Journey to the center of a Schwarzschild-de Sitter black hole using quantum computing

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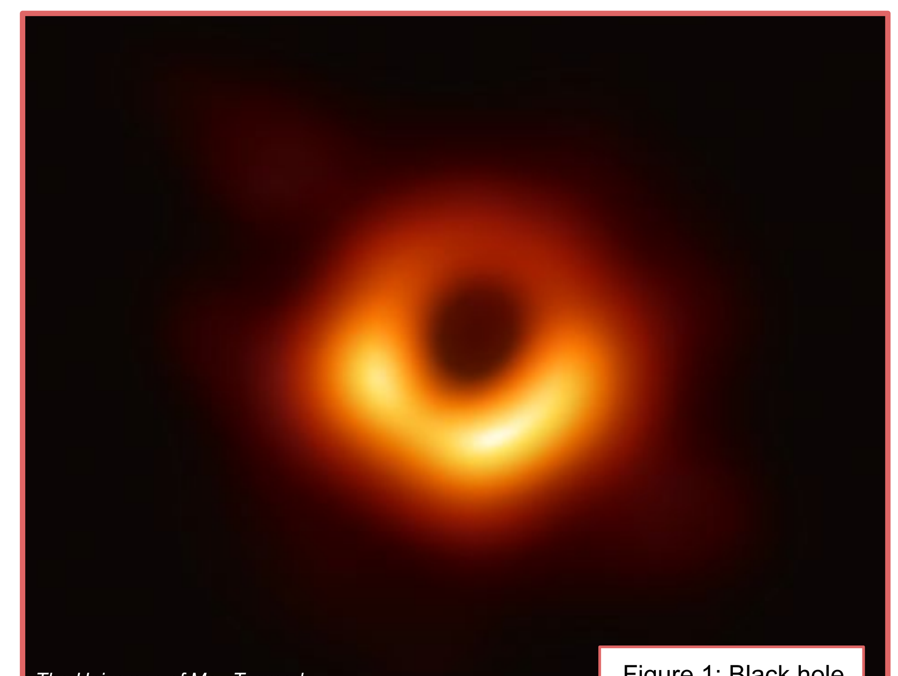


The Universes of Max Tegmark, space.mit.edu/home/tegmark/wmap.html.

Figure 1: Black hole.

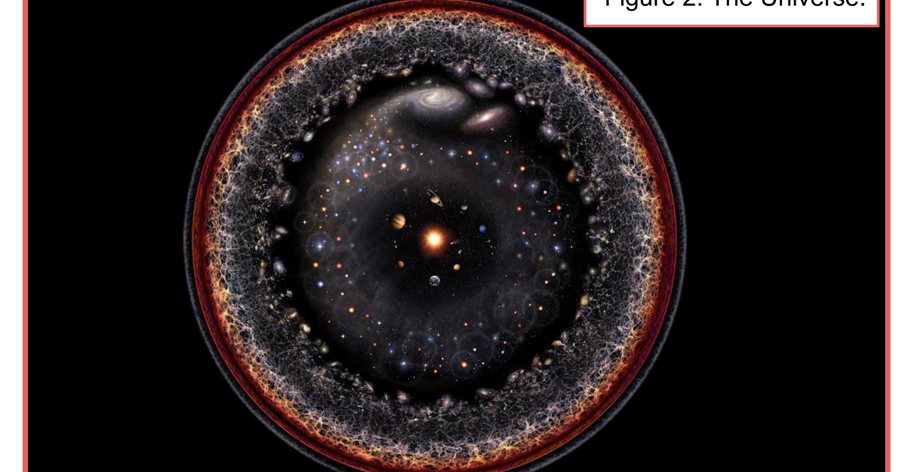


Figure 2: The Universe.

Carlos Budassi, Pablo. "Circular Map of the Universe All Versions." *Pablo Carlos Budassi,* 2018, www.pablocarlosbudassi.com/2021/02/the-infographic-and-artistic-work-named.html.

Abstract

The intricacies and features of de Sitter space are explored using both classical and quantum computing methods. de Sitter space is the maximally symmetric solution to Einstein's field equations in a vacuum with a positive cosmological constant, Λ. At the same time, a large amount of information is becoming available about the properties of black holes both from stellar collapse and from supermassive black holes at the centers of galaxies. In this case, the solution to Einstein's equation is the Schwarzschild solution. The way to reconcile these two phenomena is through the de Sitter-Schwarzschild solution. However, there are difficulties in understanding the connection between the entropy of de Sitter space and that of black holes. We will use quantum computing to analyze several aspects of the de Sitter-Schwarzschild solution in order to obtain a fresh perspective on this problem. We compare the results from the classical computer to the quantum computer.

