Quantum computations of inflationary, higher dimensional, and dark energy cosmology

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Abstract

The early universe, expansion rate of the universe, and dark energy were explored using both classical methods and quantum computing algorithms. The main program used for classical calculations was Mathematica, while the quantum computing program was IBM's open-source software, Qiskit. We primarily focused on the effects of dark energy on the universe and the overall expansion rate - beginning with the Friedmann equation, followed by the Starobinsky potential, and building up to the dark energy potential along with its respective Hamiltonian. The dark energy model and Hamiltonian were derived and studied using classical computers, and then referenced for the quantum computer calculations. We determined the classical computer was more accurate, whereas the quantum computer suffered when using higher matrice size and greater number of qubits (quantum bits). Lastly, we briefly touched on a similar dark energy theory known as quintessence - which differs from its companion in that it is time-varying whereas dark energy is typically a constant. We were able to successfully calculate a promising approximation for the value of dark energy (four dimensional lambda) and showed, by comparison, the quantum computer was less precise.

1 Introduction

Since the 1920s, the expansion of the universe has been known and accepted - although initially assumed to be decreasing in its rate of expansion towards equilibrium. Albert Einstein famously attempted to describe this in his theory of general relativity, where he assumed the universe to be isotropic and homogeneous. Building off of this, he defined gravity as an intrinsic quality of space-time and used a term that accounts for the expansion - known as the cosmological constant, Λ - that would counteract gravity and keep the universe at a static state. It wasn't until about 70 years later - in the late 1990s - that the universe was observed to have an increasing expansion rate. This revelation of an accelerating expansion sparked a fire of intrigue and curiosity in the scientific community, leading to the discovery of dark energy.

Dark energy is the energy density of space itself and is the most popular theory that describes the driving mechanism for the accelerating expansion of the universe. It's a repulsive force, acting against the attractiveness of gravity, which contrasts our initial assumptions of a static universe. It's considered to be a constant value (contrary to quintessence, which will be discussed later). One staggering fault in the exploration of dark energy, is the discrepancy between the theoretical value and the observed value; being around 120 orders of magnitude off, this inconsistency is considered the worst prediction in physics. It's actually measured to have an astonishingly small value (on the order of 10^{-120}). Thus, in order to accurately describe the universe we observe today, astrophysicists have incorporated higher dimensions with a much larger constant value, known as higher dimensional lambda. This method involves a big vacuum energy that is "hidden" and the extra dimensional lambda is there to hold everything together and keep the extra dimensions small. In other words, the small difference between the large higher dimensional lambda and the energy used to keep the extra dimensions small is what we perceive as four dimensional lambda. For the sake of having two different Λ's, we will be defining the four dimensional lambda as Λ_4 and the higher dimensional lambda as Λ_8. In this paper, we use classical and quantum computers to estimate the value of Λ_4.

Aside from exploring the mysteries surrounding dark energy and it's four dimensional value, another main motivation is to try to better understand how this relates to other models, i.e. the Standard Model of Physics. The Standard Model can accurately describe three of the fundamental forces - strong nuclear, weak nuclear, and electromagnetic - but it fails to describe the other integral forces - gravity, dark energy, and dark matter - which are likely all related. The latter forces are also the main drivers of the Universe, making up approximately 95% of its composition.[1] Our lack of understanding of what makes up the majority of our Universe is another driving inspiration for research.

2 Models of Dark Energy

We began by exploring the Friedmann equation and Starobinsky potential in order to create some groundwork for the ultimate goal: the dark energy potential. These provided insight into the expansion of the Universe, as well as assisted us in discovering initial conditions and ideas for how certain potentials behave.

The Friedmann Equation

Considered to be one of the most important equations, the Friedmann equation describes the expansion of the Universe. It chronicles the life of the Universe, predicting where it began and where it will end up, simply by inputting a few crucial values. Here, the Friedmann equation is written as,

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

where $a(t)$ is the scale factor (radius) of the Universe at time t, k is the curvature of the Universe (defined as -1, 0, or 1 for closed, flat, and open Universe, respectively), G is the gravitational constant, ρ is the energy/matter density ($\frac{C}{a^4}$ for radiation, $\frac{m}{a^3}$ for matter, respectively), and Λ is the cosmological constant. With this equation we were able to create an arsenal of cosmological models for a number of different scenario universes. The most interesting being the Benchmark for an expanding cold dark matter model (Figure 1).

