

# Parcial #1 - Señales y Sistemas 2025 - 2S

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1. La distancia media entre dos señales periódicas  $x_1(t) \in \mathbb{R}, \mathbb{C}$  y  $x_2(t) \in \mathbb{R}, \mathbb{C}$ ; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1-x_2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt.$$

Sea  $x_1(t)$  y  $x_2(t)$  dos señales definidas como:

$$x_1(t) = A e^{-j n w_0 t}$$

$$x_2(t) = B e^{j m w_0 t}$$

con  $w_0 = \frac{2\pi}{T}$ ;  $T, A, B \in \mathbb{R}^+$  y  $n, m \in \mathbb{Z}$ . Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

Entonces:

$$d(x_1, x_2) = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt}$$

$$x_1(t) = A e^{-j n \frac{2\pi}{T} t}, \quad x_2(t) = B e^{j m \frac{2\pi}{T} t}$$

$$d(x_1, x_2) = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T |A e^{-j n \frac{2\pi}{T} t} - B e^{j m \frac{2\pi}{T} t}|^2 dt} \quad | a \in \mathbb{C} \rightarrow |a|^2 = a \cdot a^*$$

$$= \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T (A e^{-j n \frac{2\pi}{T} t} - B e^{j m \frac{2\pi}{T} t})(A e^{-j n \frac{2\pi}{T} t} - B e^{j m \frac{2\pi}{T} t})^* dt} \quad | (a-b)^* = (a^* - b^*)$$

$$= \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T (A e^{-j n \frac{2\pi}{T} t} - B e^{j m \frac{2\pi}{T} t})(A e^{j n \frac{2\pi}{T} t} - B e^{-j m \frac{2\pi}{T} t}) dt}$$

$$= \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T A^2 \cdot |e^{j n \frac{2\pi}{T} t}|^2 + B^2 \cdot |e^{j m \frac{2\pi}{T} t}|^2 - AB (e^{j(n+m)\frac{2\pi}{T}t} + e^{-j(n+m)\frac{2\pi}{T}t}) dt}$$

$$\cdot |e^{j k \frac{2\pi}{T} t}|^2 : k \in \mathbb{Z} = \cos^2(k \frac{2\pi}{T} t) + \sin^2(k \frac{2\pi}{T} t) = 1$$

$$\cdot e^{-j(n+m)\frac{2\pi}{T}t} + e^{j(n+m)\frac{2\pi}{T}t}$$

$$= \cos((n+m)\frac{2\pi}{T}t) - j \sin((n+m)\frac{2\pi}{T}t) + \cos((n+m)\frac{2\pi}{T}t) + j \sin((n+m)\frac{2\pi}{T}t)$$

$$= 2 \cos((n+m)\frac{2\pi}{T}t)$$

$$= \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_T A^2 + B^2 - 2AB \cos((n+m)\frac{2\pi}{T}t) dt}$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - \frac{2AB}{T} \int_T \cos((n+m)\frac{2\pi}{T}t) dt}$$

Sea  $K := n+m \in \mathbb{Z}$ , se tienen 2 casos:

$$\bullet k=0 \rightarrow m=-n$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - \frac{2AB}{T} \int_T \cos(0) dt}$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - 2AB}$$

$$\therefore d(x_1, x_2) = \sqrt{A^2 + B^2 - 2AB}$$

$$\bullet k \neq 0 \rightarrow m \neq -n$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - \frac{2AB}{T} \int_T \cos(k \frac{2\pi}{T} t) dt}$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - \frac{2AB}{T} \left[ \frac{\sin(k \frac{2\pi}{T} T)}{k \frac{2\pi}{T}} \right]}$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2 - AB \cdot \frac{0}{k\pi}}$$

$$= \sqrt{\lim_{T \rightarrow \infty} A^2 + B^2}$$

$$\therefore d(x_1, x_2) = \sqrt{A^2 + B^2}$$

Resumiendo:

$$d(x_1, x_2) = \begin{cases} \sqrt{A^2 + B^2 - 2AB} & \text{si } m = -n \\ \sqrt{A^2 + B^2} & \text{en otro caso.} \end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de  $5\text{kHz}$  y 4 bits de capacidad de representación, aplicado a la señal continua:

$$x(t) = 3 \cos(1000\pi t) + 5 \sin(3000\pi t) + 10 \cos(11000\pi t).$$

Realizar la simulación del proceso de discretización (incluyendo al menos tres períodos de  $x(t)$ ). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$X(t)$  es el resultado de la superposición de 3 señales:

$$X(t) = X_1(t) + X_2(t) + X_3(t) \text{ respectivamente.}$$

Entonces:

$$X_1(t) = 3 \cos(1000\pi t) \rightarrow \omega_1 = 1000\pi \text{ rad/s}$$

$$X_2(t) = 5 \sin(3000\pi t) \rightarrow \omega_2 = 3000\pi \text{ rad/s}$$

$$X_3(t) = 10 \cos(11000\pi t) \rightarrow \omega_3 = 11000\pi \text{ rad/s}$$

**Nota:** "Frecuencia angular digital" :=  $\Omega$  [rad/sample]  
 $\Omega$  es periódica con periodo  $2\pi \rightarrow \Omega \in (-\pi, \pi]$

Po: (o tanto):

$$\Omega_1 = 2\pi \cdot \frac{f_1}{F_s} = \frac{\omega_1}{F_s} = \frac{1000\pi}{5000} = \frac{\pi}{5} \in (-\pi, \pi] \quad \checkmark$$

$$\Omega_2 = 2\pi \cdot \frac{f_2}{F_s} = \frac{\omega_2}{F_s} = \frac{3000\pi}{5000} = \frac{3\pi}{5} \in (-\pi, \pi] \quad \checkmark$$

$$\Omega_3 = 2\pi \cdot \frac{f_3}{F_s} = \frac{\omega_3}{F_s} = \frac{11000\pi}{5000} = \frac{11\pi}{5} \notin (-\pi, \pi] \quad \times$$

$$\text{Entonces } \Omega_3 = \underbrace{\frac{11\pi}{5}}_{(\text{mod } 2\pi)} = \frac{11\pi}{5} - 2\pi = \frac{\pi}{5}$$

$X_3(t)$  alias de  $X_1(t)$

Evaluando:

$$\chi_1(nT_s) = 3 \cos(1000\pi nT_s) = 3 \cos\left(\frac{\pi}{5}n\right) = \chi_1[n]$$

$$\chi_2(nT_s) = 5 \sin(3000\pi nT_s) = 5 \sin\left(\frac{3\pi}{5}n\right) = \chi_2[n]$$

$$\chi_3(nT_s) = 10 \cos(11000\pi nT_s) = 10 \cos\left(\frac{11\pi}{5}n\right) = \chi_3[n]$$

$$\therefore \chi(nT_s) = \chi[n] = \chi_1[n] + \chi_2[n] + \chi_3[n]$$

$$\chi[n] = 3 \cos\left(\frac{\pi}{5}n\right) + 5 \sin\left(\frac{3\pi}{5}n\right) + 10 \cos\left(\frac{11\pi}{5}n\right)$$

$$\boxed{\chi[n] = 13 \cos\left(\frac{\pi}{5}n\right) + 5 \sin\left(\frac{3\pi}{5}n\right)}$$

Período de  $\chi(t)$ :

$$\exists T_0 > 0 : \chi(t + T_0) = \chi(t) \quad \forall t \longleftrightarrow \frac{\omega_i}{\omega_j} \in \mathbb{Q} \quad \forall i, j$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Ahora:

$$\omega_0 = \gcd(\omega_1, \omega_2, \omega_3) = \gcd(1000\pi, 3000\pi, 11000\pi)$$

$$\omega_0 = 1000\pi \rightarrow T_0 = \frac{2\pi}{1000\pi} = \boxed{\frac{1}{500}s}$$

3. Sea  $x''(t)$  la segunda derivada de la señal  $x(t)$ , donde  $t \in [t_i, t_f]$ . Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(t_i - t_f)n^2 w_o^2} \int_{t_i}^{t_f} x''(t) e^{-j n w_o t} dt; \quad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes  $a_n$  y  $b_n$  desde  $x''(t)$  en la serie trigonométrica de Fourier?

