = 20.78\angle 30° \text{ A}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{207.8\angle{-90°}}{j5} = -41.4\angle 0° \text{ A}$$

and

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{207.8\angle 150°}{-j10} = 20.78\angle{-120°} \text{ A}$$

We can now find the required line currents,

(*a*) $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 20.78\angle 30° - 20.78\angle{-120°} = \mathbf{40.07\angle 45° \text{ A}}$

(*b*) $\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = -41.4\angle 0° - 20.78\angle 30° = \mathbf{60.22\angle{-170.1°} \text{ A}}$

(*c*) $\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 20.78\angle{-120°} - (-41.4\angle 0°) = \mathbf{35.85\angle{-30°} \text{ A}}$

(*d*) The total complex power is given as

$$\begin{aligned}\mathbf{S} &= \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = \mathbf{V}_{AB}\mathbf{I}^*_{AB} + \mathbf{V}_{BC}\mathbf{I}^*_{BC} + \mathbf{V}_{CA}\mathbf{I}^*_{CA} \\ &= (207.8\angle 30°)\,(20.78\angle{-30°}) + (207.8\angle{-90°})\,(-41.4\angle 0°) + (207.8\angle 150°)\,(20.78\angle 120°) \\ &= (4318 + j0) + (0 + j8603) + (0 - j4318) \\ &= \mathbf{(4318 + j4285) \text{ VA}}\end{aligned}$$

COMMENTS

Note that, as it should be, the active power is contributed only by the impedance $\mathbf{Z}_{AB}$ (= 10 Ω); both the impedances $\mathbf{Z}_{BC}$(= $j5$ Ω) and $\mathbf{Z}_{CA}$(= $-j10$ Ω) contribute to the reactive power.

C. 2. PRACTICE PROBLEMS

(A) SIMPLE PROBLEMS

C-1. Find the instantaneous value of $i = 70 \sin 400\pi t$ A at $t = 3$ ms. [**Ans.** –41.1 A]

C-2. A current wave has a peak value of 58 mA and a radian frequency of 90 rad/s. Find the instantaneous current at $t = 23$ ms. [**Ans.** 50.9 mA]

C-3. What is the shortest time required for a 2.1 krad/s sinusoid to increase from zero to four-fifth of its peak value? [**Ans.** 0.442 ms]

C-4. Evaluate (*a*) $v = 200 \sin(339t + \pi/7)$ V and (*b*) $i = 67 \cos(301t - 42°)$ mA at $t = 11$ ms.

[**Ans.** (*a*) –172 V; (*b*) –56.9 mA]

C-5. Find the angle by which i_1 lags v_1, if $i_1 = 4.5 \sin(\omega t - 20°)$ A and v_1 is equal to

(*a*) $v_1 = 125 \sin(\omega t + 60°)$ V;

(*b*) $v_1 = 125 \sin(\omega t - 120°)$ V

[**Ans.** (*a*) 80°; (*b*) –100°]

C-6. A 30-Ω resistor has a voltage of $v = 170 \sin(314t - 20°)$ V across it. Find the average power dissipation of the resistor. [**Ans.** 482 W]

C-7. Find the effective value of a periodic voltage that has a value of 20 V for one half-period and a value of –10 V for the other half-period. [**Ans.** 15.8 V]

C-8. Find the frequencies at which a 250-mH inductor has reactances of 30 Ω and 50 kΩ.

[**Ans.** 19.1 Hz, 31.8 kHz]

C-9. Find the voltage across a 2-H inductor for the following currents, assuming passive-component references for voltage and current:

(*a*) 10 A,

(*b*) $10 \sin(314.2t + 10°)$ A, and

(*c*) $10 \cos(10^4 t - 20°)$ A

[**Ans.** (*a*) 0 V; (*b*) $6.284 \cos(314.2t + 10°)$ kV; (*c*) $0.2 \cos(10^4 t + 70°)$ MV]

C-10. Find the capacitances of capacitors that have a reactance of 500 Ω at (*a*) 377 rad/s, (*b*) 10 kHz, and (*c*) 22.5 MHz.

[**Ans.** (*a*) 5.31 µF; (*b*) 31.8 nF; (*c*) 14.1 pF]

C-11. What is the voltage across a 30-mH inductor that has a 40-mA, 50-Hz current flowing through it?

[**Ans.** 0.377 V]

C-12. What current flows through a 0.1-µF capacitor that has 200 V at 400 Hz across it? [**Ans.** 50.3 mA]

C-13. Find the average power absorbed by a circuit component that has a voltage $v = 20.3 \sin(314t - 10°)$ V across it when a current $i = 15.6 \sin(314t - 30°)$ flows through it.

[**Ans.** 148.8 mW]

C-14. What current flows through a 2-µF capacitor for voltages of (*a*) $v = 5 \sin(314t - 30°)$ V and (*b*) $v = 75 \cos(10^4 t + 60°)$ V?

[**Ans.** (*a*) $i = 3.14 \sin(314t + 60°)$ mA; (*b*) $i = -15.6 \sin(10^4 t + 60°)$ A]

C-15. In the circuit given in Fig. C-41, find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

[**Ans.** 28.3∠45° A, 20∠90° A, 20∠0° A]

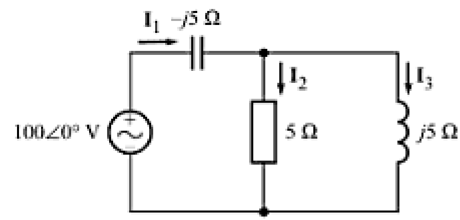


Fig. C-41

C-16. Use voltage division to find $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in the circuit shown in Fig. C-42.

[**Ans.** 100∠30° V, 5000∠120° V, 5000∠–60° V]

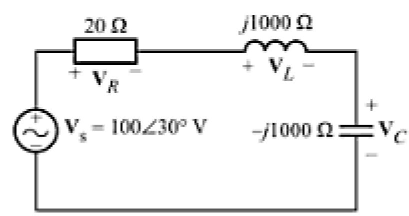


Fig. C-42

C-17. A coil is energised by a 120-V, 50-Hz source. It draws a current of 2 A that lags the voltage by 40°. What are the coil resistance and inductance?

[**Ans.** 46 Ω, 0.123 H]

C-18. A load has a voltage of $\mathbf{V} = 120\angle 30°$ V and a current of $\mathbf{I} = 30\angle 50°$ A at a frequency of 400 Hz. Find the two-element series circuit that could represent the load.

[**Ans.** $R = 3.76\ \Omega$, $C = 291\ \mu$F]

C-19. A capacitor is in series with a coil that has 1.5-H of inductance and 5 Ω of resistance. Find the capacitance that makes the combination purely resistive at 50 Hz. [**Ans.** 6.755 μF]

C-20. Find the total impedance at 1 krad/s of a 1-H inductor and a 1-μF capacitor connected (*a*) in series, and (*b*) in parallel. [**Ans.** (*a*) 0 Ω; (*b*) ∞ Ω]

C-21. Find the impedance across the terminals A and B for the circuit shown in Fig. C-43. The operating frequency is 50 Hz. [**Ans.** $(10 - j30.87)\ \Omega$]

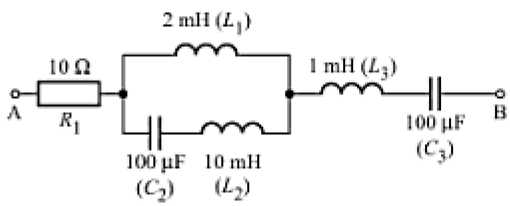


Fig. C-43

C-22. Use current division concept to find $\mathbf{I}_L$ for the circuit shown in Fig. C-44.

[**Ans.** $-2.22\angle -36.3°$ A]

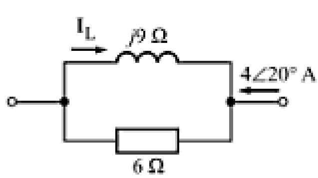


Fig. C-44

C-23. Three alternating currents:

$$i_1 = 150\sin(\omega t + \pi/4)\ \text{A},$$
$$i_2 = 40\sin(\omega t + \pi/2)\ \text{A}, \quad \text{and}$$
$$i_3 = 80\sin(\omega t - \pi/6)\ \text{A}$$

are fed simultaneously to a common conductor. Find the equation of the resultant current and its rms value.

[**Ans.** $i = 204.9\sin(\omega t + 31.17°)$ A, 144.9 A

C-24. Three sinusoidal voltages acting in series are given by

$$v_1 = 10\sin 400t\ \text{V},$$
$$v_2 = 10\sqrt{2}\sin(400t - 45°)\ \text{V}, \quad \text{and}$$
$$v_3 = 20\cos 400t\ \text{V}$$

Determine (*a*) an expression for the resultant voltage, and (*b*) the frequency and the rms value of the resultant voltage.

[**Ans.** (*a*) $v = 22.36\sin(400t + 26.57°)$ V; (*b*) 63.66 Hz, 15.81 V]

C-25. A circuit consists of a resistance R and a capacitive reactance of 60 Ω connected in series. Determine the value of R for which the power factor of the circuit is 0.8. [**Ans.** 80 Ω]

C-26. An iron choke takes 4 A when connected to a 20-V dc supply, and takes 5 A when connected to a 65-V, 50-Hz supply. Determine (*a*) the resistance and the inductance of the coil, (*b*) the power drawn by the coil, and (*c*) the power factor.

[**Ans.** (*a*) 5 Ω, 38.2 mH; (*b*) 125 W; (*c*) 0.3846 (lagging)]

C-27. The voltage applied to a circuit is $v = 100\sin(\omega t + 30°)$ V and the current flowing in the circuit is $i = 15\sin(\omega t + 60°)$ A. Determine the impedance, the resistance, the reactance, the power factor and the power.

[**Ans.** 6.67 Ω, 5.776 Ω, 3.335 Ω, 0.866 (leading), 649 W]

C-28. The current in a circuit is given as $(4.5 + j12)$ A, when the applied voltage is $(100 + j150)$ V. Determine (*a*) the complex expression for the impedance, stating whether it is inductive or capacitive, (*b*) the power, and (*c*) the power factor.

[**Ans.** (*a*) $(13.7 - j3.197)\ \Omega$, Capacitive; (*b*) 2.251 kW; (*c*) 0.9738 (leading)]

C-29. A series circuit consists of a noninductive resistance of 5 Ω and an inductive reactance of 10 Ω. When connected to a single-phase ac supply, it draws a current

$$i(t) = 27.89\sin(628.3t - 45°)\ \text{A}$$

Find (*a*) the voltage applied to the series circuit in the form $V_m\sin(\omega t + \theta°)$, (*b*) the inductance, and (*c*) the power drawn by the circuit.

[**Ans.** (*a*) $v = 311.8\sin(628.3t + 18.43°)$ V; (*b*) 15.92 mH; (c) 1945 W]

C-30. A coil takes 2.5 A when connected across 200-V, 50-Hz mains. The power consumed by the coil is found to be 400 W. Find the inductance and the power factor of the coil.

[**Ans.** 0.1528 H, 0.8 (lagging)]

C-31. A pure inductive coil is connected in series with a 10-Ω resistor to a 50-Hz ac source. The voltages across the resistor and the inductor are found to be 30 V and 40 V respectively. Find the value of the inductive reactance and the supply voltage.

[**Ans.** 13.33 Ω, 50 V]

C-32. A series circuit with 10-Ω resistor and 20-mH inductor has a current $i = 2\sin 500t$ A. Obtain the total voltage across the circuit and the angle by which the current lags the voltage.

[**Ans.** $v = 28.28\sin(500t + 45°)$ V, 45°]

C-33. A resistor and a capacitor are connected in series across a 150-V ac supply. When the frequency of the supply is 40 Hz, the current drawn is 5 A, and when the frequency is 50 Hz, the current is 6 A. Find the resistance and the capacitance of the circuit. [**Ans.** 11.7 Ω, 144 μF]

C-34. A circuit consists of a resistance of 8 Ω and a series capacitive reactance of 6 Ω. A voltage $v = 141\sin 314t$ V is applied to the circuit. Find (*a*) the complex impedance, (*b*) the rms and instantaneous values of the current, (*c*) the power delivered to the circuit, (*d*) the equation for the voltage appearing across the capacitor, and (*e*) the value of the capacitance.

[**Ans.** (*a*) $(8 - j6) = 10\angle -36.87°$ Ω; (*b*) 10 A, $14.1\sin(314t + 36.87°)$ A; (*c*) 800 W; (*d*) $84.85\sin(314t - 53.13°)$ V; (*e*) 530.8 μF]

C-35. A voltage $230\angle 30°$V is applied across two series components, one of which is a 20-Ω resistor and the other is a coil with an impedance of $40\angle 20°$ Ω. Find the individual component voltage drops.

[**Ans.** $\mathbf{V}_R = 77.6\angle 16.6°$ V; $\mathbf{V}_z = 155.2\angle 36.6°$ V]

C-36. A phasor-domain circuit has $200\angle 15°$ V applied across three series-connected components having impedances of $20\angle 15°$ Ω, $30\angle -40°$ Ω, and $40\angle 50°$ Ω. Use the concept of voltage division to find the voltage drop **V** across the component with impedance of $40\angle 50°$ Ω.

[**Ans.** $\mathbf{V} = 114\angle 51.3°$ V]

C-37. What is the total impedance of three parallel components that have impedances of $\mathbf{Z}_1 = 2.5\angle 75°$ Ω, $\mathbf{Z}_2 = 4\angle -50°$, and $\mathbf{Z}_3 = 5\angle 45°$ Ω?

[**Ans.** $\mathbf{Z}_T = 1.9\angle 39.7°$ Ω]

C-38. A load has a voltage of $\mathbf{V} = 120\angle 20°$ V and a current of $\mathbf{I} = 48\angle 60°$ A, both at 2 kHz. Find the two-element parallel circuit which could represent the load. [**Ans.** $R = 3.26$ Ω, $C = 20.5$ μF]

C-39. A resistor and a parallel 1-μF capacitor draw 0.48 A when a 120-V, 400-Hz source is applied. Find the admittance of the circuit in polar form.

[**Ans.** $4\angle 38.9°$ mS]

C-40. Find the total impedance of two parallel components which have identical impedances of $100\angle 60°$ Ω.

[**Ans.** $50\angle 60°$ Ω]

C-41. What is the total impedance to two components which have impedances of $80\angle -30°$ Ω and $60\angle 40°$ Ω? [**Ans.** $41.65\angle 10.71°$ Ω]

C-42. For the circuit shown in Fig. C-45, find (*a*) the open-circuit voltage $\mathbf{V}_{ab}$, (*b*) the downward current in a short-circuit between a and b, and (*c*) Thevenin's equivalent impedance $\mathbf{Z}_{ab}$ in parallel with the current source.

[**Ans.** (*a*) $16.77\angle -33.4°$ V; (*b*) $3\angle 30°$ A; (*c*) $(2.5 - j5)$ Ω]

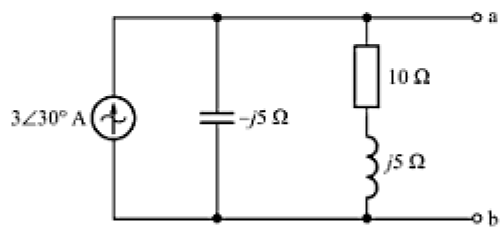


Fig. C-45

C-43. Find $\mathbf{Z}_{ab}$ for the circuit shown in Fig. C-46 if ω is equal to (*a*) 800 rad/s, and (*b*) 1600 rad/s.

[**Ans.** (*a*) $(477.9 + j175.6)$ Ω; (*b*) $(587.6 + j119.8)$ Ω]

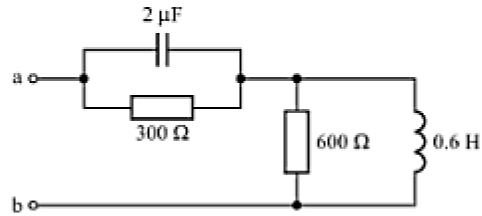


Fig. C-46

C-44. If $\omega = 100$ rad/s in the circuit shown in Fig. C-47, find (*a*) $\mathbf{Z}_{in}$, and (*b*) $\mathbf{Z}_{in}$ if a short circuit is connected between *x* and *y*.

[**Ans.** (*a*) $(22 - j6)\ \Omega$; (*b*) $(9.6 + j2.8)\ \Omega$]

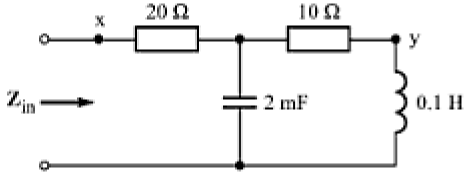


Fig. C-47

C-45. For the network of Fig. C-48, find $\mathbf{Z}_{in}$ at $\omega = 4$ rad/s if terminals *a* and *b* are (*a*) open-circuited, and (*b*) short-circuited.

[**Ans.** (*a*) $(10.56 - j1.92)\ \Omega$; (*b*) $(9.07 + j0.246)\ \Omega$]

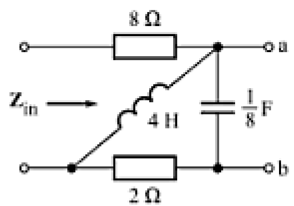


Fig. C-48

C-46. An ac circuit consists of a resistance of 5 Ω, an inductance of 0.1 H, and a capacitance of 100 µF, all in series. Determine for this circuit (*a*) the total reactance, (*b*) the impedance, (*c*) the admittance, (*d*) the susceptance, and (*e*) the conductance, at a frequency of 60 Hz.

[**Ans.** (*a*) $j11.17\ \Omega$; (*b*) $12.24\angle 65.9°\ \Omega$; (*c*) $0.0817\angle -65.9°$ S; (*d*) $j0.0746$ S; (*e*) 0.0334 S]

C-47. Find the active and reactive components of the current taken by a series circuit consisting of an inductance of 0.1 H and resistance of 8 Ω and a capacitor of 120 µF, when connected to a 240-V, 50-Hz supply. [**Ans.** 21.84 A, 13.35 A]

C-48. A coil of resistance 20 Ω and inductance 100 mH is connected in series with a capacitance of 40 µF across a 100-V, 50-Hz supply. Calculate (*a*) the magnitude of the current, (*b*) the power factor, (*c*) the phase angle, and (*d*) the voltages across each element.

[**Ans.** (*a*) 1.918 A, (*b*) 0.3835 (leading), (*c*) 67.45°, (*d*) $38.36\angle 67.45°$ V, $60.26\angle 157.5°$ V, $152.6\angle -22.55°$ V]

C-49. A current of 10 A flows in a series circuit consisting of $R = 10\ \Omega$, $L = 0.1$ H, and $C = 100\ \mu$F. Find the impedance, the power factor and the power, if the mains frequency is 50 Hz.

[**Ans.** $10.01\angle -2.348°\ \Omega$, 0.9992 (leading), 1000 W]

C-50. What power is consumed by a circuit that has an input admittance of $(0.4 + j0.5)$ S and an input current of 30 A? [**Ans.** 878 W)]

C-51. A current of 4 A flows through a load of $30\angle 40°\ \Omega$. Determine the resulting power components, i.e., the complex, the real, the reactive, and the apparent powers.

[**Ans.** 368 W, 309 kW, 3.68 kVAR, 4.8 kVA]

C-52. Find the power components of a load that draws $20\angle -30°$ A when connected to a source of $240\angle 20°$ V. [**Ans.** 3.09 kW, 3.68 kVAR, 4.8 kVA]

C-53. A coil of resistance 5 Ω and inductance 10 mH is connected in series with a condenser of 100 µF capacity across a single-phase, 230-V, 50-Hz supply. Find (*a*) the total power consumed, (*b*) the power factor, (*c*) the frequency at which resonance occurs, and (*d*) the current at resonance.

[**Ans.** (*a*) 312 W; (*b*) 0.1719 (leading); (*c*) 159.2 Hz; (*d*) 46 A]

C-54. A choke coil of resistance 5 Ω and an inductance 0.6 H, is in series with a capacitor of 10 µF. When 200 V ac is applied to it and the frequency is adjusted to give resonance, find the voltage across the capacitor. [**Ans.** 2794 V]

C-55. A circuit consisting of a coil of inductance 10 mH and resistance 300 Ω connected in parallel with a certain capacitor resonates at 48 kHz. Determine the capacitance of the capacitor and the dynamic impedance of the circuit.

[**Ans.** 1.09 nF, 30.58 kΩ]

C-56. If the bandwidth of a resonant circuit is 10 kHz and the lower half-power frequency is 110 kHz, what is its upper half-power frequency? What is the value of its quality factor? [**Ans.** 120 kHz, 11.5]

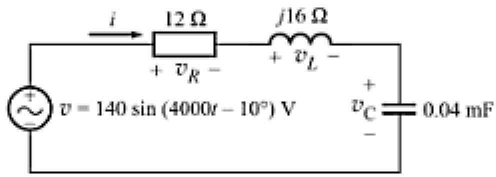


Fig. C-54

C-74. A circuit consisting mf a variable resistor in series with a capacitance of 80 μF is connected across a 120-V, 50-Hz supply, Calculate the value of the resistance so that the power absorbed is 100 W.

[**Ans.** 132 Ω or 12 Ω]

C-75. An adjustable resistor R in series with a capacitance of 25 μF draws a current of 0.8 A, when connected across a 50-Hx supply. Calculate (*a*) the value of resistor so that the voltage across the capacitor is half of the supply voltage, (*b*) the power, and (*c*) the power factor.

[**Ans.** (*a*) 220.6 Ω; (*b*) 141.2 W; (*c*) 0.866 (leading)]

C-76. A coil of 0.8 pf is connected in series with a 100-μF condenser. The frequency of supply is 50 Hz. The potential drop across the coil is found to be equal to the potential drop across the capacitor. Calculate the resistance and the inductance of the coil.

[**Ans.** 25.46 Ω, 60.8 mH]

C-77. The voltage and current for a series RLC circuit are:

$$v = 141.4 \sin (314t + 45°) \text{ V} \quad \text{and}$$
$$i = 28.28 \sin (314t - 15°) \text{ A}$$

Find (*a*) the rms values of voltage and current, (*b*) the power factor and the power consumption of the circuit, (c) the time period of the ac supply, and (*d*) the resistance of the circuit.

[**Ans.** (*a*) 100 V, 20 A; (*b*) 0.5 (lagging), 1000 W; (*c*) 0.02 s; (*d*) 2.5 Ω]

C-78. The voltage $v = 141.42 \sin (157.08t + \pi/12)$ V is applied to an ac circuit, and an ac ammeter, a wattmeter and a power-factor meter are connected to measure the respective quantities. The reading of the ammeter is 5 A and that of the power factor meter is 0.5 lagging. Find (*a*) the expression for the instantaneous value of the current, (*b*) the wattmeter reading, (*c*) the impedance of the circuit in rectangular form.

[**Ans.** (*a*) $i = 7.07 \sin (157.08t - \pi/4)$ A; (*b*) 250 W; (*c*) $(10 + j17.32)$ Ω]

C-79. When an alternating voltage $(80 + j60)$ V is applied to a circuit, the resulting current is $(-4 + j10)$ A. Find (*a*) the impedance of the circuit, (*b*) the phase angle, (*c*) the power consumed, and (*d*) the apparent power.

[**Ans.** (*a*) $(2.414 - j8.966)$ Ω; (*b*) 74.93°; (*c*) 280 W; (*d*) 1077 VA]

C-80. When an iron-core coil is connected to a 12-V dc supply, it draws a current of 2.5 A, and when it is connected to a 230-V, 50-Hz supply, it draws 2 A current and consumes 50 W power. Determine for this value of current, (*a*) the power loss in the iron core, (*b*) the inductance of the coil, (*c*) the power factor, (*d*) the value of resistance which is equivalent to the effect of iron loss.

[**Ans.** (*a*) 20 W; (*b*) 0.3657 H; (*c*) 0.1087 (lagging); (*d*) 7.7 Ω]

C-81. (*a*) Find the equation for the instantaneous current, when a voltage represented by $v = 141.4 \sin 314t$ V is applied to a circuit consisting of $R = 50$ Ω and $L = 0.2$ H. (*b*) Calculate the value of the capacitance to be connected in series with this circuit to obtain minimum impedance of the circuit. (*c*) Find the power drawn by the circuit under this condition.

[**Ans.** (*a*) $i = 1.762 \sin (314t - 51.47°)$ A; (*b*) 50.7 μF; (*c*) 200 W]

C-82. An ac load draws 2 A at a power factor of 0.8 (lagging) from a 230-V, 50-Hz mains. If a 15-μF capacitor is connected across the load terminals, what will be the net current supplied by the mains and its power factor?

[**Ans.** 1.604 A, 0.9974 (lagging)]

C-83. Two impedances, one inductive and the other capacitive, are connected in series. When a voltage of $120\angle 30°$ V and frequency of 50 Hz is impressed across the combination, the current is $3\angle -15°$A. If one of the impedances is $(10 + j48.3)$ Ω, find the other. Also, find the values of L and C in the impedances.

Ans. $\mathbf{Z}_2 = (18.28 - j20.02)$ Ω; 0.1537 H, 159 μF]

C-84. In the circuit shown in Fig. C-55, (*a*) find the current **I**, and (*b*) use voltage division twice to find $\mathbf{V}_1$. [**Ans.** (*a*) 2.32∠–28.6°A; (*b*) 77.3∠5° V]

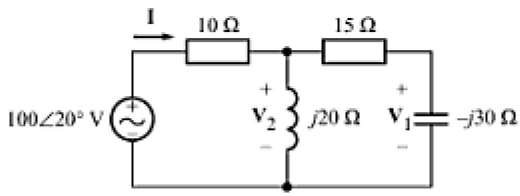


Fig. C-55

C-85. A 0.5-Ω resistor is in parallel with a 10-mH inductor. At what radian frequency, do the circuit voltage and current have a phase angle difference of 40°? [**Ans.** 59.6 rad/s]

C-86. The current source $i = 4\sqrt{2}\sin(400t - 10°)$ A in Fig. C-56. Use current division concept to find current i_L in the inductor for the circuit. [**Ans.** 6. 85∠–7° A]

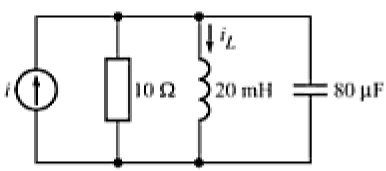


Fig. C-56

C-87. Use current division twice to find the current $\mathbf{I}_L$ for the circuit shown in Fig. C-57. [**Ans.** 6.85∠–7° A]

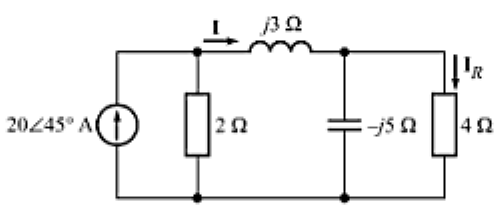


Fig. C-57

C-88. A 2-mH coil with a 10-Ω winding resistance is in parallel with a 10-μF capacitor. Find its equivalent circuit at 8 krad/s, which has two circuit-elements in series. [**Ans.** R = 13.9 Ω, C = 7.2 μF]

C-89. Use voltage division twice to find **V** for the circuit shown in Fig. C-58. [**Ans.** 81.8∠6.15° V]

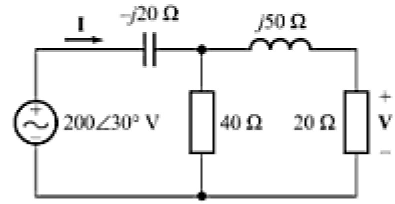


Fig. C-58

C-90. Use current division twice to find **I** for the circuit shown in Fig. C-59. [**Ans.** 1.41∠–19.5° A]

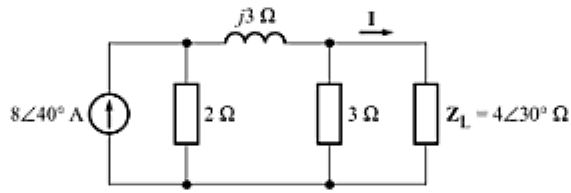


Fig. C-59

C-91. Find the admittance **Y** of the circuit shown in Fig. C-60. [**Ans.** 2.29∠–42.2° S]

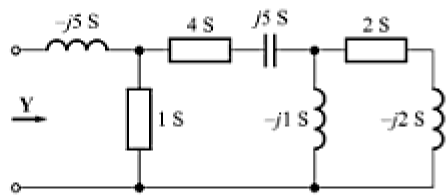


Fig. C-60

C-92. For the circuit shown in Fig. C-61, find **I**, $\mathbf{V}_R$ and $\mathbf{V}_C$, and the corresponding sinusoidal quantities if the frequency is 50 Hz. Also find the average power delivered by the source.

[**Ans. I** = 7.5∠81.3° A; $\mathbf{V}_R$ = 150∠81.3° V; $\mathbf{V}_C$ = 187∠–8.66° V; $i = 10.6\sin(314t + 81.3°)$ A; $v_R = 212\sin(314t + 81.3°)$ V; $v_C = 265\sin(314t - 8.66°)$ V]

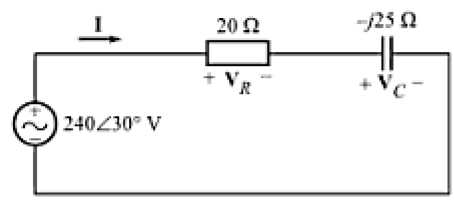


Fig. C-61

C-93. Find the input admittance $\mathbf{Y}_{ab}$ of the network shown in Fig. C-62 and draw it as a parallel combination of a resistance R and an inductance L, giving values for R and L if $\omega = 1$ rad/s.

[**Ans.** $(0.5 - j0.5)$ S, $(2\ \Omega) \parallel (2\ \text{H})$]

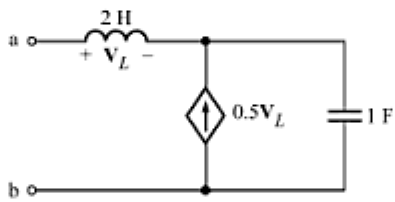


Fig. C-62

C-94. Find Thevenin's equivalent across a and b for the circuit given in Fig. C-63.

[**Ans.** $12.3\angle{-19.3°}$ V, $3.07\angle 15.7°\ \Omega$]

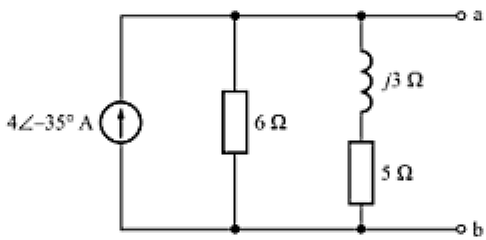


Fig. C-63

C-95. Use superposition theorem to find the average power absorbed by the 5-Ω resistor in the circuit shown in Fig. C-64. [**Ans.** 2.36 W]

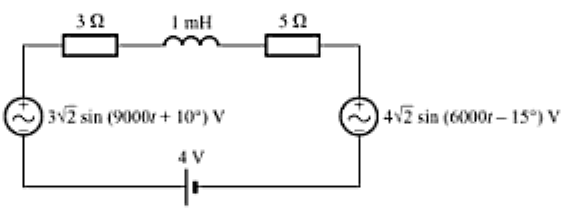


Fig. C-64

C-96. Find $\mathbf{Z}_{Th}$, $\mathbf{V}_{Th}$, and $\mathbf{I}_N$ for the Thevenin's and Norton's equivalents of the circuit shown in Fig. C-65.

[**Ans.** $1.35\angle 10.9°\ \Omega$, $4.05\angle 70.9°$ V, $3\angle 60°$ A]

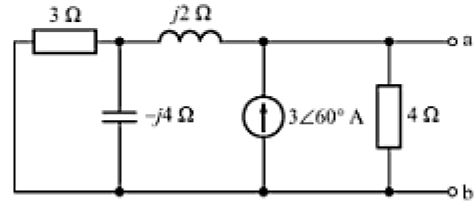


Fig. C-65

C-97. Find $\mathbf{Z}_N$, and $\mathbf{I}_N$ for the Norton's equivalent of the circuit shown in Fig. C-66.

[**Ans.** $7.92\angle{-16.48°}\ \Omega$, $2.78\angle 4.81°$ A]

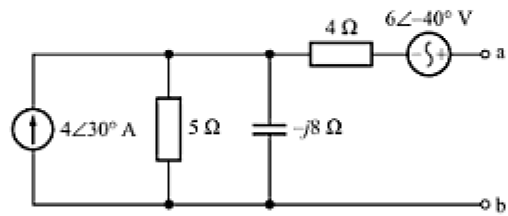


Fig. C-66

C-98. Determine the power factor of a fully loaded 10-hp induction motor that operates at 80% efficiency while drawing 28 A from 480-V line.

[**Ans.** 0.694 (lagging)]

C-99. A resistor in parallel with a capacitor absorbs 20 W when the combination is connected to a 240-V, 50-Hz supply. If the power factor is 0.7 leading, what are the resistance and capacitance?

[**Ans.** 2.88 kΩ, 1.127 μF]

C-100. A resistor in series with a capacitor absorbs 10 W when the combination is connected to a 120-V, 400-Hz supply. If the power factor is 0.6 leading, what are the resistance and capacitance?

[**Ans.** 518 Ω, 0.576 μF]

C-101. An induction motor delivers 50 hp while operating at 80% efficiency from 440-V mains. If the power factor is 0.6, what current does the motor draw? If the power factor is 0.9, instead, what current does this motor draw? [**Ans.** 176.5 A, 117.7 A]

C-102. What resistor and capacitor in parallel present the same load to a 440-V, 50-Hz supply as a fully loaded 20-hp synchronous motor that operates at 75% efficiency and 0.8 leading power factor?

[**Ans.** 9.73 Ω, 245.2 μF]

C-103. A 20-μF capacitor and a parallel 200-Ω resistor draw 4 A at 50 Hz. Find the power components.

[**Ans.** 1.99 kVA, 1.24 kW, – 1.56 kVAR]

C-104. A series *RLC* circuit is connected to 250-V, 50-Hz supply. It is found that the current in the circuit is unaltered when the capacitance is short-circuited. If R = 40 Ω and the current is 5 A, determine L and C. [**Ans.** 95.5 mH, 53 μF]

C-105. A 230-V, 50-Hz voltage is applied across a coil of L = 0.5 H and R = 200 Ω in series with a capacitor. What value must the capacitor have in order that the total voltage across the coil shall be 250-V?

[**Ans.** 89.66 μF]

C-106. The series combination of an inductive impedance Z_a consisting of R = 6 Ω and L = 25.5 mH and an unknown capacitive impedance Z_b is connected across 240-V, 50-Hz supply, as shown in Fig. C-67. It is found that the voltage V_a across Z_a is three time the voltage V_b across Z_b, and are in quadrature. Determine the values of R and C.

[**Ans.** 2.667 Ω, 1.592 mF]

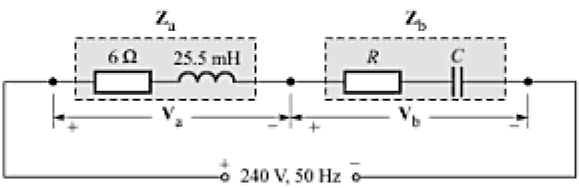


Fig. C-67

C-107. An inductor coil, when connected to a 250-V, 50-Hz supply, absorbs 160 W taking a current of 2 A. (*a*) Determine the resistance, the inductance and the power factor of the coil. (*b*) If a 50-μF is connected in series with the coil, how much current will the circuit draw from the same supply?

[**Ans.** (*a*) 40 Ω, 0.377 H, 0.32 (lagging); (*b*) 3.687 A]

C-108. Two circuits have the same numerical ohmic impedance, but the power factor of one circuit is 0.8 and the other 0.6, both lagging. Calculate the power factor of the circuit obtained by joining these two circuits in parallel.

[**Ans.** 0.707 (lagging)]

C-109. An ac circuit connected across 200-V, 50-Hz supply has two parallel branches A and B. The current in branch A is 4 A at 0.8 lagging power factor, and the total current is 5 A at unity power factor. Find for the branch B, the power consumed and the impedance. [**Ans.** 360 W, (40 – *j*53.34) Ω]

C-110. Two impedances $\mathbf{Z}_1$ = (10 + *j*15) Ω and $\mathbf{Z}_2$ = (6 – *j*8) Ω are connected in parallel. The total current supplied is 15 A. What is the power taken by each branch? [**Ans.** 738 W, 1438 W]

C-111. When two impedances $20\angle{-45°}$ Ω and $30\angle 30°$ Ω are connected in series across a certain ac supply, the resulting current is 10 A. If the supply voltage remains unaltered, calculate the supply current when the two impedances are connected in parallel. [**Ans.** $26.84\angle 17.45°$ A]

C-112. Obtain the power factor of a two-branch circuit, where the first branch has $\mathbf{Z}_1$ = (2 + *j*4) Ω and the second branch has $\mathbf{Z}_2$ = (6 + *j*0) Ω. To what value must the 6-Ω resistor be changed to result in the overall power factor of 0.9 lagging?

[**Ans.** 0.8 (lagging), 3.195 Ω]

C-113. Three impedances $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ connected in parallel have an equivalent impedance of (6 + *j*8) Ω. A sinusoidal voltage $V_m \sin(\omega t \pm \theta°)$ V is applied to it. The currents drawn by these impedances are $i_1 = 20 \sin \omega t$ A, $i_2 = 40 \sin(\omega t + 30°)$ A, and $i_3 = i_m \sin(\omega t \pm \theta°)$ A, respectively. The total current drawn is $i = 25 \sin(\omega t \pm 30°)$ A Determine (*a*) the current i_3, (*b*) expression for the supply voltage, (*c*) the total power drawn from the supply, and (*d*) the impedance $\mathbf{Z}_1$.

[**Ans.** (*a*) $33.83 \sin(\omega t - 167.2°)$ A; (*b*) $250 \sin(\omega t + 83.13°)$ V; (*c*) 1875 W; (*d*) (1.49 + *j*12.4) Ω]

C-114. For the circuit shown in Fig. C-68 find (*a*) the current delivered by the source, and (*b*) the power factor.

[**Ans.** (*a*) $6.935\angle 56.31°$ A; (*b*) 0.5547 (leading)]

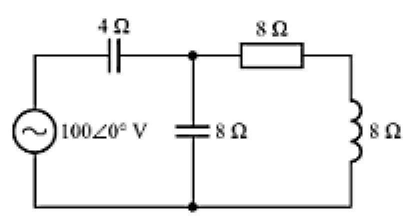


Fig. C-68

C-115. For the circuit shown in Fig. C-69 determine (*a*) the impedance, (*b*) the current $\mathbf{I}_1$, (*c*) the current $\mathbf{I}_2$, (*d*) the current $\mathbf{I}_3$, (*e*) the voltage $\mathbf{V}_{BC}$, (*f*) the power consumed by the whole circuit.

[**Ans.** (*a*) $632.4\angle 80.9°$ Ω; (*b*) $0.393\angle -80.9°$ A; (*c*) $0.776\angle -80.9°$ A; (d) $0.383\angle 99.1°$ A; (*e*) $121.9\angle 9.1°$ V; (*f*) 15.54 W]

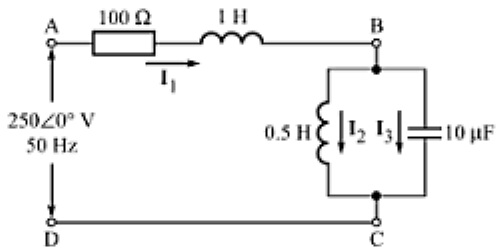


Fig. C-69

C-116. A balanced star-connected load of $(8 + j6)$ Ω per phase is connected to a three-phase, 230-V supply. Find the line current, *pf*, active, reactive and total volt-amperes.

[**Ans.** $13.28\angle -36.87°$ A; 0.8 (lagging); 4232 W; 3174 VAR; 5290 VA]

(C) CHALLENGING PROBLEMS

C-117. For the circuit shown in Fig. C-70, find voltage v_S across the current source i_S given as is = $0.234 \sin(3000t - 10°)$ A.

[**Ans.** $v_S = 95.2 \sin(3000t + 38.4°)$ V]

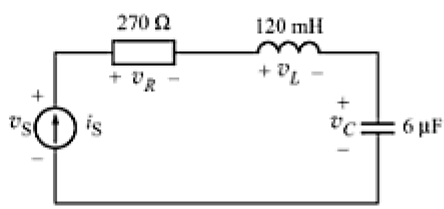


Fig. C-70

C-118. If the independent voltage source in the circuit of Fig. C-71 is $v = 10 \sin 1000t$ V, obtain the expressions for the time-domain currents i_1 and i_2. [**Ans.** $i_1 = 1.24 \sin(1000t + 29.7°)$ A, $i_2 = 2.77 \sin(1000t + 56.3°)$ A]

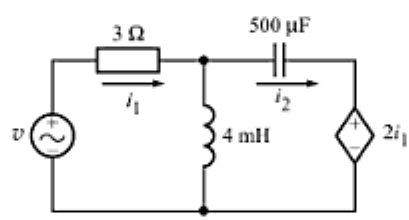


Fig. C-71

C-119. Use nodal analysis for the circuit of Fig. C-72 to find $\mathbf{V}_1$ and $\mathbf{V}_2$.

[**Ans.** $1.062\angle 23.3°$ V, $1.593\angle -50.0°$ V]

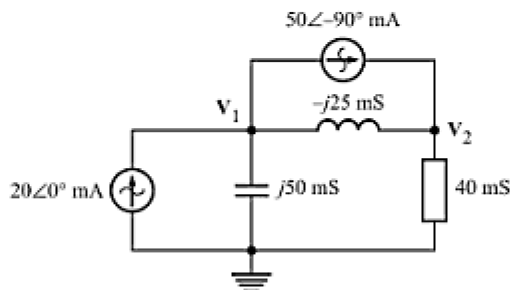


Fig. C-72

C-120. A choke coil is connected to 240-V, variable-frequency ac supply, as shown in Fig. C-73. When the frequency of the supply is 50 Hz, an ammeter

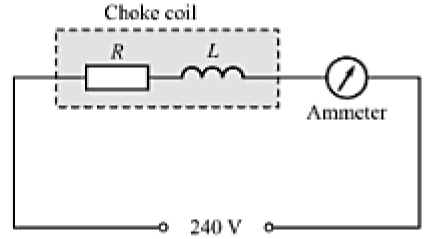


Fig. C-73

connected in series with the choke reads 60 A. On increasing the frequency to 100 Hz, the same ammeter reads 40 A. Calculate the resistance and the inductance of the choke.

[**Ans.** 3.055 Ω, 8.218 mH]

C-121. Two impedances Z_A and Z_B are connected in series across a 240-V, 50-Hz ac supply. The total current drawn is 3 A. The impedance Z_A has 0.8 lagging power factor and the voltage across it is twice that across Z_B and in quadrature to it. Analytically determine (*a*) the value of Z_A in complex form and the power consumed in Z_A. (*b*) the power consumed by Z_B and its *pf*.

[**Ans.** (*a*) $(57.25 + j42.94)$ Ω, 515.3 W; (*b*) 193.1 W; 0.6 (leading)]

C-122. An ohmic resistance is connected in series with an unknown choke coil across a 230-V, 50-Hz supply. The current drawn by the circuit is 1.5 A, and the voltages across the resistance and the choke coil are 90 V and 180 V, respectively. Calculate the resistance and the inductance of the choke coil, and the phase difference between the current and the supply voltage.

[**Ans.** 45.93 Ω, 0.353 H, 46.31°]

C-123. When a voltage of 100 V, 50 Hz is applied to a coil A, the current taken is 8 A and the power consumed is 120 W. An impedance Z_B, whose power factor is 0.8 leading at 50 Hz, is connected in series with the coil *A*, and the combination is connected across a variable-frequency source of 200 V. The current taken from the supply is maximum, when the frequency is 100 Hz. Find the components of Z_B and the value of maximum current taken by the circuit.

[**Ans.** $R_B = 16.48$ Ω, $C_B = 64.38$ μF, $I_{max} = 10.9$ A]

C-124. A 100-Ω resistance is connected in series with a choke coil. When a 440-V, 50-Hz, single-phase ac voltage is applied to this combination, the voltage across the resistance and the choke coil are 200 V and 300 V, respectively. Find the power consumed by the choke coil. Sketch a neat phasor diagram, indicating the current and all voltages.

[**Ans.** 150 W]

C-125. A sinusoidal ac voltage of 200 V is applied to a series circuit consisting of a resistor, a capacitor and an inductor choke. The voltages across the components are 170 V, 100 V and 150 V, respectively, and the current drawn by the circuit is 4 A. Determine the power factor of the inductor choke and of the circuit.

[**Ans.** 0.161 (lagging), 0.97(lagging)]

C-126. Find the input admittance at 50 krad/s of the circuit shown in Fig. C-74. [**Ans.** $2.83\angle{-135°}$ S]

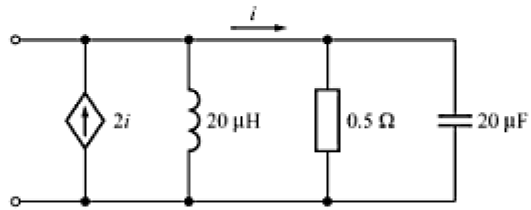


Fig. C-74

C-127. Find the input admittance at 1 krad/s of the circuit shown in Fig. C-75. [**Ans.** 4 S]

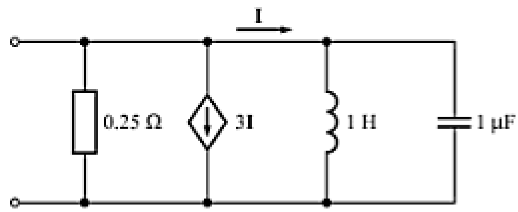


Fig. C-75

C-128. A certain industrial load has an impedance of $0.6\angle 30°$ Ω at a frequency of 50 Hz. What capacitor connected in parallel with this load causes the angle of the total impedance to decrease to 15°? Also, if the load voltage is 230 V, what is the decrease in the line current produced by adding the capacitor?

[**Ans.** 1.42 mF, 39.8 A]

C-129. Determine the current *i* through the 4-Ω resistor in the circuit shown in Fig. C-76.

[**Ans.** $i = 0.1755 \sin(2t - 20.56°) + 0.547 \sin(5t - 43.15°)$ A]

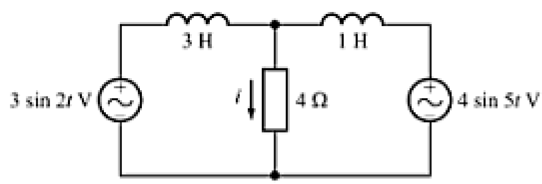


Fig. C-76

C-130. In the network shown in Fig. C-77, find the angular frequency at which (*a*) R_{in} =550 Ω, and (*b*) X_{in} = 50 Ω. [**Ans.** (*a*) 100 rad/s, (*b*) 100 rad/s]

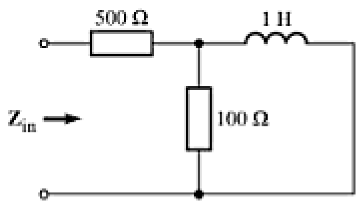


Fig. C-77

C-131. Determine the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown in Fig. C-78.

[**Ans.** 0.974∠41.5° A, −0.63∠−48.2° A]

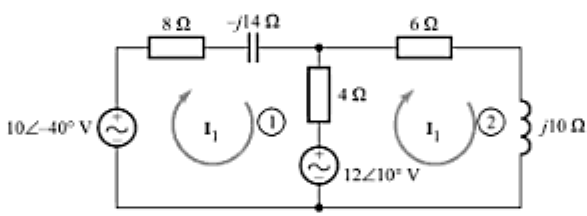


Fig. C-78

C-132. Find the node voltages in the circuit shown in Fig. C-79. [**Ans.** −3.59∠−5° V, −12∠−14° V]

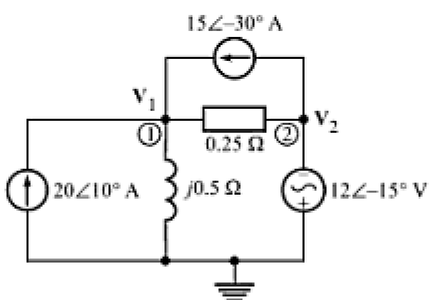


Fig. C-79

C-133. Solve the node voltages in the circuit shown in Fig. C-80. [**Ans.** 5.13∠47.3° V, 8.18∠15.7° V]

C-134. What load impedance Z_L in the circuit shown in Fig. C-81 absorbs maximum average power and what is this power?

[**Ans.** 8.46∠2.81° Ω, 1.67 kW]

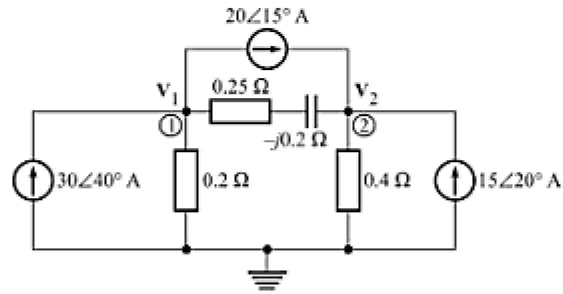


Fig. C-80

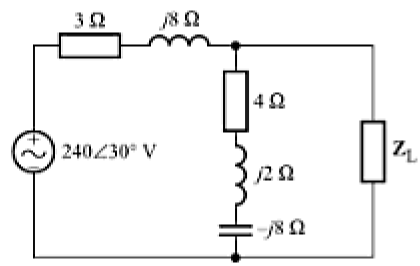


Fig. C-81

C-135. In the circuit shown in Fig. C-82, find the values of *R* and *L* so that maximum power is absorbed by the parallel resistor and capacitor load, and also find this power. [**Ans.** *R* = 0 Ω, *L* = 3.9 μH, 208 W]

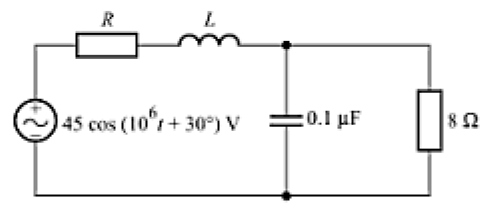


Fig. C-82

C-136. A fully loaded 10-hp induction motor operates from a 440-V, 50-Hz supply at an efficiency of 85 % and a 0.8 lagging power factor. Find the overall power factor when a 33.3 μF capacitor is connected in parallel with the motor. [**Ans.** 0.8874]

C-137. In the series circuit shown in Fig. C-83, the voltage and the current are as indicated. Find the values of *R*, *r*, *L* and the frequency of the applied voltage and its magnitude.

[**Ans.** 7.143 Ω, 1.57 Ω, 7.28 mH, 247.5 Hz, 30.97V]

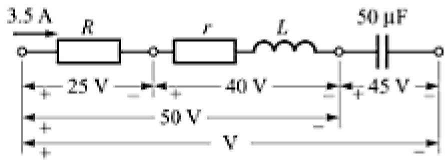


Fig. C-83

C-138. A series resonant circuit has an impedance of 500 Ω at resonant frequency and its cutoff frequencies are 10 kHz and 100 kHz. Determine the resonant frequency, the quality factor at resonant frequency, and the values of *R*, *L* and *C*.

[**Ans.** 31.62 kHz, 0.35, 500 Ω, 0.884 mH, 28.76 nF]

C-139. Two circuits, with the impedances $\mathbf{Z}_1 = (12 + j15)\ \Omega$ and $\mathbf{Z}_1 = (8 - j4)\ \Omega$, are connected in parallel. If the potential difference across the impedances is $\mathbf{V} = (230 + j0)$ V, calculate (*a*) the total current and the currents supplied to each branch, (*b*) the total power and power consumed by each branch, (*c*) the overall power factor and power factor of each branch.

[**Ans.** (*a*) $30.56\angle 4.04°$ A, $12\angle -51.34°$ A, $25.7\angle 26.56°$ A; (*b*) 7014 W, 1728 W; 5284 W; (*c*) 0.99 75 (leading) 0.6247 (lagging), 0.9021(leading)]

C-140. Two circuits *A* and *B* are connected in parallel across 200-V, 50-Hz power mains, as shown in Fig. C-84. Calculate (*a*) the, source current (*b*) the current in each branch, and (*c*) the power factor. Draw the phase diagram.

[**Ans.** (*a*) $3.27\angle -53.75°$ A; (*b*) $5.18\angle -75°$ A, $2.44\angle 75.9°$ A; (*c*) 0.59 (lagging)]

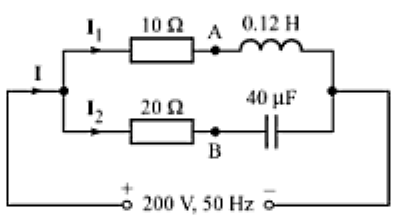


Fig. C-84

C-141. For the circuit shown in Fig. C-85, calculate (*a*) the current in each branch and the total rms current, (*b*) the power factor of the circuit, and (*c*) the total power taken from the source. Draw the phasor diagram.

[**Ans.** (*a*) $1.27\angle -60.17°$ A, $1.44\angle 90°$ A, $0.718\angle 28.47°$ A; (*b*) 0.879; (*c*) 145.2 W]

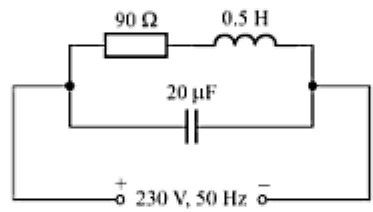


Fig. C-85

C-142. Two impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are connected in parallel across 200-V, 50-Hz, single-phase ac supply. The impedance $\mathbf{Z}_1$ carries a current of 2 A at 0.8 leading power factor, and $\mathbf{Z}_2$ consumes 668 W. If the total current is 5 A at 0.985 lagging power factor, determine (*a*) the values of $\mathbf{Z}_1$ and $\mathbf{Y}_1$, (*b*) the current through $\mathbf{Z}_2$ and its phase angle, and (*c*) the power consumed by $\mathbf{Z}_1$.

[**Ans.** (*a*) $100\angle -36.87°\ \Omega$, $0.01\angle 36.87°$ S; (*b*) $3.92\angle -31.88°$ A; (*c*) 320 W]

C-143. (*a*) The load connected to a tree-phase supply comprises three similar coils connected in star. The line currents are 25 A, and the kVA and kW inputs are 20 and 11, respectively. Find the line and the phase voltages, the resistance and the reactance of each coil. (*b*) If the coils are now connected in delta to the same three-phase supply, what would be line currents and the power taken?

[**Ans.** (*a*) 462 V, 267 V, 5.86 Ω, 8.92 Ω; (*b*) 75 A, 33 kW]

C-144. (*a*) A balanced delta-connected load consumes 2 kW of power, when connected to 400-V, 50-Hz, 3-phase supply. The same load takes 2 A at a lagging power factor when connected to a 230-V, 50-Hz, 3-phase supply. Determine the resistance and the inductance per phase and load power factor. (*b*) If the load is now connected in star to the same supply, find the power consumed.

[**Ans.** (*a*) 166.6 Ω, 0.352 H; (*b*) 667 W]

TRANSFORMERS

13

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the applications of a transformer in electrical and electronic circuits.*
- *State the principle of operation of a two-winding transformer.*
- *Draw the circuit symbol of a transformer.*
- *Derive the basic emf equation of a transformer.*
- *State the four conditions for transformer to be ideal.*
- *State the relations for transformation of voltage, current and impedance by a transformer in terms of its turns-ratio.*
- *State why a transformer should have no-load current.*
- *Define the two components of the no-load current.*
- *Explain why the hysteresis and eddy-current losses occur in the core, and how these can be reduced.*
- *Explain the construction of core-type and shell-type transformers.*
- *State what is meant by load component of primary current.*
- *State why in the equivalent circuit of a transformer we include a resistance and a leakage reactance in both the primary and secondary side.*
- *State how we obtain a simplified equivalent circuit of a transformer as referred to the primary or to the secondary.*
- *State the meaning of 'regulation down' and 'regulation up' of a transformer.*
- *Derive the condition for zero regulation and condition for maximum regulation of a transformer.*
- *Define 'commercial efficiency' and 'all-day efficiency' of a transformer.*
- *Derive the condition of maximum efficiency of a transformer.*
- *Explain how to convert a two-winding transformer into an autotransformer, and state the advantages and disadvantages of doing it.*
- *Explain how to get the 'equivalent circuit parameters' of a transformer by conducting 'open-circuit test' and 'short-circuit test'.*

13.1 INTRODUCTION

A transformer is a highly efficient* device for transferring electrical energy form one circuit to another (usually from one ac voltage level to another), without any change in its frequency. There exists no simple device that can accomplish such changes in dc voltages. Thus, the transformer has provided a feature to ac power system that lacks in dc power system.

Applications

A key application of transformers is in economically transmitting and distributing electrical power over long distances; thus, permitting generation to be located remotely from the points of demand. The general

* Some large transformers are able to transfer 99.75% of their input power to their output.

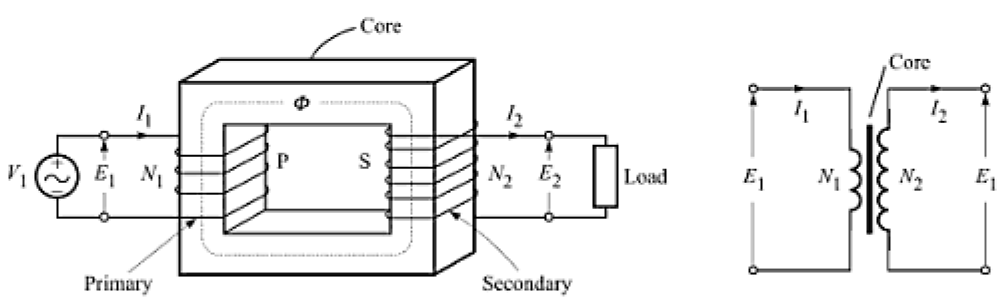
practice is to generate ac voltage at about 11 kV, then step up by means of a transformer to higher voltages of 132 kV, 220 kV and 400 kV for the transmission lines. This conversion aids the transmission of huge electrical power at low cost. High-voltage lines carry low currents, and hence the cost of lines and the power loss are tremendously reduced. At distribution points, other transformers are used to step the voltage down to 400 V or 220 V for use in industries, offices and homes. Since there are no moving parts in a transformer, it practically needs almost no maintenance and supervision. A transformer also electrically isolates the end user from contact with the supply voltage.

Apart from the above, small-size transformers are used in communication circuits, radio and TV circuits, telephone circuits, instrumentation and control systems. Audio transformers are used to couple stages of amplifier and to *match* devices such as microphones and record player cartridges to the input impedance of the amplifiers. The use of audio transformers permits to carry on two-way conversation over a single pair of wires.

13.2 PRINCIPLE OF OPERATION

A transformer operates on the principle of ***mutual induction*** between *two coils.* Figure 13.1*a* shows the general construction of a transformer. The vertical portions of the steel-core are termed *limbs*, and the top and bottom portions are called *yokes*. The two coils P and S, having N_1 and N_2 turns, are wound on the limbs. These two windings are electrically unconnected but are linked with one another through a magnetic flux in the core. The coil P is connected to the supply and is therefore called *primary*; coil *S* is connected to the load and is termed the *secondary*.

Basically, two principles are involved in the operation of a transformer. Firstly, an electric current produces a magnetic field (electromagnetism), and secondly, a changing magnetic field within a coil induces an emf across the ends of the coil (electromagnetic induction). A changing current in the primary circuit creates a changing magnetic field; in turn, this magnetic field induces a voltage in the secondary circuit. Thus, energy is transferred from one circuit to the other.



(*a*) Construction. (*b*) Circuit symbol.

Fig. 13.1 *A transformer.*

Figure 13.1*b* shows the circuit symbol of a transformer. The thick line denotes the iron core. By having different ratios N_1/N_2 of the two windings, power at lower or at higher voltage can be obtained. When $N_2 > N_1$, the transformer is called a ***step up*** transformer; and when $N_2 < N_1$, the transformer is called a ***step down*** transformer.

EMF Equation

Consider a sinusoidally varying voltage V_1 applied to the primary of the transformer shown in Fig. 13.1*a*. Due to this voltage, a sinusoidally varying magnetic flux is set up in the core, which can be represented as

$$\Phi = \Phi_m \sin \omega t = \Phi_m \sin 2\pi f t \tag{13.1}$$

where Φ_m is the peak value of the flux and f is the frequency of sinusoidal variation of flux. As per the law of electromagnetic induction, the induced emf in a winding of N turns is given as

$$e = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\Phi_m \sin \omega t) = -N\omega\Phi_m \cos \omega t = \omega N\Phi_m \sin(\omega t - \pi/2) \tag{13.2}$$

Thus, the peak value of the induced emf is $E_m = \omega N\Phi_m$. Therefore, the rms value of the induced emf E is given as

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega N\Phi_m}{\sqrt{2}} = \frac{2\pi f N\Phi_m}{\sqrt{2}} = 4.44\, fN\,\Phi_m$$

or

$$\boxed{E = 4.44\, fN\Phi_m} \tag{13.3}$$

This equation, known as ***emf equation of transformer***, can be used to find the emf induced in any winding (primary or secondary) linking with flux Φ.

Effect of Frequency

The emf of a transformer at a given flux increases with frequency (see Eq. 13.3). By operating at higher frequencies, transformers can be made physically more compact because a given core is able to transfer more power without reaching saturation, and fewer turns are needed to achieve same impedance. However, properties such as core losses and conductor skin effect* also increase with frequency. Aircraft and military equipments employ 400-Hz power supplies which reduces core and winding weight.

EXAMPLE 13.1

The primary of a 50-Hz step-down transformer has 480 turns and is fed from 6400 V supply. Find (*a*) the peak value of the flux produced in the core, and (*b*) the voltage across the secondary winding if it has 20 turns.

Solution

(*a*) Using Eq. 13.3, we get

$$\Phi_m = \frac{E}{4.44\, fN_1} = \frac{6400}{4.44 \times 50 \times 480} = 0.06 \text{ Wb} = \mathbf{60\ mWb}$$

(*b*) The voltage induced in the secondary winding is given as

$$E = 4.44\, fN_2\, \Phi_m = 4.44 \times 50 \times 20 \times 0.06 = \mathbf{266.4V}$$

13.3 IDEAL TRANSFORMER

We shall describe the physical construction and equivalent circuit of an actual transformer a little later. Here, we define the ***ideal transformer*** as a circuit element. We shall then explore its properties in voltage,

* The higher the frequency, the greater is the tendency for the current in the conductor to confine itself within its outer layer (i.e., its *skin*), thereby reducing its effective area of cross-section.

current, and impedance transformation. Primary and secondary voltage and current variables are defined in Fig. 13.1*b*, which shows the circuit model of an ideal transformer.

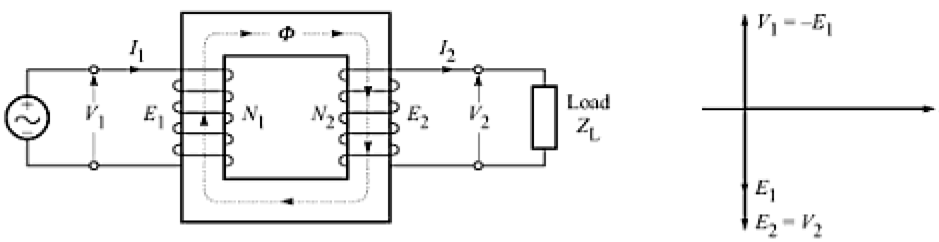
The complete behaviour of a physical transformer can be better understood by initially assuming the transformer to be ideal, and then allowing the imperfections of the actual transformer by suitably introducing some impedances.

Conditions for Ideal Transformer

(*i*) The permeability (μ) of the magnetic circuit (the core) is infinite, i.e., the magnetic circuit has zero reluctance so that no mmf is needed to set up the flux in the core.
(*ii*) The core of the transformer has no losses.
(*iii*) The resistance of its windings is zero, hence no I^2R losses in the windings.
(*iv*) Entire flux in the core links both the windings, i.e., there is no *leakage flux*.

Thus, an ideal transformer has no losses and stores no energy. However, an ideal transformer has no physical existence. But, the concept of ideal transformer is very helpful in understanding the working of an actual transformer.

Consider an ideal transformer whose secondary is connected to a load Z_L and primary is supplied from an ac source V_1 (Fig. 13.2*a*). The voltage across the load is V_2. The primary and secondary windings of the ideal transformer have zero impedance. Hence, the induced emf E_1 in the primary exactly counter balances the applied voltage V_1, that is, $V_1 = -E_1$. Also, the induced emf E_2 is the same as voltage V_2, that is, $E_2 = V_2$. Here, E_1 is called ***counter emf*** or ***back emf*** induced in the primary, and E_2 called ***mutually induced emf*** in the secondary.



(*a*) The circuit. (*b*) The phasor diagram.

Fig. 13.2 *Ideal transformer.*

Figure 13.2*b* shows the phasor diagram of the ideal transformer. We have taken flux Φ as reference phasor, as it is common to both the primary and secondary. As per Eq. 13.2, the induced emfs E_1 and E_2 lag flux Φ by 90°. The voltage V_1 is equal and opposite to emf E_1. Thus, the applied voltage V_1 leads the flux Φ by 90°. According to the first condition of ideality, the reluctance of the magnetic circuit is zero and hence the required magnetising current to produce flux Φ is also zero.

Transformation Ratio

The *ratio of secondary voltage to the primary voltage is known as* ***transformation ratio*** or ***turns-ratio***. It is denoted by letter *K*. Let N_1 and N_2 be the number of turns in primary and secondary windings, and E_1 and E_2 be the rms values of the primary and secondary induced emfs. Using Eq. 13.3, we can write

$$E_1 = 4.44\, f N_1 \Phi_m \tag{13.4}$$

and
$$E_2 = 4.44\, f N_2 \Phi_m \tag{13.5}$$
Then, the *transformation ratio* or *turns-ratio* can be expressed as
$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \tag{13.6}$$

Thus, the side of the transformer with the larger number of turns has the larger voltage. Indeed, *the voltage per turn is constant for a given transformer*. By selecting K properly, the transformation of voltage can be done from any value to any other convenient value. There can be two* cases:

(*i*) When $K > 1$ (i.e., $N_2 > N_1$), $V_2 > V_1$; the device is known as ***step-up** transformer*.
(*ii*) When $K < 1$ (i.e., $N_2 < N_1$), $V_2 < V_1$; the device is known as ***step-down** transformer*.

In general, a transformer can have more than 2 windings. The windings of a three-winding transformer are called *primary*, *secondary* and *tertiary*. The primary is connected to an ac supply. Different loads may be connected across the secondary and tertiary**. The induced emf in a winding is still proportional to its number of turns,
$$E_1 : E_2 : E_3 :: N_1 : N_2 : N_3$$

Volt-Amperes

Consider again the two-winding transformer of Fig. 13.2*a*. For an ideal transformer, the current I_1 in the primary is just sufficient to provide mmf I_1N_1 to overcome the demagnetising effect of the secondary mmf I_2N_2. Hence,
$$\therefore \qquad I_1 N_1 = I_2 N_2 \quad \text{or} \quad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{K} \tag{13.7}$$

Thus, we find that *the current is transformed in the reverse ratio of the voltage*. If the voltage is stepped up ($V_2 > V_1$), then the current is stepped down ($I_2 < I_1$) by the same factor. That is, the side of the transformer with the *larger* number of turns has the *smaller* current. For example, a step-up transformer would have a primary with few turns of thick wire (small voltage, large current) and the secondary would have many turns of thin wire (large voltage, small current).

Combining Eqs. 13.5 and 13.7, we have
$$E_1 I_1 = E_2 I_2$$
Hence, *in an ideal transformer the input VA and output VA are identical*.

Impedance Transformation

Equations 13.6 and 13.7 reveal a very useful property of transformers, called *impedance transformation*. Figure 13.3 shows an ideal transformer. It has N_1 and N_2 turns in its primary and secondary windings. A load impedance Z_L is connected across its secondary, and an equivalent impedance Z_{eq} is defined at its primary.

The equivalent impedance Z_{eq} as faced by a source V_1 is given as
$$Z_{eq} = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)} = \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{K}\right) \times \left(\frac{1}{K}\right) \times Z_L$$

* The third case, when $K = 1$ (i.e., $N_1 = N_2$) is not important. We hardly ever use a transformer with unity turns ratio. Such a transformer is used only when you need electrical isolation between two electrical circuits.

** Sometimes, the tertiary winding has a centre-tap; the two halves having same number of turns, N_3. The voltage of such a winding is then specified as $E_3/0/E_3$, or $E_3 - 0 - E_3$.

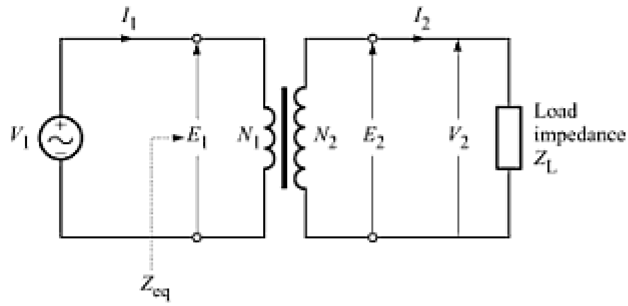


Fig. 13.3 *The transformer changes the impedance Z_L to equivalent impedance Z_{eq}.*

or $$Z_{eq} = Z_L/K^2 \tag{13.8}$$

Therefore, the impedance is transformed in inverse proportion to the *square* of the turns-ratio. The concept of impedance transformation is used for *impedance matching*. As per maximum power transfer theorem, the load impedance has to be properly matched with the source impedance, as illustrated in Example 13.3 given below.

EXAMPLE 13.2

A single-phase, 50-Hz transformer has 30 primary turns and 350 secondary turns. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to a 230-V, 50-Hz supply, calculate (*a*) the peak value of flux density in the core, (*b*) the voltage induced in the secondary winding, and (*b*) the primary current when the secondary current is 100 A. (Neglect losses.)

Solution

(*a*) The peak value of the flux in the core is given as

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 30} = 0.034534 \text{ Wb}$$

Therefore, the peak value of the flux density in the core is

$$B_m = \frac{\Phi_m}{A} = \frac{0.034534}{250 \times 10^{-4}} = \mathbf{1.3814T}$$

(*b*) The voltage induced in the secondary winding is

$$E_2 = E_1 \times \frac{N_2}{N_1} = 230 \times \frac{350}{30} = 2683.33 \text{ V} = \mathbf{2.683 \text{ kV}}$$

(*c*) The primary current is

$$I_1 = I_2\left(\frac{N_2}{N_1}\right) = 100 \times \left(\frac{350}{30}\right) = 1166.67 \text{ A} \approx \mathbf{1.167 \text{ kA}}$$

EXAMPLE 13.3

A source with an output resistance of 50 Ω is required to deliver power to a load of 800 Ω. Find the turns-ratio of the transformer to be used for maximizing the load power.

Solution For delivering maximum power to the load, the equivalent resistance must be equal to the source resistance. This requires a resistance of 50 Ω looking into the primary of the transformer. That is,

$$R_{eq} = R_L/K^2 \quad \text{or} \quad 50 = 800/K^2 \quad \Rightarrow \quad K = \sqrt{800/50} = \sqrt{16} = 4$$

Thus,

$$K = \frac{N_2}{N_1} = 4$$

EXAMPLE 13.4

Determine the load current $\mathbf{I}_L$ in the ac circuit shown in Fig. 13.4*a*.

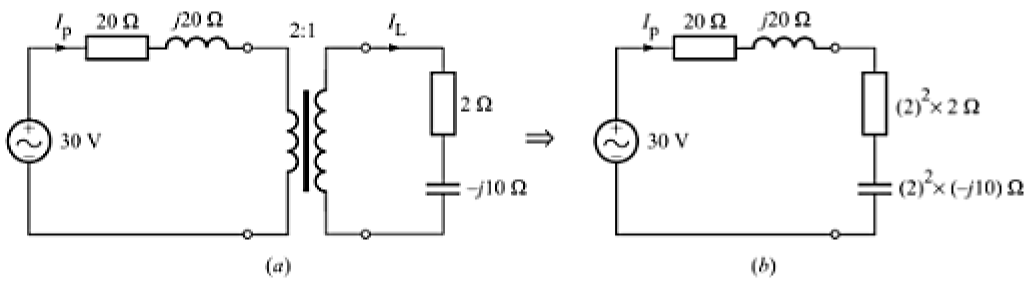


Fig. 13.4

Solution We first transform the load impedance into the primary to simplify the circuit, as shown in Fig. 13.4*b*. The primary current is then calculated as

$$\mathbf{I}_p = \frac{30\angle 0^\circ}{20 + j20 + 2^2(2 - j10)} = 0.872\angle 35.53^\circ \text{ A}$$

The load current, which is the same as the secondary current, is given by Eq. 13.7 as

$$\mathbf{I}_L = 2 \times \mathbf{I}_p = 2 \times 0.872\angle 35.53^\circ = \mathbf{1.74\angle 35.53^\circ \text{ A}}$$

EXAMPLE 13.5

A single-phase transformer has a core with cross-sectional area of 150 cm^2. It operates at a maximum flux density of 1.1 Wb/m^2 from a 50-Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of 4-Ω impedance. Neglect any voltage drop in the transformer.

Solution $\Phi_m = B_m A = 1.1 \times 0.015 = 0.0165$ Wb . Since the voltage drop in the transformer is negligible, we have

$$V_2 = E_2 = 4.44\, fN_2\Phi_m = 4.44 \times 50 \times 66 \times 0.0165 = 241.76 \text{ V}$$

The output current, $I_2 = \dfrac{V_2}{Z_L} = \dfrac{241.76}{4} = 60.44$ A

∴ Output volt-amperes = 241.76 × 60.44 = 14612 VA = **14.612 kVA**

EXAMPLE 13.6

A single-phase, 50-Hz transformer has a square core having a net cross-sectional area of 9 cm^2, and three windings designed for the following voltages:

(*i*) Primary: 230 V; (*ii*) Secondary : 110 V; and (*iii*) Tertiary : 6/0/6 V.
Find the number of turns in each winding if the flux density is not to exceed 1 T.

Solution $\Phi_m = B_m A = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb. The tertiary winding is divided into two halves; each half having a voltage $E_3 = 6$ V. Thus, the number of turns in each half of the tertiary is

$$N_3 = \frac{E_3}{4.44 f \Phi_m} = \frac{6}{4.44 \times 50 \times 9 \times 10^{-4}} = \textbf{30 turns}$$

$\therefore$ Total number of turns on the tertiary winding = 2 × 30 = **60 turns**.

We have seen that across 30 turns of the tertiary winding, the induced emf is 6 V. Therefore, the number of turns on the primary and secondary can be calculated as follows:

$$\frac{N_1}{N_3} = \frac{E_1}{E_3} \quad \text{or} \quad N_1 = \frac{N_3 E_1}{E_3} = \frac{30 \times 230}{6} = \textbf{1150 turns}$$

and

$$\frac{N_2}{N_3} = \frac{E_2}{E_3} \quad \text{or} \quad N_2 = \frac{N_3 E_2}{E_3} = \frac{30 \times 110}{6} = \textbf{550 turns}$$

13.4 PRACTICAL TRANSFORMER AT NO LOAD

In actual practice, a transformer can never satisfy any of the conditions specified above for the ideal transformer. Nevertheless, the concept of ideal transformer is helpful to understand the working of an actual transformer. We shall consider these conditions one by one, and see in what way a practical transformer deviates from the ideal transformer. In this Section, we shall consider only the first two ideality conditions. The remaining two conditions shall be considered in Section 13.7.

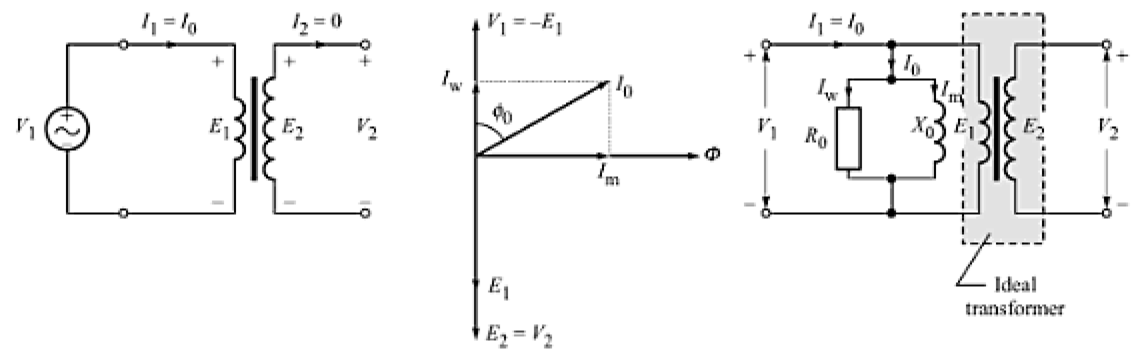
Consider a transformer with its primary connected to an alternating voltage source V_1, and no load connected across its secondary (Fig. 13.5*a*). With no closed circuit, the current in the secondary winding is zero. If the transformer were truly ideal, the primary current too would be zero, as per Eq. 13.7. But, in practice there does flow a little ***no-load current*** I_0 in the primary. This current I_0 is also called the ***exciting current*** of the transformer. Following are the two reasons why the no-load current I_0 flows in the primary.

(1) Effect of Magnetisation

Consider the *first ideality condition*. No magnetic material can have infinite permeability so as to offer zero reluctance to the magnetic circuit. Hence, in a practical transformer a finite mmf is needed to establish magnetic flux in the core. As a result, an in-phase ***magnetising current*** I_m in the primary is needed to set up flux Φ in the core. The current I_m is purely reactive and lags the voltage V_1 by 90°. This effect is modelled by an inductive reactance X_0 in parallel with the ideal transformer, as shown in the equivalent circuit of Fig. 13.5*c*.

The flux Φ induces emfs E_1 and E_2 in the primary and the secondary windings. As per Eq. 13.2, both these emfs lag flux Φ by 90°, as shown in the phasor diagram of Fig. 13.5*b*.

As the current I_2 in the secondary is zero (no load connected), the voltage drop in the secondary winding is zero. Hence, $V_2 = E_2$. The induced emf E_1 counter balances the applied voltage V_1 and establishes an electrical equilibrium. If the third and fourth ideality conditions (i.e., the effect of the resistance of the winding and the leakage of flux) are ignored, the magnitude of V_1 will be the same as that of emf E_1. Thus, $V_1 = -E_1$.



(*a*) The circuit. (*b*) The phasor diagram. (*c*) The equivalent circuit.

Fig. 13.5 *Transformer on no load.*

(2) Effect of Core Losses

Let us now consider the *second ideality condition*. There exist two reasons (*hysteresis* and *eddy current*) for the energy loss in the core of the transformer. The source must supply enough power to the primary to meet the core losses. These losses are proportional to the square of the core flux. Since the core flux is proportional to the applied voltage V_1, the iron loss can be represented by a resistance R_0 in parallel with the ideal transformer, as shown in the equivalent circuit of Fig. 13.5*c*. The ***core-loss current*** I_w flowing through R_0 is in phase with the applied voltage V_1, as shown in the phasor diagram of Fig. 13.5*b*.

