Parcial #1 - Señales y Sistemas 2025 - 2S

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1. La distancia media entre dos señales periódicas $x_1(t) \in \mathbb{R}, \mathbb{C}$ y $x_2(t) \in \mathbb{R}, \mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt.$$

Sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$x_1(t) = Ae^{-jnw_0 t}$$

$$x_2(t) = Be^{jmw_0 t}$$

con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

Entonces:

$$d(x_1, x_2) = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 dt}$$

$$x_1(t) = Ae^{-jn\frac{2\pi}{T}t}, \quad x_2(t) = Be^{jm\frac{2\pi}{T}t}$$

$$d(x_1, x_2) = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T \left|Ae^{-jn\frac{2\pi}{T}t} - Be^{jm\frac{2\pi}{T}t}\right|^2 dt} \quad \Big| \quad a \in \mathbb{C} \to |a|^2 = a \cdot a^*$$

$$= \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T \left(Ae^{-jn\frac{2\pi}{T}t} - Be^{jm\frac{2\pi}{T}t}\right)\left(Ae^{-jn\frac{2\pi}{T}t} - Be^{jm\frac{2\pi}{T}t}\right)^* dt} \quad \Big| \quad (a-b)^* = (a^* - b^*)$$

$$= \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T \left(Ae^{-jn\frac{2\pi}{T}t} - Be^{jm\frac{2\pi}{T}t}\right)\left(Ae^{jn\frac{2\pi}{T}t} - Be^{-jm\frac{2\pi}{T}t}\right) dt}$$

$$= \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T A^2 \cdot \left|e^{jn\frac{2\pi}{T}t}\right|^2 + B^2 \cdot \left|e^{jm\frac{2\pi}{T}t}\right|^2 - AB\left(e^{-j(n+m)\frac{2\pi}{T}t} + e^{j(n+m)\frac{2\pi}{T}t}\right) dt}$$

- $\left|e^{jk\frac{2\pi}{T}t}\right|^2 : k \in \mathbb{Z} \quad = \cos^2\left(k\frac{2\pi}{T}t\right) + \sin^2\left(k\frac{2\pi}{T}t\right) = 1$

- $e^{-j(n+m)\frac{2\pi}{T}t} + e^{j(n+m)\frac{2\pi}{T}t}$

$$= \cos\left((n+m)\frac{2\pi}{T}t\right) - j\cancel{\sin\left((n+m)\frac{2\pi}{T}t\right)} + \cos\left((n+m)\frac{2\pi}{T}t\right) + j\cancel{\sin\left((n+m)\frac{2\pi}{T}t\right)}$$

$$= 2\cos\left((n+m)\frac{2\pi}{T}t\right)$$

$$= \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_T A^2 + B^2 - 2AB\cos\left((n+m)\frac{2\pi}{T}t\right) dt}$$

$$= \sqrt{\lim_{T\to\infty} A^2 + B^2 - \frac{2AB}{T}\int_T \cos\left((n+m)\frac{2\pi}{T}t\right) dt}$$

Sea $k := n+m \in \mathbb{Z}$, se tienen 2 casos:

• $k=0 \longrightarrow m=-n$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-\frac{2AB}{T}\int_T \cos(0)\,dt}$$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-2AB}$$

$$\therefore\ d(x_1,x_2)=\sqrt{A^2+B^2-2AB}$$

• $k\neq 0 \longrightarrow m\neq -n$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-\frac{2AB}{T}\int_T \cos\left(k\frac{2\pi}{T}t\right)dt}$$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-\frac{\cancel{2}AB}{\cancel{T}}\left[\frac{\sin\left(k\frac{2\pi}{\cancel{T}}\cancel{T}\right)}{k\frac{\cancel{2}\pi}{\cancel{T}}}\right]}$$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-AB\cdot\frac{\sin(k2\pi)}{k}}$$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2-AB\cdot\frac{0}{k}}$$

$$=\sqrt{\lim_{T\to\infty} A^2+B^2}$$

$$\therefore\ d(x_1,x_2)=\sqrt{A^2+B^2}$$

Resumiendo:

$$d(x_1,x_2)=\begin{cases}\sqrt{A^2+B^2-2AB} & \text{si } m=-n\\ \sqrt{A^2+B^2} & \text{en otro caso.}\end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de $5kHz$ y 4 bits de capacidad de representación, aplicado a la señal continua:

$$x(t) = 3\cos(1000\pi t) + 5\sin(3000\pi t) + 10\cos(11000\pi t).$$

Realizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$x(t)$ es el resultado de la superposición de 3 señales:

$x(t) = x_1(t) + x_2(t) + x_3(t)$ respectivamente.

