

Logical Qubits and Quantum Error Correction

Team: The Ducklings

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Error Correction: A Primer

The Classical Picture

- Bits can flip due to many reasons during transmission
 - Electromagnetic interference (EMI)
 - Hardware aging
 - Cosmic rays(!)
- Leads to incorrect data being received

The Quantum Picture

- Qubits are also susceptible to errors, but...
 - More kinds of errors (bit or phase flip)
 - Extremely sensitive (more so than classical bits)
- Qubits can decohere due to any source of environmental noise, and maybe even spontaneously

Working Around Noise

WORK

$$E_{CZ} \gtrsim E_{mover} \sim E_{unpaired} > E_{sitter} \sim E_{1-qubit,local} > E_{1-qubit,global}$$

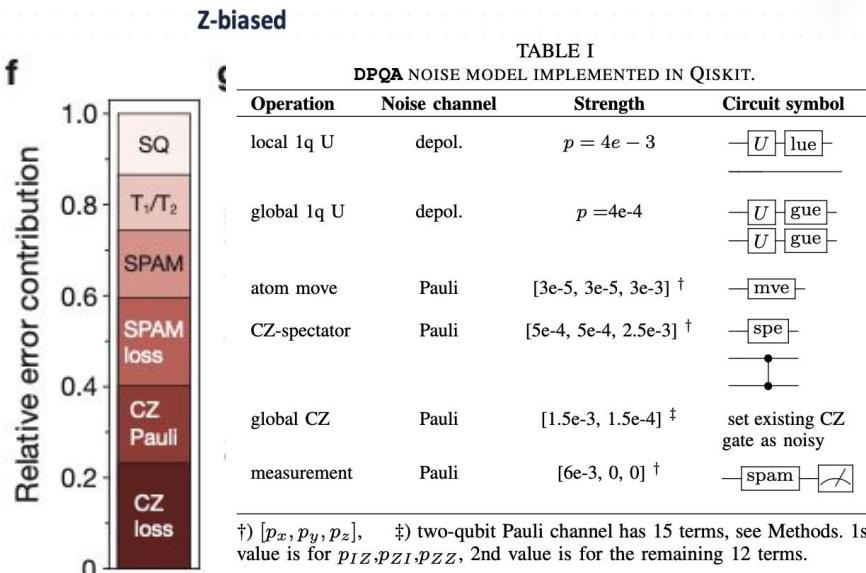


Table I summarizes the noise channels for the DPQA con-

```
github.com/QuEraComputing/bloqade-circuit/blob/14fdf7aa9f0aafe2476b23
bloqade-circuit / src / bloqade / qasm2 / dialects / noise / model.py
Code Blame 278 lines (236 loc) - 12.6 KB
class MoveNoiseModel(ABC(abc.ABC):
    ...
    local_px: float = field(default=4.10e-04, kw_only=True)
    """The error probability for a Pauli-X error during a local single qu...
    local_py: float = field(default=4.10e-04, kw_only=True)
    """The error probability for a Pauli-Y error during a local single qu...
    local_pz: float = field(default=4.112e-04, kw_only=True)
    """The error probability for a Pauli-Z error during a local single qu...
    local_loss_prob: float = field(default=0.0, kw_only=True)
    """The error probability for a loss during a local single qubit gate
    ...
    local_unaddressed_px: float = field(default=2.000e-07, kw_only=True)
    """The error probability for a Pauli-X error during a local single qu...
    local_unaddressed_py: float = field(default=2.000e-07, kw_only=True)
    """The error probability for a Pauli-Y error during a local single qu...
    local_unaddressed_pz: float = field(default=1.000e-06, kw_only=True)
    """The error probability for a Pauli-Z error during a local single qu...
    local_unaddressed_loss_prob: float = field(default=0.0, kw_only=True)
    """The error probability for a loss during a local single qubit gate
    ...
    global_px: float = field(default=6.500e-05, kw_only=True)
    """The error probability for a Pauli-X error during a global single q...
    global_py: float = field(default=6.500e-05, kw_only=True)
    """The error probability for a Pauli-Y error during a global single q...
    global_pz: float = field(default=6.500e-05, kw_only=True)
    """The error probability for a Pauli-Z error during a global single q...
    global_loss_prob: float = field(default=0.0, kw_only=True)
    """The error probability for a loss during a global single qubit gate
    ...
    cz_paired_gate_px: float = field(default=1.59e-04, kw_only=True)
    """The error probability for a Pauli-X error during CZ gate operation
    cz_paired_gate_py: float = field(default=1.59e-04, kw_only=True)
    """The error probability for a Pauli-Y error during CZ gate operation
    cz_paired_gate_pz: float = field(default=3.184e-03, kw_only=True)
    """The error probability for a Pauli-Z error during CZ gate operation
    cz_unpaired_loss_prob: float = field(default=0.0, kw_only=True)
    """The error probability for a loss during CZ gate operation when no...
    ...
    cz_unpaired_gate_px: float = field(default=5.149e-04, kw_only=True)
    """The error probability for Pauli-X error during CZ gate operation w...
    cz_unpaired_gate_py: float = field(default=5.149e-04, kw_only=True)
    """The error probability for Pauli-Y error during CZ gate operation w...
    cz_unpaired_gate_pz: float = field(default=2.185e-03, kw_only=True)
    """The error probability for Pauli-Z error during CZ gate operation w...
    cz_unpaired_loss_prob: float = field(default=0.0, kw_only=True)
    """The error probability for a loss during CZ gate operation when and
```

