

Journey to the Center of a Black Hole Using Quantum Computing

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Introduction

de Sitter Universe

- Positively curved spacetime
- Positive cosmological constant, Λ
- Empty of matter

Our Universe

- In $D = 3 + 1$ dimensions, it's positively curved
- Observed positive Λ (dark energy)
- Has matter but exponentially expanding
∴ Asymptotically de Sitter

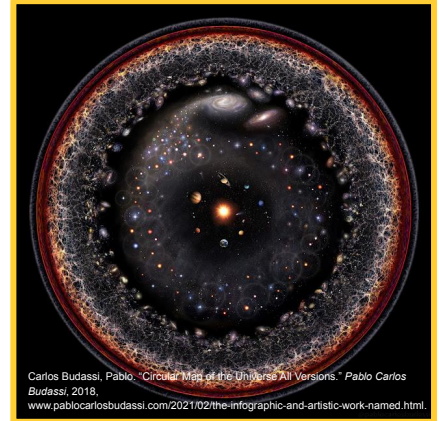
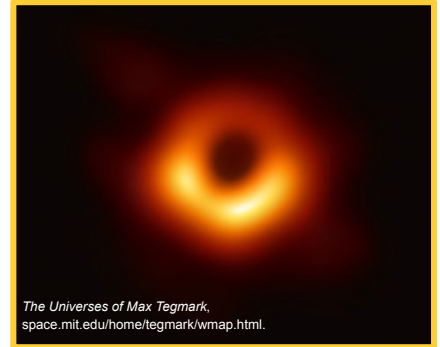
Motivation

- Existence of **dark energy** driving the observed exponential expansion of the Universe
 - ◆ Solution to Einstein's equation = de Sitter space
- New info about properties of black holes
 - ◆ Solution to Einstein's equation = Schwarzschild solution
- Cosmological horizon is similar to the event horizon of a black hole
 - ◆ **Temperature** and **entropy**

If the Universe is truly de Sitter, one can apply the quantum properties of black holes to our Universe.

Study the
abilities of
quantum
computing!

$$G_{\mu\nu} + g_{\mu\nu} \Lambda = 8 \pi G T_{\mu\nu}$$



Method

- Goal: Explore the features of SdS black holes!
- Start with classical computations using Mathematica
- Use quantum computing (QISkit) to study VQE and EOH
 - ◆ Main function was the Variational Quantum Eigensolver

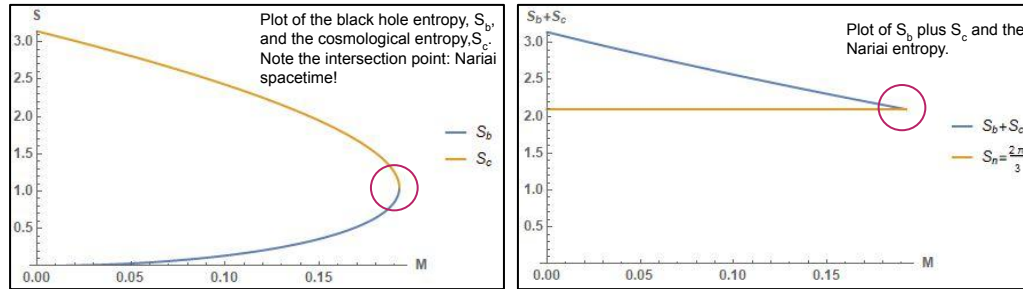
Variational Quantum Eigensolver

- Variational Quantum Eigensolver (VQE)
 - ◆ Repeatedly modifies an ansatz for a wavefunction in an effort to get as accurate of a result as possible.
 - ◆ Just like the variational method in quantum mechanics!
- Use 4 different optimizers and compare the accuracy.
 - ◆ SLSQP, COBYLA, L-BFGS-B, NELDER-MEAD

Thermodynamics

Simplified entropy equation

$$S = \frac{1}{4}A \rightarrow S_b = \pi r_b^2 \text{ and } S_c = \pi r_c^2$$

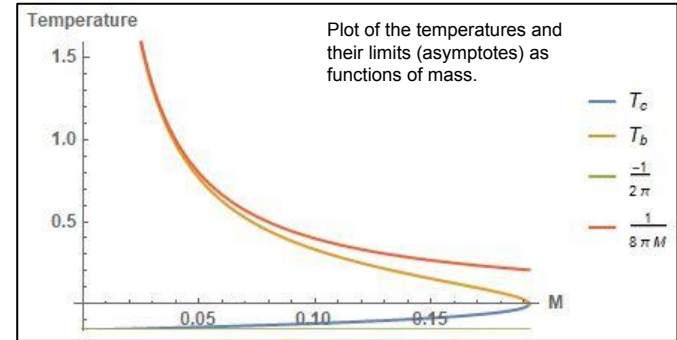


An important solution to note is the **Nariai black hole**. This is the largest possible black hole in de Sitter space. Its event horizon radius approaches that of the cosmological horizon, $r_b \rightarrow r_c$, so that the two radii coincide.

