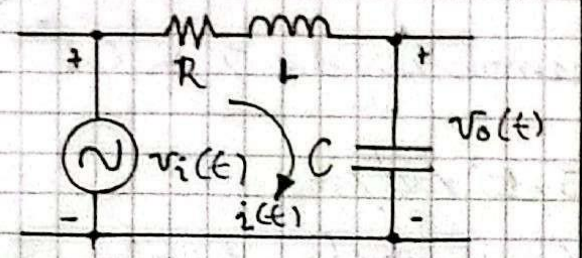


KVL: $v_i(t) = v_R(t) + v_L(t) + v_C(t)$

$$v_i(t) = RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o(t)$$

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o(t) = \frac{1}{LC}v_i(t)$$

Con $R = 1K\Omega$; $L = 180mH$; $C = 120\mu F$:

$$\frac{d^2v_o}{dt^2} + 5555{,}56\frac{dv_o}{dt} + 46296{,}30\,v_o(t) = 46296{,}30\,v_i(t) \quad // \text{ E.D.O.}$$

Suponiendo condiciones iniciales = 0:

$V_R(s) = R\,I(s)$; $V_L(s) = sL\,I(s)$; $V_C(s) = V_o(s) = \frac{1}{sC}I(s)$

∴ La impedancia total (por conexión en serie) es:

$Z(s) = R + sL + \frac{1}{sC}$; Entonces la corriente de malla será:

$$I(s) = \frac{V_i(s)}{Z(s)} \Rightarrow V_o(s) = \frac{1}{sC}\left[V_i(s)/\left(R + sL + \frac{1}{sC}\right)\right]$$

$$\therefore H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\cancel{\frac{1}{sC}}}{R + sL + \cancel{\frac{1}{sC}}} \cdot \frac{\cancel{sC}}{sC} = \frac{1}{LCs^2 + RCs + 1}$$

$$H(s) = \frac{1}{2{,}16\times10^{-5}s^2 + 0{,}12s + 1}$$

Ahora, la respuesta en frecuencia del sistema se da en el dominio de Fourier, simplemente:

$$H(\underset{\uparrow\ \sigma=0}{s=j\omega}) = H(j\omega) = \frac{1}{LC(j\omega)^2 + RC(j\omega) + 1} = \frac{1}{1 - LC\omega^2 + jRC\omega}$$

Así que $|H(j\omega)| = \left|\frac{1}{1 - LC\omega^2 + jRC\omega}\right| = \frac{|1|}{|1 - LC\omega^2 + jRC\omega|}$

$$|H(j\omega)| = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (RC\omega)^2}}$$

* Polos y ceros de $H(s)$:

Ceros: $1 \neq 0 \longrightarrow$ No hay ceros finitos

Polos: $LCs^2 + RCs + 1 = 0 \longrightarrow$ Hay dos polos.

$$s_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}$$; Reemplazando R, L, C:

$s_1 \approx -8{,}35$ rad/seg ; $s_2 \approx -5{,}55 \times 10^3$ rad/seg

tanto s_1 como s_2 están a la izquierda del plano S, por lo tanto el sistema es estable.