Thus, we find that the no-load current I_0 has two components, I_m and I_w. The magnetising current I_m lags voltage V_1 by 90° and the loss component I_w is in phase with voltage V_1. The angle ϕ_0 is the *no-load phase angle*. Thus, from the phasor diagram of Fig. 13.5*b*, we have

$$I_0 = \sqrt{I_w^2 + I_m^2}\,; \qquad \phi_0 = \tan^{-1}(I_m/I_w); \qquad \text{and} \qquad \text{Input power} = V_1 I_w = V_1 I_0 \cos\phi_0$$

In the equivalent circuit shown in Fig. 13.5*c*, the no-load current I_0 is divided into two parallel branches. The component I_w flowing through resistance R_0 accounts for the core loss, and the component I_m flowing through reactance X_0 represents magnetising current. The R_0-X_0 parallel circuit is called ***exciting circuit*** of the transformer.

EXAMPLE 13.7

A single-phase, 230-V/110-V, 50-Hz transformer takes an input of 350 volt amperes at no load while working at rated voltage. The core loss is 110 W. Find the loss component of no-load current, the magnetising component of no-load current and the no-load power factor.

Solution Given: $V_1 I_0 = 350$ VA.

$$\therefore \qquad I_0 = \frac{VA}{V_1} = \frac{350}{230} = 1.52 \text{ A}$$

The core loss = Input power at no load, $P_i = V_1 I_0 \cos\phi_0$

Therefore, the power factor at no load is given as

$$Pf = \cos\phi_0 = \frac{P_i}{V_1 I_0} = \frac{110\text{ W}}{350\text{ VA}} = \mathbf{0.314}$$

The loss component of no-load current is given as

$$I_w = I_0 \cos \phi_0 = 1.52 \times 0.314 = \mathbf{0.478\ A}$$

The magnetising component of no-load current is given as

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.52)^2 - (0.478)^2} = \mathbf{1.44\ A}$$

Since the core losses occur in the iron core, these are also called ***iron losses.*** These losses have two components: (*i*) hysteresis loss, and (*ii*) eddy-current loss.

(*i*) *Hysteresis Loss* When alternating current flows through the windings, the core material undergoes cyclic process of magnetisation and demagnetisation. It is found that there is a tendency of the flux density B to *lag behind* the field strength H. This tendency is called *hysteresis**. The effect of this phenomenon on the core material can be best understood from the B-H plot shown in Fig. 13.6.

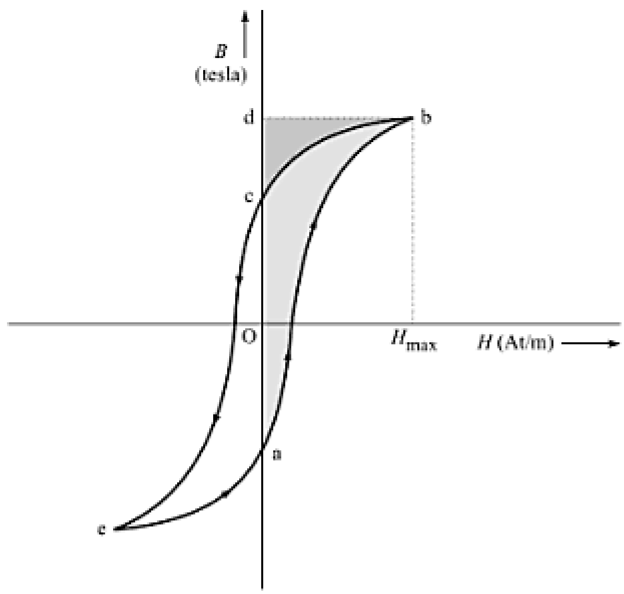


Fig. 13.6 *Hysteresis loop and energy relationship per half-cycle.*

During positive half-cycle, when H increases from zero to its positive maximum value, the energy is stored in the core. This energy is given by the area abda. However, when H decreases from its positive maximum value to zero, the energy is released which is given by the area bdcb. The difference between these two energies is the net loss and is dissipated as heat in the core. Thus, as H varies over one complete cycle, the total energy loss (per cubic metre) is represented by the area abcea of the hysteresis loop. The hysteresis loss (usually expressed in watts) is given as

$$P_h = K_h B_m^n f V \tag{13.9}$$

where K_h = hysteresis coefficient whose value depends upon the material
(K_h = 0.025 for cast steel, K_h = 0.001 for silicon steel)
B_m = maximum flux density (in tesla)

* In Greek, *hysterein* means 'to lag'.

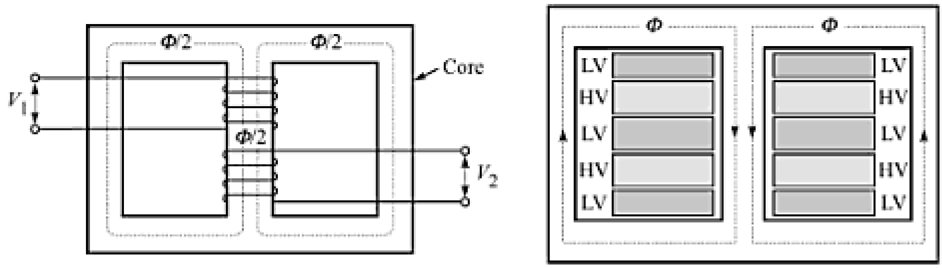


Fig. 13.9 *Shell type transformer.*

In core type transformer, the flux has single path. But in shell type transformer, the flux divides equally in the central limb and returns through the outer two legs. Since there is more space for insulation in the core type transformer, it is preferred for high voltages. On the other hand, the shell type construction is more economical for low voltages.

13.6 TRANSFORMER ON LOAD

Let us examine what happens when a load is connected to the secondary of the transformer. Note that for simplicity we are still considering a partially ideal transformer (i.e., a transformer satisfying only the *ideality conditions* (*iii*) and (*iv*) stated on page 368). Before connecting the load, there exists a flux $\boldsymbol{\Phi}$ in the core due to the no-load current I_0 flowing in the primary. On connecting the load, a current I_2 flows through the secondary, as shown in Fig. 13.10. The magnitude and phase of I_2 with respect to the secondary voltage V_2 depends upon the nature of the load.

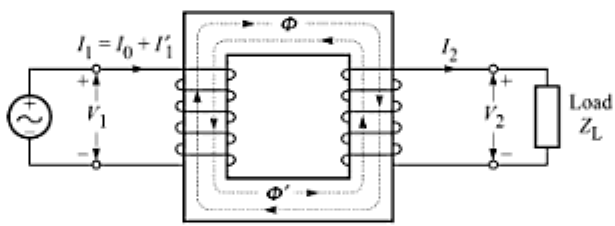


Fig. 13.10 *Transformer on load.*

The current I_2 sets up a flux $\boldsymbol{\Phi}$ in the core, which opposes the main flux $\boldsymbol{\Phi}$. This momentarily weakens the main flux, and the primary back emf E_1 gets reduced. As a result, the difference $V_1 - E_1$ increases and more current is drawn from the supply. This again increases the back emf E_1, so as to balance the applied voltage V_1. In this process, the primary current increases by I'_1. This current is known as ***primary balancing current***, or ***load component of primary current***. Under such a condition, the secondary ampere-turns must be counterbalanced by the primary ampere-turns. That is, $N_1 I'_1 = N_2 I_2$. Hence, we have

$$I'_1 = \left(\frac{N_2}{N_1}\right) I_2 = K I_2 \tag{13.12}$$

The total primary current I_1 is the phasor sum of the no-load current I_0 and the primary balancing current I'_1. That is,

$$\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}'_1 \tag{13.13}$$

The reluctance of the paths of the leakage fluxes Φ_{L_1} and Φ_{L_2} is almost entirely due to the long air paths and is therefore practically constant. Consequently, the value of the leakage flux is proportional to the current. However, the value of the useful flux Φ_u remains almost independent of the load. Furthermore, the reluctance of the paths of the leakage flux is very high. Hence, the value of this flux is relatively small even on full load.

13.8 EQUIVALENT CIRCUIT OF A TRANSFORMER

The function of an ideal transformer is to transform electric power from one voltage level to another without incurring any loss and without needing any magnetizing current. For such a transformer, the volt-amperes in the primary are exactly balanced by the volt-amperes in the secondary. An ideal transformer is supposed to operate at 100 percent efficiency.

We stated the four conditions that must be satisfied by a transformer to be ideal. We then examined the effects of each of these conditions and explored how to account for the deviations in a practical transformer. Based on this, we can now draw the *equivalent circuit* of a practical transformer (Fig. 13.15). This circuit is merely a representation of the following KVL equations for the primary and secondary sides of the transformer.

$$\mathbf{V}_1 = I_1R_1 + jI_1 X_1 - \mathbf{E}_1 = I_1 (R_1 + jX_1) - \mathbf{E}_1 \tag{13.14}$$

and

$$\mathbf{E}_2 = I_2 R_2 + jI_2 X_2 + \mathbf{V}_2 = I_2 (R_2 + jX_2) + \mathbf{V}_2 \tag{13.15}$$

Equation 13.14 states that the applied voltage $\mathbf{V}_1$ is the phasor sum of the negative of induced emf $\mathbf{E}_1$ and the voltage drops in primary *resistance*, R_1, and *leakage reactance*, X_1, due to the flow of current I_1. The induced emf $\mathbf{E}_2$ forces a current I_2 in the secondary circuit. Hence, Eq. 13.15 states that the induced emf $\mathbf{E}_2$ is phasor sum of the load voltage $\mathbf{V}_2$ and the voltage drops in secondary *resistance*, R_2, and *leakage reactance*, X_2, due to the flow of current I_2.

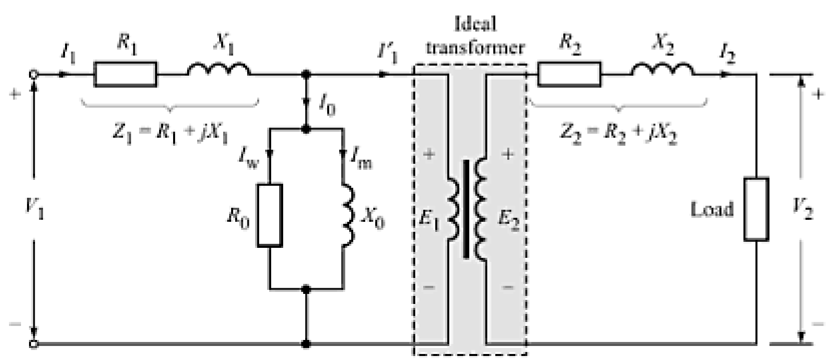


Fig. 13.15 *Equivalent circuit of a transformer.*

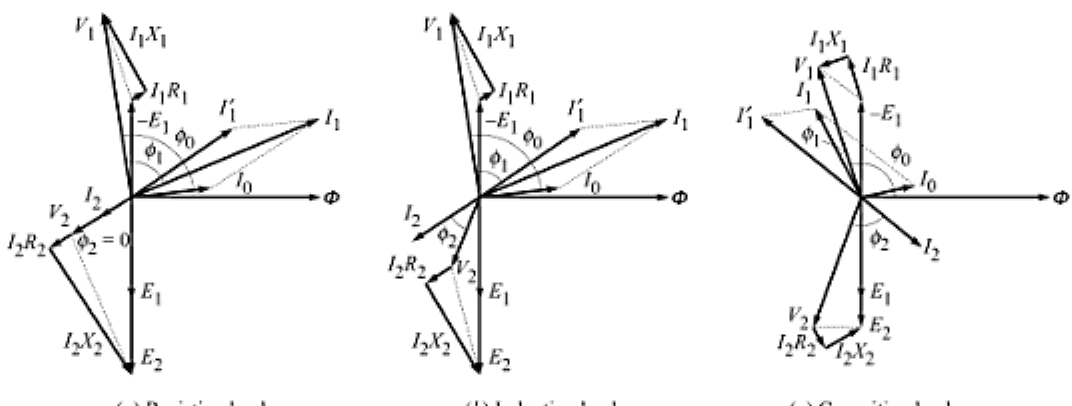
Furthermore, it can be seen from Fig. 13.15 that the primary current I_1 is composed of two components, the no-load current I_0 and the load component of primary current I_1'. Moreover, the current I_0 consists of two components I_w and I_m. The current I_w flows through resistance R_0 and accounts for the iron loss of the transformer. The current I_m, called magnetizing current, is required to establish working magnetic flux in the core. The ac voltage source connected to the primary winding does not have to supply any power in making this current flow. Hence, in the equivalent circuit, it is shown to flow through a pure reactance X_0.

Phasor Diagram

We can draw the phasor diagram of a transformer circuit with a given load, provided all its parameters (as used in the equivalent circuit of Fig. 13.15) are known. While drawing the phasor diagram, we should keep the following points in mind:

1. It is most convenient to commence the phasor diagram with the phasor representing the quantity that is common to the two windings, namely, the flux Φ.
2. The induced emfs E_1 and E_2 lag behind flux Φ by 90°.
3. The values of emfs E_1 and E_2 are proportional to the number of turns on the primary and secondary windings.
4. The magnitude and phase of the current I_2 is decided by the load.
5. The resistive voltage drops are always in phase with the respective current phasor.
6. The inductive voltage drops lead the respective current phasor by 90°.
7. The secondary induced emf E_2 is obtained by vector sum of the terminal voltage V_2 and the impedance drop I_2Z_2. Hence, V_2 must be drawn such that the phasor sum of V_2 and I_2Z_2 is E_2.
8. The primary balancing current I_1' and the secondary current I_2 are in inverse proportion to the number of turns on the primary and secondary windings.
9. The primary current I_1 is the vector sum of the no-load current I_0 and the primary balancing current I_1'.
10. The primary voltage V_1 is obtained by adding vectorially the impedance drop I_1Z_1 to the negative of E_1.
11. The phase angle f_1 between V_1 and I_1 is the power factor-angle of the transformer.

The phasor diagrams for different types of loads (resistive, inductive and capacitive) are shown in Fig. 13.16.

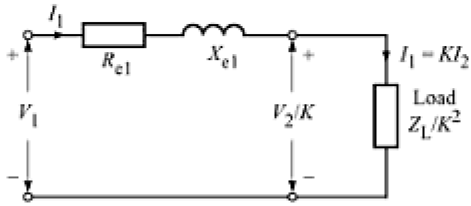


(*a*) Resistive load. (*b*) Inductive load. (*c*) Capacitive load.

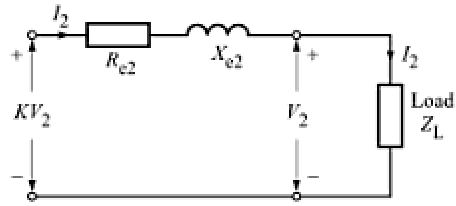
Fig. 13.16 *Phasor diagrams for different types of loads.*

Simplified Equivalent Circuit

Since the no-load current I_0 of a transformer is only about 3–5 percent of the full-load primary current, not much error will be introduced if the exciting circuit R_0-X_0 in Fig. 13.15 is shifted to the left of impedance R_1-X_1. This results in a circuit shown in Fig. 13.17*a*.



(*a*) As referred to primary side.



(*b*) As referred to secondary side.

Fig. 13.18 *Approximate equivalent circuit of a transformer.*

EXAMPLE 13.10

A single-phase, 50-kVA, 4400-V/220-V, 50-Hz transformer has primary and secondary resistances $R_1 = 3.45\ \Omega$ and $R_2 = 0.009\ \Omega$, respectively. The values of the leakage reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for this transformer (*a*) the equivalent resistance as referred to the primary, (*b*) the equivalent resistance as referred to the secondary, (*c*) the equivalent reactance as referred to the primary, (*d*) the equivalent reactance as referred to the secondary, (*e*) the equivalent impedance as referred to the primary, (*f*) the equivalent impedance as referred to the secondary, (*g*) the total copper loss first by using the individual resistances of the two windings and then by using the equivalent resistances as referred to each side.

Solution

$$\text{Full-load primary current, } I_1 = \frac{\text{kVA}}{V_1} = \frac{50000}{4400} = 11.36 \text{ A}$$

$$\text{Full-load secondary current, } I_2 = \frac{\text{kVA}}{V_2} = \frac{50000}{220} = 227.27 \text{ A}$$

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{220}{4400} = \frac{1}{20} = 0.05$$

(*a*) $R_{e1} = R_1 + (R_2/K^2) = 3.45 + [0.009/(0.05)^2] = \mathbf{7.05\ \Omega}$

(*b*) $R_{e2} = K^2 R_1 + R_2 = (0.05)^2 \times 3.45 + 0.009 = \mathbf{0.0176\ \Omega}$

(*c*) $X_{e1} = X_1 + (X_2/K^2) = 5.2 + [0.015/(0.05)^2] = \mathbf{11.2\ \Omega}$

(*d*) $X_{e2} = K^2 X_1 + X_2 = (0.05)^2 \times 5.2 + 0.015 = \mathbf{0.028\ \Omega}$

(*e*) $Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{(7.05)^2 + (11.2)^2} = \mathbf{13.23\ \Omega}$

(*f*) $Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{(0.0176)^2 + (0.028)^2} = \mathbf{0.0331\ \Omega}$

(*g*) Total copper loss $= I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (277)^2 \times 0.009 = \mathbf{909\ W}$

By considering equivalent resistances,

$$\text{Total copper loss} = I_1^2 R_{e1} = (11.36)^2 \times 7.05 = \mathbf{909.8\ W}$$

$$\text{Total copper loss} = I_2^2 R_{e2} = (227.27)^2 \times 0.0176 = \mathbf{909\ W}$$

13.9 VOLTAGE REGULATION OF A TRANSFORMER

With the increase in the load on a transformer, there is a change in its secondary terminal voltage. The voltage falls if the load power-factor is lagging. It increases if the power factor is leading. The *voltage regulation of a transformer is defined as the change in its secondary terminal voltage from no load to full load*, the primary voltage being assumed constant. Let

Here, AF is the approximate voltage drop, which we intend to determine. The error in this approximation is FG. Since the impedance voltage drop I_2Z_{e2} (= AC) is very small compared to the full-load voltage V_2 (= OA), the error committed in the determination of the approximate voltage drop would be negligibly small.

From point B, drop a perpendicular BE on OG, and draw BD parallel to AF. We can now write

$$\text{Approximate voltage drop, AF} = \text{AE} + \text{EF} = \text{AE} + \text{BD}$$

$$= I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi \tag{13.18}$$

In case of *leading power factor* of the load (instead of lagging), we can determine the approximate voltage drop either by putting $-\phi$ in place of $+\phi$ in Eq. 13.18 or by redrawing the phasor diagram for leading power factor, to get

$$\text{Approximate voltage drop, AF} = \text{AE} - \text{EF} = \text{AE} - \text{BD}$$

$$= I_2R_{e2}\cos\phi - I_2X_{e2}\sin\phi \tag{13.19}$$

Thus, in general, we can write

$$\text{Approximate voltage drop,} = I_2R_{e2}\cos\phi \pm I_2X_{e2}\sin\phi \tag{13.20}$$

in which + sign is to be used for lagging power factor and – sign for leading power factor. The voltage drop expressed in percentage is the ***percent regulation***, and is given as

$$\%\ \text{Regultion} = \frac{I_2R_{e2}\cos\phi \pm I_2X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= V_r\cos\phi \pm V_x\sin\phi \tag{13.21}$$

where $$V_r = \%\ \text{resistive drop} = \frac{I_2R_{e2}}{V_{2(0)}} \times 100 \tag{13.22}$$

and $$V_x = \%\ \text{reactive drop} = \frac{I_2X_{e2}}{V_{2(0)}} \times 100 \tag{13.23}$$

Exact Voltage Drop

Referring to Fig. 13.19, the exact voltage drop is AG. We have already determined the approximate value of voltage drop given by AF. To determine FG, we proceed as follows. For the right angle triangle OFC, we can write

$$\text{OC}^2 = \text{OF}^2 + \text{FC}^2$$

or $$\text{OC}^2 - \text{OF}^2 = \text{FC}^2$$

or $$(\text{OC} - \text{OF})(\text{OC} + \text{OF}) = \text{FC}^2$$

or $$(\text{OG} - \text{OF})(\text{OC} + \text{OF}) = \text{FC}^2$$

or $$\text{FG}\,(2\text{OC}) = \text{FC}^2\ [\text{Taking OF} \approx \text{OC}]$$

$\therefore$ $$\text{FG} = \frac{\text{FC}^2}{2\text{OC}} = \frac{(\text{DC} - \text{DF})^2}{2\text{OC}} = \frac{(\text{DC} - \text{BE})^2}{2\text{OC}} = \frac{(I_2X_{e2}\cos\phi - I_2R_{e2}\sin\phi)^2}{2V_{2(0)}}$$

Thus, for lagging power factor,

$$\text{Exact voltage factor} = \text{AF} + \text{FG}$$

$$= (I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi) + \frac{(I_2X_{e2}\cos\phi - I_2R_{e2}\sin\phi)^2}{2V_{2(0)}}$$

the full-load current, $I_2 = \dfrac{40000}{250} = 160$ A

$\therefore$ $$R_{e2} = K^2R_1 + R_2 = (0.0379)^2 \times 10 + 0.02 = 0.0343\ \Omega$$

and $$X_{e2} = K^2X_{e1} = (0.0379)^2 \times 35 = 0.0502\ \Omega$$

(*a*) For power factor, $\cos\phi = 1$; $\sin\phi = 0$. Hence,

$\therefore$ $$\%\ \text{Regulation} = \frac{I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 1 + 0}{250} \times 100 = \mathbf{2.195\,\%}$$

(*b*) For power factor, $\cos\phi = 0.8$ (lagging, ϕ positive); $\sin\phi = \sqrt{1-\cos^2\phi} = 0.6$. Hence,

$\therefore$ $$\%\ \text{Regulation} = \frac{I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 0.8 + 160 \times 0.0502 \times 0.6}{250} \times 100 = \mathbf{3.68\,\%}$$

(*c*) For power factor, $\cos f = 0.8$ (leading, f negative); $\sin\phi = -0.6$. Hence,

$\therefore$ $$\%\ \text{Regulation} = \frac{I_2R_{e2}\cos\phi - I_2X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 0.8 - 160 \times 0.0502 \times 0.6}{250} \times 100 = -\,\mathbf{0.172\,\%}$$

13.10 EFFICIENCY OF A TRANSFORMER

Like any other machine, the efficiency of a transformer is defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power output}}{\text{Power output + Power losses}} = \frac{P_o}{P_o + P_l} \tag{13.28}$$

Power Losses

An ideal transformer would have no power losses, and would be 100% efficient. In practical transformers, power is dissipated in the windings, core, and surrounding structures. Large-size transformers used in power transmission are designed to be more efficient ($\eta > 98\,\%$). But, small transformers, such as those used in power adapters for charging mobile phones, may be no more than 85 % efficient. There are some losses which are quite predominant, and some are quite insginificant.

Major Losses There are two types of major losses in a transformer:

(i) *Copper losses or I^2R losses* in the primary and secondary windings, given as

$$P_c = I_1^2R_1 + I_2^2R_2 = I_1^2R_{e1} = I_2^2R_{e2} \tag{13.29}$$

The copper losses are variable with current. Let us see by what factor the copper losses get reduced when the load on the transformer decreases. We know that the output power is given as

$$P_o = VI\cos\phi = VI \times pf \quad \Rightarrow \quad VI = \frac{P_o}{pf} \tag{13.30}$$

Thus, for a given load we can find the volt-ampere (VA) of the transformer. Assuming the voltage to remain constant, the current is proportional to the VA of the transformer. Since the copper losses are proportional to the square of current, the value of the copper losses for a given load (and hence for given VA) can be calculated from

$$P_c = \left(\frac{\text{VA}}{\text{VA}_{\text{FL}}}\right)^2 P_{c(\text{FL})} \tag{13.31}$$

(ii) *Iron losses or core losses,* due to hysteresis and eddy-currents, given by Eqs. 13.9 and 13.10, respectively. That is, $P_i = P_h + P_e$. Since the maximum value of the flux Φ_m in a normal transformer does not vary more than about 2% between no load and full load, it is usual to assume *the core losses constant at all loads*.

The copper losses dominate when the transformer is loaded. But the iron losses contribute to over 99% of no-load losses. In general, the efficiency of a transformer can be written as

$$\eta = \frac{P_o}{P_o + P_l} = \frac{P_o}{P_o + P_c + P_i} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e2} + P_i} \tag{13.32}$$

Minor Losses Following losses are negligibly small. Hence, these losses can be ignored while calculating the efficiency of a transformer.

(*i*) *Magnetostriction Losses* The alternating magnetic flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle. This produces a *buzzing sound* commonly associated with transformers*, and in turn causes losses due to friction heating.

(*ii*) *Mechanical losses* The alternating magnetic flux causes fluctuating electromagnetic forces between the primary and secondary windings. These incite vibrations within nearby metalwork, producing buzzing noise and consuming a small amount of power.

(*iii*) *Stray losses* The leakage inductance by itself is lossless, since energy supplied to its magnetic field is returned to the supply with the next half-cycle. However, any leakage flux that intercepts nearby conducting materials such as the transformer's support structure will give rise to eddy currents causing loss of power.

Condition for Maximum Efficiency

Assuming that the transformer is operating at a constant terminal voltage and a constant power factor, we are interested to know for what load (i.e., what value of I_2) the efficiency becomes maximum. To determine this, we first divide the numerator and denominator of Eq. 13.32 by I_2, to get

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_{e2} + P_i / I_2}$$

Obviously, the efficiency will be maximum when the denominator of the above equation is minimum, for which we must have

$$\frac{d}{dI_2}(V_2 \cos\phi_2 + I_2 R_{e2} + P_i/I_2) = 0 \quad \text{or} \quad R_{e2} - \frac{P_i}{I_2^2} = 0$$

or

$$I_2^2 R_{e2} = P_i \quad \text{or} \quad P_c = P_i \tag{31.33}$$

Thus, the efficiency at a given terminal voltage and load power factor is maximum for such a load current I_2 which makes the variable losses (copper losses) equal to the constant losses (iron losses).

* Similar buzzing sound is heard in chokes used in the tube-light circuits.

All-day Efficiency

The efficiency defined in Eq. 13.28 is called ***commercial efficiency.*** This efficiency is not of much use in case of a distribution transformer. The primary of a distribution transformer remains energized all the time. But the load on the secondary is intermittent and variable during the day. It means that the core losses occur throughout the day, but the copper losses occur only when the transformer is loaded. Such transformers, therefore, are designed to have minimum core losses. This gives them better ***all-day efficiency***, defined below.

$$\eta_{\text{all-day}} = \frac{\text{Output energy (in kW h) in a cycle of 24 hours}}{\text{Total input energy (in kW h)}} \tag{13.33}$$

EXAMPLE 13.12

For a single-phase, 50-Hz, 150-kVA transformer, the required no-load voltage ratio is 5000-V/250-V. Find (*a*) the number of turns in each winding for a maximum core flux of 0.06 Wb, (*b*) the efficiency at half rated kVA, and unity power factor, (*c*) the efficiency at full load, and 0.8 power factor lagging, and (*d*) the kVA load for maximum efficiency, if the full-load copper losses are 1800 W and core losses are 1500 W.

Solution

(*a*) Using the emf equation, we have

$$E_2 = 4.44\,fN_2\,\Phi_m \quad \Rightarrow \quad N_2 = \frac{E_2}{4.44\,f\Phi_m} = \frac{250}{4.44 \times 50 \times 0.06} = 18.8 \text{ (say, } \mathbf{19\ turns}\text{)}$$

and
$$N_1 = \frac{E_1}{E_2} N_2 = \frac{5000}{250} \times 19 = \mathbf{380\ turns}$$

(*b*) At half rated-kVA, the current is half the full-load current, and hence the output power too reduces by 0.5. Thus,

$$\text{Output power, } P_o = 0.5 \times (\text{kVA}) \times (\text{power factor}) = 0.5 \times 150 \times 1 = 75 \text{ kW}$$

Since copper losses are proportional to the square of current, we have

$$\text{Copper losses, } P_c = (0.5)^2 \times (\text{full-load copper loss}) = (0.5)^2 \times 1800 \text{ W} = 0.45 \text{ kW}$$

Iron losses being fixed, we have

$$\text{Iron losses, } P_i = 1500 \text{ W} = 1.5 \text{ kW}$$

$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{75}{75 + 0.45 + 1.5} \times 100 = \mathbf{97.47\%}$$

(*c*) At full load and 0.8 power factor,

$$\text{Output power, } P_o = (\text{kVA}) \times (\text{power factor}) = 150 \times 0.8 = 120 \text{ kW}$$

$$\text{Copper losses, } P_c = 1800 \text{ W} = 1.8 \text{ kW}$$

$$\text{Iron losses, } P_i = 1500 \text{ W} = 1.5 \text{ kW}$$

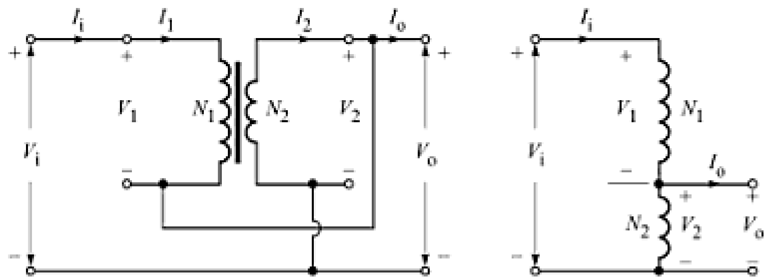
$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{120}{120 + 1.8 + 1.5} \times 100 = \mathbf{97.3\%}$$

(*d*) Let x be the fraction of full-load kVA at which the efficiency becomes maximum (that is, when the variable copper losses are equal to the fixed iron losses). Then

$$P_c = P_i \quad \text{or} \quad x^2 \times 1800 = 1500 \quad x = \sqrt{1500/1800} = 0.913$$

Therefore, the load kVA under the condition of maximum efficiency is

$$\text{Load kVA} = (\text{Full-load kVA}) \times x = 150 \times 0.913 = \mathbf{137\ kVA}$$



(*a*) Special connection of a transformer.

(*b*) Redrawn in the standard way.

Fig. 13.20 *A two-winding transformer converted into an autotransformer.*

secondary drawn in the usual position. Figure 13.20*b* shows the autotransformer (in the step-down mode) drawn in a manner that clarifies its function.

Note that the primary and secondary windings are connected in series for the new primary; the secondary is the new secondary. Also, the primary and secondary are not electrically isolated from each other. Obviously, the voltage $V_2 = V_o$. From Fig. 13.20*b*, it is obvious that

$$V_i = V_1 + V_2 = \frac{N_1}{N_2} V_2 + V_2 = \frac{N_1 + N_2}{N_2} V_o$$

or

$$V_o = \frac{N_2}{N_1 + N_2} V_i \tag{13.34}$$

Hence, the new turns-ratio becomes N_2: $(N_1 + N_2)$. Thus, we find that an autotransformer works like a ***potential divider*** circuit, except that numbers of turns are to be used instead of resistances.

The apparent power rating (kVA rating) of the transformer is increased by the special connection, as is illustrated in Example 13.14, given below.

EXAMPLE 13.14

A single phase, 12-kVA, 120-V/120-V transformer is connected as an autotransformer to make a 240-V/120-V transformer. What is the apparent power rating of the autotransformer?

Solution Figure 13.21 shows the transformer connection with rated voltage and current. The current rating on both primary and secondary windings is

$$I_1 = I_2 = \frac{12\,\text{kVA}}{120\,\text{V}} = 100\text{ A}$$

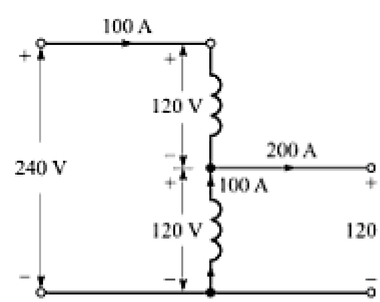


Fig. 13.21

In the autotransformer mode, the input apparent power is 240×100 = **24 kVA**, and the output apparent power is 120×200 = **24 kVA**. Thus, the apparent power capacity of the 12-kVA transformer is doubled by the autotransformer connection. In effect, half the apparent power is transformed and half is conducted directly to the secondary side.

Saving in Copper

For the same voltage ratio and capacity (volt-ampere rating), an autotransformer needs much less copper (or aluminium) material compared to a two-winding transformer. The cross-sectional area of a conductor is proportional to the current carried by it, and its length is proportional to the number of turns. Therefore,

$$\text{Weight of copper in a winding} \propto NI = kNI$$

For a two-winding transformer:

$$\text{Weight of copper in primary} = kN_1I_1$$
$$\text{Weight of copper in secondary} = kN_2I_2$$
$$\text{Total weight of copper} = k(N_1I_1 + N_2I_2)$$

For an autotransformer (see Fig. 13.22*a*):

The portion XY of the winding has $N_1 - N_2$ turns and carries current I_1. The portion YZ of the winding has N_2 turns and carries current $I_2 - I_1$. Therefore,

$$\text{Weight of copper in portion XY} = k(N_1 - N_2)I_1$$
$$\text{Weight of copper in portion YZ} = kN_2(I_2 - I_1)$$
$$\text{Total weight of copper} = k(N_1 - N_2)I_1 + kN_2(I_2 - I_1) = k[(N_1 - 2N_2)I_1 + N_2I_2]$$

Therefore, the ratio of copper-weights for the two cases is

$$\frac{k[(N_1 - 2N_2)I_1 + N_2 I_2]}{k(N_1 I_1 + N_2 I_2)} = \frac{\left[1 - 2\left(\frac{N_2}{N_1}\right)\right]\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)}{\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)} = \frac{[1 - 2K]K + K}{K + K} = 1 - K$$

Evidently, the saving is large if K is close to unity. A unity transformation ratio means that no copper is needed at all for the autotransformer. The winding can be removed all together. The volt-amperes are conductively transformed directly to the load!

Disadvantages The use of autotransformer has following disadvantages:

1. No electrical isolation between the two sides.
2. Should an open-circuit occur between points Y and Z, full primary high voltage appears across the load.
3. The short-circuit current is larger than that in two-winding transformer.

Applications Autotransformers are used in following areas:

1. Boosting or buckling of supply voltage by a small amount.
2. Starting of ac machines, where the voltage is raised in two or more steps.
3. Continuously varying ac supply as in variacs.

13.12 THREE-PHASE TRANSFORMERS

Modern large transformers are usually of the three-phase core type, schematically shown in Fig. 13.23. Three similar limbs are connected by top and bottom yokes. Each limb has primary and secondary windings arranged concentrically. In Fig. 13.23, the primary is shown star-connected and the secondary delta-connected. In actual practice, the windings may be connected star/delta, delta/star, star/star or delta/delta, depending upon the conditions under which the transformer is to be used.

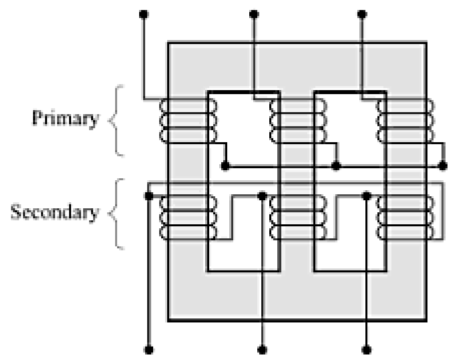


Fig. 13.23 *Three-phase core-type star/delta connected transformer.*

EXAMPLE 13.15

A three-phase, 50-Hz transformer has 840 turns on the primary and 72 turns on the secondary winding. The supply voltage is 3300 V. Determine the secondary line voltage on no load when the windings are connected (*a*) star/delta, (*b*) delta/star.

Solution

(*a*) For star/delta connection:

$$\text{Primary phase voltage, } V_{ph1} = \frac{V_{L1}}{\sqrt{3}} = \frac{3300}{\sqrt{3}} = 1905.3 \text{ V}$$

$$\text{Secondary phase voltage, } V_{ph2} = 1905.3 \times \frac{72}{840} = 163.3 \text{ V}$$

$$\therefore \quad \text{Secondary line voltage, } V_{L2} = V_{ph2} = \mathbf{163.3 \text{ V}}$$

(*b*) For delta/star connection:

$$\text{Primary phase voltage, } V_{ph1} = V_{L1} = 3300 \text{ V}$$

$$\text{Secondary phase voltage, } V_{ph2} = 3300 \times \frac{72}{840} = 283 \text{ V}$$

$$\therefore \quad \text{Secondary line voltage, } V_{L2} = V_{ph2} \times \sqrt{3} = 283 \times \sqrt{3} = \mathbf{490 \text{ V}}$$

13.13 SOME SPECIAL TRANSFORMERS

Following types of transformers are used for specific applications.

Instrument Transformers

*A **current transformer** (CT)* is a measurement device designed to provide a current in its secondary coil proportional to the current flowing in its primary coil. These transformers are commonly used in metering and protective relaying, where they facilitate the safe measurement of large currents. The current transformer isolates the measurement and control circuitry form the high voltages typically present on the circuit being measured.

A ***voltage transformer*** *(VT),* also referred to as ***potential transformer*** (PT) is used for metering and protection in high-voltage circuits. It is designed to present negligible load to the supply being measured and to have a precise voltage ratio to accurately step down high voltages so that metering and protective relay equipment can be operated at a lower potential.

Leakage Transformers

A ***leakage transformer****,* also called a ***stray-field transformer***, has a significantly higher leakage inductance than the power transformers. The leakage inductance is increased by providing loose coupling between the primary and secondary windings, or sometimes by magnetic bypass or shunt in its core. The output and input currents are low enough to prevent thermal overload under all load conditions—even if the secondary is shorted.

Leakage transformers find use in electric arc welding work, where the electrode often gets stuck on the iron piece. Other applications are short-circuit-proof extra-low voltage transformers for toys and doorbell installations.

Resonant Transformers

A ***resonant transformer*** is a kind of leakage transformer. It uses the leakage inductance of its secondary windings in combination with external capacitors, to create one or more resonant circuits. One of the applications of the resonant transformer is for the inverter used in compact fluorescent lamp (CFL). Another application of the resonant transformer is to couple between stages of a super heterodyne receiver, where the selectivity of the receiver is provided by tuned transformers in the intermediate-frequency (IF) amplifier.

13.14 TRANSFORMER TESTING

There are two simple tests conducted on a transformer to determine its efficiency and regulation. These are called open-circuit test and short-circuit test. The power required to carry out these tests is very small compared with the full-load output of the transformer.

(1) Open-Circuit Test

This test determines the no-load current and the parameters of the exciting circuit of the transformer. The transformer is connected as shown in Fig. 13.24. Generally, the low voltage (LV) side is supplied rated voltage and frequency through an autotransformer (also called a ***variac***). The high voltage (HV) side is left open. The ratio of the voltmeter readings, V_2/V_1, gives the transformation ratio of the transformer. The reading of ammeter A, I_o, gives the no-load current I_0, and its reading is a check on the magnetic quality of the ferromagnetic core and joints.

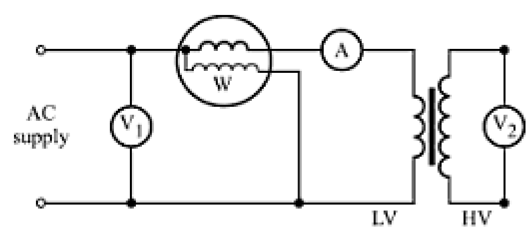


Fig. 13.24 *Open-circuit test on a transformer.*

The short-circuit test gives the equivalent resistance and reactance as referred to the primary side (high voltage winding). From the given specification of the transformer,

$$\text{The transformation ratio, } K = \frac{V_2}{V_1} = \frac{200\text{ V}}{400\text{ V}} = \frac{1}{2}$$

$$\text{The rated full-load current in the high voltage side, } I_{\text{FL}} = \frac{12\text{ kVA}}{400\text{ V}} = 30\text{ A}$$

This confirms that the short-circuit test has been done at the rated full-load. Thus,

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2} = \frac{200\text{ W}}{(30\text{ A})^2} = 0.222\ \Omega \qquad \text{and} \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{22\text{ V}}{30\text{ A}} = 0.733\ \Omega$$

$$\therefore \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(0.733)^2 - (0.222)^2} = 0.699\ \Omega$$

We can now determine the equivalent resistance and reactance as referred to the secondary side (low voltage winding), as

$$R_{e2} = K^2 R_{e1} = \left(\frac{1}{2}\right)^2 \times 0.222 = \mathbf{0.055\ \Omega}$$

and

$$X_{e2} = K^2 X_{e1} = \left(\frac{1}{2}\right)^2 \times 0.699 = \mathbf{0.175\ \Omega}$$

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 13.17

A 25-kVA transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary winding is connected to a 3-kV, 50-Hz ac source. Calculate (*a*) the secondary emf, (*b*) the primary and secondary currents on full load, and (*c*) the maximum flux in the core.

Solution

(*a*) The transformation ratio is given as

$$K = \frac{N_2}{N_1} = \frac{40}{500} = 0.08$$

$$\therefore \qquad \text{Secondary emf, } E_2 = KE_1 = KV_1 = 0.08 \times 3000 = \mathbf{240\ V}$$

(*b*) The primary and secondary full-load currents are given as

$$I_1 = \frac{\text{kVA}}{V_1} = \frac{25\text{ kVA}}{3\text{ kV}} = \mathbf{8.33\ A} \qquad \text{and} \qquad I_2 = \frac{I_1}{K} = \frac{8.33\text{ A}}{0.08} = \mathbf{104.125\ A}$$

(*c*) The maximum flux in the core is given by emf equation,

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{3000}{4.44 \times 50 \times 500} = \mathbf{0.027\ Wb}$$

EXAMPLE 13.18

A 230-V, 50-Hz, single-phase transformer has 50 turns on its primary. It is required to operate with a maximum flux density of 1 T. Calculate the active cross-sectional area of the core. Find suitable dimensions for a square core.

Solution From the emf equation, we have

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 50} = 0.02072 \text{ Wb}$$

$\therefore$ Active core area, $A = \frac{\Phi_m}{B_m} = \frac{0.02072}{1} = 0.02072 \text{ m}^2 = \mathbf{207.2\ cm^2}$

Due to the insulation of laminations from each other, the gross area is about 10% greater than the active area. Thus,

$$\text{Gross area} = 207.2 \times 1.1 = 227.92 \text{ cm}^2$$

If the core has square cross-section, the side of the square is

$$a = \sqrt{227.92} = 15.09 = \mathbf{15\ cm}$$

EXAMPLE 13.19

A single-phase transformer has a core whose cross-sectional area is 150 cm^2, operates at a maximum flux density of 1.1 Wb/m^2 from a 50-Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of 4-Ω impedance. Neglect any voltage drop in the transformer.

Solution $\Phi_m = B_m A = 1.1 \times 0.015 = 0.0165$ Wb.

$E_2 = 4.44\ \Phi_m f N_2 = 4.44 \times 0.0165 \times 50 \times 66 = 242 \text{ V} = V_2$

(Neglecting the voltage drops)

$$\text{Secondary current, } I_2 = \frac{V_2}{Z_L} = \frac{242}{4} = 60.5 \text{ A}$$

$\therefore$ output in kVA $= \frac{V_2 I_2}{1000} = \frac{242 \times 60.5}{1000} = \mathbf{14.6\ kVA}$

EXAMPLE 13.20

A 11-kV/400-V distribution transformer takes a no-load primary current of 1 A at a power factor of 0.24 lagging. Find (*a*) the core-loss current, (*b*) the magnetising current, and (*c*) the iron loss.

Solution

(*a*) The core-loss current, $I_w = I_0 \cos\phi_0 = 1.0 \times 0.24 = \mathbf{0.24\ A}$

(*b*) The magnetising current, $I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1)^2 - (0.24)^2} = \mathbf{0.971\ A}$

(*c*) The iron loss, $P_i = V_1 I_0 \cos\phi_0 = 11000 \times 1.0 \times 0.24 = \mathbf{2640\ W}$

EXAMPLE 13.21

A two-winding, step-down transformer has a turns-ratio (N_2/N_1) of 0.5. The primary winding resistance and reactance are 2.5 Ω and 6 Ω, whereas the secondary winding resistance and reactance are 0.25 Ω and 1 Ω, respectively. Its magnetising current and core-loss current are 51.5 mA and 20.6 mA, respectively. While in operation, the output voltage for a load of 25 ∠30° Ω is found to be 50 V. Determine the supply voltage, the current drawn from the supply and the power factor.

Solution Given: $\mathbf{Z}_1 = (2.5 + j6)\ \Omega = 6.5\angle 67.38°\ \Omega$; $\mathbf{Z}_2 = (0.25 + j1)\ \Omega = 1.03\angle 75.96°\ \Omega$

Let us take V_2 as the reference phasor, i.e., $\mathbf{V}_2 = 50\angle 0°$ V.

Solution

(*a*) From the short-circuit test, we have V_{SC} = 86 V, I_{SC} = 10.5 A, P_{SC} = 360 W. Then,

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{86\text{ V}}{10.5\text{ A}} = 8.19\ \Omega; \quad R_{e1} = \frac{P_{sc}}{I_{sc}^2} = \frac{360\text{ W}}{(10.5\text{ A})^2} = 3.265\ \Omega$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(8.19)^2 - (3.265)^2} = 7.51\ \Omega$$

The full-load primary current, $I_1 = \frac{\text{VA}}{V_1} = \frac{20000}{2200} = 9.09$ A

Since power factor is 0.8, we have $\cos\phi = 0.8$, and $\sin\phi = \sin[\cos^{-1} 0.8] = 0.6$

Using Eq. 13.18, we can determine regulation in terms of quantities referred to the primary side,

$$\%\text{ Regulation} = \frac{I_1(R_{e1}\cos\phi + X_{e1}\sin\phi)}{V_1} \times 100$$

$$= \frac{9.09(3.265 \times 0.8 + 7.51 \times 0.6)}{2200} \times 100 = \mathbf{2.94\,\%}$$

The short-circuit test has been conducted for a short-circuit primary current of 10.5 A, whereas full-load primary current is only 9.09 A. Therefore, the full-load copper loss is given as

$$\text{Full-load copper loss} = \left(\frac{9.09}{10.5}\right)^2 \times 360 = 269.58\text{ W.}$$

The open-circuit test gives the core loss. Hence, P_i = 148 W.

The full-load output power, $P_o = VA \times pf = 20000 \times 0.8 = 16000$ W

$$\therefore \qquad \%\text{ Efficiency, } \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{16000}{16000 + 269.8 + 148} \times 100 = \mathbf{97.45\,\%}$$

(*b*) The power factor on short-circuit is given as

$$pf = \cos\phi_{sc} = \frac{R_{e1}}{Z_{e1}} = \frac{3.265}{8.19} = \mathbf{0.399\ (lagging)}$$

EXAMPLE 13.24

A single-phase, 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarter of full-load, calculate the efficiency at half of full-load current, assuming the power factor to be 0.8 lagging.

Solution At a power factor of 0.8, the full-load output power,

$$P_o = (\text{kVA}) \times (pf) = (200\text{ kVA}) \times 0.8 = 160\text{ kW}$$

Since the efficiency is only 98 %, the input power, $P_{in} = \frac{P_o}{\eta} = \frac{160\text{ kW}}{0.98} = 163.26$ kW

Therefore, total losses (copper loss and iron loss) on full load is

$$P_c + P_i = P_{in} - P_o = 163.26 - 160 = 3.26\text{ kW} \qquad (i)$$

We know that maximum efficiency occurs when the variable loss (copper loss) equals the fixed loss (iron loss). Hence, we have

$$\left(\frac{3}{4}\right)^2 P_c = P_i \qquad (ii)$$

Solving Eqs. (*i*) and (*ii*), we get P_c = 2.09 kW and P_i = 1.17 kW

Now, at half load, the total loss is given as

$$\text{Total losses, } P_1 = \left(\frac{1}{2}\right)^2 P_c + P_i = \left(\frac{1}{2}\right)^2 \times 2.09 + 1.17 = 1.69 \text{ kW}$$

$$\therefore \quad \% \text{ Efficiency, } \eta_{\text{half-load}} = \frac{P_o}{P_o + P_l} \times 100 = \frac{(160/2)}{(160/2) + 1.69} = \mathbf{97.93\%}$$

EXAMPLE 13.25

A single-phase, 150-kVA, 5000-V/250-V, 50-Hz transformer has the full-load copper losses of 1.8 kW and core losses of 1.5 kW. Find (*a*) the number of turns in each winding for a maximum core flux of 60 mWb, (*b*) the efficiency at full rated kVA, with power factor of 0.8 lagging, (*c*) the efficiency at half the rated kVA, with unity power factor, and (*d*) the kVA load for maximum efficiency.

Solution

(*a*) Since $E = 4.44\, fN\, \Phi_m$, the number of turns on the secondary winding,

$$N_2 = \frac{E_2}{4.44\, f\, \Phi_m} = \frac{250}{4.44 \times 50 \times 0.06} = \mathbf{19\ turns}$$

$$\therefore \quad N_1 = N_2 \frac{E_1}{E_2} = 19 \times \frac{5000}{250} = \mathbf{380\ turns}$$

(*b*) At full rated kVA, the current is also full-load. Therefore,

$$\text{Power output } P_o = V_2 I_2 \times \cos\phi = (150 \text{ kVA}) \times 0.8 = 120 \text{ kW}$$

$$\text{Copper losses, } P_c = 1.8 \text{ kW} \qquad \text{and} \qquad \text{the iron losses, } P_i = 1.5 \text{ kW}$$

$$\therefore \quad \% \text{ Efficiency, } \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{120}{120 + 1.8 + 1.5} \times 100 = \mathbf{97.32\,\%}$$

(*c*) At half the rated kVA, the current is half the full-load current. Thus,

$$\text{Power output, } P_o = 0.5 \times V_2 I_2 \times \cos\phi = 0.5 \times (150 \text{ kVA}) \times 1 = 75 \text{ kW}$$

$$\text{Copper losses, } P_c = (0.5 I_2)^2 R_{e2} = 0.25 I_2^2 R_{e2} = 0.25 \times (1.8 \text{ kW}) = 0.45 \text{ kW}$$

$$\text{Iron losses, } P_i = 1.5 \text{ kW (Iron losses do not change with load)}$$

$$\therefore \quad \% \text{ Efficiency, } \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{75}{75 + 0.45 + 1.5} \times 100 = \mathbf{97.47\,\%}$$

(*d*) Let x be the fraction of the full-load kVA at which maximum efficiency occurs. Then, according to the condition for maximum efficiency, we should have

$$x^2 \text{ (copper losses at full load)} = P_i$$

or

$$x^2 \times 1.8 = 1.5 \quad \Rightarrow \quad x = \sqrt{\frac{1.5}{1.8}} = 0.913$$

Hence, the required kVA load for maximum efficiency = (150 kVA) × 0.913 = **137 kVA**

EXAMPLE 13.26

A single-phase, 50-kVA, 2400-V/240-V, 50-Hz transformer is used to step down the voltage of a distribution system. The low tension voltage is required to be kept constant at 240 V.

(*a*) What load impedance connected to the LV side will be loading the transformer fully at 0.8 power factor lagging?

(*b*) What is the value of this impedance referred to the high voltage side?
(*c*) What is the value of the current referred to the high voltage side?

Solution

(*a*) Since the secondary voltage is required to be constant, the secondary current remains the same whatever be the value of the power factor. The full-load secondary current I_2 can be calculated from the kVA ratings,

$$I_2 = \frac{50\,\text{kVA}}{240\,\text{V}} = 208.33\ \text{A}$$

Therefore, the load impedance, $Z_L = \dfrac{V_2}{I_2} = \dfrac{240\ \text{V}}{208.33\ \text{A}} = \mathbf{1.152\ \Omega}$

(*b*) Transformation ratio, $K = \dfrac{V_2}{V_1} = \dfrac{240}{2400} = 0.1$

The load impedance referred to the primary side,

$$Z_{eq} = Z_L/K^2 = 1.152/(0.1)^2 = \mathbf{115.2\ \Omega}$$

(*c*) The current referred to the high voltage side,

$$I_1' = KI_2 = 0.1 \times 208.33 = \mathbf{20.833\ A}$$

EXAMPLE 13.27

A single-phase, 10-kVA, 2300-V/230-V, 50-Hz transformer is connected as an autotransformer with LT winding in series with the HT winding as shown in Fig. 13.26. The autotransformer is excited from a 2530-V source, and it is fully loaded such that the rated currents of the windings are not exceeded. Determine (*a*) the current distribution in the windings, (*b*) the kVA output, (*c*) the volt-amperes transferred conductively and inductively from the input to the output, and (*d*) the saving in copper as compared to the two-winding transformer for the same output.

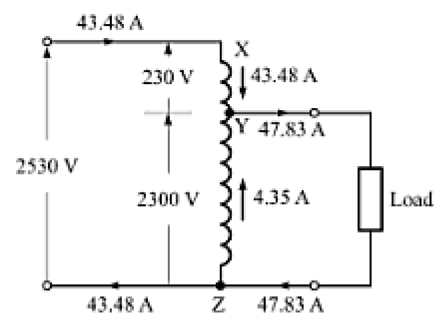


Fig. 13.26 *A two-winding transformer connected as autotransformer.*

Solution

$$\text{Current rating of HT winding} = \frac{10\,000}{2300} = 4.35\ \text{A}$$

$$\text{Current rating of LT winding} = \frac{10\,000}{230} = 43.48\ \text{A}$$

(*a*) The autotransformer can supply a load current $I_2 = 43.48 + 4.35 = 47.83$ A at 2300 V, as shown in Fig. 13.26. The input current $I_1 = 43.48$ A. The current distribution is shown in the figure.

(*b*) The kVA output $= \dfrac{2300 \times 47.83}{1000} = \mathbf{110\ kVA}$

(*c*) The volt-amperes transferred conductively $= V_2 I_1 = 2300 \times 43.48$ VA $=$ **100 kVA**

The volt-amperes transferred inductively $= V_2(I_2 - I_1) = 2300 \times 4.35$ VA $=$ **10 kVA**

(*d*) Saving in copper $= K = \dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{2300}{2530} = 0.909$ or **90.9%**

- Condition for *maximum regulation*: $\tan\phi = \dfrac{X_{e2}}{R_{e2}}$.
- Efficiency, $\eta = \dfrac{\text{Power output}}{\text{Power output + Power loss}} = \dfrac{P_o}{P_o + P_l}$.
- Condition for *maximum efficiency*: Variable losses = Fixed losses, or $P_c = P_i$.
- From OCV test: $P_i = W_0$; $I_0 = I_0$; $I_w = \dfrac{W_o}{V_1}$; $I_m = \sqrt{I_0^2 - I_w^2}$; $R_0 = \dfrac{V_1}{I_w}$; $X_0 = \dfrac{V_1}{I_m}$.
- From SC test: $R_{e1} = \dfrac{W_{sc}}{I_{sc}^2}$; $Z_{e1} = \dfrac{V_{sc}}{I_{sc}}$; $X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	When the primary winding of a transformer is connected to an ac source, an emf is induced only in the secondary winding.	☐	☐	
2.	In a step-up transformer, the current is stepped down.	☐	☐	
3.	In a core-type transformer, the two windings are put on two separate limbs.	☐	☐	
4.	The magnetizing current I_m leads the mutual flux Φ_m by 90°.	☐	☐	
5.	The no-load current in a transformer is about 2–5% of the full-load current.	☐	☐	
6.	If the power factor of the load on a transformer is lagging, it is possible to make voltage regulation zero.	☐	☐	
7.	A transformer is designed such that it has maximum efficiency at full load. If it has total losses P at full load, the total losses at half load will be 0.625 P.	☐	☐	
8.	The primary and secondary leakage fluxes are simulated by primary and secondary leakage reactances in the equivalent circuit of a transformer.	☐	☐	
9.	There is electrical isolation between primary and secondary windings of an autotransformer.	☐	☐	
10.	In a short-circuit test of a transformer, usually the high voltage side is shorted.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** True
6. False	**7.** True	**8.** True	**9.** False	**10.** False

2. A close magnetic coupling between primary and secondary windings results in
   (*a*) high commercial efficiency
   (*b*) high all-day efficiency
   (*c*) good voltage regulation
   (*d*) none of the above
3. The number of turns in the primary winding of a transformer depends on
   (*a*) the input voltage
   (*b*) the input current
   (*c*) both the input voltage and current
   (*d*) the kVA
4. Cooling of transformers is required so as to
   (*a*) increase the efficiency
   (*b*) reduce the losses
   (*c*) reduces the humming
   (*d*) dissipate the heat generated in the windings
5. The emf induced in the windings of a transformer
   (*a*) is in phase with the core flux
   (*b*) is out of phase with the core flux
   (*c*) lags the core flux by $\pi/2$
   (*d*) leads the core flux by $\pi/2$
6. A single-phase, 150-kVA, 1100-V/400-V transformer has 100 turns on the secondary winding. The number of turns on its primary winding will be
   (*a*) 5500 (*b*) 2200
   (*c*) 550 (*d*) 275
7. A single-phase, 5-kVA, 200-V/100-V transformer delivers 50 A at the rated voltage. The input current will be
   (*a*) 25 A (*b*) more than 25 A
   (*c*) less than 25 A (*d*) 100 A
8. In a single-phase, 10-kVA, 230-V/1000-V transformer, the no-load current will be about
   (*a*) 0.5 A (*b*) 2 A
   (*c*) 10 A (*d*) 15 A
9. A single-phase transformer has a turns-ratio of 4: 1. If the secondary winding has a resistance of 1 ohm, this resistance as referred to the primary will be
   (*a*) 16 Ω (*b*) 4 Ω
   (*c*) 0.25 Ω (*d*) 0.0625 Ω
10. A transformer is supplying a unity power-factor load. The power factor at the primary terminals will be
   (*a*) about 0.95 lagging
   (*b*) about 0.95 leading
   (*c*) about 0.8 lagging
   (*d*) unity
11. When the secondary winding of a transformer is short-circuited, the power factor of the input is
   (*a*) unity
   (*b*) about 0.8 leading
   (*c*) about 0.8 lagging
   (*d*) about 0.2 lagging
12. Under no-load condition, the power factor of a transformer is
   (*a*) unity
   (*b*) zero
   (*c*) about 0.4 lagging
   (*d*) about 0.4 leading
13. If the full-load copper loss of a transformer is 100 W, its copper loss at half load will be
   (*a*) 200 W (*b*) 100 W
   (*c*) 50 W (*d*) 25 W
14. If the full-load core loss of a transformer is 100 W, its core loss at half load will be
   (*a*) 200 W (*b*) 100 W
   (*c*) 50 W (*d*) 25 W
15. A single-phase transformer is supplying power to a load at a terminal voltage of 11 kV. When the load is disconnected, the terminal voltage becomes 11.5 kV. The voltage regulation of this transformer for this load is
   (*a*) 55% (*b*) 11.55%
   (*c*) 5% (*d*) 2.5%
16. A transformer operates at maximum efficiency, when
   (*a*) its hysteresis loss and eddy-current loss are minimum
   (*b*) the sum of its hysteresis loss and eddy-current loss is equal to its copper loss
   (*c*) the power factor of the load is leading
   (*d*) its hysteresis loss is equal to its eddy-current loss
17. A distribution transformer should be selected on the basis of its
   (*a*) all-day efficiency
   (*b*) regulation
   (*c*) commercial efficiency
   (*d*) all the above

across the secondary. What is now the value of the primary current? (Neglect the voltage drops in the transformer.) **[Ans.** 25.9 A]

8. A single-phase transformer has 100 turns on the primary winding and 400 turns on the secondary winding. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to a 50-Hz, 230-V supply, calculate (*a*) the emf induced in the secondary winding, and (*b*) the maximum value of the flux density in the core.

[Ans. (*a*) 920 V; (*b*) 0.414 T]

9. The no-load current of a single-phase transformer is 5.0 A at 0.3 power factor when supplied from a 240-V, 50-Hz source. The number of turns on the primary is 200. Calculate (*a*) the maximum value of the flux in the core, (*b*) the core losses, and (*c*) the magnetizing current.

[Ans. (*a*) 5.4 mWb; (*b*) 360 W; (*c*) 4.77 A]

10. A transformer on no-load takes 1.5 A at a power factor of 0.2 lagging when its primary is connected to a 50-Hz, 230-V supply. Its transformation ratio (N_2/N_1) is 1/3. Determine the primary current when the secondary is supplying a current of 300 A at a power factor of 0.8 lagging. Neglect the voltage drops in the windings. **[Ans.** 14.5 A]

11. The no-load current of a 50-Hz, 230-V transformer is 4.5 A at a power factor of 0.25 lagging. The number of turns on the primary winding is 250. Calculate (*a*) the magnetizing current, (*b*) the core loss, and (*c*) the maximum value of the flux in the core.

[Ans. (*a*) 4.35 A; (*b*) 258.75 W; (*c*) 4.14 mWb]

12. A 100-kVA transformer has 400 turns on the primary and 80 turns on the secondary winding. The primary and secondary resistances are 0.3 Ω and 0.1 Ω, respectively. The primary and secondary leakage reactances are 1.1 Ω and 0.035 Ω, respectively. Calculate the equivalent impedance referred to the primary side. **[Ans.** 3.426 Ω]

13. A single-phase, 100-kVA, 1100-V/220-V transformer has following parameters: R_1 = 0.1 Ω, X_1 = 0.3 Ω, R_2 = 0.004 Ω and X_2 = 0.012 Ω. Determine (*a*) the equivalent resistance and leakage reactance as referred to the high voltage winding, and (*b*) the equivalent resistance and leakage reactance as referred to the low voltage winding.

[Ans. (*a*) 0.2 Ω, 0.6 Ω; (*b*) 0.008 Ω, 0.024 Ω]

14. In a 50-kVA, 11-kV/400-V, single-phase transformer, the iron and copper losses are 500 W and 600 W, respectively under rated conditions. Calculate (*a*) the efficiency at unity power factor at full load, (*b*) the load for maximum efficiency, and (*c*) the iron and copper losses for this load.

[Ans. (*a*) 97.85%; (*b*) 45.64 kVA; (*c*) 500 W, 500 W]

15. A 10-kVA, 200-V/400-V, 50-Hz, single-phase transformer gave the following test-results:

OC test (HV winding open) : 200 V, 1.3 A, 120 W.

SC test (LV winding shorted) : 22 V, 30 A, 200 W.

Calculate (*a*) the magnetising current, and (*b*) the equivalent resistance and leakage reactance as referred to the low voltage side.

[Ans. (*a*) 1.15 A; (*b*) 0.0555 Ω, 0.1745 Ω]

(B) TRICKY PROBLEMS

16. A single-phase, 50-Hz transformer has 30 turns on primary and 350 turns on secondary. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to a 230-V, 50-Hz supply, calculate (*a*) peak value of the flux density in the core, (*b*) the voltage induced in the secondary winding, and (*c*) the primary current when the secondary current is 100 A. Neglect the losses.

[Ans. (*a*) 1.3814 T; (*b*) 2683.33 V; (*c*) 1166.67 A]

17. A single-phase, 50-Hz, 100-kVA, 2400-V/240-V transformer has no-load current of 0.64 A and core loss of 700 W, when its high voltage (HV) side is energised at rated voltage and frequency. Calculate the two components of the no-load current. If this transformer supplies a load current of 40 A at 0.8 lagging pf on its low voltage side, determine the primary current and its power factor. Ignore the resistance and leakage reactance drops.

[Ans. 0.5697 A, 0.2917 A; 4.584 A, 0.762 lagging]

18. An ideal 50-Hz, core-type transformer has 100 primary-turns and 200 secondary-turns. The primary rated voltage is 220 V. If the maximum permissible flux density is 1.2 T, what should be the cross-sectional area? Also, find (*a*) the secondary voltage, and (*b*) the primary current in complex form with reference to the secondary voltage vector when secondary delivers a current of 8 A at a lagging power factor of 0.8.

[**Ans.** 8.26×10^{-3} m^2; (*a*) 440 V; (*b*) $16\angle 143.1°$ A]

19. A 500-kVA, 11000-V/400-V, 50-Hz, single-phase transformer has 100 turns on the secondary winding. Calculate (*a*) the approximate number of turns on the primary winding, (*b*) the approximate values of the primary and secondary current, and (*c*) the maximum value of the flux in the core.

[**Ans.** (*a*) 2750; (*b*) 45.45 A,1250 A; (*c*) 0.018 Wb]

20. A single-phase transformer has a turns-ratio of 144/432 and operates at a maximum flux of 7.5 mWb at 50 Hz. When working on no load, it takes 0.24 kVA at a power factor of 0.26 lagging from the supply. If it supplies a load of 1.2 kVA at a power factor of 0.8 lagging, determine (*a*) the magnetising current, (*b*) the primary current, and (*c*) the primary power factor.

[**Ans.** (*a*) 0.97 A; (*b*) 5.8 A; (*c*) 0.731 (lagging)]

21. The primary and secondary windings of a 30-kVA, 6000-V/230-V, single-phase transformer have resistances of 10 Ω and 0.0016 Ω, respectively. The total reactance of the transformer as referred to the primary side is 23 Ω. Calculate the percentage regulation of the transformer when supplying full-load current at a power factor of 0.8 lagging.

[**Ans.** 2.54%]

22. A transformer with turns-ratio 8 : 1, has the resistances of the primary and secondary windings as 0.85 Ω and 0.012 Ω, respectively, and the leakage reactances as 4.8 Ω and 0.07 Ω, respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary when the secondary terminals are short-circuited. Ignore the magnetising current. [**Ans.** 176.6 V]

23. A single-phase, 50-Hz transformer has a turns ratio 6 : 1. The primary and secondary winding resistances are 0.90 Ω and 0.03 Ω, respectively, and leakage reactances are 5 Ω and 0.13 Ω, respectively. Find (*a*) the voltage to be applied to the high voltage side to get a current of 100 A in the low voltage winding on short-circuit, and (*b*) the primary power factor on short-circuit. Neglect the no-load current.

[**Ans.** (*a*) 164.67 V; (*b*) 0.2 lagging]

(C) CHALLENGING PROBLEMS

24. A single-phase transformer has $\mathbf{Z}_1 = (1.4 + j5.2)$ Ω and $\mathbf{Z}_2 = (0.0117 + j0.0465)$ Ω. The input voltage is 6600 V and turns-ratio is 10.6 : 1. The secondary feeds a load which draws 300 A at 0.8 power factor lagging. Neglecting no-load current I_0, determine the secondary voltage and the output in kW.

[**Ans.** 600 V, 144 kW)]

25. A single-phase, 10-kVA, 4000-V/400-V transformer has following parameters: $R_1 = 13$ Ω, $X_1 = 20$ Ω, $R_2 = 0.15$ Ω, $X_2 = 0.25$ Ω, $R_0 = 12000$ Ω and $X_0 = 6000$ Ω. Determine (*a*) the equivalent resistance and leakage reactance as referred to the primary winding, (*b*) the input current with secondary terminals open-circuited, and (*c*) the input current when the secondary supplies a load current of 25 A at a power of 0.8 lagging.

[**Ans.** (*a*) 28 Ω, 45 Ω; (*b*) $0.745\angle -63.5°$ A; (*c*) $3.18\angle -42.9°$ A]

26. A single-phase, 200-V/2000-V transformer is fed from a 200-V supply. The equivalent winding resistance and leakage reactance as referred to the low voltage side are 0.16 Ω and 0.7 Ω, respectively. The resistance representing core losses is 400 Ω and the magnetizing reactance is 231 Ω. A load impedance of $Z_L = (596 + j444)$ Ω is connected across the secondary terminals. Calculate (*a*) the input current, (*b*) the secondary terminal voltage, and (*c*) the primary power factor.

[**Ans.** (*a*) $25.96\angle -40.78°$ A; (*b*) 1859 V; (*c*) 0.757 lagging]

27. A single-phase, 100-kVA, 2000-V/200-V, 50-Hz transformer has impedance drop of 10% and resistance drop of 5%. (*a*) What is the regulation at full-load 0.8 power factor lagging? (*b*) At what power factor is the regulation zero?

[**Ans.** (*a*) 9.196%; (*b*) 0.866 leading]

28. The primary and secondary windings of a 500-kVA, 11-kV/415-V, single-phase transformer have resistances of 0.42 Ω and 0.0019 Ω, respectively. Its core losses are 2.9 kW. Assuming the power factor to be 0.8, calculate its efficiency on (*a*) full load, and (*b*) half load.

[**Ans.** (*a*) 98.39%; (*b*) 98.13%]

29. For the transformer in Prob. 28, assuming the power factor to be 0.8, find the output at which the efficiency is maximum and calculate its value.

[**Ans.** 447 kVA, 98.4%]

30. A single-phase transformer has percentage regulations of 4 and 4.4 for lagging power factors of 0.8 and 0.6, respectively. The full-load copper loss is equal to the iron loss. Calculate (*a*) the lagging power factor at which the full-load regulation is maximum, and (*b*) the full-load efficiency at unity power factor.

[**Ans.** (*a*) 0.4472 lagging; (*b*) 96.15%]

31. A 250-kVA, single-phase transformer has an efficiency of 96% on full load at 0.8 power factor lagging and on half load at 0.8 power factor lagging. Find iron loss and full-load copper loss.

[**Ans.** 2.78 kW, 5.56 kW]

EXPERIMENTAL EXERCISE 13.1

LOAD TEST ON A SINGLE-PHASE TRANSFORMER

Objectives

1. To determine the polarity of the primary and secondary windings.
2. To find the turns-ratio.
3. To determine the efficiency at different loads.
4. To determine the voltage regulation.

Apparatus Single-phase ac power supply 230 V; One single-phase transformer 2 kVA, 220-V/110-V; One Variac 0-270 V, 15 A; One wattmeter 10 A, 230 V; Two voltmeters (MI type) 0-230 V; Two ammeters (MI type) 0-10 A and 0-20 A; One resistive load.

Circuit Diagrams The circuit diagrams are shown in Fig. 13.27.

Brief Theory

(*i*) When a transformer is used, a terminal of primary (as well as of secondary) winding is *alternately* positive and negative with respect to other terminal. It becomes important to know the *relative polarities* of the primary and secondary terminals in situations such as follows:

(1) When two single-phase transformers are connected in parallel so as to share the total load on the system.

(2) When three single-phase transformers are connected to make a three-phase transformer.

The relative polarities can be determined by shorting one of the terminals of the primary and secondary windings (say, P_2 and S_2 in Fig. 13.27*a*), and then measuring the voltage across the other terminals of the primary and secondary windings (say, P_1 and S_1 in Fig. 13.27*a*).

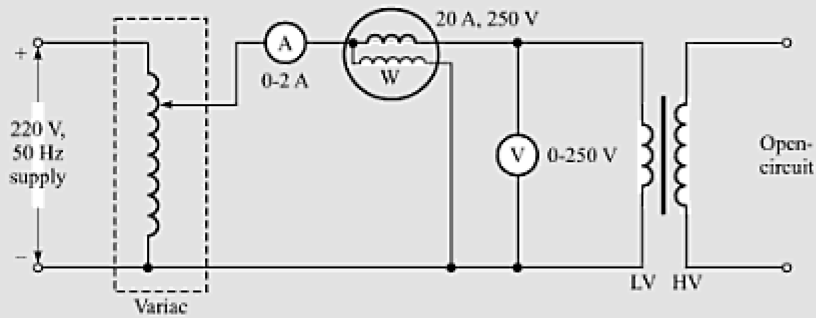
If P_1 and S_1 have same polarity (say, positive at an instant), the situation is as shown in Fig. 13.28*a*. Obviously, in such a case, the voltage $V_3 = V_1 - V_2$. On the other hand, if P_1 and S_1 have opposite polarity (say, positive and negative, respectively, at an instant), the situation becomes as shown in Fig. 13.28*b*. Obviously, in such a case, the voltage $V_3 = V_1 + V_2$.

(*ii*) The induced emf in winding is proportional to the number of turns on it. Therefore, the turns-ratio is given as

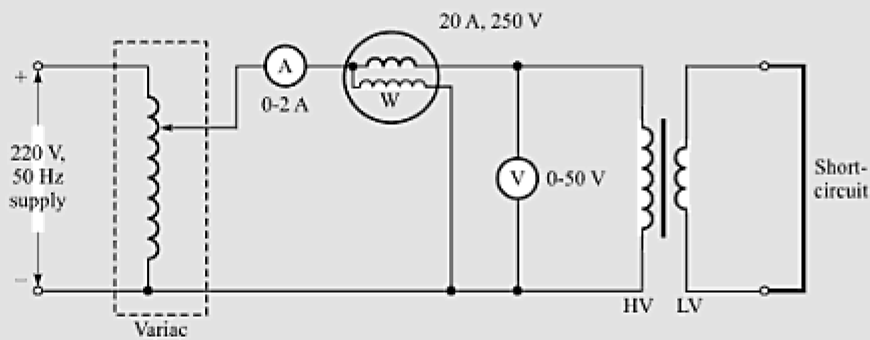
$$K = \frac{N_2}{N_1} = \frac{E_2}{E_1}; \frac{V_2}{V_1}$$

(*iii*) If P_{in} is the input power and P_o is the output power of a transformer, its efficiency is given as

$$\eta = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + P_l}$$



(*a*) Circuit for open-circuit test.



(*b*) Circuit for short-circuit test.

Fig. 13.30 *Circuit diagrams for testing a transformer.*

(V_{sc}) and the ammeter (I_{sc}) are noted. Since the applied voltage is low, the iron losses are negligibly small compared to the copper loss. Therefore, the wattmeter indicates the full-load copper loss, i.e.,

$$\text{Copper loss, } P_c = W_{sc}$$

The equivalent resistance R_{eq}, the equivalent reactance X_{eq} and the equivalent impedance Z_{eq}, as referred to the winding on which the measurements are made, can be calculated as follows:

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2}; \quad Z_{e1} = \frac{V_{sc}}{I_{sc}}; \quad \text{and} \quad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

Procedure

(*i*) *For Open-Circuit Test*

1. Connect the circuit as shown in Fig. 13.30*a*.
2. Put the variac at low voltage output.
3. Switch ON the ac supply.
4. Adjust the variac to the rated voltage of the transformer.
5. Record the ammeter, wattmeter and voltmeter readings.
6. Switch OFF the supply.

(*ii*) *For Short-Circuit Test*

1. Connect the circuit as shown in Fig. 13.30*b*.

2. Put the variac at lowest voltage output.
3. Slowly increase the applied voltage till the ammeter reading equals the rated value.
4. Record the ammeter, wattmeter and voltmeter readings.
5. Switch OFF the supply.

Observations

(*i*) *Open-Circuit Test*

Reading of the ammeter, $I_o =$

Reading of the wattmeter, $W_o =$

Reading of the voltmeter, $V_o =$

(*ii*) *Short-Circuit Test*

Reading of the ammeter, $I_{sc} =$

Reading of the wattmeter, $W_{sc} =$

Reading of the voltmeter, $V_{sc} =$

Calculations

(*i*) $I_w = \dfrac{W_o}{V_o} = \text{—} = \quad \text{A}; \quad I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{()^2 - ()^2} = \quad \text{A};$

$R_0 = \dfrac{V_o}{I_w} = \text{—} = \quad \Omega; \quad X_0 = \dfrac{V_o}{I_m} = \text{—} = \quad \Omega.$

(*ii*) $R_{e1} = \dfrac{W_{sc}}{I_{sc}^2} = \text{—} = \quad \Omega; \quad Z_{e1} = \dfrac{V_{sc}}{I_{sc}} = \text{—} = \quad \Omega;$

$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{()^2 - ()^2} = \quad \Omega.$

Results

1. The values of the equivalent circuit parameters are as follows:

$R_0 = \quad \Omega; \quad X_0 = \quad \Omega; \quad R_{e1} = \quad \Omega; \quad X_{e1} = \quad \Omega; \quad Z_{e1} = \quad \Omega.$

2. The magnetizing current, $I_m =$ A; The iron-loss current, $I_w =$ A.
3. The no-load current, $I_0 =$ A.

Precautions

1. Before switching on the supply, the zero readings of the wattmeter, voltmeters and ammeter should be checked.
2. Meters of proper range should be selected.
3. While conducting the short-circuit test, the voltage applied should be initially set at zero. It should then be increased slowly. If, by mistake, higher voltage than needed is applied, there is every possibility that the transformer would be damaged.

Viva-Voce

1. What is the importance of open-circuit and short-circuit tests on a transformer?
   Ans.: The purpose of these tests is to find the iron loss, copper loss and hence the efficiency of the transformer at different loads. This is an indirect method of testing, since the transformer is not actually loaded during the testing. Especially, for large size transformers this type of indirect testing is important as the arrangement for actual loading just cannot be done.
2. How do you justify that the reading of the wattmeter in open-circuit test indicates the iron losses?

Ans.: The iron losses (hysteresis and eddy-current) depend upon the magnetisation level of the core, which in turn depends upon the voltage applied. In open-circuit test, full rated voltage is applied to the transformer. Hence, the iron losses occur at full rated value. Furthermore, as the secondary winding is open, the secondary current is zero. Hence there is no copper loss in the secondary winding. However, in the primary winding, there is small no-load current flowing. The copper loss in the primary due to this small current is negligible. Hence, the wattmeter reading gives the iron losses.

3. What does the reading of wattmeter indicate in the short-circuit test? Justify your answer.
Ans.: It indicates the full-load copper losses of the transformer. In short-circuit test, full rated current is made to flow through both the secondary and primary windings, by applying very low voltage to the primary. Because of low voltage the magnetisation level of the core is very low. Hence the iron losses are negligibly small. Thus, the wattmeter indicates only the full-rated copper losses of the transformer.

4. Suppose that the full rated voltage is applied in the short-circuit test. What do you expect to happen?
Ans.: Since the secondary winding is shorted, a heavy current will be drawn by the transformer. As a result, the circuit breaker should trip off. If it does not, the transformer will get excessively overheated resulting in burning of the insulation.

5. How do the iron losses vary with the load on the transformer?
Ans.: Iron losses do not depend on the load. These losses depend only on the magnetisation level of the core, which in turn depend upon the voltage applied.

6. How do the copper losses vary with the load on the transformer?
Ans.: Copper losses or I^2R losses depend on the square of the currents flowing in the windings.

7. What is the phasor relationship between $\mathbf{I}_w$, $\mathbf{I}_m$ and I_0?
Ans.: The current $\mathbf{I}_0$ is the phasor sum of $\mathbf{I}_w$ and $\mathbf{I}_m$. $I_0 = \sqrt{I_w^2 + I_m^2}$. The iron-loss component I_w is in phase with the applied voltage. The magnetising component I_m lags behind the applied voltage by 90°.

8. What is the magnitude of no-load current as compared to the full-load current?
Ans.: The no-load current is about 3-5% of the full-load current.