Entonces:

$$\chi(t) = \sum_n c_n e^{j n w_o t} \rightarrow c_n = \frac{1}{T} \int_T \chi(t) \cdot e^{-j n w_o t} dt$$

$$\frac{d^2 \chi}{dt^2} = \frac{d^2}{dt^2} \left( \sum_n c_n e^{j n w_o t} \right) = \sum_n \frac{d^2}{dt^2} (c_n e^{j n w_o t})$$

$$\chi''(t) = \sum_n c_n \cdot \frac{d^2}{dt^2} (e^{j n w_o t}) = \sum_n c_n \cdot (j n w_o)^2 e^{j n w_o t}$$

Analizando:

$$c_n \cdot (j n w_o)^2 = c_n \cdot j^2 \cdot n^2 \cdot w_o^2 = -c_n n^2 w_o^2 = k_n$$

$$\therefore \chi''(t) = \sum_n k_n e^{j n w_o t} \rightarrow \text{Otra serie de Fourier.}$$

$$\therefore k_n = \frac{1}{T} \int_T \chi''(t) \cdot e^{-j n w_o t} dt$$

$$-c_n n^2 w_o^2 = \frac{1}{T} \int_T \chi''(t) \cdot e^{-j n w_o t} dt$$

$$c_n = \frac{1}{-T n^2 w_o^2} \int_T \chi''(t) \cdot e^{-j n w_o t} dt$$

$$c_n = \frac{1}{-(t_f - t_i) n^2 w_o^2} \int_{t_i}^{t_f} \chi''(t) e^{-j n w_o t} dt$$

$$c_n = \frac{1}{(t_i - t_f) n^2 w_o^2} \int_{t_i}^{t_f} \chi''(t) e^{-j n w_o t} dt, \quad n \in \mathbb{Z} \setminus \{0\}$$

**Nota:** los coeficientes de la serie trigonométrica de Fourier ( $a_n, b_n$ ), se relacionan con  $C_n$  de la siguiente manera:

$$a_n = C_n + C_{-n}, \quad b_n = j(C_n - C_{-n})$$

$$a_n = \frac{1}{(t_i - t_f) n^2 \omega_0^2} \left( \int_{t_i}^{t_f} x''(t) e^{-j n \omega_0 t} dt + \int_{t_i}^{t_f} x''(t) e^{j n \omega_0 t} dt \right)$$

$$a_n = \frac{1}{(t_i - t_f) n^2 \omega_0^2} \left( \int_{t_i}^{t_f} x''(t) [e^{-j n \omega_0 t} + e^{j n \omega_0 t}] dt \right)$$

$$\text{Nota: } \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$a_n = \frac{1}{(t_i - t_f) n^2 \omega_0^2} \left( \int_{t_i}^{t_f} x''(t) [2 \cos(n \omega_0 t)] dt \right)$$

$$a_n = \frac{2}{(t_i - t_f) n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t) \cos(n \omega_0 t) dt \quad n \in \mathbb{Z} \setminus \{0\}$$

$$b_n = j \left( \frac{1}{(t_i - t_f) n^2 \omega_0^2} \left[ \int_{t_i}^{t_f} x''(t) e^{-j n \omega_0 t} dt - \int_{t_i}^{t_f} x''(t) e^{j n \omega_0 t} dt \right] \right)$$

$$b_n = j \left( \frac{1}{(t_i - t_f) n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t) [e^{-j n \omega_0 t} - e^{j n \omega_0 t}] dt \right)$$