Entonces:

$x_1(t) = 3\cos(1000\pi t) \rightarrow \omega_1 = 1000\pi$ rad/s

$x_2(t) = 5\sin(3000\pi t) \rightarrow \omega_2 = 3000\pi$ rad/s

$x_3(t) = 10\cos(11000\pi t) \rightarrow \omega_3 = 11000\pi$ rad/s

Nota: "Frecuencia angular digital" $:= \Omega$ [rad/sample]
Ω es periódica con periodo $2\pi \rightarrow \Omega \in (-\pi, \pi]$

Por lo tanto:

$$\Omega_1 = 2\pi \cdot \frac{f_1}{F_s} = \frac{\omega_1}{F_s} = \frac{1000\pi}{5000} = \frac{\pi}{5} \in (-\pi, \pi] \checkmark$$

$$\Omega_2 = 2\pi \cdot \frac{f_2}{F_s} = \frac{\omega_2}{F_s} = \frac{3000\pi}{5000} = \frac{3\pi}{5} \in (-\pi, \pi] \checkmark$$

$$\Omega_3 = 2\pi \cdot \frac{f_3}{F_s} = \frac{\omega_3}{F_s} = \frac{11000\pi}{5000} = \frac{11\pi}{5} \notin (-\pi, \pi] \times$$

$$\hookrightarrow \Omega_3 = \underbrace{\frac{11\pi}{5} \equiv \frac{11\pi}{5} - 2\pi}_{(\mathrm{mod}\ 2\pi)} = \frac{\pi}{5} \rightarrow \boxed{x_3(t) \text{ alias de } x_1(t)}$$

Evaluando:

$$x_1(nT_s) = 3\cos(1000\pi nT_s) = 3\cos\left(\frac{\pi}{5}n\right) = x_1[n]$$

$$x_2(nT_s) = 5\sin(3000\pi nT_s) = 5\sin\left(\frac{3\pi}{5}n\right) = x_2[n]$$

$$x_3(nT_s) = 10\cos(11000\pi nT_s) = 10\cos\left(\frac{\pi}{5}n\right) = x_3[n]$$

$$\therefore x(nT_s) = x[n] = x_1[n] + x_2[n] + x_3[n]$$

$$x[n] = 3\cos\left(\frac{\pi}{5}n\right) + 5\sin\left(\frac{3\pi}{5}n\right) + 10\cos\left(\frac{\pi}{5}n\right)$$

$$\boxed{x[n] = 13\cos\left(\frac{\pi}{5}n\right) + 5\sin\left(\frac{3\pi}{5}n\right)}$$

Periodo de $x(t)$:

$$\exists T_0 > 0 : x(t+T_0) = x(t)\ \forall t \longleftrightarrow \frac{\omega_i}{\omega_j} \in \mathbb{Q}\ \forall i,j$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Ahora:

$$\omega_0 = \gcd(\omega_1, \omega_2, \omega_3) = \gcd(1000\pi, 3000\pi, 11000\pi)$$

$$\omega_0 = 1000\pi \longrightarrow T_0 = \frac{2\pi}{1000\pi} = \boxed{\frac{1}{500}\,s}$$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(ti - tf)n^2 w_o^2} \int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt; \quad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?.

Entonces:

$$x(t) = \sum_n C_n e^{jnw_o t} \longrightarrow C_n = \frac{1}{T}\int_T x(t) \cdot e^{-jnw_o t} dt$$

$$\frac{d^2 x}{dt^2} = \frac{d^2}{dt^2}\left(\sum_n C_n e^{jnw_o t}\right) = \sum_n \frac{d^2}{dt^2}(C_n e^{jnw_o t})$$

$$x''(t) = \sum_n C_n \cdot \frac{d^2}{dt^2}(e^{jnw_o t}) = \sum_n C_n \cdot (jnw_o)^2 e^{jnw_o t}$$

