Quantum Computations of Inflationary, Higher Dimensional, and Dark Energy Cosmology

Amy Joseph, School of Earth and Space Exploration, Arizona State University

Mentor: Michael McGuigan, Computational Science Initiative, Brookhaven National Laboratories

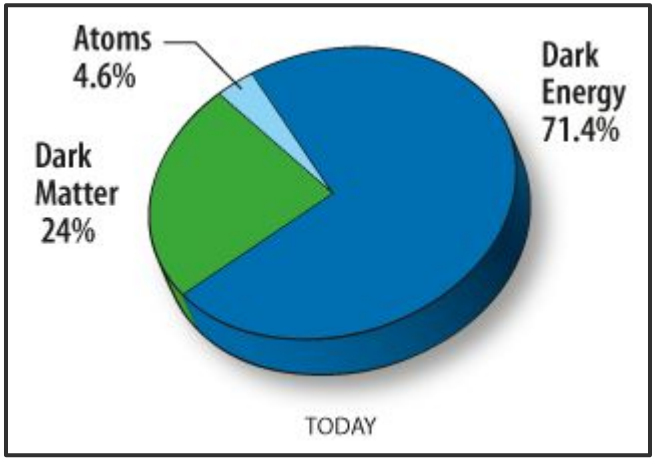
School of Earth and Space Exploration
Arizona State University

U.S. DEPARTMENT OF ENERGY | Office of Science

BROOKHAVEN NATIONAL LABORATORY

Introduction

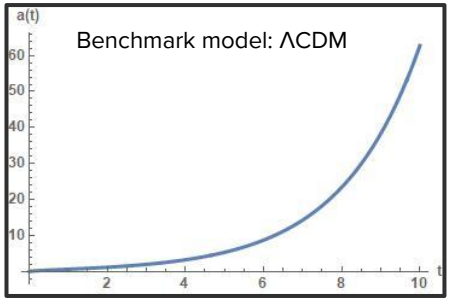
- Dark energy
  - Driving mechanism for the accelerating expansion of the universe
    - Repulsive force, acting against the attractiveness of gravity
- Higher dimensions with a much larger lambda
  - Use classical and quantum computers to estimate the value for 4 dimensional lambda



Source: NASA/WMAP Science Team
"WMAP 9 Year Mission Results." *NASA*, NASA, 2013, map.gsfc.nasa.gov/news/index.html.

- Motivation
  - The Standard Model: 3 of the fundamental forces - strong, weak, and electromagnetic ☺
  - Other integral forces - dark energy, gravity, and dark matter ☹

Dark Energy

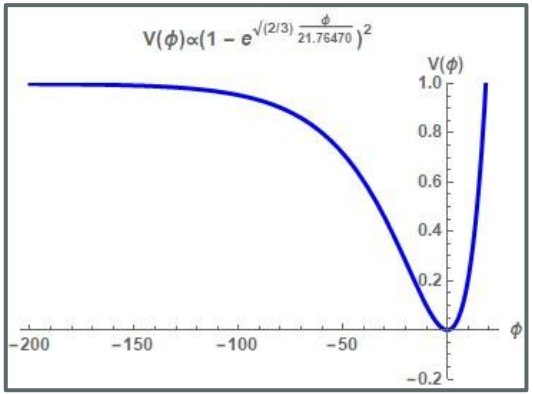


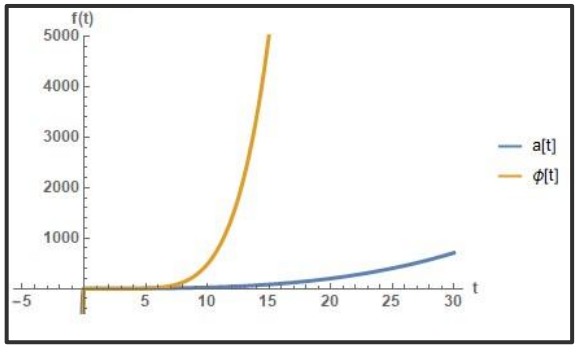
- Friedmann equation - describes the expansion of the universe

$$\frac{\dot{a}^2+kc^2}{a^2} = \frac{8\pi G\rho+\Lambda c^2}{3}$$

- Starobinsky potential - model for cosmological inflation [1],

$$V(\Phi) = \Lambda^4(1 - e^{-\sqrt{\frac{2}{3}}\frac{\Phi}{M_{Pl}}})^2 \longrightarrow V(\Phi) = \Lambda^4(1 - e^{\Phi})^2$$