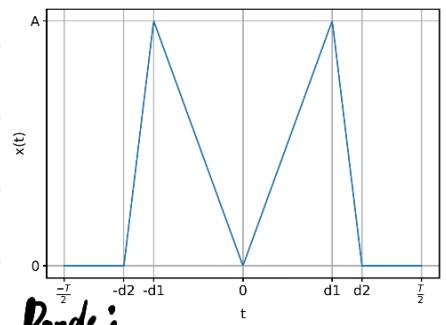
$$\text{Nota: } \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$b_n = j \left( \frac{1}{(t_i - t_f) n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t) [-2j \sin(n \omega_0 t)] dt \right)$$

$$b_n = \frac{j^2 \cdot (-2)}{(t_i - t_f) n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t) \sin(n \omega_0 t) dt$$

$$b_n = \frac{2}{(t_i - t_f) n^2 \omega_0^2} \int_{t_i}^{t_f} x''(t) \sin(n \omega_0 t) dt \quad n \in \mathbb{Z} \setminus \{0\}$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para  $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ , a partir de  $x''(t)$  para la señal  $x(t)$  en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de  $x(t)$ . Presente las simulaciones de Python respectivas.



Donde:

$$m_1 = \frac{A-0}{-d_2 - (-d_1)} = \frac{A}{d_2 - d_1}, b_1 = m_1 d_1$$

$$m_2 = \frac{A-0}{d_1 - 0} = \frac{A}{d_1}, b_2 = 0$$

$$T, d_2, d_1, A \in \mathbb{R}^+$$

$$\frac{T}{2} > d_2 > d_1$$

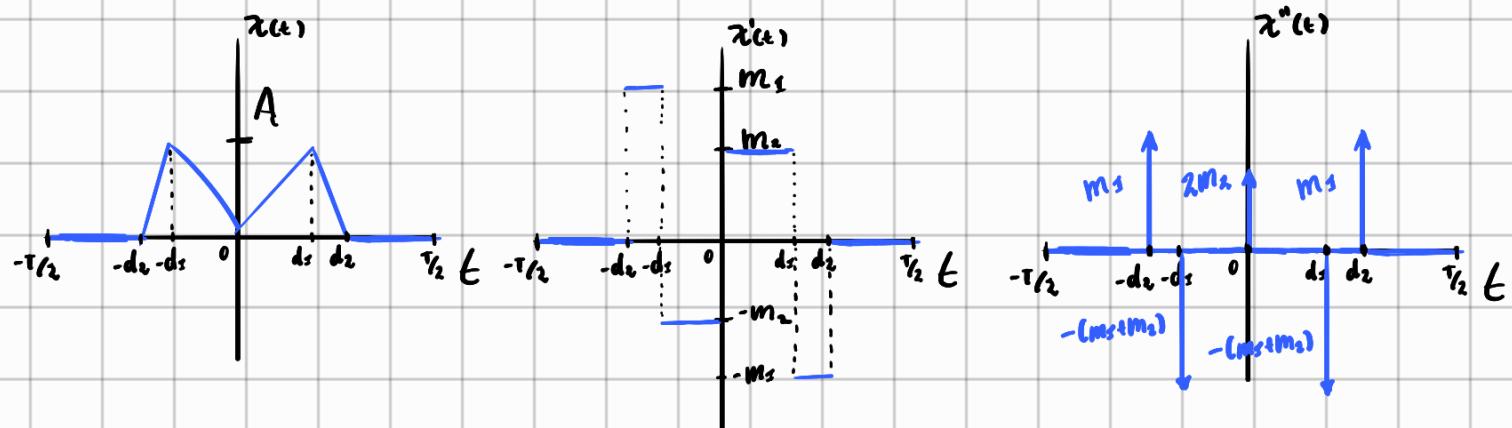
$$x(t) = \begin{cases} 0 & \text{si } t \in [-\frac{T}{2}, -d_2] \cup [d_2, \frac{T}{2}] \\ m_1 t + b_1 & \text{si } t \in [-d_2, -d_1] \\ -m_2 t + b_2 & \text{si } t \in [-d_1, 0] \\ m_2 t + b_2 & \text{si } t \in [0, d_1] \\ -m_1 t + b_1 & \text{si } t \in [d_1, d_2] \end{cases}$$

