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Quantum Computations of Inflationary, Higher Dimensional, and Dark Energy Cosmology

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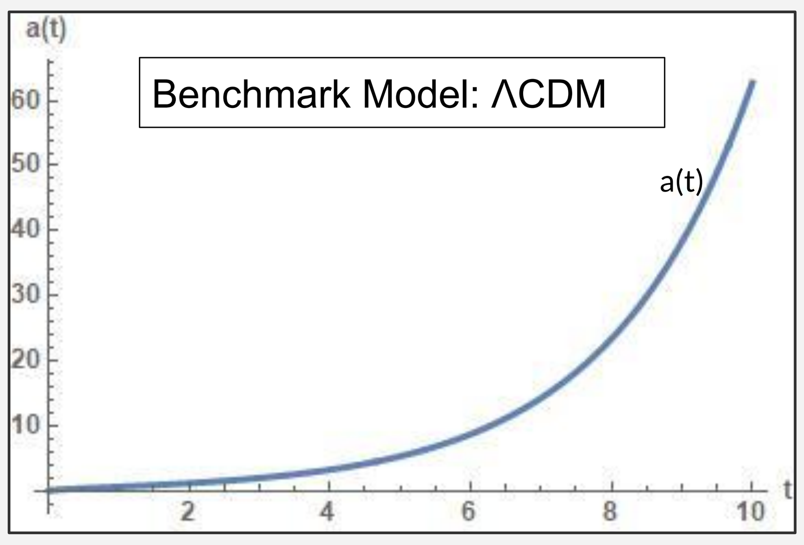
Abstract

The early universe, expansion rate of the universe, and dark energy are explored using both classical methods and quantum computing algorithms. The main program used for classical calculations is Mathematica, while the quantum computing program is IBM's open-source software, QISKIT. We mainly focus on the effects of dark energy on the universe and the overall expansion rate - beginning with the Friedmann equation, followed by the Starobinsky potential, and building up to dark energy potentials along with its respective Hamiltonian. The dark energy models and Hamiltonian were derived and studied using classical computers, then referenced for the quantum computer calculations. We determined the classical computer was more accurate, but the quantum computer was still very close.

Dark Energy

- The **Friedmann equation** describes the expansion of the universe; it chronicles the life of the universe, predicting where it began and where it will end up, simply by inputting a few crucial values.

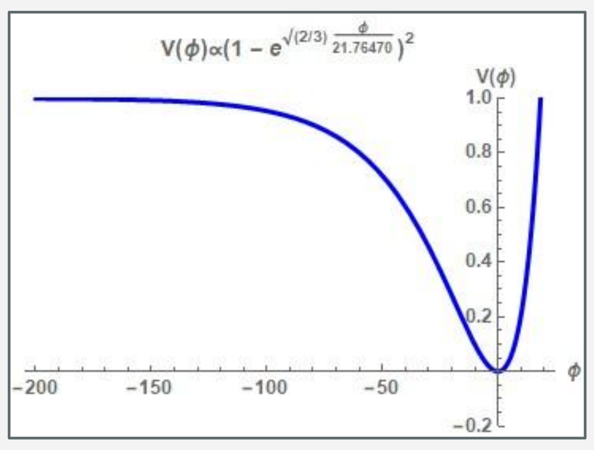
$$\frac{\dot{a}^2+kc^2}{a^2} = \frac{8\pi G\rho+\Lambda c^2}{3}$$



  - Where a = the scale factor, Λ = dark energy term (cosmological constant), k = curvature of the universe, G = gravitational constant, ρ = matter/energy density, and c = the speed of light.
  - This is the starting equation for the project, by solving this (along with its solutions from Wiltshire) for simple cases, we can determine the initial conditions for the universe and then expand into more complex potentials.