9. Of what order is the power factor of a transformer under no-load condition?
Ans.: It is about 0.2 lagging.

ALTERNATORS AND SYNCHRONOUS MOTORS 14

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the meaning of electro-mechanical energy-conversion (EMEC) machines.*
- *Explain how the power flows from a prime-mover to an electrical load by means of a generator.*
- *Explain how the power flows from electric supply to a mechanical load by means of a motor.*
- *State how a generator differs from a motor.*
- *Explain the two ways (alignment and interaction) in which a mechanical force is generated in an electromechanical system.*
- *State the general characteristics of a synchronous machine.*
- *Derive the expression for synchronous speed in terms of number of poles and frequency of induced emf.*
- *Describe the construction of stator and rotor of a synchronous machine.*
- *Explain how a rotating magnetic flux is produced by three-phase currents in three windings on the stator in a synchronous machine.*
- *Explain why a synchronous machine has armature on its stator and field on its rotor.*
- *Derive the expressions for pitch factor and distribution factor.*
- *Derive the expression for induced emf in a generator.*
- *Explain the meaning of 'armature reaction' and its effect on generated induced emf.*
- *Draw the equivalent circuit for the stator of an alternator.*
- *Explain the meaning of synchronous reactance (X_s) and synchronous impedance (Z_s).*
- *Derive expressions of real power and reactive power generated by a generator.*
- *Describe how to determine the synchronous impedance of a synchronous machine by conducting open-circuit and short-circuit tests on it.*
- *Determine voltage regulation for lagging and leading power factors.*
- *Explain in what way the operation of a synchronous motor differs from that of a generator, with the help of suitable phasor diagrams.*
- *Justify that a synchronous motor is a constant-speed motor in true sense.*
- *State different methods of starting a synchronous motor.*
- *State how the operation of a synchronous motor is affected by changing the mechanical load and by changing the excitation current.*
- *Explain how an overexcited synchronous motor works as a condenser.*

14.1 ELECTRO-MECHANICAL ENERGY-CONVERSION MACHINES

In the previous Chapters, we saw how electrical power is transmitted by means of three-phase circuits and how transformers are used to raise and to lower the system voltages. Next, we should consider to what purpose the electrical power is used at the consumer's end.

A large amount of electrical power is used to provide heat during winters, either for heating a room or for heating water in a geyser. During summers, we use electrical power to provide cooling, as in air-conditioners or refrigerators. Some heat conversion is used to provide light as in the common light bulb. But a lot of electrical power is used to drive machines in industry and homes. Useful gadgets like fans, tape-recorders, mixies, etc., all run with the help of a ***motor*** fixed inside them. Basically, these motors take energy from electrical power mains and ***convert*** it into mechanical energy which is used to turn the blades of the fan or mixie. Reverse conversion takes place in a machine called ***generator***. You have a generator (also, called *dynamo*) in your car. While the car is running, the dynamo converts mechanical energy (drawn from the engine) into electrical energy which keeps on charging the battery.

Motors and generators are electro-mechanical energy-conversion (EMEC) devices or machines*. The process of electro-mechanical energy conversion is essentially *reversible* (except for a small amount which is lost as heat energy). In both the motor and the generator, it is the *magnetic field* that ***couples*** the electrical system and the mechanical system, as illustrated in Fig. 14.1. The magnetic field provides the mutual link between the two systems. The energy to be transferred from one system to the other is temporarily stored in the magnetic field and then released in the other system.

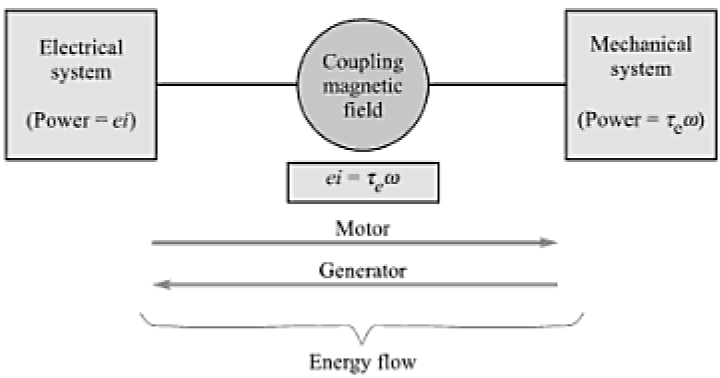


Fig. 14.1 *Representation of electro-mechanical energy-conversion (EMEC) machines.*

The interchange of energy between the electrical system and mechanical system takes place through the conductors placed in a magnetic field. The primary quantities in the electrical system are *induced emf* (e) and *current* (i). The analogous quantities in the mechanical system are *electromagnetic torque* (τ_e) and *angular speed* (ω), respectively.

When the electrical system is characterized by direct current, the machines are called *dc motors* and *dc generators*. Similarly, if the electrical system is characterized by alternating current, the machines are called *ac motors* and *ac generators*. Basically, the ac machines are not different from the dc machines. They differ merely in the constructional details. Only for our convenience, we discuss different type of machines in different Chapters.

* Not all machines deal with large amount of energy. Some devices operate at very low power levels and are termed *transducers* (such as a microphone or a loudspeaker), which deal with 'signals' to operate electronic circuits.

Essentially, an electrical rotary machine has two parts—the fixed part, called the ***stator***, and the moving part, called the ***rotor.*** Following are the two phenomena associated with the operation of a machine:

1. Whenever the conductors of a coil cut across the magnetic flux (or are cut by it), an emf e is induced in it. This emf can supply a current i to an electrical load. Thus, an electrical power ei is generated. This is the *generator action.*
2. Whenever a current-carrying conductors of a coil are placed (perpendicularly) in a magnetic flux, a force is experienced by each conductor giving rise to an electromagnetic torque τ_e. This torque can rotate a mechanical load at an angular speed ω. Thus, a mechanical power $\tau_e\omega$ is generated. This is the *motor action.*

Both the generator action and motor action go hand in hand in an electro-mechanical energy-conversion machine. Only the direction of power flow decides whether the machine is a generator or a motor, as illustrated in Fig. 14.1. In a generator, the magnetic field converts the mechanical power $\tau_e\omega$ into electrical power ei. Reverse is the conversion in a motor. Conservation of energy dictates that we must have

$$\boxed{ei = \tau_e\,\omega} \tag{14.1}$$

Power Considerations for a Generator

Consider a generator (Fig. 14.2) whose shaft is rotated by a prime-mover (such as a water-turbine or a steam-turbine for large-size generators*, and a petrol-engine or a diesel-engine for small-size generators). The prime-mover applies a mechanical torque τ_m on the rotor of the generator. The armature, placed in a magnetic field, is made to rotate at a speed ω. An emf e is induced in the coil. If the coil terminals AB are connected to an external electrical load R_L, a current i flows.

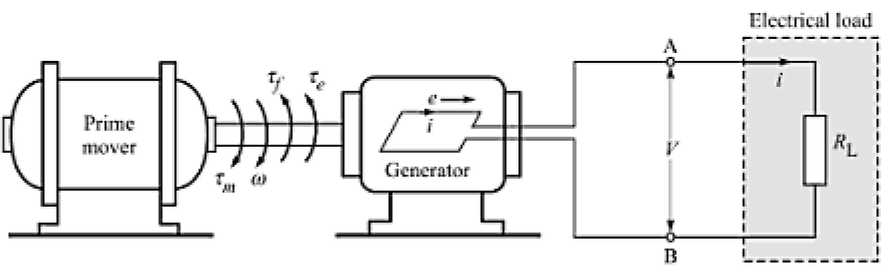


Fig. 14.2 *A generator supplying power to an electrical load.*

The current-carrying coil in the armature kept in magnetic field experiences a torque, called electromagnetic torque τ_e. This is the *reaction torque* and it opposes the applied torque τ_m. In addition, there is also a frictional torque τ_f that opposes the applied torque τ_m. Thus, the applied torque τ_m overcomes both the electromagnetic torque τ_e and the frictional torque τ_f, as shown in Fig. 14.2. That is,

$$\tau_m = \tau_e + \tau_f$$

Multiplying both sides by ω, we get

$$\tau_m\omega = \tau_e\omega + \tau_f\omega \tag{14.2}$$

* Large-size generators are usually called ***alternators***.

This equation simply states that out of the total mechanical power $\tau_m\omega$ supplied by the prime-mover, only the power $\tau_e\omega$ is converted by the generator into electrical power ei. The remaining power $\tau_f\omega$ is wasted as frictional loss.

Now, let us see what happens at the output side. If r is the resistance of the coil, the current i in the circuit is given as

$$i = \frac{e}{r + R_L} \quad \Rightarrow \quad e = ir + iR_L$$

Multiplying both sides by i, we get

$$ei = i^2r + i^2R_L \quad \text{or} \quad ei = i^2r + iV \tag{14.3}$$

where, $V = iR_L$ is the terminal voltage. This equation simply states that out of the electrical power ei generated by the generator, only the power Vi is delivered to the electrical load R_L and the remaining power i^2r is wasted as copper-loss in the coil. Thus, the power flow diagram for a generator can be represented as shown in Fig. 14.3.

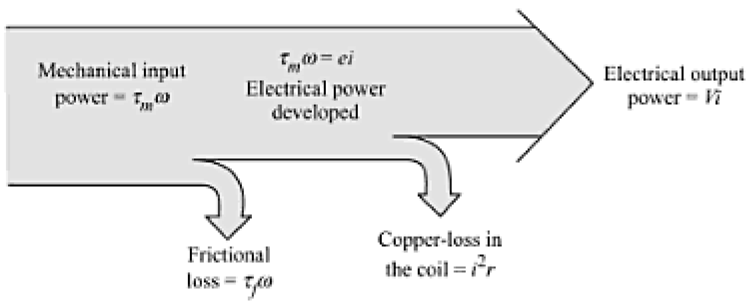


Fig. 14.3 *Power flow diagram for a generator.*

Power Considerations for a Motor

The same rotary machine can work as a motor. An electrical source of voltage V is connected to the terminals AB of the armature-coil (Fig. 14.4). Current i flows through the conductors of the coil. Since these current-carrying conductors of the coil are placed in a magnetic field, an electromagnetic torque τ_e is developed. This causes the coil (and hence the rotor) to rotate at a speed ω. As a result, the mechanical load (say, a water pump) connected to the shaft of the motor also rotates at a speed ω.

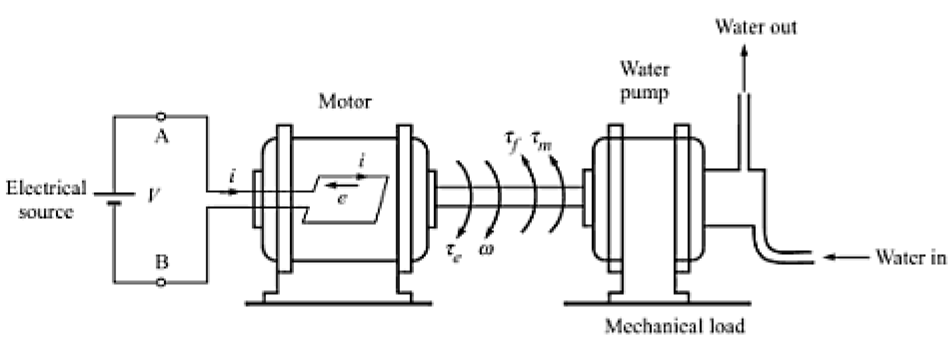


Fig. 14.4 *A motor supplying power to a mechanical load.*

As the conductors of the coil cut through the magnetic field, an emf e is induced. This is the *reaction emf* and it opposes the applied voltage V. The difference of the applied voltage V and the induced emf e is responsible for causing the current i to flow in the coil. That is,

$$i = \frac{V - e}{r} \quad \Rightarrow \quad V = e + ir \tag{14.4}$$

where, r is the resistance of the coil. Note that in a motor the direction of the induced emf e is opposite to that of the current i; whereas in a generator, the emf e and the current i are in the same direction. Multiplying both sides of Eq. 14.4 by i, we get

$$Vi = ei + i^2 r \tag{14.5}$$

This equation simply states that out of the total power Vi supplied by the electrical source, only the power ei is made available for conversion to mechanical power, the remaining power i^2r is wasted as copper-loss in the coil.

Now, next let us see what happens at the output side. The motor converts electrical power ei into mechanical power $\tau_e\omega$ (see Eq. 14.1). Out of this mechanical power $\tau_e\omega$ developed by the motor, a portion $\tau_f\omega$ is lost in overcoming the friction; the remaining part $\tau_m\omega$ is supplied to the mechanical load. That is,

$$\tau_e\omega = \tau_f\omega + \tau_m\omega \tag{14.6}$$

Thus, the power flow diagram for an electrical motor can be represented as shown in Fig. 14.5.

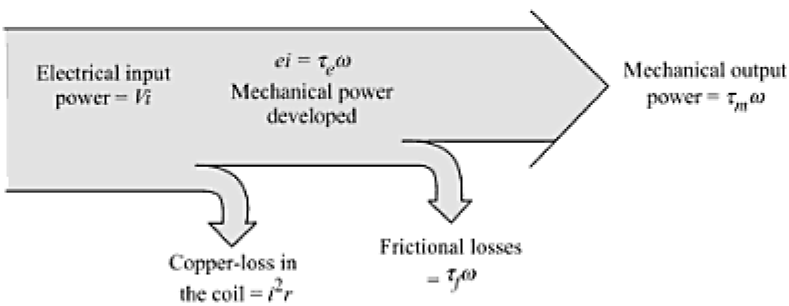


Fig. 14.5 *Power flow diagram for a motor.*

How a Generator Differs from a Motor

With reference to Figs. 14.2 and 14.4, we can bring out the essential differences between a generator and a motor as follows:

1. The directions of induced emf e and current i are same in a generator but opposite in a motor*.
2. The directions of electromagnetic torque τ_e and rotation ω of shaft are opposite in a generator but same in a motor.
3. The directions of electromagnetic torque τ_e and frictional torque τ_f are same in a generator but opposite in a motor.

* In fact, we can conclude that a circuit having e and i in the same direction acts as a source of electrical energy. On the other hand, a circuit with e and i in the opposite direction acts as a consumer of electrical energy.

2. The power supplied to the dc field circuit supplies the ohmic losses in the field winding but does not enter directly into the energy-conversion process.
3. The armature circuit is placed on the stator and carries three-phase currents.
4. The flow of real power through the system is determined by the mechanical input because the mechanical system exchanges real power only. For *generator action*, the mechanical input power, minus the losses, becomes three-phase output power. For *motor action*, the real power from the electrical input minus the losses becomes mechanical output power.
5. When the load on a generator is a large power system, the reactive power flow is controlled by the dc field current. When the machine is operated as a motor, the reactive power required, and thus the power factor, is controlled by the field current. *The direct control of reactive power flow is a surprising, and extremely useful property of the synchronous machines.*

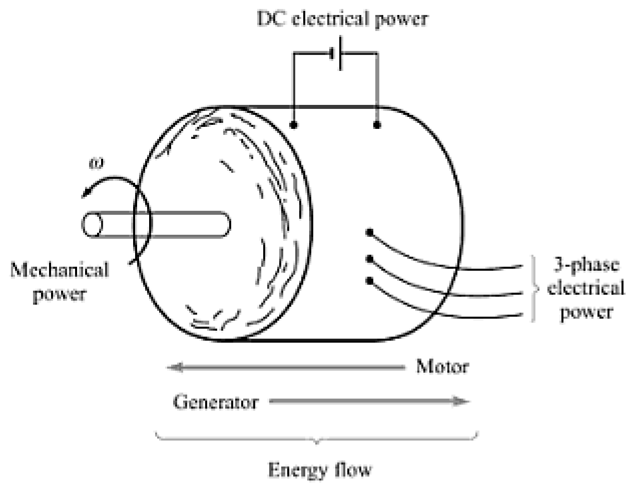


Fig. 14.8 *A synchronous machine can act as a generator or a motor.*

Synchronous Speed

In a generator, if the armature windings on the stator are designed for two poles, the induced emf passes through one complete cycle in one revolution of the rotor. One revolution of rotor represents 360 *mechanical degrees*, and one cycle of emf represents 360 *electrical degrees*. Thus, in a two-pole machine, the mechanical and electrical angles are identical. Now, suppose that the machine has 4 poles. Then one cycle of emf would be generated when the field structure (on the rotor) rotates through one-half revolution only. Thus, in a 4-pole machine two cycles of emf is generated when the rotor completes one revolution. It means that if the machine has P poles, the number of cycles of emf in one revolution will be $P/2$. Therefore, in general the electrical angle θ_e and the mechanical angle θ_m in a machine are related as

$$\theta_e = \left(\frac{P}{2}\right) \theta_m \tag{14.7}$$

If the rotor has a speed of n_s revolutions per second, the frequency f of the induced emf would be

$$f = \frac{P}{2} \cdot n_s$$

For frequency to remain constant, the speed n_s must remain constant. Therefore, a synchronous generator must run at a constant speed known as ***synchronous speed.*** Alternatively, the *synchronous speed*, N_s (expressed in revolutions per minute*) is given as

$$N_s = \frac{2}{P} \times f \times 60 = \frac{120f}{P} \tag{14.8}$$

In India, the frequency of ac supply is 50 Hz. Therefore, for a two pole machine, the synchronous speed is

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

Note that, as a machine cannot have less than 2 poles, the *maximum speed* a synchronous machine can have is 3000 rpm.

EXAMPLE 14.1

A six-pole ac generator is running and producing voltage at a frequency of 60 Hz. Calculate the revolutions per minute of the generator. If the frequency of the generated voltage is required to be decreased to 20 Hz, how many poles would be needed on the generator, if it still runs at the same speed?

Solution The speed of rotation is given as

$$N_s = \frac{120f}{P} = \frac{120 \times 60}{6} = \mathbf{1200\ rpm}$$

If the frequency is decreased to 20 Hz, required number of poles,

$$P = \frac{120f}{N} = \frac{120 \times 20}{1200} = \mathbf{2}$$

14.3 CONSTRUCTION OF ALTERNATORS

Alternators are generally constructed in large sizes (11 kVA, 500 MW or more). Because of a number of advantages, the field system is placed on the rotor and the armature winding is placed on the stator. The main parts are described below.

Stator

The core of the stator is made of CRGO (cold rolled grain oriented) sheet steel or silicon steel. This material has high permeability and low hysteresis loss. To minimize eddy-current loss, the core is made of laminations (about 0.35 mm thick), insulated from each other by varnish or paper. The laminations are stamped out in complete rings (for smaller machines) or in segments (for larger machines). These stampings have uniformly distributed open or semi-closed slots on its inner periphery to accommodate the armature conductors (Fig. 14.9). Also, these stampings have openings which make axial and radial ventilating ducts meant for air cooling. The whole structure is held in a cast iron frame. A general view of the stator is shown in Fig. 14.10.

* The speed of rotation of a machine is normally expressed in revolutions per minute (rpm).

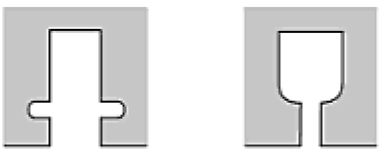


Fig. 14.9 *Slots in stampings.*

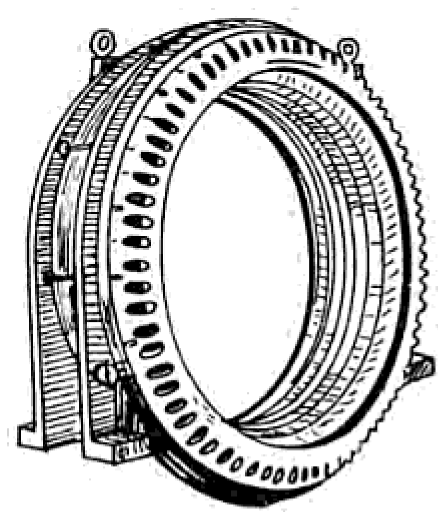


Fig. 14.10 *General view of the stator.*

The open slots are usually preferred as the coils can be form-wound and insulated prior to being packed in the slots. Also, removal and replacement of defective coils becomes easy. However, such slots have the disadvantage of making distribution of the air-gap flux non-smooth. This would result in ripples in the induced emf wave. Semi-closed slots give better results, but do not permit the use of form-wound coils.

Rotor

It is a cylindrical structure which can rotate inside the stator leaving a very small air gap. It houses the windings to produce dc magnetic field. This winding is excited by a separate dc generator, called *exciter**. The exciting current is supplied to the rotor windings through two slip rings and carbon brushes. The brush is pressed against the slip-ring by a spring whose tension can be adjusted.

The voltage rating of the exciter is usually between 125 V and 250 V. Its power rating is about 0.3 to 1% of the power ratings of the alternator.

Depending upon the type of prime-mover (a steam turbine, or a water turbine) used to drive the alternator, there are two types of rotors: (*i*) Cylindrical or non-salient type, and (*ii*) Salient** or projected-pole type.

1. *Cylindrical or Non-Salient Type* In a thermal or nuclear power station, thermal energy (obtained by burning coal or gas) or nuclear energy is used to heat water so as to produce steam at high pressure. The steam is then utilized to run a steam-turbine which in turn drives the alternator. As the steam turbines have high efficiency when operated at high speeds, these are designed to run at 3000 rpm. For this speed, the rotor needs only two poles. For such high-speed alternators***, cylindrical type of rotor construction (Fig. 14.11*a*) is preferred.

* Often the exciter is a dc shunt or compound generator which is mounted on the shaft of the alternator itself.

** 'Salient' means outstanding or jutting out.

*** Also called ***turbo-alternators*** or ***turbo-generators***.

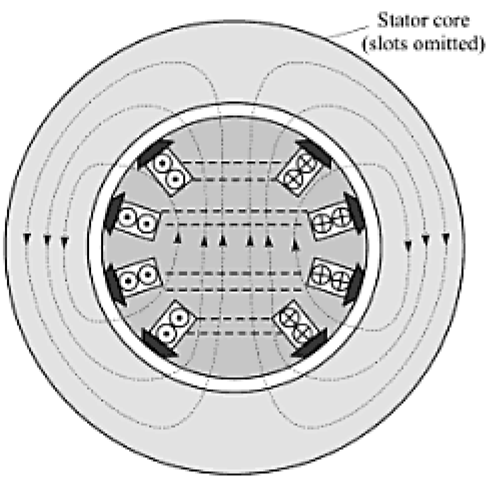
The centrifugal force on a high-speed rotor is enormous. For instance, a mass of 1 kg on the outside of a rotor of 1 m diameter, rotating at 3000 rpm, has a centrifugal force (= $mv^2/r = mr\omega^2$) of about 50 kN acting on it. To withstand such a force, the rotor is usually made of solid steel forging with longitudinal slots cut as shown in Fig. 14.11*a*. The figure shows a two-pole rotor with eight slots and two conductors per slot. In an actual rotor there are more slots and more conductors per slot. The winding is in the form of insulated copper strip held securely in position by phosphor-bronze wedges. The regions forming the centres of the poles are left unslotted. The horizontal dotted lines joining the conductors in Fig. 14.11*a* represent the end connections. The flux distribution in the air gap due to the current in the rotor conductors is made sinusoidal by tapered distribution of rotor mmf.

To reduce the centrifugal force on the conductors, the diameter of the cylindrical rotor is made small and axial length is made long. Because of the smooth surface, this rotor has less windage loss compared to the salient pole type rotor.

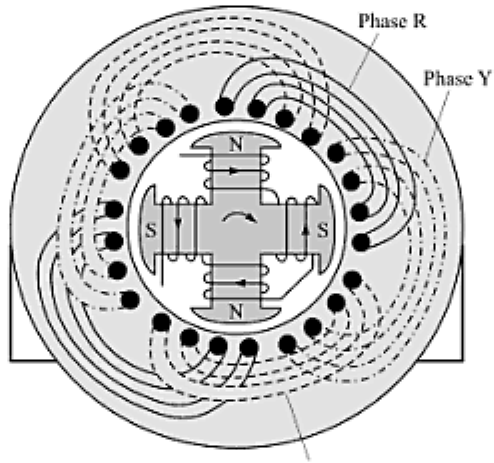
2. *Salient or Projected Pole Type* In a hydroelectric power station, the alternators are driven by water turbines. These hydraulic turbines operate best at relatively low speeds (50 rpm to 500 rpm). For such low speeds, the rotor is required to have large number of poles. For example, in Bhakra Hydroelectric Power Station (in Punjab), the alternators are driven by water turbines running at 166.7 rpm and have 36 poles on their rotors.

It is more convenient to build a rotor having large number of poles in projected or salient form (as shown in Fig. 14.11*b* for 4 poles). To accommodate such large number of poles, the rotor-diameter is made larger and consequently its length is made smaller. The field winding is arranged so that alternate poles are made N and S.

Unlike the cylindrical type rotor, the mmf in salient type is concentrated by the coils wound around the pole pieces. The pole tips are well rounded so as to make the flux distribution around the periphery nearer to a sine wave and thus improve the waveform of the generated emf.



(*a*) Cylindrical type.



(*b*) Salient type.

Fig. 14.11 *Construction of rotor of an alternator.*

14.4 ROTATING MAGNETIC FLUX DUE TO THREE-PHASE CURRENTS

Figure 14.12*a* shows a cylindrical magnetic structure with one winding excited by a single-phase current. The current is entering into the bottom conductors and coming out of the top conductors. If the current i is dc, the flux density in the air gap will also be dc and it will have a maximum value along the horizontal plane, $B_m = Ki$. However, it decreases sinusoidally as we move away in the air gap from this direction, and reduces to zero along the vertical plane, as shown in the figure. This sinusoidal distribution is accomplished by having heavy concentration of conductors in the centre of the coil, tapered to few conductors at the edge of the winding. Thus, the flux density is a function of angle θ_m, given as $B = B_m \cos \theta_m$.

If the current is alternating ($i = I_m \cos \omega t$), so will be the flux density. Therefore, the flux density along the air gap is function of time t as well as angle θ_m,

$$B(t, \theta_m) = B_m \cos \omega t \cos \theta_m \tag{14.9}$$

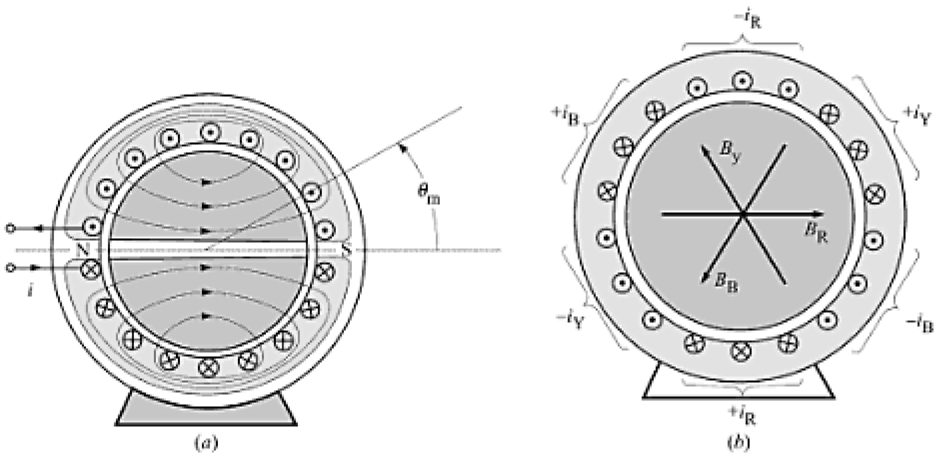


Fig. 14.12 *Magnetic flux produced by current(s) in the coil(s).*

Next, we consider Fig. 14.12*b* which shows a two-pole magnetic structure wound with three coils separated by 120° in space. These coils are supplied three-phase (RYB) currents. Note that the coil R corresponds to the coil of Fig. 14.12*a*. The current i_R enters into the bottom conductors and returns from the top. The current i_Y enters in the first quadrant and current i_B enters in the second quadrant. The coil mmfs are tapered sinusoidally and share slots between their most dense regions. As shown in Fig. 14.12*b*, the maximum flux density from coil R lies in the horizontal plane, and the maxima from coils Y and B are displaced 120° and 240° in space, respectively.

Analysis of Three-Phase System

Three-phase voltages are shown in Fig. 12.2, and are represented by Eqs. 12.2 to 12.4. Similarly, the three-phase currents supplied to the three-phase structure of Fig. 14.12*b* can be represented as

$$i_R(t) = I_m \cos(\omega t)$$
$$i_Y(t) = I_m \cos(\omega t - 120°)$$

electrical degrees. The ***distribution factor*** (or ***breadth factor***) k_d for q slots per pole per phase is defined as

$$k_d = \frac{\text{phasor sum of component emfs}}{\text{arithmetic sum of component emfs}} = \frac{\text{AD}}{q \times \text{BC}} = \frac{2 \times \text{AF}}{q \times 2 \times \text{BG}} = \frac{2R\sin(q\alpha/2)}{q \times 2R\sin(\alpha/2)}$$

or
$$k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} \tag{14.17}$$

If q (the number of slots per pole per phase) is very large, the angle α becomes very small so that $\sin(\alpha/2) \approx \alpha/2$, and Eq. 14.17 simplifies to

$$k_d = \frac{\sin(q\alpha/2)}{q\alpha/2} \tag{14.18}$$

The total angle $q\alpha$ (expressed in electrical radians) is called the ***phase spread.***

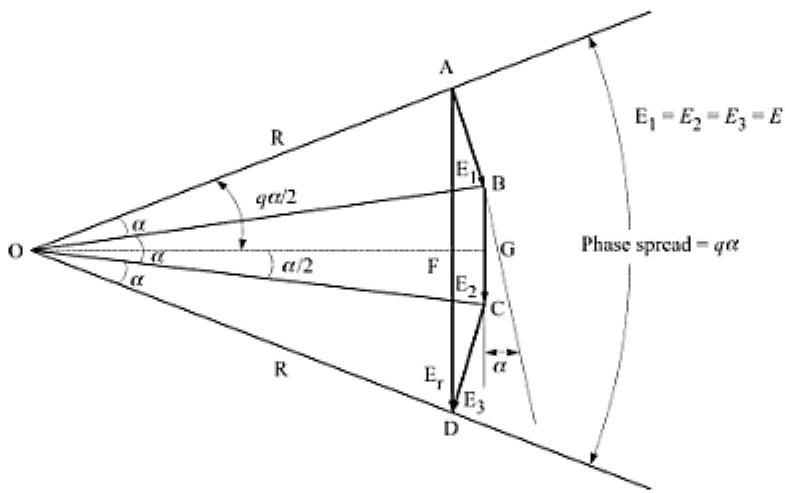


Fig. 14.14 *Calculation of distribution factor, for q = 3.*

14.6 EMF EQUATION

We know that if a conductor of length l moves perpendicularly to a magnetic field of average flux density B with a relative speed v, the magnitude of the induced emf is given as (see Eq. 5.15),

$$e = Blv$$

Let d be the diameter of the armature (on the stator) and n_s be the rotational speed of the rotor in revolutions per second. Then, the relative speed $v = \pi d n_s$. If Φ is the total flux per pole and P is the total number of poles, the average flux density is given as

$$B = \frac{\text{Flux per pole} \times \text{Number of poles}}{\text{Cylindrical area of armature surface}} = \frac{\Phi P}{\pi d l}$$

Thus, the *average* value of the induced emf in a conductor is

$$e = \frac{\Phi P}{\pi dl} \cdot l \cdot \pi d n_s = \Phi P n_s = \Phi \times 2f$$

To get the *rms* value, we must multiply the *average* value by the form factor (which is 1.11 for a sinusoidal waveform). Moreover, if the winding consists of T concentrated turns (i.e., $2N$ conductors), the rms value of the net induced emf in a concentrated winding is given as

$$E_c = (2f\Phi) \times 1.11 \times 2N = 4.44\, f\Phi T \qquad (14.19)$$

In practice, the coils are short pitched and the winding is distributed. Hence, the rms value of the induced emf is reduced by the *pitch factor* k_p and *distribution factor* k_d, to give

$$E = (2f\Phi) \times 1.11 \times 2T \times k_p \times k_d = 4.44\, f\Phi T\, k_p k_d \qquad (14.20)$$

EXAMPLE 14.2

Determine the distribution factor for a machine having 9 slots per pole for the following cases: (*a*) a three-phase winding with 120° phase group, and (*b*) a three-phase winding with 60° phase group.

Solution The slot angle, $\alpha = \frac{180°}{9} = 20°$

(*a*) Since one phase occupies 120°, the number of slots in one phase group, $q = \frac{120°}{20°} = 6$.

$$\therefore \qquad k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(6 \times 20°/2)}{6 \times \sin(20°/2)} = \mathbf{0.831}$$

(*b*) Since one phase occupies 60°, the number of slots in one phase group, $q = \frac{60°}{20°} = 3$.

$$\therefore \qquad k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(3 \times 20°/2)}{3 \times \sin(20°/2)} = \mathbf{0.960}$$

EXAMPLE 14.3

A 3-phase, 50-Hz, 20-pole, salient-pole alternator with star-connected stator winding has 180 slots on the stator. There are 8 conductors per slot and the coils are full-pitch. The flux per pole is 25 mWb. Assuming sinusoidally distributed flux, calculate (*a*) the speed, (*b*) the generated emf per phase, and (*c*) the line emf.

Solution Total number of armature conductors, $Z = 180 \times 8 = 1440$.

Therefore, the number of turns per phase, $T = \frac{1440/2}{3} = 240$.

(*a*) The speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{20} = \mathbf{300\ rpm}$

(*b*) Since the coils are full-pitch, the pitch factor, $k_p = 1$. Next we calculate the value of distribution factor, k_d.

$$\text{No. of slots per pole} = \frac{180}{20} = 9$$

$$\therefore \qquad \text{Solt angle, } \alpha = \frac{\text{Electrical angle per pole}}{\text{Number of slots per pole}} = \frac{180°}{9} = 20°$$

$$\text{Number of slots per pole per phase, } q = \frac{9}{3} = 3$$

$$\therefore \qquad k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(3\times 20°/2)}{3\times\sin(20°/2)} = \mathbf{0.960}$$

Therefore, the rms value of the generated emf per phase,

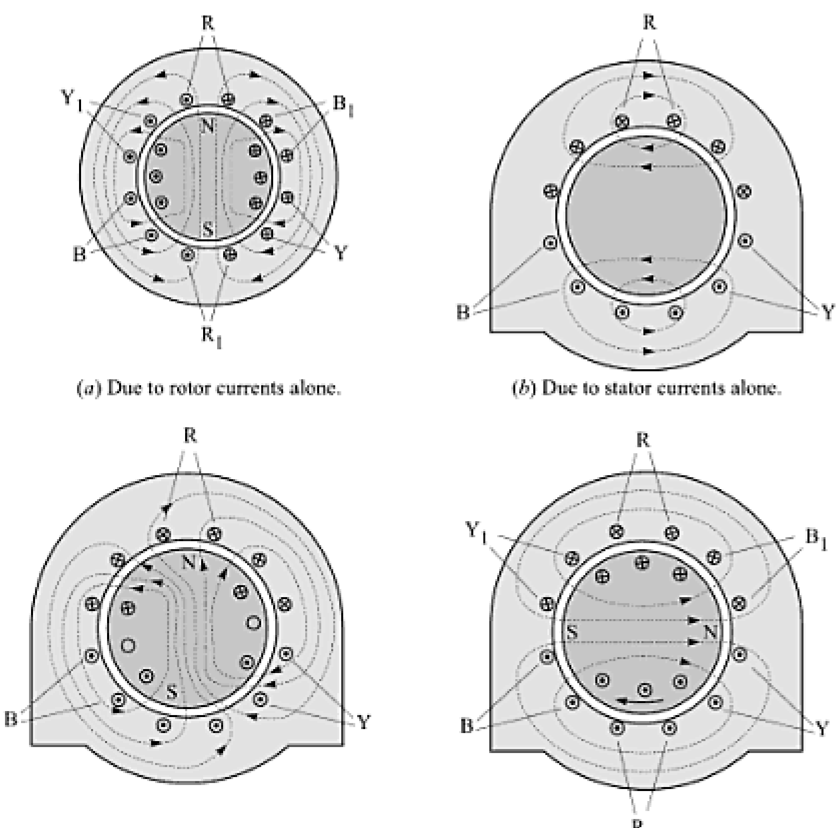
$$E = 4.44\, f\Phi T k_p k_d = 4.44 \times 50 \times 0.025 \times 240 \times 1 \times 0.960 = \mathbf{1278.7\ V}$$

(*c*) Since, the stator winding is star-connected, the line emf,

$$E_L = \sqrt{3} \times 1278.7 = \mathbf{2214.8\ V}$$

14.7 ARMATURE REACTION

Armature reaction in a synchronous generator means the influence of the stator mmf upon the value and the distribution of the magnetic flux in the air gap. Consider a three-phase, two-pole synchronous generator with two slots per pole per phase. Suppose its rotor is excited by dc current and is rotated clockwise at synchronous speed by a prime mover. An emf is induced in the stator winding and if the generator is on no load the stator current is zero. The magnetic flux in the air gap is entirely due to rotor current, as shown in Fig. 14.15*a*. This flux is symmetrically distributed.



(*a*) Due to rotor currents alone. (*b*) Due to stator currents alone.

(*c*) Resultant flux for case (*i*). (*d*) Resultant flux for case (*ii*) and case (*iii*).

Fig. 14.15 *Magnetic flux in the air gap of a synchronous generator.*

Now, suppose the generator is connected to an electrical load resulting in stator-currents. The magnetic flux produced by the stator currents rotates clockwise at synchronous speed and therefore is stationary relative to the rotor. At the instant shown in Fig. 14.15*a*, the induced emf in phase RR_1 is maximum and is towards the paper in conductor R and coming out of the paper in R_1. We can find the distribution of the resultant magnetic flux in the air gap by first considering the two fluxes (due to rotor currents and due to stator currents) separately and then superimposing them. We shall consider three cases of the electrical load: (*i*) when the current and the generated emf are in phase, (*ii*) when the current lags the generated emf by 90°, and (*iii*) when the current leads the generated emf by 90°.

Case (i) When the stator current is in phase with the generated emf, the distribution of magnetic flux due to stator current in phase R alone is shown in Fig. 14.15*b* at the instant when the rotor (unexcited) is in the position shown in Fig. 14.15*a*. Comparison of these figures shows that over the leading half of each pole face two fluxes are in opposition, whereas over the trailing half they are in the same direction. Hence, the resultant flux becomes distorted as shown in Fig. 14.15*c*.

Note that the direction of most of the lines of flux in the air-gap has been skewed and thereby lengthened. Since the lines of flux behave like stretched elastic cords, they exert a backward pull on the rotor. To overcome the tangential component of this pull, the prime-mover driving the rotor has to exert a larger torque than that required on no load. Since the magnetic flux due to stator currents rotate synchronously with the rotor, for all positions of the rotor the flux distortion is the same as shown in Fig. 14.15*c*.

Case (ii) When the stator current lags the generated emf by quarter cycle, by the time the current in phase R reaches its maximum value, the poles of the rotor will have moved forward by 90° to the position as shown in Fig. 14.15*d*. At this instant, the stator mmf acting alone would still be as shown in Fig. 14.15*b*. It means the flux due to stator mmf is in direct opposition to the flux produced by the rotor mmf. Hence, the net flux is reduced. The resultant flux, however, is symmetrical over the two halves of the pole face, as shown in Fig. 14.15*d*. Hence, no additional torque is required to drive the rotor, except that needed to overcome losses.

Case (iii) When the stator current leads the generated emf by a quarter cycle, at the instant the current in phase R reaches its maximum value, the poles of the rotor are behind the position as shown in Fig. 14.15*a* by 90°. In other words, at this instant the pole position is just opposite to that shown in Fig. 14.15*d*. It means that the flux due to the stator mmf is now in the same direction as that due to the rotor mmf. As a result, the net flux is increased and remains symmetrically distributed, as shown in Fig. 14.15*d*.

Variation in Terminal Voltage

The influence of armature reaction upon the variation of the terminal voltage with load current is shown in Fig. 14.16*a*. Here, it has been assumed that the field current is maintained constant at a value that gives emf E when the generator is on no load. Let us see how the terminal voltage changes with increase in load current. For unity power-factor load (i.e., resistive load), the fall in terminal voltage is comparatively small. However, with inductive load (lagging power factor of, say, 0.8) the terminal voltage falls more rapidly due to the *demagnetisation effect of armature reaction*. If the load is purely inductive ($pf = 0$), the demagnetisation effect of armature reaction becomes much more predominant, and hence the fall in terminal voltage is also much more rapid.

With capacitive load (power factor leading), the effect of armature reaction is to increase the magnetic field in the air gap of the generator. Due to this *magnetising effect of the armature reaction*, the terminal voltage increases with an increase in the load current.

As in transformers, not all flux couples the rotor and stator. It means that the synchronous machine will have, in addition to the synchronous reactance, a ***leakage reactance*** (X_l) due to leakage of flux. However, usually $X_l << X_s$, hence this leakage reactance is normally ignored.

We can combine the resistance and the synchronous reactance of the winding to get its ***synchronous impedance***, Z_s. Thus, if R is the resistance per phase and X_s is the synchronous reactance per phase, then the synchronous impedance per phase is given as

$$\mathbf{Z}_s = Z_s\angle\theta = R + jX_s = \sqrt{R^2 + X_s^2}\ \angle\tan^{-1}(X_s/R) \approx X_s\angle 90° \tag{14.23}$$

(For a synchronous generator, usually R is very small compared with X_s. Hence, for practical purposes, Z_s can be assumed to be the same as X_s.)

Thus, Fig. 14.17 represents the equivalent circuit of a synchronous generator for one phase. Here, S represents *one* phase of the stator winding, and R and X_s represent the resistance and synchronous reactance of that phase, respectively. The synchronous impedance, $\mathbf{Z}_s = R + jX_s$. The voltage drop across $\mathbf{Z}_s$ is then given as $\mathbf{V}_z = \mathbf{IZ}_s$. The induced emf $\mathbf{E}$ is obviously the phasor sum of terminal voltage $\mathbf{V}$ and the voltage drop $\mathbf{V}_z$. That is,

$$\mathbf{E} = \mathbf{V} + \mathbf{V}_z = \mathbf{V} + \mathbf{IZ}_s = \mathbf{V} + \mathbf{I}(R + jX_s) \tag{14.24}$$

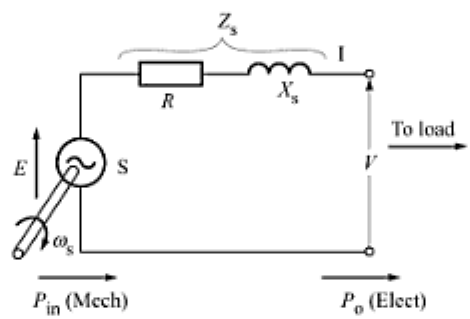


Fig. 14.17 *Equivalent circuit of a synchronous generator.*

Phasor Diagram

Using Eq. 14.24, we can draw phasor diagram for the stator circuit of a synchronous generator. Since the voltage drop across Z_s is determined by the current I, we shall take the current I as the reference phasor. Suppose that the load takes a current I at a power factor angle ϕ. Note that for lagging power factor, the angle ϕ is negative (see Fig. 14.18*a*) and for leading power factor, it is positive (see Fig. 14.18*b*). Various quantities, as described below, are represented by phasors in Fig. 14.18.

OA = current per phase, I (*reference phasor*)

OB = terminal voltage per phase, V

OC = component of the generated emf E absorbed in sending current through R
= IR

OD = emf per phase induced by the rotating flux due to currents in the stator; it lags current I by 90°

$\therefore$ Phase current, $I = I_{ph} = I_L = \dfrac{600 \times 10^6}{\sqrt{3} \times 22 \times 10^3} = 15.7$ kA

The terminal voltage per phase on full load $= \dfrac{22\text{ kV}}{\sqrt{3}} = 12.7$ kV

Voltage drop per phase on full load due to synchronous impedance is

$$V_z = IZ_s = (15.7 \text{ kA}) \times (0.16\ \Omega) = 2.512 \text{ kV}$$

With reference to Fig. 14.18*a*, we have

$$\cos\theta = \frac{OC}{OG} = \frac{0.014}{0.16} = 0.0875 \qquad \text{and} \qquad \theta = \cos^{-1} 0.0875 = 84.98°$$

(*a*) ***Voltage regulation for pf* = 0.8 *lagging* :** $\phi = \cos^{-1} 0.8 = 36.87°$, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 84.98° - 36.87° = 48.11° \qquad \text{and} \qquad \cos\alpha = 0.6677$$

Putting all the voltages in kV, we get

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$

$$= \sqrt{(12.7)^2 + (2.512)^2 + 2 \times 12.7 \times 2.512 \times 0.6677} = 14.5 \text{ kV}$$

Therefore, the voltage regulation is

$$\frac{E - V}{V} = \frac{14.5 - 12.7}{12.7} = 0.1417 \text{ per unit} = \mathbf{14.17\ per\ cent}$$

(*b*) ***Voltage regulation for pf* = 1 :** $\phi = \cos^{-1} 1 = 0°$, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 84.98° \qquad \text{and} \qquad \cos\alpha = 0.0875$$

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$

$$= \sqrt{(12.7)^2 + (2.512)^2 + 2 \times 12.7 \times 2.512 \times 0.0875} = 13.16 \text{ kV}$$

Therefore, the voltage regulation is

$$\frac{E - V}{V} = \frac{13.16 - 12.7}{12.7} = 0.036 \text{ per unit} = \mathbf{3.6\ per\ cent}$$

Note the drastic reduction in voltage regulation when the power factor is improved from 0.8 lagging to unity.

(*c*) ***Voltage regulation for pf* = 0.8 *leading* :** $\phi = \cos^{-1} 0.8 = 36.87°$, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta + \phi) = 84.98° + 36.87° = 121.85° \qquad \text{and} \qquad \cos\alpha = -0.5277$$

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$

$$= \sqrt{(12.7)^2 + (2.512)^2 + 2 \times 12.7 \times 2.512 \times (-0.5277)} = 11.57 \text{ kV}$$

Therefore, the voltage regulation is

$$\frac{E - V}{V} = \frac{11.57 - 12.7}{12.7} = -0.0889 \text{ per unit} = \mathbf{-8.89\ per\ cent}$$

Note that the voltage regulation for leading power-factor load is negative. It means that on removing the load, the terminal voltage decreases.

Hence,

$$\text{Real power, } P = \frac{EV}{Z_s}\cos(\theta - \delta_R) - \frac{V^2}{Z_s}\cos\theta \tag{14.27}$$

and
$$\text{Reactive power, } Q = \frac{EV}{Z_s}\sin(\theta - \delta_R) - \frac{V^2}{Z_s}\sin\theta \tag{14.28}$$

The above expressions for two types of power delivered by the alternator can further be simplified by considering a practical case. We know that in practice, $R \ll X_s$ so that $Z_s \approx X_s$ and $\theta \approx 90°$. Equations 14.27 and 14.28 then modify to

$$\text{Real power, } P = \frac{EV}{Z_s}\cos(90° - \delta_R) - \frac{V^2}{Z_s}\cos 90° = \frac{EV}{Z_s}\sin\delta_R \tag{14.29}$$

and
$$\text{Reactive power, } Q = \frac{EV}{Z_s}\sin(90° - \delta_R) - \frac{V^2}{Z_s}\sin 90° = \frac{V}{Z_s}(E\cos\delta_R - V) \tag{14.30}$$

Equation 14.29 shows that when δ_R is 90°, real power is maximum and is given as

$$P_m = \frac{EV}{Z_s} \tag{14.31}$$

Thus, for any angle δ_R, the real power can be written as

$$P = P_m \sin\delta_R \tag{14.32}$$

If we increase the mechanical drive to the alternator, the rotor power angle δ_R increases and as a result the real power, given by Eq. 14.29, delivered to the infinite bus increases. However, the reactive power delivered by the alternator can be controlled by controlling the dc exciting current I_f. If we increase I_f, the magnitude of excitation voltage E increases. If we keep the mechanical drive constant (i.e., the angle δ_R unaltered), we can get the following three conditions by just varying the dc excitation current I_f.

(*i*) For low value of I_f, we have $E\cos\delta_R < V$. Hence, according to Eq. 14.30 the reactive power Q is negative. It means the alternator absorbs reactive power from the infinite bus. This condition is described by saying that the alternator is ***under-excited.***

(*ii*) For such a value of I_f that makes $E\cos\delta_R = V$, according to Eq. 14.30 the reactive power Q becomes zero. It means the alternator delivers only the real power. The current I is in phase with the voltage V. The alternator is operating at unity power factor. The alternator is said to be ***normally excited.***

(*iii*) For high value of I_f, we have $E\cos\delta_R > V$. Hence, according to Eq. 14.30 the reactive power Q is positive. It means the alternator delivers reactive power to the infinite bus. This condition is described by saying that the alternator is ***over-excited.***

14.10 MEASUREMENT OF SYNCHRONOUS IMPEDANCE

The synchronous impedance Z_s of an alternator can be determined by plotting its open-circuit and short-circuit characteristics.

Open-Circuit Characteristic

To plot the open-circuit characteristic (OCC) of an alternator, it is run at synchronous speed and its terminals RYB are left open. The field winding on rotor is supplied exciting current I_f from a dc supply. The current I_f can be varied by varying the rheostat connected in the field circuit. A voltmeter connected across any two terminals gives the open-circuit voltage V_{oc} (see Fig. 14.20*a*).

is obtained. If the armature reactance is 0.9 Ω/phase, determine the synchronous reactance per phase. Also determine the voltage regulation for (*a*) 0.8 *pf* lagging, and (*b*) 0.8 *pf* leading.

Solution Since the stator winding is star-connected, the terminal voltage during the open-circuit test is

$$V_{oc} = \frac{V_L}{\sqrt{3}} = \frac{900}{\sqrt{3}} = 519.6 \text{ V}$$

Synchronous impedance per phase is given as

$$Z_s = \left.\frac{V_{oc}}{I_{sc}}\right|_{\text{same } I_f} = \frac{519.6 \text{ V}}{100 \text{ A}} = 5.196\ \Omega$$

Therefore, the synchronous reactance per phase is given as

$$X_s = \sqrt{Z_s^2 - R^2} = \sqrt{(5.196)^2 - (0.9)^2} = \mathbf{5.117\ \Omega}$$

The synchronous impedance angle is given by

$$\cos\theta = \frac{R}{Z_s} = \frac{0.9}{5.196} = 0.1732 \quad \Rightarrow \quad \theta = \cos^{-1} 0.1732 = 80.026°$$

$$\text{Rated phase voltage, } V = \frac{3.3 \times 1000}{\sqrt{3}} = 1905.3 \text{ V}$$

The voltage drop across synchronous impedance, $V_Z = IZ_s = 100 \times 5.196 = 519.6$ V

(*a*) ***Voltage regulation for pf = 0.8 lagging***: $\phi = \cos^{-1} 0.8 = 36.87°$, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 80.026° - 36.87° = 43.156° \quad \text{and} \quad \cos\alpha = 0.729$$

$$\therefore \quad E = \sqrt{V^2 + V_Z^2 + 2V \cdot V_Z \cdot \cos\alpha}$$

$$= \sqrt{(1905.3)^2 + (519.6)^2 + 2 \times 1905.3 \times 519.6 \times 0.729} = 2311.6 \text{ V}$$

Therefore, the voltage regulation is

$$\frac{E - V}{V} = \frac{2311.6 - 1905.3}{1905.3} \times 100 = \mathbf{21.32\ \%}$$

(*b*) ***Voltage regulation for pf = 0.8 leading***: $\phi = \cos^{-1} 0.8 = 36.87°$, so that from Eqs. 14.25 and 14.26,

$$\alpha + \phi = 80.026° + 36.87° = 116.896° \quad \text{and} \quad \cos(\alpha + \phi) = -0.452$$

$$E = \sqrt{V^2 + V_Z^2 + 2V \cdot V_Z \cdot \cos\alpha}$$

$$= \sqrt{(1905.3)^2 + (519.6)^2 + 2 \times 1905.3 \times 519.6 \times (-0.452)} = 1733.5 \text{ V}$$

Therefore, the voltage regulation is

$$\frac{E - V}{V} = \frac{1733.5 - 1905.3}{1905.3} \times 100 = \mathbf{-9.0\ \%}$$

14.11 SYNCHRONOUS MOTORS

A synchronous machine can work as a generator as well as a motor. Compare the equivalent circuit of a synchronous motor shown in Fig. 14.21 with that of a synchronous generator shown in Fig. 14.17. The motor

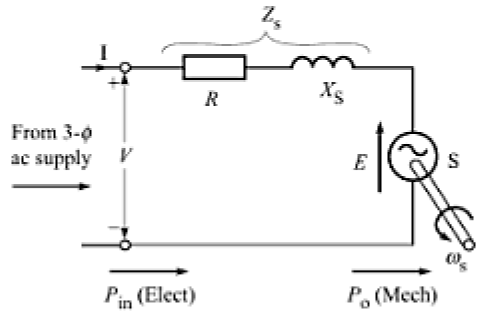


Fig. 14.21 *Equivalent circuit of a synchronous motor.*

draws power from a three-phase ac supply and generates output mechanical power at its shaft. Note the reference directions of voltage and current. The motor works as an electrical load (the current entering the positively marked voltage terminal).

Phasor Diagrams for Generator and Motor

If the armature resistance R is ignored (as $R << X_s$), Eq. 14.24 (the KVL equation for a ***generator***) reduces to

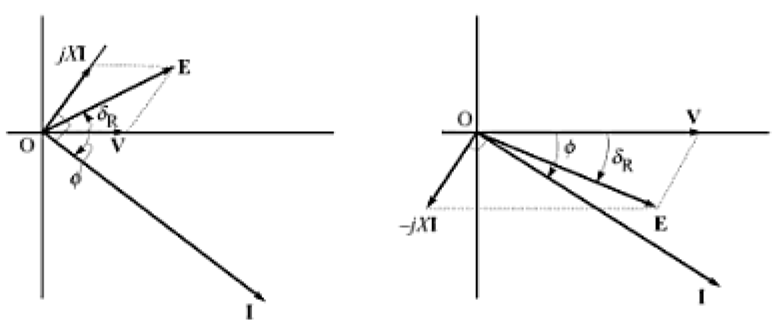
$$\mathbf{E} = \mathbf{V} + \mathbf{I}\mathbf{Z}_s = \mathbf{V} + \mathbf{I}(R + jX_s) \approx \mathbf{V} + jX_s\mathbf{I} \tag{14.35}$$

The phasor diagram reflecting this equation is shown in Fig. 14.22*a*, where Φ is the phase angle between per-phase voltage and current (lagging power factor). Note that the voltage across the synchronous reactance is shown leading the current by 90°. The rotor power-angle δ_R is positive.

The KVL equation for the stator circuit of the ***motor*** (Fig. 14.21) is

$$\mathbf{E} = \mathbf{V} - \mathbf{I}\mathbf{Z}_s = \mathbf{V} - \mathbf{I}\,(R + jX_s) \approx \mathbf{V} - jX_s\mathbf{I} \tag{14.36}$$

The phasor diagram reflecting this equation is shown in Fig. 14.22*b*. Note that here the voltage across the synchronous reactance is shown lagging the current by 90°. This is the consequence of reversing the direction of current in the equivalent circuit. The rotor power angle δ_R is negative.



(*a*) For a generator.

(*b*) For a motor.

Fig. 14.22 *Comparison of per-phase diagrams.*

Constant Speed Operation

A synchronous motor is a truly constant speed* motor. This is the speciality of this motor. Yet, it has very limited applications. To develop a steady torque, its rotor must be rotating at synchronous speed, N_s. This is the major defect of synchronous motors. Either it runs at synchronous speed, or it does not run at all.

The stator field rotates at synchronous speed due to the three-phase currents supplied to its windings. At zero speed or at any other speed lower than synchronous speed, the rotor poles rotate slower than the stator field. Therefore, in one cycle of rotation of the stator field, the *N*-pole of the rotor is for some time nearer to *N*-pole of the stator and for some other time nearer to the *S*-pole of the stator. As a result, the torque developed is for some time clockwise and for some other time anticlockwise. Consequently, the average torque developed remains zero.

In order to develop a continuous unidirectional torque it is necessary that the stator and rotor poles do not move with respect to each other. This is possible only if the rotor also rotates at the synchronous speed.

A synchronous motor is therefore not a self-starting motor. We got to use an *auxiliary device* to start the rotor and to bring it to synchronous (or almost synchronous) speed before it can be left to run itself. For this, following methods are used.

(1) *Starting by Using Damper Winding* As shown in Fig. 14.23, bars of copper, aluminium or bronze are inserted in slots on the pole faces. These bars are short-circuited by end-rings on each side of the poles. These shorted turns work like a squirrel cage rotor of an induction motor** and develop torque.

Initially, no dc supply is given to field winding. On applying three-phase supply to the stator winding, the machine starts running as an induction motor. When the machine attains near synchronous speed, the field winding is excited by dc supply. The rotor is then pulled up into synchronous speed. Once the motor starts running at synchronous speed, the shorted turns have no effect on the steady-state operation of the motor. In fact, these shorted turns improve the transient performance of the motor by damping out the oscillations in the power angle caused by variation in mechanical load. For this reason, these shorted turns are called *damper winding*.

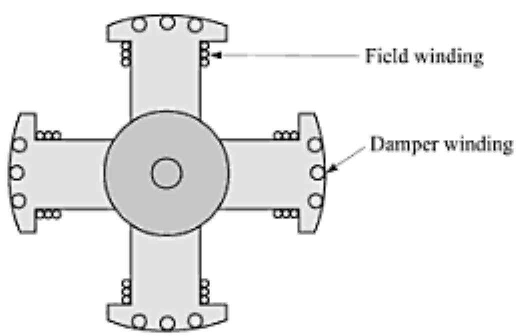


Fig. 14.23 *Damper winding placed on the pole faces of a synchronous motor.*

There is one problem with this method. Since a squirrel-cage induction motor does not develop enough starting torque, large capacity synchronous motors cannot be started on full load by this method.

* As we shall see in latter Chapters, an induction motor or a dc motor is not a constant speed motor.

** We shall study about induction motors in next Chapter.

where P_{max} (= $3EV/X_s$) is the maximum power for which δ_R must be 90°. If the motor is running at n_s rotations per second (rps), the total torque developed by the motor is given as

$$\tau = \frac{P_m}{\omega_s} = \frac{P_{max}\sin\delta_R}{\omega_s} = \frac{3EV}{X_s}\cdot\frac{\sin\delta_R}{2\pi n_s} = \tau_{max}\sin\delta_R \tag{14.40}$$

where
$$\tau_{max} = \frac{3EV}{X_s}\cdot\frac{1}{(2\pi n_s)} \tag{14.41}$$

Loading of a Synchronous Motor *The load on a synchronous motor should never be increased suddenly with a jerk; it may pull out of synchronism.* However, we can increase the load gradually up to a maximum of P_{max} (or τ_{max}) so as to maintain the static stability of the motor. In case, the load on the motor becomes more than P_{max}, the angle δ_R becomes greater than 90° and the motor loses synchronism. For this reason, the maximum torque τ_{max} is also known as *pull-out torque*. Normally, the applied voltage V to a synchronous motor remains constant. If ever a need arises to load the motor excessively, we may increase the value of the pull-out torque by simply increasing the field current I_f and hence the excitation voltage E.

Output Power The net power output P_o at the shaft of the synchronous motor is given as

$$P_o = P_m - \text{rotational losses}$$

Thus, the net shaft torque developed by the motor can then be determined as

$$\tau_{sh} = \frac{P_o}{\omega_s} = \frac{P_o}{2\pi n_s} \tag{14.42}$$

EXAMPLE 14.6

A three-phase, 9-kW, 400-V, star-connected synchronous motor has synchronous impedance per phase, $Z_s = (0.4 + j3)\ \Omega$. Determine the angle of retard of the rotor and the excitation emf E to which it must be excited to give a full-load output at 0.8 leading power factor. Assume the efficiency of the machine to be 90%.

Solution The input power to the motor is given as

$$P_{in} = \frac{P_o}{\eta} = \frac{9000\text{ W}}{0.9} = 10000\text{ W}$$

Since $P_{in} = \sqrt{3}V_L I_L \cos\phi$, the input current per phase is given as

$$I = I_L = \frac{P_{in}}{\sqrt{3}\times V_L\times\cos\phi} = \frac{10\,000}{\sqrt{3}\times 400\times 0.8} = 18\text{ A}$$

Power-factor angle, $\phi = \cos^{-1} 0.8 = 36.87°$ (leading)

$$Z_s = \sqrt{R^2 + X_s^2} = \sqrt{(0.4)^2 + 3^2} = 3.03\ \Omega;\ \theta = \tan^{-1}(X_s/R) = \tan^{-1}(3/0.4) = 82.4°$$

Supply voltage per phase, $V = \dfrac{400\text{ V}}{\sqrt{3}} = 231\text{ V}$

Voltage drop per phase across the synchronous impedance is

$$E_r = IZ_s = 18\times 3.03 = 54.54\text{ V}$$

For a motor, we have

$$\mathbf{V} = \mathbf{E} + \mathbf{IZ}_s \quad\text{or}\quad \mathbf{V} = \mathbf{E} + \mathbf{E}_r \;\Rightarrow\; \mathbf{E} = \mathbf{V} - \mathbf{E}_r$$

emf **E** is such that the current **I** is in phase with voltage **V.** It means that *the power factor is unity*. This condition is described as ***normal excitation***.

Now, suppose that we increase the excitation current I_f beyond its normal excitation value. This is described as ***over-excitation***. The induced emf **E** will now have larger magnitude (but same angle δ_R), as shown in Fig. 14.27*b*. We find that the resultant voltage $\mathbf{E}_r$ is such that the current $\mathbf{I}_1$ leads the voltage **V** by an angle ϕ_1; and its magnitude adjusts itself so as to make $I_1 \cos \phi_1 = I$. Obviously, *the power factor is leading* and its value is less than unity. The magnitude of current **I** has also increased. This fact is shown in Fig. 14.28, which depicts the variation of both the power factor (*pf*) and the armature current *I* with excitation current I_f.

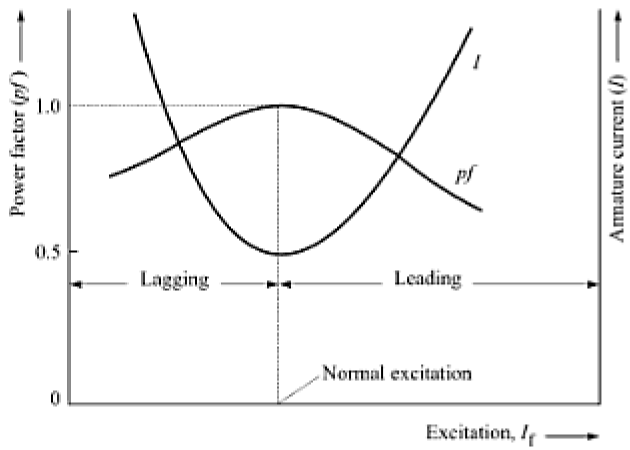


Fig. 14.28 *Effect of change in excitation current I_f on power factor (pf) and armature current (I) of a synchronous motor.*

Next, let us see what happens if the machine is ***under-excited*** (Fig. 14.27*c*). The induced emf **E** now has a smaller magnitude. The resultant voltage $\mathbf{E}_r$ assumes a value such that the armature current $\mathbf{I}_2$ lags the voltage **V** by some angle ϕ_2. However, its component along **V**, $I_2 \cos \phi_2 = I$ still remains the same. *The power factor is lagging* and has a value less than unity. The magnitude of **I** becomes more than that in normal excitation case. This is shown in Fig. 14.28. Since the curve for *I* versus I_f has V-shape, this characteristics is often called **V-*curve*** of the synchronous motor. A series of V-curves for different loads can be plotted.

Hunting in Synchronous Motors

For smooth operation of a synchronous motor, the mechanical speed of the rotor should remain in synchronism with the rotating magnetic field of the stator. Any departure from this equilibrium condition produces forces that tend to bring the rotor back to its equilibrium position. As a result, whenever a departure takes place the rotor slightly oscillates about its equilibrium position. This is called ***hunting*** of the motor. The hunting may occur due to sudden changes in mechanical load or in excitation current. We can reduce hunting of the motor by using fly-wheels or by providing *damper windings* on the rotor pole faces, as shown in Fig. 14.23.

14.14 SYNCHRONOUS CONDENSER

We have seen that a change in the excitation current I_f of a synchronous motor changes the power factor at which the motor operates. As the excitation is increased, the power factor passes from a lagging, through unity, to a leading power factor. This unique ability to draw leading current allows the synchronous motor to improve overall power factor of a collection of loads that tend to draw lagging current. For example, in a factory with many induction motors, using an overexcited synchronous motor (as shown in Fig. 14.29) becomes desirable to improve power factor of overall load. Such overexcited synchronous motors with no mechanical load (in fact, having no output shaft) are employed as ***synchronous condensers**** to correct the power factor.

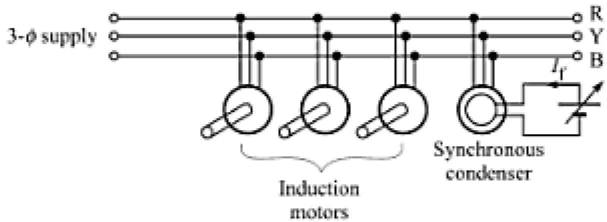


Fig. 14.29 *Employing synchronous condenser to correct the load power factor.*

In factories, mostly three-phase induction motors are used. These motors have power factor of about 0.8 (lagging) at full load. On lower loads, the power factor gets further reduced. For efficient utilisation of power supply systems, it is necessary that the power factor remains as near to unity as possible. The electric supply companies therefore impose a regulation that the power factor in a factory does not fall below a certain minimum value, say, 0.85 lagging; otherwise the factory has to pay a heavy penalty. Synchronous condensers are, therefore, employed as ***phase corrector*** or ***phase modifier***.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 14.7

A 3-phase, 4-pole, star-connected alternator has 24 slots with 12 conductors per slot and the flux per pole of 0.1 Wb. Calculate the line emf generated when the alternator is run at 1500 rpm.

Solution The number of conductors in series per phase, $Z_{ph} = 24 \times 12/3 = 96$.

$\therefore$ Number of turns per phase, $T = Z_{ph}/2 = 48$.

Number of slots/pole $= 24/4 = 6$; Slot angle, $\alpha = 180°/6 = 30°$.

Number of slots/pole per phase, $q = 6/3 = 2$.

Thus, the distribution factor is given as

$$k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(2\times 30°/2)}{2\sin(30°/2)} = 0.966$$

The pitch factor (for full-pitched coils), $k_p = 1$. The frequency of generated emf is

$$f = \frac{PN_s}{120} = \frac{4\times 1500}{120} = 50 \text{ Hz}$$

Generated emf per phase is

$$E = 4.44\, f\Phi T k_p k_d = 4.44 \times 50 \times 0.1 \times 48 \times 1 \times 0.966 = 1029.36 \text{ V}$$

* 'Condenser' is an antique word for 'capacitor'.

Therefore, for star-connected winding, the line emf generated is

$$E_L = \sqrt{3} \times E_{ph} = \sqrt{3} \times 1029.36 = \mathbf{1783\ V}$$

EXAMPLE 14.8

A part of alternator-winding consists of 6 coils in series, each having an emf of 10 V (rms) induced in it. The coils are placed in successive slots and the electrical angle between two consecutive slots is 30°. Calculate the net emf induced in six coils in series.

Solution Given: $\beta = 30°$; $q = 6$

$$\therefore \quad \text{Distribution factor, } k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(6 \times 30°/2)}{6\sin(30°/2)} = 0.644$$

The arithmetic sum of voltages induced in 6 coils = 6 × 10 = 60 V. Therefore, the vector sum of the voltages induced is

$$E_r = k_d \times \text{arithmetic sum} = 0.644 \times 60 = \mathbf{38.64\ V}$$

EXAMPLE 14.9

A water-turbine-driven, 3-phase, star-connected, 11-kV, 100-MVA, unity-*pf*, 50-Hz alternator is run at 120 rpm. Determine (*a*) the number of poles it has, (*b*) its current rating, (*c*) the input power at rated kW load if the efficiency is 97% (excluding the field loss), and (*d*) the prime-mover torque applied to the generator shaft.

Solution

(*a*) The number of poles, $P = \dfrac{120f}{N} = \dfrac{120 \times 50}{120} = \mathbf{50}$

(*b*) The kW-rating = VA-rating × *pf* = 100 MVA × 1 = 100 MW

(*c*) The current rating, $I_L = \dfrac{\text{VA-rating}}{\sqrt{3}V_L} = \dfrac{(100\ \text{MVA})}{\sqrt{3} \times (11\ \text{kV})} = \mathbf{5249\ A}$

(*d*) The input power, $P_{in} = \dfrac{P_o}{\eta} = \dfrac{100\ \text{MW}}{0.97} = \mathbf{103\ MW}$

(*e*) The prime-mover torque, $\tau = \dfrac{P_{in}}{2\pi N/60} = \dfrac{103 \times 10^6}{2\pi \times 120/60} = \mathbf{8.2 \times 10^6\ Nm}$

EXAMPLE 14.10

The armature of a 3-phase, 800-kVA, 11-kV, star-connected alternator has a resistance of 1.5 Ω per phase and synchronous reactance of 25 Ω per phase. Determine the percentage regulation for a load of 600 kW at a power factor of 0.8 leading.

Solution The voltage per phase, $V = V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{11\ \text{kV}}{\sqrt{3}} = 6.35\ \text{kV} = 6350\ \text{V}$

$$\text{The full-load current, } I = I_{ph} = I_L = \frac{P_o}{\sqrt{3} \times V_L \times pf} = \frac{600\ \text{kW}}{\sqrt{3} \times (11\ \text{kV}) \times 0.8} = 39.36\ \text{A}$$

The synchronous impedance, $Z_s = \sqrt{1.5^2 + 25^2} = 25.045\ \Omega$;

$$\text{with } \theta = \tan^{-1}\frac{X_s}{R} = \tan^{-1}\frac{25}{1.5} = 86.57°$$

The voltage drop across synchronous impedance, $V_z = IZ_s = 39.36 \times 25.045 = 985.77$ V

The power-factor angle, $\phi = \cos^{-1} 0.8 = 36.87°$ leading, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta + \phi) = 86.57° + 36.87° = 123.44° \quad \text{and} \quad \cos\alpha = -0.551$$

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$

$$= \sqrt{6350^2 + 985.77^2 + 2 \times 6350 \times 985.77 \times (-0.551)} = 5864.8 \text{ V}$$

Therefore, percentage regulation is

$$\% \text{ Regn.} = \frac{E - V}{V} \times 100 = \frac{5864.8 - 6350}{6350} \times 100 = -\mathbf{7.648 \text{ per cent}}$$

EXAMPLE 14.11

A three-phase, star-connected, 6000-kVA, 6-kV alternator has the winding resistance per phase of 0.2 Ω. The tests conducted on it gave following results:

OC Test :	Field current = 10 A;	Line voltage = 480 V
SC Test :	Field current = 5 A;	Line current = 105 A

Determine the full-load voltage regulation of this alternator at 0.8 power factor lagging.

Solution The terminal voltage per phase, $V = V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{6000}{\sqrt{3}} = 3464$ V

Since the total kVA-rating = $3 \times I_{ph} \times V_{ph}$, the full-load phase current is given as

$$I = I_{ph} = \frac{\text{kVA-rating}}{3 \times V_{ph}} = \frac{6000 \times 1000}{3 \times 3464} = 577.35 \text{ A}$$

In the short-circuit test, the currents are small compared to the full-load current. Hence, the magnetisation characteristic is linear. Since the field current of 5 A gives an armature current of 105 A, a field current of 10 A will give armature current of 105 × 2 = 210 A. Therefore, from the test-results, we get

$$Z_s = \frac{V_{ph(OCT)}}{I_{ph}} = \frac{480/\sqrt{3}}{210} = 1.32\ \Omega$$

$$\therefore \quad X_s = \sqrt{Z_s^2 - R^2} = \sqrt{1.32^2 - 0.2^2} = 1.305\ \Omega \quad \text{and} \quad \theta = \tan^{-1}(1.305/0.2) = 81.28°$$

The voltage drop across synchronous impedance, $V_z = IZ_s = 577.35 \times 1.32 = 762.1$ V

The power factor angle, $\phi = \cos^{-1} 0.8 = 36.87°$ lagging, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 81.26° - 36.87° = 44.41° \quad \text{and} \quad \cos\alpha = 0.714$$

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$

$$= \sqrt{3464^2 + 762.1^2 + 2 \times 3464 \times 762.1 \times 0.714} = 4043.5 \text{ V}$$

Therefore, percentage regulation is

$$\frac{E - V}{V} \times 100 = \frac{4043.5 - 3464}{3464} = \mathbf{16.73\ \%}$$

EXAMPLE 14.12

A three-phase, 50-Hz, star-connected generator has 96 conductors per phase and a flux per pole of 0.1 Wb. The stator winding has a synchronous reactance of 5 Ω/phase and negligible resistance. The distribution factor for the stator winding is 0.96. Calculate the terminal voltage when three non-inductive resistors, of 10 Ω/phase, are connected in star across the terminals. Sketch the phasor diagram for one phase.

Solution $\mathbf{Z}_s = jX_s = j5\ \Omega$; $Z_s = 5\ \Omega$ and $\theta = 90°$; $\Phi = 0.1$ Wb; $k_d = 0.96$; $k_p = 1$.

The number of turns per phase, $T = 96/2 = 48$

Therefore, per phase emf generated,

$$E = 4.44 f\Phi k_p k_d = 4.44 \times 50 \times 0.1 \times 48 \times 1 \times 0.96 = 1023 \text{ V}$$

Taking **E** as reference phasor, that is, $\mathbf{E} = 1023\angle 0°$ V we can write

$$\mathbf{V} = \mathbf{E} - \mathbf{I}\mathbf{Z}_s \quad \text{or} \quad \mathbf{V} = \mathbf{E} - \frac{\mathbf{V}}{\mathbf{Z}_L}\mathbf{Z}_s$$

$$\Rightarrow \qquad \mathbf{V} = \frac{\mathbf{E}}{(1 + \mathbf{Z}_s/\mathbf{Z}_L)} = \frac{1023\angle 0°}{(1 + j5/10)} = \frac{1023}{\sqrt{1 + (0.5)^2}}\angle -\tan^{-1}(0.5) = 915\angle -26.57° \text{ V}$$

Therefore, the terminal line voltage, $V_L = \sqrt{3}V = \sqrt{3} \times 915 = \mathbf{1584\ V}$

The phasor diagram for one phase is shown in Fig. 14.30.

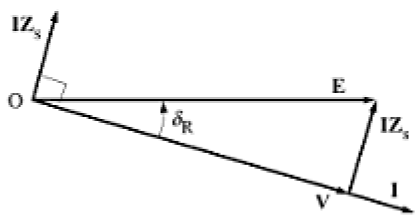


Fig. 14.30 *Phasor diagram for one phase (Example 14.12).*

EXAMPLE 14.13

A three-phase, 1500-kVA, 6.6-kV, star-connected synchronous generator has a resistance of 0.5 Ω/phase and a synchronous reactance of 5 Ω/phase. Calculate the percentage change of voltage when the rated output of 1500 kVA at 0.8 lagging power factor is switched off. Assume that the speed and the exciting-current remain unaltered.

Solution The voltage per phase, $V = V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{6.6 \text{ kV}}{\sqrt{3}} = 3810.6$ V

The full-load current, $$I = I_{ph} = \frac{\text{kVA-rating}}{3 \times V_{ph}} = \frac{1500 \times 1000}{3 \times 3810.6} = 131.2 \text{ A}$$

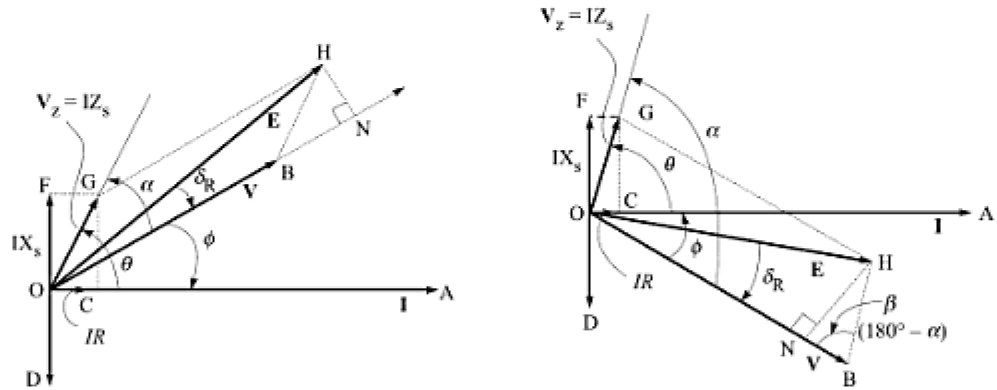
The synchronous impedance,

$$Z_s = \sqrt{0.5^2 + 5^2} = 5.025\ \Omega;\ \theta = \tan^{-1}\frac{X_s}{R} = \tan^{-1}\frac{5}{0.5} = 84.29°$$

The voltage drop across synchronous impedance, $V_z = IZ_s = 131.2 \times 5.025 = 659.28$ V

Therefore, percentage regulation is

$$\frac{E-V}{V} \times 100 = \frac{3810-4264}{4264} = \mathbf{-10.6\,\%}$$



(*a*) For 0.8 power factor lagging. (*b*) For 0.6 power factor lagging.

Fig. 14.31 *Phasor diagram (for Example 14.14).*

EXAMPLE 14.15

A three-phase, star-connected synchronous generator is rated at 1.5 MVA, 11 kV. It has armature resistance of 1.2 Ω and a synchronous reactance of 25 Ω per phase. Calculate the percentage regulation for a load of 1.4375 MVA at (*a*) 0.8 power factor lagging, and (*b*) 0.8 power factor leading.

Solution The terminal voltage per phase, $V = V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{11\text{ kV}}{\sqrt{3}} = 6351\text{ V}$

The full-load current, $I = I_{ph} = \dfrac{1.4375 \times 10^6}{3 \times 6351} = 75.45\text{ A}$

The synchronous impedance, $Z_s = \sqrt{1.2^2 + 25^2} = 25.0288\ \Omega;\ \theta = \tan^{-1}\dfrac{25}{1.2} = 87.25°$

The voltage drop across synchronous impedance, $V_Z = IZ_s = 75.45 \times 25.0288 = 1888.42\text{ V}$

The power factor angle, $\phi = \cos^{-1} 0.8 = 36.87°$

(*a*) ***For 0.8 pf lagging*:** From Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 87.25° - 36.87° = 50.38° \quad \text{and} \quad \cos\alpha = 0.637$$

$$E = \sqrt{V^2 + V_Z^2 + 2V \cdot V_Z \cdot \cos\alpha}$$

$$= \sqrt{6351^2 + 1888.42^2 + 2 \times 6351 \times 1888.42 \times 0.637} = 7692.9\text{ V}$$

Therefore, percentage regulation is

$$\frac{E-V}{V} \times 100 = \frac{7692.9 - 6351}{6351} \times 100 = \mathbf{21.13\,\%}$$

(*b*) ***For 0.8 pf leading*:** From Eqs. 14.25 and 14.26,

$$\alpha = (\theta + \phi) = 87.25° + 36.87° = 124.12° \quad \text{and} \quad \cos\alpha = -0.561$$

$$E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$$
$$= \sqrt{6351^2 + 1888.42^2 - 2 \times 6351 \times 1888.42 \times 0.561} = 5517.68 \text{ V}$$

Therefore, percentage regulation is

$$\frac{E - V}{V} \times 100 = \frac{5517.68 - 6351}{6351} \times 100 = \mathbf{-13.12\,\%}$$

EXAMPLE 14.16

A three-phase, star-connected, 2200-V alternator has an effective resistance of 0.5 Ω per phase. In an experiment, it is found that a field current of 30 A produces the full-load current of 200 A and a line-to-line voltage of 1100 V on open-circuit. Determine the torque angle of the alternator when it delivers full load at 0.8 power factor lagging.