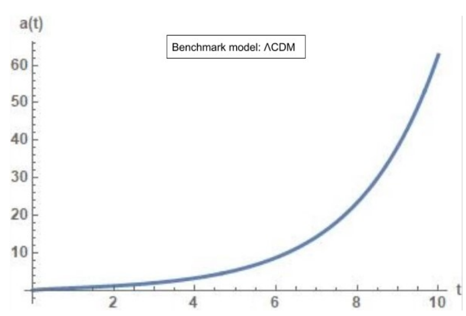

Figure 1: Plot of the Benchmark: ΛCDM model, a fitting theory for the Universe we observe. This particular model has parameters: $k = C = 0$, $\Lambda = 0.73$, $m = 0.27$, baryonic matter $= 0.04$, and dark matter $= 0.23$.

In addition to plotting various expansion models, we were able to input scalar potentials into the Friedmann equation solutions provided by Wiltshire[2]. These set of solutions include a scalar potential parameter and create a set of two equations with two unknowns which can be solved using Mathematica to explore scalar potential cosmological expansion models.

The Starobinsky Potential

The Starobinsky potential is a model for cosmological inflation. It is a type of scalar potential which closely resembles the Morse potential from chemistry - the noticeable difference being that the Starobinsky is reflected over the vertical axis. The equation can be found as follows[3],

$$V(\phi) = \Lambda(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Planck}}})^2$$

where ϕ is the scalar field for the potential and M_{Planck} is the Planck mass. A simple plot of the Starobinsky potential is found in Figure 2.

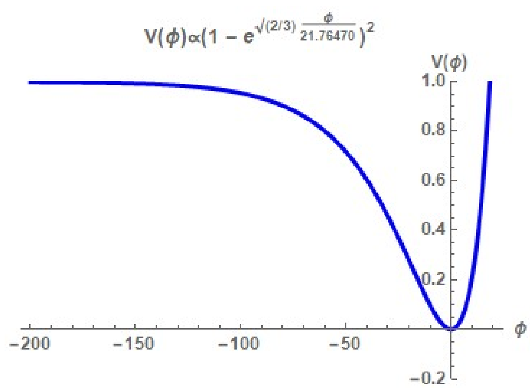

Figure 2: Plot of the Starobinsky potential as a function of the scalar field for the potential. Note the resemblance to the Morse potential simply reflected across the vertical axis.

This potential strongly mimics the shape of the dark energy potential, making it an incredibly useful building block. From here, we were able to explore the dark energy potential using classical and quantum computing.

The Dark Energy Potential

Recall, the measly size of the cosmological constant in present day. Although it is puzzling and quite difficult to create a model universe that has such a small value for lambda, there are some strong theories. One of these theories - which we will be using here - involves higher dimensional lambda, Λ_8, which is significantly larger than the four dimensional lambda, Λ_4, we perceive. In order to model the dark energy potential, it was necessary to incorporate higher dimensions and thus use a dilaton potential[4] of the form,

$$V(\phi) = e^{\frac{4}{d-2}\phi}(a - be^{\phi} + ce^{2\phi})$$

where a, b, c, and d are all constant parameters consisting of defining features of the Universe. A series expansion provides the values for these terms and creates the following dark energy potential: $a = 1 =$ curvature term, $b = 2$, and $c = 1.1845575279846237 = \Lambda_8$,

$$V(\phi) = \frac{7}{48(0.005)}\left(e^{\frac{-2\phi}{7.637626158259733}} - 2e^{\frac{-\phi}{7.637626158259733}} + 1.1845575279846237e^{\frac{-2\phi}{7.637626158259733}}\right)$$