A partir de  $x''(t)$ :

$$x(t) = C_0 + \sum_n C_n e^{jn\omega_0 t} \rightarrow C_n = \frac{-1}{T n^2 \omega_0^2} \int_T x''(t) e^{-jn\omega_0 t} dt, n \in \mathbb{Z} \setminus \{0\}$$

**Nota:** Según la gráfica de  $x(t)$ ,  $m_1 > m_2$

Entonces:



De modo que:

$$x''(t) = m_1 \delta(t+d_2) - (m_1+m_2) \delta(t+d_1) + 2m_2 \delta(t) - (m_1+m_2) \delta(t-d_1) + m_1 \delta(t-d_2)$$

$$x''(t) = m_1 [\delta(t+d_2) + \delta(t-d_2)] - (m_1+m_2) [\delta(t+d_1) + \delta(t-d_1)] + 2m_2 \delta(t)$$

Evaluando  $\mathcal{X}''(t)$ :

$$C_n = \frac{-1}{T n^2 w_0^2} \left[ \int_T m_1 [\delta(t+d_2) + \delta(t-d_2)] e^{-jnw_0 t} dt - \int_T (m_1 + m_2) [\delta(t+d_1) + \delta(t-d_1)] e^{-jnw_0 t} dt + \int_T 2m_2 \delta(t) e^{-jnw_0 t} dt \right]$$

**Nota:**  $\int f(t) \delta(t \pm t_0) dt = f(\pm t_0)$

$$C_n = \frac{-1}{T n^2 w_0^2} \left[ m_1 \left( \int_T \delta(t+d_2) e^{-jnw_0 t} dt + \int_T \delta(t-d_2) e^{-jnw_0 t} dt \right) - (m_1 + m_2) \left( \int_T \delta(t+d_1) e^{-jnw_0 t} dt + \int_T \delta(t-d_1) e^{-jnw_0 t} dt \right) + 2m_2 \left( \int_T \delta(t) e^{-jnw_0 t} dt \right) \right]$$

$$C_n = \frac{-1}{T n^2 w_0^2} \left[ m_1 \left( e^{jnw_0(d_2)} + e^{-jnw_0(d_2)} \right) - (m_1 + m_2) \left( e^{jnw_0(d_1)} + e^{-jnw_0(d_1)} \right) + 2m_2 \left( e^{-jnw_0(0)} \right) \right]$$

$$C_n = \frac{-1}{T n^2 w_0^2} \left[ m_1 \left( \cos(d_2 n w_0) + j \sin(d_2 n w_0) + \cos(d_2 n w_0) - j \sin(d_2 n w_0) \right) - (m_1 + m_2) \left( \cos(d_1 n w_0) + j \sin(d_1 n w_0) + \cos(d_1 n w_0) - j \sin(d_1 n w_0) \right) + 2m_2 \left( \cos(0) - j \sin(0) \right) \right]$$

$$C_n = \frac{-1}{T n^2 \omega_0^2} \left[ 2m_2 \cos(d_2 n \omega_0) - 2(m_1 + m_2) \cos(d_1 n \omega_0) + 2m_2 \right] \quad n \in \mathbb{Z} \setminus \{0\}$$

∴  $C_n$  es puramente real  $\rightarrow \chi(t)$  es par ✓

Por lo tanto :

$$\operatorname{Re}\{C_n\} = C_n, \operatorname{Im}\{C_n\} = 0 \rightarrow \chi(t) = \begin{cases} 0 & \text{si } C_n > 0 \\ \pi & \text{si } C_n < 0 \end{cases}$$

$$|C_n| = \left| \frac{-1}{T n^2 \omega_0^2} \left[ 2m_2 \cos(d_2 n \omega_0) - 2(m_1 + m_2) \cos(d_1 n \omega_0) + 2m_2 \right] \right|$$