WORK

Solutions to Quantum Decoherence

Increase Coherence Time

- Pros:
 - Reduces need for physical qubits
 - Can work with physical qubits more directly
- Cons:
 - Extremely difficult
 - Difficulty scales quickly with number of qubits
 - Extreme conditions need to be maintained

Quantum Error Correction

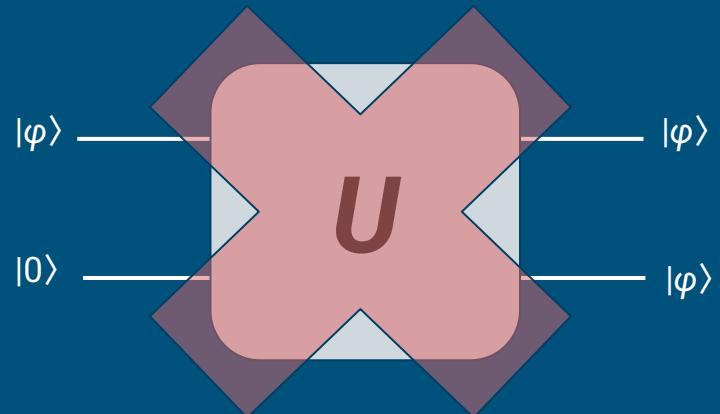
- Pros:
 - Much easier than increasing coherence time
 - Creates dependable logical qubits
- Cons:
 - Requires more physical qubits as overhead
 - Still pretty difficult

Quantum Complications

You can just copy classical bits...



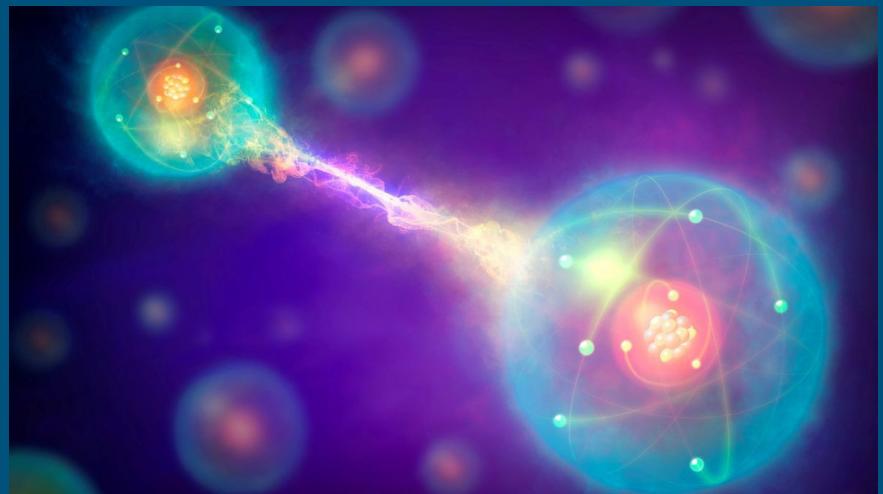
...but you can't copy qubits!



No-Cloning Theorem

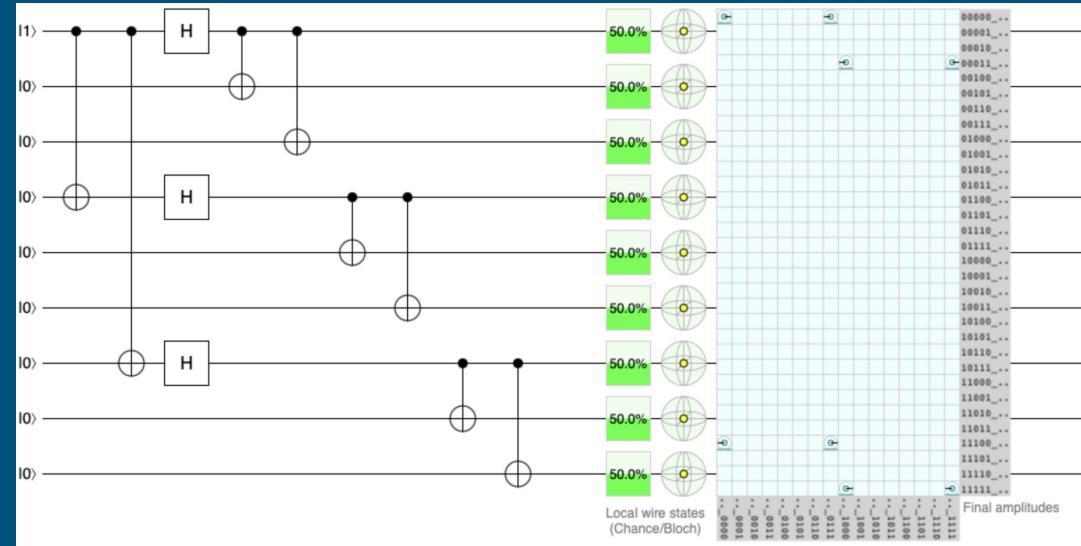
Now What?