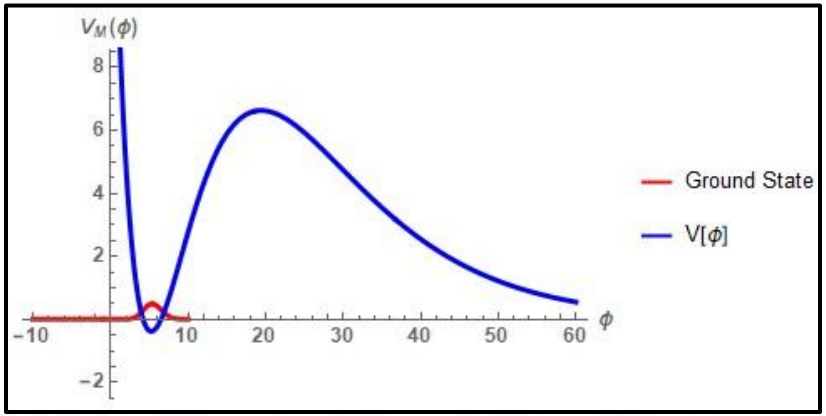


Plot of the Starobinsky potential plugged in to Wiltshire's Friedmann equation solutions, to give the resulting inflation model

Dark Energy Potential

- Finally, below is the equation for the dark energy potential (determined using series expansions), the plot of the potential and its ground state energy, and the exact result of the energy determined by Mathematica.

$$V(\Phi) = 100(e^{-2\frac{\Phi}{7.637626158259733}} - 2e^{\frac{-\Phi}{7.637626158259733}} + 1.18455752798462 37e^{\frac{-2\Phi}{3(7.637626158259733)}})$$

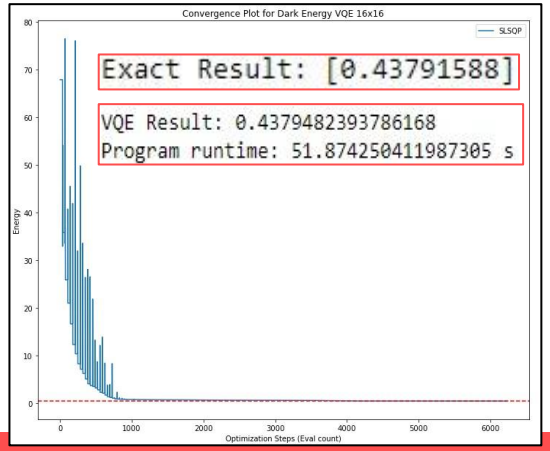


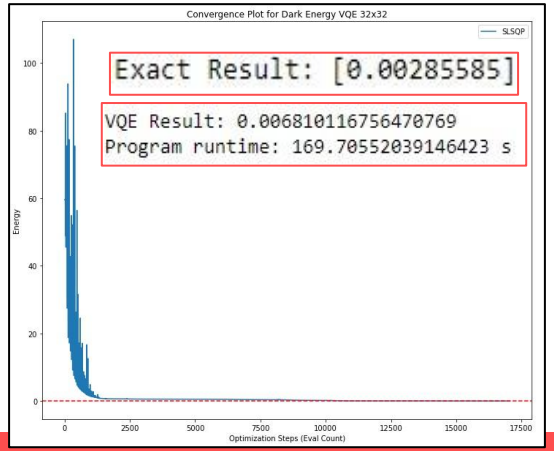
Ground State Energy Exact Result: 1.11637×10^{-6}