No spacial singularity! It appears to be a singularity in time.

Temperature equation

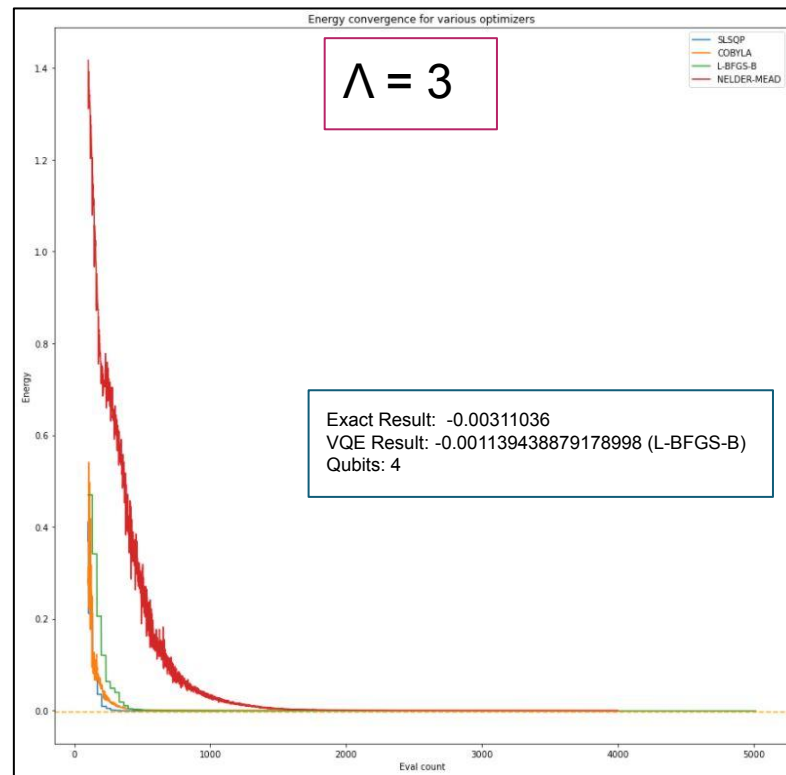
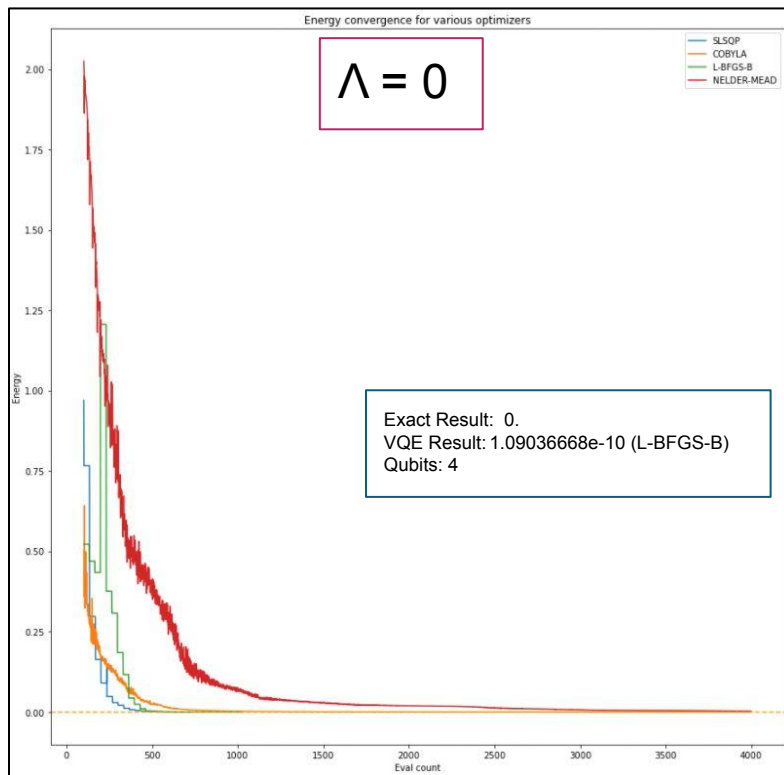
$$\beta = T^{-1} = \frac{\partial S}{\partial M}$$