The dark energy potential has the following plot, including its determined ground state energy value, Λ_4,

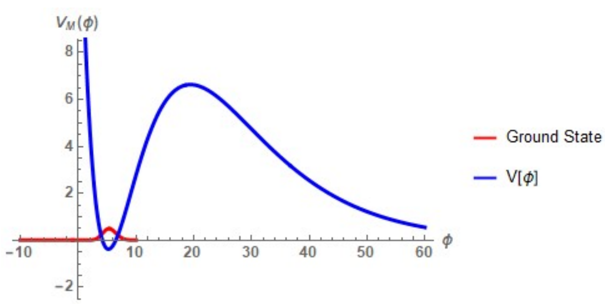

Figure 3: Plot of V[ϕ] as a function of ϕ, depicting the dark energy potential (blue) and the ground state energy for the potential (red). This red line is the value of the four dimensional cosmological constant, Λ_4. The bottom of the potential well is the classical value which gets raised by quantum effects. The value for the four dimensional cosmological constant was found to be $\Lambda_4 = 1.11637 \times 10^{-6} M_{Planck}^{-4}$. Given the actual value is approximately $\Lambda_4 \approx 10^{-120} M_{Planck}^{-4}$, this result is quite promising. Although still far too large, it's the closest we can achieve given the constraints of our classical computers and it is definitely heading in the right direction.

Quintessence

Alternative to dark energy, is quintessence - another theory for the driving mechanism of the expansion of the universe. While dark energy is typically considered to be a constant energy density, quintessence is a time-varying energy field. If we look at the solutions to the Friedmann equation derived by Wiltshire[2], we can plot the scalar field of the potential as a function of time to see how it varies,

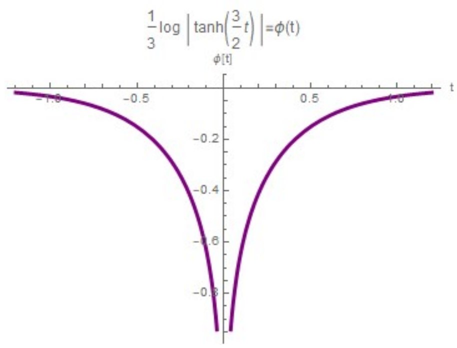

Figure 4: Plot of $\phi(t)$ vs time. Notice that as time approaches 0 from both the left and right sides, ϕ approaches negative infinity. This is very similar to the effects we observe with quintessence.

A satisfactory potential model for this is the exponential oscillator potential shown here,

$$V(\phi) = 100e^{0.01\sqrt{\phi^2}} - 100$$

where ϕ is the scalar field of the potential. This is a reliable potential that allows us to illustrate the concept of a potential which over time eventually drops to small values - such as what we perceive as dark energy, Λ_4. As a preview into the later work in this paper, we wanted to compare the results for the ground state energy from Mathematica (classical computer) to that of Qiskit (quantum computer). We created a Hamiltonian for the exponential potential as follows,

$$H = \frac{p^2}{2} + 100e^{0.01\sqrt{\phi^2}} - 100$$

where p is the canonical momentum of the system. In Figure 5, the convergence plot, exact results (for both Mathematica and Qiskit), and VQE results can be found. More details behind the code and method for quantum computing can be found in Section 3.

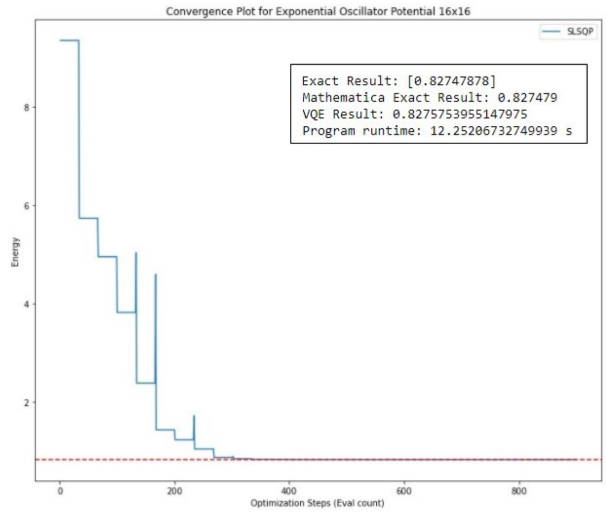

Figure 5: Convergence Plot for the Hamiltonian of the exponential oscillator potential using a 16x16 matrix. Note the accuracy of the quantum computer when using a small matrix size.