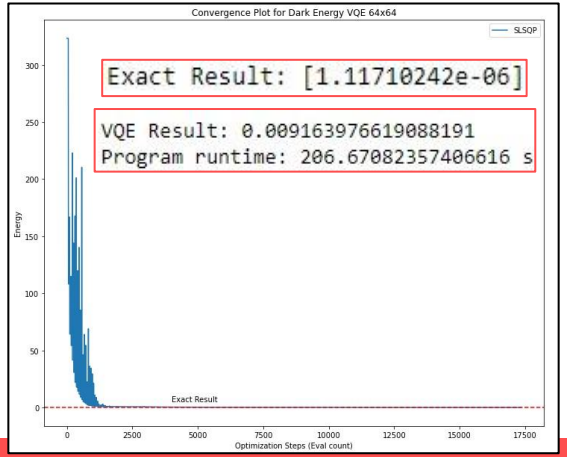
Quantum Computing

Method

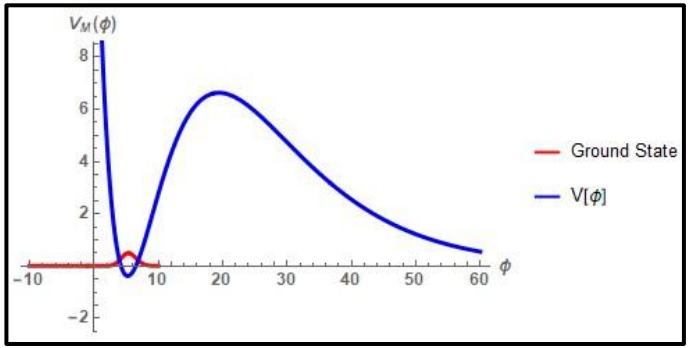
- IBM's **Variational Quantum Eigensolver (VQE)**
  - Variational Method to approximate the ground state energy
    - Repeatedly modifies the ansatz for a wavefunction to get as accurate a result as possible
- Notice: As the number of qubits and matrix size increases, the exact result becomes more accurate, but the VQE results are still lackluster







Metastable Vacua



- DEP is metastable - stable unless disturbed by small upsets
  - Probability that a Universe can tunnel through the barrier
- Series expansion about the min $V(\Phi) - V_{min} + \frac{M^2}{2}\Phi^2 - \frac{\delta\Phi^3}{3} + \ldots$
- Apply tunnel action & tunnel probability [4] $S_E \simeq 205(\frac{M^2}{\delta^2}) \longrightarrow \frac{S_E}{4} = \frac{205}{4}(\frac{M^2}{\delta^2})$
- The Universe will decay in $e^{\frac{S_E}{4}}\ Planck\ times = 4.2823 \times 10^{615}\ years$
- BUT we will be okay! Universe age = $13.82 \times 10^9\ years$

Conclusion

- The Starobinsky potential is a good approximation for inflationary cosmological models.
- The dark energy potential is metastable, giving rise to the notion of a tunneling Universe - but we'll be fine.
- In the future, quantum computers will be an incredible resource and will provide groundbreaking insights into our world but for right now, they still have their setbacks.
    - Not as accurate for the ground state energy of the DEP as a classical computer

Acknowledgements

- Thank you to Dr. McGuigan for providing all the support in the world and being such a great guide through the research process, and to my fellow interns - Molly and Yuan- for becoming such a close and supportive research group.
- This project was supported in part by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists (WDTS) under the Science Undergraduate Laboratory Internships Program (SULI).

References

[1] Collaboration, Planck, et al. “Planck 2018 Results. X. Constraints on Inflation.” *ArXiv.org*, 2 Aug. 2019, arxiv.org/abs/1807.06211.

[2] Wiltshire, D. L. “An Introduction to Quantum Cosmology.” *ArXiv.org*, 3 Sept. 2003, arxiv.org/abs/gr-qc/0101003.

[3] E. W. Kolb, “Vacuum leaks in extra dimensions,” FERMILAB-PUB-87-104-A.

[4] E. W. Kolb, “Cosmology and Extra Dimensions,” FERMILAB-PUB-86-138-A.