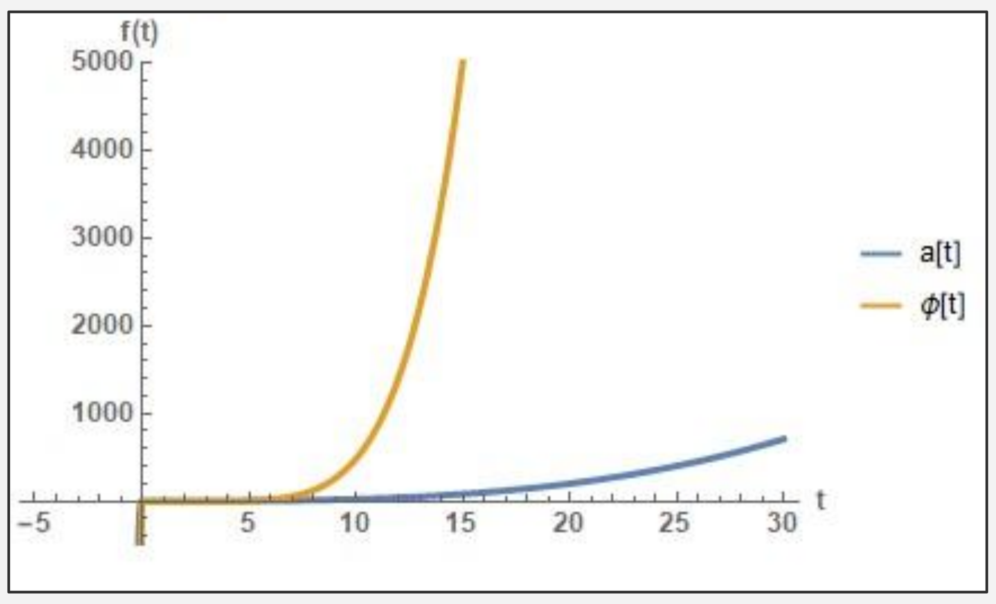
- Building off of the Friedmann equation, brings the **Starobinsky potential** - a model for cosmological inflation which has this equation,

$$V(\Phi) = \Lambda^4\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\Phi}{M_{Pl}}}\right)^2$$