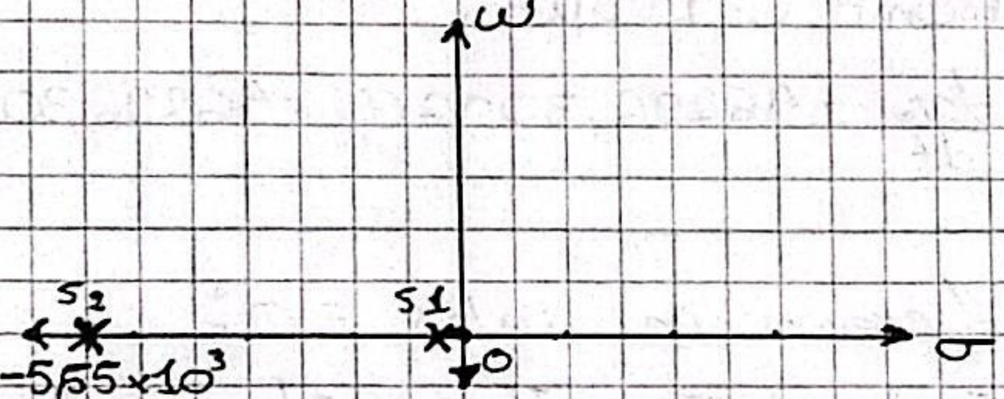


*Diagrama de Bode: $B(f) := 20\log|H(jf \cdot 2\pi)|$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-LC\omega^2)^2 + (RC\omega)^2}} \rightarrow 20\log|H(j\omega)| = 20\log\left(\frac{1}{\sqrt{(1-LC\omega^2)^2 \ldots}}\right)$$

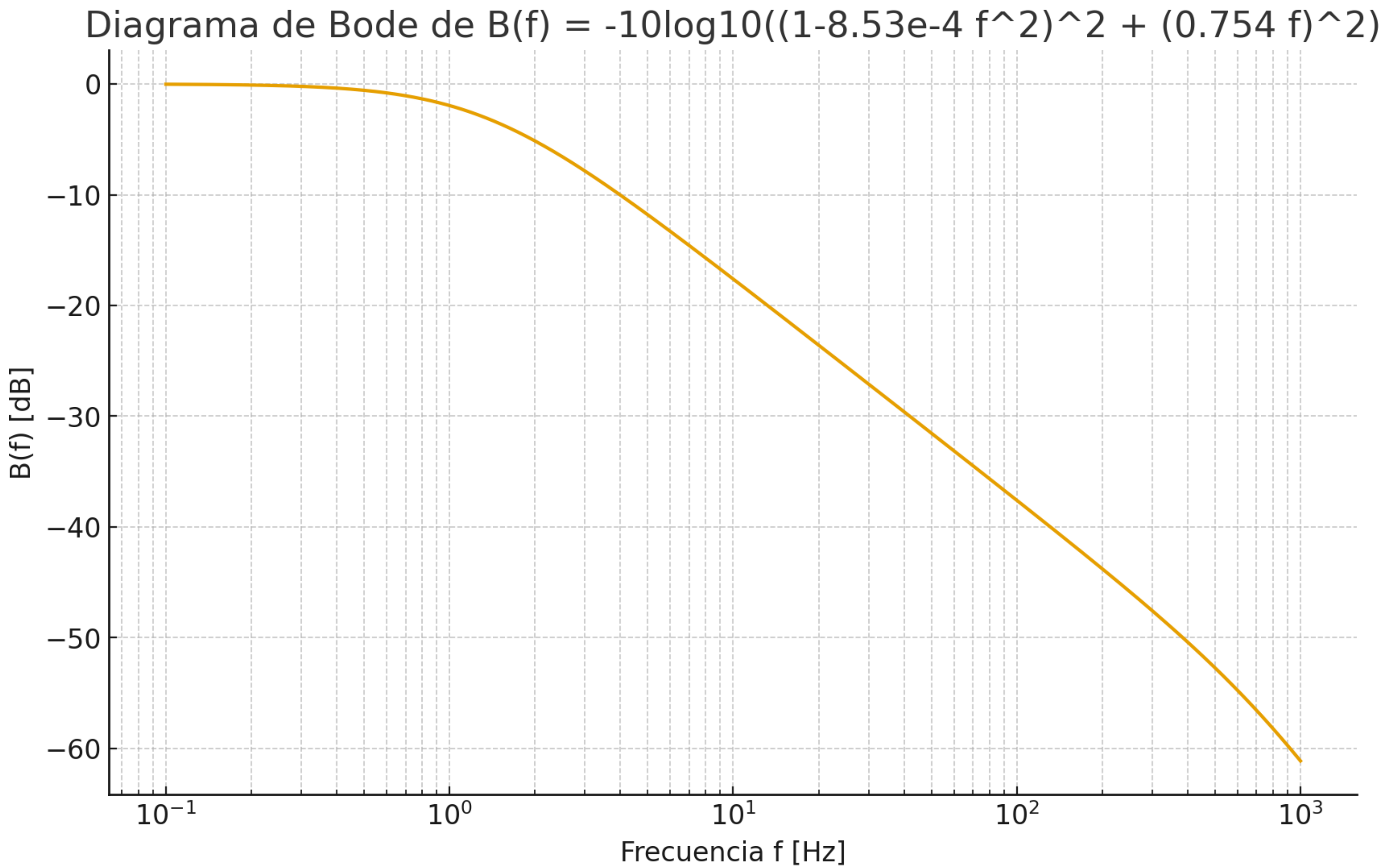
$$= \cancel{20\log(1)}^{\,0} - 20\log\left(\sqrt{(1-LC\omega^2)^2 + (RC\omega)^2}\right)$$