Solution The synchronous impedance per phase, $Z_s = \dfrac{1100/\sqrt{3}}{200} = 3.1754\ \Omega$

$$\therefore \qquad \theta = \cos^{-1}\frac{R}{Z_s} = \cos^{-1}\frac{0.5}{3.1754} = 80.9°;\ V_z = IZ_s = 200 \times 3.1754 = 635.08 \text{ V}$$

The voltage per phase, $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{2200}{\sqrt{3}} = 1270.2$ V

The power factor angle, $\phi = \cos^{-1} 0.8 = 36.87°$ lagging, so that from Eqs. 14.25 and 14.26,

$$\alpha = (\theta - \phi) = 80.9° - 36.87° = 44.03° \quad \text{and} \quad \cos\alpha = 0.719$$

The emf induced per phase, $E = \sqrt{V^2 + V_z^2 + 2V \cdot V_z \cdot \cos\alpha}$

$$= \sqrt{1270.2^2 + 635.08^2 + 2 \times 1270.2 \times 635.08 \times 0.719} = 1782.34 \text{ V}$$

The power output per phase, $P_o = V_{ph} I_{ph} \cos\phi = 1270.2 \times 200 \times 0.8 = 203227$ W

Neglecting losses, this must be the same as power developed per phase due to field excitation. Hence, using Eq. 14.27, we have

$$\text{Real power, } P = \frac{EV}{Z_s}\cos(\theta - \delta_R) - \frac{V^2}{Z_s}\cos\theta$$

or
$$203227 = \frac{1782.34 \times 1270.2}{3.1754}\cos(80.9° - \delta_R) - \frac{1270.2^2}{3.1754} \times \cos 80.9°$$

$$\Rightarrow \qquad \cos(80.9° - \delta_R) = 0.3978 \quad \text{or} \quad 80.9° - \delta_R = 66.56° \quad \text{or} \quad \delta_R = \mathbf{14.34°}$$

EXAMPLE 14.17

A three-phase, star-connected, 6-pole, 3-MVA alternator running at 1000 rpm feeds 3.3-kV bus bars. The synchronous-reactance voltage drop at full load is 25% of the rated terminal voltage and the rotor displacement is 1 electrical-degree, when the alternator supplies full load at 0.8 power factor lagging. Calculate the synchronizing power and the torque per mechanical-degree of displacement of the rotor.

Solution For star-connected winding, the voltage per phase, $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{3300}{\sqrt{3}} = 1905.26$ V

The full-load current, $I = \dfrac{3 \times 10^6}{\sqrt{3} \times 3.3 \times 10^3} = 524.9$ A

Solution

(*a*) The supply voltage per phase, $V = \dfrac{6600}{\sqrt{3}} = 3810.5$ V

The induced emf per phase, $E = \dfrac{8942}{\sqrt{3}} = 5162.67$ V

Since the induced emf is greater than the supply voltage, the motor must be running with a leading power factor, say, cos ϕ. Since the power input to the motor is given as

$$P_{in} = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3}\ V_L I \cos\phi$$ (For star connection, the phase current is same as line current.)

$$\therefore \qquad I\cos\phi = \frac{P_{in}}{\sqrt{3}V_L} = \frac{915\times 10^3}{\sqrt{3}\times 6600} = 80\ \text{A} \qquad (i)$$

Since, the resistance is negligible, the impedance voltage drop $E_r = IX_s = 20I$, and it leads the current by 90°, as shown in phasor diagram of Fig. 14.32.

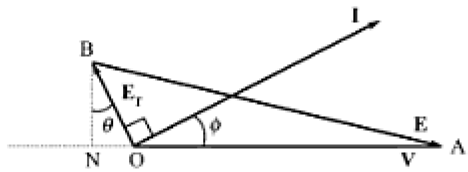


Fig. 14.32 *Phasor diagram* (*for Example* 14.19).

From the geometry of the phasor diagram, $\theta = \phi$. In right-angle ΔBNO,

$$BN = E_r\cos\phi = IX_s\cos\phi = X_s(I\cos\phi) = 20\times 80 = 1600 \quad [\text{from Eq. } (i)] \qquad (ii)$$

Now, from right-angle ΔABN, $AB^2 = BN^2 + NA^2$, or

$$NA = \sqrt{AB^2 - BN^2} = \sqrt{E^2 - (1600)^2} = \sqrt{(5162.67)^2 - (1600)^2} = 4908.8\ \text{V}$$

But $$NO = NA - OA = 4908.8 - 3810.5 = 1098.3\ \text{V} \qquad (iii)$$

Using Eqs. (*ii*) and (*iii*), we get

$$E_r = OB = \sqrt{NO^2 + BN^2} = \sqrt{(1098.3)^2 + (1600)^2} = 1940.7\ \text{V}$$

Thus, the line current is given as

$$I_L = I = \frac{E_r}{X_s} = \frac{1940.7}{20} = \mathbf{97\ A}$$

(*b*) The power factor is given as

$$pf = \frac{I\cos\phi}{I} = \frac{80}{97} = \mathbf{0.8247\ leading}$$

EXAMPLE 14.20

A three-phase, star-connected, 6600-V, synchronous motor operates at a constant voltage and a constant excitation. It has synchronous reactance of 15 Ω per phase and negligible resistance. When the input power is 500 kW, it operates at 0.8 power factor leading. Find the power factor when the load on the motor increases such that the input power becomes 800 kW.

Solution The supply voltage per phase, $V = \dfrac{6600}{\sqrt{3}} = 3810.5$ V

(*i*) *When the input is* 500 kW *at* 0.8 *pf leading*:

$$\text{The current per phase, } I = I_L = \frac{500\,000}{\sqrt{3} \times 6600 \times 0.8} = 54.67 \text{ A}$$

$$\text{The phase angle, } \phi = \cos^{-1} 0.8 = 36.87°$$

$$E_r = IZ_s = 54.67 \times 15 = 820.1 \text{ V}$$

Since $\mathbf{E} = \mathbf{V} - \mathbf{E}_r$, referring to phasor diagram of Fig. 14.25, the angle between **V** and $-\mathbf{E}_r$ is

$$\alpha = 180° - (\theta + \phi) = 180° - (90° + 36.87°) = 53.13°$$

$$\therefore \quad E = \sqrt{V^2 + E_r^2 + 2VE_r \cos\alpha}$$

$$= \sqrt{(3810.5)^2 + (820.1)^2 + 2 \times 3810.5 \times 820.1 \cos 53.13°} = 4352.3 \text{ V}$$

(*ii*) *When the input changes to* 800 kW:

The supply voltage and the excitation remain constant. Therefore,

$$I_1 \cos\phi_1 = \frac{P_{in}}{\sqrt{3} \times V_L} = \frac{800\,000}{\sqrt{3} \times 6600} = 70 \text{ A}$$

Using Eq. 14.38, we get

$$\sin\delta_R = \frac{P_{in} X_s}{EV} = \frac{800\,000 \times 15}{4352.3 \times 3810.5} = 0.7236$$

$$\therefore \quad \delta_R = 46.35° \quad \text{and} \quad \cos\delta_R = 0.69$$

Referring to phasor diagram of Fig. 14.33,

$$\text{AN} = V \sin\delta_R = 3810.5 \times 0.7236 = 2757.3$$

$$\text{and} \quad \text{NB} = \text{OB} - \text{ON} = E - V\cos\delta_R = 4352.3 - 3810.5 \times 0.69 = 1723.1$$

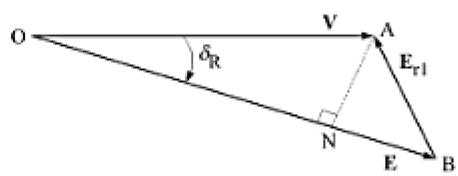


Fig. 14.33 *Phasor diagram* (*for Example* 14.20).

Thus, the new value of voltage drop across synchronous impedance is

$$E_{r1} = \text{AB} = \sqrt{\text{AN}^2 + \text{NB}^2} = \sqrt{(2757.3)^2 + (1723.1)^2} = 3251.4 \text{ V}$$

$$\therefore \quad I_1 = \frac{E_{r1}}{X_s} = \frac{3251.4}{15} = 216.7 \text{ A}$$

Therefore, the power factor is given as

$$pf = \frac{I_1\cos\phi_1}{I_1} = \frac{70}{216.7} = \mathbf{0.323\ (leading)}$$

EXAMPLE 14.21

A 400-V, three-phase system supplies a 500 kVA load at 0.5 power factor lagging. A synchronous motor supplying an active load of 100 hp at an efficiency of 0.87 is used to improve the overall power factor to 0.9 lagging. Find (*a*) the power factor of the synchronous motor, and (*b*) the kVA rating of the synchronous motor.

Solution

(*a*) Let the subscript i represent the original inductive load, the subscript s represent the synchronous motor, and the subscript t represent the overall total load.

(*i*) *For the inductive load*:

$$S_i = 500 \text{ kVA}; \cos \phi_i = 0.5 \quad \Rightarrow \quad \phi_i = 60° \quad \text{and} \quad \sin \phi_i = 0.866$$

$$\therefore \qquad \text{Active power, } P_i = S_i \cos \phi_i = 500 \times 0.5 = 250 \text{ kW}$$

$$\text{Volt ampere reactive, } Q_i = S_i \sin \phi_i = 500 \times 0.866 = 433 \text{ kVAR (inductive)}$$

(*ii*) *For the synchronous motor*:

$$\text{The active power, } P_s = \frac{100 \times 746}{0.87} = 85\,747 \text{ W} = 85.7 \text{ kW}$$

(*iii*) *For the overall load*:

$$\text{The active power, } P_t = P_i + P_s = 250 + 85.7 = 335.7 \text{ kW}$$

$$\cos \phi_t = 0.9 \quad \Rightarrow \quad \phi_t = 25.84° \quad \text{and} \quad \sin \phi_t = 0.436$$

$$\text{The volt amperes, } S_t = \frac{P_t}{\cos \phi_t} = \frac{335.7}{0.9} = 373 \text{ kVA}$$

$$\text{The volt ampere reactive, } Q_t = S_t \sin \phi_t = 373 \times 0.436 = 162.63 \text{ kVAR (inductive)}$$

Therefore, the volt ampere reactive of the synchronous motor is given as

$$Q_s = Q_t - Q_i = 162.63 - 433 = -270.37 \text{ kVAR (capactive)}$$

$$\therefore \qquad \phi_s = \tan^{-1} \frac{Q_s}{P_s} = \tan^{-1} \frac{270.37}{85.7} = 72.41°$$

$$\Rightarrow \qquad pf = \cos \phi_s = \cos 72.41° = \mathbf{0.3\ (leading)}$$

(*b*) The kVA rating of the synchronous motor is given as

$$S_s = \sqrt{P_s^2 + Q_s^2} = \sqrt{(85.7)^2 + (270.37)^2} = \mathbf{283.63\ kVA}$$

SUMMARY

TERMS AND CONCEPTS

- An *electromechanical energy conversion* (EMEC) *device* can convert mechanical energy into electrical energy (generator action), or vice versa (motor action).
- The directions of induced emf *e* and current *i* are the same in a generator but opposite in a motor.
- The directions of electromagnetic torque τ_e and friction torque τ_f are the same in a generator but opposite in a motor.
- A synchronous machine has armature on its stator and field poles on its rotor.
- In thermal power stations, the alternators use a cylindrical or non-salient type rotor as it runs at high speed. In hydroelectric power stations, the alternators, driven by water-turbines (low speed) use salient-pole rotors.
- Three-phase currents flowing in three-phase armature winding (on the stator) produce a magnetic field that rotates with synchronous speed.

- The ***pitch factor*** (or ***coil span factor***) k_p is used to account for the short-pitched coils.
- The phase angle between the excitation emf E and the terminal voltage V is the ***rotor power angle***, δ_R.
- The effect of armature flux on the main field flux is known as ***armature reaction.*** The armature reaction has *distorting effect* at unity power factor, wholly *demagnetising effect* at zero power factor lagging, and wholly *magnetising effect* at zero power factor leading.
- The rotating magnetic flux due to the stator currents can be regarded as generating an emf lagging the current by a quarter cycle. This effect is accounted in the equivalent circuit by including an inductive reactance, called ***synchronous reactance*** (X_s).
- A synchronous motor develops torque only at synchronous speed.
- A synchronous motor is a doubly excited motor. The armature on the stator is excited by a three-phase ac supply and the field winding on the rotor is excited by a dc supply.
- As the excitation current to a synchronous motor is increased, the power factor passes from a lagging, through unity, to a leading power factor. Hence, a synchronous motor is used as a power factor modifier (or corrector) in factories.

IMPORTANT FORMULAE

- Energy conversion from electrical to mechanical, $\boxed{ei = \tau_e \omega}$.
- Relation between electrical and mechanical angles, $\theta_e = \left(\frac{P}{2}\right)\theta_m$.
- ***Synchronous speed***, $N_s = \frac{120f}{P}$.
- $k_p = \frac{\text{phasor sum of the coil-side emfs}}{\text{arithmetic sum of the coil-side emfs}} = \frac{2E\cos(\beta/2)}{2E} = \cos(\beta/2)$
- $k_d = \frac{\text{phasor sum of component emfs}}{\text{arithmetic sum of component emfs}} = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)}$, where α is ***slot angle***.
- The emf equation, $E = 4.44 f\Phi T k_p k_d$.
- Per-unit voltage regulation $= \frac{E - V}{V}$.
- The output power per phase, $P = (EV/Z_s)\sin\delta_R$.
- $E^2 = V^2 + (IZ_s)^2 + 2V\cdot IZ_s\cos(\theta \mp \phi)$ with minus sign for lagging *pf* and plus sign for leading *pf*.
- The mechanical power (per phase) developed, $P = \frac{EV}{X_s}\sin\delta_R$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	A cylindrical magnetic structure cannot have an odd number of magnetic poles.	☐	☐	

2.	For maximum torque in an electrical machine, the rotor and stator fluxes should be aligned.	☐	☐	
3.	The vector magnetic flux density in the air gap of a machine changes only in magnitude, not in direction.	☐	☐	
4.	If we change the phase sequence of a three-phase cylindrical structure, the direction of rotation of the magnetic flux will reverse.	☐	☐	
5.	The alternators in thermal power stations have 2 or 4 poles.	☐	☐	
6.	The rotor of a three-phase synchronous generator has only two slip-rings.	☐	☐	
7.	A synchronous motor develops an alternating torque at a speed other than synchronous speed.	☐	☐	
8.	If a synchronous motor develops a torque of 1000 Nm at its rated speed, it will develop only 500 Nm torque at half the rated speed.	☐	☐	
9.	The value of excitation for which the induced emf E in the stator winding is in phase with the applied voltage V is called normal excitation.	☐	☐	
10.	The synchronous condenser draws a lagging current from the three-phase supply.	☐	☐	

Your Score

ANSWERS

1. True	**2.** False	**3.** True	**4.** True	**5.** True
6. True	**7.** True	**8.** False	**9.** False	**10.** False

REVIEW QUESTIONS

1. How do you justify that a rotary machine can work as a generator as well as a motor?

2. Compare a salient-pole alternator with a cylindrical-pole alternator on the basis of the following points: (*i*) speed, (*ii*) diameter, (*iii*) axial length, (*iv*) number of conductors, (*v*) number of poles, and (*vi*) type of prime-mover.

3. What is the meaning of 'electrical angle'? How is it different from the mechanical angle? Can these two angles be equal in a machine?

4. Explain the constructional details of a synchronous machine, giving reasons for making two different types of rotors—salient-pole and cylindrical-pole rotors.

5. State why salient-type of rotors are not used in alternators driven by steam turbines.

6. Explain why cylindrical-rotor alternators have small diameters and large lengths of the core, whereas salient-pole alternators have large diameters and small core lengths.

7. What is 'breadth factor'? What is its effect? Derive the expression for breadth factor of a winding having q slots per pole per phase and a slot angle α.

8. Derive an expression for the induced emf in an alternator. Discuss the role of different factors which appear in this expression.

9. Explain how a rotating magnetic field is produced by a 3-phase stator winding supplied with a 3-phase ac supply.

10. State the advantages of having rotating field system rather than a rotating armature system in a synchronous machine.

11. Discuss the concept of replacing the armature reaction by a reactance. What is the necessity of making this simplification?
12. What is meant by voltage regulation of an alternator? Explain how you will determine the value of voltage regulation for an alternator, by using its equivalent circuit.
13. Draw the phasor diagrams of an alternator at unity, lagging and leading power factors.
14. What is meant by synchronous impedance? How is it determined experimentally?
15. Explain why a synchronous motor does not develop a starting torque. Explain one method of starting a synchronous motor.
16. Explain the effect of change of excitation of a synchronous motor on (*a*) its armature current, and (*b*) its power factor.
17. Explain why an overexcited synchronous motor is called a synchronous condenser. How is this used in industries to correct the power factor?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. Salient-pole machines usually have
   (*a*) large number of poles
   (*b*) small number of poles
   (*c*) small diameters
   (*d*) long cores
2. The full-pitch coil in an alternator has a span of 18 slots. If it is desired to eliminate the third harmonic, the coil span should be
   (*a*) 18 slots (*b*) 15 slots
   (*c*) 12 slots (*d*) 9 slots
3. A synchronous machine can operate
   (*a*) only as a generator
   (*b*) only as a motor
   (*c*) both as a generator and as a motor
   (*d*) none of the above
4. The current from the stator of an alternator is taken out to the external load circuit through
   (*a*) commutator segments
   (*b*) slip-rings
   (*c*) carbon brushes
   (*d*) solid connections
5. The stator core of a synchronous machine is built of laminations of
   (*a*) stainless steel (*b*) silicon steel
   (*c*) cast steel (*d*) cast iron
6. The machine that supplies dc power to the rotor of a synchronous machine is called
   (*a*) the rectifier (*b*) the inverter
   (*c*) the converter (*d*) the exciter
7. Turbo-alternators are usually driven at
   (*a*) 500 rpm (*b*) 750 rpm
   (*c*) 1000 rpm (*d*) 3000 rpm
8. Hydro-alternators are usually driven at
   (*a*) 500 rpm (*b*) 750 rpm
   (*c*) 1000 rpm (*d*) 3000 rpm
9. The salient-pole rotors are not suitable for high speed turbo-alternators due to
   (*a*) undesirable mechanical vibrations
   (*b*) non-sinusoidal flux-distribution and windage loss
   (*c*) excessive bearing friction
   (*d*) large eddy-current loss
10. In turbo-alternators, smooth cylindrical rotors have very long length because
    (*a*) the number of conductors being less, the length has to be necessarily long to generate required emf
    (*b*) it reduces friction and windage loss
    (*c*) it reduces centrifugal force
    (*d*) none of the above
11. The maximum possible speed at which an alternator can be driven to generate an emf of 50 Hz is
    (*a*) 1500 rpm (*b*) 3000 rpm
    (*c*) 3600 rpm (*d*) 4000 rpm
12. The armature-reaction effect in a synchronous machine depends on
    (*a*) the load current only
    (*b*) the load power factor only
    (*c*) the speed of the rotor only
    (*d*) both (*a*) and (*b*)

13. A 4-pole, 1200 rpm alternator will generate an emf at a frequency of

(*a*) 60 Hz (*b*) 50 Hz
(*c*) 40 Hz (*d*) 25 Hz

14. In an alternator, the armature reaction is considered to be equivalent to a fictitious

(*a*) reactance (*b*) resistance
(*c*) impedance (*d*) admittance

15. A synchronous generator is supplying power to an inductive load. If the load is thrown off, the terminal voltage will

(*a*) decrease slightly
(*b*) decrease appreciably
(*c*) increase slightly
(*d*) remain unchanged

16. The percentage regulation of an alternator is defined as

(*a*) $\frac{V-E}{V} \times 100$ (*b*) $\frac{E-V}{E} \times 100$
(*c*) $\frac{V}{E} \times 100$ (*d*) $\frac{E-V}{V} \times 100$

17. Synchronous generator has its armature winding on the stator and field winding on the rotor. When running under steady-state conditions, the magnetic field in its air gap is

(*a*) stationary with respect to the stator
(*b*) rotating at synchronous speed with respect to the rotor
(*c*) rotating at synchronous speed against the direction of rotation
(*d*) rotating at synchronous speed in the direction of rotation

18. A 3-phase synchronous motor needs

(*a*) dc supply only
(*b*) 3-phase ac supply only
(*c*) the dc supply as well as the 3-phase ac supply
(*d*) both the single-phase ac supply and the 3-phase ac supply

19. The number of slip rings in a synchronous motor is

(*a*) 5 (*b*) 3
(*c*) 2 (*d*) nil

20. In a synchronous motor,

(*a*) the rotor mmf and the stator mmf are stationary with respect to each other
(*b*) the stator mmf rotates slightly faster than the rotor mmf
(*c*) the rotor mmf rotates slightly faster than the stator mmf
(*d*) none of the two mmfs rotate at all

21. For steady production of torque by a synchronous motor, the relative speed of the magnetic field of the stator with respect to that of the rotor should be

(*a*) equal to the synchronous speed
(*b*) slightly more than the speed of rotating magnetic field
(*c*) slightly less than the speed of rotating magnetic field
(*d*) zero

22. A synchronous motor may fail to pull into synchronism owing to

(*a*) excessive load (*b*) high friction
(*c*) low excitation (*d*) any of these

23. The speed of a synchronous motor depends upon

(*a*) supply voltage (*b*) supply frequency
(*c*) number of poles (*d*) both (*b*) and (*c*)

24. For a 3-phase, 4-pole, 50-Hz synchronous motor, the frequency, the number of poles and the load torque are all halved. The motor speed will be

(*a*) 500 rpm (*b*) 750 rpm
(*c*) 1500 rpm (*d*) 3000 rpm

25. The power factor of an under-excited synchronous motor will be

(*a*) lagging (*b*) leading
(*c*) unity (*d*) zero

26. A synchronous motor can operate at

(*a*) lagging *pf* only (*b*) leading *pf* only
(*c*) unity *pf* only (*d*) any *pf*

27. A synchronous motor is connected to a 3-phase supply having a constant voltage and constant frequency. The motor is operating at lagging power factor. When the field current is increased,

(*a*) the load angle increases
(*b*) the power input increase
(*c*) the power factor decreases
(*d*) the armature current decreases up to a certain limit and then again increases

ANSWERS

1. *a*	**2.** *c*	**3.** *c*	**4.** *d*	**5.** *b*	**6.** *d*	**7.** *d*	**8.** *a*	**9.** *b*	**10.** *a*
11. *b*	**12.** *d*	**13.** *c*	**14.** *a*	**15.** *c*	**16.** *d*	**17.** *d*	**18.** *c*	**19.** *c*	**20.** *a*
21. *d*	**22.** *d*	**23.** *d*	**24.** *c*	**25.** *a*	**26.** *d*	**27.** *d*			

PROBLEMS

(A) SIMPLE PROBLEMS

1. A 3-phase, 50-Hz synchronous generator runs at a speed of 166.67 rpm. How many poles does it have? [**Ans.** 36]

2. Determine the rms value of the induced emf per phase for a three-phase, 20-pole, 50-Hz alternator. It has 180 slots, each having 6 conductors. The armature winding is single layer with full-pitch coils which are connected in 60° phase groups. All the coils in each phase are connected in series. Flux per pole is 25 mWb. [**Ans.** 959 V]

3. A three-phase, 50-Hz, 6-pole, star-connected alternator has 972 conductors distributed in 54 slots. The coils are short pitched by one slot. The fundamental flux per pole is 0.01 Wb. Calculate the breadth factor, the pitch factor and the line voltage on no load. [**Ans.** 0.959, 0.985, 588 V]

4. A 4-pole, 3-phase, 50-Hz, star-connected alternator has a single layer winding in 36 slots with 30 conductors per slot. The flux per pole is 0.05 Wb and the winding is full-pitched. Find the synchronous speed and the line voltage on no load. [**Ans.** 1500 rpm, 3319 V]

5. Calculate the no-load terminal voltage of a 3-phase, 8-pole, star-connected alternator running at 750 rpm. It has 72 slots, each slot having 10 conductors. It has sinusoidally distributed flux per pole of 55 mWb, and full-pitched coils with a distribution factor of 0.96. [**Ans.** 2436 V]

6. A three-phase, star-connected, 1-MVA, 11-kV alternator has rated current of 52.5 A. The resistance of the winding per phase is 0.45 Ω. The test-results are given below:

OC Test: Field current = 121.5 A
Voltage between the lines = 422 V
SC Test: Field current = 121.5 A
Line current = 52.5 A

Determine the full-load voltage regulation of the alternator for (*a*) 0.8 power factor lagging, and (*b*) 0.8 power factor leading.
[**Ans.** (*a*) 2.63%; (*b*) –1.94%]

(B) TRICKY PROBLEMS

7. A 3-phase, star-connected alternator is rated at 1600 kVA, 13.5 kV. The effective resistance and synchronous reactance of its armature, per phase, are 1.5 Ω and 30 Ω, respectively. Determine the percentage regulation for a load of 1280 kW at *pf* of 0.8 leading. [**Ans.** –11.98%]

8. Calculate the voltage regulation at full load 0.8 power factor lagging for a three-phase, 2500-kVA, 6.6-kV, star-connected alternator having an armature resistance of 0.073 ohm per phase and a synchronous reactance of 10.7 ohms per phase. [**Ans.** 45.63%]

9. A three-phase, star-connected alternator is excited to give 5196 V between lines on open circuit. It has a resistance of 0.05 Ω and a synchronous reactance of 7.5 Ω per phase. Determine the terminal voltage and regulation at full load current of 150 A when the load power factor is 0.8 lagging.
[**Ans.** 2301 V, 30.38%]

INDUCTION MOTORS

15

OBJECTIVES

After completing this Chapter, you will be able to:

- *Explain why an induction motor cannot run at synchronous speed.*
- *Describe the construction of two types of rotors for an induction motor.*
- *Determine slip for a given speed of the motor.*
- *Derive an expression for the emf induced in the rotor.*
- *State how the induction motor differs from a transformer.*
- *Draw and explain the power flow diagram for an induction motor.*
- *Draw the equivalent circuit of an induction motor.*
- *Draw and explain the torque-slip characteristic of an induction motor.*
- *Explain how the starting torque can be increased in a phase-wound induction motor by inserting more resistance.*
- *Derive the condition for obtaining maximum torque.*
- *Derive an expression for the ratio of torque τ for any s to the maximum torque τ_m.*

15.1 INTRODUCTION

The ac synchronous motor has limited practical applications. An alternative is the ***asynchronous motor*** which we usually call ***induction motor***. Most motors that we meet in the home and in industry are induction motors. These motors are more rugged, they need less maintenance, and they are less expensive than the synchronous motors or dc motors*.

Induction motors are available both for three-phase and single-phase operation. Three-phase induction motors are used for high power and industrial applications such as lifts, cranes, pumps, exhaust fans, lathes, etc. Single-phase induction motors** find use in domestic electric appliances such as fans, refrigerators, washing machines, pumps, hair-driers, etc.

15.2 PRINCIPLE OF WORKING

The *stator* of an induction motor (Fig. 15.1*a*) is similar to that of a synchronous machine. It has three-phase windings, P poles, sinusoidal mmf and flux distribution. For simplicity, the stator slots and windings are omitted in the figure. When three-phase currents flow through the stator windings, a magnetic flux is

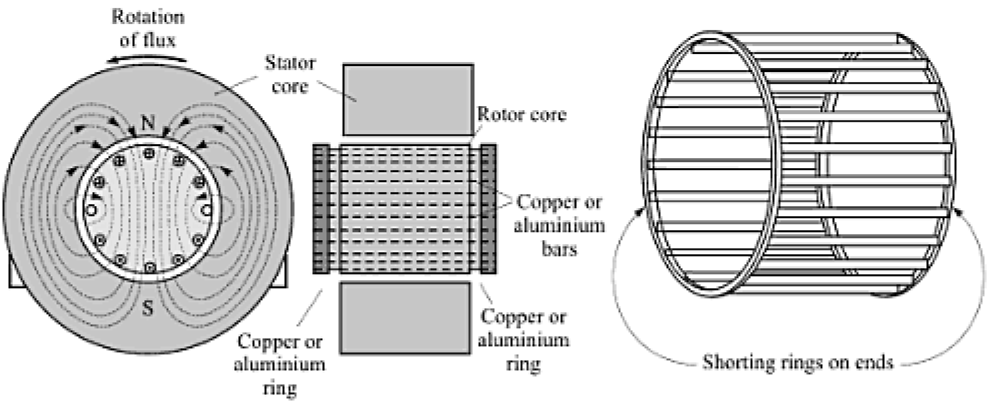
* We shall discuss dc motors in the next Chapter.

** We shall discuss single-phase induction motors in Chapter 17.

produced that rotates at *synchronous speed* given by Eq. 14.8, repeated here for convenience,

$$N_s = \frac{120f}{P} \tag{15.1}$$

The *rotor* is an iron laminated cylinder with large embedded conductors in the form of copper or aluminium bars in the semi-closed slots. The slots are usually not made parallel to the axis, but are given a slight twist. The rotor is then known as *skewed rotor**. The bars are short-circuited at each end by a conducting ring or plate (Fig. 15.1*b*). The bars and the shorting rings look like a ***squirrel cage***, as shown in Fig. 15.1*c*. The air-gap between the rotor and the stator is uniform and is made as small as possible mechanically.



(*a*) Front view. (*b*) Side view. (*c*) Squirrel-cage rotor.

Fig. 15.1 *Induction motor.*

Suppose that the stator is wound for two poles. Let the distribution of magnetic flux due to stator currents at a particular instant be as shown in Fig. 15.1*a*. Assume that the stator flux rotates anticlockwise. With respect to this flux, the rotor conductors move in clockwise direction. The emfs are thus induced** in the rotor conductors, whose directions can be determined by Fleming's right hand rule, as indicated by crosses and dots in Fig. 15.1*a*. The emf generated in the rotor conductors is a maximum in the region of maximum flux density.

Now, consider conductors A and B of the rotor which face the *N*-pole and *S*-pole of the stator, as shown in Fig. 15.2. The emf generated in these conductors circulates a current, which in turn produces its own flux. The resultant of these two fluxes is such that it strengthens the flux density on the right-side and weakens that on the left side for the conductor A. Consequently, the conductor A experiences a force F_1 left-ward. Similar action takes place for the conductor B, so that it experiences a force F_2 right-ward. These two forces create a torque τ_e that tends to rotate the rotor in the direction of the rotating flux.

Once the rotor starts rotating, the relative movement between the stator's rotating field and the rotor-conductors is reduced. As a result, the induced emf, the current, and its frequency are all reduced. If the motor shaft is not loaded, the machine has to rotate to meet the mechanical losses. The rotor speed can approach

* The skewing of the rotor helps in reducing noise, in increasing the starting torque and in eliminating clogging.

** This is why this type of motor is called an *induction motor*. In fact, an induction motor is like a transformer with its secondary winding short-circuited. The only difference is that in an induction motor, the secondary winding is free to rotate.

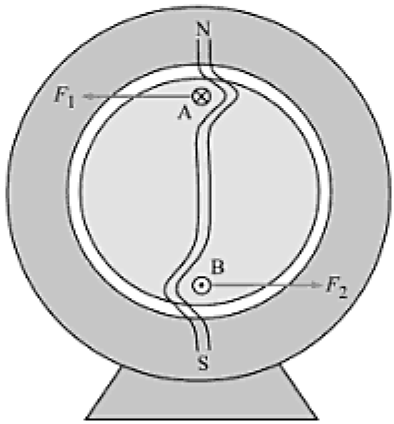


Fig. 15.2 *Torque generated on the rotor.*

very close to the synchronous speed. But, it can never be the same as the synchronous speed. If it does, the induced emf in the rotor-conductors would become zero and there would be no torque produced. Hence, *the rotor speed always remains slightly less than the synchronous speed.*

Now, suppose we put a mechanical load on the shaft. Its immediate reaction is to slow down the rotor. As a result, the relative speed with respect to the rotating field increases. The induced emf in the rotor-conductors increases and hence the torque τ exerted on the rotor increases. Ultimately, an equilibrium state is attained. The rotor speed adjusts itself to make the torque τ sufficient to balance the mechanical-loss torque and the load torque. Obviously, the speed of the motor running under full-load is less than the no-load speed.

Note that unlike a synchronous machine, *the induction motor has field on the stator and armature on the rotor.*

Slip of Induction Motor

As discussed above, the rotor speed must always remain less than the synchronous speed N_s given by Eq. 15.1. The difference between the synchronous speed N_s and the actual speed N of the rotor is known as *slip speed*, $N_\Delta = N_s - N$. This term is descriptive of the manner the rotor *slips back* from the exact synchronous speed. The ***normalized slip speed***, or simply the ***slip*** *s* is usually expressed as per-unit or fraction of the synchronous speed,

$$s = \frac{N_\Delta}{N_s} = \frac{N_s - N}{N_s} \tag{15.2}$$

For a given slips, the rotor speed is given as

$$N = N_s(1 - s) \tag{15.3}$$

The slip can also be expressed as a percentage of the synchronous speed, as

$$s = \frac{N_s - N}{N_s} \times 100\% \tag{15.4}$$

When the motor is at *standstill* (that is, it is not running), the rotor speed N is zero, and hence $s = 1$. The value of s can never be zero. Because this would mean that the rotor is rotating at synchronous speed which is impossible.

In practice, the value of slip is very small. At no load, the slip is around 1% only and at full load, it is around 3 %. For large size, efficient motors, the slip at full load may be around 1% only. Thus, an induction motor almost has a constant speed.

Thus, we find that $0 < s \leq 1$. Is it possible to make the slip s have a negative value? Yes, if the rotor is made to rotate by a prime-mover at a speed higher than the synchronous speed. The negative slip corresponds to the *generator action*.

Frequency of Rotor Currents When the induction motor is at standstill, the frequency of the currents induced in the rotor winding is the same as the supply frequency. However, when the motor runs, the frequency of rotor currents depends upon the relative speed or slip-speed. If the rotor-speed N and the synchronous-speed N_s are expressed in rpm (revolutions per minute), the frequency f_r of the rotor currents is given by an expression similar to that in Eq. 15.1, as

$$N_s - N = \frac{120 f_r}{P}$$

Dividing the above equation by Eq. 15.1, we get

$$\frac{N_s - N}{N_s} = \frac{f_r}{f} \quad \text{or} \quad s = \frac{f_r}{f}$$

$$\therefore \qquad \boxed{f_r = s \cdot f} \tag{15.5}$$

Speed of Rotation of Rotor-Field The rotor currents produce their own rotating magnetic field. Since the frequency f_r of the rotor currents is $s \cdot f$, the speed of this rotating field is $s \cdot N_s$ with respect to the rotor winding. However, the rotor itself is running at a speed N with respect to the stator. Hence,

The speed of rotor field in space = Speed of rotor field relative to the rotor + Speed of rotor relative to the stator

$$= sN_s + N = sN_s + N_s(1 - s) = N_s$$

Thus, we find that even though the rotor is not rotating at synchronous speed, the rotor field rotates at the synchronous speed. In fact, the rotor field remains locked with the stator field, irrespective of the rotor speed.

15.3 CONSTRUCTION OF INDUCTION MOTOR

Like any other rotating machine, an induction motor has a stator and a rotor. The stator has three-phase windings which receive energy form three-phase ac supply. The rotor carries windings in which the working currents are induced.

Stator

The stator core is a hollow cylindrical structure. It is made of sheet-steel laminations, each about 0.4 mm thick, slotted on its inner surface. The slots in large size motors are open type to facilitate the insertion of form-wound coils, which are well insulated before they are slipped into the slots. In small size motors, the slots are partially closed type. This helps in reducing the effective length of the air gap between the stator and the rotor. The coils are externally wound and are inserted through the narrow openings one wire at a time.

EXAMPLE 15.3

A 6-pole induction motor is fed from 50-Hz supply. If the frequency of rotor emf at full-load is 2 Hz, find the full-load slip and speed.

Solution The synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = \mathbf{1000\ rpm}$

The slip at full load, $s = \frac{f_r}{f} = \frac{2}{50} = 0.04 = \mathbf{4\ \%}$

The full-load speed, $N = N_s(1 - s) = 1000(1 - 0.04) = \mathbf{960\ rpm}$

EXAMPLE 15.4

A three-phase induction motor is wound for four poles and is supplied from a 50-Hz supply. Calculate (*a*) the synchronous speed, (*b*) the speed of the rotor when the slip is 4 %, and (*c*) the rotor frequency when the speed of the rotor is 600 rpm.

Solution

(*a*) The synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = \mathbf{1500\ rpm}$

(*b*) The speed of the rotor when the slip is 4 %,

$$N = N_s(1 - s) = 1500(1 - 0.04) = \mathbf{1440\ rpm}$$

(*c*) When the speed of the rotor is 600 rpm, the slip is

$$s = \frac{1500 - 600}{1500} = 0.6$$

∴ The rotor frequency, $f_r = s \cdot f = 0.6 \times 50 = \mathbf{30\ Hz}$

15.4 ROTOR EMF, CURRENT AND POWER FACTOR

The analysis of induction motor performance is done under the assumption that the applied voltage V_1 per phase is constant and purely sinusoidal of constant frequency f. It is further assumed that the flux per pole Φ is sinusoidally distributed in the space around the air gap and is rotating at synchronous speed N_s. We define following quantities:

V_1 = applied voltage to stator per phase (V)
N_1 = number of turns in series per phase of the stator
N_2 = number of turns in series per phase of the rotor
Φ = flux per pole (Wb)
f = frequency of the supply (Hz)
E_1 = emf induced per phase in the stator (V)
E_2 = emf induced per phase in the rotor (V) (when the motor is running)
E_{20} = emf induced per phase in the rotor (V) (when the motor is standstill)
R_2 = resistance of the rotor winding per phase (Ω)
X_2 = reactance of the rotor winding per phase (Ω) (when the motor is running)

Rotor Current The induced emf per phase in the rotor circuit, under running condition, is given by Eq. 15.8. The magnitude of the rotor current is therefore given as

$$I_2 = \frac{E_2}{Z_2} = \frac{sE_{20}}{\sqrt{R_2^2 + (sX_{20})^2}} \tag{15.10}$$

For small values of s, on increasing slip speed, the induced emf E_2 increases at a faster rate than the rotor impedance Z_2. As a result, on increasing slip speed s, the rotor current I_2 initially increases, and then tends to approach a maximum value, where the increase in E_2 is offset by the corresponding increase in Z_2.

Power Factor The phase angle between the induced emf E_2 and the rotor current I_2 is same as the impedance angle θ_2, as given by Eq. 15.9. Thus, the power factor of the rotor circuit, under running condition, is given as

$$pf = \cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + (sX_{20})^2}} \text{ (lagging)} \tag{15.11}$$

As the slip speed increases, the rotor circuit becomes more inductive and the power factor becomes poorer.

Induction Motor as Transformer

The working of an induction motor resembles in many respects with that of a transformer. The stator winding works as primary. The rotor winding works as secondary. Under standstill condition, when a 3-phase supply is connected to the stator, emfs are induced in both the stator and the rotor, as given by Eqs. 15.6 and 15.7, respectively. These equations are similar to Eqs. 13.4 and 13.5 for a transformer. In a transformer, the primary and secondary coils are concentrated. But in an induction motor, the stator and rotor windings are distributed. Hence, the distribution factor k_d and pitch factor k_p are included in Eqs. 15.6 and 15.7 for an induction motor.

When the rotor circuit is closed, a current flows in the rotor winding. This current creates an mmf. The rotor mmf and stator mmf can be combined just in the same way as in a transformer. In a transformer, the primary and secondary fields remain stationary. But in an induction motor, the stator and rotor fields keep rotating in space. However, this does not make any difference; these two fields, though rotating, remain stationary with respect to each other.

In a transformer, a greater secondary current causes an increased primary current. Similarly, in an induction motor, a greater load on the shaft causes an increased stator current to balance the rotor mmf. If the shaft is held stationary with rotor circuit closed (a condition known as ***blocked rotor*** or ***locked rotor***), the situation becomes same as in a transformer with its secondary short-circuit. Under such blocked rotor condition, excessive heating of the induction motor occurs as it draws a very heavy current from the ac supply.

Differences from a Transformer An induction motor differs from a transformer in following ways:

1. Magnetic leakage and hence the leakage reactances of stator and rotor are much higher than those in a transformer.
2. Because of the presence of the air gap, the magnetising current required is much greater than that in a transformer.
3. Because of the distributed windings, the ratio of the stator and the rotor currents is not equal to the ratio of turns.

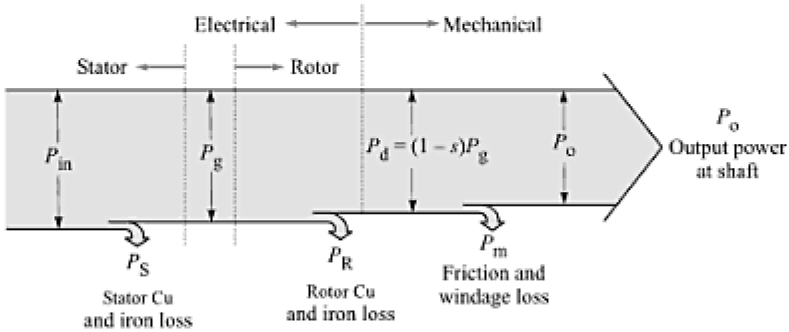


Fig. 15.3 *Power flow diagram for an induction motor.*

Let τ (in Nm) be the electromagnetic torque exerted on the rotor by the rotating magnetic field at synchronous speed N_s (in rpm). Then the air-gap power P_g (in W) transferred from the stator to the rotor is given as

$$P_g = \frac{2\pi\tau N_s}{60} \tag{15.13}$$

This is the input power to the rotor. If the rotor rotates at a speed N (in rpm), the total mechanical power developed by the rotor is given as

$$P_d = \frac{2\pi\tau N}{60} \tag{15.14}$$

Putting Eqs. 15.13 and 15.14 in Eq. 15.12, we get

$$P_R = P_g - P_d = \frac{2\pi\tau}{60}(N_s - N) \tag{15.15}$$

Dividing the above equation by Eq. 15.13, we get

$$\frac{P_R}{P_g} = \frac{N_s - N}{N_s} = s \quad \text{(according to the definition of slip)}$$

or The rotor copper loss, $P_R = s \times P_g$ (15.16)

Therefore, the power P_d developed by the rotor, using Eq. 15.12, is then given as

$$\text{The power developed by the rotor, } P_d = P_g - P_R = P_g - sP_g = (1-s)P_g \tag{15.17}$$

We can put the air-gap power as

$$P_g = P_g - sP_g + sP_g = \underbrace{(1-s)P_g}_{P_d} + \underbrace{sP_g}_{P_R} \quad \Rightarrow \quad \frac{P_d}{P_R} = \frac{1-s}{s} \tag{15.18}$$

This shows that the air-gap power divides between the developed power and rotor-copper loss in a ratio that depends only on the slip speed.

EXAMPLE 15.6

A three-phase, four-pole, 50-Hz induction motor has a full-load output power of 5 hp at 1470 rpm. The efficiency of the motor at full load is 87.5 %. The mechanical losses are 5 % of the total losses. Determine the developed power, air-gap power, rotor copper loss, and stator loss.

Solution The output power, $P_o = 5 \text{ hp} = 5 \times 746 = 3730 \text{ W}$

$\therefore$ The input power, $P_{in} = \dfrac{P_o}{\eta} = \dfrac{3730}{0.875} = 4263 \text{ W}$

Total losses, $P_l = 4263 - 3730 = 533 \text{ W}$

$\therefore$ Mechanical losses, $P_m = 0.05 \times 533 = 26.65 \text{ W}$

and Electrical losses, $P_e = 533 - 26.65 = 506.35 \text{ W}$

Developed power, $P_d = P_o + P_m = 3730 + 26.65 = \mathbf{3756.65\ W}$

To determine the air-gap power P_g from the developed power P_d, we need the slip s. First we calculate synchronous speed,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, the slip,

$$s = \frac{N_s - N}{N_s} = \frac{1500 - 1470}{1500} = 0.02$$

From Eq. 15.17, the air-gap power,

$$P_g = \frac{P_d}{1 - s} = \frac{3756.65}{1 - 0.02} = \mathbf{3833.3\ W}$$

From Eq. 15.16, the rotor copper loss,

$$P_R = s \times P_g = 0.02 \times 3833.3 = \mathbf{76.7\ W}$$

Hence, the stator loss, $P_S = 506.35 - 76.7 = \mathbf{429.65\ W}$

Note that the stator loss could also be determined by subtracting air-gap power from the input power,

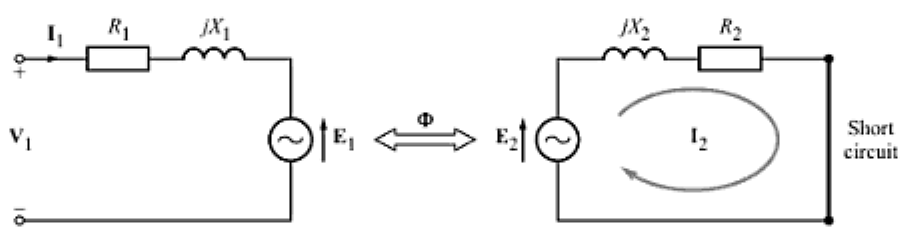
$$P_S = 4263 - 3833.3 = \mathbf{429.7\ W}$$

15.6 EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR

An induction motor is similar to a transformer, except that its secondary (i.e., the rotor) is not stationary but rotating at a speed N. Also, the load on the secondary is not electrical but mechanical.

Stator Equivalent Circuit

In the per-phase stator circuit in Fig. 15.4, R_1 accounts for the stator copper loss, X_1 accounts for the stator leakage magnetic flux, and E_1 is the induced emf in the stator winding due to rotating air-gap magnetic flux Φ. $\mathbf{V}_1$ is the per-phase stator voltage, and $\mathbf{I}_1$ the per-phase current drawn by the stator from the three-phase power supply. Obviously, the frequency of stator voltage and current is the same as that of the supply.



(*a*) Stator (Frequency = f). (*b*) Rotor (Frequency = $f_r = s.f$).

Fig. 15.4 *Per-phase equivalent circuit for stator and rotor.*

which accounts for the *rotor electrical loss*, and (2) $[(1 - s)/s] \times R_2$ which accounts for the *developed mechanical power*. The result is shown in Fig. 15.7. The figure also shows resistor R_1 to account for the stator copper loss, R_i for iron loss, and X_1 as the leakage-flux reactance.

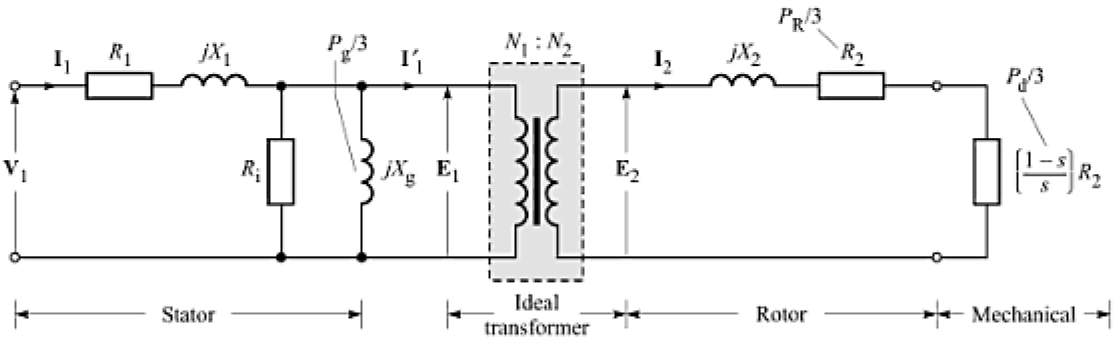


Fig. 15.7 *Equivalent circuit accounting for developed mechanical power.*

We can further simplify the circuit of Fig. 15.7 using the properties of an ideal transformer. If $K = (N_2/N_1)$ is the transformation ratio, the equivalent impedances as referred to the primary (stator) are given by Eq. 13.8 as $R'_2 = R_2/K^2$ and $X'_2 = X_2/K^2$. Thus, we get the equivalent circuit of Fig. 15.8.

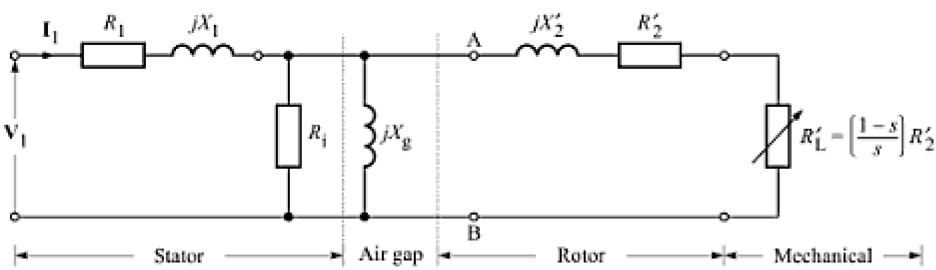


Fig. 15.8 *Equivalent circuit with rotor impedance transformed to stator-side.*

As for a transformer, we could have further simplified the circuit of Fig. 15.8 by shifting the shunt branches R_i and X_g to the right of rotor impedance. However, this simplification cannot be done in an induction motor without incurring considerable error, as the no-load current is about 25 % to 40 % of the rated current.

We can make use of the equivalent circuit of Fig. 15.8 by finding Thevenin's equivalent of the circuit on the left of terminals A-B. In practice, the core losses in an induction motor are quite small compared to the other powers. We can therefore ignore the resistance R_i. Thevenin's voltage is given as

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{oc}} = \mathbf{V}_{\text{AB}} = \mathbf{V}_1 \times \frac{jX_g}{R_1 + j(X_1 + X_g)} \tag{15.27}$$

Thevenin's equivalent impedance is given as

$$\mathbf{Z}_{\text{Th}} = \frac{jX_g(R_1 + jX_1)}{R_1 + j(X_1 + X_g)} \tag{15.28}$$

Thus, $$\tau \propto \frac{1}{s} \quad [R_2 \text{ and } X_{20} \text{ being constant}]$$

Hence, for large values of s, torque is seen to be inversely proportional to slip s. The torque-slip curve should be a rectangular hyperbola, as shown in Fig. 15.10.

The overall torque-slip characteristic curve has a shape as shown in Fig. 15.10. We can interpret the values of s in terms of the rotor speed N. When $s = 0$, the rotor speed N is same as the synchronous speed N_s. When $s = 1$, the rotor speed N becomes zero (i.e., the motor is standstill).

Three Modes of Operation

Depending on the value of slip s, there can be following three modes of operation of an induction motor (Fig. 15.11): (1) Motor action ($0 < s \leq 1$), (2) Brake action ($s > 1$), and (3) Generator action ($s < 0$).

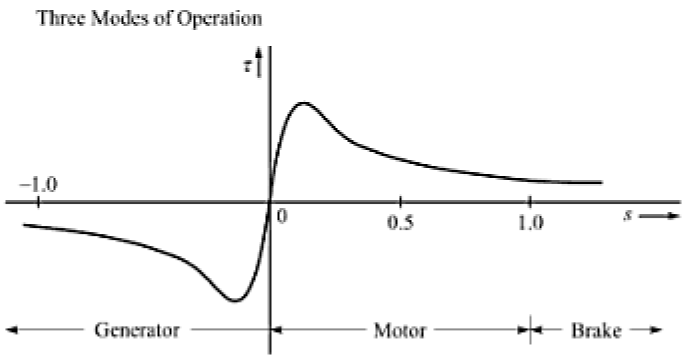


Fig. 15.11 *Three modes of operation of an induction motor.*

(1) *Motor Action* ($0 < s \leq 1$) In this mode, the rotor rotates in the same direction as the stator field. The speed is less than the synchronous speed.

When $s = 1$, the rotor speed N is zero corresponding to point C on the curve (Fig. 15.10). The torque at zero speed is called ***starting torque*** τ_{st}. Point B on the characteristic curve corresponds to a value of slip s for which the torque developed by the motor is ***maximum torque*** τ_m.

The shaded portion (for $0.01 < s < 0.06$) shows the normal working-range of the induction motor. Obviously, if the motor is to start running, the load torque at the shaft must be less than the starting torque τ_{st}. The motor will accelerate from standstill, until the torque developed and the load torque comes to equality at a speed close to but less than the synchronous speed. In the region AB, the machine has a *stable motor action*. If the load torque increases (but still remains below the value τ_m), the motor develops increased torque at a slightly reduced speed.

If the load torque is increased beyond τ_m, the speed decreases and the point of operation goes beyond B. The motor further decelerates and ultimately comes to a standstill. Thus, the region BC represents an *unstable motor action.*

(2) *Brake Action* ($s > 1$) There are two ways of making s greater than unity. First, the rotor can be driven by a prime mover in a direction opposite to the rotating magnetic field. Second, we can reverse any two of the phase supplies while operating the machine as a motor. The effect of reversing two supply-phases is to make the stator field rotate in the opposite direction. Thus, at the time of switch-over, the rotor is rotating almost at synchronous speed in one direction and the stator field is rotating at synchronous speed in the opposite direction. The difference is almost twice the synchronous speed and hence the slip s is almost 2.

The effect is that the rotor now attempts to reverse its direction of rotation. This amounts to braking of the rotor in order to bring it to a standstill prior to commencing rotation in the opposite direction. This braking effect is known as ***plugging***. As soon as the machine stops, the power supply is switched off and the machine remains at standstill.

(3) ***Generator Action* ($s < 0$)** The slip s can be made negative if with the help of a prime mover the rotor is made to rotate at a speed higher than the synchronous speed. In such cases, the machine works as a generator. However, induction generators are rarely used as the most significant generators are synchronous machines.

Condition for Maximum Torque

We have seen that the torque developed depends on the value of slip s. To determine the value of s that gives maximum torque, we differentiate the expression for τ (Eq. 15.36) with respect to s and equate the differential to zero.

$$\therefore \qquad \frac{d\tau}{ds} = K_1 \cdot \frac{(R_2^2 + s^2 X_{20}^2) R_2 - s R_2 \cdot 2s X_{20}^2}{(R_2^2 + s^2 X_{20}^2)^2} = K_1 \cdot \frac{R_2 (R_2^2 - s^2 X_{20}^2)}{(R_2^2 + s^2 X_{20}^2)^2}$$

For τ to have a maximum value, we should have

$$(R_2^2 - s^2 X_{20}^2) = 0 \quad \text{or} \quad R_2 = s X_{20} \tag{15.37}$$

This means that *the torque is maximum when the rotor resistance is equal to the rotor reactance under running condition.*

For a given machine (i.e., for given R_2 and X_{20}), the value of slip s at which the torque is maximum is given as

$$s_m = \frac{R_2}{X_{20}} \tag{15.38}$$

Maximum Torque

Putting the value of s from Eq. 15.38 in Eq. 15.36, we get the maximum value of the torque as

$$\tau_m = K_1 \frac{(R_2 / X_{20}) R_2}{R_2^2 + (R_2 / X_{20})^2 X_{20}^2} = K_1 \frac{R_2^2}{2 R_2^2 X_{20}} = \frac{K_1}{2} \cdot \frac{1}{X_{20}} \tag{15.39}$$

Thus, we find that the maximum torque of an induction motor is *inversely proportional to the leakage reactance at standstill* X_{20} and *is independent of the value of the rotor resistance* R_2.

The ratio of torque τ for any slip s to the maximum torque τ_m can be determined by dividing Eq. 15.36 by Eq. 15.39,

$$\frac{\tau}{\tau_m} = \frac{K_1 \dfrac{s R_2}{R_2^2 + s^2 X_{20}^2}}{\dfrac{K_1}{2} \cdot \dfrac{1}{X_{20}}} = \frac{2 s R_2 X_{20}}{R_2^2 + s^2 X_{20}^2} = \frac{2s (R_2 / X_{20})}{(R_2 / X_{20})^2 + s^2} = \frac{2s \cdot s_m}{s_m^2 + s^2} \tag{15.40}$$

Solution The synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm

$$\therefore \qquad \text{Slip, } s = \frac{N_s - N}{N_s} = \frac{1000 - 940}{1000} = 0.06$$

Maximum torque occurs at a slip so as to satisfy the condition: $R_2 = sX_{20}$. Therefore,

$$X_{20} = \frac{R_2}{s} = \frac{0.1}{0.06} = \mathbf{1.66\ \Omega}$$

15.8 STARTING OF INDUCTION MOTORS

There are two problems in starting an induction motor: (1) Low starting torque, and (2) Heavy starting current, though for a short duration.

As can be seen from Fig. 15.12, an induction motor having a low-resistance rotor, such as the usual type of squirrel-cage rotor, the starting torque is small compared to the maximum torque available. However, if the bars of the cage rotor were made with sufficiently high resistance to give high starting torque, the slip for full-load torque would be quite large. This would have two adverse effects. *First*, the I^2R loss in the rotor would be high causing excessive heating and reduced efficiency of the motor. *Second*, the variation of speed with load would be quite large.

Another problem is that the motor draws about four to seven times the full-load current, if it is directly switched to the power supply. At the instant of starting, the slip is unity, the frequency of induced emf in the rotor is $f_r = sf = 50$ Hz, and hence a large emf is induced. The induction motor works as a transformer with its secondary short-circuited. This results in a large circulating current in the rotor winding. Such a large current causes large voltage drops in the lines and thereby produces an undesirable dimming of the lamps in the vicinity. Therefore, the direct-on-line starting of induction motors is not desirable. Rules laid down by electric supply companies do not permit the direct-on-line starting of 3-phase induction motors above 5 hp. For such heavy-duty motors, we use some method by which either more resistance is included in the rotor circuit in the starting, or a reduced voltage is applied at the starting.

Starting of Wound-Rotor Induction Motor

The best (but costly) solution is to use a phase-wound rotor induction motor, with a starting arrangement shown in Fig. 15.13. The motor is designed to have low rotor resistance. External variable resistance is

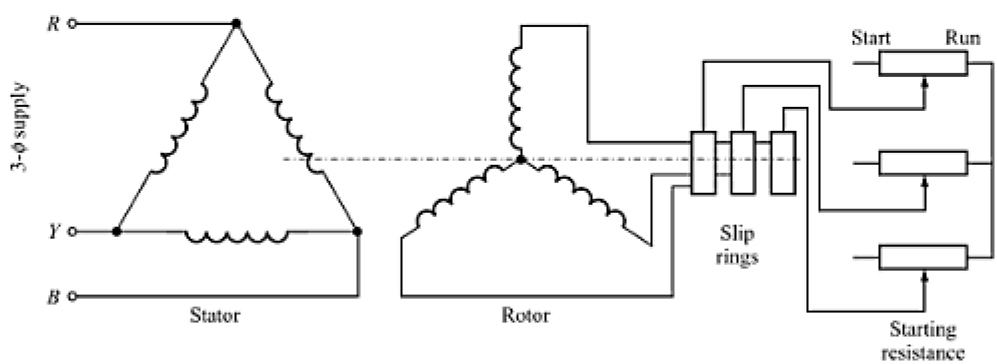


Fig. 15.13 *Starting of wound-rotor induction motor.*

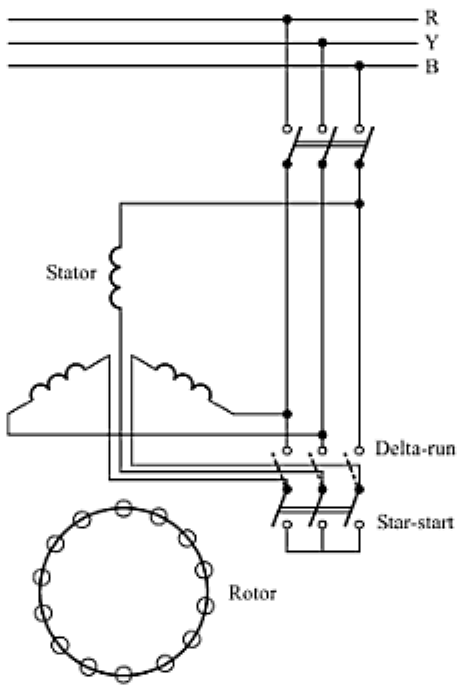
connected across the slip rings. The motor is started with all the resistance included in the rotor circuit. This gives a high starting torque. As the motor picks up speed, the resistance is slowly reduced. When the motor attains full speed, all the resistance is cut out.

Large motors are often fitted with a short-circuiting and brush-lifting device. On attaining full speed, first the three-rings are short-circuited and then the brushes are lifted off the rings. This eliminates the losses due to the brush-contact resistance and the brush friction. Also, the wear of the brushes and slip-rings is reduced.

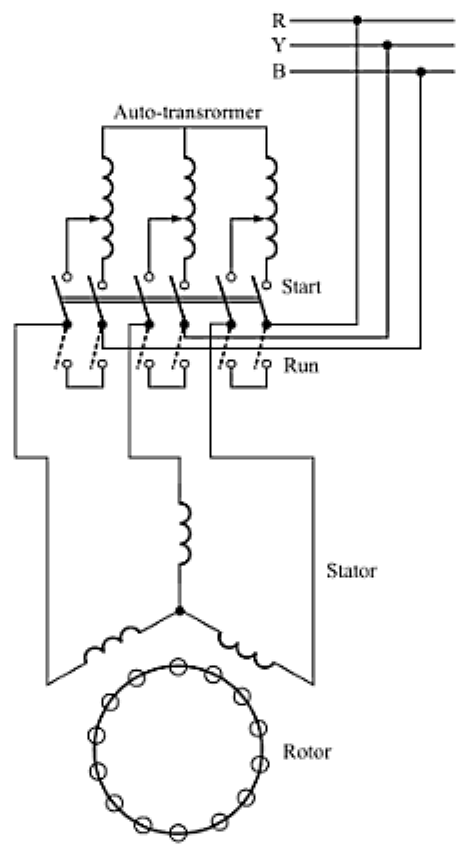
Starting of Cage-Rotor Induction Motor

Most induction motors have squirrel-cage type rotor. It is usual to start cage-rotor motors—except small machines—with a reduced voltage, using one of the methods given below.

Star-Delta Starter It can be used only for those motors with stator winding designed for delta-connection during normal operation. All the six terminals of the three-phase stator windings are brought out and connected as shown in Fig. 15.14*a*. Using the double-throw triple-pole switch, in the starting, the stator windings are connected in star, so that the voltage across each phase is $1/\sqrt{3}$ times the normal value. As the



(*a*) Star-delta starter.



(*b*) Auto-transformer starter.

Fig. 15.14 *Two ways of starting a cage-rotor induction motor.*

Therefore, using Eq. 15.41, we have

$$0.8 = \frac{2s_m}{s_m^2 + 1} \quad \text{or} \quad s_m^2 + 1 = 2.5s_m \quad \Rightarrow \quad s_m = 2,\ 0.5$$

Obviously, $s_m = 2$ is not possible. Hence, $s_m = 0.5$. Now, using Eq. 15.40,

$$\frac{\tau_{fl}}{\tau_m} = \frac{2s_{fl} \cdot s_m}{s_m^2 + s_{fl}^2} \qquad \text{or} \quad 0.5 = \frac{2 \times s_{fl} \times 0.5}{(0.5)^2 + s_{fl}^2}$$

or

$$s_{fl}^2 - s_{fl} + 0.125 = 0 \quad \Rightarrow \quad s_{fl} = 0.8535,\ 0.1465$$

Since, for stable motor action, s_{fl} must be less than s_m, we have $s_{fl} = 0.1465$. The synchronous speed is

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, full-load speed, $N_{fl} = N_s(1 - s_{fl}) = 1500 \times (1 - 0.1465) = \mathbf{1280.25\ rpm}$

(*b*) The speed at maximum torque, $N_m = N_s(1 - s_m) = 1500 \times (1 - 0.5) = \mathbf{750\ rpm}$

EXAMPLE 15.16

A 3-phase, 4-pole, 50-Hz, 18.65-kW induction motor has friction and windage losses of 2.5 % of the output, and full-load slip of 4 %. Determine (*a*) the rotor copper loss, (*b*) the rotor input, (*c*) the output torque, and (*d*) the gross torque.

Solution The friction and windage losses, $P_m = 0.025 \times 18\,650 = 466.25$ W.

The rotor gross output, $P_d = P_o + P_m = 18\,650 + 466.25 = 19\,116.25$ W

(*a*) Using Eq. 15.18, the rotor copper loss is

$$P_R = P_d\left(\frac{s}{1-s}\right) = 19\,116.25\left(\frac{0.04}{1 - 0.04}\right) = \mathbf{796.5\ W}$$

(*b*) The rotor input, $P_g = \dfrac{P_R}{s} = \dfrac{796.5}{0.04} = \mathbf{19\,912.5\ W}$

(*c*) The synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

The rotor speed, $N = N_s(1 - s) = 1500 \times (1 - 0.04) = 1440$ rpm

The shaft torque, $\tau_{sh} = \dfrac{P_o}{2\pi(N/60)} = \dfrac{18650}{2\pi \times (1440/60)} = \mathbf{123.7\ Nm}$

(*d*) The gross torque, $\tau_d = \dfrac{P_d}{2\pi(N/60)} = \dfrac{19116.25}{2\pi \times (1440/60)} = \mathbf{126.8\ Nm}$

EXAMPLE 15.17

A 3-phase, 4-pole, 1100-V, 50-Hz, delta-connected induction motor has a star-connected rotor with a phase transformation ratio of 3.8 : 1. The rotor resistance and standstill reactance are 0.012 Ω and 0.25 Ω per phase, respectively. The motor runs at 1440 rpm at full load. Neglecting the stator impedance and magnetizing current, determine (*a*) the rotor current at starting with slip-rings shorted, (*b*) the rotor power factor at starting with slip-rings shorted, (*c*) the rotor current while running at full load with slip-rings shorted, (*d*) the rotor power factor while running at full load with slip-rings shorted, and (*e*) the external resistance per phase required to limit the starting current to 100 A in the stator supply lines.

Solution The synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

The slip at full load, $s = \dfrac{1500 - 1440}{1500} = 0.04$

The transformation ratio, $K = \dfrac{N_2}{N_1} = \dfrac{1}{3.8}$

The induced voltage in rotor per phase at standstill, $E_{20} = KE_1 = \dfrac{1}{3.8} \times 1100 = 289.5$ V

The rotor impedance at standstill, $Z_{20} = \sqrt{R_2^2 + X_{20}^2} = \sqrt{(0.012)^2 + (0.25)^2} = 0.25\ \Omega$

The rotor impedance at full load,

$$Z_2 = \sqrt{R_2^2 + (sX_{20})^2} = \sqrt{(0.012)^2 + (0.04 \times 0.25)^2} = 0.0156\ \Omega$$

(*a*) The rotor current at starting with slip-rings shorted,

$$I_{20} = \frac{E_{20}}{Z_{20}} = \frac{289.5}{0.25} = \mathbf{1158\ A}$$

(*b*) The rotor power factor at starting with slip-rings shorted,

$$pf_{20} = \cos\left(\tan^{-1}\frac{0.25}{0.012}\right) = \cos 87.25° = \mathbf{0.048\ (lagging)}$$

(*c*) The rotor current while running at full load with slip-rings shorted,

$$I_2 = \frac{E_2}{Z_2} = \frac{sE_{20}}{Z_2} = \frac{0.04 \times 289.5}{0.25} = \mathbf{46.32\ A}$$

(*d*) The rotor power factor while running at full load with slip-rings shorted,

$$pf = \frac{R_2}{Z_2} = \frac{0.012}{0.0156} = \mathbf{0.769\ (lagging)}$$

(*e*) The stator phase current corresponding to starting line-current of 100 A is

$$I_1 = \frac{100}{\sqrt{3}} = 57.73\ \text{A}$$

$\therefore$ Rotor current per phase, $I_2 = \dfrac{I_1}{K} = 57.73 \times 3.8 = 219.4$ A

$\therefore$ Rotor total impedance, $Z_2' = \dfrac{E_{20}}{I_{20}} = \dfrac{289.5}{219.4} = 1.32\ \Omega$

Thus, the rotor total resistance required is

$$R_2 + r = \sqrt{Z_2'^2 - X_{20}^2} = \sqrt{(1.32)^2 - (0.25)^2} = 1.296\ \Omega$$

Therefore, the external rotor resistance required is

$$r = 1.296 - 0.012 = \mathbf{1.284\ \Omega}$$

SUMMARY

TERMS AND CONCEPTS

- An induction motor has a distributed three-phase field winding on the stator and armature on the rotor.
- The speed of the rotor of an induction motor always remains slightly less than the synchronous speed.

10. If an induction motor is made to run at synchronous speed, then
(*a*) the rotor emf would be zero
(*b*) the rotor current would be zero
(*c*) the torque developed would be zero
(*d*) all the above

11. The rotor circuit of a three-phase induction motor under running condition
(*a*) is always closed
(*b*) is always open
(*c*) is sometimes closed and sometimes open
(*d*) is neither completely closed nor completely open

12. The number of slip rings in a 3-phase wound rotor induction motor is
(*a*) 3 (*b*) 4
(*c*) 9 (*d*) 12

13. A 3-phase, 50-Hz induction motor at standstill, when energized, has a voltage of 50 V between its slip rings. While running at full load with a slip of 0.04, the voltage between its slip rings would be
(*a*) 50 V (*b*) 20 V
(*c*) 4 V (*d*) 2 V

14. At standstill, the power factor of the rotor circuit of an induction motor is found to be 0.2 lagging. While running at full load, the power factor of the rotor circuit will be
(*a*) 0.2 lagging
(*b*) about 0.8 lagging
(*c*) almost unity
(*d*) any of the above

15. If the rotor circuit resistance of a three-phase induction motor is increased,
(*a*) both the starting torque and the maximum value of the torque developed increase
(*b*) both the starting torque and the maximum value of the torque developed remain unchanged
(*c*) the starting torque increases but the maximum value of the torque developed decreases
(*d*) the starting torque increases but the maximum value of the torque developed remains unchanged

16. An induction motor has a starting torque of 600 Nm when started by direct switching. If it is started through an auto-transformer with 50 % tapping, the starting torque will be
(*a*) 1200 Nm (*b*) 600 Nm
(*c*) 300 Nm (*d*) 150 Nm

17. An induction motor has a starting torque of 600 Nm when started by direct switching. If a star-delta starter is used for starting this motor, the starting torque will be
(*a*) 1200 Nm (*b*) 600 Nm
(*c*) 300 Nm (*d*) 200 Nm

ANSWERS

1. *a*	**2.** *d*	**3.** *c*	**4.** *c*	**5.** *d*	**6.** *c*	**7.** *d*	**8.** *c*	**9.** *a*	**10.** *d*
11. *a*	**12.** *a*	**13.** *d*	**14.** *c*	**15.** *d*	**16.** *d*	**17.** *d*			

PROBLEMS

(A) SIMPLE PROBLEMS

1. A 6-pole induction motor is fed from a 50-Hz supply. If the frequency of the rotor emf at full load is 2 Hz, find the full-load slip and speed.
[**Ans.** 0.04, 960 rpm]

2. A 3-phase, 6-pole, 50-Hz induction motor has a slip of 1 % at no load and 3 % at full load. Find (*a*) the synchronous speed, (*b*) the no-load speed, (*c*) the full-load speed, (*d*) the frequency of rotor current at standstill, and (*e*) the frequency of rotor current at full load.
[**Ans.** (*a*) 1000 rpm; (*b*) 990 rpm; (*c*) 970 rpm; (*d*) 50 Hz; (*e*) 1.5 Hz]

3. A 10-pole induction motor is supplied by a 6-pole alternator which is driven by a prime mover at 1200 rpm. If the motor runs at slip of 3 %, what is its speed? [**Ans.** 698.4 rpm]

4. A 4-pole induction motor is energized from a 50-Hz supply system. If the machine runs on full-load at 3 % slip, determine the running speed and the frequency of the rotor currents.

[**Ans.** 1455 rpm, 1.5 Hz]

5. If the electromotive force in the stator of an 8-pole induction motor has a frequency of 50 Hz, and that in the rotor is 1.5 Hz, at what speed is the motor running? What is the slip?

[**Ans.** 727.5 rpm, 0.03]

6. A 3-phase, 440-V, 50-Hz, 4-pole induction motor the standstill rotor emf per phase of 115 V. If the motor is running at 1440 rpm, calculate for this speed (*a*) the slip, (*b*) the frequency of the rotor induced emf, and (*c*) the value of the rotor induced emf per phase.

[**Ans.** (*a*) 4 %; (*b*) 2 Hz; (*c*) 4.6 V]

7. A 3-phase, 440-V, 50-Hz, 4-pole, phase-wound induction motor at standstill has the induced emf of 157 V between the slip-ring terminals. The rotor windings are star-connected and have a resistance of 0.6 Ω per phase. Calculate the rotor current when the slip-ring terminals are short-circuited and the rotor is rotating at a speed of 1455 rpm.

[**Ans.** 4.53 A]

8. A 6-pole, 3-phase, 50-Hz motor with star-connected rotor has the rotor resistance of 0.3 Ω per phase and the rotor standstill reactance of 1.5 Ω per phase. When the motor is at standstill, the emf between the slip-rings on open circuit is 175 V. If the motor runs at a speed of 950 rpm, find (*a*) the slip, (*b*) the rotor emf per phase, and (*c*) the rotor frequency and reactance.

[**Ans.** (*a*) 0.05; (*b*) 5.05 V; (*c*) 2.5 Hz, 0.075 Ω per phase]

9. A 500-V, 3-phase, 6-pole, 50-Hz induction motor, working at a power factor of 0.87, develops 20 hp inclusive of windage and friction losses when running at 975 rpm. Calculate (*a*) the slip, (*b*) the rotor copper loss, (*c*) the total input if the stator losses are 1500 W, (*d*) the current drawn by the motor from the supply, and (*e*) the frequency of rotor current.

[**Ans.** (*a*) 2.5 %; (*b*) 382.6 W; (*c*) 16.8 kW; (*d*) 22.3 A; (*e*) 1.25 Hz]

(B) TRICKY PROBLEMS

10. A 3-phase, 12-pole alternator is driven by an engine running at 500 rpm. The alternator supplies an induction motor which has a full-load speed of 1455 rpm. Find the slip and the number of poles of the motor. [**Ans.** 3 %, 4 poles]

11. A 3-phase, 400-V, 50-Hz, 30-hp, 2-pole induction motor operates at an efficiency of 85 % with a power factor of 0.75 lag. Calculate the current drawn by the motor from the power mains.

[**Ans.** 50.67 A]

12. The power input to a 3-phase induction motor is 60 kW. The stator losses total to 1 kW. Find the total mechanical power developed and the rotor copper losses per phase, if the motor is running with a slip of 3 %. [**Ans.** 57.23 kW, 1.77 kW]

13. A 3-phase, 50-Hz, 4-pole, induction motor has a rotor resistance of 0.024 ohm per phase and stand-still reactance of 0.6 ohm per phase. Determine the speed at which maximum torque is developed.

[**Ans.** 1440 rpm]

14. A 3-phase, 4-pole induction motor runs at a speed of 1440 rpm on 500-V, 50-Hz mains. The mechanical power available at the shaft is 20.3 hp. The mechanical losses are 2.23 hp. The stator copper loss and iron loss add up to 1 kW. Calculate (*a*) the slip, (*b*) the rotor copper losses, and (*c*) the efficiency.

[**Ans.** (*a*) 4 %; (*b*) 701 W; (*c*) 81.8 %]

15. The power input to a three-phase induction motor is 50 kW and the corresponding stator losses are 2 kW. Calculate (*a*) the total mechanical power developed and the rotor I^2R loss when the slip is 3 %, (*b*) the output horse power of the motor, if the friction and windage losses are 1.0 kW, and (*c*) the efficiency of the motor.

[**Ans.** (*a*) 46.56 kW, 1.44 kW; (*b*) 61 hp; (*c*) 91.1 %]

16. The rotor of a 6-pole, 50-Hz, slip-ring induction motor has a resistance of 0.2 ohm per phase. It is running at 960 rpm. Calculate the resistance to be added to the rotor circuit to reduce the speed to

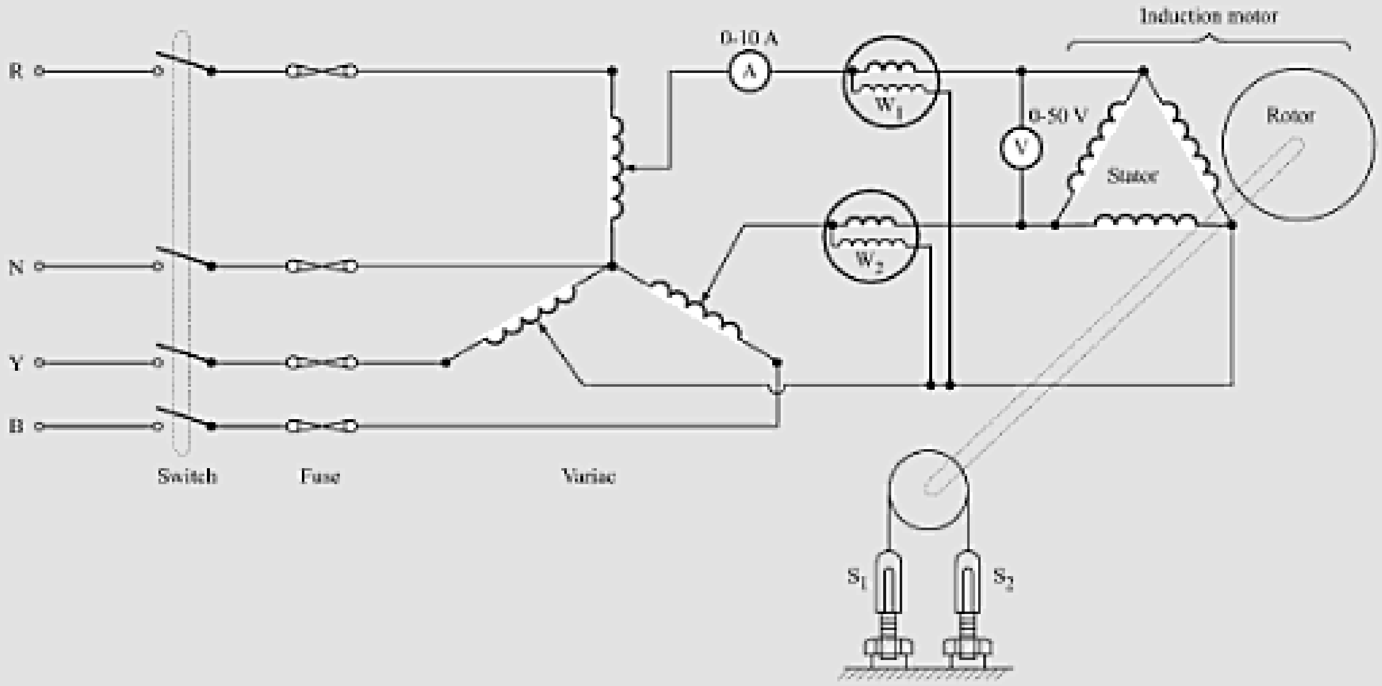


Fig. 15.15 *Circuit arrangement to conduct load test on a 3-phase, squirrel-cage induction motor.*

Normal rated voltage at normal frequency is supplied to the motor and a variable load is applied to its shaft.