Introduction

de Sitter space is defined to have positively curved space-time with a positive cosmological constant and energy density. It is considered to be empty of matter - both dark and ordinary. Despite our Universe being filled with matter currently, it's expanding, foreshadowing its future as a de Sitter universe (i.e. it is asymptotically de Sitter) [1]. Since de Sitter space possesses a cosmological constant which is static everywhere in the Universe, the curvature is thus homogeneous throughout.

We can draw comparisons between the de Sitter black hole and the Universe because of their convenient similarities. The Universe's cosmological horizon is similar to the event horizon of a black hole, in that they both possess temperature and entropy. Therefore, if the Universe is truly de Sitter, one can apply the quantum properties of black holes to our Universe.

Methods and Results

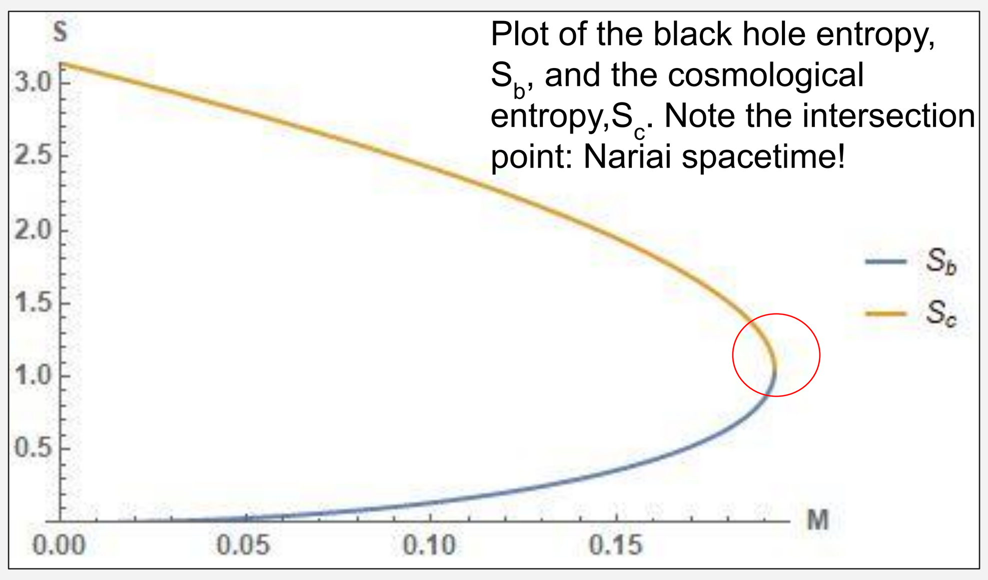
Methods

- Started with classical computations using Mathematica
- Used quantum computing, QISkit, to study VQE, EOH, and TFD
  - Main function was the Variational Quantum Eigensolver