$$= -10\log\left((1-LC\omega^2)^2 + (RC\omega)^2\right)$$

Entonces:

$$B(f) = -10\log\left(\left(1-LC(2\pi f)^2\right)^2 + \left(RC(2\pi f)\right)^2\right)$$

$$B(f) = -10\log\left(\left(1-8{,}53\times10^{-4} f^2\right)^2 + \left(0{,}754 f\right)^2\right)$$



* Respuesta al impulso $h(t)$:

$$H(s) = \frac{1}{LC} \cdot \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$ / Parámetros estándar $\omega_0 = \frac{1}{\sqrt{LC}}$; $2\zeta\omega_0 = \frac{R}{L}$

$\therefore \zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \rightarrow H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \rightarrow$ Sistema de 2° orden estándar.

Ahora: $H(s) = \mathcal{L}\{h(t)\} \longrightarrow h(t) = \mathcal{L}^{-1}\{H(s)\}$

Escribiendo $H(s)$ con sus polos factorizados:

$H(s) = \frac{1}{LC(s-s_1)(s-s_2)}$; Por medio de fracciones parciales.

$H(s) = \frac{A}{s-s_1} + \frac{B}{s-s_2} \rightarrow \frac{A(s-s_2) + B(s-s_1)}{(s-s_1)(s-s_2)}$

Entonces: $A(s-s_2) + B(s-s_1) = \frac{1}{LC}$

$\therefore A = \frac{1}{LC(s_1-s_2)}$; $B = -\frac{1}{LC(s_1-s_2)}$

$\therefore H(s) = \frac{1}{LC(s_1-s_2)}\left(\frac{1}{s-s_1} - \frac{1}{s-s_2}\right)$

Entonces:

$$\mathcal{L}^{-1}\{H(s)\} = h(t) = \frac{1}{(s_1-s_2)LC}\mathcal{L}^{-1}\left\{\frac{1}{s-s_1} - \frac{1}{s-s_2}\right\}$$

Siguiendo la forma estándar $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at} \cdot u(t)$:

$$h(t) = \frac{1}{LC(s_1-s_2)}\left(e^{s_1 t} - e^{s_2 t}\right) \cdot \quad ; \quad t \geq 0.$$

Evaluando: $h(t) \approx 8{,}36\left(e^{-8{,}35t} - e^{-5{,}59 \times 10^3 t}\right) \cdot \quad t \geq 0.$

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* Respuesta al escalón unitario $(x(t) = u(t))$:

$\mathcal{L}\{u(t)\} = \frac{1}{s} = U(s)$

Como el sistema es SLIT: $Y(s) = H(s)U(s) = \frac{H(s)}{s}$ ← Porque $y(t) = u(t) * h(t)$

$Y(s) = \frac{1}{s(LC(s-s_1)(s-s_2))}$, Ahora ;

$Y = \frac{A}{s} + \frac{B}{s-s_1} + \frac{C}{s-s_2} \longrightarrow A = 1$; $B = \frac{s_2}{s_1-s_2}$; $C = \frac{s_1}{s_2-s_1}$

$$Y(s) = \frac{1}{s} + \frac{s_2}{s_1 - s_2} \cdot \frac{1}{s - s_1} + \frac{s_1}{s_2 - s_1} \cdot \frac{1}{s - s_2}$$