Load Torque The mechanical load of the shaft can be varied by using brake and pulley arrangement. A pulley or a drum (called *brake drum*) is attached to the shaft. A belt of suitable length rides over this drum. Two spring balances S_1 and S_2 are attached at each end of the belt. The other end of the spring balance is fixed to a nut and bolt. If the nut is tightened, the belt presses harder on to the rotating drum thereby increasing the mechanical load. If S_1 and S_2 are the readings of the two spring balances, the net restraining force applied to the drum is

$$F = (S_1 - S_2)\text{ kgf} = (S_1 - S_2)g = (S_1 - S_2) \times 9.8\text{ N}$$

If D is the effective diameter of the brake drum, the torque applied to the shaft is given as

$$\tau = F \times \frac{D}{2} = (S_1 - S_2) \times 9.8 \times \frac{D}{2}\text{ Nm} \qquad (i)$$

Slip The speed at which the stator magnetic field rotates is called *synchronous speed* and is given by

$$N_s = \frac{120 f}{P}$$

Here, the frequency f of the 3-phase ac supply is 50 Hz, and the number of poles P can be seen on the name plate of the induction motor. The actual speed N of the rotor is always less than N_s. The difference between the two is called *slip*, which can be defined in percentage as

$$s = \frac{N_s - N}{N_s} \times 100\ \% \qquad (ii)$$

The slip at full-load normally is 2 to 5 %.

Output Power The output power P_o of the motor is given as

$$P_o = \frac{2\pi N \tau}{60}\text{ watts} \qquad (iii)$$

Input Power The input power P_{in} to the motor can be determined from the readings of the two wattmeters connected in the input circuit (see Fig. 15.15). If P_1 and P_2 are the readings of the two wattmeters, the input power is given as

$$P_{in} = P_1 + P_2 \qquad (iv)$$

Note that at low power factors, one of the wattmeters may give negative deflection. If it happens, the connections of one of the coils (either *current coil* or *pressure coil*) should be reversed and the readings should be treated negative.

Input Power Factor Using the readings P_1 and P_2 of the two wattmeters, the input power factor angle can be calculated from following:

$$\phi = \tan^{-1} \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}$$

The power factor is then given as

$$pf = \cos \phi \qquad (v)$$

Efficiency The efficiency of the induction motor is given as

$$\eta = \frac{P_o}{P_{in}} \times 100\ \% \qquad (vi)$$

Procedure

1. Make the connections as shown in Fig. 15.15.
2. Keep the three-phase variac at its minimum position.
3. Release the brake and pulley system so that the shaft is not loaded.
4. Switch ON the three-phase supply.
5. Gradually increase the position of the variac towards higher voltage. Note the direction of deflection in the wattmeters. If the deflection is found negative in a wattmeter, bring the variac back to its minimum and reverse the connections of either the current coil or pressure coil of that wattmeter. The reading of this wattmeter should be recorded negative.
6. Slowly increase variac position to apply rated voltage to the induction motor.

7. Note the readings of all the meters. Measure the speed of the rotor by attaching the tachometer to the shaft. These are no-load readings.
8. Increase the mechanical load by tightening the nut of the springs and note the readings. Repeat this for 6–8 settings of the load.
9. At some stage of the load, one wattmeter may again start reading negative. If so, again repeat step 5.
10. Switch OFF the three-phase supply.

Observations

S. No.	*Line voltage, V_L* (*in* V)	*Line current, I_L* (*in* A)	*Speed, N* (*in* rpm)	*Input*		*Output*	
				P_1 (*in* W)	P_2 (*in* W)	S_1 (*in* kgf)	S_2 (*in* kgf)
1							
2							
3							
4							
5							
6							
7							
8							

Calculations

S. No.	*Slip, s* (*in* %) *Eq.* (*ii*)	*Input power, P_{in}* (*in* W) *Eq.* (*iv*)	*Torque, τ* (*in* Nm) *Eq.* (*i*)	*Output power, P_o* (*in* W) *Eq.* (*iii*)	*Power factor* *Eq.* (*v*)	*Efficiency, η* *Eq.* (*vi*)
1						
2						
3						
4						
5						
6						
7						
8						

Results

For eight different observations, different characteristic curves can be plotted.

Precautions

1. Before switching ON the supply, the zeroes of the ammeter, voltmeter, and wattmeters should be checked.
2. The readings in the ammeter should not exceed the current ratings of the wattmeter.
3. Special care should be taken about the sign of the readings of wattmeters.
4. During the experiment, the mechanical power developed by the motor is dissipated as frictional loss (heat) at the brake and pulley arrangement. Cooling of the brake pulley should be done properly; otherwise, the belt may wear away in a short time.

Viva-Voce

1. What do you mean by slip?
Ans.: The difference between the synchronous speed and the actual speed of the rotor, expressed as percentage, is known as *slip*.

2. Usually, what is the order of slip for an induction motor, while running at no load and running at full load?
   Ans.: At no load, it is about 1-2 %, and at full-load it is about 2-5 %.
3. At what voltage should the load test on an induction motor be performed?
   Ans.: At its rated voltage.
4. Suppose that an induction motor is running on no load. What do you think its power factor will be?
   Ans.: At no load, the motor draws very small active power. Therefore, its power factor is very low, of the order of 0.1 to 0.3 lagging.
5. And when it is running at full load, what would be the power factor?
   Ans.: At full load, the power factor is about 0.85 lagging.
6. Under what conditions, one of the wattmeters may give negative reading?
   Ans.: If the reading of one wattmeter, say P_2, is negative, the value of ratio $(P_1 - P_2)/(P_1 + P_2)$ will always be greater than unity. Thus,

$$\tan\phi = \sqrt{3} \times (\text{greater than unity}) \quad \Rightarrow \quad \tan\phi > \sqrt{3} \quad \Rightarrow \quad \phi > 60^\circ$$

or $$\cos\phi < 0.5$$

   Hence, if one of the wattmeters reads negative, the power factor has to be less than 0.5.
7. Approximately, how much is the efficiency of an induction motor while running at full load?
   Ans.: It is about 0.75 to 0.85.
8. What do you mean by '*pull-out torque*' of an induction motor?
   Ans.: The maximum torque the motor can develop is also called pull-out torque.
9. Why should it be called pull-out torque?
   Ans.: If the load on an induction motor is increased beyond the maximum torque it can develop, the motor slows down, the slip increases and the operation becomes unstable. Ultimately, the machine stops. It *pulls out* of the motion.
10. What is meant by *plugging* of an induction motor?
    Ans.: In case of a running motor, if suddenly two of the three-phase supply connections to the stator are interchanged, the stator magnetic field reverses its direction of rotation. As a result, the machine comes to a stop quickly. This amounts to *braking* or *plugging* of the machine.
11. What do you mean by single-phasing of a 3-phase induction motor?
    Ans.: If one of the lines of the three-phase supply gets open due to some fault, the 3-phase motor continues running as a single-phase motor. This phenomenon is called single-phasing. When this happens, the current drawn by the two remaining lines increases and the motor gets overheated, and it may even get damaged.
12. Do you know any other machine whose output-speed characteristic is similar to that of a three-phase induction motor?
    Ans.: Yes, dc shunt motor.
13. In what way, an induction motor is similar to a transformer?
    Ans.: (*i*) Both work on the principle of electromagnetic induction.
    (*ii*) The stator of the induction motor is analogous to the primary of the transformer and the rotor to the secondary of the transformer.
    (*iii*) Both can be represented by similar equivalent circuits.
    (*iv*) Both machines can be tested through similar tests. The no-load test on an induction motor is similar to the open-circuit test on a transformer.
14. In what way, an induction motor differs from a transformer?
    Ans.: (*i*) The windings of a transformer are cylindrical, whereas those of an induction motor are distributed.
    (*ii*) The induction motor has an air gap in its magnetic path and hence the magnetising current of an induction motor is much higher than that of a transformer.
    (*iii*) Due to mechanical losses (frictional and windage losses), the efficiency of an induction motor is lower than that of a transformer.
    (*iv*) An induction motor converts electrical energy into mechanical energy, whereas a transformer transforms electrical energy from one voltage level to another.

DC MACHINES

16

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the importance of dc machines.*
- *Describe the basic construction of a dc machine.*
- *Describe the principles of working of a dc machine.*
- *Explain the process of commutation in dc machines.*
- *State the difference between lap winding and wave winding and state the number of parallel paths in these two types of windings.*
- *Derive the emf equation for a dc machine.*
- *State the difference between the dc generator and dc motor with respect to the induced emf and the terminal voltage.*
- *State different ways of establishing magnetic field in dc machines.*
- *State the two effects of armature reaction on magnetic flux.*
- *State different types of losses occurring in a dc machine and the factors on which these losses depend.*
- *State three types of efficiency of a dc generator.*
- *Derive the condition for maximum efficiency.*
- *State and explain the open-circuit characteristic (OCC) and load characteristic of a dc generator.*
- *Explain how the voltage builds up in a self-excited generator.*
- *State the meaning of critical resistance and critical speed in relation to voltage build-up in a dc generator.*
- *Draw the load characteristic of a compound dc generator, and explain how it depends on the relative ampere-turns of shunt and series fields.*
- *Draw the equivalent circuit of a dc motor.*
- *Derive the expression for the torque developed in a dc motor.*
- *Draw the torque and speed characteristics of shunt, series, and compound motors.*
- *State the need of a starter in a dc motor, and explain the working of the three-point starter.*

16.1 IMPORTANCE

The dc machines were the first electrical machines invented. An elementary dc motor drove an electric locomotive in Edinburgh in 1839, although it took another forty years before dc motors were commercially used. It is still the best motor to drive trains and cranes.

The dc machine can be used either as a motor or a generator. However, because semiconductor rectifiers can easily convert ac into dc, dc generators are not needed except for remote operations. Even in the automobiles, the dc generator has been replaced by the *alternator* plus diodes for rectification. Nevertheless, generator operations must be discussed because motors operate as generators in braking and reversing.

Portable devices powered by batteries require dc motors, such as portable tape players, walkman, window-lifters, etc. Also, the dc motor is readily controlled in speed and torque and hence is useful for control systems. Examples are robots, elevators, machine tools, rolling mills, etc.

16.2 CONSTRUCTION OF A DC MACHINE

Figure 16.1 shows the basic structure of a dc machine (a motor or a generator). The machine has following important parts.

Stator Magnetic Structure

Figure 16.1 shows the magnetic structure of a four-pole dc machine. Its main components are described below.

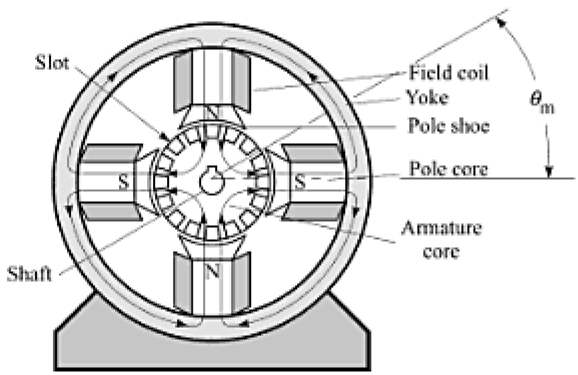


Fig. 16.1 *Main parts of a dc machine.*

(*i*) *Yoke* It is the outermost cylindrical part which serves two purposes. First, it acts as a supporting frame for the machine, and second, it provides a path for the magnetic flux. It is made of cast iron, cast steel, or forged steel. Usually, small machines have cast-iron yokes.

(*ii*) *Poles* The machine has salient poles. The *pole cores* are fixed inside the yoke, usually by bolts. The cross-section of the pole core is rectangular. By attaching a *pole shoe,* the end of the pole is made to have a cylindrical surface. The cross-sectional area of the pole shoe is considerably larger than that of the pole core to leave as little inter-pole space as possible. This is done to reduce the leakage flux. The poles are made of cast steel, or forged steel. Each pole carries a *field coil* (or exciting coil). Small machines generally use permanent magnets.

(*iii*) *Field Coils* The field coils are wound on the pole cores and are supported by the pole shoes. All coils are identical and are connected in series such that on excitation by a dc source, alternate N and S poles are made. Thus, a machine always has even number of poles. The magnetic flux distribution approximates a square wave, as shown in Fig. 16.2 for the four-pole structure shown in Fig. 16.1. The flux is taken positive in the radially inward direction. Note that the yoke carries one-half of the pole flux Φ. Therefore, the cross-section of the yoke should be selected accordingly.

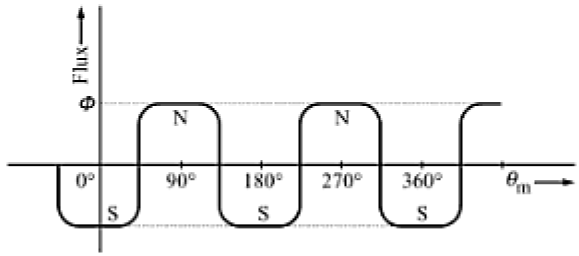


Fig. 16.2 *Magnetic flux distribution for four-pole dc machine.*

Rotor

The rotor is the inner cylindrical part having armature and commutator-brush arrangement. It is mounted on the shaft of the motor.

(i) Armature The armature core consists of steel laminations, each about 0.4-0.6 mm thick, insulated from one another. The purpose of laminating the core is to reduce the eddy-current loss. Slots are stamped on the periphery of the laminations to accommodate the armature winding. The top of the slot has a groove in which a wedge can be fixed. After the winding *conductors* are put into the slots, the wedge is inserted. The wedge prevents the conductors from flying out due to the centrifugal force when the armature rotates. The axial length of the armature is the same as that of the poles on the yoke. The term *conductor* refers to the active portion of the winding, namely that part which cuts the flux when the rotor rotates, thereby generating an alternating emf.

(ii) Commutator It consists of a large number of wedge-shaped copper segments or bars, assembled side by side to form a ring. The segments are insulated from one another by thin mica sheets. Each segment is connected to a coil-end of the armature winding, as shown schematically in Fig. 16.3. The radial lines represent the active lengths of the rotor conductors. The commutator is a part of the rotor and participates in its rotation.

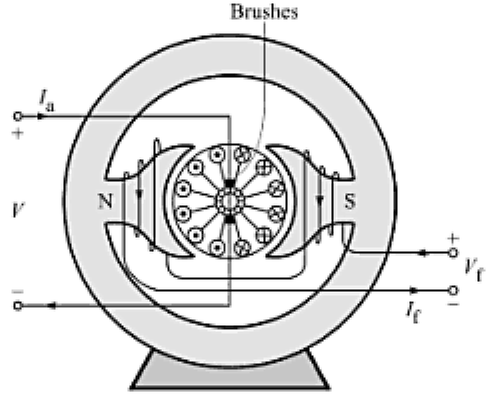


Fig. 16.3 *A two-pole dc motor with a brush-commutator system.*

***(iii)* Brushes** Two stationary brushes, made of carbon, are pressed against the commutator with the help of a spring fitted in a brush-gear. The brush-commutator system provides two related functions: (*i*) electrical connection is made with the moving rotor, and (*ii*) a steady or direct voltage is obtained from the alternating emf generated in the rotating conductors.

Process of Commutation

The width of a brush is made a little more than the width of a commutator segment and the mica insulation. Whenever a brush spans two commutator segments, it short-circuits the two coils connected to these segments. On the two sides of the *magnetic neutral axis* (MNA), the conductors of the armature winding carry currents in opposite directions. The brushes are aligned along the MNA, so that they make contact with conductors which are moving midway between the poles and therefore have no emf induced in them. Thus, the reversal of current directions in the two short-circuited coils can take place with least sparking.

Commutation means the process of current collection by a brush, or the changes that take place in the coils during the period of short-circuit by a brush. The reversal of current in a coil during the commutation period sets up a self-induced emf in the coil undergoing commutation. This emf, called *reactance voltage*, opposes the reversal of current.

16.3 ARMATURE CURRENT AND FLUX

In the two-pole dc motor shown in Fig. 16.3, I_f is the *field current* (or *exciting current*) supplied to the field winding from the source V_f. Current I_a is the current supplied to the armature from the dc mains of voltage V. Because of the brush-commutator system, the currents in the conductors on the right side are going into the paper and the currents in the conductors on the left side are coming out of the paper.

The currents in the armature conductors produce their own flux. According to right-hand thumb rule, the flux produced will be upwards. This is equivalent to making the bottom of the rotor a south pole and the top a north pole. These poles are attracted to their opposites on the stator. Thus, a clockwise torque is produced on the rotor.

16.4 ARMATURE WINDING

When the armature rotates, a small emf is induced in each conductor. A larger emf can be obtained if a number of conductors are connected in series such that their emfs add up as we travel along the circuit. Thus, a conductor under N-pole has to be connected to a conductor under S-pole. To make all the coils identical, it is most convenient and practical to connect conductors housed in slots one pole-pitch* apart.

Double-Layer Armature Winding

Normally, the armature winding is arranged in double layer, as shown in Fig. 16.4*a* for a four-pole armature with 11 slots. First, a coil is wound in the correct shape and then it is assembled on the core. For making all the coils similar in shape, it is necessary that if side 1 of a coil occupies the outer half of a slot under N_1 pole, the other side 1′ occupies the inner half of another slot in similar position under S_1 pole. This brings in a kink in the end connections so that the coils may overlap one another as they are assembled. Figure 16.4*b* shows three coils 1-1′, 2-2′ and 3-3′, arranged in the slots so that their end connections overlap one another.

* Pole-pitch is the number of conductors per pole.

EXAMPLE 16.2

A 4-pole, 1200-rpm dc generator has a lap-wound armature having 65 slots and 12 conductors per slot. If the flux per pole is 0.02 Wb, determine the emf induced in the armature.

Solution The total number of conductors, $Z = 65 \times 12 = 780$

For lap winding, the number of parallel paths, $A = P = 4$

Therefore, using Eq. 16.2, the total emf induced is given as

$$E = \frac{\Phi ZNP}{60\,A} = \frac{0.02 \times 780 \times 1200 \times 4}{60 \times 4} = \mathbf{312\ V}$$

EXAMPLE 16.3

The induced emf in a dc machine while running at 500 rpm is 180 V. Assuming constant magnetic flux per pole, calculate the induced emf when the machine runs at 600 rpm.

Solution The induced emf is given by Eq. 16.2 as

$$E = \frac{\Phi ZNP}{60\,A} = KN$$

where K is a constant for the machine. Therefore, we have

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{or} \quad E_2 = \frac{N_2}{N_1} E_1 = \frac{600}{500} \times 180 = \mathbf{216\ V}$$

EXAMPLE 16.4

The induced emf in a dc generator running at 750 rpm is 220 V. Calculate (*a*) the speed at which the induced emf is 250 V (assume the flux to be constant), and (*b*) the required percentage increase in the field flux so that the induced emf is 250 V, while the speed is only 600 rpm.

Solution

(*a*) From Eq. 16.2, if the flux is constant, we have

$$E = KN$$

where K is a constant for the machine. Therefore, we have

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{or} \quad N_2 = \frac{E_2}{E_1} N_1 = \frac{250}{220} \times 750 = \mathbf{852\ rpm}$$

(*b*) Here, neither the speed nor the flux remains constant. Therefore, from Eq. 16.2, we can write

$$E = K'\Phi N$$

where K' is a constant. Thus, we have

$$\frac{E_2}{E_1} = \frac{\Phi_2 N_2}{\Phi_1 N_1} \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{E_2}{E_1} \times \frac{N_1}{N_2} = \frac{250}{220} \times \frac{750}{600} = 1.42$$

Thus, the required percentage increase in flux is

$$(1.42 - 1.00) \times 100 = \mathbf{42\ \%}$$

16.6 TYPES OF DC MACHINES

There are several ways of exciting the stator field winding of a dc machine. Each method of field connections gives different characteristics.

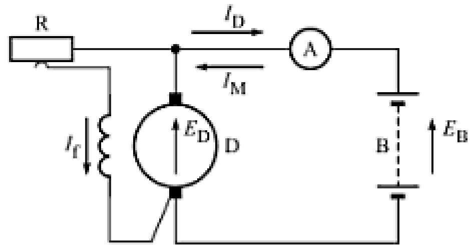


Fig. 16.7 *Shunt-wound machine as generator or motor.*

emf is available to circulate a current I_D through the resistance R_a of the armature circuit, and the battery. Since the current I_D is in the same direction as the emf E_D, the machine D is working as a ***generator*** of electrical energy. Note that the battery B is getting charged and hence working as a load on the generator.

Next, suppose that we cut off the supply of oil to the engine driving machine D. The speed of the machine falls, the emf E_D decreases, current I_D gets reduced, until when $E_D = E_B$, there is no circulating current I_D. But E_D continues to decrease and becomes less than E_B. Therefore, the current I_M through the ammeter A flows in the reverse direction. The battery B is now supplying electrical energy to drive machine D as an electric ***motor***.

Note that the direction of field current I_f is the same whether the machine D works as a generator or as a motor. The relationship between the emf, the current and the terminal voltage can now be expressed when the machine D works as a generator or as a motor. Let E be the emf generated in the armature of the machine*, V the terminal voltage, R_a the resistance of the armature circuit, and I_a the armature current.

As Generator The current I_a flows in the same direction as the generated emf E, and the terminal voltage V is less than the emf E due to the armature-circuit voltage-drop. Thus, we have

$$V = E - I_a R_a \tag{16.3}$$

As Motor The current I_a flows in the opposite direction to that of the generated emf E, and the terminal voltage V is more than the emf E due to the armature-circuit voltage-drop. Thus, we have

$$V = E + I_a R_a \tag{16.4}$$

Types of DC Generators

The type of dc machine depends on the way the magnetic flux is established in it. Though equally applicable to motors, let us describe different types of dc generators. There can be three ways of establishing magnetic flux in a dc generator:

(1) Using a permanent magnet,
(called ***permanent magnet generators***).

(2) Using some external source to excite the field coils, and
(called ***separately excited generators***).

(3) Using the armature supply to excite the field coils.
(called ***self-excited generators***).

* Note that the emf in the armature is generated both in the generator as well as in the motor. In the generator, it is due to this emf that the current flows in the external electric circuit. In a motor, this emf opposes the applied voltage V, and hence it is called ***back emf***.

In describing various relations for a dc generator, following notations for different quantities are used:

I_a = armature current
R_a = net resistance of the armature circuit
E = emf generated in the armature winding
I_{se} = current through series field coil
R_{se} = resistance of the series field coil
I_{sh} = current through shunt field coil
R_{sh} = resistance of the shunt field coil
I_L = current supplied to the load
V = terminal voltage across the load
R_L = load resistance

(1) *Permanent Magnet Generators* These do not find many applications in the industry, because of their low efficiency. However, low-power, low-cost, small size machines do use permanent magnet.

(2) *Separately Excited Generators* As shown in Fig. 16.8, the field coils are excited from a storage battery or from a separate dc source.

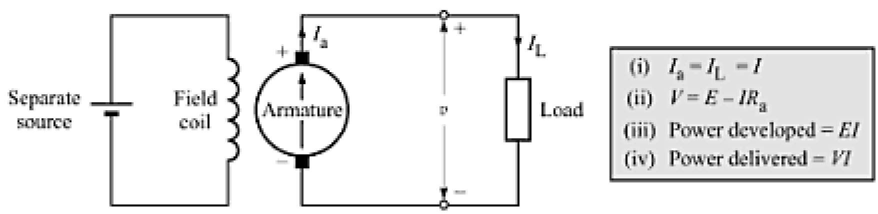


Fig. 16.8 *Separately excited dc generator.*

(3) *Self-Excited Generators* The field coils are excited by the dc voltage generated by the generator itself. Such generators are further subdivided into following three categories:

(*a*) *Series-Wound Generators* The field coils are connected in series with the armature circuit (Fig. 16.9*a*).

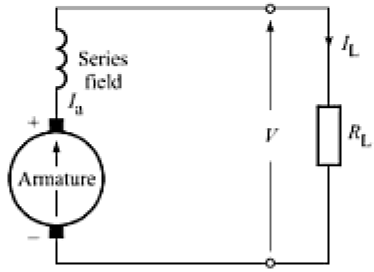
(*b*) *Shunt-Wound Generators* The field coils are connected across the armature circuit (Fig. 16.9*b*).

(*c*) *Compound-Wound Generators* There are two windings on each pole, one connected in series and the other in parallel with the armature circuit. The compound-wound generators may again be of two types:

(*i*) ***Short-Shunt*** in which the shunt field winding is connected in parallel with the armature (Fig. 16.9*c*).

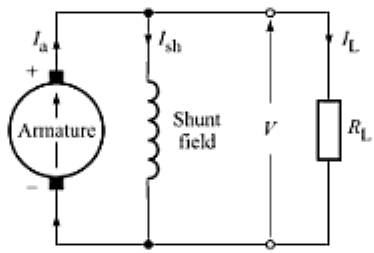
(*ii*) ***Long-Shunt*** in which the shunt field winding is connected in parallel with both the armature and series winding (Fig. 16.9*d*).

The compound-wound generators can also be classified, from another point of view, in two classes, viz., *differential compound generators* and *cumulative compound generators* depending on the fact whether the series field opposes or supports the shunt field, respectively.



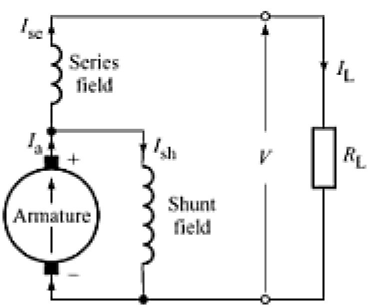
(ii) $I_a = I_{se} = I_L = I$ (say)
(iii) $V = E - I(R_a + R_{se})$
(iv) Power delivered = EI
(v) Power delivered = VI

(*a*) Series wound.



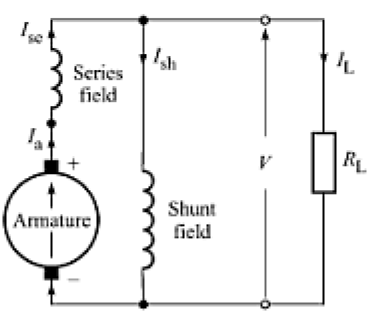
(i) $I_{sh} = V/R_{sh}$
(ii) $I_a = I_{sh} + I_L$
(iii) $V = E - I_a R_a$
(iv) Power delivered = EI_a
(v) Power delivered = VI_L

(*b*) Shunt wound.



(i) $I_{se} = I_L$
(ii) $I_{sh} = (V + I_{se} R_{se})/R_{sh}$
(iii) $I_a = I_{sh} + I_L$
(iv) $V = E - I_a R_a - I_{se} R_{se}$
(v) Power delivered = EI_a
(vi) Power delivered = VI_L

(*c*) Short-shunt compound wound.



(i) $I_a = I_{se} = I_{sh} + I_L$
(ii) $I_{sh} = V/R_{sh}$
(iii) $V = E - I_a R_a - I_a R_{se}$
$= E - I_a(R_a + R_{se})$
(iv) Power delivered = EI_a
(v) Power delivered = VI_L

(*d*) Long-shunt compound wound.

Fig. 16.9 *Self-excited dc generators.*

EXAMPLE 16.5

A shunt-wound dc generator delivers 496 A at 440 V to a load. The resistance of the shunt filed coil is 110 Ω and that of the armature winding is 0.02 Ω. Calculate the emf induced in the armature.

Solution The current through the shunt-field coil is given as

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{110} = 4 \text{ A}$$

∴ Armature current, $I_a = I_L + I_{sh} = 496 + 4 = 500$ A

Therefore, the generated emf is

$$E_g = V + I_a R_a = 440 + (500 \times 0.02) = \mathbf{450\ V}$$

EXAMPLE 16.6

A 4-pole shunt generator with lap connected armature has armature and field resistances of 0.2 Ω and 50 Ω, respectively. It supplies power to 100 lamps, each of 60 W, 200 V. Calculate the total armature current, the current per path and the generated emf. Allow a brush drop of 1 V at each brush.

Solution The current taken by each lamp, $I_1 = \frac{P}{V} = \frac{60}{200} = 0.3$ A

Since all the lamps are connected in parallel, the total load current is

$$I_L = 100 \times I_1 = 100 \times 0.3 = 30 \text{ A}$$

Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{200}{50} = 4$ A

∴ Armature current, $I_a = I_{sh} + I_L = 30 + 4 = \mathbf{34\ A}$

For lap winding, the number of parallel paths, $A = P = 4$. Thus,

The current per path, $I_c = \dfrac{I_a}{A} = \dfrac{34}{4} = \mathbf{8.5\ A}$

The generated emf, $E_g = V + I_a R_a + \text{brush-drop} = 200 + 34 \times 0.2 + 2 \times 1 = \mathbf{208.8\ V}$

EXAMPLE 16.7

A short-shunt compound-wound dc generator supplies a load current of 100 A at 250 V. The generator has following winding resistances:

Shunt field = 130 Ω, armature = 0.1 Ω, and series field = 0.1 Ω

Find the emf generated, if the brush drop is 1 V per brush.

Solution Refer to Fig. 16.9*c*. The series-field current, $I_{se} = I_L = 100$ A

The voltage drop across the series field, $V_{se} = I_{se} R_{se} = 100 \times 0.1 = 10$ V

The voltage drop across the shunt field, $V_{sh} = V + V_{se} = 250 + 10 = 260$ V

The shunt-field current, $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{260}{130} = 2$ A

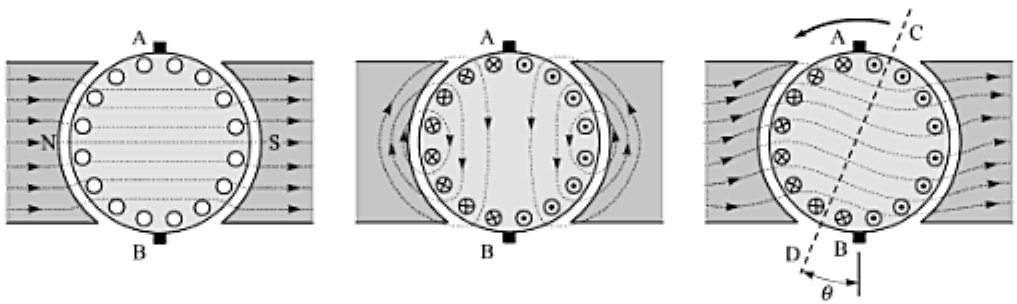
∴ The armature current, $I_a = I_L + I_{sh} = 100 + 2 = 102$ A

The generated emf, $E_g = V + V_{se} + I_a R_a + \text{brush-drop}$

$$= 250 + 10 + 102 \times 0.1 + 2 \times 1 = \mathbf{272.2\ V}$$

16.7 ARMATURE REACTION

The effect of armature ampere-turns upon the value and distribution of the magnetic flux entering and leaving the armature core is called *armature reaction.* Let us, for simplicity, consider a two-pole dc machine, as shown in Fig. 16.10*a*. The brushes A and B are placed in the Geometric Neutral Plane (GNP). For the sake of clarity, we have omitted the slots on the armature and shown the conductors uniformly distributed. The figure shows the flux distribution due to the field current alone (i.e., when there is no armature current). Note that the flux in the air gap is practically radial and uniformly distributed.



(*a*) Flux due to field current alone.

(*b*) Flux due to armature current alone.

(*c*) Resultant flux due to field and armature currents.

Fig. 16.10 *Flux distribution in a dc machine.*

Now, suppose that the dc machine is to work as a motor rotating in counterclockwise direction. To produce a counterclockwise torque, the current is made to flow through the armature conductors in direction shown in Fig. 16.10*b*. The figure also shows the flux distribution due to this current alone (assumed no flux due to the field winding). Note that at the centre of the armature and in the pole shoes, the direction of this flux is at right angles to that due to the field winding. For this reason, the flux due to the armature current is called ***cross flux***.

The pole tip which is first met by a point on the armature during its revolution is known as the *leading tip* and the other as *trailing tip.*

Figure 16.10*c* shows the resultant distribution of the flux due to the combination of the fluxes in Figs. 16.10*a* and *b*. We find that over the trailing halves of the pole faces the cross flux is in opposite direction to the main flux, thereby reducing the flux density. Now, over the leading halves of the pole faces the cross flux is in the same direction as the main flux, thereby strengthening the flux density. However, if the teeth are strongly saturated under no load, the strengthening of the flux at the leading pole tips would not be as much as the weakening of the flux at the trailing pole tips. Therefore, the total flux would be somewhat reduced. Hence, the ***demagnetisation effect*** is one of the consequences of the armature reaction.

Another important consequence of the armature reaction is to ***distort the flux distribution***. As shown in Fig. 16.10*c*, the Magnetic Neutral Plane (MNP) is shifted through an angle θ from AB to CD, in a direction opposite to rotation*.

Thus, the armature reaction has two components, namely, the *demagnetising component* and the *distorting component.* With the increase in the armature current (or load), both these components increase. At times,

* If the machine works as a generator, the magnetic neutral plane shifts by angle θ in the direction of rotation.

when the machine is working as a generator and if the 'short-circuit' or 'excessive-overload' condition occurs, the demagnetizing component may even reverse the polarity of the main poles.

REMEDY

The adverse effect of armature reaction can be neutralized by shifting the brushes to the magnetic neutral plane and by increasing the air gap at pole tips.

16.8 LOSSES IN A DC MACHINE

Various losses occurring in a dc machine are as follows.

(1) Copper losses

Copper loss occurs in armature winding, in field winding and brush contacts.

(*i*) *Armature Copper Loss* It is given as $I_a^2 R_a$. This loss amounts to about 30 to 40% of the full-load losses.

(*ii*) Field Copper Loss It is given as $I_{sh}^2 R_{sh}$ for shunt-wound machine and as $I_{se}^2 R_{se}$ for series wound machine. This loss amounts to about 20 to 30% of the full-load losses. For shunt-wound machine, it remains practically constant; but for a series-wound machine, it increases with the load.

(*iii*) Brush Contact Loss This loss occurs due to the resistance of the brush contact with the commutator. This is usually included in armature copper loss.

(2) Magnetic (or Iron) losses

Since the current in the armature winding is alternating at a frequency f, the flux produced is also alternating. Some of this flux also enters the pole cores. The magnetic loss, therefore, mainly occurs in the armature core. This loss amounts to about 20 to 30% of the full-load losses. There can be two types of magnetic (or iron) losses :

(*i*) *Hysteresis Loss* $= B_{max}^{1.6} f$

(*ii*) *Eddy-current Loss* $= B_{max}^{2} f^2$

(3) Mechanical Losses

There are two types of mechanical losses.

(*i*) *Air Friction (or Windage) Loss* It occurs due to rotation of the armature.

(*ii*) *Bearing Friction Loss* It occurs at the ball-bearing fixed on the rotor.

Mechanical losses are about 10 to 20% of the full-load losses. Mechanical losses taken together are also called ***stray losses.***

16.9 EFFICIENCY OF A DC GENERATOR

Following types of efficiencies can be defined for a dc generator.

(1) ***Mechanical Efficiency***, $\eta_m = \dfrac{\text{Total watts generated in armature}}{\text{Mechanical power supplied at the input}}$

$$= \frac{EI}{\text{hp} \times 746} \qquad (16.5)$$

(2) ***Electrical Efficiency***, $\eta_e = \dfrac{\text{Total watts available to the load}}{\text{Total watts generated}}$

$$= \frac{VI}{EI} \qquad (16.6)$$

(3) ***Commercial or Overall Efficiency***, $\eta_c = \dfrac{\text{Total watts available to the load}}{\text{Mechanical power supplied}}$

$$= \frac{VI}{\text{hp} \times 746} \qquad (16.7)$$

It is obvious that $\eta_c = \eta_m \times \eta_e$.

Condition for Maximum Efficiency

Due to the losses occurring in the generator, its efficiency is not cent percent. The total losses P_{tl} can be divided in to two categories: (*i*) constant losses, P_c, and (*ii*) variable losses, P_v. The copper loss in armature winding (i.e., $I_a^2 R_a$) is the only loss that varies with the load current. Other losses remain almost constant. Now,

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Output + Total losses}} = \frac{VI}{VI + (P_v + P_c)} = \frac{VI}{VI + I_a^2 R_a + P_c}$$

$$= \frac{VI}{VI + I^2 R_a + P_c} \qquad (\text{Since, } I_a \approx I)$$

$$= \frac{1}{1 + \left(\dfrac{IR_a}{V} + \dfrac{P_c}{VI}\right)}$$

For efficiency to be maximum, the denominator of the above expression should be minimum, for which we must have

$$\frac{d}{dI}\left\{1 + \left(\frac{IR_a}{V} + \frac{P_c}{VI}\right)\right\} = 0 \quad \text{or} \quad \frac{R_a}{V} - \frac{P_c}{VI^2} = 0$$

or
$$I^2 R_a = P_c \qquad (16.8)$$

This shows that maximum efficiency is obtained when the variable loss equals constant loss. Thus, the load current corresponding to the maximum efficiency is given by

$$I^2 = P_c/R_a \quad \text{or} \quad I = \sqrt{P_c/R_a} \qquad (16.9)$$

EXAMPLE 16.8

A shunt generator gives full-load output of 30 kW at a terminal voltage of 200 V. The armature and shunt-field resistances are 0.05 Ω and 50 Ω, respectively. The iron and friction losses are 1000 W. Calculate (*i*) the emf generated, (*ii*) the copper losses, and (*iii*) the efficiency.

Solution

(*i*) $I_L = \dfrac{30\text{ kW}}{200\text{ V}} = 150\text{ A}$; $I_{sh} = \dfrac{200\text{ V}}{50\ \Omega} = 4\text{ A}$; $I_a = I_L + I_{sh} = 150 + 4 = 154\text{ A}$

$$\text{The emf generated, } E_g = V + I_a R_a = 200 + 154 \times 0.05 = \mathbf{207.7\ V}$$

(*ii*) The copper losses $= I_{sh}^2 R_{sh} + I_a^2 R_a = 4^2 \times 50 + 154^2 \times 0.05 = \mathbf{1985.8\ W}$

(*iii*) The efficiency, $\eta = \dfrac{\text{Output}}{\text{Output + Losses}} = \dfrac{30\,000}{30\,000 + (1000 + 1985.8)} = 0.9095\text{ pu} = \mathbf{90.95\,\%}$

EXAMPLE 16.9

A dc shunt generator, with shunt-field resistance of 52.5 Ω, supplies full-load current of 195 A at 210 V. Its full-load efficiency is 90 % and it has stray losses of 710 W. Determine its armature resistance and the load current corresponding to maximum efficiency.

Solution The output power of the generator, $P_o = V_o \times I_L = 210 \times 195 = 40.95\text{ kW}$

$$\therefore \quad \text{The input power, } P_{in} = \frac{P_o}{\eta} = \frac{40.95\text{ kW}}{0.90} = 45.5\text{ kW}$$

$$\therefore \quad \text{Total losses} = P_{in} - P_o = 45.5 - 40.95 = 4.55\text{ kW}$$

$$\text{Shunt-field current, } I_{sh} = \frac{V}{R_{sh}} = \frac{210\text{ V}}{52.5\,\Omega} = 4\text{ A}$$

$$\therefore \quad \text{Armature current, } I_a = I_L + I_{sh} = 195 + 4 = 199\text{ A}$$

$$\text{Shunt-field copper loss} = I_{sh}^2 R_{sh} = 4^2 \times 52.5 = 840\text{ W}$$

$$\therefore \quad \text{Constant losses} = 840 + 710 = 1550\text{ W}$$

$$\text{Thus, the armature copper loss, } I_a^2 R_a = 4550 - 1550 = 3000\text{ W}$$

$$\text{Hence, the armature resistance, } R_a = \frac{\text{Armature copper loss}}{I_a^2} = \frac{3000}{199^2} = \mathbf{0.0757\ \Omega}$$

For maximum efficiency, we must have

$$\text{Variable losses} = \text{Constant losses}$$

or

$$I_a^2 R_a = P_c$$

$$\therefore \quad I_a = \sqrt{\frac{P_c}{R_a}} = \sqrt{\frac{1550}{0.0757}} = \mathbf{143.1\ A}$$

$$\therefore \quad \text{Load current, } I_L = I_a - I_{sh} = 143.1 - 4 = \mathbf{139.1\ A}$$

16.10 CHARACTERISTICS OF DC GENERATORS

There are following three important characteristics of a dc generator.

1. ***Open-Circuit, Magnetisation, or No-Load Characteristic*** It provides the relationship between the no-load emf E generated in the armature and the field (or exciting) current I_f.

2. ***Load (or External) Characteristic*** It shows the relationship between the terminal voltage V and the load current I_L. It is also called the ***performance characteristic*** or ***voltage regulation curve***.

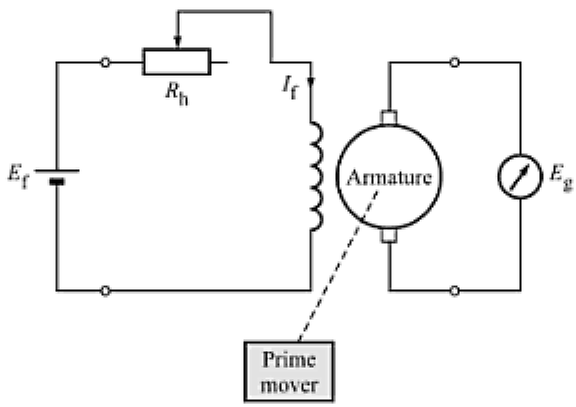
3. ***Internal Characteristic*** It gives the relationship between the emf E generated in the armature (after considering the demagnetising effect of armature reaction) and the armature current I_a.

The first two characteristics, which we shall be discussing, are more important in order to know the performance of the generator.

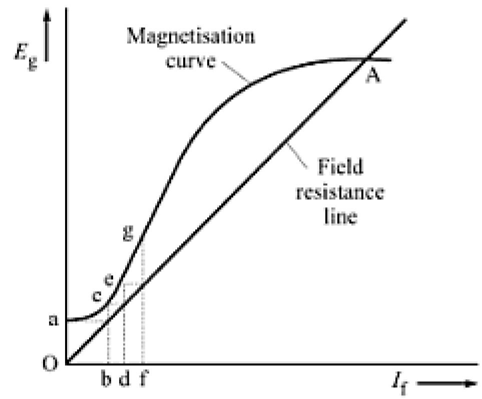
Open-Circuit Characteristic (OCC)

To understand how the self-excitation process takes place, we must know the magnetisation curve of the machine. This curve is sometimes called the *saturation curve*. Strictly speaking, the magnetisation curve represents a plot of magnetic flux (in the air gap) versus field winding mmf. However, if the speed N is fixed, the magnetisation curve represents a plot of the open-circuit induced emf E_g (in the armature) as a function of field-winding current I_f. This is why this curve is called *open-circuit characteristic* (*OCC*) of the machine.

For plotting the OCC of a self-excited generator, the generator is separately excited by a battery of emf E_f. The generator is driven by a motor or any other prime-mover at a fixed speed and its armature terminals are left open. A voltmeter (of high resistance) is used for measuring the induced emf E_g, as shown in Fig. 16.11*a*.



(*a*) The circuit arrangement.



(*b*) The characteristic curve.

Fig. 16.11 *Open-circuit characteristic (OCC) of a dc generator.*

Figure 16.11*b* shows a typical magnetization curve or OCC of a generator, for a constant speed of rotation of the armature. Note that the emf E_g is not necessarily zero for $I_f = 0$. It happens because the machine has been previously used and some *residual magnetism* is left. If that was not the case, the magnetisation curve would start from the origin. As the exciting current I_f is increased (by decreasing the rheostat R_h in the field circuit), the flux per pole increases and consequently the induced emf E_g increases.

The magnetic path in a dc generator consists of partly the air gap and partly iron (the pole shoes and the armature core). For low flux density, the iron has high permeability and therefore offers negligible reluctance. Hence, the total reluctance of the magnetic path is almost that of the air gap. Consequently, the flux (and hence the induced emf E_g) varies *linearly* with the exciting current I_f. The OCC curve is a straight line. However, for high flux densities, the permeability of iron reduces due to magnetic saturation and hence its reluctance is no longer negligible. A stage is reached when the flux does not proportionately increase with increase in the current I_f. The curve starts levelling off.

The Field Resistance Line In Fig. 16.11*b*, the straight line OA represents the *field resistance line*. It is a plot of the current caused by the voltage E_f applied to the field circuit. Since, we are drawing the OCC of a self-excited generator, when the generator is actually put to use, the voltage E_f would be the same as the armature voltage E_g. The slope of this line is E_g/I_f is a constant and is equal to the total resistance R_F of the field circuit. Note that the resistance R_F represents the sum of the field winding resistance R_f and the active portion of the rheostat resistance R_h.

Building Up of Voltage Let us now examine how voltage is built up in the self-excited generator. Assume that the generator has been used previously and hence has some residual magnetism left at its poles. If the machine is running at constant speed, a small emf Oa is induced in the armature due to the residual magnetism, even if the field current I_f is zero in the beginning. The small emf Oa causes a feeble current Ob in the field winding, as given by the field resistance line OA. This field current produces more flux and a larger emf bc is induced. This increased emf causes an even larger field current Od. This produces more emf de, which in turn causes more field current Of, and so on. This process of voltage build up continues until the induced emf is just enough to produce a field current to sustain it. This corresponds to the point A, the point of intersection of the OCC curve and the field resistance line.

Note that for the voltage to build up, following three conditions must be satisfied:

(*i*) There must be residual magnetism.
(*ii*) The field winding mmf must act to aid this residual flux.
(*iii*) The field resistance line must intersect the OCC curve at some point.

Critical Field Resistance Let us consider the third point given above, in some detail. Corresponding to field resistance line OA, the emf induced is E_1. For a larger value of the field resistance, the slope of the line increases (line OB in Fig. 16.12*a*). This line cuts the OCC curve at a lower voltage E_2. Hence, the larger the field resistance, the smaller is the emf generated. Now suppose that the field resistance is increased to a value corresponding to line OC, which just touches the initial straight part of the OCC curve. When the generator is run, the final emf induced will be low, as the voltage build-up process cannot start. Thus, we conclude that voltage build-up takes place only if the field resistance is *less than* that given by line OC. This resistance is called the *critical field resistance*.

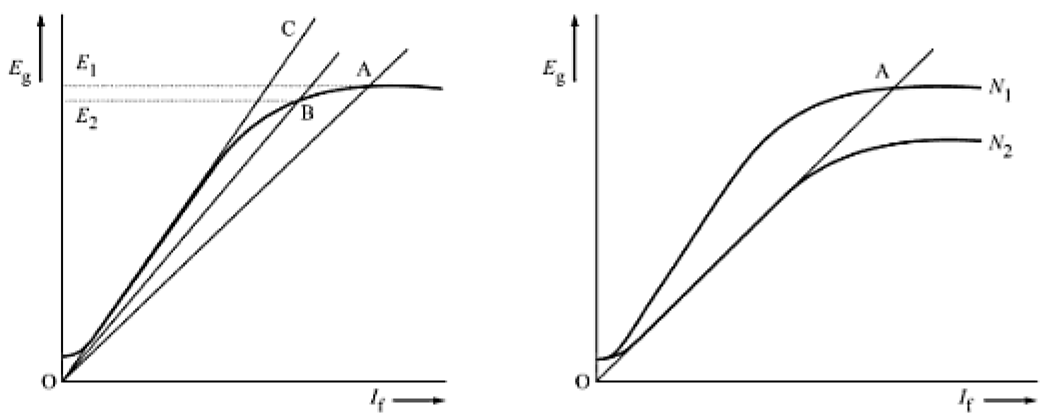


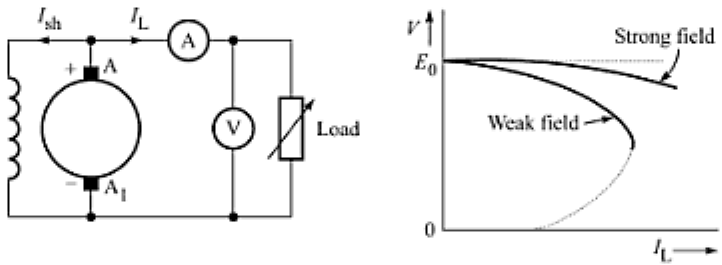
Fig. 16.12 *The OCC curves for a dc generator.*

Critical Speed We know that the emf induced in a dc generator is directly proportional to the speed N. Therefore, a generator has different OCC curves for different speeds. Figure 16.12*b* shows two OCC curves—

indicates the ***unstable*** region of operation of the generator. There are two reasons why the voltage falls on increasing the load:

(*i*) Due to the armature resistance voltage drop, and
(*ii*) Due to the demagnetising effect of the armature reaction.

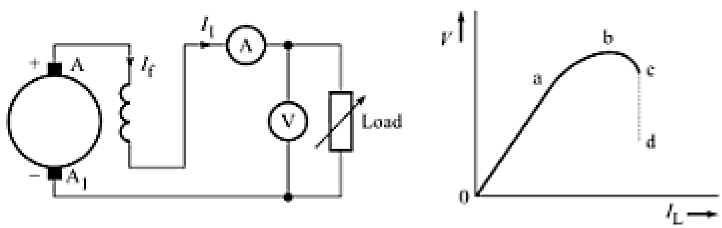
As shown in Fig. 16.14*b*, if the shunt generator is designed with a strong field, the load characteristic curve becomes comparatively flat.



(*a*) The circuit arrangement. (*b*) The load characteristic.

Fig. 16.14 *Shunt dc generator.*

(3) *Series Generator* The circuit connection is shown in Fig. 16.15*a*, and the load characteristic is shown in Fig. 16.15*b*. Here, the field current I_f is the same as the load current I_L. Therefore, at no load ($I_L = 0$), the field current and hence the flux is zero. As a result, the emf E induced in the armature too is zero. Up to a point a, *the terminal voltage V increases proportionately to the load current* I_L. This property makes a series generator suitable to work as a *booster*, which boosts up the supply voltage. From point a to point b, the increase in terminal voltage with load current is much less due to the magnetic saturation. Beyond point b, the terminal voltage starts falling due to the demagnetising effect of the armature reaction. After point c, the voltage *falls steeply* as the armature reaction becomes prominent. In this region, the series generator may be used as a *constant current but variable voltage source.*



(*a*) The circuit arrangement. (*b*) The load characteristic.

Fig. 16.15 *Series dc generator.*

(4) *Compound-Wound Generators* *Ideally*, we would like to have load characteristic of a generator as shown by the horizontal straight line A, in Fig. 16.16. This is possible neither from a shunt generator (Fig. 16.14*b*) nor from a series generator (Fig. 16.15*b*). However, a compound generator, either short-shunt or long-shunt as shown in Figs. 16.9*c* and *d*, respectively, can be designed to achieve a characteristic very near to ideal. It utilises opposing effects of both (*i*) the falling characteristic of a shunt generator, and (*ii*) the rising characteristic of a series generator.

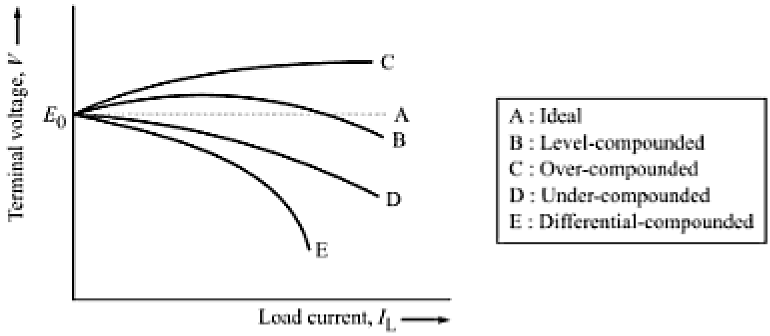


Fig. 16.16 *Load characteristics of compound dc generators.*

Case I We can have a combination of shunt and series excitations in such a way that the resultant terminal voltage varies very little over a range of load current (curve B in Fig. 16.16). The generator is then said to be ***flat*** or ***level-compounded***. The terminal voltage V remains almost constant between the no-load and full-load.

Case II In case the series field supports the shunt field (i.e., if the generator is *cumulative compounded*), and the series ampere turns are more than the shunt ampere turns, the terminal voltage V can be made to rise with load current (curve C in Fig. 16.16). Such a generator, known as ***over-compounded***, can be used for supplying power over long distances. Whenever the load increases, the terminal voltage falls due to large voltage drops in transmission lines. The terminal voltage at load end can easily be re-adjusted by the over compounded generator.

Case III In case the series ampere turns are less than the shunt ampere turns, the terminal voltage V falls as the load increases (curve D in Fig. 16.16). Such generators are said to be ***under-compounded.*** Under-compounding is useful where a short might occur, e.g., in an arc welding machine.

Case IV If the series ampere turns oppose the shunt ampere turns, the generator is said to be ***reverse*** or ***differential-compounded***. For such generators, the terminal voltage V falls very rapidly as the load current increases (curve E in Fig. 16.16).

EXAMPLE 16.10

A dc shunt generator is to be converted into a level-compounded generator by adding a series field winding. From a test on the machine with shunt excitation only, it is found that the shunt current is 4 A to give 440 V on no load and 6 A to give the same voltage when the machine is supplying its full load of 100 A. The shunt winding has 1500 turns per pole. Find the number of series turns required per pole.

Solution Ampere turns per pole required on no load = 4 × 1500 = 6000 At

Ampere turns per pole required on full load = 6 × 1500 = 9000 At

Hence, ampere turns per pole to be provided by the series winding

= 9000 – 6000 = 3000 At

Since, the full-load current is 100 A, the number of turns per pole needed in the series winding is given as

$$N_{se} = \frac{3000}{100} = \mathbf{30}$$

16.11 DC MOTORS

In construction, a dc motor is no different from a dc generator. As in case of dc generators, there are three types of dc motors : (*i*) shunt, (*ii*) series, and (*iii*) compound. Unlike the series generators, the dc series motors find wide applications, especially for traction type of loads.

When the motor terminals are connected to dc mains supply, a current flows in the field winding as well as in the armature winding. In a shunt motor, the two currents have different values. But in a series motor, the two currents are the same.

Equivalent Circuit of a DC Motor

Like a dc generator, a dc motor too has induced emf E in the armature, given by the same equation (Eq. 16.4):

$$E = \frac{\Phi ZNP}{60\ A} \tag{16.10}$$

However, this induced emf opposes the supply voltage V and hence it is treated as *counter* or ***back emf***, and is usually designated as E_b. The equivalent circuit of a dc shunt motor is depicted in Fig. 16.17. Note that the terminal voltage V must be equal to the sum of induced emf E_b and voltage drop in the armature. Similarly, the line current I_L is equal to the sum of the armature current I_a and field current I_f. That is,

$$V = E_b + I_a R_a \tag{16.11}$$

and

$$I_L = I_a + I_f \tag{16.12}$$

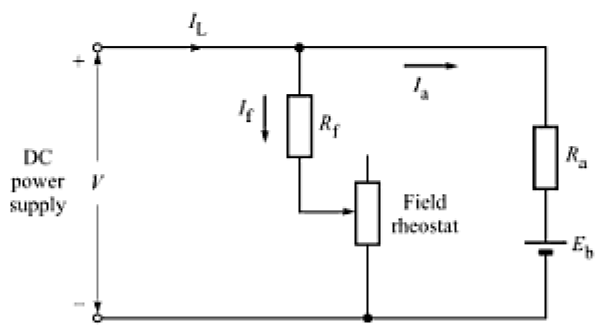


Fig. 16.17 *Equivalent circuit of a dc shunt motor.*

Speed Regulation of a DC Motor

For a given machine, A, Z and P are fixed, so that the expression for induced emf E_b (Eq. 16.10) can be written as

$$E_b = kN\Phi$$

where

$$k = \frac{ZP}{60\ A} \quad \text{(a constant)}$$

Substituting for E_b in Eq. 16.11, we get

$$V = kN\Phi + I_a R_a \quad \Rightarrow \quad N = \frac{V - I_a R_a}{k\Phi} \tag{16.13}$$

The value of the voltage drop $I_a R_a$ is usually less than 5% of the terminal voltage V, so that the above equation can be written as

$$N \approx \frac{V}{k\Phi} \quad \text{or} \quad N \propto \frac{V}{\Phi} \tag{16.14}$$

It means that the speed of a dc motor is approximately proportional to the applied voltage V and inversely proportional to the flux Φ. All methods of controlling the speed involve the use of either or both of these relationships.

When a motor is mechanically loaded, its speed decreases. If N_0 represents the no-load speed and N_f the full-load speed, the *percentage speed regulation* is defined as

$$\%\ \text{speed regulation} = \frac{N_0 - N_f}{N_f} \times 100\% \tag{16.15}$$

EXAMPLE 16.11

A 250-V dc shunt motor takes 41 A current while running at full load. The resistances of motor armature and of field windings are 0.1 Ω and 250 Ω, respectively. Determine the back emf generated in the motor.

Solution The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{250} = 1\text{ A}$

Therefore, the armature current, $I_a = I_L - I_{sh} = 41 - 1 = 40\text{ A}$

$\therefore$ Back emf, $E_b = V - I_a R_a = 250 - 40 \times 0.1 =$ **246 V**

EXAMPLE 16.12

A 4-pole, 440-V dc motor takes an armature current of 50 A. The resistance of the armature circuit is 0.28 Ω. The armature winding is wave-connected with 888 conductors and the useful flux per pole is 23 mWb. Calculate the speed of the motor.

Solution From Eq. 16.11, the generated emf is given as

$$E_b = V - I_a R_a = 440 - 50 \times 0.28 = 426\text{ V}$$

Using Eq. 16.10, we get the speed of the motor as

$$N = \frac{60AE_b}{\Phi ZP} = \frac{60 \times 2 \times 426}{0.023 \times 888 \times 4} = \mathbf{626\ rpm}$$

EXAMPLE 16.13

A dc motor runs at 900 rpm from a 460-V supply. Calculate the approximate speed when the machine is connected across a 200-V supply. Assume the new flux to be 0.7 times the original flux.

Solution If Φ is the original flux, then from Eq. 16.14 we have

$$900 = \frac{460}{k\Phi} \quad \text{or} \quad k\Phi = 0.511$$

When the supply voltage changes to 200 V, the new speed is given as

$$N' = \frac{V'}{k\Phi'} = \frac{V'}{k(0.7\Phi)} = \frac{V'}{0.7k\Phi} = \frac{200}{0.7 \times 0.511} \approx \mathbf{559\ rpm}$$

16.12 TORQUE DEVELOPED BY A DC MOTOR

If we multiply each term of Eq. 16.11 by I_a, namely, the total armature current, we get

$$VI_a = E_b I_a + I_a^2 R_a$$

Here, VI_a represents the total electric power supplied to the armature, and $I_a^2 R_a$ represents the loss due to the armature resistance. The difference between these two quantities, namely $E_b I_a$, represents the electrical power that is converted to mechanical power by the armature. If τ_d is the torque, in newton-metres, exerted on the armature to develop the mechanical power ($= E_b I_a$), and N is the speed of rotation in rpm, then we have

$$\text{Mechanical power developed, } P_m = \frac{2\pi\tau_d N}{60} \text{ watts}$$

Hence, we have

$$\frac{2\pi\tau_d N}{60} = E_b I_a$$

$$= \frac{\Phi ZNP}{60\,A} \times I_a \qquad \text{(replacing } E_b \text{ by the expression of Eq. 16.10)}$$

Thus, the torque developed by the armature is given as

$$\tau_d = \frac{\Phi Z}{2\pi}\left(\frac{P}{A}\right) I_a \tag{16.16}$$

Since, for a given machine, Z, P and A are fixed, we can write

$$\tau_d \propto I_a \times \Phi \tag{16.17}$$

It means that *the torque developed in a given dc motor is proportional to the product of the armature current and the flux per pole.*

Note that all of the mechanical power developed, namely $E_b I_a$, by the armature is not available externally. Some of it is absorbed as friction loss at the bearing and at the brushes and some is wasted as hysteresis loss and in circulating eddy currents in the core. The useful torque available at the shaft, namely τ_{sh}, is less than the torque developed τ_d, because of these losses.

EXAMPLE 16.14

A 6-pole, dc motor takes an armature current of 110 A at 480 V. The resistance of the armature circuit is 0.2 Ω, and flux per pole is 50 mWb. The armature has 864 lap-connected conductors. Calculate (*a*) the speed, and (*b*) the gross torque developed by the armature.

Solution

(*a*) The generated emf, $E_b = V - I_a R_a = 480 - 110 \times 0.2 = 458$ V

Using Eq. 16.10, we have

$$E_b = \frac{\Phi ZNP}{60\,A} \quad \text{or} \quad N = \frac{60\,AE_b}{\Phi ZP} = \frac{60 \times 6 \times 458}{0.05 \times 864 \times 6} = \mathbf{636\ rpm}$$

(*b*) Torque developed by the armature,

$$\tau_d = \frac{\Phi Z}{2\pi}\left(\frac{P}{A}\right) I_a = \frac{0.05 \times 864}{2\pi} \times \left(\frac{6}{6}\right) \times 110 = \mathbf{756\ Nm}$$

EXAMPLE 16.15

A dc generator runs at 900 rpm when a torque of 2 kNm is applied by a prime mover. If the core, friction and windage losses in the machine are 8 kW, calculate the power generated in the armature winding.

Solution The power required to drive the generator,

$$P_{in} = \frac{2\pi\tau N}{60} = \frac{2\pi \times 2000 \times 900}{60} = 188\,495 \text{ W} = 188.5 \text{ kW}$$

Therefore, the power generated in the armature,

$$P_d = P_{in} - P_{losses} = 188.5 - 8 = \mathbf{180.5\ kW}$$

16.13 TORQUE AND SPEED CHARACTERISTICS OF A DC MOTOR

When no load is connected to the shaft of a dc motor, it develops only that much torque which overcomes the rotational (frictional) losses and the iron losses. How does the motor react to the application of a shaft load? To answer this question, we require to know the performance characteristics of the motor.

Speed Characteristics of DC Motors

The speed characteristic of a motor represents the variation of speed with the input current. Its shape can be easily derived from the expression of Eq. 16.13, namely

$$N = \frac{V - I_a R_a}{k\Phi}$$

(1) *Shunt Motor* The field winding of a shunt motor consists of many turns of thin wire and is connected in parallel with the armature. The flux Φ, therefore, remains constant. Since the drop $I_a R_a$ at full load rarely exceeds 5 % of V, the speed N is almost constant. Its speed characteristic may be represented by curve A in Fig. 16.18*a*. Thus, *a dc shunt motor is a* ***constant speed*** *motor.* In actual practice, the drop in speed with current is even less than that shown in figure. This is because as the armature current increases, the armature reaction tends to slightly reduce the main flux Φ. This reduction in flux causes an increase in speed that partially compensates the drop due to $I_a R_a$.

(2) *Series Motor* In a series motor, the field winding is made of a few turns of thick wire and is connected in series with the armature. If R_{se} represents the resistance of this winding, the back emf is given as

$$E_b = V - I_a (R_a + R_{se})$$

Therefore, Eq. 16.13 modifies to

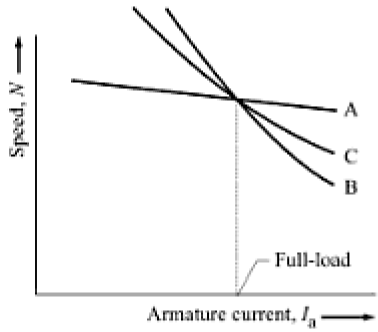
$$N = \frac{V - I_a (R_a + R_{se})}{k\Phi}$$

The flux Φ increases first in direct proportion to the armature current I_a and then less rapidly due to the magnetic saturation. Hence, the speed is roughly inversely proportional to the current, as indicated by the curve B in Fig. 16.18*a*. Thus, *a dc series machine is a* ***variable speed*** *motor.*

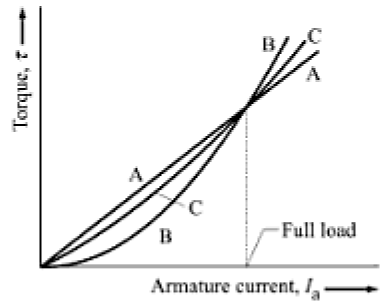
Note that if a dc motor is started with no load, the current (and hence the flux) is very low, and the speed may become dangerously high. It may fly to pieces due to such high speed. For the same reason, a series motor should never be used when there is a risk of the load becoming very low. For instance, the load should never be belt-connected, as it has the risk of breaking or slipping. The load to a dc series motor is either directly connected or geared to the shaft.

(3) *Compound Motor* A compound motor has both a shunt winding and a series winding. The flux due to shunt field remains fixed, but that due to the series field increases with the current. Therefore, the total flux increases with the current, but not as rapidly as in a series motor. Hence, the speed characteristic (curve C in Fig. 16.18*a*) is in between those of the shunt and series motors. Depending upon the ratio of the shunt and series ampere-turns, any desired characteristic can be obtained.

In many cases, enough shunt field is provided to guarantee a safe no-load speed. Such motors are called *stabilized series motors*. Large shunt motors operating at high speeds face large fluctuations in speed due to the line-voltage fluctuations. This problem can be reduced by adding a few series-field turns to the machine.



(*a*) Speed characteristic.



(*b*) Torque characteristic.

Fig. 16.18 *Performance characteristics of dc motors.*

Torque Characteristics of DC Motors

The torque characteristic of a motor represents the variation of the developed torque τ_d with the input current. Its shape can be easily derived from the expression of Eq. 16.17, namely

$$\tau_d \propto I_a \times \Phi \quad \text{or} \quad \tau_d = k_t I_a \times \Phi$$

where, k_t is a constant for a machine.

(1) *Shunt Motor* Since the flux Φ in a shunt motor is practically independent of the armature current, $\tau_d \propto I_a$, and hence the torque characteristic is represented by the straight line A in Fig. 16.18*b*.

(2) *Series Motor* In a series motor, the flux Φ is approximately proportional to the armature current up to full load, so that $\tau_d \propto I_a^2$. Above full-load, magnetic saturation becomes more prominent and the torque does not increase so rapidly. The torque characteristic is represented by curve B in Fig. 16.18*b*.

(3) *Compound Motor* The torque characteristic of a compound motor is in between those of the shunt and series motors, and is represented by curve C in Fig. 16.18*b*. The exact shape of curve C depends upon the relative value of the shunt and series ampere-turns at full-load.

From Fig. 16.18*b*, it is evident that for a given current below the full-load value, the shunt motor exerts the largest torque. But for a current above the full-load value, the series motor exerts the largest torque.

The maximum permissible current at starting is usually about 1.5 times the full-load value. Therefore, where a large starting torque is required such as for hoists, cranes, electric trains, etc., the series motor is the most suitable choice.

(*d*) When $E_b = -250$ V (the negative sign is taken because the machine is connected to the supply with reversed polarities).

$$I_a = \frac{V - E_b}{R_a} = \frac{250 - (-250)}{0.2} = \mathbf{2500\ A}$$

EXAMPLE 16.18

A 6-pole, lap-connected dc series motor, with 864 conductors, takes a current of 110 A at 480 V. The armature resistance and the series-field resistance are 0.18 Ω and 0.02 Ω, respectively. The flux per pole is 50 mWb. Calculate (*a*) the speed, and (*b*) the gross torque developed by the armature.

Solution

(*a*) The generated emf is given as

$$E_b = V - I_a(R_a + R_{se}) = 480 - 110 \times (0.18 + 0.02) = 458 \text{ V}$$

$$\therefore \qquad N = \frac{60\,AE_b}{\Phi ZP} = \frac{60 \times 6 \times 458}{0.05 \times 864 \times 6} = \mathbf{636\ rpm}$$

(*b*) Since, $\frac{2\pi\tau_d N}{60} = E_b I_a$, we have

$$\tau_d = \frac{60\,E_b I_a}{2\pi N} = \frac{60 \times 458 \times 110}{2 \times \pi \times 636} = \mathbf{756\ Nm}$$

EXAMPLE 16.19

A 220-V, shunt motor, running at 700 rpm, has an armature resistance of 0.45 Ω and takes an armature current of 22 A. What resistance should be placed in series with the armature to reduce the speed to 450 rpm?

Solution For speed, $N_1 = 700$ rpm:

$$E_1 = V - I_a R_a = 220 - 22 \times 0.45 = 210.1 \text{ V}$$

In a shunt motor, the flux remains constant. Hence,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{or} \quad \frac{E_2}{210.1} = \frac{450}{700} \quad \Rightarrow \quad E_2 = 135.06 \text{ V}$$

If R is the additional resistance placed in series with the armature, we have

$$E_2 = V - I_a(R_a + R)$$

or

$$135.06 = 220 - 22 \times (0.45 + R)$$

On solving the above, we get

$$R = \mathbf{3.411\ \Omega}$$

EXAMPLE 16.20

A 230-V, dc series motor has an armature circuit resistance of 0.2 Ω and series field resistance of 0.1 Ω. At rated voltage, the motor draws a line current of 40 A and runs at a speed of 1000 rpm. Find the speed of the motor for a line current of 20 A at 230 V. Assume that the flux at 20 A line current is 60% of the flux at 40 A line current.

Solution The back emf when the line current is 40 A is given as

$$E_1 = V - I_a(R_a + R_{se}) = 230 - 40 \times (0.2 + 0.1) = 218 \text{ V}$$

The back emf when the line current is 20 A is given as

$$E_2 = V - I_a(R_a + R_{se}) = 230 - 20 \times (0.2 + 0.1) = 224 \text{ V}$$

The flux, $\Phi_2 = 0.6\Phi_1$. Here, in the two cases, only the flux and speed change. Hence,

$$E = k\Phi N$$

$$\therefore \qquad \frac{E_2}{E_1} = \frac{N_2\,\Phi_2}{N_1\,\Phi_1} = \frac{N_2 \times 0.6\Phi_1}{N_1\,\Phi_1} = \frac{0.6N_2}{N_1}$$

$$N_2 = \frac{E_2\,N_1}{E_1 \times 0.6} = \frac{224 \times 1000}{218 \times 0.6} \approx \mathbf{1713\ rpm}$$

16.14 STARTING OF DC MOTORS

When a dc motor is at rest, there is no back emf generated. Hence, if the motor is directly connected to the supply mains, a heavy current flows through the armature. This may result in a damage to the armature. It therefore becomes necessary to include a high resistance in series with the armature at the start. Once the motor picks up speed, the back emf is generated and then the series resistance can be gradually cut out. Ultimately, when the motor attains its normal (rated) speed, the entire resistance may be disconnected from the circuit. The device that provides this facility is called a ***starter***.

Only small dc motors (say, up to 2-3 watts) can be started by directly connecting to the supply mains, because of the following reasons:

(*i*) The resistance and inductance of the armature winding are generally quite high to limit the rush of the current.

(*ii*) Because of the low inertia of the rotor, the motor picks up speed quickly. Hence, the high current does not last long to cause any damage.

For heavy duty motors, we have to use either a *three-point starter* or a *four-point starter*. Here, we discuss only the three point starter

Three-Point Starter

It consists of a series starting resistance divided into several sections, each connected to a brass stud. As shown in Fig. 16.19, the starter is also provided with *no-volt release* and *overload release* facilities.

Initially, the starter arm is at OFF position towards the left. After switching ON the dc supply, the starter arm is moved towards right. When connected to stud 1, the field circuit is directly connected to the supply through the brass arc. At the same time, the entire resistance is inserted in the armature circuit. Some current flows in the armature, developing a torque. The motor starts running, and an emf is generated. As the motor picks up speed, the starter arm is slowly moved towards right across studs 2, 3 …, etc. cutting out parts of the series resistance. Finally, when the arm is brought to ON position the motor attains the rated speed.

No-Volt Release When brought to ON position, the starter arm is held by the no-volt release magnet against the pull of the spiral spring. The magnet is energised by the field current. Whenever the pull of this magnet weakens or becomes zero, the arm is released to go back to OFF position, automatically.

If this arrangement were not provided, the starter arm would remain in ON position even when the supply goes off. When the supply is restored, the armature gets directly connected to the supply. This may cause heavy damage.

Solution Total number of conductors, $Z = 260 \times 2 = 520$

For lap-wound, number of parallel paths, $A = P = 4$

Therefore, the emf generated,

$$E_g = \frac{\Phi ZNP}{60A} = \frac{0.08 \times 520 \times 1000 \times 4}{60 \times 4} = 693.34 \text{ V}$$

The total resistance of the winding, $R_w = 260 \times 0.006 = 1.56\ \Omega$

There are four parallel paths, and the resistance of one path, $R_1 = \frac{R_w}{4} = \frac{1.56}{4} = 0.39\ \Omega$

Thus, the net armature resistance, $R_a = \frac{R_1}{4} = \frac{0.39}{4} = 0.0975\ \Omega$.

Hence, the terminal voltage, $V = E_g - I_a R_a = 693.34 - 55 \times 0.0975 =$ **687.98 V**

EXAMPLE 16.22

An eight-pole, dc shunt generator has 778 wave-connected conductors on its armature. While running at 500 rpm, it supplies power to a load of 12.5 Ω at 250 V. The armature and the shunt-field resistances are 0.24 Ω and 250 Ω, respectively. Determine the armature current, the emf induced, and the flux per pole.

Solution The load current, $I_L = \frac{V}{R_L} = \frac{250}{12.5} = 20 \text{ A}$

The shunt-field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}$

$\therefore$ Armature current, $I_a = 20 + 1 =$ **21 A**

The emf induced, $E_g = V + I_a R_a = 250 + 21 \times 0.24 =$ **255.04 V**

Since, $$E_g = \frac{\Phi ZNP}{60 \text{ A}}$$

Therefore, $$\Phi = \frac{60AE_g}{ZNP} = \frac{60 \times 2 \times 255.04}{778 \times 500 \times 8} = 0.00983 \text{ Wb} = \mathbf{9.83\ mWb}$$

EXAMPLE 16.23

Estimate the percentage reduction in speed of a dynamo working with constant excitation on 500-V bus bars to decrease its load from 500 kW to 250 kW. The resistance between the terminals is 0.015 Ω. Neglect the armature reaction.

Solution For the *first* case, the armature current, $I_a = \frac{P_o}{V} = \frac{500 \times 1000}{500} = 1000 \text{ A}$

Therefore, the induced emf is given as

$$E_1 = V + I_a R_a = 500 + 1000 \times 0.015 = 515 \text{ V}$$

In the *second* case, the armature current, $I_a = \frac{P_o}{V} = \frac{250 \times 1000}{500} = 500 \text{ A}$

Therefore, the induced emf is given as

$$E_2 = V + I_a R_a = 500 + 500 \times 0.015 = 507.5 \text{ V}$$

Since the excitation remains constant in the two cases, we have

$$E = \frac{\Phi ZNP}{60 \text{ A}} \quad \text{or} \quad N = KE, \qquad \text{where } K \text{ is a constant}$$

Hence, the fractional reduction in speed is given as

$$\frac{N_1 - N_2}{N_1} = \frac{K(E_1 - E_2)}{KE_1} = \frac{515 - 507.5}{515} = 0.01456 \text{ pu} = \mathbf{1.456\,\%}$$

EXAMPLE 16.24

A dc series generator has external characteristic given by a straight line through zero to 50 V at 200 A. It is connected as a booster between a station bus-bar and a feeder of 0.3 Ω resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (*a*) 160 A, and (*b*) 50 A.

Solution The electrical energy is transmitted from station bus-bar to the feeder through a transmission line or an underground cable. A dc generator can work as a booster to compensate for the voltage drop in the transmission lines.

(*a*) *For* $I_L = 160$ A:

Voltage drop in the transmission lines, $V_t = I_L R_t = 160 \times 0.3 = 48$ V

From the external characteristic of the dc generator, the boost-in voltage supplied at 160 A is

$$V_b = (50/200) \times 160 = 40 \text{ V}.$$

Hence, the voltage difference between the far end of the feeder and bus-bar is

$$V_d = V_t - V_b = 48 - 40 = \mathbf{8\ V}$$

(*b*) *For* $I_L = 50$ A:

Voltage drop in the transmission lines, $V_t = I_L R_t = 50 \times 0.3 = 15$ V

The boost-in voltage supplied by the booster at 50 A is $V_b = (50/200) \times 50 = 12.5$ V. Hence, the voltage difference between the far end of the feeder and bus-bar is

$$V_d = V_t - V_b = 15 - 12.5 = \mathbf{2.5\ V}$$

EXAMPLE 16.25

A dc long-shunt compound generator delivers a load current of 50 A at 500 V, and has armature, series-field and shunt-field resistances of 0.05 Ω, 0.03 Ω and 250 Ω, respectively. Calculate the generated emf and the armature current. Allow 1.0 V per brush for contact drop.

Solution The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{500}{250} = 2$ A

∴ Armature current, $I_a = I_L + I_{sh} = 50 + 2 = \mathbf{52\ A}$

The generated emf is given as

$$E_g = V + I_a(R_a + R_{se}) + \text{brush drop} = 500 + 52\,(0.05 + 0.03) + 2 = \mathbf{506.16\ V}$$

EXAMPLE 16.26

An 8-pole, dc generator has 500 conductors on its armature, and is designed to have 0.02 Wb of magnetic flux per pole crossing the air gap with normal excitation.

(*a*) What voltage will be generated at a speed of 1800 rpm, if the armature is (*i*) wave-wound, (*ii*) lap-wound?
(*b*) If the allowable current is 5 A per path, what will be the kW generated by the machine in each case?

Solution

(*a*) (*i*) For wave-wound, $A = 2$.

EXAMPLE 16.30

A 500-V, dc shunt motor takes 4 A on no load and runs at 1000 rpm. The armature resistance (including that of the brushes) is 0.2 Ω, and the field current is 1 A. On loading, if the motor takes a current of 100 A, determine its speed and estimate the efficiency at which it is working.

Solution At no load, $I_{a1} = I_{L1} - I_{sh} = 4 - 1 = 3$ A

$$\text{Back emf, } E_1 = V - I_{a1}R_a = 500 - 3 \times 0.2 = 499.4 \text{ V}$$

$$\text{Under loaded condition, } I_{L2} = 100 \text{ A; and } I_{a2} = I_{L2} - I_{sh} = 100 - 1 = 99 \text{ A}$$

$$\therefore \quad \text{Back emf, } E_2 = V - I_{a2}R_a = 500 - 99 \times 0.2 = 480.2 \text{ V}$$

For a shunt motor, the flux remains constant and hence $E \propto N = kN$. Therefore,

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{480.2}{499.4} \times 1000 = \mathbf{962\ rpm}$$

At no load, the power taken by the motor mainly meets the constant losses (iron and frictional losses). Hence,

$$\text{Constant losses, } P_c = VI_{L1} = 500 \times 4 = 2000 \text{ W}$$

On loading, the copper loss in shunt field winding is negligible compared to the copper loss in armature winding. Thus,

$$\text{The variable losses, } P_v = I_{a2}^2 R_a = 99^2 \times 0.2 = 1960 \text{ W}$$

$$\text{The total input power, } P_{in} = VI_{L2} = 500 \times 100 = 50\,000 \text{ W}$$

$$\therefore \quad \text{Efficiency, } \eta = \frac{P_{in} - (P_v + P_c)}{P_{in}} = \frac{50\,000 - (2000 + 1960)}{50\,000} = 0.92 \text{ pu} = \mathbf{92\ \%}$$

EXAMPLE 16.31

A dc shunt generator running at 500 rpm delivers 50 kW at 250 V. It has an armature resistance of 0.02 Ω and a field-winding resistance of 50 Ω. Calculate the speed of the machine running as a shunt motor and taking a power of 50 kW at 250 V.