Analizando:

$$C_n \cdot (jnw_o)^2 = C_n \cdot j^2 \cdot n^2 \cdot w_o^2 = -C_n n^2 w_o^2 = K_n$$

$$\therefore x''(t) = \sum_n K_n e^{jnw_o t} \longrightarrow \text{Otra serie de Fourier.}$$

$$\therefore K_n = \frac{1}{T}\int_T x''(t) \cdot e^{-jnw_o t} dt$$

$$-C_n n^2 w_o^2 = \frac{1}{T}\int_T x''(t) \cdot e^{-jnw_o t} dt$$

$$C_n = \frac{1}{-T n^2 w_o^2}\int_T x''(t) \cdot e^{-jnw_o t} dt$$

$$C_n = \frac{1}{-(t_f - t_i) n^2 w_o^2}\int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt$$

$$\boxed{C_n = \frac{1}{(t_i - t_f) n^2 w_o^2}\int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt, \quad n \in \mathbb{Z} \setminus \{0\}}$$

Nota: Los coeficientes de la serie trigonométrica de Fourier (a_n, b_n), se relacionan con C_n de la siguiente manera:

$$a_n = C_n + C_{-n} \quad , \quad b_n = j(C_n - C_{-n})$$

$$a_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\left(\int_{t_i}^{t_f} x''(t)e^{-jn\omega_0 t}\,dt + \int_{t_i}^{t_f} x''(t)e^{jn\omega_0 t}\,dt\right)$$

$$a_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\left(\int_{t_i}^{t_f} x''(t)\left[e^{-jn\omega_0 t} + e^{jn\omega_0 t}\right]dt\right)$$

Nota: $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$$a_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\left(\int_{t_i}^{t_f} x''(t)\left[2\cos(n\omega_0 t)\right]dt\right)$$

$$\boxed{a_n = \frac{2}{(t_i - t_f)n^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\cos(n\omega_0 t)\,dt} \quad n \in \mathbb{Z}\setminus\{0\}$$

$$b_n = j\left(\frac{1}{(t_i - t_f)n^2\omega_0^2}\left[\int_{t_i}^{t_f} x''(t)e^{-jn\omega_0 t}\,dt - \int_{t_i}^{t_f} x''(t)e^{jn\omega_0 t}\,dt\right]\right)$$

$$b_n = j\left(\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\left[e^{-jn\omega_0 t} - e^{jn\omega_0 t}\right]dt\right)$$

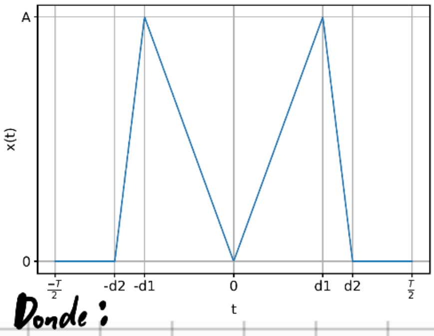
Nota: $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

$$b_n = j\left(\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\left[-2j\sin(n\omega_0 t)\right]dt\right)$$

$$b_n = \frac{j^2 \cdot (-2)}{(t_i - t_f)n^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\sin(n\omega_0 t)\,dt$$

$$\boxed{b_n = \frac{2}{(t_i - t_f)n^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\sin(n\omega_0 t)\,dt} \quad n \in \mathbb{Z}\setminus\{0\}$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.



$$x(t) = \begin{cases} 0 & \text{si } t \in [-\frac{T}{2}, -d_2] \cup [d_2, \frac{T}{2}) \\ m_1 t + b_1 & \text{si } t \in [-d_2, -d_1) \\ -m_2 t + b_2 & \text{si } t \in [-d_1, 0) \\ m_2 t + b_2 & \text{si } t \in [0, d_1) \\ -m_1 t + b_1 & \text{si } t \in [d_1, d_2) \end{cases}$$

Donde:

$m_1 = \frac{A-0}{-d_1-(-d_2)} = \frac{A}{d_2-d_1}$, $b_1 = m_1 d_2$

$m_2 = \frac{A-0}{d_1-0} = \frac{A}{d_1}$, $b_2 = 0$

$T, d_2, d_1, A \in \mathbb{R}^+$

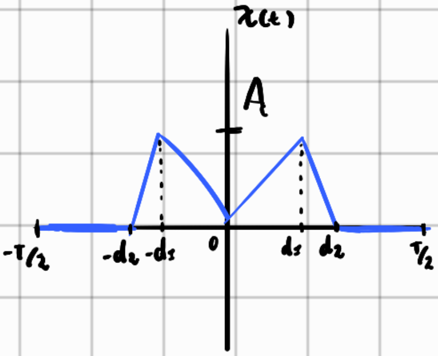
$\frac{T}{2} > d_2 > d_1$

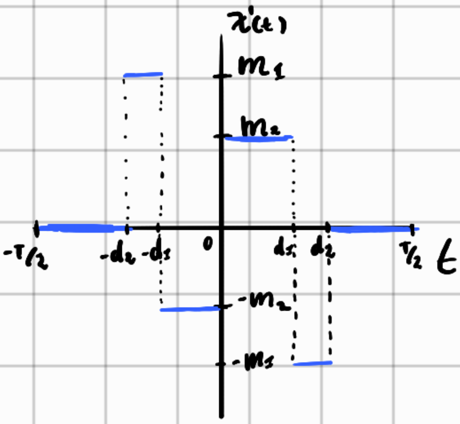
• A partir de $x''(t)$:

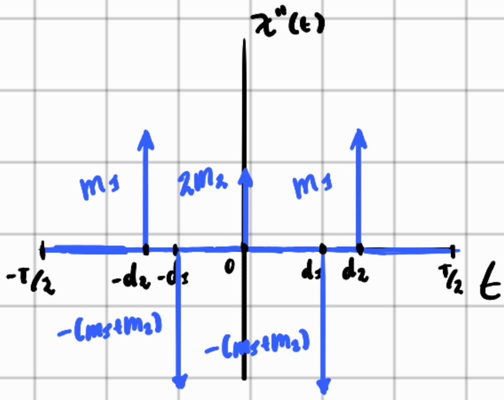
$$x(t) = C_0 + \sum_n C_n e^{jnw_0 t} \rightarrow C_n = \frac{-1}{T n^2 w_0^2} \int_T x''(t) e^{-jnw_0 t} dt, \; n \in \mathbb{Z} \setminus \{0\}$$

$\boxed{\text{Nota: Según la gráfica de } x(t), \; m_1 > m_2}$

Entonces:







De modo que:

$$x''(t) = m_1\delta(t+d_2) - (m_1+m_2)\delta(t+d_1) + 2m_2\delta(t) - (m_1+m_2)\delta(t-d_1) + m_1\delta(t-d_2)$$

$$x''(t) = m_1[\delta(t+d_2) + \delta(t-d_2)] - (m_1+m_2)[\delta(t+d_1) + \delta(t-d_1)] + 2m_2\delta(t)$$

Evaluando $x''(t)$:

$$C_n = \frac{-1}{T n^2 \omega_0^2}\left[\int_T m_1\left[\delta(t+d_2)+\delta(t-d_2)\right]e^{-jn\omega_0 t}dt \right.$$
$$-\int_T (m_1+m_2)\left[\delta(t+d_1)+\delta(t-d_1)\right]e^{-jn\omega_0 t}dt$$
$$\left.+\int_T 2m_2\delta(t)e^{-jn\omega_0 t}dt\right]$$

$$\boxed{\text{Nota: } \int f(t)\,\delta(t\pm t_0)\,dt = f(\mp t_0)}$$

$$C_n = \frac{-1}{T n^2 \omega_0^2}\left[m_1\left(\int_T \delta(t+d_2)e^{-jn\omega_0 t}dt + \int_T \delta(t-d_2)e^{-jn\omega_0 t}dt\right)\right.$$
$$-(m_1+m_2)\left(\int_T \delta(t+d_1)e^{-jn\omega_0 t}dt + \int_T \delta(t-d_1)e^{-jn\omega_0 t}dt\right)$$
$$\left.+2m_2\left(\int_T \delta(t)e^{-jn\omega_0 t}dt\right)\right]$$

$$C_n = \frac{-1}{T n^2 \omega_0^2}\left[m_1\left(e^{jn\omega_0(d_2)}+e^{-jn\omega_0(d_2)}\right)\right.$$
$$-(m_1+m_2)\left(e^{jn\omega_0(d_1)}+e^{-jn\omega_0(d_1)}\right)$$
$$\left.+2m_2\left(e^{-jn\omega_0(0)}\right)\right]$$

$$C_n = \frac{-1}{T n^2 \omega_0^2}\left[m_1\left(\cos(d_2 n\omega_0)+\cancel{j\sin(d_2 n\omega_0)}+\cos(d_2 n\omega_0)-\cancel{j\sin(d_2 n\omega_0)}\right)\right.$$
$$-(m_1+m_2)\left(\cos(d_1 n\omega_0)+\cancel{j\sin(d_1 n\omega_0)}+\cos(d_1 n\omega_0)-\cancel{j\sin(d_1 n\omega_0)}\right)$$
$$\left.+2m_2\left(\overset{1}{\cancel{\cos(0)}}-j\overset{0}{\cancel{\sin(0)}}\right)\right]$$

$$C_n = \frac{-1}{T n^2 \omega_0^2}\left[2m_1\cos(d_2 n\omega_0) - 2(m_1+m_2)\cos(d_1 n\omega_0) + 2m_2\right] \quad n\in\mathbb{Z}\setminus\{0\}$$