Thermodynamics

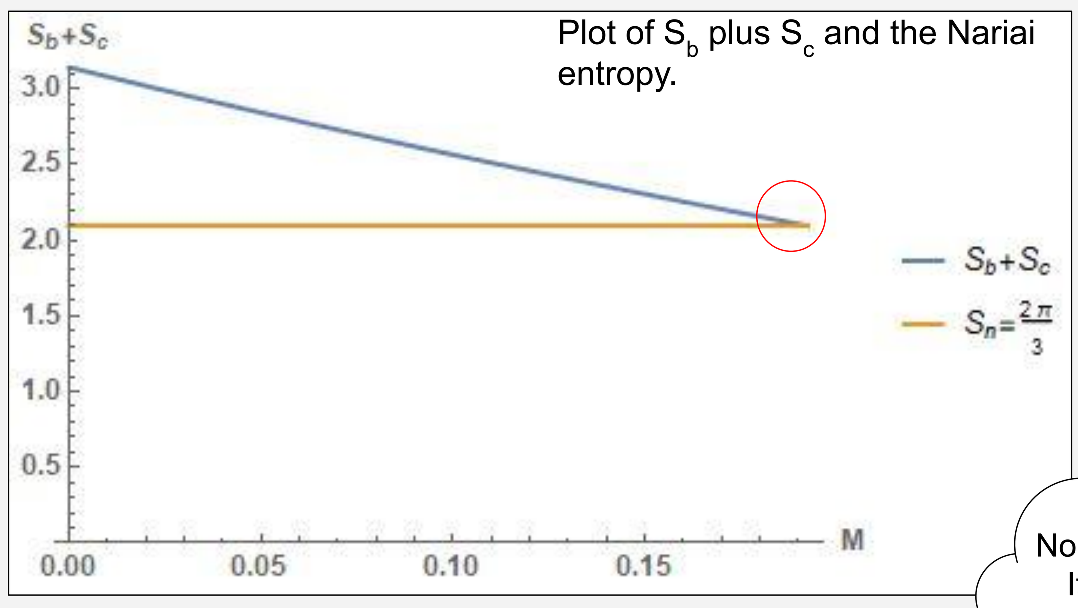
Entropy equation for SdS

$$S_{SdS} = \frac{\pi}{k_{bh}^2} + \frac{\pi}{k_c^2} + \frac{2\pi}{k_c k_{bh}}$$



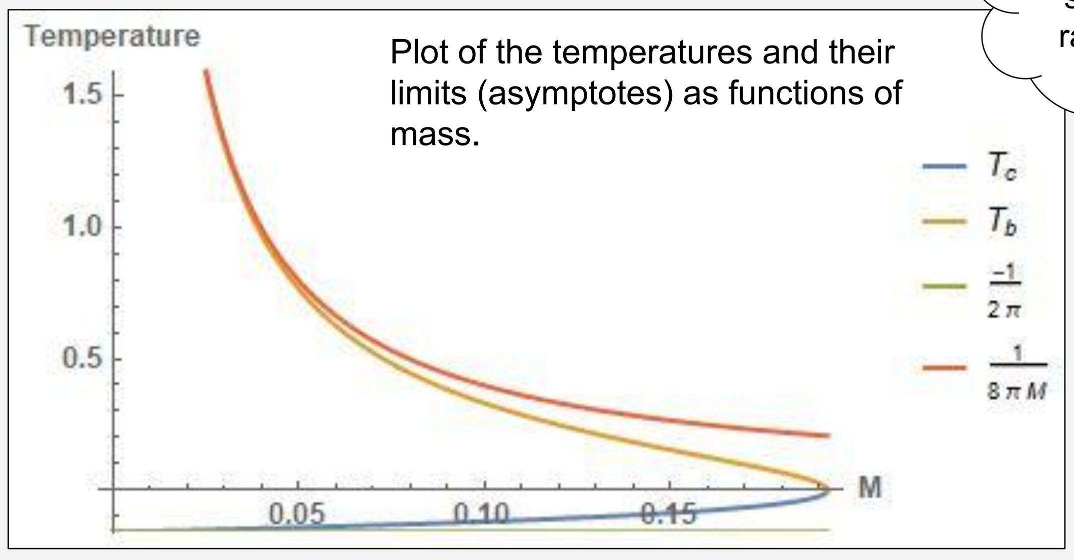
Plot of the black hole entropy, S_b, and the cosmological entropy, S_c. Note the intersection point: Nariai spacetime!

An important solution to study is the **Nariai black hole**. This is the largest possible black hole in de Sitter space. Its event horizon radius approaches that of the cosmological horizon, $r_b \rightarrow r_c$, so that the two radii coincide.