Solution The field current in both cases, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{50} = 5$ A

When working as a generator, the machine supplies a load of 50 kW at 250 V. Therefore, the load current,

$$I_L = \frac{50 \text{ kW}}{250 \text{ V}} = 200 \text{ A}; \qquad \text{and} \qquad I_{a1} = I_L + I_{sh} = 200 + 5 = 205 \text{ A}$$

Hence, the induced emf, $E_{a1} = V + I_{a1}R_a = 250 + 205 \times 0.02 = 254.1$ V

When working as a motor, the machine takes a power of 50 kW at 250 V. The line current I_L is still 200 A. Out of this current, 5 A goes to the shunt field winding. Therefore, the armature current,

$$I_{a2} = I_L - I_{sh} = 200 - 5 = 195 \text{ A}$$

Hence, the induced emf, $E_{a2} = V - I_{a2}R_a = 250 - 195 \times 0.02 = 246.1$ V

As the field current and hence the flux per pole is the same in the two cases, we should have

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \quad \text{or} \quad N_2 = \frac{E_2}{E_1} \times N_1 = \frac{246.1}{254.1} \times 500 = \mathbf{484\ rpm}$$

EXAMPLE 16.32

A series motor takes 20 A at 400 V and runs at 250 rpm. The armature and field resistances are 0.6 Ω and 0.4 Ω, respectively. Find the applied voltage and the current to run the motor at 350 rpm, if the torque required varies as the square of the speed.

- A dc shunt motor is almost a constant speed motor.
- A dc series motor is a variable speed motor.
- For loads requiring large starting torque, the series motor is the best choice. For example, hoists, cranes, electric trains, etc.

IMPORTANT FORMULAE

- The **emf** generated, $E = \dfrac{\Phi ZNP}{60\,A}$.
- For a ***generator***, $V = E_g - I_a R_a$.
- For a ***motor***, $V = E_g + I_a R_a$.
- ***Torque*** developed, $\tau_d = \dfrac{\Phi Z}{2\pi}\left(\dfrac{P}{A}\right) I_a$, or $\tau_d \propto I_a \Phi$.
- For a dc shunt motor, $\tau_d \propto I_a$.
- For dc series motor, $\tau_d \propto I_a^2$.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The yoke of a dc machine has two-fold purposes — first, it acts as a supporting frame, and secondly it provides a path for the magnetic flux.	☐	☐	
2.	In lap winding, the number of parallel paths is always two, irrespective of the number of poles the machine has.	☐	☐	
3.	The curve drawn between the induced emf and the armature current of a dc generator is known as its open-circuit characteristic.	☐	☐	
4.	In order that the voltage may build up in a dc shunt generator, the value of the field resistance should be more than the critical value.	☐	☐	
5.	In a dc shunt generator, for a given value of field-circuit resistance the lowest speed at which a generator can just build up the voltage, is called the critical speed.	☐	☐	
6.	The copper loss in a dc machine occurs only in the armature winding and not in the field winding.	☐	☐	
7.	In a series motor, the field flux does not remain constant but it depends on the mechanical load coupled to the motor.	☐	☐	
8.	The commutator segments of a dc machine are insulated from each other by a thin layer of mica.	☐	☐	
9.	For low values of armature current I_a, the torque produced by a dc series motor is proportional to I_a^2.	☐	☐	
10.	A torque is developed not only in a dc motor but also in a dc generator.	☐	☐	

Your Score

ANSWERS

1. True	**2.** False	**3.** False	**4.** False	**5.** True
6. False	**7.** True	**8.** True	**9.** True	**10.** True

REVIEW QUESTIONS

1. Explain the principle of working of a dc generator.
2. Explain the constructional features of a dc machine.
3. Name the main parts of a dc machine and state the material of which each part is made.
4. Explain why a commutator and brush arrangement is necessary for the operation of a dc machine.
5. Derive the emf equation for a dc generator.
6. Explain why the armature core of a dc machine is made of laminated silicon sheet steel.
7. Explain why the air gap between the pole-pieces and the armature is kept very small.
8. What will be the effect on the emf induced in the armature conductors, if the armature core of a dc generator is made of wood instead of iron?
9. What do you understand by no-load saturation curve for a dc generator?
10. Describe how a self-excited dc shunt generator builds up its terminal voltage as it is run by a prime mover. Clearly bring out the importance of (*a*) the residual magnetism in the field core, (*b*) the value of the shunt field resistance, and (*c*) the speed of rotor, in the process of building the terminal voltage.
11. Explain the terms "critical resistance" and "critical speed" of a dc shunt generator with reference to its relevant characteristics.
12. Explain in brief the different methods of excitation of a dc generator. Write the expression for the terminal voltage in each case.
13. A dc generator fails to build up voltage when it is run at rated speed. What may be the possible reasons?
14. Explain the principle of operation of a dc motor.
15. Derive the expression for the back emf in a dc motor. Briefly explain the role it plays in starting and running of the motor.
16. Explain why the induced emf in a dc motor is called back or counter emf.
17. Explain how the torque is produced in a dc motor.
18. Explain the speed and torque characteristics of (*a*) a dc shunt motor, and (*b*) a dc series motor.
19. Explain why a series dc motor is best suited for electric traction service.
20. Explain what the term "speed regulation" of a dc motor means.
21. Enumerate the various losses that occur in a dc machine.
22. Explain why the starting current is high in a dc motor.
23. Why is it necessary to use a starter for starting a dc motor? Draw a diagram of a three-point starter and explain the function of each component.
24. Explain why the no-voltage release coil is provided in the starter of a dc motor.
25. How will you reverse the direction of rotation of a dc motor?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. The armature of a dc machine is made up of laminated sheets in order to
(*a*) reduce armature copper-loss
(*b*) reduce eddy-current loss
(*c*) reduce hysteresis loss
(*d*) increase the dissipation of heat from the armature surface

2. The purpose of having the commutator-brush arrangement in a dc motor is
(*a*) to produce a unidirectional torque
(*b*) to produce a unidirectional current in the armature

21. A 250-V, dc shunt motor has a shunt-field resistance of 250 Ω and armature resistance of 0.5 ohm. While running at a speed of 1500 rpm on no load, it takes 5 A current from the mains. Calculate its speed when loaded so that it takes 50 A current from the mains. **[Ans.** 1364 rpm]

22. A 220-V, dc shunt motor takes a full-load current of 32 A while running at 850 rpm. It has an armature resistance of 0.5 Ω and shunt field resistance of 110 Ω. Calculate the speed at which the machine runs, if (*a*) a 1.5-Ω resistor were introduced in series with the armature, (*b*) a 30-Ω resistor were connected in series with the field winding. Assume that the torque remains constant throughout and the field flux is proportional to the field current. **[Ans.** (*a*) 663 rpm; (*b*) 1061 rpm]

23. A 200-V, dc series motor runs at 1000 rpm and takes 20 A. Combined resistance of armature and series field windings is 0.4 Ω. Calculate the resistance to be inserted in series to reduce the speed to 800 rpm, assuming that the torque varies as square of the speed. **[Ans.** 4.42 Ω]

EXPERIMENTAL EXERCISE 16.1

MAGNETISATION CHARACTERISTICS OF A DC SHUNT GENERATOR

Objectives

1. To plot the magnetisation characteristics of a dc shunt generator.
2. To plot the field resistance line.

Apparatus A dc shunt generator (with armature and field terminals); A three-phase induction motor; 220-V dc supply; Three-phase, 440-V ac supply; One 80-Ω, 5-A rheostat; Two dc voltmeters (0-300 V); One dc ammeter (0-2 A).

Circuit Diagram The circuit arrangement is shown in Fig. 16.20.

Brief Theory Strictly speaking, the magnetisation curve represents a plot of magnetic flux Φ (per pole) versus the field winding mmf. However, as the emf generated in a dc machine is given as

$$E_g = \frac{\Phi ZNP}{60A}$$

Since Z, P and A are constant for a machine, if we keep the speed N constant, the emf $E_g \propto \Phi$. Further, if the number of turns in the field winding remains constant, its mmf is directly proportional to the field current I_f. Hence, a plot between the induced emf E (i.e., the open-circuit terminal voltage) and the field current I_f represents the magnetisation characteristic. That is why this curve is often called *Open Circuit Characteristic* (OCC) curve.

For plotting the OCC of a shunt generator, its field winding is separately excited, as shown in Fig. 16.20. The generator is driven by a three-phase induction motor (or any other prime-mover) at a fixed speed and its armature terminals are left open to measure the induced emf E by a high-resistance voltmeter V_2.

The field resistance line (the line OA in Fig. 16.21) can be plotted by noting the readings of ammeter A and voltmeter V_1.

Procedure

1. Make the connections as shown in Fig. 16.20. The dc generator is mechanically coupled to the 3-phase induction motor. Field winding terminals F-F_1 are connected to 220-V dc supply through a rheostat. The voltmeter V_2 is connected across the armature terminals A-A_1.
2. By putting on the switch S_2, connect the induction motor to 3-phase ac supply. The induction motor drives the generator at a constant speed.

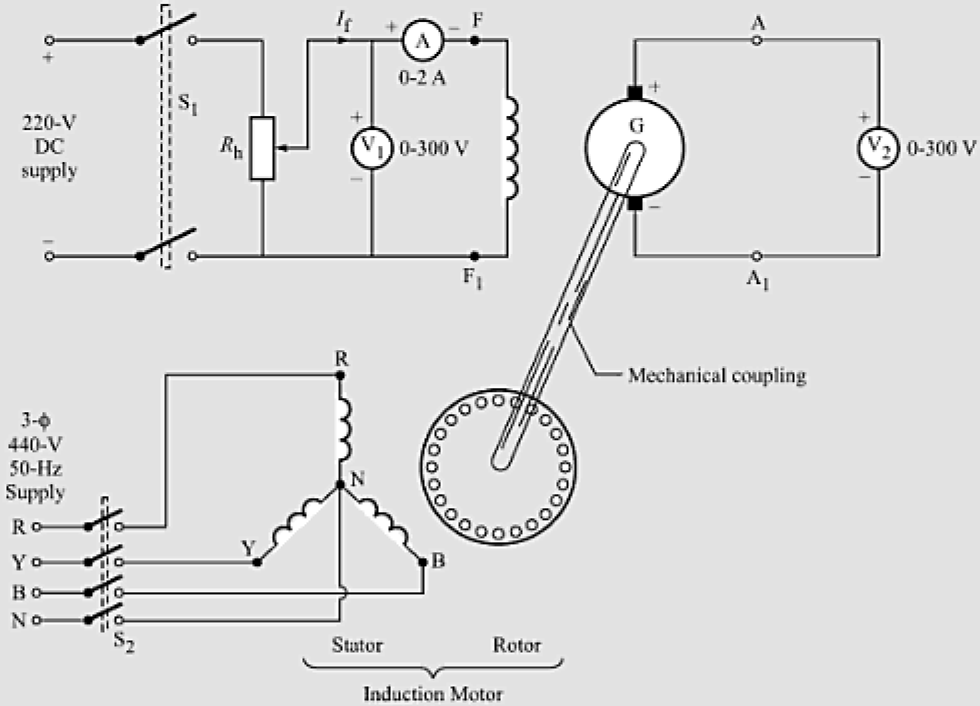


Fig. 16.20

3. Note the reading of voltmeter V_2. This small voltage gives the measure of the residual magnetism.
4. Now, put on switch S_1 by keeping the rheostat R_h at the minimum.
5. Gradually increase in steps the field current I_f (as measured by ammeter A), by increasing the rheostat R_h, and note the corresponding values of induced emf E (as given by the voltmeter V_2) and the voltage across the field-winding (as given by the voltmeter V_1).
6. Plot the magnetisation curve between V_2 and I_f, and the field-resistance line between V_1 and I_f.
7. Switch OFF first the dc supply and then the 3-phase ac supply to the induction motor.

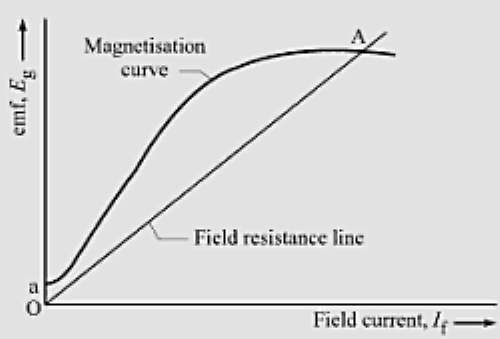
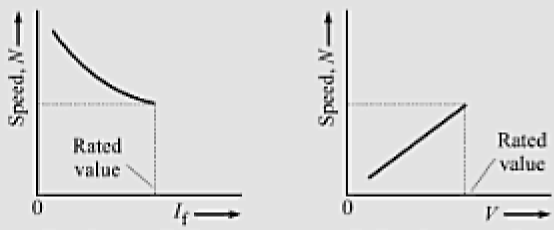


Fig. 16.21 *Magnetisation curve and field resistance line for a dc shunt generator.*

2. Keep the armature-control rheostat R_{ac} to its maximum and the field control rheostat R_{fc} to its minimum value.
3. Switch ON the dc supply. The motor starts running at slow speed.
4. Bring the armature-control rheostat R_{ac} to its minimum value so that the armature terminal voltage is at its rated value.
5. Gradually increase the rheostat R_{fc} to decrease the field current I_f in steps up to a level where speed does not become exorbitantly high. Measure the corresponding speed using the tachometer. Note down the values.
6. Draw the speed versus field-current characteristic curve.
7. Bring the field-control rheostat R_{fc} to its minimum value so that the field current I_f is at its rated value.
8. Now, gradually increase the armature-control rheostat R_{ac} to decrease the terminal voltage V in steps. Measure the corresponding speed using the tachometer. Note down the values.
9. Draw the speed versus terminal-voltage characteristic curve.
10. Switch OFF the dc supply.



(*a*) Speed versus field-current characteristic.
(*b*) Speed versus voltage characteristic.

Fig. 16.23 *Speed control of dc shunt motor.*

Observations

S. No.	*Flux Control Method*		*Armature Control Method*	
	I_f (*in* A)	N (*in* rpm)	V (*in* V)	N (*in* rpm)
1				
2				
3				
4				
5				

Results

1. The speed versus field-current characteristic curve and the speed versus armature-voltage characteristic curve are plotted in Fig. 16.23.
2. As the field current I_f is decreased (and hence the flux Φ is decreased), the speed N increases (Fig. 16.23*a*).
3. As the armature voltage V is decreased, the speed N also decreases almost proportionately (Fig. 16.23*b*).

Precautions

1. Before switching ON the supplies, the zero readings of the ammeters and voltmeter should be checked.
2. The terminals of the rheostat should be connected properly.
3. The dc ammeter and voltmeters should be connected with correct polarity.
4. Before starting the motor, the field-control rheostat should be at its minimum and the armature-control rheostat should be at its maximum.

FRACTIONAL HORSE POWER MOTORS

17

OBJECTIVES

After completing this Chapter, you will be able to:

- *Explain why the starting torque in a single-phase motor is zero.*
- *Explain the double-field revolving theory for a pulsating field.*
- *Explain how a rotating field is produced by a two-phase motor.*
- *State and explain following techniques of converting a single-phase motor into almost a two-phase motor: (i) Split-phase motor, (ii) Capacitor-start motor, (iii) Capacitor-start capacitor-run motor, (iv) Permanent-capacitor motor, and (v) Shaded-pole motor.*
- *State and explain the modifications needed in a dc series motor so that it can also run satisfactory on ac supply.*
- *Explain the working and applications of a universal motor.*
- *Explain the working and applications of a Geneva cam.*
- *Explain the function and applications of a stepper motor.*
- *Describe the working of three types of stepper motors: (i) Variable reluctance (VR) stepper motor, (ii) Permanent-magnet (PM) stepper motor, and (iii) Hybrid stepper motor.*
- *Describe the working of a VR motor in one-phase ON mode, two-phase ON mode, and half-step operation mode.*
- *Describe the working and the applications of microstepping.*
- *Describe the working of a multi-stack VR stepper motor.*
- *Define 'step angle', 'resolution', and 'shaft speed' in reference to a stepper motor.*

17.1 INTRODUCTION

In previous Chapters, we have considered the three-phase motors at some length. But it is not always convenient to have three-phase supplies available. The most common situation is in a house where almost universally we have single-phase supplies available.

Single-Phase Motors

Single-phase motors are widely used for various equipments in homes and offices. These are mostly used in fractional horse-power ranges. In 1/8 hp to 1/4 hp range, these motors are widely used in fans, washing machines, refrigerators, blowers, centrifugal pumps, etc. Motors of very small sizes (1/300 hp to 1/20 hp) used in toys, hair dryers, vending machines, etc., are also single-phase motors. The ac series motors, also known as *universal motors* are widely used in portable tools, vacuum cleaners, and kitchen equipments. Single-phase motors operate at low power factor and low efficiency relative to the three-phase motors.

Control System Motors

In this Chapter, we shall also discuss the motors that are used in *control systems*. The control systems can be divided into two main categories:

1. Regulators.
2. Remote position control (RPC) systems.

Regulators Here, we seek to control the speed of the motor with high degree of accuracy. A dc shunt motor, described in Chapter 16, has good torque-speed characteristic for such applications. For better control, we can have separately excited dc motor whose field winding is excited by a control amplifier, as shown in Fig. 17.1.

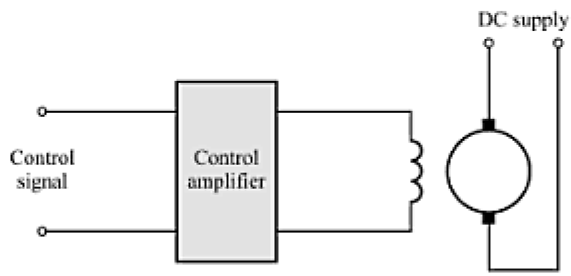


Fig. 17.1 *Separately excited dc motor in a regulator.*

The dc motor is relatively more expensive and less robust than the ac induction motor. Therefore, some engineers consider using a cage-rotor three-phase induction motor a better alternative for regulators. Its speed can be electronically controlled by just varying the frequency of the supply.

There are other regulator motors such as the brushless dc motor, the switched reluctance motor, and the variable-frequency synchronous motor. But none of these proves to be better than the two considered above.

RPC *System* An RPC motor moves its load to a position determined by the control system. Following two limitations are experienced by this system.

1. There is a limit to the mechanical ability of a motor to just move its load by the required angle. This limit occurs when the required angle of rotation becomes too small.
2. When the motor is almost aligned to the desired position, the error-signal is so small that the amplifier can no longer drive the motor.

These limitations have been overcome by following two arrangements, which we shall discuss in this Chapter:

1. Geneva cam.
2. Stepper motor.

17.2 PROBLEM WITH SINGLE-PHASE MOTOR

In physical appearance, a single-phase induction motor is similar to a three-phase squirrel cage induction motor. The stator has distributed single-phase winding. The rotor has a short-circuited squirrel cage winding. There is uniform air gap between the stator and the rotor.

The problem with such a motor is that *it has no starting torque*. This can be seen from Fig. 17.2. For clarity of understanding, the stator field winding is shown as concentrated, instead of distributed. When terminals A-B are connected to an ac supply, the magnetic flux is produced along the vertical axis, and it varies in a sinusoidal manner. Consider an instant when terminal B is positive with respect to terminal A and the current i is increasing. The resulting magnetic field is from top to bottom and is increasing. Because of the transformer action, emfs are induced in the rotor bars. Since the bars are shorted at the ends, according to Lenz's law the currents flow in them in the directions shown in Fig. 17.2. As per Fleming's left hand rule, the bars in the left half experience rightward forces. Moreover, the forces on the bars in the right half are leftward. As a result, the net torque developed is zero.

Due to the ac current in the stator winding, the field pulsates sinusoidally. That is, the field poles alternate in polarity and vary in strength sinusoidally.

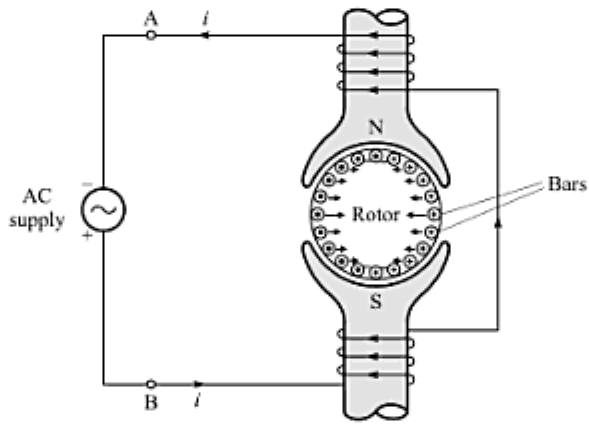


Fig. 17.2 *The starting torque in a single-phase induction motor is zero.*

What Can be Done to Run a Single-Phase Induction Motor

If we could somehow start the rotor, we would find that it continues to rotate in much the same way as a three-phase induction motor does.

Let us consider an explanation for this action of the motor. The sinusoidally pulsating flux density B (along the vertical axis in Fig. 17.2) can be shown by a sinusoidal curve (Fig. 17.3*a*) and can be represented as

$$B = -B_m \sin \omega t$$

where B_m is the maximum value of the flux density.

Double-Field Revolving Theory The surprising thing is that this field B, sinusoidally pulsating along a fixed direction in space, can be represented as the combined effect of two rotating fields B_{r1} and B_{r2}, each of magnitude $B_m/2$, but rotating in opposite directions. This is illustrated in Fig. 17.3. As shown in Fig. 17.3*b*, at $\omega t = 0°$, the two fields B_{r1} and B_{r2} are shown by two equal and oppositely directed arrows along horizontal direction. Their resultant is zero. The field B_{r1} is rotating in anticlockwise direction and field B_{r2} in clockwise direction, both at angular speed ω. At later instants, when $\omega t = 30°$, 90° and 225°, the positions of B_{r1} and B_{r2} are shown in Figs. 17.3*c*, *d* and *e*, respectively. In each case, the horizontal components are equal and opposite and therefore cancel each other. The resultant is always along the vertical direction and has the same magnitude as the pulsating field B.

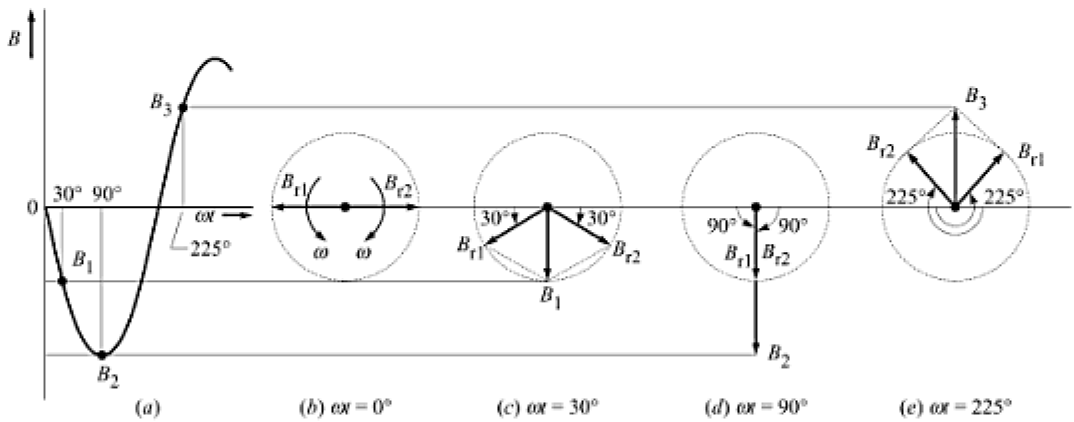


Fig. 17.3 *A sinusoidally pulsating field can be represented by two equal rotating fields in opposite directions.*

Therefore, *the pulsating field of the stator can be considered to be composed of two fields, equal in magnitude but rotating in opposite directions with synchronous speed.* Each of these component fields will produce induction-motor action in the opposite direction. If the motor is moving, it must be moving in the direction of one of these fields (we can call this *forward* direction) and rotating in the opposite direction to that of the other field (in *backward* direction). In the forward direction, we have a motor action with a value of slip between 0 and 1. However, in the reverse direction, we have a plugging or braking action with a value of slip between 2 and 1. The total action is sum of these two actions. Fortunately, the motor action is stronger than the plugging action and therefore the device acts as a motor.

The dotted curves in Fig. 17.4 show the torque-speed characteristics for the two rotating fields. These are similar to that of a 3-phase induction motor, except that these are extended beyond the synchronous speed. The resultant of these two components is shown by the solid-line curve. It is seen to have zero value at the starting, but a definite non-zero value at any other speed.

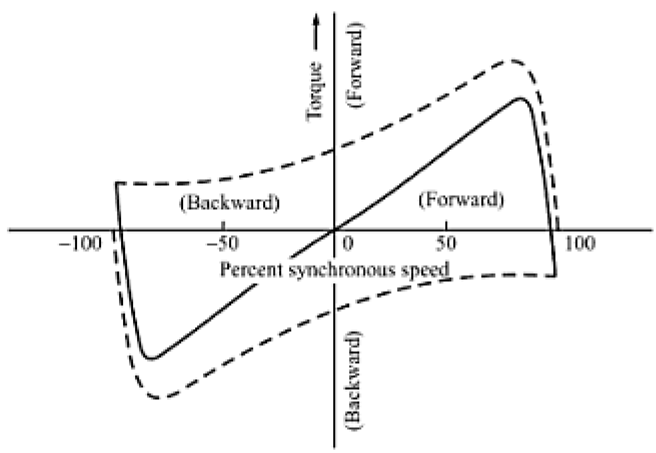
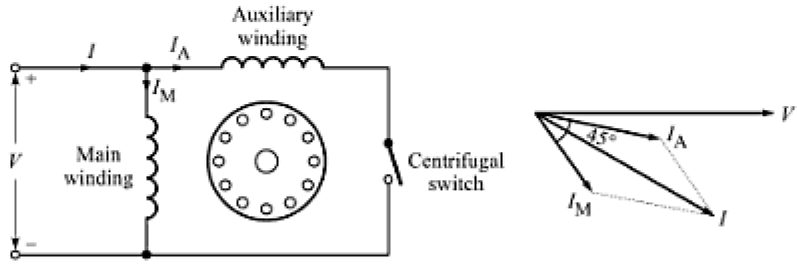


Fig 17.4 *Resultant torque-speed curve (solid line) and its two components (dotted line) in a single-phase motor.*



(*a*) Schematic diagram. (*b*) Phasor diagram at the starting.

Fig. 17.6 *Split-phase induction motor.*

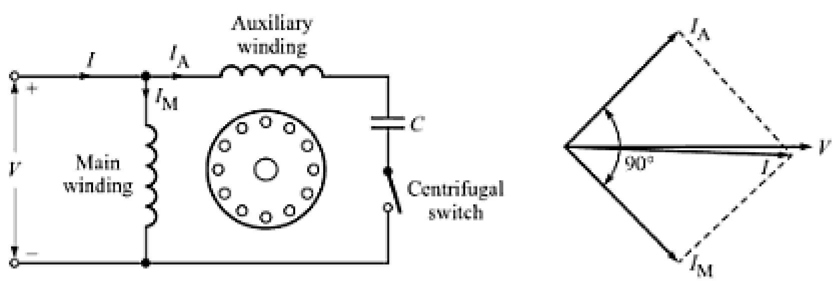
way, the motor becomes equivalent to an unbalanced two-phase motor, and a starting torque is developed. Because of the high resistance of the auxiliary winding, this motor is also called ***resistance-start motor***.

When the motor picks up about 75 % of the synchronous speed, the auxiliary winding is disconnected by a centrifugal switch. This is done to avoid noisy and inefficient performance of the motor. If not done, it may result in overheating which may even damage the motor.

Applications These motors have moderate starting torque with starting current 6 to 8 times the full-load current. Their ratings are from 1/20 hp to 1/2 hp. They are cheap and are used in washing machines, fans, blowers, centrifugal pumps, refrigerators, duplicating machines, grinders, etc.

(2) Capacitor-Start Motor

This is an improved form of split-phase motor. Here, the time displacement between the currents of the main and the auxiliary windings is achieved by connecting a capacitor in series with the auxiliary winding (see Fig. 17.7*a*). The current I_A in the auxiliary winding leads the applied voltage by some angle, and the current I_M in the main winding lags the applied voltage by some angle, as shown in Fig. 17.7*b*.



(*a*) Schematic diagram. (*b*) Phasor diagram at the starting.

Fig. 17.7 *Capacitor-start induction motor.*

Starting Torque By selecting the capacitor of suitable value, the current I_A can be made to lead the current I_M by about 90°. Thus, this motor develops a much larger starting torque than that developed by a resistance-start motor. The capacitor used is of electrolytic type, which is smaller and cheaper than a paper capacitor. However, an electrolytic capacitor and the auxiliary winding are designed for short time duty. Therefore, when the motor attains a speed of about 75 % of the rated speed, the auxiliary winding circuit is disconnected by a centrifugal switch.

(*c*) The line current,

$$\mathbf{I}_L = \mathbf{I}_M + \mathbf{I}_A = 17.7\angle{-67.38°} + 17.7\angle{-22.62°} = 32.73\angle{-45°}\ \text{A}$$

$$\therefore \qquad I_L = \mathbf{32.72\ A}$$

(*d*) The phase displacement between the two winding currents,

$$\phi = \phi_A - \phi_M = -22.62° - (-67.38°) = 44.76° \approx \mathbf{45°}$$

(*e*) The power factor, $pf = \cos\phi = \cos 45° = \mathbf{0.707\ (lagging)}$

EXAMPLE 17.3

A 230-V, 50-Hz, split-phase induction motor has the main-winding resistance of 2 Ω and inductive reactance of 20 Ω, and the auxiliary-winding resistance of 25 Ω and inductive reactance of 5 Ω. Determine the value of capacitance to be connected in series with the auxiliary winding to obtain maximum starting torque.

Solution For obtaining maximum torque, the phase angle between the main winding current I_M and auxiliary winding current I_A should be 90°. That is, $\phi_M - \phi_A = 90°$. Here,

$$\phi_M = \tan^{-1}\frac{X_M}{R_M} = \tan^{-1}\frac{20}{2} = 84.3°$$

$$\therefore \qquad \phi_A = \phi_M - 90° = 84.3° - 90° = -5.7°$$

Now, $$\tan\phi_A = \frac{X_A - X_C}{R_A} \quad \text{or} \quad \tan(-5.7°) = \frac{5 - X_C}{25} \quad \text{or} \quad -0.0998 = \frac{5 - X_C}{25}$$

Or $$-2.495 = 5 - X_C \quad \Rightarrow \quad X_C = 7.495\ \Omega$$

$$\therefore \qquad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 7.495} = \mathbf{424.7\ \mu F}$$

17.4 AC SERIES MOTOR

A dc shunt or series motor can operate on both the dc and ac supplies. When ac is fed to a series motor, the stator and the rotor fields vary in exact time phase. Both reverse simultaneously to produce torque in the same direction*. However, in a shunt motor, due to its high inductance the field winding causes the field current to lag the armature current by such a large angle that a very low torque is developed. Hence, compared to a dc series motor, a dc shunt motor is **not** very suitable for ac operation.

If an ordinary dc series motor is connected to an ac supply, it runs, but very unsatisfactorily because of the following reasons:

1. Due to reversal of armature and field currents every half cycle, a pulsating torque is developed.
2. Due to alternating flux, excessive eddy-current loss occurs in the yoke and field cores, causing excessive heat.
3. Due to large voltages and currents induced in the short-circuited armature coils, vicious sparking occurs at the brushes during commutation period.
4. There occurs an abnormal voltage drop and a low power factor due to high inductance of the field and armature circuits.

Hence, some modifications are needed if we are to use a dc series motor on ac supply. To improve the efficiency, the stator yoke and field cores are also laminated**. To reduce the reactance of the armature

* However, the torque developed is not of constant magnitude (as in a dc series motor) but pulsates between zero and maximum value each half-cycle.

** The dc series motor has only its rotor laminated.

17.6 GENEVA CAM

This is a simple 'remote position control (RPC)' mechanism used for moving the load to a position determined by the control system. It permits the motor to move without moving the load beyond the desired position.

The basic Geneva cam is shown in Fig. 17.12. Here, it is assumed that the striker arm attached to the motor rotates clockwise. In order to see the action of the mechanism, we have marked one of the edges of the cam by a star. In Fig. 17.12*a*, the striker arm has just engaged in one of the slots on the cam. As the motor rotates clockwise, the cam rotates anticlockwise as long as the striker remains engaged in the slot. In Fig. 17.12*b*, the striker arm has rotated through 60° and the cam too has rotated through 60°. Further rotation of the striker brings the system to the position shown in Fig. 17.12*c*. Here, the striker has disengaged, leaving the cam rotated through *exactly* 90°.

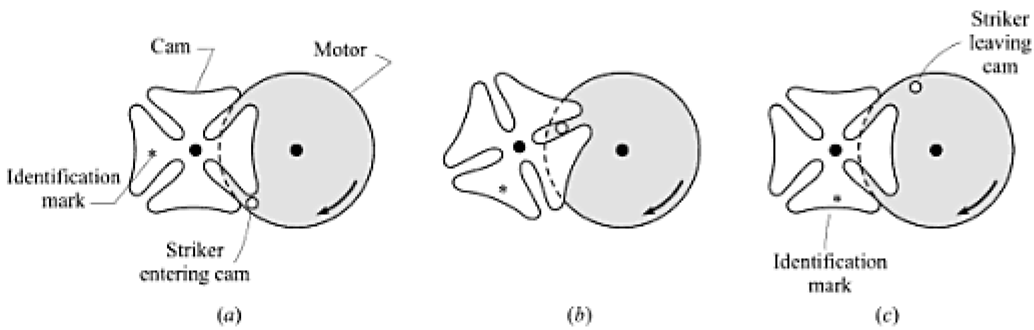


Fig. 17.12 *Geneva cam mechanism.*

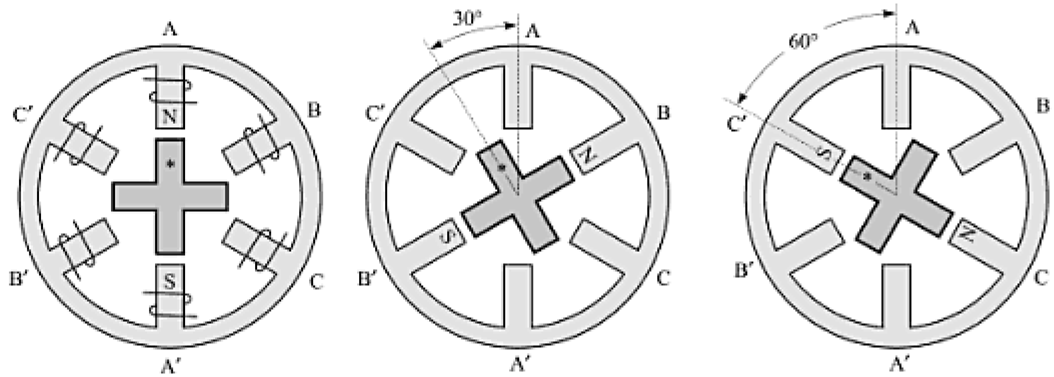
Thus, we find that during a quarter of every rotation of the motor, the striker arm engages with one of the slots of the cam and rotates the load through exactly 90°. For the remaining part of the rotation of the motor, there is no engagement. So, here it does not matter if we cannot stop the motor in exactly the required position. Rather we can stop the motor anywhere when it is not engaged. Thus, the Geneva cam provides mechanical equivalent of a digital bit.

The Geneva cams have now been largely superseded by the stepper motors except in systems that require large torques, and where the positioning of the load in steps of quite large angles is acceptable.

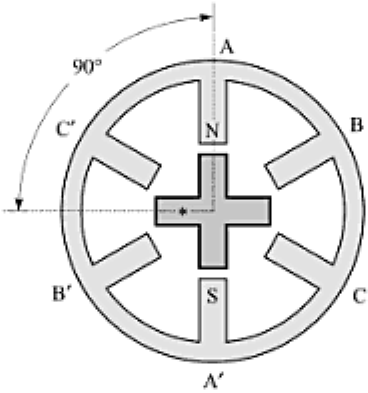
17.7 STEPPER MOTORS

These motors, also called ***stepping motors*** or ***step motors***, are so named because they rotate through a fixed angular *step* in response to each input current pulse from its ***controller***. The industrial motors cannot be used for precise positioning of an object or precise control of speed without using closed-loop feedback system. The stepper motors are able to provide very precise position and/or speed control without needing any feedback loop.

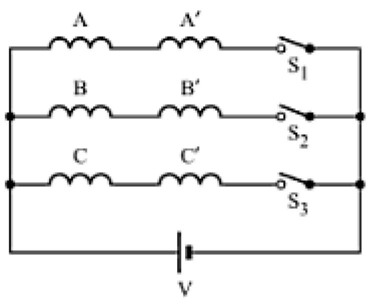
A stepper motor has salient poles with field coils. The rotor is usually salient pole but has no exciting coil. The unique feature of a stepper motor is that its shaft rotates in a series of discrete steps in response to command pulses received from an electronic control circuit. By counting the pulses, we can determine the exact rotation achieved. This basic control is illustrated in Fig. 17.13. Note that the control has two elements—the number of pulses which determine the angle of rotation, and the direction data which determines the order in which the stator poles are excited.



(a) (b) (c)



(d)



(e)

Truth Table 1

	A	B	C	θ
A	+	0	0	0°
B	0	+	0	30°
C	0	0	+	60°
A	+	0	0	90°

(f) One-phase ON mode.

Fig. 17.14 *Full-step operation of a* VR *stepper motor.*

Now, let us open the switch S_1 and close the switch S_2 to energise the next phase B. Note that the next phase is selected by going *clockwise* around the stator. The rotor poles of unmarked pair are now nearest to the new position of alignment. The rotor therefore rotates by 30° *anticlockwise* to align along the path of least reluctance (see Fig. 17.14*b*).

Again, let us open the switch S_2 and close switch S_3 to energise the phase C. Once again, the nearest rotor poles to align are from the marked pair. These poles align themselves by rotating a further 30° *anticlockwise* (see Fig. 17.14*c*).

If we again close switch S_1 to energise phase A, the rotor further rotates by another 30° *anticlockwise* (see Fig. 17.14*d*), the total rotation being 90° from the initial position. Thus, we find that by repetitively closing the switches in sequence 1-2-3-1-2-…, that is, by energising the phases in sequence A-B-C-A-B-…, the rotor rotates anticlockwise in steps of 30°. Now, if the phases are energised in the reverse sequence, A-C-B-A-C-…, the rotor will rotate *clockwise* in steps of 30°. Since only one phase is energized at a time, this mode of operation is called one-phase ON operation. The stator phase switching truth table for this operation is given in Fig. 17.14*f*.

(*c*) The shaft speed, $n = \dfrac{\beta f}{360°}$ rps $= \dfrac{2.5° \times 3600}{360°} = \mathbf{25\ rps}$

EXAMPLE 17.5

A 3-phase VR motor has a step angle of 15°. Find the number of its rotor and stator poles.

Solution Using Eq. 17.2, we have

$$\beta = \frac{360°}{mN_r} \quad \text{or} \quad N_r = \frac{360°}{m\beta} = \frac{360°}{3 \times 15°} = \mathbf{8}$$

For determining the number of stator poles, we use Eq. 17.1. Two possibilities, as given below, must be considered.

(*i*) When $N_s > N_r$: $\beta = \dfrac{(N_s - N_r)}{N_s \cdot N_r} \times 360°$ or $15° = \dfrac{(N_s - 8)}{N_s \times 8} \times 360° \Rightarrow N_s = \mathbf{12}$

(*ii*) When $N_s < N_r$: $\beta = \dfrac{(N_r - N_s)}{N_s \cdot N_r} \times 360°$ or $15° = \dfrac{(8 - N_s)}{N_s \times 8} \times 360° \Rightarrow N_s = \mathbf{6}$

(2) Two-Phase ON Mode, Full-Step Operation

In this mode of operation, two stator phases are energised simultaneously. When phases A and B are energised together, the rotor comes to rest at a point mid-way between the two adjacent full-step positions (say, the marked pair will be at 15° anticlockwise from the reference given in Fig. 17.14). Now, if the stator phases are switched in the sequence AB, BC, CA, AB…, etc., the rotor will take full steps of 30° anticlockwise each time (as in one-phase on mode), as shown in the truth table No. 2 in Fig. 17.15*a*. Note that the equilibrium positions in this case are interleaved between those of the one-phase ON mode.

Truth Table 2

	A	B	C	θ
AB	+	+	0	15°
BC	0	+	+	45°
CA	+	0	+	75°
AB	+	+	0	105°

(*a*) Two-phase ON mode.

Truth Table 3

	A	B	C	θ
A	+	0	0	0°
AB	+	+	0	15°
B	0	+	0	30°
BC	0	+	+	45°
C	0	0	+	60°
CA	+	0	+	75°
A	+	0	0	90°

(*b*) Half-step operation.

Fig. 17.15 *Truth tables.*

(3) Alternate One-Phase ON Mode & Two-Phase ON Mode, Half-Step Operation

It is possible to have 'half-stepping' or 'half-step operation' by alternately having 'one-phase ON' and 'two-phase ON' modes. That is, by energising the three phases in the sequence A, AB, B, BC, C, CA, A…, etc. This operation is illustrated in Fig. 17.16, where the rotor moves by only 15° *clockwise* with each switching. Note that here the phase sequence is selected by going *anticlockwise* around the stator. The truth table for this operation is given in Fig. 17.15*b*.

The stator has four salient poles which can be excited by two pairs of coils. The cross-sectional views of the motor along the planes x-x′ and y-y′ are shown in Figs. 17.20*a* and *c*, respectively. Note that both the end-caps have 5 teeth, which are magnetised by the respective polarities of the axial permanent magnet. Thus, all the teeth of the end-cap on the left are N-poles, and all the teeth of the end-cap on the right are S-poles. While fitting the two end-caps, it is ensured that the teeth of one end-cap are offset by half the tooth-pitch with respect to the other. It means the tooth of one end-cap coincides with the slot of the other. The step angle of such a motor is

$$\beta = \frac{(N_r \sim N_s)}{N_s \cdot N_r} \times 360° = \frac{(5-4)}{5 \times 4} \times 360° = 18°$$

Working Here, we shall discuss one-phase ON mode only*, for which Truth Table No. 7 is given in Fig. 17.21. When phase A is excited positively (refereed to as A^+), the top stator pole becomes S-pole and the bottom pole becomes N-pole. The rotor aligns so that one of the N-teeth of left-hand end-cap and one of the S-teeth of right-hand end-cap aligns along the axis A-A′, as shown in Fig. 17.20. Now, when phase A is de-energised and phase B is energised positively, the left-hand stator pole becomes S-pole and the right-hand pole becomes N-pole. The rotor has to rotate *anticlockwise* by an angle of 18° so that one teeth of each end-cap is aligned *along* B-B′ *axis*.

Truth Table 7

	A	B	θ
A^+	+	0	0°
B^+	0	+	18°
A^-	−	0	36°
B^-	0	−	54°
A^+	+	0	72°

(For anticlockwise rotation)

Fig. 17.21 *Truth table for one-phase* ON *mode operation of the hybrid stepper motor of Fig.* 17.20.

Next, de-energising phase B and energising phase A negatively further rotates the rotor by another 18°. Thus, we can rotate the rotor in full steps, either clockwise or anticlockwise by the phase sequences as follows:

(*i*) For *anticlockwise* rotation: $A^+, B^+, A^-, B^-, A^+, B^+, \ldots$

(*ii*) For *clockwise* rotation: $A^+, B^-, A^-, B^+, A^+, B^-, A^-, \ldots$

Practical hybrid stepper motors are built with more rotor poles than shown in Fig. 17.20 to give higher resolution. The resolution is further increased by providing more stacks.

Advantages The hybrid motor achieves much smaller step sizes easily with a simple magnetic structure. Compared to a VR motor, a hybrid motor needs lower excitation current to produce same torque. Like a PM motor, this motor also develops good detent torque provided by the permanent-magnet flux.

* You can easily extend this discussion for the two-phase ON mode and the alternate one-phase ON & two-phase ON mode.

CHECK YOUR UNDERSTANDING

Before you proceed to the next Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	A single-phase induction motor can be thought of as a composite of two motors mechanically coupled. If one of these has a slip of s, the slip of other motor is $1 - s$.	☐	☐	
2.	In double revolving field theory, the speeds of both the fields are synchronous speeds.	☐	☐	
3.	The cheapest motor in the fractional horse power motors is the shaded-pole motor.	☐	☐	
4.	In a split-phase single-phase induction motor, the main winding and the auxiliary winding occupy equal number of stator slots.	☐	☐	
5.	The auxiliary winding in a split-phase induction motor has higher X/R ratio as compared to the main winding.	☐	☐	
6.	A split-phase motor is also known as a resistance-start motor.	☐	☐	
7.	A capacitor-start capacitor-run motor has two capacitors, one is electrolytic capacitor, and the other is oil-impregnated paper capacitor.	☐	☐	
8.	The starting torque of a shaded-pole motor is low.	☐	☐	
9.	To provide very precise position control of an object, the stepper motors require a feedback loop.	☐	☐	
10.	While switching the phases of a VR stepper motor, if the selection of phases is done clockwise around the stator, the rotor moves anticlockwise with each switching.	☐	☐	

Your Score

ANSWERS

1. False	**2.** True	**3.** True	**4.** False	**5.** True
6. False	**7.** True	**8.** True	**9.** False	**10.** True

REVIEW QUESTIONS

1. Explain why a single-phase motor does not develop a starting torque.
2. What are the main differences between a 3-phase and single-phase induction motors? With the help of neat diagrams, explain the working principle of split-phase induction motor.
3. Briefly discuss different methods of starting single-phase motors.
4. Prepare a table showing the horse power rating and applications of each type of single-phase induction motor.
5. Explain why the auxiliary winding in a capacitor-start motor should be disconnected after the motor has picked up speed.
6. Describe the construction and principle of working of a capacitor-start capacitor-run single-phase motor.

7. Explain in what ways does a capacitor-start motor differs from a capacitor-start capacitor-run motor.
8. Explain the working principle and application of single-phase shaded-pole motor. Is it possible to reverse the direction of rotation of such a motor? If yes, how? If not, why not?
9. Explain why ac series motors are preferred for electric locomotives.
10. What is a universal motor? Where is it used?
11. Explain the working of a universal motor. What modifications should be carried out in a dc series motor so that it can operate as a universal motor?
12. What is a stepper motor? Why is it named so? Does it run on ac or dc?
13. State the different types of stepper motors and briefly explain the difference between them.
14. Explain the working of a VR stepper motor in its half-step operation. Is there any other name given to this type of operation?
15. What is a hybrid stepper motor? In what way is it better than the VR stepper motor?

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. In a resistance split-phase single-phase induction motor, a time-phase difference between the currents in the main and auxiliary windings is achieved by
   (*a*) placing the two windings at an angle of 90° electrical in the stator slots
   (*b*) applying two-phase supply across the two windings
   (*c*) having different ratio of resistance to inductive reactance for the two windings supplied from a single-phase supply
   (*d*) connecting the two windings in series across a single-phase supply
2. The direction of rotation of an ordinary shaded-pole single-phase induction motor
   (*a*) can be reversed by interchanging the supply terminals connected to the stator winding
   (*b*) can be reversed by open-circuiting the shading rings
   (*c*) can be reversed by short-circuiting the shading rings
   (*d*) cannot be reversed
3. The direction of rotation of split-phase single-phase induction motor can be reversed by
   (*a*) interchanging the supply terminals connected to the stator winding
   (*b*) reversing the terminals of only the auxiliary winding across the supply
   (*c*) reversing the terminals of only the main winding across the supply
   (*d*) reversing the terminals of either the main winding or the auxiliary winding across the supply
4. If the centrifugal switch of a resistance split-phase single-phase induction motor fails to close when de-energised, then
   (*a*) no starting torque will be developed when an attempt is made to restart
   (*b*) a dangerously high current will flow through the main winding when an attempt is made to restart
   (*c*) the starting torque may not be sufficient to enable the motor to run when an attempt is made to restart
   (*d*) the motor will develop extremely high torque when an attempt is made to restart
5. The motor used in the ceiling fans in homes is
   (*a*) the split-phase motor
   (*b*) capacitor-start motor
   (*c*) the shaded-pole motor
   (*d*) the ac series motor
6. In a capacitor-start capacitor-run motor, the two capacitors
   (*a*) have similar construction
   (*b*) are of different type
   (*c*) have equal capacitance
   (*d*) are disconnected when the motor attains its rated speed
7. According to the double revolving field theory, a pulsating field can be resolved into two revolving fields. The amplitude of each of these component fields is equal to
   (*a*) half the amplitude of the pulsating field

SUPPLEMENTARY EXERCISES

D

D.1 Solved Problems
D.2 Practice Problems

PART D: ELECTRICAL MACHINES

Assemblage of

- Chapter 13: Transformers
- Chapter 14: Alternators and Synchronous Motors
- Chapter 15: Induction Motors
- Chapter 16: D.C. Machines
- Chapter 17: Fractional Horse Power Motors

D. 1. SOLVED PROBLEMS

PROBLEM D-1

A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current is 3 A at a *pf* of 0.2 lagging. Calculate the primary current and the power factor when the secondary current is 280 A at a *pf* of 0.8 lagging.

Solution The no-load phase angle, $\phi_0 = \cos^{-1} 0.2 = 78.46°$ (lagging). Therefore, taking the primary voltage as the reference, the no-load current is given as

$$\mathbf{I}_0 = 3\angle{-78.46°}\ \text{A} = (0.6 - j2.94)\ \text{A}$$

When load is connected across the secondary, the secondary current is 280 A at a *pf* of 0.8 lagging. The phase angle of the secondary current,

$$\phi_2 = \cos^{-1} 0.8 = 36.87° \text{ (lagging)}$$

The corresponding primary balancing current is

$$\mathbf{I}_1' = \{I_2(N_2/N_1)\}\angle\phi_2 = \{280(200/1000)\}\angle{-36.87°} = 56\angle{-36.87°}\ \text{A}$$
$$= (44.8 - j33.6)\ \text{A}$$

Hence, the total primary current,

$$\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_1' = (0.6 - j2.94) + (44.8 - j33.6) = (45.4 - j36.54)\ \text{A}$$
$$= \mathbf{58.28\angle{-38.83°}\ A}$$

The power factor, $pf = \cos 38.83° =$ **0.779 (lagging)**

PROBLEM D-2

A single-phase, 100-kVA, 2000-V/200-V, 50-Hz transformer has impedance drop of 10 %, and resistance drop of 5 %. Calculate (*a*) the regulation at full-load 0.8 *pf* lagging, and (*b*) the value of the *pf* at which the regulation is maximum.

Solution $\cos\phi = 0.8$, $\sin\phi = \sqrt{1 - \cos^2\phi} = 0.6.$

Given: the % impedance drop $= \dfrac{I_2 Z_{e2}}{E_2} \times 100 = 10$

$$\therefore \quad I_2 Z_{e2} = \frac{10\,E_2}{100} = \frac{10 \times 200}{100} = 20\ \text{V}$$

Again, given the % resistance drop $= \dfrac{I_2 R_{e2}}{E_2} \times 100 = 5$

$$\therefore \quad I_2 R_{e2} = \frac{5\,E_2}{100} = \frac{5 \times 200}{100} = 10\ \text{V}$$

Therefore, the reactance drop is given as

$$I_2 X_{e2} = \sqrt{(I_2 Z_{e2})^2 - (I_2 R_{e2})^2} = \sqrt{20^2 - 10^2} = 17.32\ \text{V}$$

(*a*) The percentage voltage regulation is given as

$$VR = \frac{(I_2 R_{e2} \cos\phi + I_2 X_{e2} \sin\phi)}{E_2} \times 100 = \frac{(10 \times 0.8 + 17.32 \times 0.6)}{200} \times 100 = \mathbf{9.2\ \%}$$

(*b*) For regulation to be zero, the power factor must be leading, and we must have

$$\frac{(I_2 R_{e2} \cos\phi - I_2 X_{e2} \sin\phi)}{E_2} \times 100 = 0 \quad \Rightarrow \quad \tan\phi = \frac{I_2 R_{e2}}{I_2 X_{e2}} = \frac{10}{17.32} = 0.5774$$

$\therefore$ $\phi = \tan^{-1} 0.5774 = 30°$

Therefore, the power factor $= \cos\phi = \cos 30° =$ **0.866 (leading)**

PROBLEM D-3

A 50-kVA transformer has an efficiency of 98 % at full load, 0.8 power factor, and an efficiency of 96.9 % at one-fourth full-load, unity power factor. Determine the iron loss and full-load copper loss.

Solution Let P_o be the full-load output power at unity power factor, P_i be the iron loss and P_c be the full-load copper loss. Then, the efficiency of the transformer at full load, unity power factor is given as

$$\eta = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + P_i + P_c}$$

In the first case, the output power, P_o = (kVA) (*pf*) = 50 000 × 0.8 = 40 000 W. Hence, we have

$$0.98 = \frac{40000}{40000 + P_i + P_c} \quad \Rightarrow \quad P_i + P_c = 40\,000\left\{\frac{1}{0.98} - 1\right\} = 816.3\text{ W}$$

In the second case, the output power, $P_o = (1/4)\,(kVA)\,(pf) = 0.25 \times 50\,000 \times 1 = 12\,500$ W.

The copper loss, $P'_c = (1/4)^2 P_c = 0.0625 P_c$

Hence, we have

$$0.969 = \frac{12\,500}{12\,500 + P_i + 0.0625\,P_c} \quad \Rightarrow \quad P_i + 0.0625 P_c = 12\,500\left\{\frac{1}{0.969} - 1\right\} = 399.9\text{ W}$$

Solving the above two equations, we get

$$P_i = \mathbf{372.1\ W} \qquad \text{and} \qquad P_c = \mathbf{444.2\ W}$$

PROBLEM D-4

A 40-kVA transformer has core loss of 450 W, and full-load copper loss of 850 W. If the power factor of the load is 0.8, calculate (*a*) the full-load efficiency, (*b*) the maximum efficiency, and (*c*) the load in kVA at which the maximum efficiency occurs.

Solution

(*a*) The full-load output power at 0.8 pf P_o = (kVA) (*pf*) = 40 000 × 0.8 = 32 000 W.

Hence, the efficiency is given as

$$\eta = \frac{P_o}{P_o + P_i + P_c} = \frac{32000}{32000 + 450 + 850} = \mathbf{96.1\ \%}$$

(*b*) Let the maximum efficiency occur at *x* times the full-load. Since, the copper loss must be equal to the iron loss at maximum efficiency, we have

$$P_c = P_i \quad \text{or} \quad x^2 \times 850 = 450 \quad \Rightarrow \quad x = \sqrt{450/850} = 0.7276$$

(c) The iron loss, P_i = no-load losses = 38.4 W. When the efficiency of the transformer is maximum, the copper loss equals the iron loss. Hence, $P_c = P_i = 38.4$ W.

Since, $P_c = I_2^2 R_{e2}$, load current at maximum efficiency is given as

$$I_2 = \sqrt{\frac{P_c}{R_{e2}}} = \sqrt{\frac{38.4}{0.6}} = \mathbf{8\ A}$$

And the maximum efficiency is given as

$$\eta_{max} = \frac{P_o}{P_o + P_i + P_c} = \frac{500 \times 8 \times 0.8}{500 \times 8 \times 0.8 + 38.4 + 38.4} = \mathbf{97.66\ \%}$$

(d) When OC test is carried out on LV side, $E_1 = V_1 = 250$ V; $P_i = 38.4$ W; $\cos \phi_0 = 0.22$ (lag);

and $$\sin \phi_0 = \sqrt{1 - (0.22)^2} = 0.9755.$$

∴ The no-load current, $$I_0 = \frac{P_{oc}}{V_1 \cos \phi_0} = \frac{38.4}{250 \times 0.22} = \mathbf{0.698\ A}$$

The iron-loss component, $I_w = I_0 \cos \phi_0 = 0.698 \times 0.22 = \mathbf{0.1536\ A}$

The magnetising component, $I_m = I_0 \sin \phi_0 = 0.698 \times 0.9755 = \mathbf{0.68\ A}$

(e) $R_{01} = \dfrac{V_1}{I_w} = \dfrac{250}{0.1536} = \mathbf{1628\ \Omega}$; $X_{01} = \dfrac{V_1}{I_m} = \dfrac{250}{0.68} = \mathbf{368\ \Omega}$

$R_{e1} = R_{e2}(N_1/N_2)^2 = 0.6 \times (1/2)^2 = \mathbf{0.15\ \Omega}$; $X_{e1} = X_{e2}(N_1/N_2)^2 = 1.28 \times (1/2)^2 = \mathbf{0.32\ \Omega}$

Thus, the equivalent circuit is shown in Fig. D-2.

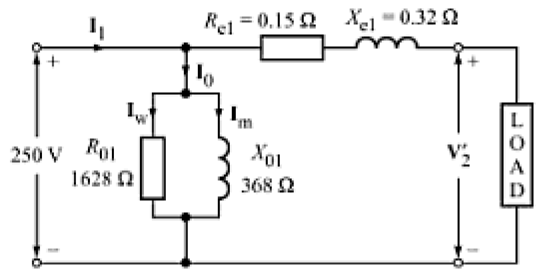


Fig. D-2

PROBLEM D-9

A 5-kVA, 200-V/400-V, 50-Hz, single-phase transformer has a leakage impedance of (0.12 + *j*0.32) Ω as referred to the low voltage side. Calculate the per unit values of resistance, reactance and impedance taking the rated quantities as the base values. Check that the per unit values are the same on the two sides.

Solution Base volt-ampere, $VA_{base} = 5000$ VA; $\mathbf{Z}_{lv} = (0.12 + j0.32)\ \Omega$.

For low voltage side:

$$V_{base} = 200\ \text{V}; \quad I_{base} = \frac{VA}{V} = \frac{5000}{200} = 25\ \text{A}; \quad Z_{base} = \frac{V}{I} = \frac{200}{25} = 8\ \Omega;$$

Therefore,

Per unit resistance, $$R_{pu} = \frac{R_{lv}}{Z_{base}} = \frac{0.12\ \Omega}{8\ \Omega} = \mathbf{0.015\ pu}$$

Since nothing is mentioned about the pitch factor, we can take $k_p = 1$. Now, the voltage per phase is given as

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6351 \text{ V}$$

However, the emf induced per phase is given as

$$E_{ph} = 4.44 k_p k_d f \Phi T$$

$$\therefore \quad T = \frac{E_{ph}}{4.44\, k_p k_d f \Phi} = \frac{6351}{4.44 \times 1 \times 0.9598 \times 50 \times 0.16} = 186.3$$

Thus, the number of conductors in series per phase, $Z_{ph} = 2T = 2 \times 186.3 = \mathbf{373}$

PROBLEM D-12

A 3-phase, 20-pole, 50-Hz, salient-pole alternator with star-connected stator winding has 180 slots on the stator, each slot having 8 conductors. The flux per pole is 25 mWb and is sinusoidally distributed. The coils are full pitch. Calculate (*a*) the speed, (*b*) the generated emf per phase, and (*c*) the line emf.

Solution

(*a*) The speed, $N = \frac{120 f}{P} = \frac{120 \times 50}{20} = \mathbf{300 \text{ rpm}}$

(*b*) Number of slots per pole is 180/20 = 9.

Number of slots per pole per phase, $q = 9/3 = 3$

Slot angle, $\alpha = 180°/9 = 20°$

$$\therefore \quad k_d = \frac{\sin(q\alpha/2)}{q\sin(\alpha/2)} = \frac{\sin(3 \times 20°/2)}{3\sin(20°/2)} = 0.9598$$

Total turns per phase, $T = \frac{Z_{ph}}{2} = \frac{(180 \times 8)/3}{2} = 240$

The generated emf per phase,

$$E_{ph} = 4.44 k_p k_d f \Phi T = 4.44 \times 1 \times 0.9598 \times 50 \times 0.025 \times 240 = \mathbf{1278.5 \text{ V}}$$

(*c*) The line emf, $E_L = \sqrt{3}\, E_{ph} = \sqrt{3} \times 1278.5 = \mathbf{2214.3 \text{ V}}$

PROBLEM D-13

A 400-V, 10-kVA, 50-Hz, star-connected alternator has an effective armature resistance of 1.0 Ω. It generates an open circuit voltage per phase of 90 V with a field current of 1.0 A. During the short-circuit test, the short-circuit current in the armature is 15 A when the field current is 1.0 A. Calculate (*a*) the synchronous impedance, (*b*) the synchronous reactance. (*c*) If the alternator is supplying a load current of 15 A at 0.8 power factor lagging, to what value would the terminal voltage rise if the load is thrown off. (*d*) Calculate the percentage regulation at (*i*) 0.8 *pf* lagging, (*ii*) 0.8 *pf* leading, and (*iii*) unity *pf*.

Solution

(*a*) Synchronous impedance, $Z_s = \left.\frac{V_{oc}}{I_{sc}}\right|_{I_f \text{ same}} = \frac{90 \text{ V}}{15 \text{ A}} = \mathbf{6\ \Omega}$

(*b*) Synchronous reactance, $X_s = \sqrt{Z_s^2 - R^2} = \sqrt{6^2 - 1.0^2} = \mathbf{5.92\ \Omega}$

(*c*) Terminal voltage per phase, $V = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$

Solution The power output, $P_o = 20 \text{ hp} = 20 \times 746 \text{ W} = 14\,920 \text{ W}$

The mechanical losses, $P_m = 1 \text{ hp} = 1 \times 746 \text{ W} = 746 \text{ W}$

Rotor gross output or power developed = Rotor shaft output + Mechanical losses

$$P_d = P_o + p_m = 14\,920 + 746 = 15\,666 \text{ W}$$

Therefore, the rotor input power, $P_g = \dfrac{P_d}{(1-s)} = \dfrac{15666}{1-0.05} = 16\,491 \text{ W}$

The input power to the stator, P_{in} = Rotor input power + Stator loss

or $P_{in} = 16\,491 + 1000 = 17\,491 \text{ W}$

Hence, the efficiency of the motor,

$$\eta = \frac{P_o}{P_{in}} = \frac{14920}{17491} = 0.853 = \mathbf{85.3\ \%}$$

PROBLEM D-20

A 3-phase, 400-V, 50-Hz, 100-hp, star-connected induction motor has a star-connected slip-ring rotor with a transformation ratio T_1/T_2 of 2.5. The rotor has 0.02 Ω resistance and 0.6 mH inductance per phase. Assuming the stator losses to be negligible, calculate (*a*) *At starting* ($s = 1$): (*i*) the rotor starting current per phase with slip-rings short-circuited when switched on normal voltage, (*ii*) the rotor power factor, and (*b*) *At a slip of* 3 % ($s = 0.03$): (*i*) the rotor current per phase, and (*ii*) the rotor power factor.

Solution As the stator winding is star-connected, the normal applied voltage per phase,

$$V_1 = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

(*a*) *At standstill* ($s = 1$):

(*i*) Since the turns ratio $T_1/T_2 = 2.5$, the induced emf per phase in the rotor winding is given as

$$E_{20} = V_1 \times \frac{T_2}{T_1} = 231 \times \frac{1}{2.5} = 92.4 \text{ V}$$

Rotor reactance per phase, $X_{20} = 2\pi sfL = 2\pi \times 1 \times 50 \times 0.6 \times 10^{-3} = 0.1885\ \Omega$

Rotor impedance per phase, $Z_{20} = \sqrt{R_2^2 + X_{20}^2} = \sqrt{(0.02)^2 + (0.1885)^2} = 0.1896\ \Omega$

Rotor current, $I_{20} = \dfrac{E_{20}}{Z_{20}} = \dfrac{92.4}{0.1896} = \mathbf{487\ A}$

(*ii*) The rotor power factor, $pf = \dfrac{R_2}{Z_{20}} = \dfrac{0.02}{0.1896} = \mathbf{0.105}$

(*b*) *At a slip of* 3 % ($s = 0.03$):

(*i*) The voltage induced per phase in the rotor,

$$E_2 = sE_{20} = 0.03 \times 92.4 = 2.77 \text{ V}$$

Rotor reactance, $X_2 = sX_{20} = 0.03 \times 0.1885 = 0.00566\ \Omega$

Rotor impedance, $Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(0.02)^2 + (0.00566)^2} = 0.0208\ \Omega$

Rotor current, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{2.77}{0.0208} = \mathbf{133\ A}$

(*ii*) Rotor power factor, $pf = \dfrac{R_2}{Z_2} = \dfrac{0.02}{0.0208} = \mathbf{0.96}$

where K_1 is another constant. The maximum torque occurs when $R_2 = sX_{20}$. Therefore, the slip at which maximum torque occurs is given as

$$s_m = \frac{R_2}{X_{20}} = \frac{0.02}{0.3} = 0.0667$$

Hence, the speed at which the maximum torque occurs is given as

$$N = (1 - s_m)N_s = (1 - 0.0667) \times 1000 = \mathbf{933.3\ rpm}$$

(*b*) The maximum torque is obtained by putting the condition $R_2 = sX_{20}$ into Eq. (*i*),

$$\tau_m = \frac{K_1 sR_2}{R_2^2 + R_2^2} = \frac{0.0667\,K_1}{2\,R_2} = 1.6675K_1 \qquad (ii)$$

Given that full-load torque is obtained at a speed of 970 rpm or a slip of 0.03. Hence, from Eq. (*i*), we have

$$\tau_{FL} = \frac{K_1 sR_2}{R_2^2 + s^2X_{20}^2} = \frac{K_1 \times 0.03 \times 0.02}{(0.02)^2 + (0.03 \times 0.3)^2} = 1.2474K_1 \qquad (iii)$$

Hence, using Eqs. (*ii*) and (*iii*), the ratio,

$$\frac{\tau_m}{\tau_{FL}} = \frac{1.6675K_1}{1.2474K_1} = \mathbf{1.337}$$

(*c*) At starting, $s = 1$. Hence, from Eq. (*i*), the starting torque is given as

$$\tau_{st} = \frac{K_1 R_2}{R_2^2 + X_{20}^2} = \frac{0.02\,K_1}{(0.02)^2 + (0.3)^2} = 0.221K_1 \qquad (iv)$$

Hence, using Eqs. (*iii*) and (*iv*), the ratio,

$$\frac{\tau_{st}}{\tau_{FL}} = \frac{0.221\,K_1}{1.2474\,K_1} = \mathbf{0.177}$$

PROBLEM D-23

A 3-phase, 220-V, 50-Hz, 4-pole, star-connected, squirrel-cage induction motor has rotor resistance of 0.1 Ω per phase and rotor standstill reactance of 0.8 Ω per phase. The ratio of rotor to stator turns is 0.65.

(*a*) At a slip of 5 %, calculate (*i*) the speed, (*ii*) the rotor current, (*iii*) the rotor input power, (*iv*) the total mechanical power developed, and (*v*) the total torque exerted on the rotor.

(*b*) Determine the slip at which maximum torque occurs and then at this slip, calculate (*i*) the speed, (*ii*) the rotor current, (*iii*) the rotor input power, (*iv*) the total maximum mechanical power developed, and (*v*) the total maximum torque exerted on the rotor.

Solution

Stator per phase voltage, $V_1 = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127$ V

The emf induced in rotor per phase at standstill, $E_{20} = V_1 \times \frac{T_2}{T_1} = 127 \times 0.65 = 82.55$ V

The synchronous speed, $N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500$ rpm

(*a*) *At 5 % slip*:

(*i*) The actual speed, $N = (1 - s)N_s = (1 - 0.05) \times 1500 =$ **1425 rpm**

(*ii*) Rotor impedance, $\mathbf{Z}_2 = (R_2 + jsX_{20}) = (0.1 + j0.05 \times 0.8) = (0.1 + j0.04)\ \Omega$

$= 0.1077\angle 21.8°\ \Omega$

The rotor current, $I_2 = \frac{E_2}{Z_2} = \frac{sE_{20}}{Z_2} = \frac{0.05 \times 82.55}{0.1077} = \mathbf{38.32\ A}$

Solution

(*a*) The voltage across the load terminals is 220 V and power supplied to it is 10 kW. Therefore, the load current,

$$I_L = \frac{P_L}{V_L} = \frac{10 \times 10^3}{220} = 45.5 \text{ A}$$

The voltage drop due to this load current flowing through the feeders is 45.5 × 0.1 = 4.55 V. Hence, the voltage across the terminals of the generator,

$$V = V_L + 4.55 = 220 + 4.55 = \mathbf{224.55\ V}$$

(*b*) The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{224.55}{100} = \mathbf{2.25\ A}$

(*c*) The armature current, $I_a = I_L + I_{sh} = 45.5 + 2.25 = 47.75$ A

The generated emf, $E = V + I_a R_a = 224.55 + 47.75 \times 0.05 = \mathbf{226.94\ V}$

PROBLEM D-27

A 4-pole, dc generator has 60 slots with 6 conductors per slot on its armature. (*a*) Find the useful flux per pole, if the generator gives an open-circuit voltage of 230 V, when the conductors are wave-connected and the generator is run at 750 rpm. (*b*) Keeping the flux constant, suggest a suitable modification in the armature so that the generator is able to produce an open-circuit voltage of 115 V, when it is run at the same speed.

Solution

(*a*) For wave-connected armature, $A = 2$.

Total number of conductors, $Z = 60 \times 6 = 360$

Since, $$E_g = \frac{\Phi ZNP}{60\,A} \Rightarrow \Phi = \frac{60\,E_g A}{NZP} = \frac{60 \times 230 \times 2}{750 \times 360 \times 4} = \mathbf{0.0256\ Wb}$$

(*b*) To meet the requirement of producing 115 V at the same speed, the simplest way is to change the number of conductors per path (Z/A), if it is feasible. As P, N and Φ are to remain constant, from the expression for induced emf, we have

$$(Z/A) \propto E_g \quad \Rightarrow \quad (Z/A)_2 = (Z/A)_1 \times \frac{E_2}{E_1} = (360/2) \times \frac{115}{230} = 90$$

It is possible to get 90 conductors per path on the armature if the 360 conductors are connected in lap winding (for which $A = P = 4$). Hence, the suggested modification is to connect the armature in lap winding.

PROBLEM D-28

The armature of a 4-pole, lap-wound, dc shunt generator has 120 slots with 4 conductors per slot. The flux per pole is 0.05 Wb. The armature resistance and the shunt-field resistance are 0.05 Ω and 50 Ω, respectively. Determine the speed of the machine when supplying a load current of 450 A at a terminal voltage of 250 V.

Solution Since the terminal voltage of the generator is 250 V and shunt-field resistance is 50 Ω, the shunt-field current is given as

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \text{ A}$$

Therefore, the armature current, $I_a = I_{sh} + I_L = 5 + 450 = 455$ A

∴ The generated emf, $E = V + I_a R_a = 250 + 455 \times 0.05 = 272.75$ V

The induced emf in the dc machine (a generator or a motor) is given

$$E = \frac{\Phi ZNP}{60\,A} \quad \text{or} \quad E \propto N = kN \quad \text{(as other parameters are constant)}$$

$$\therefore \qquad \frac{E_m}{E_s} = \frac{N_m}{N_g} \quad \Rightarrow \quad N_m = N_g \frac{E_m}{E_g} = 450 \times \frac{233.12}{247.12} = \mathbf{424.5\ rpm}$$

PROBLEM D-34

The open-circuit characteristic (OCC) of a separately excited dc generator driven at 750 rpm is given below.

Field current, I_f (A) :	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Generated emf, E_g(V) :	40	66	86	101	112	121	128	133

If this dc machine is connected as a dc shunt generator and is driven at 750 rpm, find (*a*) the open-circuit voltage when the field resistance is 94 Ω, (*b*) the additional resistance needed in the field circuit so as to reduce the open-circuit voltage to 110 V, and (*c*) the critical value of the shunt-field resistance.

Solution

(*a*) Using the given data, we first plot the open-circuit characteristic curve, as shown by thick line in Fig. D-7. We then plot the field-resistance line corresponding to the resistance of 94 Ω. For this, we take a suitable value of the field-current (say, 1 A) and find the corresponding shunt-field voltage, $V_{sh} = I_f R_{sh} = 1 \times 94 = 94$ V. When the machine is used as a shunt generator, the voltage V_{sh} is the same as the open-circuit voltage E_g. We mark the point A with coordinates (1.0 A, 94 V) in Fig. D-7. Joining point A with the origin O gives us the required field-resistance line, OA. This line intersects the OCC curve at point B. By projecting point B on E_g-axis, we get point C which gives the open-circuit voltage as **126 V**.

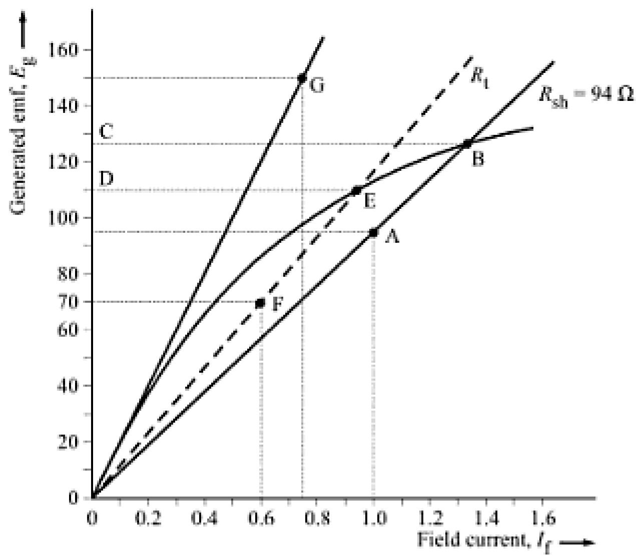


Fig. D-7 *The OCC curve for the dc generator.*

(*b*) Corresponding to E_g = 110 V (point D), a horizontal line is drawn which intersects OCC curve at point E. The line OE corresponds to the total field-resistance R_1 needed to generate E_g of 110 V. The resistance R_1 can be determined

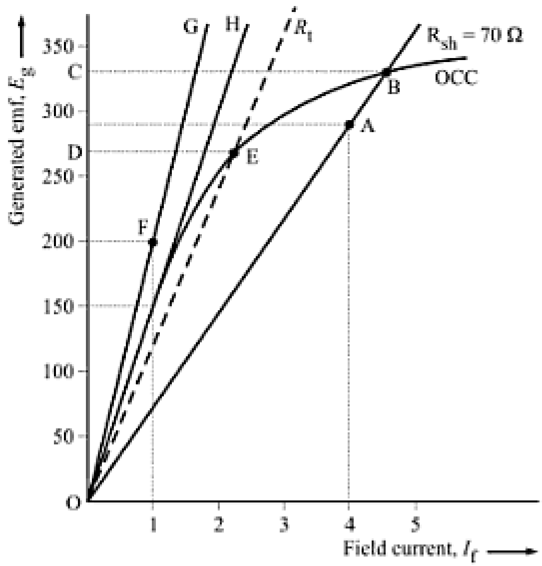


Fig. D-8 *The OCC curve for the dc generator running at 1250 rpm.*

Next, we mark the point F (1 A, 200 V) and join it to the origin to get the resistance line for R_{t1} = 200 Ω. We find that this line does not intersect the OCC curve anywhere. It means that in this case, the generator fails to build up voltage. So, $E_g \approx$ **0 V**.

(*d*) Starting from the origin, we draw the tangent OH to the OCC curve. This line corresponds to the critical field-resistance. Thus,

$$R_{fc} = \frac{155\text{ V}}{1\text{ A}} = \mathbf{155\ \Omega}$$

PROBLEM D-36

A 200-V dc shunt motor takes a total current of 100 A and runs at 750 rpm. The resistance of the armature winding and of shunt-field winding are 0.1 Ω and 40 Ω, respectively. The friction and iron losses amount to 1500 W. Calculate (*a*) the torque developed by the armature, (*b*) the copper losses, (*c*) the shaft power, (*d*) shaft torque, and (*e*) the efficiency.

Solution

(*a*) The shunt-field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{200}{40} = 5\text{ A}$

The armature current, $I_a = I_L - I_{sh} = 100 - 5 = 95\text{ A}$

The back emf, $E_b = V - I_a R_a = 200 - 95 \times 0.1 = 190.5\text{ V}$

Now, we have

Electrical input power = Mechanical power developed

or $$E_b I_a = \frac{2\pi N \tau_d}{60}$$

$\therefore$ Torque developed, $$\tau_d = \frac{60 E_b I_a}{2\pi N} = \frac{60 \times 190.5 \times 95}{2\pi \times 750} = \mathbf{230.4\ Nm}$$

(*b*) The back emf developed is given as $E_b = V - I_a R_a$. Multiplying this equation by I_a, and then rearranging the terms, we get

$$\text{The armature copper loss} = I_a^2 R_a = VI_a - E_b I_a = 200 \times 95 - 190.5 \times 95 = 902.5 \text{ W}$$

$$\text{The shunt-field copper loss} = I_{sh}^2 R_{sh} = 5^2 \times 40 = 1000 \text{ W}$$

$$\therefore \quad \text{Total copper losses} = 902.5 + 1000 = \mathbf{1902.5\ W}$$

(*c*) Total losses = Total copper losses + (Friction losses + Iron losses)

$$= 1902.5 + 1500 = 3402.5 \text{ W}$$

$$\text{Input power, } P_{in} = VI = 200 \times 100 = 20\,000 \text{ W}$$

$$\text{Output shaft power, } P_o = P_{in} - \text{losses} = 20\,000 - 3402.5 = 16\,598 \text{ W} \approx \mathbf{16.6\ kW}$$

(*d*) The shaft torque, $\tau_{sh} = \dfrac{60P_o}{2\pi N} = \dfrac{60 \times 16598}{2\pi \times 750} = \mathbf{211.25\ Nm}$

(*e*) The efficiency, $\eta = \dfrac{P_o}{P_{in}} = \dfrac{16598}{20000} = 0.8299 = \mathbf{82.99\ \%}$

PROBLEM D-37

A 100-V, dc series motor takes 45 A when running at 750 rpm. The armature resistance and series-field resistance are 0.22 Ω and 0.13 Ω, respectively. Windage loss, friction loss and iron losses amount to 750 W. Find (*a*) the shaft power, (*b*) the total torque, and (*c*) the shaft torque.