Metastable Vacua

Recall the dark energy potential from Figure 3 above, and how it decreases as ϕ increases (dissimilar to the Starobinsky potential which plateaued). This is a key feature that leads to the conclusion that the dark energy potential is metastable. In essence, this means that the potential is stable unless perturbed by small disturbances. Therefore, there is a probability that a Universe sitting at the bottom of the potential well can tunnel out through the barrier. When this happens the Universe will decompactify and become higher dimensional. Thus, one must check that the time required for this to happen is longer than the age of the Universe for the dark energy model to be compatible to what we see. We can use the methods of Kolbe[7,8] to do the necessary calculation. First, starting with a series expansion of the dark energy potential about it's minimum we find it is of the form,

$$V(\phi) = V_{\min} + \frac{M^2}{2}\phi^2 - \frac{\delta}{3}\phi^3 + \ldots$$

where $V_{min} = -0.378498$, $M^2 = 0.584376$ and $\delta = 0.139707$, and they are all defining qualities of the expansion (M is the frequency at the bottom of the well, whereas δ is taking control at the top of the well). The analysis of Kolbe[7,8] shows that for potentials of this form the tunnel action is given by,

$$S_E = 205\frac{M^2}{\delta^2}$$

And the tunneling probability for the potential is thus,

$$\frac{S_E}{4} = 51.25\frac{M^2}{\delta^2}$$

For the dark energy potential this is,

$$\frac{S_E}{4} = 51.25\frac{M^2}{\delta^2} = 1534.44$$

and the Universe will decay after $e^{S_E/4} = 2.505098 \times 10^{666}$ Planck times ($5.391 \times (10^{-44} sec)$. Converting this to years, we get approximately 4.2823×10^{615} years. This is far greater than the age of the Universe which is 13.82×10^9 billion years old. Thus, the dark energy potential is consistent with the stability of our Universe.

3 Quantum Computing for Dark Energy Models

In this section, we will be exploring the dark energy potential using IBM's open-source quantum computing software, Qiskit. First, we will use classical methods to derive the Hamiltonian for the dark energy potential, and then apply quantum computing algorithms to solve for the ground state energy (the value for dark energy, Λ_4). The main purpose of utilizing the quantum computational methods for our research is ultimately to help us understand the current limits and benefits of quantum computing.

The main procedure used for this exploration was the Variational Quantum Eigensolver (VQE). The VQE applies quantum mechanics' variational method to approximate the ground state energy of a system. This method repeatedly modifies an ansatz for a given wavefunction in an effort to get as accurate a result as possible. It attempts to create an upper bound for the ground state energy, giving an approximation for its actual value.

The Hamiltonian for Dark Energy

To begin, we derived the Hamiltonian for the dark energy potential and output the ground state energies using a classical computer (Mathematica software) and then imported them into the quantum computer code. To do so, we started by defining the number of qubits, which establishes the size of the matrix for the Hamiltonian as 2^{qubits}. For example, when we have 4 qubits, the matrix size will be $2^4 = 16$, i.e. a 16x16 matrix. We then created the matrix in the oscillator basis using Mathematica's SparseArray function. Lastly, we applied the Fourier transpose to the matrix and were able to create the Hamiltonian and output the lowest eigenvalues. Below is the model of the Hamiltonian used,

$$H = \frac{p^2}{2} + V(\phi)$$

where p is the momentum of the system. Thus, in simplest forms, the Hamiltonian for the dark energy potential was determined to be,

$$H = \frac{p^2}{2} + \left[\frac{7}{48(0.005)}\left(e^{\frac{-2\phi}{7.637626158259733}} - 2e^{\frac{-\phi}{7.637626158259733}} + 1.1845575279846237e^{\frac{-2\phi}{7.637626158259733}}\right)\right]$$

We repeated the above process for 4, 5, and 6 qubits, thereby creating three separate sized matrices (16x16, 32x32, and 64x64, respectively) to test using the quantum computer. In the following subsection, we compare the results of the classical computer Λ_4 value to the quantum computer's value.