$\therefore$ C_n es puramente real $\rightarrow$ $x(t)$ es par ✓

Por lo tanto:

$$\mathrm{Re}\{C_n\} = C_n \,,\; \mathrm{Im}\{C_n\} = 0 \rightarrow \measuredangle C_n = \begin{cases} 0 & \text{si } C_n > 0 \\ \pi & \text{si } C_n < 0 \end{cases}$$

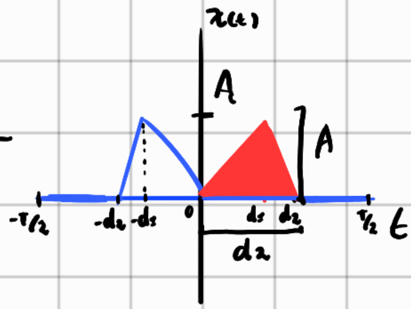
$$|C_n| = \left|\frac{-1}{T n^2 \omega_0^2}\left[2m_1\cos(d_2 n\omega_0) - 2(m_1+m_2)\cos(d_1 n\omega_0) + 2m_2\right]\right|$$

$$|C_n| = \frac{|-1|}{|T n^2 \omega_0^2|}\left|2m_1\cos(d_2 n\omega_0) - 2(m_1+m_2)\cos(d_1 n\omega_0) + 2m_2\right|$$

$$|C_n| = \frac{1}{T n^2 \omega_0^2}\left|2m_1\cos(d_2 n\omega_0) - 2(m_1+m_2)\cos(d_1 n\omega_0) + 2m_2\right|$$

Coeficiente C_0: Se seguirá calculando de manera usual.

$$C_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{T}\left(2\cdot\frac{A\cdot d_2}{2}\right) = \frac{A\cdot d_2}{T}$$



- A partir de $x(t)$:

$$x(t) = C_0 + \sum_n C_n e^{jn\omega_0 t} \longrightarrow C_n = \frac{1}{T}\int_T x(t)\cdot e^{-jn\omega_0 t}\,dt$$

Como $x(t)$ es par y real $\rightarrow$ $C_n = \frac{2}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt$

Evaluando:

$$C_n = \frac{2}{T}\left[\int_0^{d_1} m_2 t \cos(n\omega_0 t)\,dt + \int_{d_1}^{d_2} (-m_1 t + b_1)\cos(n\omega_0 t)\,dt + \int_0^{T/2} \cancelto{0}{0\cdot\cos(n\omega_0 t)dt}\right]$$

$$C_n = \frac{2}{T}\left[m_2\left(\frac{t\sin(n\omega_0 t)}{n\omega_0}\Big|_0^{d_1} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\Big|_0^{d_1}\right) - m_1\left(\frac{t\sin(n\omega_0 t)}{n\omega_0}\Big|_{d_1}^{d_2} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\Big|_{d_1}^{d_2}\right) + b_1\left(\frac{\sin(n\omega_0 t)}{n\omega_0}\Big|_{d_1}^{d_2}\right)\right]$$

$$C_n = \frac{2}{T}\left[m_2\left(\frac{d_1\sin(d_1 n\omega_0)}{n\omega_0} + \frac{\cos(d_1 n\omega_0)-1}{n^2\omega_0^2}\right) - m_1\left(\frac{d_2\sin(d_2 n\omega_0)}{n\omega_0} - \frac{d_1\sin(d_1 n\omega_0)}{n\omega_0} + \frac{\cos(d_2 n\omega_0)}{n^2\omega_0^2} - \frac{\cos(d_1 n\omega_0)}{n^2\omega_0^2}\right) + b_1\left(\frac{\sin(d_2 n\omega_0)}{n\omega_0} - \frac{\sin(d_1 n\omega_0)}{n\omega_0}\right)\right]$$

$$C_n = \frac{-1}{T n^2\omega_0^2}\left[2m_1\cos(d_2 n\omega_0) - 2(m_1+m_2)\cos(d_1 n\omega_0) + 2m_2\right] \quad n \in \mathbb{Z}\setminus\{0\}$$