Solution

(*a*) The armature current, $I_a = I_L = 45$ A. Therefore, the back emf is given as

$$E_b = V - I_a(R_a + R_{se}) = 100 - 45 \times (0.22 + 0.13) = 84.25 \text{ V}$$

The mechanical power developed, $P_d = E_b I_a = 84.25 \times 45 = 3791.3$ W

The output power or shaft power is given as

$$P_{sh} = P_d - P_{losses} = 3791.3 - 750 = \mathbf{3041.3\ W}$$

(*b*) If the total torque developed at the armature is τ_d, then we can write

$$P_d = \frac{2\pi N \tau_d}{60} \quad \Rightarrow \quad \tau_d = \frac{60\,P_d}{2\pi N} = \frac{60 \times 3791.3}{2\pi \times 750} = \mathbf{48.27\ Nm}$$

(*c*) In terms of the shaft torque (τ_{sh}), the output power or shaft power is given as

$$P_{sh} = \frac{2\pi N \tau_{sh}}{60} \quad \Rightarrow \quad \tau_{sh} = \frac{60\,P_{sh}}{2\pi N} = \frac{60 \times 3041.3}{2\pi \times 750} = \mathbf{38.72\ Nm}$$

PROBLEM D-38

A 200-V, 4-pole, wave-wound, dc series motor having 1200 conductors on its armature and 20 mWb flux per pole, draws a current of 46 A for a given load. The armature resistance and series-field resistance are 0.25 Ω and 0.15 Ω, respectively. The iron and friction losses amount to 900 W. Find (*a*) the speed, (*b*) the total torque developed, (*c*) the shaft power, (*d*) the shaft torque, and (*e*) the efficiency.

Solution

(*a*) The armature current, $I_a = I_L = 46$ A. Therefore, the back emf is given as

$$E_b = V - I_a(R_a + R_{se}) = 220 - 46 \times (0.25 + 0.15) = 201.6 \text{ V}$$

Since, $$E_b = \frac{\Phi ZNP}{60A} \Rightarrow N = \frac{60AE_b}{\Phi ZP} = \frac{60 \times 2 \times 201.6}{20 \times 10^{-3} \times 1200 \times 4} = \mathbf{252\ rpm}$$

(*b*) The total torque developed is given as

$$\tau_d = \frac{\Phi ZPI_a}{2\pi A} = \frac{20 \times 10^{-3} \times 1200 \times 4 \times 46}{2\pi \times 2} = \mathbf{351.4\ Nm}$$

(*c*) The mechanical power developed, $P_d = E_b I_a = 201.6 \times 46 = 9273.6$ W

The output power or shaft power is given as

$$P_{sh} = P_d - P_{losses} = 9273.6 - 900 = 8373.6 \text{ W} = \mathbf{8.374\ kW}$$

(*d*) The shaft torque, $\tau_{sh} = \dfrac{60 P_{sh}}{2\pi N} = \dfrac{60 \times 8373.6}{2\pi \times 252} = \mathbf{317.3\ Nm}$

(*e*) The efficiency, $\eta = \dfrac{P_o}{P_{in}} = \dfrac{P_{sh}}{VI_L} = \dfrac{8373.6}{220 \times 46} = 0.8274 = \mathbf{82.74\ \%}$

PROBLEM D-39

A 230-V, dc series motor has an armature-circuit resistance of 0.2 Ω and the field resistance of 0.1 Ω. At rated voltage, the motor draws a line current of 40 A and runs at a speed of 1000 rpm. Find the speed of the motor for a line current of 20 A at 230 V. Assume that the flux at 20 A line current is 60 % of the flux at 40 A line current.

Solution *For line current of* 40 A: The back emf induced in the motor is given as

$$E_{b1} = V - I_{a1}(R_a + R_{se}) = 230 - 40 \times (0.2 + 0.1) = 218 \text{ V}$$

For line current of 20 A: The back emf induced,

$$E_{b2} = V - I_{a2}(R_a + R_{se}) = 230 - 20 \times (0.2 + 0.1) = 224 \text{ V}$$

Given that the flux per pole at 20 A line current is 60 % of the flux at 40 A line current, we have $\Phi_2 = 0.6\Phi_1$. Now, the back emf induced for a motor is given as

$$E_b = \frac{\Phi ZNP}{60A} \Rightarrow E_b \propto \Phi N = k\Phi N \quad \text{(as other parameters are constant)}$$

Therefore, taking the ratio of emfs in the two cases, we get

$$\frac{E_{b2}}{E_{b1}} = \frac{k\Phi_2 N_2}{k\Phi_1 N_1} = \frac{0.6\Phi_1 N_2}{\Phi_1 N_1} = \frac{0.6 N_2}{N_1}$$

$$\Rightarrow \quad N_2 = \frac{N_1}{0.6}\frac{E_{b2}}{E_{b1}} = \frac{1000}{0.6} \times \frac{224}{218} = \mathbf{1712.5\ rpm}$$

PROBLEM D-40

A 200-V, dc shunt motor has an armature resistance of 0.4 Ω and a field resistance of 200 Ω. The motor runs at 750 rpm and takes an armature current of 25 A. Assuming that the load torque remains constant, find the reduction in field-resistance necessary to reduce the speed to 500 rpm.

Solution The shunt-field current, $I_{sh1} = \dfrac{V}{R_{sh1}} = \dfrac{200}{200} = 1$ A

The back emf at 750 rpm, $E_{b1} = V - I_{a1}R_a = 200 - 25 \times 0.4 = 190$ V and shunt-field resistances are 0.04 Ω, 0.03 Ω and 60 Ω, respectively. Determine the emf generated.

[**Ans.** 114.07 V]

D-61. A 4-pole, 500-V, dc shunt motor has 720 wave-connected conductors on its armature. The full-load armature current is 60 A and the flux per pole is 0.03 Wb. The armature resistance is 0.2 Ω and the contact drop is 1 V per brush. Calculate the full-load speed of the motor. [**Ans.** 675 rpm]

D-62. Calculate the ohmic value of the starting resistance for the following dc shunt motor:

$$\begin{aligned} \text{Supply} &= 2240 \text{ V;} \\ \text{Output} &= 14\,920 \text{ W;} \\ \text{Armature resistance} &= 0.25\ \Omega; \\ \text{Efficiency at full load} &= 86\ \%. \end{aligned}$$

The starting current is to be limited to 1.5 times the full-load current. Ignore the current in the shunt winding. [**Ans.** 1.9632 Ω]

D-63. A dc series motor with series-field and armature resistances of 0.06 Ω and 0.04 Ω, respectively, is connected across 220 V mains. The motor runs at 900 rpm and the armature takes 40 A current. Determine its speed when the armature takes 75 A, assuming the field is increased by only 15 % due to saturation. [**Ans.** 770 rpm]

D-64. A 4-pole, 250-V, dc series motor has a wave-connected armature with 1254 conductors. The flux per pole is 22 mWb when taking 50 A. If the armature resistance and series-field resistance are each 0.2 Ω, calculate the speed. [**Ans.** 250 rpm]

(B) TRICKY PROBLEMS

D-65. A single-phase, 4-kVA transformer has 400 primary-turns and 1000 secondary-turns. The net cross-sectional area of the core is 60 cm^2. When the primary winding is connected to 500-V, 50-Hz supply, calculate (*a*) the maximum value of flux density in the core, (*b*) the voltage induced in the secondary winding, and (*c*) the secondary full-load current.

[**Ans.** (*a*) 0.938 T; (*b*) 1250 V; (*c*) 3.2 A]

D-66. A 4600-V/230-V, 60-Hz step-down transformer has core dimensions of 76.2 mm by 111.8 mm. A maximum flux density of 0.93 Wb/m^2 is to be used. Assuming 9 % loss of area due to stacking factor of laminations, calculate (*a*) the primary turns required, (*b*) the turns per volt, (*c*) the secondary turns required, and (*d*) the transmission ratio.

[**Ans.** (*a*) 2422; (*b*) 0.5265; (*c*) 121; (*d*) 0.05]

D-67. A 400-V/200-V, single-phase transformer is supplying a load of 25 A at a power factor of 0.855 lagging. On no load, the current and *pf* are 2.0 A and 0.208, respectively. Calculate the current taken from the supply and specify its phase.

[**Ans.** 13.92 A, 36.13° (lagging)]

D-68. An iron-cored transformer has 200 turns on the primary and 100 turns on the secondary. A supply of 400 V, 50 Hz is given to the primary and an impedance of $(4 + j3)\ \Omega$ is connected across the secondary. Assume ideal behaviour and calculate (*a*) the voltage and current through the load, (*b*) the primary current, (*c*) the power taken from the supply, and (*d*) the input impedance of the transformer.

[**Ans.** (*a*) 200 V, $40\angle{-36.87°}$ A; (*b*) $20\angle{-36.87°}$ A; (*c*) 6.4 kW; (*d*) $(16 + j12)\ \Omega$]

D-69. The iron loss of a 80-kVA, 1000-V/250-V, 1-phase, 50-Hz transformer is 800 W. When the primary winding carries a current of 50 A, the copper loss of the transformer is 400 W. Estimate (*a*) the area of cross-section of the limb, if the working flux density is 1 tesla and there are 1000 turns on primary, (*b*) the current ratio (primary to secondary), (*c*) the efficiency at full load 0.8 *pf*, and (*d*) the efficiency for a load when the copper loss becomes equal to the iron loss and *pf* remains 0.8 lagging.

[**Ans.** (*a*) 45 cm^2; (*b*) 0.25; (*c*) 97.23 %; (*d*) 97.25 %]

D-70. The SC test on a single-phase transformer with primary winding short-circuited and 30 V applied to the secondary gives a wattmeter reading of 60 W and secondary current of 10 A. If the normal primary voltage is 200 V, the transformer ratio is 1 : 2 and the full-load secondary current is 10 A, calculate the secondary potential difference at full-load current for (*a*) unity *pf*, (*b*) 0.8 *pf* lagging.

[**Ans.** (*a*) 394 V; (*b*) 377.56 V]

synchronous reactance of 30 Ω. Find the power supplied to the motor and the induced emf.

[**Ans.** 914.5 kW, 13 041 V]

D-85. A 3-phase, 400-V, 50-Hz line supplies power to a 3-phase, star-connected induction motor which runs at almost 1000 rpm on no load and at 960 rpm on full load. The motor has stator to rotor turns ratio of 1 : 0.4. Find (*a*) the number of poles, (*b*) the slip at full load, (*c*) the frequency of rotor emf, (*d*) the speed of rotor mmf with respect to the rotor, (*e*) the speed of rotor mmf with respect to the stator, (*f*) the speed of rotor mmf with respect to the stator mmf, (*g*) the rotor induced emf at standstill, and (*h*) the rotor induced emf while the motor is running on full load.

[**Ans.** (*a*) 6; (*b*) 0.04; (*c*) 2 Hz; (*d*) 40 rpm; (*e*) 1000 rpm; (*f*) 0 rpm; (*g*) 92.4 V; (*h*) 3.69 V]

D-86. A 6-pole, 410-V, 50-Hz, 3-phase induction motor has a speed of 950 rpm on full load. Calculate the slip. How many complete alternations will the rotor-voltage make per minute? [**Ans.** 5 %, 150 c/min]

D-87. A 3-phase, 12-pole generator driven at a speed of 500 rpm, supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor is 3 % on full load, calculate the full-load speed of the motor. [**Ans.** 727.5 rpm]

D-88. A 3-phase, 400-V, 6-pole, 50-Hz induction motor develops 5 hp at 950 rpm. What is the stator input and efficiency, if the stator loss is 300 W?

[**Ans.** 4226.3 W; 88.27 %]

D-89. The power input to a 3-phase induction motor is 60 kW. The stator losses total to 1 kW. Find the total mechanical power developed and the rotor copper loss per phase, if the motor is running at 3 % slip. [**Ans.** 57.231 kW; 590 W]

D-90. A 3-phase, 4-pole, 400-V, 50-Hz, 30-hp induction motor operates at an efficiency of 0.8 with a power factor of 0.85 lagging. Calculate the current drawn by the induction motor from the mains.

[**Ans.** 47.5 A]

D-91. In a certain power station, the alternator is a 6-pole machine driven at 1000 rpm. A 3-phase, 140-hp, 16-pole induction motor supplied from this alternator runs with a slip of 2.5 %. What is the actual speed of the motor? If the motor delivers full load at a speed of 360 rpm, what is the percentage slip of the motor? [**Ans.** 365.6 rpm, 4 %]

D-92. A 3-phase, 6-pole, 400-V, 50-Hz induction motor develops 3.85 kW (including friction and windage losses) at a speed of 950 rpm. If the stator loss is 260 W, find the stator input power.

[**Ans.** 4.313 kW]

D-93. A 3-phase, 6-pole induction motor develops maximum torque when running at 1000 rpm and fed from a 60-Hz supply. The rotor resistance per phase is 1.1 Ω. Calculate the speed at which the motor will develop maximum torque when operated from a 50-Hz supply. [**Ans.** 800 rpm]

D-94. An 8-pole, 3-phase alternator is coupled to an engine running at 750 rpm. The alternator supplies power to an induction motor which has a full-load speed of 1425 rpm. Find the percentage slip and the number of poles on the motor. [**Ans.** 5 %, 4]

D-95. A 110-V, 6-pole, lap-wound, dc shunt generator delivers a load current of 50 A. The armature resistance is 0.2 Ω, and the field-circuit resistance is 55 Ω. The armature has 360 conductors and is rotating at 1800 rpm. Calculate (*a*) the no load voltage at the armature, and (*b*) the flux per pole.

[**Ans.** (*a*) 120.4 V; (*b*) 11.148 mWb]

D-96. The armature of a 4-pole, lap-wound dc shunt generator has 120 slots with 4 conductors per slot. The flux per pole is 0.05 Wb. The armature resistance is 0.05 Ω and shunt-field resistance is 50 Ω. Find the speed of the machine when supplying 450 A at a terminal voltage of 250 V.

[**Ans.** 681.88 rpm]

D-97. A 4-pole, dc generator has a total of 500 conductors on its armature and a magnetic flux per pole of 0.01 Wb crossing the air gap with normal excitation. (*a*) What voltage will be generated at a speed of 900 rpm, if the armature is (*i*) wave-wound, (*ii*) lap-wound. (*b*) If the allowable current is 5 A per path, what will be the power in kW generated in each case?

[**Ans.** (*a*) (*i*) 150 V, (*ii*) 75 V; (*b*) (*i*) 1.5 kW, (*ii*) 1.5 kW]

D-98. A 20-kW, 440-V, short-shunt compound, dc generator has a full-load efficiency of 81 %. If the resistance of the armature, series field and shunt-field are 0.4 Ω, 0.25 Ω and 240 Ω, respectively, calculate the combined bearing, windage and core calculate the speed of the generator.

[**Ans.** 1041 rpm]

D-136. A 440-V, compound generator has armature, series-field and shunt-field resistances of 0.5 Ω, 1 Ω and 200 Ω, respectively. Calculate the generated voltage while delivering 40 A to the external circuit for (*a*) the long-shunt, and (*b*) short-shunt connections. [**Ans.** (*a*) 503.3 V; (*b*) 501.2 V]

D-137. A short-shunt compound, dc generator supplies a current of 100 A at a voltage of 220 V. The resistances of the shunt-field, series-field and armature are 50 Ω, 0.025 Ω and 0.05 Ω, respectively. The total brush drop is 2 V, and the iron and friction losses amount to 1 kW. Determine (*a*) the generated emf, (*b*) the copper losses, (*c*) the output of the prime-mover driving the generator, and (*d*) the generator efficiency.

[**Ans.** (*a*) 229.72 V; (*b*) 1785.6 W; (*c*) 24.786 kW; (*d*) 88.76 %]

D-138. A dc long-shunt compound generator has an armature resistance of 0.03 Ω, a series-field resistance of 0.04 Ω and a shunt-field resistance of 55 Ω. It supplies a load at 110 V through a pair of feeders of total resistance 0.04 Ω. The load consists of five motors, each taking 30 A and a lighting load of 150 bulbs each of 60 W. Calculate (*a*) the load current, (*b*) the terminal voltage, and (*c*) the emf generated.

[**Ans.** (*a*) 231.8 A; (*b*) 119.27 V; (*c*) 135.65 V]

D-139. The open-circuit characteristic (OCC) of a dc generator at 1000 rpm is given below:

Field current, I_f(A)	: 0.5	1.0	1.5	2.0	2.5	3.0	3.5
Generated emf, E_g(V)	: 60	120	138	145	149	151	152

The machine is connected as a dc shunt generator and is driven at 1000 rpm. The armature resistance and the shunt-field resistance are 0.1 Ω and 60 Ω, respectively. Find (*a*) the open-circuit voltage, (*b*) the critical value of the field resistance, (*c*) the terminal voltage when a load of 4.0 Ω is connected, and (*d*) the load current for which the terminal voltage is 100 V.

[**Ans.** (*a*) 150 V; (*b*) 120 Ω; (*c*) 146.1 V; (*d*) 498.3 A]

D-140. A 4-pole, wave-wound, 60-kW, 400-V, dc shunt motor has 450 conductors on its armature and 45 mWb flux per pole. The armature resistance and shunt-field resistance are 0.1 Ω and 200 Ω, respectively. The full-load efficiency of the motor is 90.5 %. Determine for full load, (*a*) the speed, (*b*) the armature torque, and (*c*) the useful torque.

[**Ans.** (*a*) 568.34 rpm; (*b*) 1055.5 Nm; (*c*) 1008.1 Nm]

D-141. A 220-V, dc shunt motor draws 4.5 A on no load and runs at 1000 rpm. The resistances of the armature and shunt-field are 0.3 Ω and 157 Ω, respectively. Calculate the speed when loaded and drawing a current of 30 A. Assume that the armature reaction weakens the field by 3 %. [**Ans.** 995 rpm]

D-142. A dc shunt motor draws 230 A at 110 V when running at a speed of 450 rpm. It has an armature resistance of 0.03 Ω and a shunt-field resistance of 45 Ω. (*a*) Assuming that the flux is proportional to the field current, what will be its speed when drawing 85 A at 210 V? (*b*) Determine the speed at which it would have to run as a generator to give 140 A at 200 V.

[**Ans.** (*a*) 465.8 rpm; (*b*) 486.5 rpm]

D-143. A 220-V, dc shunt motor has an armature resistance of 0.05 Ω and a shunt-field resistance of 110 Ω. When taking a full-load current of 32 A, it runs at 850 rpm. Assuming that the field flux is proportional to the field current, calculate the speed at which the motor runs, if (*a*) a 1.5-Ω resistor is connected in series with the armature, and (*b*) a 30-Ω resistor is connected in series with the field winding. The load torque remains the same throughout.

[**Ans.** (*a*) 663.4 rpm; (*b*) 1061 rpm]

D-144. A 20-hp, 230-V, 1150-rpm, 4-pole, dc shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature resistance of 0.2 Ω. When it delivers rated power at rated speed, it draws a line current of 74.8 A and field current of 3.0 A. Calculate (*a*) the flux per pole, (*b*) the torque developed, (*c*) the rotational losses, and (*d*) the total losses expressed as a percentage of power.

[**Ans.** (*a*) 9.073 mWb ; (*b*) 128.56 Nm; (*c*) 561.2 Nm; (*d*) 15.29 %]

D-145. A 250-V, dc shunt motor has an armature resistance of 0.5 Ω and is excited to give constant main field. At full-load, the motor runs at 400 rpm and takes an armature current pf 30 A. If a resistance of 1 Ω is placed in series with the armature, find (*a*) the

ELECTRICAL MEASURING INSTRUMENTS

18

OBJECTIVES

After completing this Chapter, you will be able to:

- *State and define the two broad categories of measuring instruments: (i) absolute, and (ii) secondary.*
- *Differentiate between the three types of secondary instruments: (i) indicating, (ii) recording, and (iii) integrating instruments.*
- *State different effects of electric current and voltage that can be used as principles of operation of different instruments.*
- *State the names of the essential torques required for satisfactory working of an indicating instrument.*
- *Explain different ways of obtaining the (i) deflecting torque, (ii) the controlling torque, and (iii) damping torque.*
- *Explain the meaning of under damped, over damped, and critically damped systems.*
- *Explain the construction and working of permanent-magnet moving coil* (PMMC) *instruments.*
- *Explain the construction and working of dynamometer type moving coil instruments.*
- *Explain the construction and working of moving-iron instruments.*
- *Explain how to convert a galvanometer into an ammeter having desired range.*
- *Explain how to use a universals shunt to extend current ranges.*
- *Explain how to convert a galvanometer into a voltmeter having desired range.*
- *Explain the construction and working of a multimeter.*
- *Explain why the resistance scale on a multimeter is inverted.*
- *Explain the construction and working of a wattmeter.*
- *Explain the construction and working of an energy meter.*

18.1 CLASSIFICATION OF INSTRUMENTS

Various measuring instruments can be classified into two broad categories:

(*i*) Absolute instruments
(*ii*) Secondary instruments

Absolute Instruments Such instruments give the magnitude of the quantity being measured in terms of the constants of the instruments. For example, a *tangent galvanometer*, as it measures current in terms of the tangent of the angle of deflection produced by the current, radius and number of turns of the galvanometer and the horizontal component of the earth's magnetic field. In practice, we seldom use an absolute instrument.

Secondary Instruments The deflection of such instruments directly give us the magnitude of the quantity being measured. These have to be calibrated by comparison with an absolute instrument or with another secondary instrument which has already been calibrated beforehand. Such instruments are in most common use in laboratories, industries and power stations, etc. Secondary instruments are further divided into three groups:

(*i*) Indicating instruments
(*ii*) Recording instruments
(*iii*) Integrating instruments

Indicating Instruments These instruments given the magnitude of the quantity at the time when it is being measured. The measurement is indicated by a pointer moving over a marked (graduated) dial. Ordinary voltmeters, ammeters, wattmeters belong to this group.

Recording Instruments These instruments give a continuous record of the quantity being measured over a given period. The variations of the quantity being measured are recorded by a pen. This inked pen lightly touches a sheet of paper wrapped over a drum. The drum rotates slowly at uniform speed. The pen is deflected by the magnitude of the quantity being measured. Thus, a curve is traced. Such an instrument is used by doctors to give ECG (Electro-Cardio-Gram) of a patient.

Integrating Instruments Such instruments give total amount of quantity being measured over a period of time. The summation given by such an instrument is the product of time and an electrical quantity. Ampere-hour meter, watt-hour (energy) meter and odometer in a car (which measures the total distance covered) are examples of this category. The summation value is generally given by a register consisting of a set of pointers and dials, or an electronic digital display.

18.2 PRINCIPLE OF OPERATION

An electrical measuring instrument depends for its operation on one of the many effects produced by current or voltage. For example, an ordinary ammeter or voltmeter utilizes magnetic effect of electric current. When the current is made to pass through a coil (placed in magnetic field due to a permanent magnet), a magnetic field is produced. This field interacts with the field already existing. Due to such an interaction, the coil rotates and the quantity of current is indicated on a dial.

Table 18.1 gives the effects utilized by various types of instruments.

Table 18.1 *Instruments using various effects of electric current.*

No.	*Effect*	*Instrument*	*Suitable for measurement of*
1.	Magnetic effect	Ammeters, voltmeters wattmeters, integrating meters	Current, voltage, power, energy (ac and dc systems)
2.	Thermal effect	Ammeters, voltmeters	Current, voltage (ac and dc systems)
3.	Chemical effect	Integrating meters	DC ampere hour
4.	Electrostatic effect	Voltmeters	Voltage (ac and dc systems)
5.	Electromagnetic induction effect	Ammeters, voltmeters, wattmeters, energy meters	Current, voltage, power, energy (ac systems only)

18.3 ESSENTIALS OF AN INSTRUMENT

An indicating instrument (such as an ordinary voltmeter) has an indicating mechanism. The moving system is usually carried by a spindle of hardened steel, having its ends tapered and highly polished to form pivots. These pivots rotate in hollow-ground bearings to reduce friction. Attached to the spindle is a light-weight (usually made of aluminium) pointer that can move over a calibrated scale. A strip of mirror is usually fitted on the plate which bears the scale to avoid the parallax error in reading the deflection on the scale.

In some instruments, the moving system is attached to thin threads of phosphor-bronze, held taut by tension springs mounted on the frame. This arrangement eliminates pivot friction and the instrument becomes less susceptible to damage by shock and vibrations. For satisfactory working of an indicating instrument, following types of torques (forces) are required.

(1) Deflecting Torque

When the instrument is connected in a circuit, the pointer should move from its zero position. The pointer is made to move because of a force or a torque produced. This deflecting torque can be produced by any of the effects of current (or of voltage) given in Table 18.1. The deflecting torque works on the moving system to which the pointer is attached. Obviously, the magnitude of deflecting torque (τ_d) produced is proportional to the magnitude of the quantity being measured, say, the current I flowing through the instrument.

(2) Controlling Torque

This torque, also called ***restraining torque*** or ***restoring torque***, opposes the deflecting torque. The magnitude of the controlling torque is proportional to the deflection of the pointer. When the pointer is at its zero position, the controlling torque is zero. But, as the deflection increases, the magnitude of the controlling torque also increases. At some position of the pointer, the controlling torque becomes equal to the deflecting torque. The pointer then stops moving further. Thus, the final position of the pointer gives the magnitude of the quantity being measured.

When the instrument is disconnected from the circuit, the deflecting torque vanishes. It is due to the controlling torque that the pointer comes back to its zero position. If the controlling torque were not there, the pointer would have remained in the deflected position even after the instrument was disconnected from the circuit.

Thus, the controlling torque serves two functions: (*i*) the pointer stops moving beyond the final deflection, (*ii*) the pointer comes back to its zero position when the instrument is disconnected.

There are following two ways of obtaining controlling torque (τ_c).

(*i*) *Spring Control* This is the most commonly used method of obtaining the controlling torque τ_c. In cheaper instruments, only one hairspring made of phosphor bronze is used (Fig. 18.1*a*). The outer end of this spring is fixed and the inner end is attached with the spindle. When the pointer is at zero of the scale, the spring is normal. As the pointer moves, the spring winds and produces an opposing torque. This torque τ_c is proportional to the angle of deflection. The figure also shows a balance weight attached to the pointer. This is meant to balance the moving system so that its centre of gravity coincides with the axis of rotation, thereby reducing the friction between the pivot and bearings.

A better arrangement utilizes two flat spiral hairsprings, A and B, as shown in Fig. 18.1*b*. The inner ends of these springs are attached to the spindle S. The outer end of B is fixed, whereas that of A is attached to one end of lever L, pivoted at P. By moving sideways the other end of lever L, the zero of the pointer can be easily adjusted.

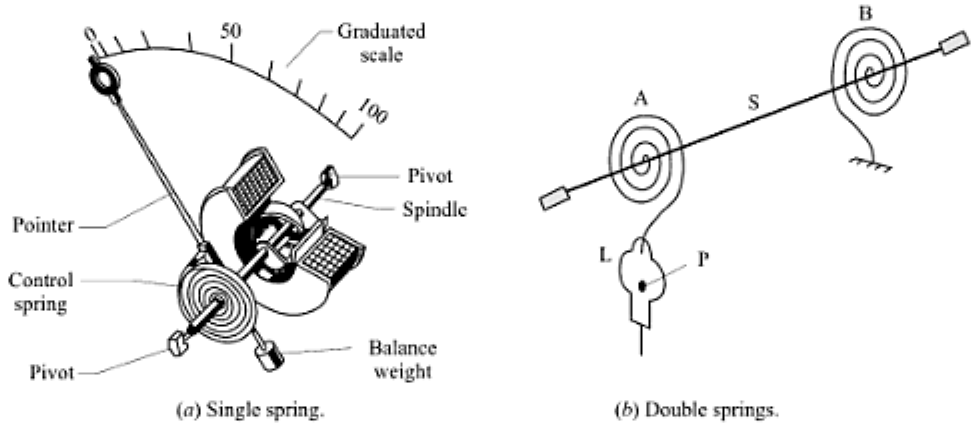


Fig. 18.1 *Spring control for obtaining controlling torque.*

The two springs A and B are wound in opposite directions so that when the moving system is deflected, one spring winds while the other unwinds. The controlling torque produced is due to the combined torsions of the two springs. To make the controlling torque directly proportional to the angle of deflection, the springs should have fairly large number of turns so that deformation per unit length remains small. The springs should be stressed well below the elastic limit at the maximum deflection of the instrument in order to avoid occurrence of zero-shift.

Since the controlling torque τ_c is directly proportional to the angle of twist θ, the deflecting torque τ_d is proportional to the current I, and $\tau_d = \tau_c$, the angle $\theta \propto I$. Hence, these instruments have uniform scale.

The spring control has some *disadvantages*. The stiffness of the spring is a function of temperature. Hence, the readings given by the instruments are temperature dependent. Furthermore, with the usage the spring develops an inelastic yield which affects the zero position of the moving system. The gravity control, discussed next, does not have these disadvantages.

(ii) Gravity Control A small control weight is attached to the moving system, as shown in Fig. 18.2*a*. In addition, an adjustable balance weight is also attached to make the centre of gravity pass through the spindle. In zero position of the pointer, this control weight is vertical. When deflected by an angle θ, the weight exerts a force $W \sin \theta$, as shown in Fig. 18.2*b*. The restraining or controlling torque thus developed is given as

$$\tau_c = (W \sin \theta) \times L = WL \sin \theta$$

If the deflecting torque, τ_d is proportional to the current I passing through the instrument, then at the final deflected position, we have

$$\tau_c = \tau_d \quad \text{or} \quad WL \sin \theta = kI \quad \Rightarrow \quad I = \left(\frac{WL}{k}\right) \sin \theta \quad \text{or} \quad I \propto \sin \theta$$

Hence, the gravity control instruments have the draw back of not having uniform scale. Another draw-back is that these instruments must be used in vertical position so that the control may operate properly.

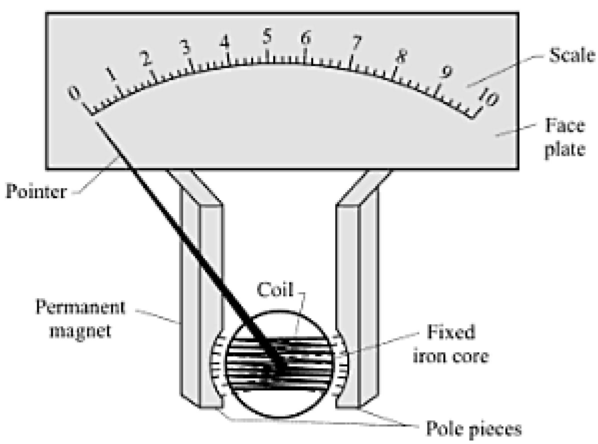
18.4 MOVING COIL INSTRUMENTS

There are two types of moving coil instruments:

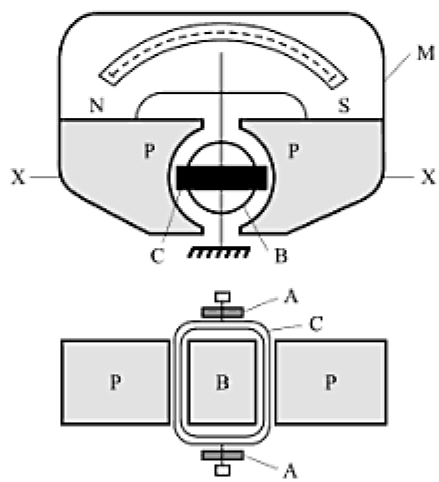
(1) ***Permanent Magnet Type*:** It is the most accurate and useful for dc measurements.
(2) ***Dynamometer Type*:** It can be used for both dc and ac measurements.

18.5 PERMANENT MAGNET MOVING COIL (PMMC) INSTRUMENTS

The basic PMMC instrument was developed in 1881 by Jacques Arsene d'Arsonval. It is popularly known as ***d'Arsonval movement***. As shown in Fig. 18.6*a*, it consists of an iron-core coil mounted on bearings between a permanent magnet. The coil uses a very fine insulated wire of many turns. It is wound on an aluminium bobbin which is free to rotate by about 90°. An aluminium pointer attached to the coil can move on a calibrated scale. There are two spiral springs (not shown in the figure) attached to the coil assembly—one at the top and the other at the bottom. These springs serve two purposes. Firstly, they provide a path for the current to reach the coil. Secondly, they keep the pointer at the low end of the scale (zero) when there is no current through the coil; and they provide controlling torque when current flows through the coil.



(*a*) The d'Arsonval movement.



(*b*) The front view and sectional plan along X-X.

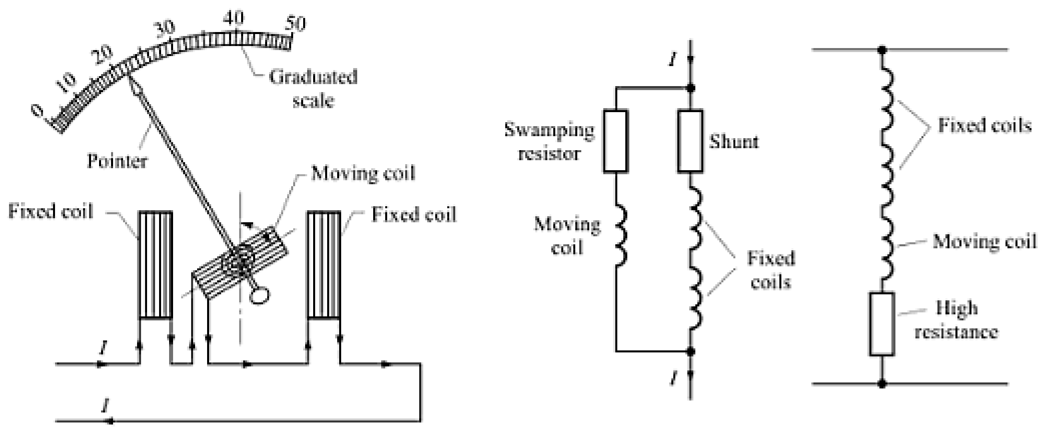
Fig. 18.6 *The permanent-magnet moving coil (PMMC) instrument.*

The poles of the magnet are usually cylindrical in shape. The core is of soft iron and is also cylindrical. This has two advantages. Firstly, the length of the air gap is reduced so that the magnetic flux linking the coil increases. Secondly, the iron core helps in making the field (of the permanent magnet) radial in the air gap. This ensures a uniform magnetic field throughout the motion of the coil. This way, the angle of deflection of the coil becomes directly proportional to the current in the coil. As a result, the scale is uniform throughout.

Due to the development of high coercive-force modern steel alloys, such as Alcomax (iron, aluminium, cobalt, nickel and copper), it has become possible to have short magnets giving variety of arrangements of the

18.6 DYNAMOMETER TYPE INSTRUMENTS

These instruments are similar to the permanent magnet type instruments, except that the permanent magnet is replaced by a fixed coil. The coil is divided into two halves, connected in series with the moving coil. These coils are usually air-cored*. The two halves of the coil are placed close together and parallel to each other, to provide uniform field within the range of the movement of moving coil, as shown in Fig. 18.9*a*.



(*a*) Construction. (*b*) An ammeter. (*c*) A voltmeter.

Fig. 18.9 *Dynamometer moving coil instrument.*

The deflecting torque depends on the fields of both fixed and moving coils. That is, in an ammeter, the torque is roughly proportional to the current squared. The instrument, therefore, has *square-law* response. The deflection is proportional to *the mean value of the square of the current*. When used on ac, it indicates rms or effective value of current (or voltage).

The moving coil of the instrument is wound using a thin wire so that it deflects easily. Therefore, it cannot carry high currents. If the instrument is to be used for the measurement of high currents, the fixed coils are wound using thick wires. The connections are made so that more current flows through the fixed coils and less current goes through the moving coils. As shown in Fig. 18.9*b*, a shunt (of low resistance) is connected in series with the fixed coil. The moving coil in series with the swamping resistor (of relatively high value) is connected across the shunt and fixed coils.

When used as a voltmeter, the coils are wound using high-resistance fine wire. Both the fixed and moving coils are connected in series along with a non-inductive resistor (of high resistance), as shown in Fig. 18.9*c*.

Though this instrument can be used as an ammeter or a voltmeter, but it works very well as a power meter (i.e., wattmeter).

The main ***advantages*** of the dynamometer moving coil instruments are :

(*i*) Can be used on both dc as well as ac systems.
(*ii*) No errors due to hysteresis or eddy currents.

* The use of iron-core is usually avoided, as the iron-core introduces hysteresis, eddy currents and other errors when the instrument is used for ac.

(*iii*) Ammeters up to 10 A and voltmeters up to 600 V can be measured with good accuracy.
(*iv*) Same calibration for dc and ac measurements, and hence can be used as ***transfer*** instruments.

The main ***disadvantages*** of the dynamometer moving coil instruments are:

(*i*) The scale is not uniform, as they have square-law response.
(*ii*) Since air-cored coils are used, the magnetic field produced is weak. Hence, a large number of ampere-turns must be used on the moving coil to give necessary deflecting torque. This results in a heavy moving system and hence the torque/weight ratio is small.
(*iii*) Heavy moving system gives more friction losses.
(*iv*) Screening against stray magnetic fields required.
(*v*) Lower sensitivity than that of PMMC instruments.
(*vi*) More expensive than the PMMC instruments.

18.7 MOVING-IRON INSTRUMENTS

These instruments can be manufactured with required accuracy, and are robust and quite inexpensive. These are commonly used in laboratories and switch-boards. They work on both the dc as well as ac systems.

The current to be measured (or a current proportional to the voltage to be measured) is passed through a coil of wire. A certain number of ampere-turns is required for the operation of the instrument. Hence, the number of turns of the coil depends upon the current to be passed through it. There are two general types of such instruments: (*i*) the *attraction type*, and (*ii*) the *repulsion type*.

Attraction (or Single-iron) Type Moving-Iron Instrument

As shown in Fig. 18.10, it consists of an air-cored coil and a moving iron piece to which a pointer is attached. When the current flows through the coil, a magnetic field is set up which attracts the iron price. It moves from the weaker magnetic field outside the coil into the stronger field inside it. The shape of the iron piece is specially designed to make the scale as nearly uniform as possible. In Fig. 18.10, gravity control has been shown. Nowadays, spring control instruments are preferred.

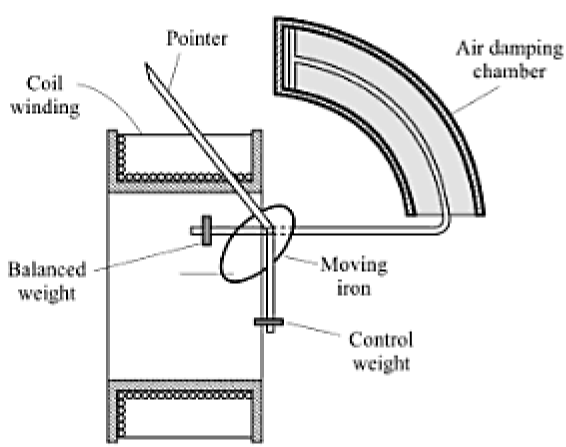


Fig. 18.10 *Attraction type moving-iron instrument.*

(*ii*) Robust and simple in construction and hence quite inexpensive.

(*iii*) Can withstand overloads momentarily.

(*iv*) High deflecting torque.

The main ***disadvantages*** of these instruments are :

(*i*) The scale is not uniform.

(*ii*) The power consumption (for low voltages) is high.

(*iii*) Errors are caused due to hysteresis in the iron.

(*iv*) Operation affected by stray magnetic fields.

(*v*) Change in frequency in case of ac measurements causes serious errors.

18.8 AMMETERS AND VOLTMETERS

Ammeters and voltmeters operate on the same principle. An ammeter carries the current (or its definite fraction) to be measured and produces the deflecting torque. A voltmeter carries a current proportional to the voltage to be measured and produces the deflecting torque.

An ***ammeter*** is connected in ***series*** with the circuit carrying the current to be measured. It must have ***very low resistance*** so that the voltage drop across the ammeter and the power absorbed from the circuit are as low as possible. *Ideally, an ammeter should have zero resistance.*

A ***voltmeter*** is connected in ***parallel*** with the circuit across which the voltage is to be measured. It must have ***very high resistance*** so that the current taken by the voltmeter and the power absorbed from the circuit are as low as possible. *Ideally, a voltmeter should have infinite resistance.*

It is impractical in most of the cases, and dangerous in some, for an instrument designed and manufactured as an ammeter to be employed as voltmeter, or vice versa. For example, the low-resistance winding of an ammeter will suffer serious damage if directly connected across a high-voltage.

The range of an electrical measurement is limited by the current which can be safely carried by the coil of the instrument. For example, the moving coil and the spiral springs used for coil-connections can be designed for a maximum current of about 100 mA. An instrument, such as a d'Arsonval movement, can be used either as an ammeter or as a voltmeter, of different ranges, just by suitably connecting pre-determined resistances.

Consider a d'Arsonval movement having ***current sensitivity*** (CS) of 0.1 mA and internal resistance (R_m) of 500 Ω. This instrument gives full scale deflection when a current of 0.1 mA flows through it. Thus, we can say that the ***full-scale deflection current***, I_m, for this instrument is 0.1 mA. When full-scale current flows, the voltage across its terminals is given as

$$V_m = I_m \times R_m = (0.1 \text{ mA}) \times (500\ \Omega) = 50 \text{ mV}$$

It means that this instrument, without any additional circuitry, can serve either as an ammeter of range 0-0.1 mA, or as a voltmeter of range 0-50 mV. However, in practice we need ammeters and voltmeters of much higher ranges. Therefore, we need to extend the range of the meter, by providing a suitable additional circuitry.

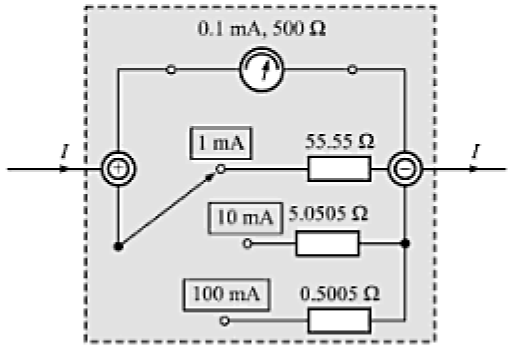


Fig. 18.13 *Construction of a multi-range ammeter.*

EXAMPLE 18.2

A meter movement with full-scale-deflection current of 100 μA and internal resistance of 100 Ω is required to measure a maximum current of 10 mA. Determine the shunt resistance needed.

Solution The current through the shunt,

$$I_{sh} = I_{fsd} - I_m = 10 \text{ mA} - 100 \text{ μA} = 9.9 \text{ mA}$$

Since the voltage drop across the meter movement should be the same as that across the shunt, we have

$$I_m R_m = I_{sh} R_{sh} \quad \text{or} \quad R_{sh} = \frac{I_m R_m}{I_{sh}} = \frac{100 \times 100 \times 10^{-6}}{9.9 \times 10^{-3}} = \mathbf{1.010101\ \Omega}$$

In the above Example, we had used a basic meter movement of 100 μA, 100 Ω. To extend its range to 10 mA, we need a shunt resistance of 1.010101 Ω. In case, we wish to extend the range to 100 mA, we would need a shunt of 0.11001001 Ω (as calculated using Eq. 18.1). It is impracticable to obtain resistors of such low values and that too so precise and accurate. A very novel idea, called *universal-shunt* or *ring-shunt* method, is used for solving this problem.

Universal Shunt for Extending Current Ranges

We have seen that for extending the range of an ammeter to high values, we need shunt resistances of impractically low values. If somehow we increase the value of R_m as we go for higher ranges, the value of the required shunt resistance need not be so low. This is exactly what we do in *universal-shunt*, as shown in Fig. 18.14. Here, the shunt resistors R_1, R_2, R_3, R_4, and R_5 form a *ring* with the basic meter.

When the range-switch is at 1-mA range, the full-scale current entering the ammeter divides into two parts at point A. The part through the meter branch is I_m and the remaining current bypasses through the shunt path consisting of five resistors in series. When the range-switch is brought at 10-mA range, the current entering divides at point B. But now the meter branch consists of R_m in series with resistor R_1. The shunt branch consists of only the remaining four resistors. Similarly, when the range-switch is at 1-A range, the current divides at point E. The meter branch now consists of R_m in series with resistors R_1, R_2, R_3 and R_4, and the shunt branch is made of only single resistor R_5.

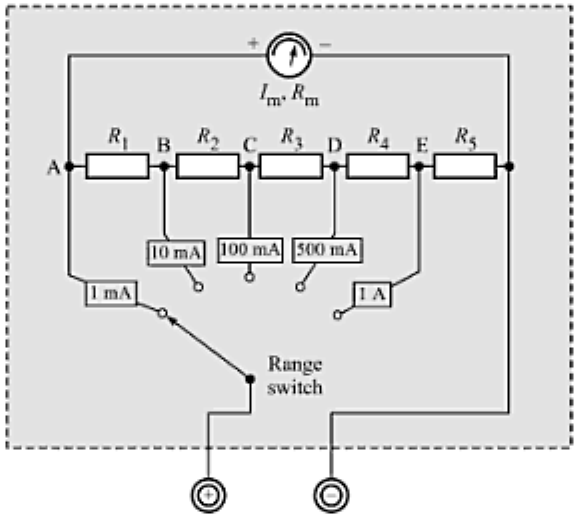


Fig. 18.14 *Universal shunt for extending the current ranges.*

Thus, we see that as we go for higher range, the switch cuts off a portion of the shunt resistance and it gets added to the meter branch.

EXAMPLE 18.3

Design a universal shunt for making a multi-range milliammeter with ranges 0-1 mA, 0-10 mA, 0-100 mA, 0-500 mA and 0-1 A. The basic meter movement has a full-scale deflection (FSD) current of 100 µA and internal resistance of 100 Ω.

Solution $I_m = 100$ µA. For 1-mA range, the required shunt resistance can be calculated by using Eq. 18.1 as

$$R_{sh} = \frac{I_m R_m}{(I_{fsd} - I_m)} = \frac{100 \times 100^{-6} \times 100}{(1 \times 10^{-3} - 0.1 \times 10^{-3})} = 11.111111\ \Omega$$

To make a resistor of this value is not practically feasible. To get a suitable value for this shunt resistor, we connect a resistor R_T of suitable value in series with the basic meter, as shown in Fig. 18.15.

Let us arbitrarily choose a resistor R_T such that the total meter resistance is 900 Ω. Connecting R_T in series with the basic meter serves another important purpose too. The meters that are manufactured may not all have exactly the same internal resistance. The resistance of a meter with a nominal $R_m = 100\ \Omega$ may actually be anywhere between 90 Ω and 110 Ω. The variable resistor R_T provides an arrangement by which the resistance of the meter can be set to a desired value. Thus, it works as a *preset adjustment*. For instance, we can use a 1-kΩ pot (potentiometer) for R_T. It can be set to give a total resistance of the meter at the predetermined value (900 Ω in our case). The meter resistance R_m now effectively becomes 900 Ω. We can now calculate the values of different resistors used in the universal shunt.

(*a*) *Range-switch at* 1-mA:

The resistance of the meter branch,	$R_{m1} = R_m = 900\ \Omega$
For full-scale, the current in the shunt path,	$I_{sh1} = 1 - 0.1 = 0.9$ mA
The shunt-path resistance,	$R_{sh1} = R_1 + R_2 + R_3 + R_4 + R_5 = R$ (say)

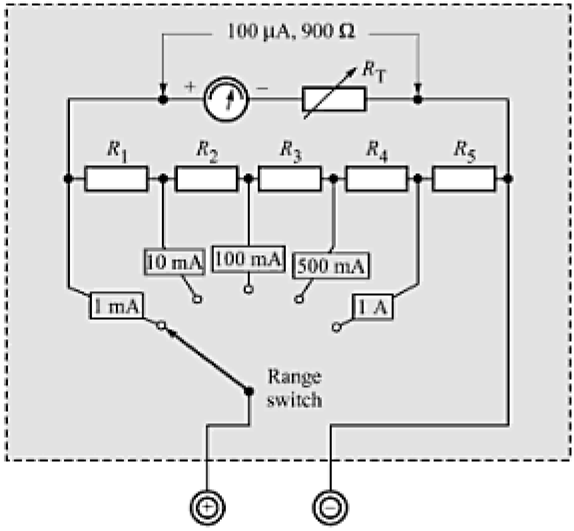


Fig. 18.15 *Design of universal shunt for multi-range milliammeter.*

The value of R_{sh1} is calculated as

$$R_{sh1} = \frac{R_{m1} I_m}{I_{sh1}} = \frac{900 \times 100 \times 10^{-6}}{0.9 \times 10^{-3}} = 100\ \Omega$$

This value of shunt resistance is quite practicable.

(*b*) *Range-switch at* 10-mA:

$$R_{m2} = R_m + R_1 = (900 + R_1)\ \Omega; \quad I_{sh2} = 10 - 0.1 = 9.9 \text{ mA}$$

$$R_{sh2} = R_2 + R_3 + R_4 + R_5 = R - R_1 = 100 - R_1$$

But, $$R_{sh2} = \frac{R_{m2} I_m}{I_{sh2}} = \frac{(900 + R_1) \times 100 \times 10^{-6}}{9.9 \times 10^{-3}} = \frac{900 + R_1}{99}$$

$\therefore$ $$100 - R_1 = \frac{900 + R_1}{99} \quad \text{or} \quad 9900 - 99R_1 = 900 + R_1$$

$\Rightarrow$ $$R_1 = \frac{9900 - 900}{100} = \mathbf{90\ \Omega}$$

(*c*) *Range-switch at* 100-mA:

$$R_{m3} = R_m + R_1 + R_2 = (900 + 90 + R_2)\ \Omega = (900 + R_2)\ \Omega$$

$$I_{sh2} = 100 - 0.1 = 99.9 \text{ mA}$$

$$R_{sh3} = R_3 + R_4 + R_5 = R - R_1 - R_2 = 100 - 90 - R_2 = 10 - R_2$$

But, $$R_{sh3} = \frac{R_{m3} I_m}{I_{sh3}} = \frac{(990 + R_2) \times 100 \times 10^{-6}}{99.9 \times 10^{-3}} = \frac{990 + R_2}{999}$$

$\therefore$ $$10 - R_2 = \frac{990 + R_2}{999} \quad \text{or} \quad 9990 - 999R_2 = 990 + R_2$$

$\Rightarrow$ $$R_2 = \frac{9990 - 990}{1000} = \mathbf{9\ \Omega}$$

Ideally, a voltmeter should have infinite series resistance. Such an *ideal voltmeter*, when connected across two points of an electric circuit, will not draw any current. Hence, the circuit will not be disturbed. The reading of the voltmeter will indicate the true value of the voltage existing across those two points of the circuit. However, we know that no voltmeter can give any reading unless some current flows through it. A practical voltmeter will be nearer to the ideal, if it draws very little current (i.e., if its *resistance is very high*).

There are two important points about the use of a voltmeter. First, a voltmeter is always connected in parallel with the portion of the circuit across which the voltage is to be measured. *Second*, always connect the voltmeter with the correct polarity. Normally, the positive test lead is red and the negative test lead is black.

AC Voltage Measurement A dc meter can easily be converted into an ac meter, by simply putting a rectifier in the circuit. Figure 18.18 shows the circuit of an ac multi-range voltmeter. It uses a full-wave bridge rectifier circuit to convert ac voltage to be measured into a dc voltage. The dc voltage is then read on the dc meter. The *range-switch* selects different voltage ranges.

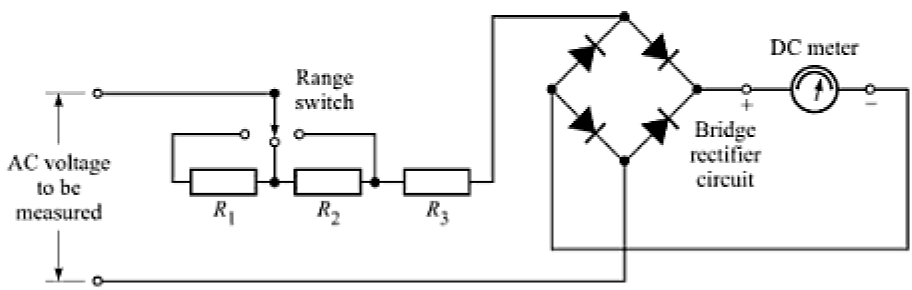


Fig. 18.18 *An ac multi-range voltmeter.*

EXAMPLE 18.4

You are provided with a d'Arsonval meter movement having a current sensitivity of 100 μA and coil resistance of 100 Ω. What will you do to convert it into a voltmeter of range 100 V?

Solution Here, I_m = 100 μA, and R_m = 100 Ω. To convert it into a voltmeter of range 100 V, we connect a resistor R_s in series. From Eq. 18.3,

$$R_s = \frac{V_{fsd}}{I_m} - R_m = \frac{100\,\text{V}}{100\,\mu\text{A}} - 100 = \mathbf{999.9\ k\Omega}$$

EXAMPLE 18.5

A 50-μA meter movement with an internal resistance of 1 kΩ is to be used as a dc voltmeter of range 50 V. Calculate (*a*) the multiplier resistance needed, and (*b*) the voltage multiplying factor.

Solution Here, I_m = 50 μA, and R_m = 1 kΩ.

(*a*) The series resistance needed is given as

$$R_s = \frac{V_{fsd}}{I_m} - R_m = \frac{50\,\text{V}}{50\,\mu\text{A}} - 1000 = \mathbf{999\ k\Omega}$$

(*b*) The voltage multiplying factor is given as

$$n = \frac{V_{fsd}}{V_m} = \frac{V_{fsd}}{I_m R_m} = \frac{50}{50 \times 10^{-6} \times 1 \times 10^3} = \mathbf{1000}$$

18.9 RESISTANCE MEASUREMENT

The basic meter movement can be converted into an instrument that measures resistance. Such an instrument is appropriately called ***ohmmeter.*** There are following three types of ohmmeters commonly used for the measurement of resistances.

(*i*) *Shunt-Type Ohmmeter* The unknown resistor R_x is connected in shunt (i.e., in parallel) with the meter, as shown in Fig. 18.19. We then determine the ability of the unknown resistor R_x to bypass current through this shunt path. When $R_x = 0$ (i.e., when the test-leads X and Y are shorted), all the current from the internal battery passes through this short and no current passes through the meter. On the other extreme, when $R_x = \infty$ (i.e., when the test-leads X and Y are open), the entire current from the internal battery finds a path only through the meter. By proper selection of resistance R_1, the pointer can be made to read full scale. Thus, this ohmmeter has the 'zero' mark at the left-hand side of the scale (no current) and the 'infinite' mark at the right-hand side of the scale (full-deflection current). Such ohmmeters are suitable for the measurement of *low-value* resistors.

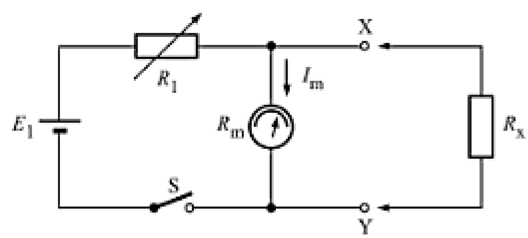


Fig. 18.19 *Basic construction of shunt-type ohmmeter.*

(*ii*) *Series-Type Ohmmeter* The unknown resistor R_x is connected in series with the meter, as shown in Fig. 18.20*a*. We then determine the ability of the unknown resistor R_x to prevent current flow in the meter path. Such ohmmeters are suitable for the measurement of *medium-value* resistors.

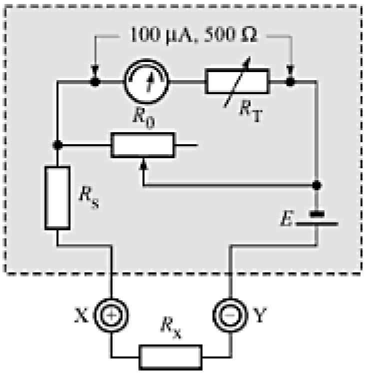
(*iii*) *Meggar-Type Ohmmeter* We apply a known voltage across the unknown resistor R_x and then determine the resulting current. The ratio of the voltage to the current gives the resistance. Such ohmmeters are suitable for the measurement of *high-value* resistances, such as the insulation of a cable.

In electric and electronic circuits, we usually come across medium-value resistors. We would hardly need a resistor more than 10 MΩ and less than 10 Ω. We therefore use series-type ohmmeters, discussed below in detail.

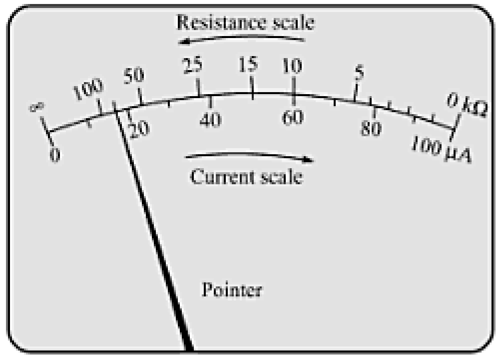
The Series-Type Ohmmeter

As shown in Fig. 18.20*a*, it has a battery E to energize the meter. The resistor R_T, called a ***preset resistor,*** is meant to compensate the individual differences in the meter resistances, R_m. The variable resistor R_0 works as a ***zero-adjust***. It compensates for any decrease in battery voltage E with ageing. The series resistor R_s limits the current to full-scale deflection when the test leads X and Y are shorted.

Shorting the test leads X and Y simulates 0 Ω, and as a result, the current in the circuit is maximum. The series resistor R_s is suitably selected so that with X-Y shorted, the current through the meter is same as its full-deflection current (100 μA, in this case). The deflection of the pointer should be full scale. If not, it can be adjusted to full-scale by adjusting the *zero-adjust* resistor R_0.



(*a*) The basic construction.



(*b*) The current and resistance scales.

Fig. 18.20 *Series-type ohmmeter.*

Opening the test leads X and Y simulates infinite resistance. The resulting current in the circuit is zero. The pointer does not deflect at all. When an unknown resistor R_x is connected across the test leads, the deflection will be somewhere between no-deflection and full-scale deflection. Thus, unlike a shunt-type ohmmeter, this ohmmeter has the 'zero' mark at the right-hand side of the scale (full-scale deflection current) and the 'infinite' mark at the left-hand side of the scale (no current). That is, as shown in Fig. 18.20*b*, the scale for resistance is *inverted* as compared to the current scale.

Although the series-type ohmmeter is a popular design, it has some ***disadvantages***:

(*i*) As explained above, the scale is inverted.

(*ii*) The scale is nonlinear. It is expanded at the right (near zero ohm) and crowded at the left (near infinite ohms). This nonlinearity is due to the reciprocal function, $I = V/R$. Due to this nonlinearity, it becomes difficult to accurately read the large value resistances on the left side of the scale. So, we avoid using the left end of the scale by providing a range switch. Different ranges are obtained by switching in different series resistors, R_s.

(*iii*) The accuracy of measurement depends on the internal battery whose voltage decreases gradually with time. The variable shunt resistor R_0 provides an adjustment to counteract the effect of battery change.

A word of ***caution*** is necessary about the use of ohmmeter. *The test leads of an ohmmeter are never connected to an energised circuit.* If you do so, you may send excessive current through the meter causing damage to it. Furthermore, even if the ohmmeter is not damaged, the reading on resistance scale becomes meaningless. Another important point is to make sure that there is no parallel branch across the component whose resistance you are measuring. When in doubt, just disconnect one terminal of the component under test.

Wattmeter Errors

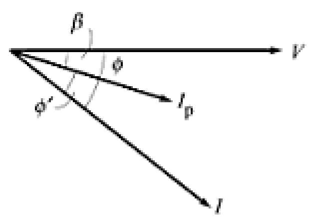
There are many factors which may cause errors in the measurement of power, as given below.

(i) Voltage-Coil Inductance For the wattmeter to give correct measurement, it is necessary that the current in voltage coil is in phase with the applied voltage. If the voltage coil (and the series resistor R) has some inductance, the current I_p will lag the voltage V by an angle β.

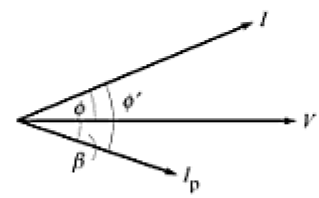
For lagging phase angle of the load circuit (Fig. 18.23*a*), the angle ϕ' between the voltage-coil current I_p and the current-coil current I becomes less than the actual phase angle ϕ,

$$\phi' = \phi - \beta$$

The reading of the wattmeter is proportional to cos ϕ' and not cos ϕ. Hence, the wattmeter reads higher than the actual power going to the load.



(*a*) Lagging power factor.



(*b*) Leading power factor.

Fig. 18.23 *Effect of voltage-coil inductance on power factor.*

For leading phase angle of the load circuit (Fig. 18.23*b*), the angle ϕ' becomes more than the actual phase angle ϕ,

$$\phi' = \phi + \beta$$

Hence, the wattmeter now reads lower than the actual load power.

If the power factor is very low (i.e., the angler ϕ is nearer to 90°), a serious error may be caused due to the inductance of the voltage coil. Special precautions are taken to reduce this effect. Some compensation for this error may be provided by connecting a capacitor in parallel with a portion of the series resistor R.

(ii) Voltage-Coil Capacitance The voltage-coil circuit may possess some capacitance. It may come into the circuit due to the inter-turn capacitance of the series resistor R. The effect of this capacitance on wattmeter reading is opposite to that of the inductance. Usually, the capacitive reactance of the voltage-coil circuit is smaller than its inductive reactance.

(iii) Mutual Inductance Errors may be caused due to mutual inductance between the two coils of the wattmeter. Such errors are quite low at power frequency. But, these errors become important as frequency is increased.

(iv) Eddy Currents When ac current flows through the current coil, an alternating magnetic field is set up. This induces eddy currents within the thickness of the conductors and also in the solid metal parts of the wattmeter. These currents produce a field of their own. This may change the magnitude and phase of the field produced by the current-coil. This causes errors.

(v) *Stray Magnetic Fields* The wattmeter has air-core coils. Their magnetic fields are relatively weak. These can easily be influenced by stray magnetic fields, causing errors. Hence, the instrument should be shielded against the effects of stray magnetic fields.

18.13 MEASUREMENT OF ENERGY

Energy is the total power delivered or consumed over a time-interval, that is,

$$\text{Energy} = \text{Power} \times \text{Time}$$

Usually, ***motor meters*** are used for the measurement of energy on both dc and ac circuits. For dc circuits, the meter may be an *ampere-hour meter* (if the voltage is assumed to remain constant) or a *watt-hour meter*.

Energy meters are *integrating* instruments. Unlike the indicating instruments, their moving system rotates continuously. The speed of rotation is proportional to the power (in case of watt-hour meter) and to the current (in case of ampere-hour meter). Thus the total number of rotations made by the moving system is proportional to the energy supplied to the load in a given time interval*. Every energy meter is marked for its *meter constant*. It gives the number of rotations per kilowatt hour (kWh) of energy.

Single-Phase Induction Type Energy Meter

A schematic constructional diagram of a single-phase induction type energy meter is given in Fig. 18.24. Essentially, it consists of following systems.

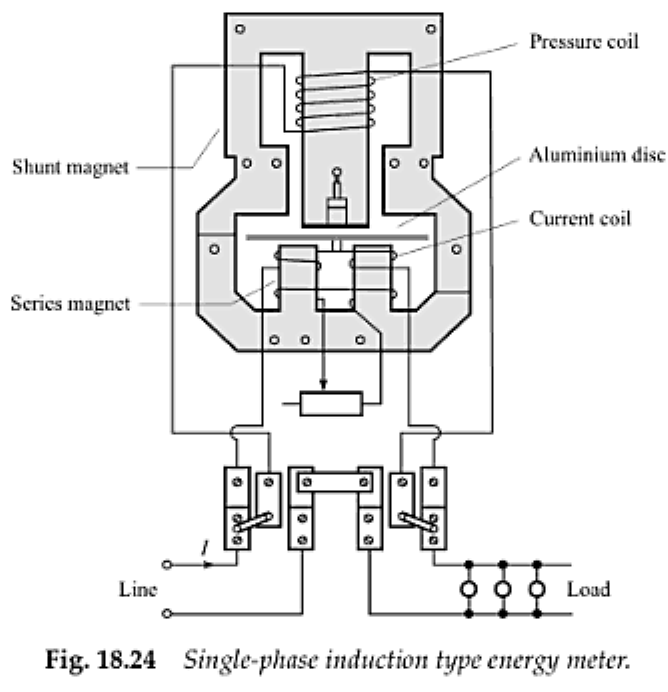


Fig. 18.24 *Single-phase induction type energy meter.*

* However, it does not give any idea about the variation in the rate of energy consumption during the period.

(i) Driving System It consists of two electromagnets, called *series* and *shunt* magnets. The cores of these electromagnets are made up of silicon steel laminations riveted together to form a rigid mechanical structure. The shunt magnet consists of M-shaped iron laminations and is wound on the middle limb with large number of turns of fine wire. This coil is called *voltage* or *pressure coil* and is connected to the supply mains. This coil is highly inductive.

The series magnet consists of U-shaped iron laminations and is wound on the two limbs with a few turns of heavy wire. This coil is known as *current coil* and is connected in one of the lines in series with the load.

Copper shading bands are provided on the central limb of shunt magnet. Their position is adjusted to provide a phase displacement of 90° between the magnetic field and the applied voltage. These copper shading bands are also called *power factor compensator* or *compensating loop*.

(ii) Moving System It consists of a rotating aluminium disc mounted on a vertical spindle supported on a saphire cap at the bottom and held in position by a pivot at the top. A pinion engages the shaft with the *registering* or *counting system*.

The rotating disc cuts through the pulsating magnetic field produced by the shunt electromagnet and hence eddy currents are set up in it. The interaction between the two magnetic fields and these eddy currents produce a driving torque in the disc.

(iii) Braking System It consists of a C-shaped permanent magnet forming a complete magnetic circuit except a narrow air gap between its poles. This magnet is mounted such that the aluminium disc rotates in this narrow air gap having magnetic field of the permanent magnet. The eddy currents induced in the disc interact with the magnetic field and exert a braking (retarding) torque. Thus, the speed of rotation of the disc can be adjusted by moving the permanent magnet radially in or out. The braking torque is proportional to the speed of the disc. We prefer aluminium disc to a copper disc since the resistance per unit weight of aluminium is smaller.

(iv) Registering or Counting System It consists of a train of gears driven by worm and pinion fixed on the spindle of the rotating disc. The pointers on the dials indicate the total kW h consumed. The rightmost dial gives the readings in steps of 1 kW h. The pointer on this dial moves one step when the disc has made the required number of revolutions. The gearing is so arranged that each division on this dial represents 1 kW h, on the second dial 10 kW h, on the third dial 100 kW h, on the fourth dial 1000 kW h, and so on.

ADDITIONAL SOLVED EXAMPLES

EXAMPLE 18.7

The deflecting torque in an ammeter is directly proportional to the current flowing through it. If a current of 20 A produces a deflection of 60°, what deflection will occur for a current of 12 A when the instrument is (*a*) spring controlled, and (*b*) gravity controlled?

Solution Given that $\tau_d \propto I = k_1 I$.

(*a*) For spring-control, the controlling torque, $\tau_c \propto \theta = k_2\theta$. At final steady state deflection,

$$\tau_d = \tau_c \quad \text{or} \quad k_1 I = k_2\theta \quad \Rightarrow \quad \theta \propto I \quad \text{or} \quad \theta = kI$$

For 20-A current, $\theta = 60°$. Therefore,

$$k = \frac{60°}{20\text{ A}}$$

Hence, the angle of deflection for 12-A current,

$$\theta = kI = \frac{60°}{20\text{ A}} \times 12\text{ A} = \mathbf{36°}$$

(*b*) For gravity-control, the controlling torque, $\tau_c \propto \sin\theta = k_2 \sin\theta$. At final steady-state deflection,

$$\tau_d = \tau_c \quad \text{or} \quad k_1 I = k_2 \sin\theta \quad \Rightarrow \quad \sin\theta \propto I \quad \text{or} \quad \sin\theta = kI$$

For 20-A current, $\theta = 60°$. Therefore,

$$k = \frac{\sin 60°}{20\text{ A}} = \frac{0.866}{20\text{ A}}$$

Hence, the angle of deflection for 12-A current is given by

$$\sin\theta = kI = \frac{0.866}{20\text{ A}} \times 12\text{ A} = 0.5196 \quad \Rightarrow \quad \theta = \sin^{-1} 0.5196 = \mathbf{31.3°}$$

EXAMPLE 18.8

In a gravity-controlled instrument, the controlling weight is 0.005 kg and acts at a distance of 2.4 cm from the axis of the moving system. Determine the defection in degrees corresponding to deflecting torque of 1.05×10^{-4} kg m.

Solution For the steady-state deflection angle θ, we have

$$\text{Deflecting torque, } \tau_d = \text{Controlling torque, } \tau_c$$

$$\tau_d = Wl \sin\theta$$

or

$$\theta = \sin^{-1}\frac{\tau_d}{Wl} = \sin^{-1}\frac{1.05 \times 10^{-4}}{0.005 \times 0.024} = \mathbf{61°}$$

EXAMPLE 18.9

The deflecting torque in an ammeter varies as the square of the current through it. If a current of 10 A produces a deflection of 90°, what deflection will occur for a current of 5 A when the instrument is (*a*) spring-controlled, and (*b*) gravity-controlled.

Solution The deflecting torque, $\tau_d \propto I^2$.

(*a*) For spring-control, the controlling torque, $\tau_c \propto \theta$.

Since, for the steady-state deflection, $\tau_d = \tau_c$, we have $\theta \propto I^2$, or $\dfrac{\theta_2}{\theta_1} = \left(\dfrac{I_2}{I_1}\right)^2$

Therefore, the deflection for 5-A current is given as

$$\theta_2 = \left(\frac{I_2}{I_1}\right)^2 \times \theta_1 = \left(\frac{5}{10}\right)^2 \times 90° = \mathbf{22.5°}$$

(*b*) For gravity-control, the controlling torque, $\tau_c \propto \sin\theta$.

Since, for the steady-state deflection, $\tau_d = \tau_c$, we have $\sin\theta \propto I^2$, or $\dfrac{\sin\theta_2}{\sin\theta_1} = \left(\dfrac{I_2}{I_1}\right)^2$

Therefore, the deflection for 5-A current is given as

$$\theta_2 = \sin^{-1}\left[\left(\frac{I_2}{I_1}\right)^2 \times \sin\theta_1\right] = \sin^{-1}\left[\left(\frac{5}{10}\right)^2 \times \sin 90°\right]$$

$$= \sin^{-1}(0.25) = \mathbf{14.5°}$$

- There are three types of ***ohmmeters***: (*i*) Shunt-Type Ohmmeter (for low-value resistances), (*ii*) Series-Type Ohmmeter (for medium-value resistances), and (*iii*) Meggar-Type Ohmmeter (for high-value resistances).
- Usually, we use series-type ohmmeters, which have inverted and non-uniform scale.
- The test leads of an ohmmeter are never connected to an energised circuit.
- For a given range, the internal resistance of the voltmeter (or of the ammeter) remains the same irrespective of the deflection of the pointer.
- A meter with higher sensitivity (or ohms-per-volt rating) gives more accurate results, since it produces less loading effect on the circuit.
- A ***multimeter*** is an instrument which can be used to measure current (in **A**mperes), voltage (in **V**olts), and resistance (in **O**hms). Hence, it is also called an **'AVO meter'**.
- A dynamometer type instrument is used as a ***wattmeter***. It has two coils. The *current coil* made of thick wire is fixed and connected in series. The *pressure coil* made of fine wire is moving and connected in parallel.
- An ***energy meter*** is an integrating instrument consisting of (*i*) *Driving System*, (*ii*) *Moving System*, (*iii*) *Braking System*, and (*iv*) *Registering* or *Counting System*.

IMPORTANT FORMULAE

- Conversion of a galvanometer into a voltmeter, $R_{sh} = \dfrac{I_m R_m}{(I_{fsd} - I_m)}$.
- Conversion of a galvanometer into an ammeter, $R_s = \dfrac{V_{fsd}}{I_m} - R_m$.
- Ohms-per-volt rating $= \dfrac{1}{\text{current sensitivity}}$.

CHECK YOUR UNDERSTANDING

To check your understanding of this Chapter, take this Test. Give yourself ***two*** marks for each correct answer and ***minus one*** for each wrong answer. If your score is **12** or more, go to the next Chapter; otherwise study this Chapter again.

S. No.	*Statement*	*True*	*False*	*Marks*
1.	The two spiral springs attached to the coil assembly of a d'Arsonval movement provide a path for the current to reach the coil.	☐	☐	
2.	The higher the current sensitivity of a moving-coil meter movement used, the better is the multimeter.	☐	☐	
3.	The resistance scale of a multimeter has 'zero' on the right end, and it linearly increases towards left.	☐	☐	
4.	Whatever be the deflection of the pointer, the internal resistance of a voltmeter is determined by its full-scale voltage.	☐	☐	
5.	Both the moving-coil and moving-iron instruments depend for their operation upon the magnetic effect of current.	☐	☐	
6.	A dynamometer type moving-coil instrument cannot be used on ac circuits.	☐	☐	
7.	The deflection produced in a dynamometer type instrument is proportional to the current flowing through the instrument.	☐	☐	
8.	The moving-iron instruments are unpolarized.	☐	☐	

9.	Due to the presence of inductance in the pressure-coil, the wattmeter reads higher than the actual power going to a lagging power-factor load.	☐	☐	
10.	In an energy meter, the retarding torque produced due to eddy currents in the aluminium disc is proportional to the angle by which the disc moves from its zero position.	☐	☐	

Your Score

ANSWERS

1. True	**2.** False	**3.** False	**4.** True	**5.** True
6. False	**7.** False	**8.** True	**9.** True	**10.** False

REVIEW QUESTIONS

1. Differentiate between absolute and secondary instruments.
2. What is the basic difference between indicating instruments and integrating instruments?
3. What is meant by (*a*) an indicating type instrument, (*b*) a recording type instrument, and (*c*) an integrating type instrument? Give at least one example of each type.
4. What are the different effects of electric current or voltage that are used as basic principle of operation of various electrical measuring instruments?
5. What are the essential components of an indicating instrument?
6. Explain why the controlling mechanism is provided in an indicating instrument.
7. Explain the need of a zero-adjuster in an indicating instrument.
8. What are the different methods of obtaining the controlling torque in an indicating instrument? Discuss briefly bringing out the advantages and disadvantages of each.
9. Explain the need of providing damping torque in electrical measuring instruments. What is meant by critically damping?
10. Explain briefly the different methods by which damping can be produced in an indicating instrument.
11. An indicating instrument depends for its operation on a deflecting force, a controlling force, and a damping force. Explain the various techniques by which these forces are produced in an indicating instrument.
12. Explain in brief the construction and working of the permanent magnet moving coil (PMMC) instrument.
13. Explain why an iron core is placed inside the deflecting coil in the permanent magnet moving coil (PMMC) instrument.
14. Explain the working principle of moving-iron indicating instrument. Show that this type of instrument can be used for both dc and ac measurements. Indicate the errors involved.
15. Describe the construction and working of a moving-iron instrument. Give its advantages and disadvantages over the moving-coil instrument.
16. Describe the operating principle of a dynamometer type instrument. Explain how the deflecting torque is proportional to the product of the currents in the fixed and moving coils. Discuss its advantages and disadvantages.
17. What are the various quantities that can be measured with a multimeter? Briefly explain the principle of measurement of each.
18. Explain what you understand by (*a*) the current sensitivity (CS), and (*b*) the voltage sensitivity (VS) of a d'Arsonval movement. How are these two quantities inter-related?
19. The current sensitivity or full-scale deflection current for a meter movement is known to be 0.5 mA. Can we extend its current range to as low a value as 50 μA? If yes, explain the procedure. If not, why?

20. Explain why the resistance scale in a multimeter is usually inverted. Also, explain why the ohmmeter scale is not linear throughout the range.

21. Explain what you understand by ohms-per-volt rating of a multimeter. How is it related to current sensitivity?

22. The internal resistance of a voltmeter is 100 kΩ in its 50-V range. If the meter is measuring a voltage of 25 V in this range, will its internal resistance change to 50 kΩ? Explain your answer in brief.

23. What do you mean by loading effect of a meter? Will the loading effect be more for greater ohms-per-volt rating of the meter? Explain your answer in brief.

24. Explain why you cannot measure power in an ac circuit, by using an ammeter and a voltmeter.

25. Explain the principle of working of an energy meter.

26. Explain why the braking torque in an energy meter should be proportional to the speed of the moving system, and not to its deflection.

MULTIPLE CHOICE QUESTIONS

Here are some incomplete statements. Four alternatives are provided below each. Tick the alternative that completes the statement correctly:

1. Which one of the following is *not* a secondary instrument?
(*a*) voltmeter (*b*) ammeter
(*c*) energy meter (*d*) tangent galvanometer

2. The electric energy meter installed near the mains switch in a home is
(*a*) an indicating instrument
(*b*) an integrating instrument
(*c*) a recording instrument
(*d*) an absolute instrument

3. The magnitude of the controlling torque produced in an instrument
(*a*) should be proportional to the deflection of the moving system
(*b*) should be proportional to the velocity of the moving system
(*c*) should be proportional to the acceleration of the moving system
(*d*) should remain constant for any deflection of the moving system

4. Which one of the following torques does not act on the moving system, when an indicating instrument shows a steady non-zero reading?
(*a*) the deflecting torque
(*b*) the controlling torque
(*c*) the restraining torque
(*d*) the damping torque

5. The reason why the pole faces and the iron core are made cylindrical in the PMMC instrument is that
(*a*) it is easy to manufacture
(*b*) the coil can easily move in the air-gap
(*c*) damping torque is produced
(*d*) the magnetic field is made radial

6. The main reason why we use moving-iron instruments in laboratories and panels, is that
(*a*) they are not affected by stray magnetic fields
(*b*) they are inexpensive
(*c*) in case of ac measurements, the reading is not affected if the supply frequency changes
(*d*) they have low deflecting torque

7. The voltage sensitivity of a multimeter is 20 mV. Its internal resistance is 40 Ω. Then, its current sensitivity is
(*a*) 1 mA (*b*) 5 μA
(*c*) 500 μA (*d*) 0.8 A

8. A meter movement having full-scale deflection current of 50 μA and internal resistance of 40 Ω is used to measure current up to 50 mA. The value of the shunt resistance should be
(*a*) 0.040 04 Ω (*b*) 0.5 Ω
(*c*) 0.002 Ω (*d*) 1.00 Ω

9. The current sensitivity of a meter is 100 μA and its internal resistance is 50 Ω. Its ohm-per-volt rating should be
(*a*) 10 kΩ/V (*b*) 50 kΩ/V
(*c*) 20 kΩ/V (*d*) 50 Ω/V

10. It is found that the internal resistance of a voltmeter is 100 kΩ when it is reading full-scale voltage of 100 V. On the same range, it reads another voltage of 40 V, its internal resistance will be
(*a*) 400 kΩ (*b*) 4000 kΩ
(*c*) 40 kΩ (*d*) 100 kΩ

ELECTRICAL INSTALLATION AND ILLUMINATION

19

OBJECTIVES

After completing this Chapter, you will be able to:

- *State the factors that must be considered to select the system of wiring for a building.*
- *Explain how electrical energy from a supply company is distributed in a building.*
- *Describe various types of wires and cables used for internal wiring.*
- *Describe different types of switches used in various circuits.*
- *Draw circuit diagrams of staircase wiring, godown wiring, and verandah wiring.*
- *Explain following types of wiring: (i) cleat wiring, (ii) wooden-batten wiring, (iii) wooden-casing wiring, (iv) Lead-sheathed wiring, and (v) conduit wiring.*
- *Give brief description of (i) fuses and* MCBs, *(ii) socket outlets, (iii) plugs, (iv) male-female connectors, (v) ceiling roses, (vi) junction boxes, and (vii) lamp holders.*
- *Describe a typical control circuit for a house.*
- *Explain what is meant by earthing of an electrical installation and why it is required.*
- *Describe plate earthing and pipe earthing methods.*
- *Describe the method and the objective of different tests to be conducted on an electrical installation.*
- *Describe the construction, working and usages of (i) incandescent lamp, (ii) fluorescent tube light, (iii)* CFL *(compact fluorescent light), (iv) mercury vapour lamp, (v) sodium vapour lamp, and (vi) neon lamp.*

19.1 INTRODUCTION

A building, whether used as a home, as an office, as a godown, as a factory, as a cafeteria, as a hotel, as a research laboratory, or as an educational institute, needs electrical installation. To get proper lighting and to run various appliances, equipments and machinery, we need electrical power.