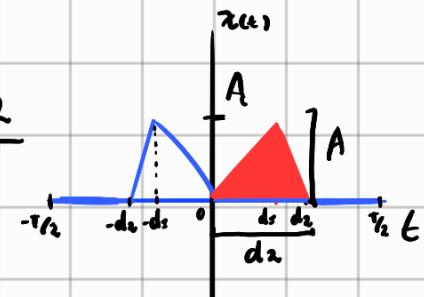
$$|C_n| = \frac{1}{|T n^2 \omega_0^2|} \left| 2m_2 \cos(d_2 n \omega_0) - 2(m_1 + m_2) \cos(d_1 n \omega_0) + 2m_2 \right|$$

$$|C_n| = \frac{1}{T n^2 \omega_0^2} \left| 2m_2 \cos(d_2 n \omega_0) - 2(m_1 + m_2) \cos(d_1 n \omega_0) + 2m_2 \right|$$


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Coeficiente  $C_0$ : Se seguirá calculando de manera usual.

$$C_0 = \frac{1}{T} \int_T x(t) dt = \frac{1}{T} \left( 2 \cdot \frac{A \cdot d_2}{2} \right) = \frac{A \cdot d_2}{T}$$



• A partir de  $\chi(t)$ :

$$\chi(t) = C_0 + \sum_n C_n e^{j n \omega_0 t} \longrightarrow C_n = \frac{1}{T} \int_T \chi(t) \cdot e^{-j n \omega_0 t} dt$$

$$\text{Como } \chi(t) \text{ es par y real} \rightarrow C_n = \frac{2}{T} \int_0^{T/2} \chi(t) \cos(n \omega_0 t) dt$$

Evaluando:

$$C_n = \frac{2}{T} \left[ \int_0^{d_2} m_2 t \cos(n\omega_0 t) dt + \int_{d_2}^{d_1} (-m_1 t + b_1) \cos(n\omega_0 t) dt + \int_{d_1}^{T/2} 0 \cdot \cos(n\omega_0 t) dt \right]$$

$$C_n = \frac{2}{T} \left[ m_2 \left( \frac{t \sin(n\omega_0 t)}{n\omega_0} \Big|_0^{d_2} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2} \Big|_0^{d_2} \right) - m_1 \left( \frac{t \sin(n\omega_0 t)}{n\omega_0} \Big|_{d_2}^{d_1} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2} \Big|_{d_2}^{d_1} \right) + b_1 \left( \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_{d_1}^{T/2} \right) \right]$$

$$C_n = \frac{2}{T} \left[ m_2 \left( \frac{d_2 \sin(d_2 n\omega_0)}{n\omega_0} + \frac{\cos(d_2 n\omega_0) - 1}{n^2 \omega_0^2} \right) - m_1 \left( \frac{d_2 \sin(d_2 n\omega_0)}{n\omega_0} - \frac{d_1 \sin(d_1 n\omega_0)}{n\omega_0} + \frac{\cos(d_1 n\omega_0) - \cos(d_2 n\omega_0)}{n^2 \omega_0^2} \right) + b_1 \left( \frac{\sin(d_1 n\omega_0)}{n\omega_0} - \frac{\sin(d_2 n\omega_0)}{n\omega_0} \right) \right]$$

$$C_n = \frac{-1}{T n^2 \omega_0^2} \left[ 2m_1 \cos(d_2 n\omega_0) - 2(m_1 + m_2) \cos(d_1 n\omega_0) + 2m_2 \right] \quad n \in \mathbb{Z} \setminus \{0\}$$

→ Misma expresión ✓

• Error relativo de la estimación:

Recordando:

$$\chi(t) = \sum_{n=-\infty}^{\infty} C_n \cdot \phi_n(t) \rightarrow \hat{\chi}(t) = \sum_{n=-N}^N C_n \cdot \phi_n(t)$$

$$\bar{P}_x = \frac{1}{T} \int_T |\chi(t)|^2 dt \rightarrow \bar{P}_{\hat{x}} = \frac{1}{T} \int_T |\hat{\chi}(t)|^2 dt$$

$$\begin{aligned} \bar{P}_{\hat{x}} &= \frac{1}{T} \int_T |\sum C_n \cdot \phi_n(t)|^2 dt = \frac{1}{T} \int_T \sum |C_n \cdot \phi_n(t)|^2 dt \\ &= \frac{1}{T} \int_T \sum |C_n|^2 |\phi_n(t)|^2 dt = \sum |C_n|^2 \underbrace{\frac{1}{T} \int_T |\phi_n(t)|^2 dt}_{P_n} \\ &= \sum_{n=-N}^N |C_n|^2 \cdot P_n \end{aligned}$$

$$e_r = \left( 1 - \frac{\bar{P}_{\hat{x}}}{\bar{P}_x} \right) = \left( 1 - \frac{\sum_{n=-N}^N |C_n|^2 \cdot P_n}{\bar{P}_x} \right)$$

$$e_r [\%] = \left( 1 - \frac{\sum_{n=-N}^N |C_n|^2 \cdot P_n}{\bar{P}_x} \right) \cdot 100$$

Entonces:

$$P_n = \frac{1}{T} \int_T |e^{jn\omega_0 t}|^2 dt = \frac{1}{T} \int_T (\cos^2(n\omega_0 t) + \sin^2(n\omega_0 t)) dt$$
$$= \frac{1}{T} \int_T 1 \cdot dt = \frac{1}{T} T = 1 \quad \forall n$$

$$\bar{P}_x = \frac{1}{T} \int_T |x(t)|^2 dt \quad // \text{como } x(t) \geq 0 \quad \forall t$$

$$\bar{P}_x = \frac{1}{T} \int_T (x(t))^2 dt$$

Como  $x(t)$  es par  $\rightarrow (x(t))^2$  también lo es:

$$\bar{P}_x = \frac{2}{T} \int_0^{T/2} (x(t))^2 dt = \frac{2}{T} \left[ \int_0^{d_3} (m_2 t)^2 dt + \int_{d_3}^{d_2} (-m_3 t + b_3)^2 dt \right]$$

$$\bar{P}_x = \frac{2}{T} \left[ m_2^2 \cdot \frac{t^3}{3} \Big|_0^{d_3} + \int_{d_3}^{d_2} (b_3^2 - 2b_3 m_3 t + m_3^2 t^2) dt \right]$$

$$\bar{P}_x = \frac{2}{T} \left[ \frac{m_2^2 \cdot d_3^3}{3} + b_3^2 \cdot t \Big|_{d_3}^{d_2} - 2b_3 m_3 \cdot \frac{t^2}{2} \Big|_{d_3}^{d_2} + m_3^2 \cdot \frac{t^3}{3} \Big|_{d_3}^{d_2} \right]$$

$$\bar{P}_x = \frac{2}{T} \left[ \frac{m_2^2 d_3^3}{3} + b_3^2 (d_2 - d_3) - 2b_3 m_3 \left( \frac{d_2^2}{2} - \frac{d_3^2}{2} \right) + m_3^2 \left( \frac{d_2^3}{3} - \frac{d_3^3}{3} \right) \right]$$

$$\bar{P}_x = \frac{2}{T} \left[ \frac{A^2 d_3}{3} + \frac{A^2 (d_2 - d_3)}{3} \right] = \frac{2}{T} \left[ \frac{A^2 d_2}{3} \right]$$

$$\bar{P}_x = \frac{2 A^2 d_2}{3 T}$$

$$\therefore C_F[\%] = \left( 1 - \frac{\sum_{n=-N}^N |C_n|^2 \cdot 1}{\frac{2A^2 d_2}{3T}} \right) \cdot 100$$

$$C_F[\%] = \left( 1 - \frac{3T \sum_{n=-N}^N |C_n|^2}{2A^2 d_2} \right) \cdot 100$$

Para  $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

$$C_F[\%] = \left( 1 - \frac{3T \sum_{n=-5}^5 |C_n|^2}{2A^2 d_2} \right) \cdot 100$$