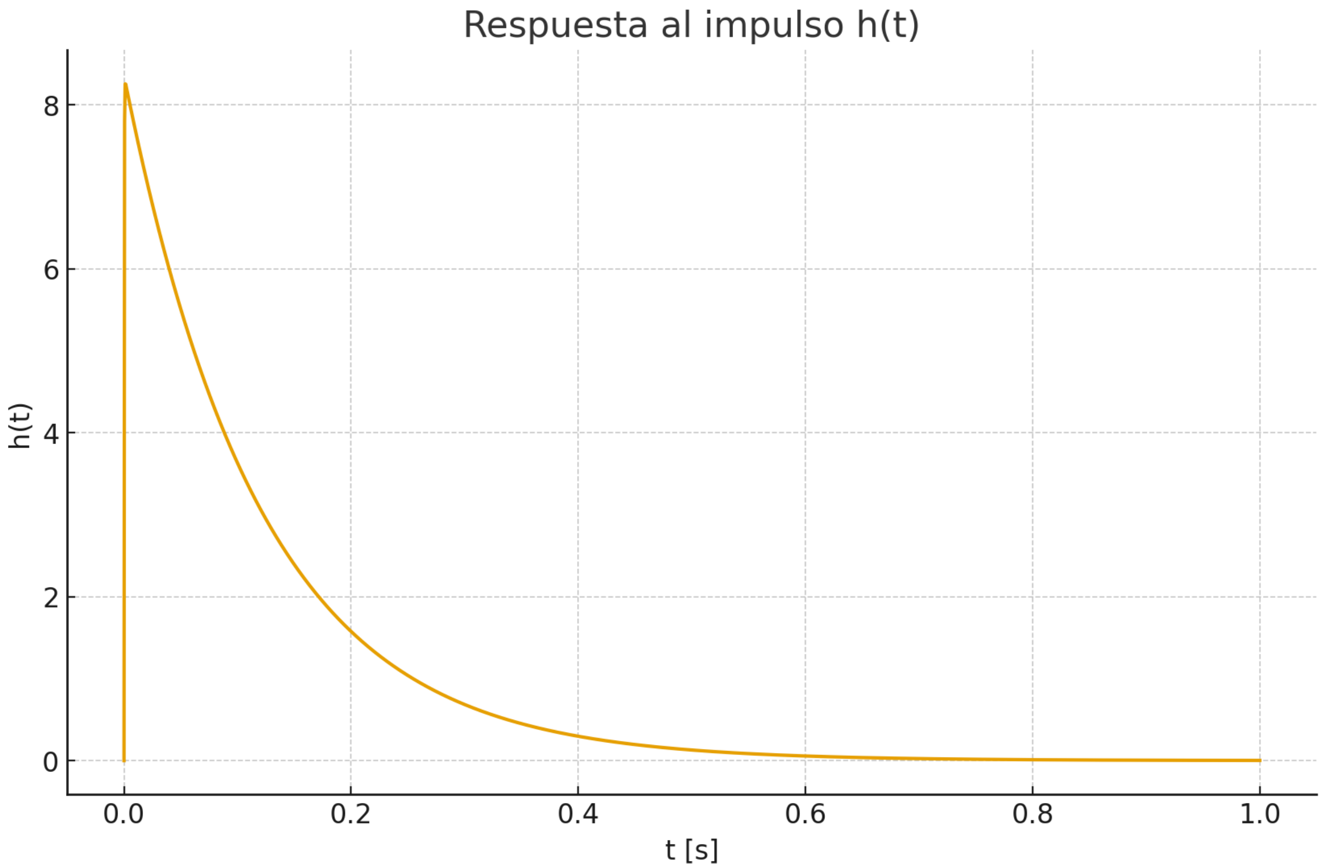
Entonces:

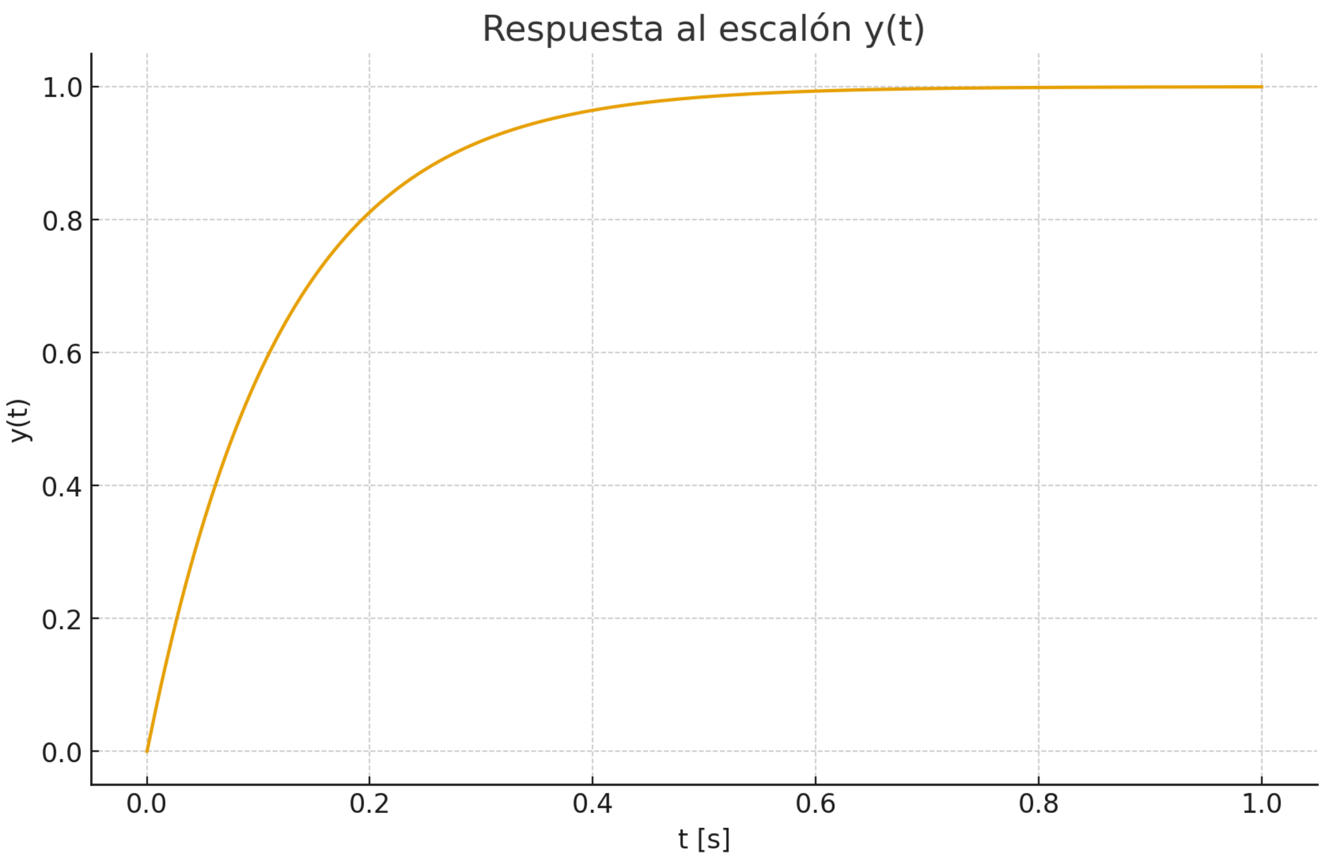
$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{s_2}{s_1 - s_2} \mathcal{L}^{-1}\left\{\frac{1}{s - s_1}\right\} + \frac{s_1}{s_2 - s_1} \mathcal{L}^{-1}\left\{\frac{1}{s - s_2}\right\}$$

$$y(t) = 1 + \frac{s_2}{s_1 - s_2} e^{s_1 t} + \frac{s_1}{s_2 - s_1} e^{s_2 t} \; ; \; t \geq 0.$$

Evaluando:

$$y(t) \approx 1 + 1{,}0015 e^{-8{,}35 t} + 0{,}0015 e^{-555 \times 10^{3} t} \; ; \; t \geq 0.$$


Respuesta al impulso h(t)
h(t)
t [s]
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0


Respuesta al escalón y(t)
y(t)
t [s]
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0