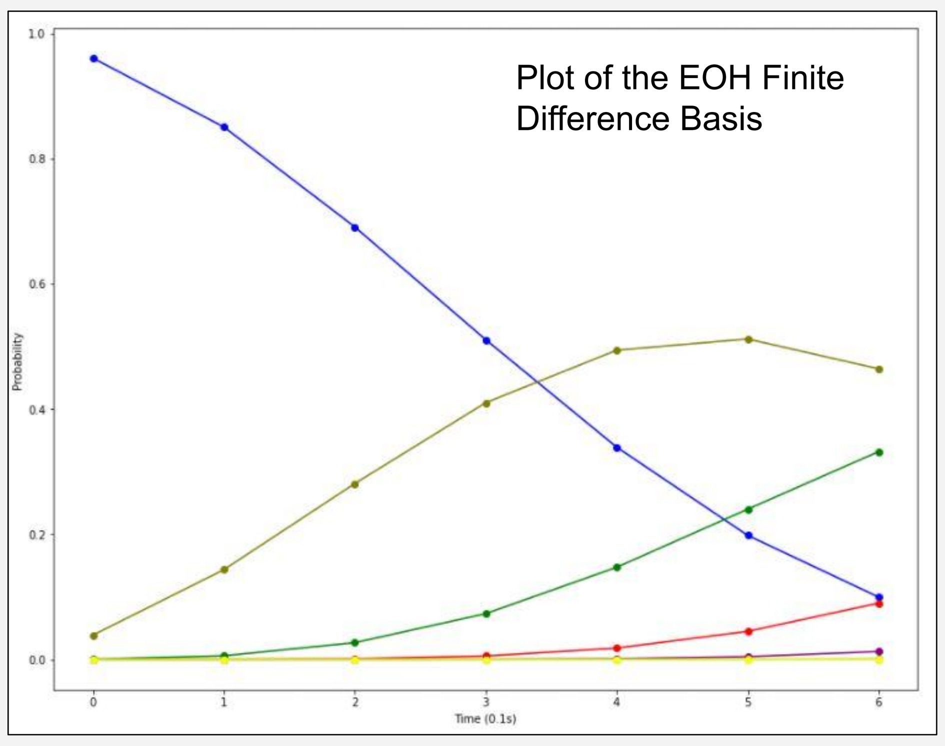
Plot of S_b plus S_c and the Nariai entropy.

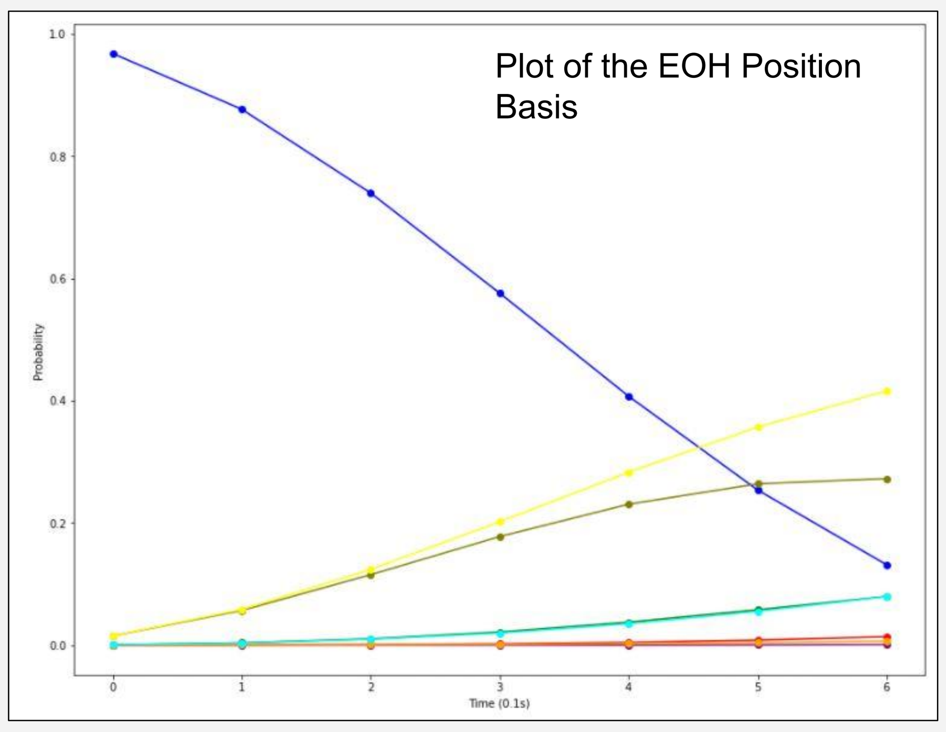
No spacial singularity! It appears to be a singularity in time rather than space.



Plot of the temperatures and their limits (asymptotes) as functions of mass.