- Which, according to [1], can be approximated with,

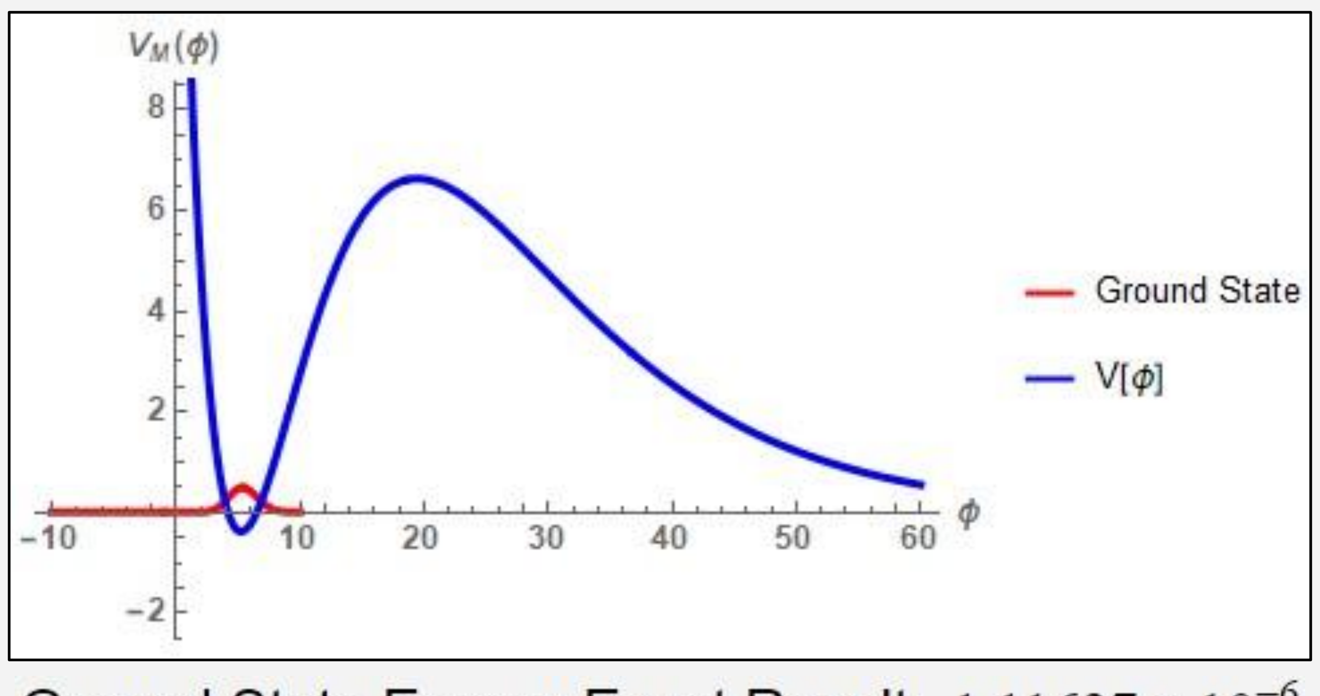
$$V(\Phi) = \Lambda^4(1 - e^{\Phi})^2$$



- Above is the plot of the Starobinsky potential
- To the left is a plot of the Starobinsky potential plugged in to Wiltshire's Friedmann equation solutions, to give the resulting inflation model
- Where a(t) = scale factor and Φ(t) = scalar field for the potential

- Finally, below is the equation for the dark energy potential (determined using series expansions), the plot of the potential and its ground state energy, and the exact result of the energy determined by Mathematica.

$$V(\Phi) = 100\left(e^{-2\frac{\Phi}{7.637626158259733}} - 2e^{\frac{-\Phi}{7.637626158259733}} + 1.1845575279846237e^{\frac{-2\Phi}{3(7.637626158259733)}}\right)$$



Ground State Energy Exact Result: 1.11637×10^{-6}

Introduction

Dark energy is the (constant) energy density of space itself and is the most popular theory that describes the driving mechanism for the accelerating expansion of the universe. It's a repulsive force, acting against the attractiveness of gravity, which contrasts our initial assumptions of a static universe. One staggering fault in the exploration of this constant, is the discrepancy between the theoretical value for dark energy (lambda, Λ) and the observed value; being on the order of 10^{120}, this inconsistency is considered the worst prediction in physics. It's actually measured to have an astonishingly small value (on the order of 10^{-35}). In order to accurately describe the universe we observe today, astrophysicists have incorporated higher dimensions with a much larger lambda, known as higher dimensional lambda (string theory). This method involves a big vacuum energy that is "hidden" and the extra dimensional lambda is there to hold everything together and keep the extra dimensions small. In this paper, we use classical and quantum computers to estimate the value for 4 dimensional lambda (what we observe today). One main motivation for this study of dark energy is to try to better understand how everything ties together. The Standard Model can accurately describe three of the fundamental forces - strong, weak, and electromagnetic - but it fails to describe the other integral forces - dark energy, gravity, and dark matter - which are likely all related.