The Ground State

In order to determine the ground state energy, Λ_4, of the above equation using the classical computer, we simply applied Mathematica's Eigenvalues function and corrected it so that the values would appear from smallest to largest (see Table 1). For the quantum computer, the procedure is more complex. It required importing the Hamiltonian into Python (the coding engine we used to run the quantum computing algorithms) and then creating the variational form using the Qiskit function, EfficientSU2 (Special Unitary Group of degree 2). This is essentially the VQE ability in Qiskit. We used the "ry" and "rz" variational forms to allow for more accuracy in lieu of speedy runtime. The qubits were in full entanglement, meaning each qubit was entangled with all the other qubits with a CNOT gate. The depth of the circuit - or "reps" - was set to 3, so that each variational form was repeated 3 times. As a visual of the variational form, we displayed the effect below,

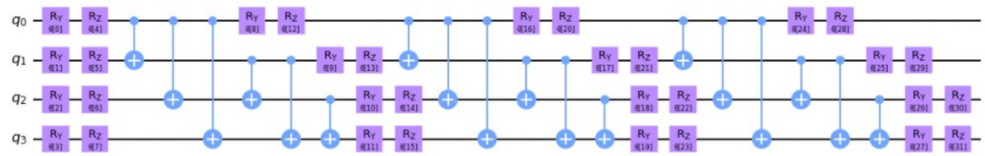

Figure 6: Image depicting the "ry" and "rz" variational form of an EfficientSU2 function applied to the 16x16 Hamiltonian for the dark energy potential. The qubits were in full entanglement using a CNOT gate and the repetitions were set to 3.

Next, we wrote a line of code to output the exact result for the ground state energy using NumPyEigensolver (see Figure 8). Then, we set up for the VQE code using the backend, "statevector_simulator", the SLSQP optimizer (with maximum iterations at 600). To run the VQE code, we created an object which is an instance of the VQE class. We also created convergence plots for all 3 matrices,