↳ Misma expresión ✓

- Error relativo de la estimación:

Recordando:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot \phi_n(t) \rightarrow \hat{x}(t) = \sum_{n=-N}^{N} C_n \cdot \phi_n(t)$$

$$\overline{P_x} = \frac{1}{T}\int_T |x(t)|^2 dt \longrightarrow \overline{P_{\hat{x}}} = \frac{1}{T}\int_T |\hat{x}(t)|^2 dt$$

$$\overline{P_{\hat{x}}} = \frac{1}{T}\int_T \left|\sum C_n \cdot \phi_n(t)\right|^2 dt = \frac{1}{T}\int_T \sum |C_n \cdot \phi_n(t)|^2 dt$$

$$= \frac{1}{T}\int_T \sum |C_n|^2 |\phi_n(t)|^2 dt = \sum |C_n|^2 \underbrace{\frac{1}{T}\int_T |\phi_n(t)|^2 dt}_{P_n}$$

$$= \sum_{n=-N}^{N} |C_n|^2 \cdot P_n$$

$$e_r = \left(1 - \frac{\overline{P_{\hat{x}}}}{\overline{P_x}}\right) = \left(1 - \frac{\sum_{n=-N}^{N} |C_n|^2 \cdot P_n}{\overline{P_x}}\right)$$

$$e_r[\%] = \left(1 - \frac{\sum_{n=-N}^{N} |C_n|^2 \cdot P_n}{\overline{P_x}}\right) \cdot 100$$

Entonces:

$$P_n = \frac{1}{T}\int_T |e^{jnw_0 t}|^2 dt = \frac{1}{T}\int_T \cos^2(nw_0 t) + \operatorname{sen}^2(nw_0 t)\, dt$$

$$= \frac{1}{T}\int_T 1 \cdot dt = \frac{1}{T} T = 1 \quad \forall n$$

$$\overline{P_x} = \frac{1}{T}\int_T |x(t)|^2 dt \quad // \text{como } x(t) \geq 0 \ \forall t$$

$$\overline{P_x} = \frac{1}{T}\int_T (x(t))^2 dt$$

Como $x(t)$ es par $\longrightarrow (x(t))^2$ también lo es:

$$\overline{P_x} = \frac{2}{T}\int_0^{T/2} (x(t))^2 dt = \frac{2}{T}\left[\int_0^{d_1} (m_2 t)^2 dt + \int_{d_1}^{d_2} (-m_1 t + b_1)^2 dt\right]$$

$$\overline{P_x} = \frac{2}{T}\left[m_2^2 \cdot \frac{t^3}{3}\Big|_0^{d_1} + \int_{d_1}^{d_2} (b_1^2 - 2b_1 m_1 t + m_1^2 t^2) dt\right]$$

$$\overline{P_x} = \frac{2}{T}\left[\frac{m_2^2 \cdot d_1^3}{3} + b_1^2 \cdot t\Big|_{d_1}^{d_2} - 2b_1 m_1 \cdot \frac{t^2}{2}\Big|_{d_1}^{d_2} + m_1^2 \cdot \frac{t^3}{3}\Big|_{d_1}^{d_2}\right]$$

$$\overline{P_x} = \frac{2}{T}\left[\frac{m_2^2 d_1^3}{3} + b_1^2 (d_2 - d_1) - 2b_1 m_1 \left(\frac{d_2^2}{2} - \frac{d_1^2}{2}\right) + m_1^2 \left(\frac{d_2^3}{3} - \frac{d_1^3}{3}\right)\right]$$

$$\overline{P_x} = \frac{2}{T}\left[\frac{A^2 d_1}{3} + \frac{A^2 (d_2 - d_1)}{3}\right] = \frac{2}{T}\left[\frac{A^2 d_2}{3}\right]$$

$$\overline{P_x} = \frac{2A^2 d_2}{3T}$$

$$\therefore\ e_r[\%] = \left(1 - \frac{\sum_{n=-N}^{N} |C_n|^2 \cdot 1}{\frac{2A^2 d_2}{3T}}\right) \cdot 100$$

$$e_r[\%] = \left(1 - \frac{3T \sum_{n=-N}^{N} |C_n|^2}{2A^2 d_2}\right) \cdot 100$$

Para $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

$$\boxed{e_r[\%] = \left(1 - \frac{3T \sum_{n=-5}^{5} |C_n|^2}{2A^2 d_2}\right) \cdot 100}$$