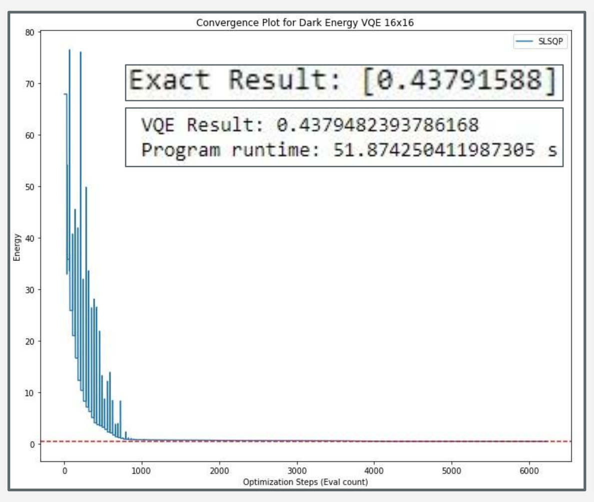
Quantum Computing

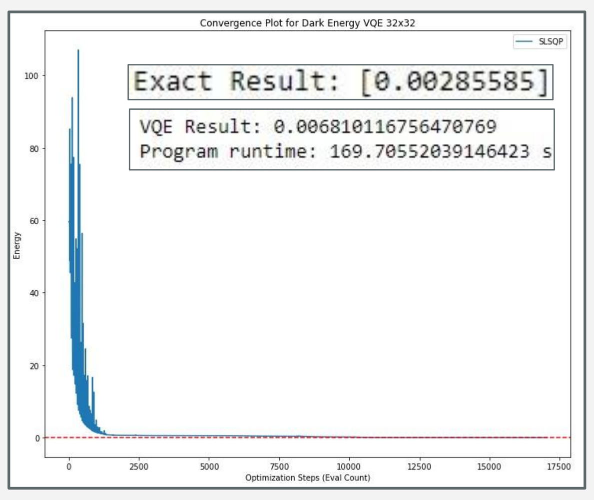
Motivation

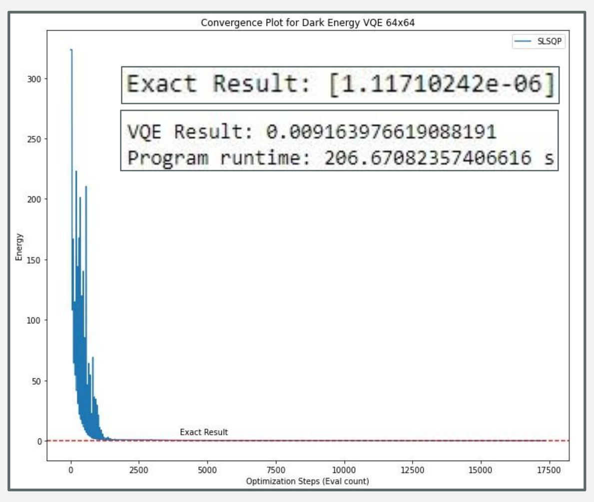
- A powerful alternative to classical computers (for specific reasons)
  - **Classical** computers use bits - 0's and 1's - to store information and calculate programs
  - **Quantum** computers use qubits, which are not limited to a binary system - they allow for the famous superposition of states (it is exponentially faster than a classical computers in several scenarios)
- Useful for crunching large amounts of data in less time than classical computers
- Can help us understand the current limits and benefits of quantum computing.

Method

- The quantum computing mechanism used was IBM's **Variational Quantum Eigensolver (VQE)**
  - Incorporates the Variational Method of quantum mechanics to approximate the ground state energy of a system
  - Repeatedly modifies the ansatz for a wavefunction in an effort to get as accurate a result as possible
- Below are the results for the ground state energy of the Hamiltonian for the dark energy potential (4-D lambda)
- Notice: As the number of qubits and matrix size increases, the exact result becomes more accurate, but the VQE results are still lackluster







Metastable Vacua

- The dark energy potential (DEP) to the left is metastable - meaning it is stable unless disturbed by small upsets.
  - Probability that a Universe sitting at the bottom of a potential well can tunnel through the barrier.
- As a slight deviation but still building off of the previous work, we can calculate the probability.
- First, a series expansion about the minimum of the DEP $V(\Phi) - V_{min} + \frac{M^2}{2}\Phi^2 - \frac{\delta\Phi^3}{3} + \ldots$
- Applying the tunnel action and the tunnel probability [4] $S_E \simeq 205(\frac{M^2}{\delta^2}) \longrightarrow \frac{S_E}{4} = \frac{205}{4}(\frac{M^2}{\delta^2})$
- The Universe will decay $e^{\frac{S_E}{4}}$ $Planck\ times$ = 4.2823×10^{615} $years$
- We will be okay!
  (Universe current age = 13.82×10^9 $years$)

Conclusion

- The Starobinsky potential is a good approximation for inflationary cosmological models.
- The dark energy potential is metastable, giving rise to the notion of a tunneling Universe - but we'll be fine.
- In the future, quantum computers will be an incredible resource and will provide groundbreaking insights into our world but for right now, they still have their setbacks.
  - Not as accurate for the ground state energy of the DEP as a classical computer

References

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[2] Wiltshire, D. L. "An Introduction to Quantum Cosmology." *ArXiv.org*, 3 Sept. 2003, arxiv.org/abs/gr-qc/0101003.

[3] E. W. Kolb, "Vacuum leaks in extra dimensions," FERMILAB-PUB-87-104-A.

[4] E. W. Kolb, "Cosmology and Extra Dimensions," FERMILAB-PUB-86-138-A.

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