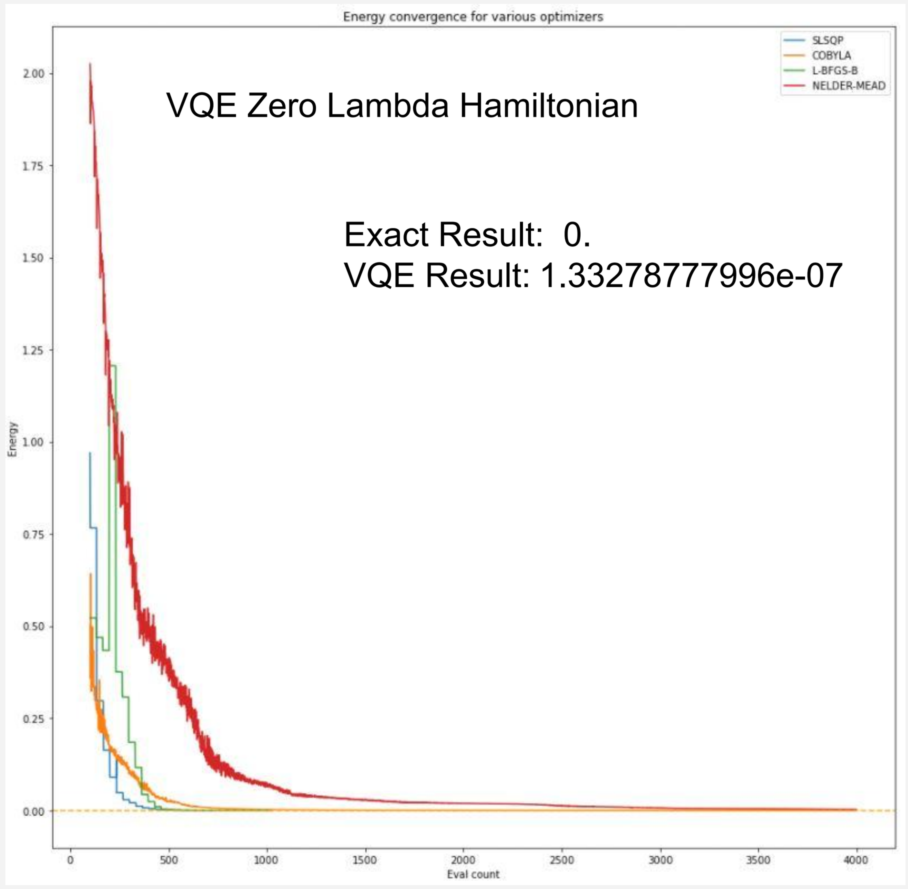
Evolution of the Hamiltonian



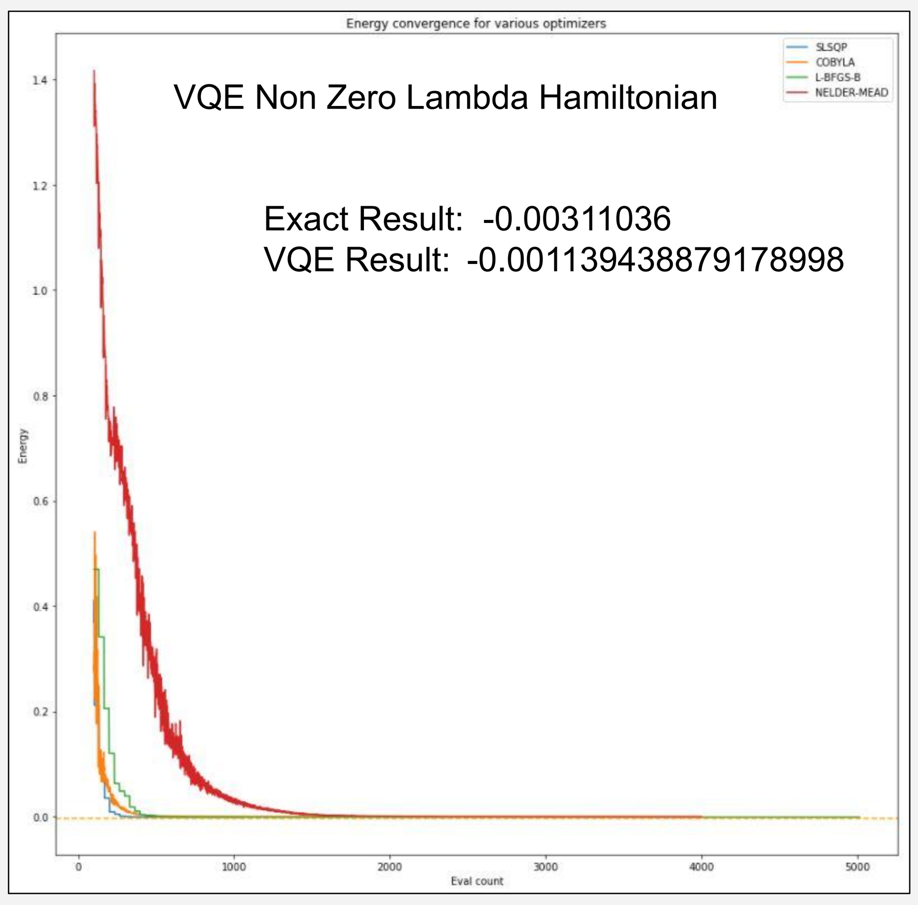


The EOH shows how the system's energy changes over time. Each line is a different state of the Hamiltonian and this shows the probability of the system being in that state. The total adds up to 1.

Variational Quantum Eigensolver



- The VQE repeatedly modifies the ansatz for a wavefunction in an effort to get as accurate a result as possible.
- Just like variational method in quantum mechanics!

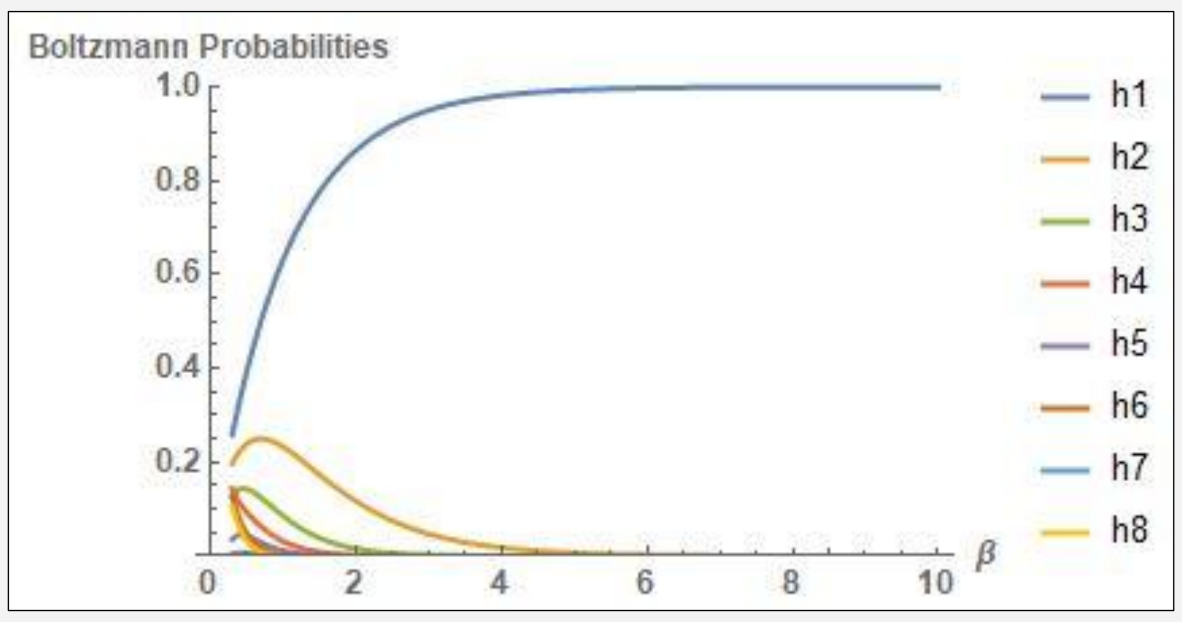


Thermal Field Double

The TFD is similar to the EOH, except we evolve in temperature rather than time. It is a way to describe a system by introducing a "double" particle (in our case, boson).

$$G = -i\theta[\beta]\left(a_{Bd}\, a_B - a_B^{+} a_{Bd}^{+}\right)$$

To evolve in temperature: $e^{-i\theta G}$



Conclusion

- The thermodynamics of SdS black holes and cosmology show the Nariai solution.
  - The Nariai black hole (spacetime) occurs when the dS black hole horizon radius approaches that of the cosmological horizon (around M = 1/Sqrt[27]) - it is the largest possible bh.
- The connections that can be drawn between a dS black hole and the Universe play important roles in the study of both cosmology and black holes

Acknowledgements

Thank you to Dr. McGuigan for allowing me to work with him again during this summer and continue our research progress. I appreciate all of the support, guidance, and knowledge shared. This project was supported in part by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists (WDTS) under the Science Undergraduate Laboratory Internships Program (SULI).

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