Notice the convergence point of the two temperatures at the Nariai point,

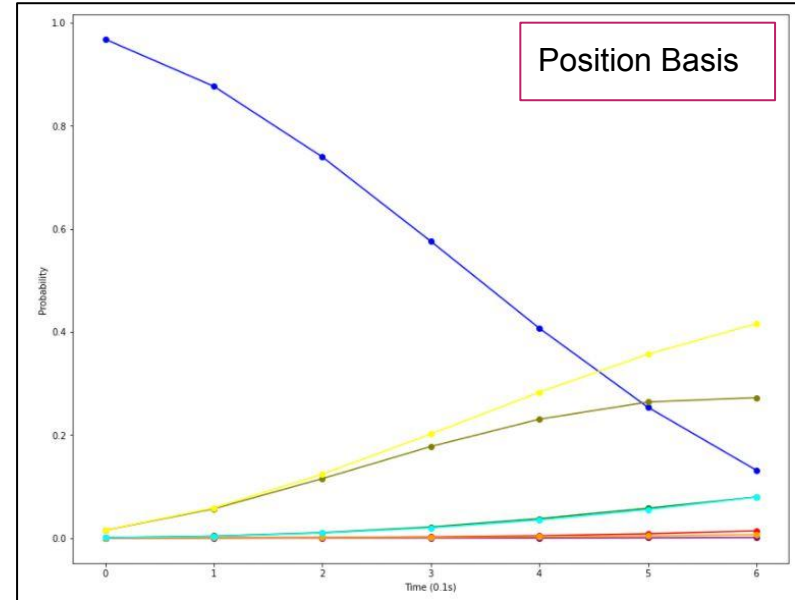
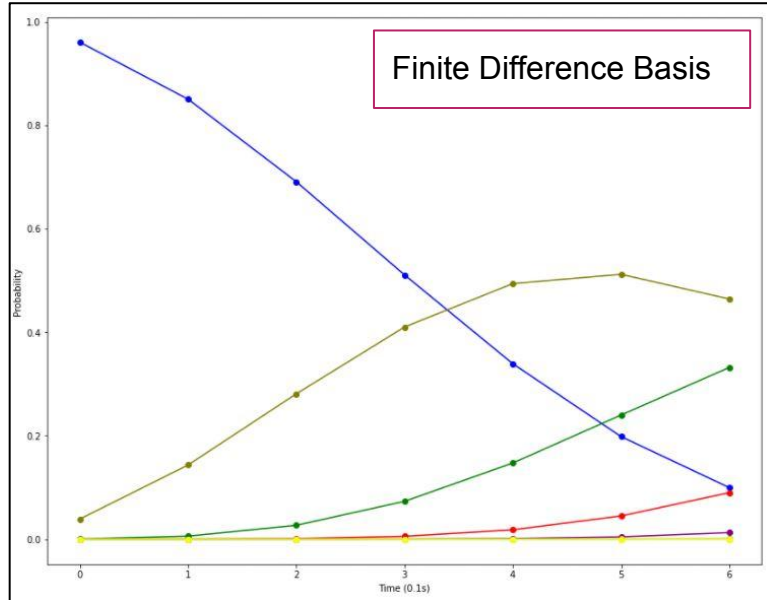
$$M = \frac{1}{\sqrt{27}}$$

Hamiltonian VQE



Evolution of the Hamiltonian

The Hamiltonian represents the quantum version of the Friedmann equation which describes the evolution of the Universe.



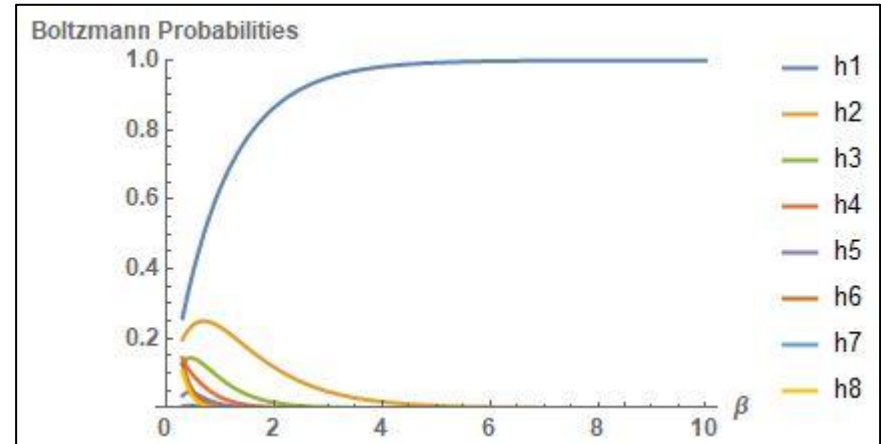
Thermofield Double

- Essentially, the TFD is a special entangled state that cannot be denoted as a mixture of states.
- Similar to the EOH, except we evolve in temperature rather than time.
- Way to study a thermal system by introducing a “double” particle (in our case, boson) into the TFD state.

$$G = -i\theta[\beta] \left(a_{Bd} a_B - a_B^+ a_{Bd}^+ \right)$$

$$e^{-i\theta G}$$

Evolve in temperature



Conclusion

- The Nariai black hole is the largest possible bh.
- Useful connections can be drawn between a dS black hole and the Universe.
- The TFD state is useful in the study of black holes since it involves a special entangled state which can be applied to two sides of the black hole.

Acknowledgements

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