Following *supply systems* are normally used for distributing electrical power to consumers:

(*i*) Single-phase two-wire system.
(*ii*) Three-phase four-wire system.
(*iii*) Three-phase three-wire system.

For domestic consumers, the most commonly used system is 1-phase 2-wire system, which is often derived from a 400-V, 3-phase 4-wire system. Groups of consumers are connected between one phase line and the neutral conductor, thus providing 230-V, 50-Hz, 1-phase supply. The consumer-groups are arranged such that the loads on the three phases remain balanced.

For large industrial consumers, particularly those using heavy motor-loads and drawing more than 1 MVA, the power is supplied from a 3-phase system at high voltage, such as 6.6 kV, 11 kV, or 33 kV. The consumer has his own substation to distribute electrical power at appropriate voltages at different locations within his premises.

There are different *systems of wiring* available, some of which require to be taken care of during construction of the building. This necessitates advance planning for the location and the number of light-points and power-sockets to be provided in each room. Due consideration must be given to following points while selecting a system of wiring for a building:

(*i*) Overall cost of the installation.
(*ii*) Conditions of service.
(*iii*) Life of the installation.
(*iv*) Projected extensions and alterations.
(*v*) Fire hazards.
(*vi*) Dampness.
(*vii*) Corrosive fumes.

19.2 DISTRIBUTION OF ELECTRICAL ENERGY

The schematic diagram for the distribution of electrical energy from the supply company to a domestic consumer is shown in Fig. 19.1. An insulated cable coming from the feeder pole is first connected to a panel having cut-outs and energy meter. This panel is termed as *service connection*, and is properly sealed by the supply company to prevent any tempering by the consumer. The maintenance of the supply up to this point is the responsibility of the supply company.

Now-a-days, the electric supply companies prefer installing *electronic energy meters* in place of the ordinary meters whose function depends upon the rotation of an aluminium disc. These modern meters provide digital display of readings and are temper-proof.

After the energy meter, the consumer's main switch is installed. The distribution of energy to the lighting and power circuits in the premises of the consumer is done through this *consumer's terminal*. The main switch is 'double-pole single-throw type' so that it operates simultaneously on the live as well as neutral line. This switch is known as *double-pole iron-clad* (DPIC) switch and is enclosed in an iron-box.

The consumer's main switch feeds power to a distribution fuse board. The total load is divided into smaller loads called sub-circuits. Each sub-circuit should have not more than eight light-points and a total load not exceeding 800 W. In the distribution board, the live phase-wire of each sub-circuit is connected through a suitable fuse and the neutral wire is directly connected through a copper strip, as shown in Fig. 19.1. Whenever a sub-circuit is overloaded or excessive current flows through it due to a fault or due to an accidental short-circuit, the corresponding fuse blows off. Other sub-circuits remain protected from any interruption to supply. This helps the consumer to locate the fault in a smaller area of the premises. After correcting the fault, the fuse is replaced and the supply to the sub-circuit restored.

In modern practice, complete consumer units housed in sheet-steel or plastic enclosure are available for making the installation simpler. Such units have one double-pole miniature circuit breaker (MCB) of 32 A or 63 A rating to work as the main switch, and many MCBs of 6 A and 16 A ratings to be used in individual sub-circuits.

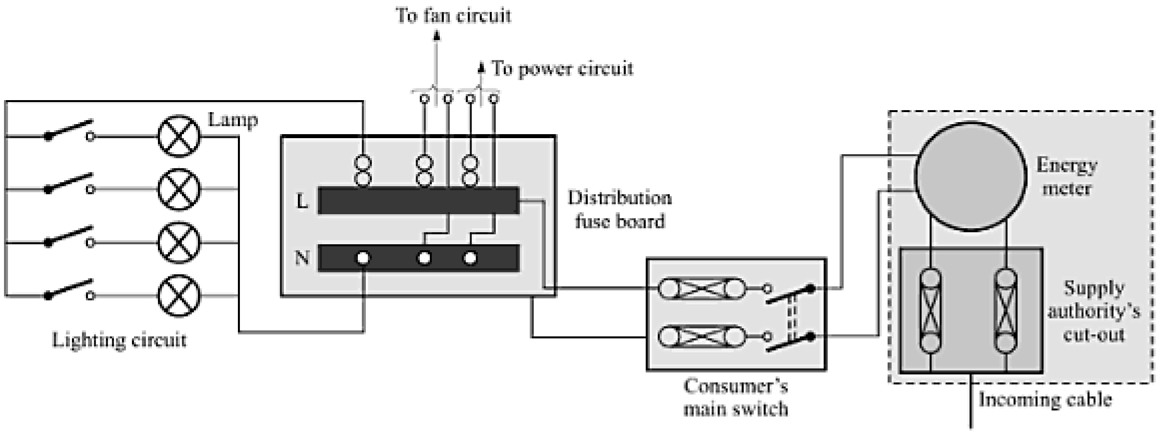


Fig. 19.1 *Distribution of electrical energy to a domestic consumer.*

19.3 WIRES AND CABLES FOR INTERNAL WIRING

For domestic wiring, the most extensively used conductor material is *copper* or *aluminium**. To prevent any leakage of current from the conductor and also to provide mechanical strength, it is surrounded by an insulation and sheath. Normally, the cables are classified according to the insulation used over the conductor. The selection of suitable cable for an installation depends upon the following considerations:

(*i*) The nature of conditions under which the cable is to be used (for example, underground, hanging in air, in damp conditions, etc.).
(*ii*) The operating voltage.
(*iii*) The current capacity of the installation.

The operating conditions decide the type of insulation and other protection needed around the conductor of the cable. The operating voltage determines the thickness of the insulation. The current capacity of the installation determines the cross-sectional area or size of the cable-conductor. Following types of cables are available in market:

(1) Elastomer Insulated Cables

These cables have insulation coating of following different rubber-like materials:

(*i*) *Vulcanised Indian Rubber* (*VIR*) The insulation covering has 36 % natural rubber and balance materials such as sulphur, carbon, wax, etc. The insulation is applied either as a single coating by extrusion or in two or more layers. It is then vulcanised.

(*ii*) *Butyl Rubber Coatings* Such coatings are more resistant to oxidation and ozone than the natural rubber. These can be used for maximum conductor temperature of 85 °C.

* Aluminium is much cheaper than copper, but its electrical conductivity is about 60 % that of copper.

(iii) Ethylene-Propylene Rubber Coatings These coatings possess good electrical insulation properties, and are resistant to heat, humidity, chemicals and ozone. Hence, these cables can be directly buried underground. These are suitable for hazardous industries, oil refineries, power plants, etc.

(iv) Silicon Rubber Coatings These coatings can safely withstand conductor temperature of 150 °C for long durations and of 200 °C for short durations.

(v) Polychloroprene Coatings These coatings are superior to natural rubber in resistance to outdoor weather. Furthermore, these coatings possess self-extinguishing property, if ignited externally. These cables can safely withstand conductor temperature of 60 °C.

(2) Polyvinyl Chloride (PVC) Insulated Cables

The PVC coverings are chemically inert towards oxygen, many alkalis and oils. These are non-inflammable, insoluble in common liquids, and possess good mechanical strength, high dielectric strength and long life. However, these coatings have a tendency of softening when subjected to even moderate temperature; hence these are used only if the conductor temperature does not exceed 65 °C. Because of many advantages, the PVC cables have replaced VIR cables for low voltage installation.

(3) Lead Sheathed Cables

These are mainly employed for underground laying of service mains. In such cables, paper insulation is provided by helically wrapping paper tapes of desired thickness around the conductor. The insulated cover is vacuum dried and then impregnated under pressure with mineral oil or some other suitable liquid compound. Finally, the insulation is covered with a lead (or aluminium) sheath to protect it from moisture, soil, etc.

(4) Flexible Cables (or Chords)

Flexible wires are made up of a large number (say, 17 or 21) of very fine wires, called *strands*, twisted together to make a composite wire or cable. This is insulated by PVC, rubber, or plastic. Normally, two such wires having different colour insulation-coatings are twisted together to make a flexible chord. Such chords are used for the socket outlet to portable apparatus (such as table lamp, radio, battery charger, etc.) or from ceiling roses to a lamp holder or a ceiling fan. The chords used for hanging the pendant light-fittings from a ceiling rose must have sufficient mechanical strength to support the weight of the light-fittings.

The flexible chords used for connecting appliances such as microwave oven, OTG, refrigerator, washing machine, etc. must incorporate an earth continuity conductor so as to avoid any hazard of electric shock. The advantage of using flexible chords is the easiness of handling due to their flexibility. However, care must be taken to protect the insulation cover over the conductors against mechanical damage.

19.4 SWITCHES AND CIRCUITS

Different electrical loads are required to be *switched in* and *switched off* quite frequently. We use a *switch* to make and break the electrical connection to a load. This could be achieved by inserting a switch anywhere in the circuit. However, it is the standard practice in an electrical installation to connect the switches in the live wire (and NOT in the neutral wire) in the circuits of different loads. This ensures no electric shock to the person handling the load (say, replacing a fused bulb) after the switch has been put OFF.

This type of wiring has longer life than the batten-wiring since the wires are not exposed to weather. It provides better protection on the conductor-insulation, but is costlier than the batten-wiring. However, this type of wiring is not perfectly damp-proof and is risky in respect of fire hazards.

(4) Lead-Sheathed Wiring

It is similar to batten wiring, except that the cable used is different. The conductors used are insulated with VIR and then covered with an outer sheath of an alloy containing 95 % lead and 5 % aluminium. The metal sheath protects the wiring from mechanical injury, dampness and atmospheric adverse effects.

(5) Conduit Wiring

This type of wiring system is most modern for domestic and commercial installations. It gives good appearance and provides best protection to the installation against fire hazards, electrical shock, mechanical damage and dampness. It uses VIR or PVC insulated cables carried through steel, iron or PVC pipes, commonly called ***conduits***.

Normally, the conduits are made of mild steel, specially annealed so that it can easily be bent or set without breaking. The conduits are available in standard length of about 4 metres. Each length is threaded on both ends, and a coupler is attached to each length. Whenever needed, the conduit can be extended in length by joining two or more pieces. The size of the conduit is specified in terms of its outer diameter. Standard sizes available are 1/2, 5/8, 3/4, 1, $1^1/_4$, $1^1/_2$, 2 and $2^1/_2$ inches. The choice of the size depends on the number of conductors to be drawn through it. The conduits and other accessories (such as L-coupler, T-coupler and switch boxes) are coated with black enamel to give them long life. For exposed positions, damp or humid atmosphere, the conduits and accessories are galvanized or zinc-coated.

Complete conduit system for each circuit is built before putting any cables through it. For the ease of wiring, sufficient number of inspection-boxes and draw-boxes are provided at suitable points along the run of conduits. The metal conduits must be made mechanically and electrically continuous across all the junctions and joints. For the sake of safety, the entire conduit system is solidly *earthed* by running an earth wire by its side. Before the construction of a building starts, layout plan of the conduit system is made based on the number and location of the lighting and power points. Accordingly, the conduits are laid making joints using L and T-couplers and fixed with the help of clamps on shuttering, and then filled with concrete to make ceiling-slab.

For *concealed conduit wiring system*, it is very important to accurately fix the positions of ceiling-roses, lighting points, power points, switches, socket outlets, etc.; it becomes almost impossible to make any alterations later. Covers should be fitted over all the inspection and draw boxes to prevent any ingress of concrete or plaster into the conduit system.

After the builder's work is complete, the conduit system is thoroughly swabbed out. Using the ***fish wire****, a bunch of requisite number of wires is gently drawn from one draw-box to the other. The wires must be looped from terminal to terminal; wire-joints are not permitted in between inside the conduit system.

In case the building is already constructed, concealed conduit system can be made by chiselling channels in the walls and ceilings and then sinking the conduits into the surface. Or else, a cheaper and less bothersome option is to fix the conduits on the surface of the walls and ceilings using clips, as shown in Fig. 19.8.

* A GI (galvanized iron) wire of 18 SWG which had been left in the conduit system during its erection.

lamp holder is provided with porcelain interior for making electrical connections. Its exterior may be of either brass or bakelite. Brass holders are costlier but more durable compared to bakelite holders. Following types of lamp holders are available.

(i) Batten Holder This holder is screwed directly on a wooden board fixed on a wall.

(ii) Pendant Holder This type of holder is used when the bulb is to be suspended from the ceiling rose.

(iii) Bracket Holder This type of holder is used, normally along with a fancy shade, when direct light is needed in a room. The brackets used are made of brass or some other good-looking material.

(iv) Water-Tight Bracket Holder This type of holder has a water-tight cover. Such holders are used for the bulbs fixed outside the house premises, or for the coloured dancing lights under water in the fountains installed in amusement parks.

19.8 A TYPICAL CONTROL CIRCUIT

A typical room of a residential building is provided with a 3-pin socket outlet, a ceiling fan, a tube light and a lamp, each controlled by their individual switches, as shown in Fig. 19.9. Note that the switches are inserted in live (red coloured) wire, so that after switching OFF, repair at the holder or ceiling rose can be done without any shock hazard. Switch S_1 controls the 3-pin socket outlet. The top hole of this socket is connected to the earth (green) wire. Switch S_2 is used to switch ON or OFF the ceiling fan. The fan regulator in this circuit introduces appropriate resistances for its different settings to control the speed of the ceiling fan. Switch S_3 controls the tube-light and switch S_4 controls the lamp.

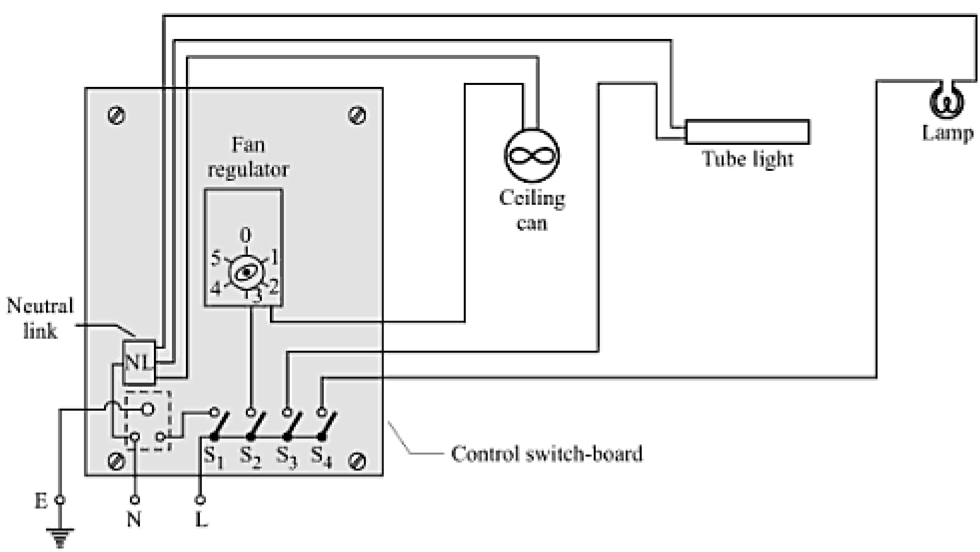


Fig. 19.9 *Control circuit for a typical room.*

2. Broken switches, plugs, etc., should be replaced immediately.
3. Use a 'line tester' to check whether a terminal is live. Still better is to use a 'test lamp', as the line tester can show a glow even with a small voltage.
4. Before replacing a broken switch, plug or blown fuse, always *put off* the main supply.
5. Never use equipments and appliances with *damaged* or *frayed* lead wires.
6. Never insert bare wires in the holes of a socket, for taking a connection. Always use a proper plug.
7. Use *rubber-sole* shoes while repairing/testing electrical equipments. If this is not possible, use some *dry-wooden* support under your feet, so that your body has no direct contact with earth.
8. Use *rubber gloves* while touching any terminal or while removing insulation layer from a conductor.
9. Always use well *insulated tools* (such as screw-drivers, pliers, cutters, etc.).
10. Never touch *two different terminals* at the same time.
11. Be careful that your body does not touch the wall or any other metallic frame having contact with earth.
12. Use correct *rating of fuse wire.*
13. While repairing an electrical appliance (such as table fan, iron, heater, geyser, etc.), be sure that its plug has been taken out from the socket. Switching OFF may not be sufficient, since *leaky insulation can give serious shock.*
14. Never try to connect machines or equipments to a voltage supply other than the rated one.
15. Strictly follow all the precautions and instructions given on the 'name plate' of the machine you are working.
16. In case of electric fire, use only 'soda-acid' fire-extinguisher. *Do not throw water on live conductors or equipments.* Best remedy is to first disconnect the electric supply and then *throw sand* on fire.
17. While working on an electric pole or tower, use safety-belt and a rubber padded ladder.
18. It is preferable to work in the presence of an 'assistant', so that he can immediately disconnect the supply whenever needed. The assistant should have the knowledge of providing first-aid in case of an electric shock.

19.11 TESTING OF ELECTRICAL INSTALLATION

After completing the electrical installation or after extension work of existing installation, the supply should not be connected unless following tests have been successfully conducted. The installation should be put into service only after no defect is detected in the tests.

(1) Testing of Insulation Resistance of Complete Installation to Earth

The objective of this test is to ascertain whether complete wiring is sound enough to keep the leakage current within prescribed limits. As per Indian Electricity Rules, the leakage current of an installation should not exceed 1/5000 of the maximum supply demand of the consumer. As such the insulation resistance of complete installation to the earth should not be less than 1 MΩ.

For measuring such high resistances, we use an instrument called ***megger***. The insulation resistance is measured on dc voltage generated in the megger itself. The testing dc voltage should not be less than twice

(4) Testing of Polarity of Single-Pole Switches

In an electrical installation, all switches (controlling lamps, fans, sockets, etc.) should be inserted in the live wire, so that the concerned circuit becomes dead on putting the switch OFF. This test is performed to verify whether the switches have been connected correctly. For this test, we need a *simple test lamp*. Before conducting this test, we must ensure that

(*i*) all lamps are removed from their holders,
(*ii*) the main fuse is inserted, or main MCB is put ON,
(*iii*) all other fuses are in their position, or all other MCBs are ON, and
(*iv*) all switches are in OFF position.

Now, one end of the test lamp is connected to the earth and the other end is touched to the switch-connection. If the lamp lights, it indicates that the switch is rightly connected. In case the lamp does not light, it indicates that the switch is not connected correctly; the connections must be changed.

19.12 INCANDESCENT OR FILAMENT LAMP

Since the time electricity was invented, the incandescent lamps (commonly called *light bulbs*) have been in use for general lighting. It is the cheapest type of lamp, costing about Rs. 10.

Principle of Working

When a body is heated, it emits radiation whose wavelength is inversely proportional to its absolute temperature T. When the temperature becomes very high, the radiant energy is mostly in the visible light range (λ = 4000 Å - 8000 Å). Since the amount of light emitted is proportional to T^{12}, the lamp should be run at as high temperature as possible.

Material for Heating Filament

The material for making the filament of an incandescent lamp should have following properties: (*i*) To obtain high operating temperature, it should have high melting point. (*ii*) It should have low vapour pressure. (*iii*) It should have high ductility so that it can be drawn into thin wires. (*iv*) Its specific resistance should be high, so that lamps of low wattage can be made without requiring a long length of filament. (*v*) Its temperature coefficient of resistance should be low, so that resistance of the filament does not change much with rise in temperature. (*vi*) It should possess sufficient mechanical strength to withstand vibrations, jerks, etc.

Since ***tungsten*** (melting point 3655 K) possesses practically all the above properties, it is almost invariably used in incandescent lamps.

Construction

Figure 19.11 shows the construction of a light bulb. Various parts are: (*i*) the *filament* in coiled form, (*ii*) the *support wires* (made of molybdenum) for providing mechanical support to the filament, (*iii*) the *button* (made of glass) in which the support wires are fused, (*iv*) the *glass rod* meant for supporting the button, (*v*) the *stem tube* through which the lead-wires come out, (*vi*) the *exhaust tube* through which air is exhausted and then inert gas is introduced, (*vii*) the *fuse*, the portion of the lead-in-wires inside the stem tube, (*viii*) the *lead-in-wire* (made of nickel or nickel-copper alloy) meant for carrying current to the filament, (*ix*) the *gas*,

Fig. 19.13 *Two designs of CFL.*

The purchase price of a CFL is higher (about Rs. 100) than that of an incandescent lamp (about Rs. 10) of the same luminous output. But this cost is recovered in energy savings and replacement costs over the lamp's lifetime.

Due to the potential to reduce electric consumption and hence pollution, various organizations have undertaken measures to encourage the adoption of CFLs. Efforts range from publicity to encourage awareness and make CFLs more widely available, to direct measures to provide CFLs to the public. Some state governments have subsidized CFLs or provided them free to customers as a means of reducing electric demand (and thereby delaying additional investments in generation).

19.15 MERCURY VAPOUR LAMP

It is an *electric discharge lamp* in which light is produced by electrical conduction through mercury vapour. It emits a very bright *greenish-blue light*, in which true colours of objects cannot be seen. It has an average life of about 9000 working hours and high efficiency of about 50 lumens per watt. Mercury lamps are available in various sizes upto 2 kW. Compared to an incandescent lamp, the cost of a mercury lamp is quite high (about Rs. 850). Due to their long life and high efficiency, these are widely used in shopping centres, industrial installations, railway yards, ports, flood-lighting in stadium, street lighting, highway lighting, etc.

Construction

As shown in Fig. 19.14, it consists of two tubes, one inside the other. The smaller inner tube is made of hard glass (or quartz) and is surrounded by the larger glass tube or bulb. The space between the two tubes is *completely evacuated* to prevent heat loss. The outer tube also absorbs harmful ultraviolet radiations.

The inner tube contains argon gas and a small quantity of mercury. Since the arc is confined to this inner tube, it is also called ***arc tube***. It houses two *main electrodes* A and B, and an *auxiliary electrode* C. The main electrodes are made of tungsten and are coated with barium oxide. The auxiliary electrode C is placed near the main electrode A, and it is connected to the other main electrode B through a resistance R. A choke is connected in series to limit the current. The capacitor C_1 is connected across the main lines to improve the power factor.

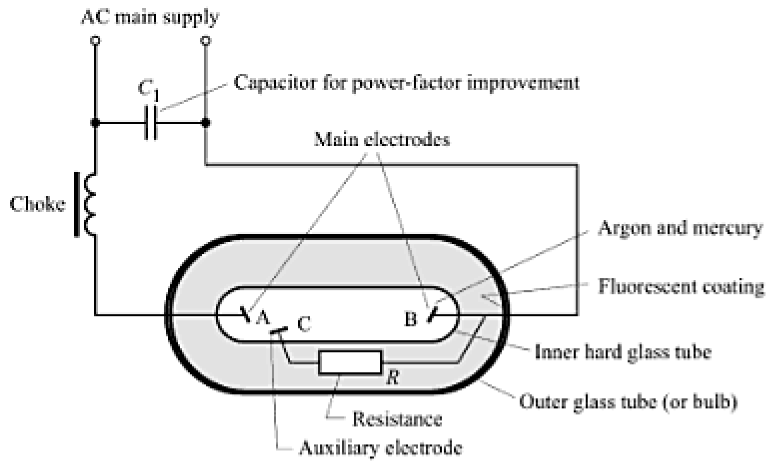


Fig. 19.14 *A mercury vapour lamp.*

Operation

On switching on the supply, an initial discharge is established in argon gas between the main electrode A and the auxiliary electrode C. After a few seconds, the discharge takes place between the two main electrodes and thus spreads throughout the inner tube. The heat produced by this gaseous discharge causes mercury to get vapourized. Slowly, the vapour pressure increases; the discharge column becomes brighter and narrower. The potential across the electrodes A and B rises from about 20 V to 150 V. It takes about 4-5 minutes for the mercury arc to build up to full brilliancy.

The mercury vapour lamps must be operated in its *vertical position only*. If used in horizontal position, convections cause the discharge to touch the glass affecting the operation of the lamp.

After the lamp is switched off, before it can restart it must first be cooled to reduce the vapour pressure. The cooling down also takes about 4-5 minutes. Even if there is a momentary interruption of power supply, the lamps cannot be made to give enough light before 8-10 minutes. Therefore, along with the mercury vapour lamps it is customary for safety reasons to install a few incandescent lamps which can be used for this intervening period of 8-10 minutes.

19.16 SODIUM VAPOUR LAMP

It emits its characteristic *yellow light* due to discharge through sodium vapour at a temperature of about 270 °C. It has an average life of 6000 working hours and has very high efficiency of about 110 lumens per watt. Compared to a mercury vapour lamp, it has more than double efficiency, but is slightly costlier (about Rs. 1400). Because of their high efficiency, these lamps are used for street-lighting, park lighting, railway yard lighting, etc.

Construction

As shown in Fig. 19.15, sodium vapour lamp consists of an inner U-tube made of a sodium-vapour-resisting glass. It has two tungsten electrodes coated with barium oxide. It contains *neon gas* at a pressure of about

SUPPLEMENTARY EXERCISES

E

E.1 Solved Problems
E.2 Practice Problems

PART E : MISCELLANEOUS

Assemblage of

- Chapter 8: DC Transients
- Chapter 18: Electrical Measuring Instruments
- Chapter 19: Electrical Installation and Illumination

E. 1. SOLVED PROBLEMS

PROBLEM E-1

The capacitors in the circuit shown in Fig. E-1 are initially uncharged. Find the indicated voltages and currents at $t = 0^+$, immediately after the switch closes. Also, find these voltages and currents a long time after the switches closes.

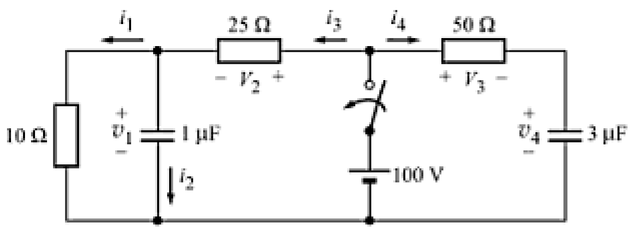


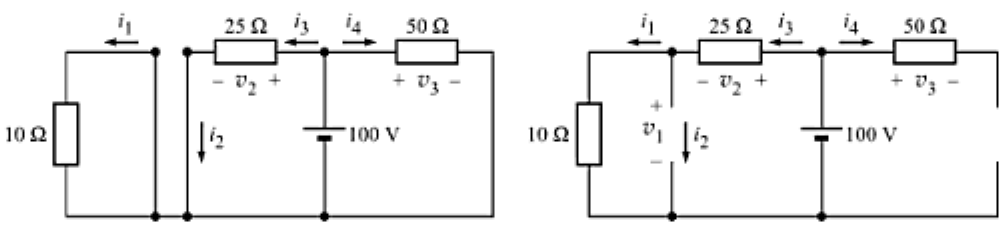
Fig. E-1

Solution Since the voltages across capacitors cannot change suddenly, we have

$$v_1(0^+) = v_1(0^-) = \mathbf{0\ V} \qquad \text{and} \qquad v_4(0^+) = v_4(0^-) = \mathbf{0\ V}$$

Further, with 0 V across them, the capacitors act like short circuits at $t = 0^+$. The equivalent circuit at $t = 0^+$ is as shown in Fig. E-2*a*. It means that the 100-V of the source appears across both the 25-Ω and 50-Ω resistors. Therefore,

$$v_2(0^+) = v_3(0^-) = \mathbf{100\ V}$$



(*a*) (*b*)

Fig. E-2 *The equivalent circuits.*

Obviously, the currents at $t = 0^+$ are given as

$$i_1(0^+) = \frac{0}{10} = \mathbf{0\ A}; \quad i_2(0^+) = i_3(0^+) = \frac{100}{25} = \mathbf{4\ A}; \qquad \text{and} \qquad i_4(0^+) = \frac{100}{50} = \mathbf{2\ A}$$

A long time after the switch closes, the capacitors are fully charged and their voltages are constant; they act like open circuits. The equivalent circuit at $t = \infty$ is as shown in Fig. E-2*b*. It means that $i_2(\infty) = i_4(\infty) = \mathbf{0\ A}$. The 10-Ω and 25-Ω resistors are seen in series across 100-V source. Hence, the currents,

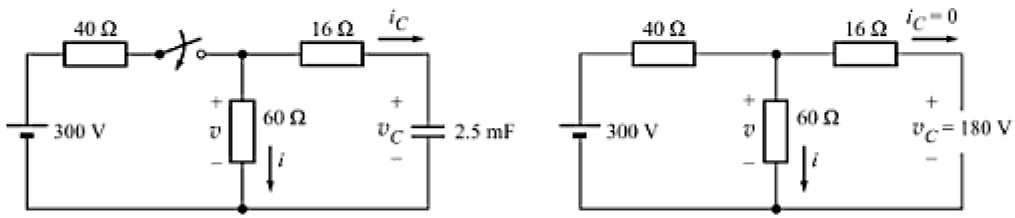
$$i_1(\infty) = i_3(\infty) = \frac{100}{10 + 25} = \mathbf{2.86\ A}$$

Now, the voltages are given as

$$v_1(\infty) = i_1(\infty) \times 10 = 2.86 \times 10 = \mathbf{28.6\ V}; \qquad v_2(\infty) = i_3(\infty) \times 25 = 2.86 \times 25 = \mathbf{71.5\ V};$$
$$v_3(\infty) = i_4(\infty) \times 50 = 0 \times 50 = \mathbf{0\ V}; \qquad v_4(\infty) = 100 - v_3(\infty) = 100 - 0 = \mathbf{100\ V}$$

PROBLEM E-2

In the circuit shown in Fig. E-3*a*, the capacitor is charged to 100 V. The switch closes at $t = 0$ s. Find v_C and i for $t > 0$ s.



(*a*) For $t < 0$. (*b*) For $t = \infty$.

Fig. E-3

Solution All that we need for finding the expression for v and i are $v_C(0^+)$, $v_C(\infty)$, $i(0^+)$, $i(\infty)$ and $\tau = R_{Th}C$. Because the capacitor voltage cannot jump, we have $v_C(0^+) = 100$ V. A long time after the switch is put on, the capacitor gets fully charged and the current i_C reduces to zero because the capacitor acts like an open circuit, as shown in Fig. E-3*b*. The voltage $v_C(\infty)$ across the capacitor is the same as the voltage across the 60-Ω resistor. So, by voltage division,

$$v_C(\infty) = 300 \times \frac{60}{60 + 40} = 180 \text{ V}$$

Also, writing KVL for the first loop, we have

$$i(\infty) = \frac{300}{40 + 60} = 3 \text{ A}$$

We can easily get $i(0^+)$ from $v(0^+)$. For finding $v(0^+)$, we write a node voltage equation for the middle top node,

$$\frac{v_C(\infty) - 300}{40} + \frac{v_C(\infty)}{60} + \frac{v_C(\infty) - 100}{16} = 0$$

Solving the above equation, we get $v(0^+) = 132$ V. Therefore, $i(0^+) = 132/60 = 2.2$ A. Now, the Thevenin resistance at the capacitor terminals is $16 + 60 \| 40 = 40\ \Omega$. Therefore, the time constant for charging of the capacitor is

$$\tau = R_{Th}C = 40 \times 2.5 \times 10^{-3} = 0.1 \text{ s}$$

We can now write the formulas for v_C and i,

$$v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{-t/\tau} = 180 + (100 - 180)e^{-10t} = \mathbf{180 - 80}\, e^{-10t}\ \mathbf{V}$$

and
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = 3 + (2.2 - 3)e^{-10t} = \mathbf{3 - 0.8}\, e^{-10t}\ \mathbf{A}$$

PROBLEM E-3

In the circuit shown in Fig. E-4 the capacitor is initially uncharged and the switch is closed at $t = 0$ s. Find the current i for $t > 0$ s.

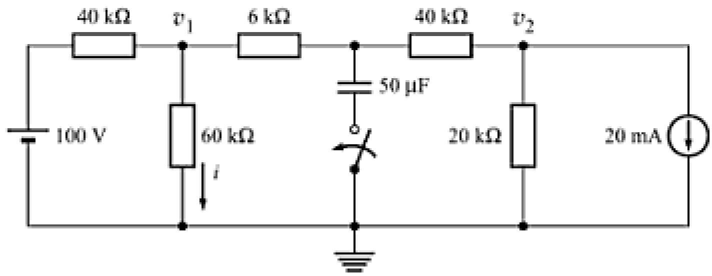


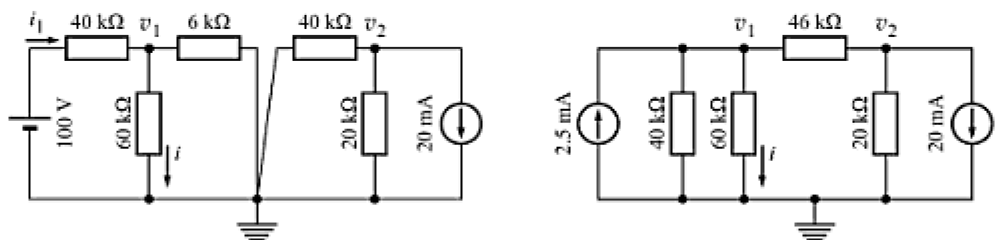
Fig. E-4

Solution For writing the expression of i for $t > 0$ s, we need $i(0^+)$, $i(\infty)$ and τ. At $t = 0$ s, the uncharged capacitor acts as a short circuit, and hence the equivalent circuit becomes as shown in Fig. E-5*a*. Obviously, the 20-mA current does not affect $i(0^+)$ Furthermore, the 60-kΩ and 6-kΩ resistors are placed in parallel. The current $i_1(0^+)$ is given as

$$i_1(0^+) = \frac{100}{R_{eq}} = \frac{100\ \text{V}}{(40 + 60\|6)\,\text{k}\Omega} = 2.2\ \text{mA}$$

By current division, we get

$$i(0^+) = i_1(0^+)\left(\frac{6}{60+6}\right) = (2.2\ \text{mA}) \times \frac{6}{66} = 0.2\ \text{mA}$$



(*a*) (*b*)

Fig. E-5

After 5 time-constants, the capacitor no longer conducts current. It acts as an open circuit. The 6-kΩ and 40-kΩ resistors come in series. Transforming the 100-V source in series with 40-kΩ resistor into a 2.5-mA [= (100 V)/(40 kΩ)] source in parallel with 40-kΩ resistor, we get the circuit as shown in Fig. E-5*b*. Writing nodal analysis equations, by inspection, we get

$$\begin{bmatrix} \left(\frac{1}{40}+\frac{1}{60}+\frac{1}{46}\right) & -\left(\frac{1}{46}\right) \\ -\left(\frac{1}{46}\right) & \left(\frac{1}{20}+\frac{1}{46}\right) \end{bmatrix} \begin{bmatrix} v_1(\infty) \\ v_1(\infty) \end{bmatrix} = \begin{bmatrix} 2.5 \\ -20 \end{bmatrix}$$

Note that, for convenience, we have taken volt-milliampere-kilohm units. Using calculator, the above equations can be solved to give $v_1(\infty) = -62.67$ V. So,

$$i(\infty) = \frac{v_1(\infty)}{60} = \frac{-62.67}{60} = -1.04\ \text{mA}$$

Now, Thevenin resistance across the capacitor terminals is

$$R_{Th} = (6 + 40 \,||\, 60) \,||\, (40 + 20) = 20 \text{ k}\Omega$$

so that $\tau = R_{Th}C = (20\text{k}\Omega)\,(50\ \mu\text{F}) = 1 \text{ s}$

Having known $i(0^+)$, $i(\infty)$, and τ, we can now write the expression for the current i as

$$i = i(\infty) + [i(0^+) - i(\infty)]\, e^{-t/\tau} = -1.04 + [0.2 - (-1.04)]\, e^{-t/1} = \mathbf{-1.04 + 1.24}\, e^{-t} \textbf{ mA}$$

PROBLEM E-4

Find the voltage across the inductor in the circuit of Fig. E-6a, after the switch is opened at $t = 0$ s.

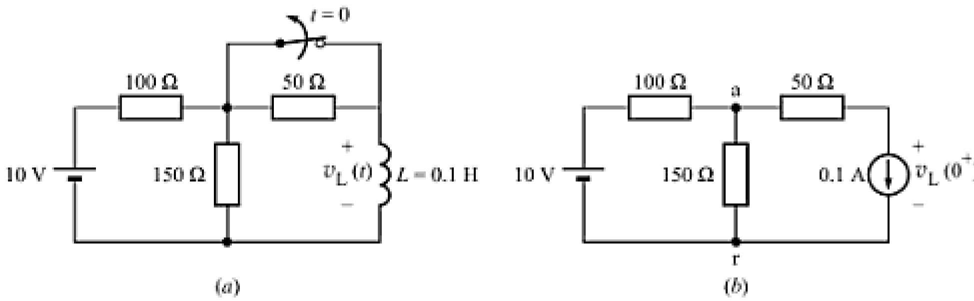


Fig. E-6

Solution The current in an inductor cannot change abruptly. Hence, we must have $i_L(0^+) = i_L(0^-)$. To determine $i_L(0^-)$, we note that the switch had been closed for a time long enough to establish steady-state. Hence, it acts as a short circuit. Furthermore, the closed switch short circuits the 50-Ω resistor, and the closed switch and the inductor short circuits the 150-Ω resistor. Thus, the current from the battery is (10 V)/(100 Ω) = 0.1 A, and this is the current $i_L(0^-)$ through the closed switch and the inductor. Hence, $i_L(0^+) = i_L(0^-) = 0.1$ A. But, this does not give us the initial value of $v_L(t)$. For finding $v_L(0^+)$, we resort to nodal analysis as follows.

Immediately after the switch is opened, the inductor acts as current source of 0.1 A. Therefore, the equivalent circuit at $t = 0^+$ is as shown in Fig. E-6b. Note that we have not shown the switch in this equivalent circuit as it is now open. Writing nodal equation for the node a, we get

$$\frac{v_a - 10}{100} + \frac{v_a}{150} = -0.1 \quad \Rightarrow \quad v_a = 0 \text{ V}$$

Therefore, $v_L(0^+) = -(50\ \Omega)\,(0.1\text{ A}) = -5 \text{ V}$

Now, for determining the time constant we need to find R_{Th}. As can be seen from the circuit of Fig. E-16,

$$R_{Th} = 50 + 150 \,||\, 100 = 110\ \Omega$$

∴ $\tau = L/R_{Th} = 0.1/110 = 909\ \mu\text{s}$

Since the final value of the inductor voltage is zero, we get the full solution as

$$v_L(0^+) = v_L(\infty) + [v_L(0^+) - v_L(\infty)]\, e^{-t/\tau} = 0 + [(-5) - 0]\, e^{-t/909\mu s} = \mathbf{-5}\, \boldsymbol{e^{-t/909\mu s}} \textbf{ V}$$

PROBLEM E-5

Find the time constant of the circuit shown in Fig. E-7, andtermine the energy stored in the inductor.

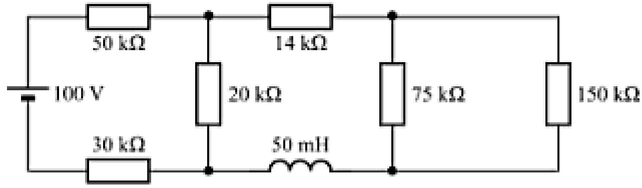


Fig. E-7

Solution Thevenin resistance of the circuit across the inductor terminals,

$$R_{Th} = (50 + 30) || 20 + 14 + 75 || 150 = 80\ k\Omega$$

Hence, the time constant of the circuit,

$$\tau = L/R_{Th} = (50 \times 10^{-3})/(80 \times 10^{3})s = \mathbf{0.625\ \mu s}$$

When the inductor terminals are open circuited, no current flows through the 14-kΩ resistor. Thevenin's voltage is the voltage across the 20-kΩ resistor. By voltage division, we have

$$V_{Th} = (100\ V) \times \frac{20}{20 + 50 + 30} = 20\ V$$

Since the inductor behaves as a short circuit, the inductor current is

$$I_L = \frac{V_{Th}}{R_{Th} + 0} = \frac{20\ V}{80\ k\Omega} = 0.25\ mA$$

Therefore, the stored energy in the inductor,

$$W = \frac{1}{2} L I_L^2 = \frac{1}{2} \times 50 \times 10^{-3} \times (0.25 \times 10^{-3})^2\ J = \mathbf{1.56\ nJ}$$

PROBLEM E-6

In the circuit shown in Fig. E-8*a*, the switch has been in position 1 for a long time. (*a*) Find the indicated currents. (*b*) Find the indicated currents and voltage immediately after the switch has been thrown to position 2.

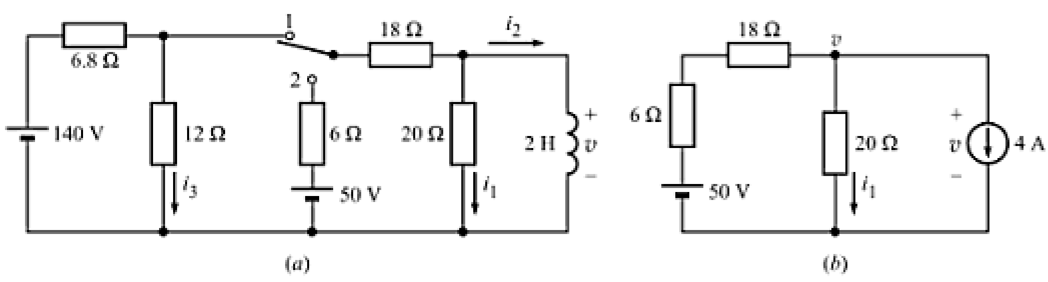


Fig. E-8

Solution

(*a*) The inductor acts as a short circuit, and shorts out the 20-Ω resistor. As a result, the current through 20-Ω resistor, $i_1 = 0$ A. This short circuit also places the 18-Ω resistor in parallel with the 12-Ω resistor. Thus, the total resistance across the 140-V source is

$$R = 6.8 + 12 || 18 = 14\ \Omega$$

Hence, the current drawn from the 140-V source, $i = \dfrac{140}{14} = 10$ A

By current division, we get

$$i_2 = 10 \times \frac{12}{12+18} = \mathbf{4\ A} \qquad \text{and} \qquad i_3 = 10 \times \frac{18}{12+18} = \mathbf{6\ A}$$

(*b*) As soon as the switch leaves position 1, the left-hand side of the circuit is isolated. It becomes just a series circuit in which the current,

$$i_3 = \frac{140}{6.8+12} = \mathbf{7.45\ A}$$

In the right-hand part of the circuit, the current in the inductor cannot suddenly change, and is **4 A,** as it was found above. Hence, $i_2 = 4$ A. Since this is a known current, it can be considered to be from a current source, as shown in Fig. E-8*b*. By nodal analysis,

$$\frac{v}{20} + \frac{v-50}{6+18} + 4 = 0 \quad \Rightarrow \quad v = -20.9 \text{ V}$$

$$\therefore \qquad i_1 = \frac{v}{20} = \frac{-20.9}{20} = \mathbf{-1.05\ A}$$

PROBLEM E-7

A moving-coil instrument gives full-scale deflection with 15 mA and has a resistance of 5 Ω. Calculate the resistance required (*a*) in parallel to enable the instrument to read up to 1 A, (*b*) in series to enable it to read up to 10 V.

Solution Here, $I_m = 15$ mA = 0.015 A, $R_m = 5\ \Omega$. The voltage drop across the instrument for full-scale deflection,

$$V_m = I_m R_m = (15 \times 10^{-3}) \times 5 = 75 \text{ mV}$$

(*a*) The current through the shunt, $\quad I_{sh} = I_{fsd} - I_m = 1 - 0.015 = 0.985$ A

The resistance of the shunt required, $\quad R_{sh} = \dfrac{V_m}{I_{sh}} = \dfrac{0.075}{0.985} = \mathbf{0.07614\ \Omega}$

(*b*) The required full-scale voltage, $\quad V_{fsd} = 10$ V

The voltage drop across the series resistor, $V_s = V_{fsd} - V_m = 10 - 0.075 = 9.925$ V

$\therefore$ The series resistance, $R_s = \dfrac{V_s}{I_m} = \dfrac{9.925}{0.015} = \mathbf{661.7\ \Omega}$

PROBLEM E-8

A voltage of 100 V is applied to a circuit comprising two 50-kΩ resistors in series. A voltmeter, with an *fsd* of 50 V and a figure of merit of 1 kΩ/V, is used to measure voltage across the 50-kΩ resistors. Calculate the percentage error in this measurement.

Solution By potential division, the true value of the voltage across the 50-kΩ resistor (when the voltmeter is not connected),

$$V_t = 100 \times \frac{50 \text{ k}\Omega}{(50+50)\text{k}\Omega} = 50 \text{ V}$$

The resistance of the voltmeter, $R_m = (50 \text{ V}) \times (1 \text{ k}\Omega/\text{V}) = 50 \text{ k}\Omega$

When the voltmeter is connected in the circuit, it shunts the 50-kΩ resistor. The equivalent resistance of this parallel combination,

$$R_e = (50 \text{ k}\Omega) || (50 \text{ k}\Omega) = 25 \text{ k}\Omega$$

closed. **[Ans.** 78.7 V, 60.7 μA]

E-3. For the circuit used in Prob. E-2, find the time required for the capacitor voltage to reach 50 V. Then find the time required for the capacitor to increase another 50 V, from 50 V to 100 V. **[Ans.** 57.5 ms, 81.1 ms]

E-4. Find the time constant of the circuit shown in Fig. E-11. **[Ans.** 60 μs]

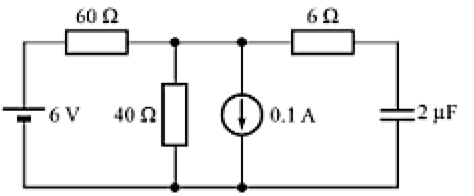


Fig. E-11

E-5. The switch in the circuit shown in Fig. E-12 is closed at $t = 0$ s. Determine (*a*) the voltage v_L across the inductor, and (*b*) the initial rate of increase of the current $i(t)$. **[Ans.** (*a*) $10\,e^{-t/0.4}$ V; (*b*) 5 A/s]

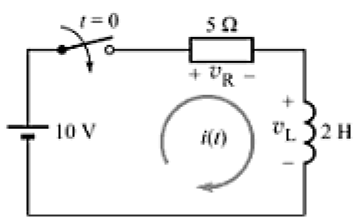


Fig. E-12

E-6. A 50-mH inductor is connected to a current source of $i_s(t) = 0$, for $t < 0$, and of $i_s(t) = 150t^3$ A, for $t > 0$. Calculate the voltage across the inductor, $v_L(t)$, with the polarity shown in Fig. E-13. **[Ans.** $22.5t^2$ V, for $t > 0$]

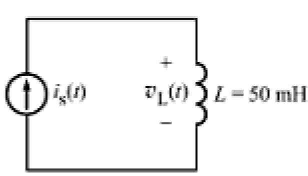


Fig. E-13

E-7. Figure E-14 shows a 20-μF capacitor that has a voltage $v_s(t) = 0$ V, $t < 0$; $v_s(t) = 10^4t^2$ V, $0 < t < 0.1$ s; $v_s(t) = 100$ V, $t > 0.1$ s. (*a*) Find the current, $i_C(t)$. (*b*) At what time is the energy in the capacitor 50 mJ? (*c*) At what time is the power into the capacitor 1 W?

[Ans. (*a*) $0.4t$ A for $0 < t < 0.1$ s, 0 elsewhere; (*b*) 0.0841 s; (*c*) 0.063 s]

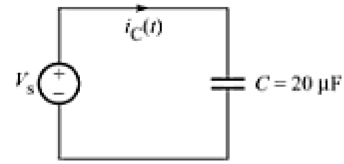


Fig. E-14

E-8. To move the bright spot of a CRT smoothly across its screen, the voltage across a pair of deflection plates must be increased linearly, as shown in Fig. E-15 If the capacitance of the plates is 1 pF, find the resulting current through the capacitor. **[Ans.** 4 μA when increasing, – 20 A when decreasing]

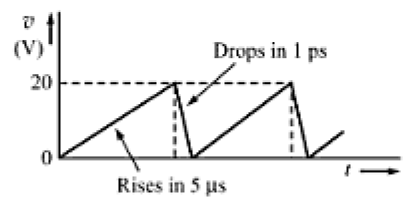


Fig. E-15

E-9. A 0.1-μF capacitor, initially charged to 230 V, is discharged through a 3-MΩ resistor. (*a*) Find the capacitor voltage 0.2 s after the capacitor starts to discharge. (*b*) How long does it take the capacitor to discharge to 40 V? **[Ans.** (*a*) 118 V; (*b*) 0.525 s]

E-10. A current $i = 0.32t$ A flows through a 150-mH inductor. Find the energy stored at $t = 4$ s. **[Ans.** 0.123 J]

E-11. Closing a switch connects in series a 20-V source, a 2-Ω resistor and a 3.6-H inductor. How long does it take the current to get to its maximum value, and what is this value? **[Ans.** 9 s, 10 A]

E-12. A short is placed across a coil carrying a current of 0.5 A at that instant. If the coil has an inductance of 0.5 H and a resistance of 2 Ω, what is the coil current 0.1 s after the short is applied? **[Ans.** 0.335 A]

E-13. A moving-coil instrument gives full-scale deflection with 15 mA and has a resistance of 5 Ω. Calculate the resistance of the necessary components in order that the instrument may be used as (*a*) a 2-A ammeter, and (*b*) a 100-V voltmeter.

[**Ans.** (*a*) $R_{sh} = 0.0378\ \Omega$; (*b*) $R_s = 6662\ \Omega$]

E-14. A moving-coil milliammeter has a coil of resistance 15 Ω, and full-scale deflection is given by a current of 5 mA. This instrument is to be adapted to operate (*a*) as a voltmeter with a full-scale deflection of 100 V, and (*b*) as an ammeter with a full- scale deflection of 2 A. Calculate the value of any components to be used to meet the above requirements.

[**Ans.** (*a*) $R_s = 19.985$ kΩ; (*b*) $R_{sh} = 0.0376\ \Omega$]

E-15. A moving-coil galvanometer of resistance 5 Ω, gives full-scale reading when a current of 15 mA passes through it. Explain how its range could be altered so as to read up to (*a*) 5 A, and (*b*) 150 V.

[**Ans.** (*a*) $R_{sh} = 0.015045\ \Omega$; (*b*) $R_s = 9.995$ kΩ]

E-16. If a rectifier type voltmeter has been calibrated to read the rms value of a sinusoidal voltage, by what factor must the scale readings be multiplied when it is used to measure the rms value of (*a*) a square-wave voltage, and (*b*) a voltage having a form factor of 1.15? [**Ans.** (*a*) 0.9; (*b*) 1.036]

E-17. A dc voltmeter has a resistance of 28 600 Ω. When connected in series with an external resistor across a 480 V dc supply, the instrument reads 220 V. What is the value of the external resistor?

[**Ans.** 33.8 kΩ]

(B) TRICKY PROBLEMS

E-18. How long does it take a 10-μF capacitor charged to 200 V to discharge through a 160-kΩ resistor, and what is the total energy dissipated in the resistor?

[**Ans.** 8 s, 0.2 J]

E-19. Closing of a switch at $t = 0$ s connects in series a 150-V source, a 1.6-kΩ resistor, and the parallel combination of a 1-kΩ resistor and an uncharged 0.2-μF capacitor. Find (*a*) the initial capacitor current, (*b*) the initial and final 1-kΩ resistor currents, (*c*) the final capacitor voltage, and (*d*) the time required for the capacitor to reach its final voltage.

[**Ans.** (*a*) 93.8 mA; (*b*) 0 A, 57.7 mA; (*c*) 57.7 V; (*d*) 0.615 ms]

E-20. Repeat Prob. E-19 for a 200-V source and an initial capacitor voltage of 50 V opposed in polarity to that of the source.

[**Ans.** (*a*) 43.8 mA; (*b*) 50 mA, 76.9 mA; (*c*) 76.9 V; (*d*) 0.615 ms]

E-21. The switch in the circuit shown in Fig. E-16 is closed at $t = 0$ s. Determine the current $i(t)$ through the capacitor. [**Ans.** $0.183e^{-t/1.1\ \mu s}$]

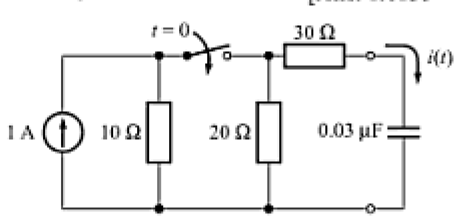


Fig. E-16

E-22. Assuming that the switch in the circuit shown in Fig. E-17 has been in position *a* for a long time before it is switched to position *b* at $t = 0$, find $v_L(t)$, the voltage across the inductor with the polarity shown. [**Ans.** $-10\,e^{-t/1.5\text{ ms}}$ V]

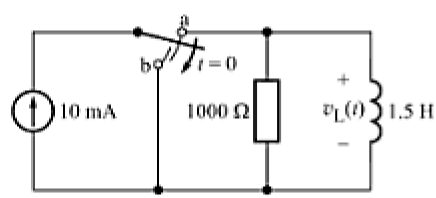


Fig. E-17

E-23. A current source is placed in series with a resistor and an inductor, as shown in Fig. E-18. During this period, the switch is open. The switch is closed at $t = 0$ s, so that the circuit is separated into two independent loops that share a common short circuit but do not interact. Find the current in the right-hand loop after the switch closes.

[**Ans.** $10\,e^{-t/0.02\ \mu s}$ mA]

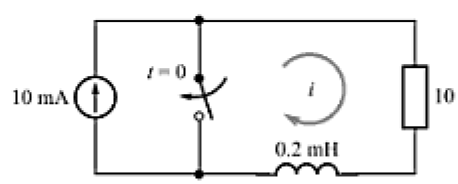


Fig. E-18

GLOSSARY

A : An abbreviation for *ampere*.

Absolute instrument : It measures a quantity directly in terms of absolute parameters, without needing prior calibration.

Absolute permeability : It is the ratio of the magnetic flux density produced to the magnetic field strength producing it; $\mu = B/H$ henry/meter.

AC or **ac : 1**. Abbreviation for *alternating current*. Electric current which alternates at regular intervals. Its direction is reversed 50 times per second in India. **2**. Pertaining to, or utilising **ac**.

AC generator : Also called *alternator*. It produces alternating emf that delivers power to circuits.

Accuracy of measurement : The ratio of the error in the indicated value to the true value.

Active component : Also called *active element*. An independent source which can deliver or absorb energy continuously.

Active current : The component of the current phasor along the voltage phasor.

Active power : Also called *actual power, real power,* or *true power*. In an ac circuit, the product of the voltage and the current that is in phase with the voltage. It is measured in watts.

Adapter : A device used to connect different types of electrical terminals.

Admittance : A measure of the ease with which ac current flows through a circuit. It is a complex quantity written as $Y = G + jB$, and it is expressed in siemens (S). Its real number component (G) is called *conductance*, and its imaginary number component (B) is called *susceptance*. It is the reciprocal of *impedance* (Z).

Air capacitor : A capacitor in which the dielectric is nearly all air. Used for tuning electrical circuits with minimum energy loss.

All-day efficiency of a transformer : The ratio of energy output in 24 hours to the energy output plus energy losses in 24 hours.

Alternating current : Same as *ac*.

Alternator : A device which produces AC. Also called synchronous generator.

Ammeter : Abbreviation of *ampere meter*. An instrument used for measuring electric current.

Ampere : The fundamental SI unit of electric current. It is defined as the constant current that produces a force of 0.2 micronewtons per metre between two infinitely long parallel conductors placed one metre apart in vacuum.

Ampere-hour meter : A meter that records the product of current and time (ampere-hour) for a given circuit. If the voltage is constant, it can be calibrated as an energy meter (indicating kilowatt-hour).

Ammeter shunt : A resistor connected in parallel to an ammeter, in order to increase the current range that can be measured.

Ampere-turn : Unit of magneto-motive force (MMF); equivalent to ampere times turns.

Amplitude : The maximum value of a periodically varying quantity during a cycle, e.g., the maximum value of an alternating current.

Amplitude factor : Also *called peak factor*. It is the ratio of the maximum (peak) value to the rms value of an alternating quantity.

Angular velocity : Also called angular speed or angular frequency, represented by the symbol ω (Greek letter omega), and measured in radians per second (rad/s).

Apparent power : In an ac circuit, the product of the rms voltage and the rms current. It is measured in volt-ampere (V/A).

Armature : 1. The rotating, or moving part, in an electric generator or motor. **2.** in an electromagnetic device, the moving element. For instance, the moving contact in an electromagnetic relay.

Armature coil : A coil of insulated copper wire wrapped around the armature core. It is a part of the armature winding.

Armature core : The assembly of laminations forming the magnetic circuit of the armature of an electric machine.

Armature reactance : A reactance associated with the armature winding of a machine. It is caused by the armature leakage-flux.

Armature reaction : The interaction between the magnetic field produced by the field coils of an electric motor or generator, and the magnetic field produced by the current flowing through the armature.

Asynchronous machine : An ac machine which does not run at a speed which is synchronised to the frequency of the power supply, as opposed to a *synchronous machine* which does.

Autotransformer : A transformer that has a single-tapped winding, so that one part serves as the primary winding, and the other part as the secondary winding.

Auxiliary winding : A special winding on a machine, in addition to the main winding.

Average value : The adding together of many instantaneous values of an amplitude taken at equal time intervals divided by the number of measurements taken. For a sine wave, the average value is 0.637 times its peak amplitude value.

Ayrton shunt : A high-resistance shunt utilised to reduce the sensitivity of a measuring instrument, such as a galvanometer. This increases its range. It is also called *universal shunt*.

B : Symbol for *susceptance* and *magnetic flux density*.

Back emf : The emf which arises in an inductance (because of the rate of change in current) or in an electric motor (because of flux-cutting).

Balanced current : A term used in connection with a polyphase circuit to denote the current which is equal to the sum of currents in all phases.

Balanced load : A load connected to a polyphase system such that each phase current has the same magnitude and all phase currents add up to zero.

Balanced voltage : A term used in polyphase circuits to denote voltages which are equal in all phases.

Battery : A source of dc power that incorporates two or more cells.

B-H curve : The relationship between the flux density and the field density for a ferromagnetic material.

Bimetallic strip : Bonded strip composed of two metals with differing thermal expansion coefficients. Used as a thermal switch.

Blowing current : The current (ac or dc) which will cause a fuse link to melt.

Branch : Part of a circuit that lies between two nodes.

Breadth factor : A factor used in the calculation of the emf generated in the winding of an ac machine to allow for the fact that the emfs in each of the individual coils are not in phase with one another. Also called *breadth coefficient* or *distribution factor*.

Brush : A conductor which slides to maintain contact between stationary and moving parts of an electrical device, such as a motor. It is usually made of graphite, or a metal.

Bus bar: A length of a constant-voltage conductor in a power circuit. Also called power line.

C: Symbol for *capacitance* and *coulomb*.

Cable: An insulated electrical conductor, often in strands. Or a combination of electrical conductors insulated from one another.

Cage rotor: A form of rotor which is used in induction motors.

Calibration: The method of adjusting an instrument so that it conforms to a specified standard.

Candela: The fundamental SI unit of luminous intensity. If, in a given direction, a source emits monochromatic radiations of frequency 540×10^{12} Hz, and the radiant intensity in that direction is 1/683 watt per steradian, the luminous intensity of the source is 1 candela (Cd).

Capacitance: The ability of a *capacitor* to store charge. It is equal to the quantity of charge which is required to be given to the capacitor so as to increase the potential difference across its terminals by one volt. Its symbol is C, and it is measured in farads (F).

Capacitive reactance: The impedance associated with a capacitance. Its symbol is X_C, and is measured in ohms (Ω). Its value is frequency dependent and is given as $X_C = 1/\omega C = 1/2\pi f C$

Capacitor: A component having capacitance. Also called a *condenser*.

Carbon brush: A small block of carbon used in electrical equipment to make contact with a moving surface.

CFL (Compact Fluorescent Lamp): It is a type of fluorescent lamp, which can fit in the existing light fixtures (bulb holders).

Charge: Its symbol is Q, and it is measured in coulombs (C). A surplus of electrons in a body creates *negative* charge, whereas a deficiency of electrons creates a *positive* charge.

Choke: Usually refers to a coil having high self-inductance. It is used to limit the flow of current with little power dissipation.

Circuit: An arrangement of passive and active components joined by conductors so as to provide path or paths for electric current.

Circulating current: Also called *mesh current*. It is an imaginary current that is assumed to flow around a loop in a circuit.

Coefficient of coupling: The ratio of flux linkages between the primary and the secondary coil to the total flux produced by the primary.

Coil: A length of an insulated conductor wound around a core of iron, ferrite, or air.

Coil span: The distance between one side of a coil and the other, measured around the periphery of the armature.

Commutation: **1.** The transfer of current from one path of a circuit to another path within the same circuit. **2.** The reversals of current through the windings of an armature, to provide the dc at the brushes.

Commutator: **1.** A device, such as a switch, used to reverse current. **2.** In a dc motor or generator, the section that causes the direction of the electric current to be reversed in the armature windings. **3.** In a dc motor of generator, the section which maintains electrical continuity between the rotor and the stator.

Compound generator: A dc generator having both series and shunt windings.

Compound motor: A dc motor having both series and shunt windings.

Compound winding: A winding which has both series and shunt windings.

Conductivity: The ease with which an electric current can flow through a body. Its symbol is σ, and is measured in S/m. It is the reciprocal of *resistivity*.

Conductor: A medium which allows electric current to flow easily, such as silver, copper and aluminum wire.

Conduit: A pipe (metallic or plastic) containing electric wires or cables so as to protect them from any damage due to external cause.

Conduit box: A box connected to the metal conduit used in some electric wiring schemes.

Conduit fittings: All the auxiliary items, such as boxes, elbows, pipes, etc. needed for the conduit system of wiring.

Constant current source: A source having very high internal resistance compared to the external load.

Constant voltage source: A source having very low internal resistance compared to the external load.

Continuity: A continuous electrical connection between two points.

Controlling torque: Also called *restoring torque*. Its magnitude increases with deflection and it opposes the deflecting torque in an instrument.

Copper loss: Also called I^2R loss. It occurs in windings (usually made of copper) as a result of current flowing through them, in transformers and electrical machines.

Core losses: The losses occurring in the iron core of the transformers and electrical machines due to hysteresis and eddy currents caused by alternating flux.

Core-type transformer: A transformer in which most of the core is enclosed by the windings.

Crest factor: It is the ratio of the peak value to the rms value of an ac wave. It is also called **peak factor** or **amplitude factor**. In the specific case of sinusoidal ac, the crest factor is 1.414.

Critical field-resistance of a shunt generator: The maximum value of field-resistance beyond which the generator cannot build up voltage.

Critical speed of a shunt generator: The lowest speed below which the generator fails to build up voltage.

Cumulatively compound machine: A compound machine in which the shunt field and series field assist each other.

Current: It is the rate of flow of electric charge through a conductor. Its symbol is i or I, and is expressed in amperes (A).

Current coil: A term used with wattmeter, energy meters, etc. to denote the coil connected in series and hence carrying main current.

Cycle: One complete oscillation.

Damped oscillation: Oscillation that dies always from initial maximum amplitude, with each successive oscillation diminishing until the amplitude becomes zero. Also called *damped vibration*.

Damping torque: This torque acts on the moving system of an instrument, and it comes into action only when the system is moving so as to oppose the motion.

D'Arsonval movement: A common meter movement using a moving coil in a fixed magnetic field.

DC or dc: Abbreviation of *direct current*.

DC generator: A machine which converts mechanical energy into dc electrical energy.

DC machine: A machine which converts mechanical energy into dc electrical energy and vice versa.

DC meter: An instrument which responds only to dc component of a signal, e.g., moving coil instrument.

DC motor: A machine which converts dc electrical energy into mechanical energy.

Delta connection: A three terminal having a combination of three circuit elements, such as resistors, connected in series, and arranged in the form of triangle, similar to shape of the Greek letter Δ.

Demagnetisation: Reducing the residual magnetism of a previously magnetised material by applying a magnetising force in the opposite direction.

Dielectric: Insulating material which separates the two plates of a capacitor.

Dielectric Constant: It denotes the ratio of the insulating quality of a material to that of air.

Differentially wound motor: A dc motor with both the series and shunt field windings arranged such that their fields oppose each other.

Direct Current: A current which flows only in one direction, although it may have pulsations in its magnitude.

Distribution factor: A factor used in ac machines which accounts for the fact that the emfs produced in each of the individual coils are not in phase. Also called *breadth factor*, or *breadth coefficient*.

Induction: **1.** The generation or modification of electric fields, magnetic fields, voltages, or current, through the influence of nearby entities. **2.** The generation of an electromotive force in a circuit or conductor caused by a change in the magnetic flux.

Induction motor: An ac motor in which the electric current flowing through the rotor is induced by a current flowing in its stator.

Inductive reactance: The opposition to the flow of an ac current due to the inductance of a component or circuit. Its symbol is X_L, and is measured in ohms. $X_L = \omega L = 2\pi f L$.

Inductor: A conductor wound in a series of turns, so as to introduce inductance, or to produce a magnetic field in an electric circuit. Used in transformers, electromagnets, solenoids, motors, speakers, and so on.

In-phase: A state in which two or more periodic quantities having the same frequency and wave shape pass through corresponding values, such as maximas and minimas, at the same instant at all times. In an out-of-phase state, the periodic quantities do not pass through corresponding values at the same instant at all times.

Inertia: The tendency of a body to preserve its state of rest or of uniform motion.

Instantaneous value: The value of varying quantity, such as voltage or current, at a particular instant within a cycle.

Insulated neutral: A term used to denote the neutral point of a star-connected generator or transformer when it is not connected to earth directly or through low impedance.

Insulated wire: A solid conductor insulated throughout its length.

Insulator: A material having high electrical resistivity.

Integrating instrument: An instrument whose indications are the total over time of a measured quantity, such as power.

Iron core: A core, such as that of a transformer or armature, which is made of solid or laminated iron.

Iron loss: The energy dissipated by an iron core, due to eddy currents and hysteresis loss, as found in a transformer or inductor.

***j*:** Symbol for $\sqrt{-1}$, used by electrical engineers in place of mathematician's i. It denotes a 90° counterclockwise phasor rotation.

J: Symbol of joule, the unit of energy.

Junction: A point in a circuit where two or more conductors are connected.

k: Symbol for a prefix kilo (10^3 times).

Killowatt-hour: A unit of energy equal to 10^3 watt-hours. Abbreviated as kW h.

kV: Abbreviation for *kilovolt*.

kVA: Abbreviation for *kilovolt-ampere*.

kVAR: Abbreviation for *kilovolt-ampere-reactive*.

kW: Abbreviation for *kilowatt*.

kW h: Abbreviation for *kilowatt-hour*.

L: Symbol of *inductance*.

Lag: The amount, measured as a time interval or the angle in electric degrees, by which one periodically varying wave is delayed in phase with respect to another wave of same period.

Lagging current: An alternating current which reaches its maximum value later in the cycle than the voltage which produces it.

Lagging load: A reactive load in which inductive reactance exceeds the capacitive reactance and therefore carries a lagging current with respect to the voltage across its terminals.

Lamination: A sheet steel stamping so shaped that a number of them can be put together to form the magnetic circuit of electric machines, transformers, etc.

Lap winding : A form of two-layer winding of electric machines in which each coil is connected in series with the one adjacent to it.

Lead : 1. The amount, measured as a time interval or the angle in electric degree, by which one periodically varying wave is advanced in phase with respect to another wave of same period. **2.** A term used to denote an electric wire or cable connected to a terminal.

Leading current : An alternating current which reaches its maximum value earlier in the cycle than the voltage which produces it.

Leading load : A reactive load in which capacitive reactance exceeds the inductive reactance and therefore carries a leading current with respect to the voltage across its terminals.

Leakage : The flow of electric current along a path other than the one intended, due to faulty insulation in a circuit.

Leakage coefficient : The ratio of the total flux in the magnetic circuit of an electric machine or transformer to the useful flux which actually links the armature or the secondary winding. Also called *leakage factor*.

Leakage flux : Magnetic flux which does not pass through a useful or intended part of a magnetic circuit.

Leakage inductance : Self-inductance especially in a transformer, due to *leakage flux*.

Leakage reactance : Unwanted reactance in a transformer or in an electric machine, caused by leakage flux cutting one coil but not the other.

Lenz's law : It states that the induced emf in a circuit opposes the very cause due to which it has been induced.

Lifting magnet : A large electromagnet on a crane or hoist, used to lift iron objects.

Lightning conductor : A metal rod which sticks out at the top of a high-rise building; the other end of the rod is earthed.

Linear resistor : A resistor which obeys ohm's law. Also called *ohmic resistor*.

Linear element : An element whose volt-ampere characteristic is at all times a straight line through the origin.

Line current : The current flowing through a line

Line voltage : The voltage between the lines of an electric power supply.

Live : Connected to a voltage source.

Live wire : The wire which is at a higher potential than the *neutral* wire of the supply.

Load : The rate at which energy is fed into a process or removed from it.

Load characteristic : Instantaneous voltage-current characteristic at the output of a generator under loading condition.

Load factor : The ratio of the average load to the peak load over a period.

Loss : Dissipation of energy without accomplishing any useful work.

Loss angle : The difference between 90° and the angle of lead of current over the voltage in a capacitor.

Lumen : The unit of luminous flux defined as the amount of light emitted which falls on unit area when the surface area is at a unit distance from the source of one candela.

Lux : The unit of *illumination*, symbolised as lx. It is equal to 1 lumen/m^2

M : Symbol for prefix *mega* (10^6 times).

m : Symbol for prefix *milli* (10^{-3} times).

***M* :** Symbol for *mutual inductance*.

Magnetic circuit : Complete path for magnetic flux, excited by a permanent magnet or an electromagnet.

Magnetic field : The region around a magnet where its influence can be felt.

Magnetic field strength (H) : The measure of the strength of magnetic field. Measured in ampere-turns (At). Also called *magnetic intensity*.

Magnetic flux : The total lines of force in a magnetic field. Symbolised as Φ, and measured in weber (Wb).

Network: An interconnection of electrical components. If it contains only passive components, such as resistances, inductances, and capacitances, it is called *passive network*. If it contains source(s) of energy in addition to the passive components, it is called *active network*.

Network analysis: The process of theoretically determining the currents and voltages at different points of a network.

Node: The point at which a branch of a circuit or a network terminates. Also, a point at which a junction occurs.

Nodal analysis: The method in which we define the voltages at each node (with respect to some reference) and then determine these voltages by writing KCL equations.

Nonlinear circuit: A circuit whose output is not linearly proportional to its input.

Nonplanar circuit: A circuit which cannot be drawn on a plane without crossovers.

Ohm: The SI unit of resistance, impedance, and reactance. Its symbol is Ω (omega).

Ohmic contact: A contact between two materials such that the electric current flows with equal ease in both directions.

Ohmic heating: Heating of a material conductor through which an electric current passes, due to the resistance of the material or conductor.

Ohmic loss: The power dissipation in an electrical circuit due to the current flow through the circuit resistance.

Ohmic resistance: The resistance to dc, offered by a device, circuit, or material.

Ohm metre: The SI unit of *resistivity*.

Ohms per volt: A measure of the sensitivity of an instrument, such as a voltmeter. It refers to the resistance, in ohm, divided by the full-scale voltage value for a given range.

Open circuit: Its abbreviation is OC. A circuit which is broken, or which otherwise does not have a complete and uninterrupted path for the flow of current.

Open-circuit characteristic: The term used for the curve obtained by plotting the emf generated by an electric generator on an open circuit against the field current. Also called *no-load characteristic*.

Output: The power, voltage, or current delivered by any circuit or machine.

Output impedance: The impedance presented by the device to the load and which determines the regulation (voltage drop) of source when current is drawn.

Output regulation: Of the power supply, the variation of voltage with load current.

Overload: Any load delivered at the output of an electrical machine that exceeds its rated output.

***P*:** Symbol of power.

P: Symbol for the prefix *pico* meaning 10^{-12} times.

Pd: Abbreviation for potential difference.

Panel: A sheet of metal, plastic or other material upon which instruments, switches, etc. are mounted.

Paper capacitor: A capacitor in which thin paper acts as dielectric separating aluminium foil electrodes, rolled together.

Parallel circuit: A circuit whose components are connected such that each has the same terminal voltage.

Passive component: A component or device, such as a resistor or a transformer, that cannot operate on an applied electrical signal, as in amplifying, rectifying, or switching, and is not a source of energy.

Peak factor: In a periodically-varying function, such as an ac voltage or current, the ratio of the peak amplitude to the rms value. Also known as *crest factor* or *amplitude factor*.

Peak-to-peak value: Its abbreviation is p-p. For a waveform of alternating quantity, such as that of ac, it is the difference between the maximum positive peak and the maximum negative peak.

Peak value: The maximum instantaneous value (positive or negative) of a voltage, current, signal or other quantity.

Periodic: Occurring, appearing, or characterised by regular and repetitive intervals or cycles.

Reactive load : A load in which the current lags or leads the voltage applied to its terminals.

Reactive power : The power in an ac circuit which cannot perform work. It is given by $V \cdot I \cdot \sin \phi$.

Real power : Same as *active power,* or *true power.*

Relative permeability : The ratio of magnetic flux density produced in a material to the value in free space produced by the same magnetic field strength, i.e., $\mu_r = \mu/\mu_0$.

Relay : An electromagnetic switch.

Reciprocity theorem : A theorem stating that if a voltage source at one point produces a given current at another point in an electric network, then the source and current may be interchanged.

Reluctance : The opposition a material or magnetic circuit presents to the passage of magnetic flux. It is expressed as the ratio of the magnetomotive force to the magnetic flux, and is analogous to electrical resistance. Its reciprocal is *permeance.*

Residual magnetism : The magnetism which remains in a material after the magnetising force is removed.

Resistance : The opposition a material offers to the flow of current. Its symbol is R, and it is measured in ohms (Ω).

Resistance box : A box containing carefully constructed precision resistors which can be introduced into a circuit by switches or keys.

Resistive load : The terminating impedance which is nonreactive, so that the load current is in phase with the source emf.

Resistivity : The resistance between the opposite faces of a cube of the material having side of 1 meter. It is symbolised by ρ, and measured in *ohm meters* (Ωm). It is the reciprocal of conductivity.

Resistor : Electric component designed to introduce known resistance into a circuit.

Resonance : The condition in an ac circuit when $X_L = X_C$.

Resonant frequency : A frequency at which a circuit body, or system exhibits enhanced vibration, oscillation, response, etc.

Retentivity : The ability of a material to hold magnetism.

Rheostat : A variable resistor designed for the control of current in a circuit.

Root mean square value : Also called *effective value.* A value obtained by squaring multiple instantaneous measurements, averaging these over a given time interval, and taking the square root of this average.

Rotor : In an electrical and/or mechanical device or machine, the member which can *rotate.* For instance, the moving part or armature of a motor. A stationary part in such a device is called *stator.*

Rotating field : The resultant field produced by the 3-phase currents flowing in the three-phase windings of an ac machine. Its magnitude remains constant but its direction keeps on rotating at *synchronous speed.*

S : Symbol for siemens, the SI unit for admittance or conductance.

Saturation : The condition in a magnetic material when an increase of the magnetising field does not increase the flux density.

Sawtooth : A triangular waveshape, resembling saw tooth.

Self-excited : A component, circuit, device, piece of equipment, system, process, or mechanism which provides its own excitation signal.

Self-inductance : The property of a coil that induces an emf in itself when the current through it changes.

Series circuit : A circuit whose components carry the same current.

Series motor : An electric motor whose main excitation is derived from a field winding connected in series with the armature.

Shell type transformer : A transformer in which most of the windings are enclosed by the core.

Short circuit : A low resistance connection which is established between two points in a circuit, bypassing any paths with higher resistance.

Superposition theorem : It states that in a linear circuit containing more than one source, the resultant current in any branch is the algebraic sum of the currents that would be produced by taking each source at a time.

Surge : A sudden change in current or voltage.

Susceptance (B) : The imaginary part of the admittance (**Y**), which is given by $\mathbf{Y} = G + jB$.

Switch : The device for opening and closing an electrical circuit.

Switch board : An assembly of switch panel.

Switch box : An enclosure housing one or more switches operated by an external handle.

Switch panel : An insulating panel on which a switch is mounted.

Symmetrical winding : A term applied to an armature winding which fulfills the conditions of electrical symmetry.

Synchronous motor : An ac motor which runs at a speed which is synchronous to the frequency of the power supply.

Synchronous speed : For an ac motor, the rate at which the magnetic field rotates.

T : Symbol for prefix *tera* meaning 10^{12} times. Also, symbol for *tesla*, the SI unit of flux density. 1 T = 1 Wb/m^2.

Temperature coefficient of resistance : An incremental change in the resistance of a component, device, or material as a function of temperature.

Tera : A prefix meaning 10^{12} times.

Terminal : A point in an electrical circuit at which an electrical element may be connected.

Terminal voltage : The voltage at the terminals of a piece of electrical equipment or machine.

Test board : A switch board carrying instruments and sometimes an electric lamp, used for testing the equipments and appliances.

Thermistor : A resistor having a negative temperature coefficient of resistance.

Thevenin's theorem : A theorem stating that the combinations of a two-terminal ac voltage sources and impedances can be represented by a single voltage source and a series impedance.

Three-phase four-wire system : A system of three-phase ac distribution making use of three outgoing conductors (*lines*) and a common return conductor (*neutral*).

Three-phase induction motor : Commonest form of industrial electric motor because of its simplicity, robustness and ability to start without requiring any additional device.

Three-way switch : A rotary-type single-pole switch having three independent contact positions.

Time constant : The time required for an electrical quantity, such as a current or a voltage, to rise from zero or another value to a given proportion (say, 63.2 % = 1 – 1/e) of the final value.

Toroid : Magnetic component (a coil or transformer) made in the shape of an anchor ring.

Torque : The moment exerted by a force acting on a body which tends to cause rotation about an axis.

Total loss : The power loss in an electrical machine, equal to the difference between the input and the output powers.

Transformer : A device in which ac power is transferred from one voltage level to another, without any change in its frequency.

Transformer core : The structure, usually of laminated iron or ferrite, forming the magnetic circuit.

Transformer oil : A mineral oil of high dielectric strength, forming the cooling and insulating medium of electric power transformers.

Transient response : The speed with which a component, circuit, device, piece of equipment, or system changes its output in response to a sudden and intense change in its input.

Transmission line : General name for any conductor used for transmission of electric power.

Transmission voltage : The nominal voltage at which electric power is transmitted from one place to another.

Tree : A circuit or network with multiple branches, but no meshes.

True power : Same as *real power*. It is the average power consumed by a circuit.

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