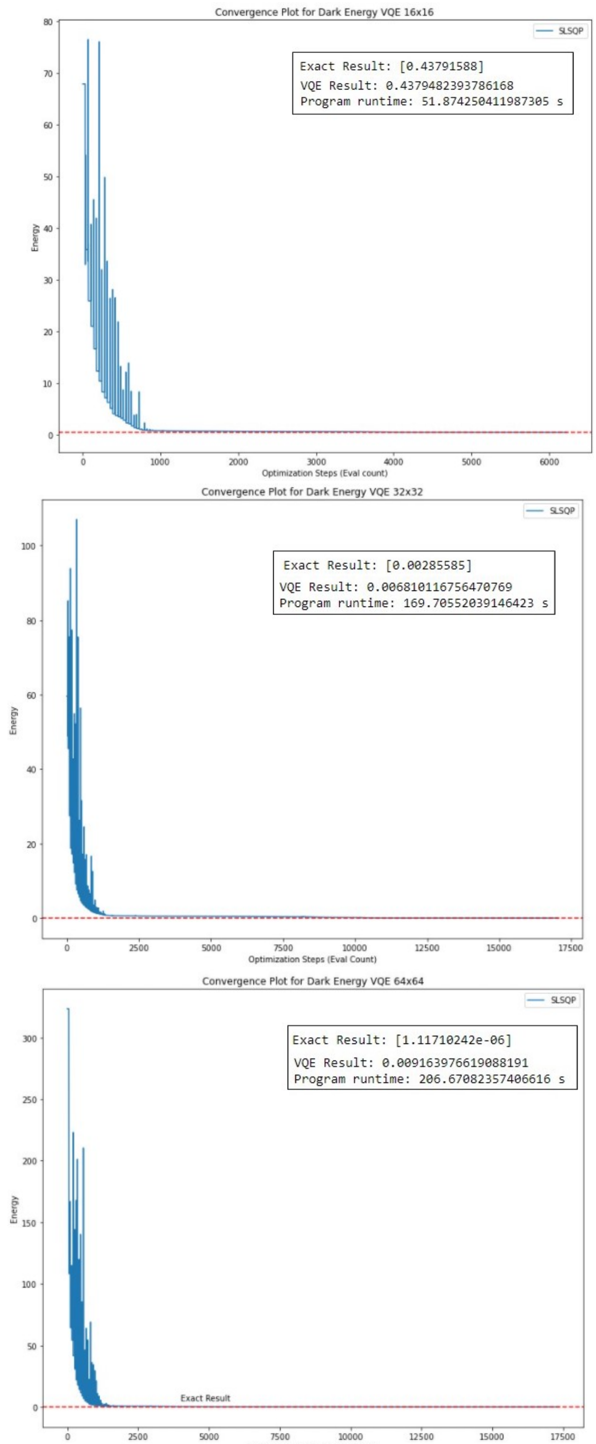

Figure 7: Convergence plot for the 16x16, 32x32, and 64x64 matrices of VQE results on the quantum computer for Λ_4 in the dark energy potential's Hamiltonian. In the upper right hand corner of each plot is a legend containing the quantum computer's exact result for Λ_4, the VQE result, and the program runtime.

The table comparing the results of the quantum computer to the classical computer is shown in Figure 8. As suggested in the table, quantum computing still doesn't quite stand up to classical computing. Although it will, without a doubt, be an incredible method in the future, quantum computing hasn't quite surpassed classical computers just yet.

Matrix Size	CC, Λ_4 Exact Result	QC, Λ_4 Exact Result	QC, Λ_4 VQE Result	Abs Percent Error, CC Exact : QC Exact	Abs Percent Error, CC Exact : QC VQE
16x16	0.437916	0.43791588	0.4379482394	0.0000274%	0.007362%
32x32	0.00285585	0.00285585	0.00681011676	0%	138.462%
64x64	1.11711×10^{-6}	$1.11710242 \times 10^{-6}$	0.00916397662	0.00067854%	820228.94%

Figure 8: Comparison of the classical computer (CC) results for the ground state energy, (Λ_4), of the Hamiltonian for dark energy potential and the quantum computer (QC) results (both exact and VQE). All units for Λ_4 values are in M_{Planck}^{-4}. Recall the actual known value is $\Lambda_4 \approx 10^{-120} M_{Planck}^{-4}$. Note, the quantum computer exact results are impeccably close to those of the classical computer. The issue arises with the VQE results, particularly when increasing the number of qubits and in turn, the matrix size. As the number of qubits increases, the accuracy of the quantum computer declines significantly.

4 Conclusion

We were able to conduct copious simulations and experiments on cosmological models throughout the research period. From the ample Friedmann equation models, to the various scalar potentials, and ending with comparisons to quantum computational methods, we derived a potluck of knowledge. It is clear the Starobinsky potential is an adequate approximation for inflationary cosmological models; bearing close resemblance to the dark energy potential. In addition, the dark energy potential is metastable, giving rise to the notion of a tunneling Universe - although it is nothing we should be worried about in our lifetime - or any lifetime for that matter. Finally, in the future, quantum computers will be an incredible resource and will provide groundbreaking insights into our Universe, but for now, they still have their drawbacks. The accuracy for larger matrices and greater qubits is not quite where we need it to be. Quantum computers have yet to replace our need for classical computers. Overall, cosmology has come a long way from its early beginnings; with our knowledge growing exponentially every day. In less than a hundred years, our theories have moved from a static and ”safe” Universe, to one that's expanding so unfathomably fast that it will eventually devour itself. At least we can take a small bit of comfort in knowing that our Universe is still infantile and has countless years ahead of it.

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