Idea:
Use entanglement.



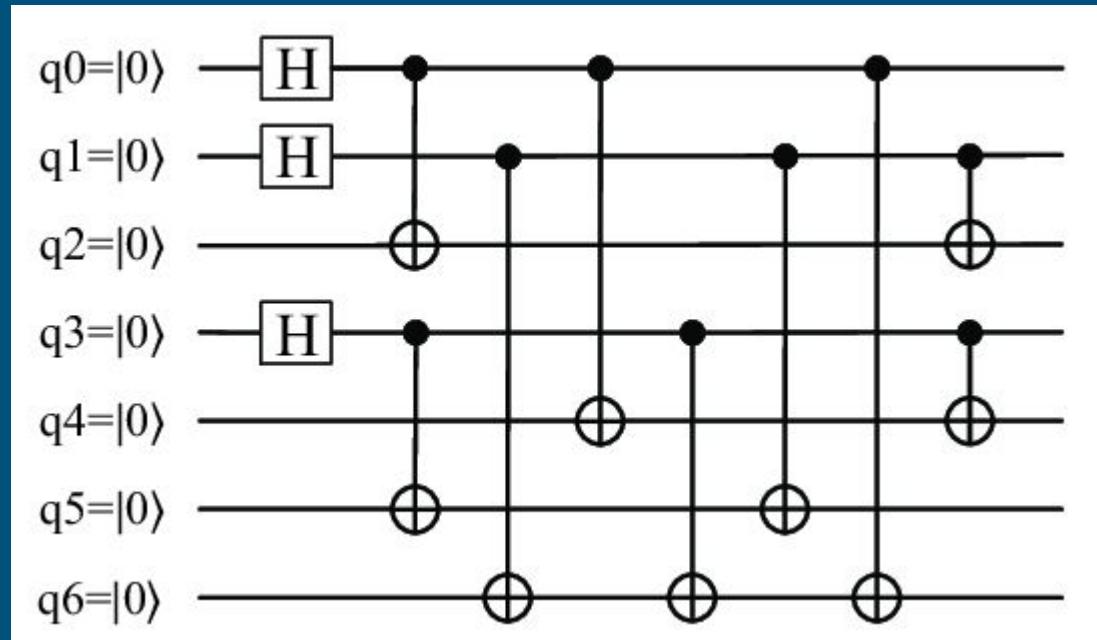
A Simple Example

- The Shor code was the first quantum error-correcting code.
- Uses nine physical qubits to encode one logical qubit
- Can correct both bit-flip errors and phase-flip errors
 - Sufficient to correct arbitrary bit errors



Can We Do Better?

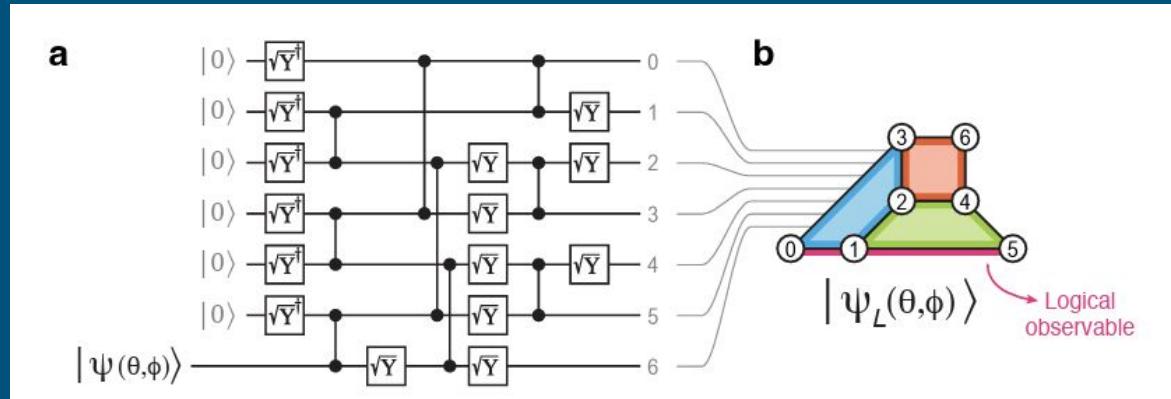
- The Steane code is also able to encode one logical qubit and fix arbitrary errors, but with only *seven* physical qubits instead of nine!
- Constructed from [7, 4, 3] classical Hamming code
- Considered a “color code”



Can We Do Better...Better?

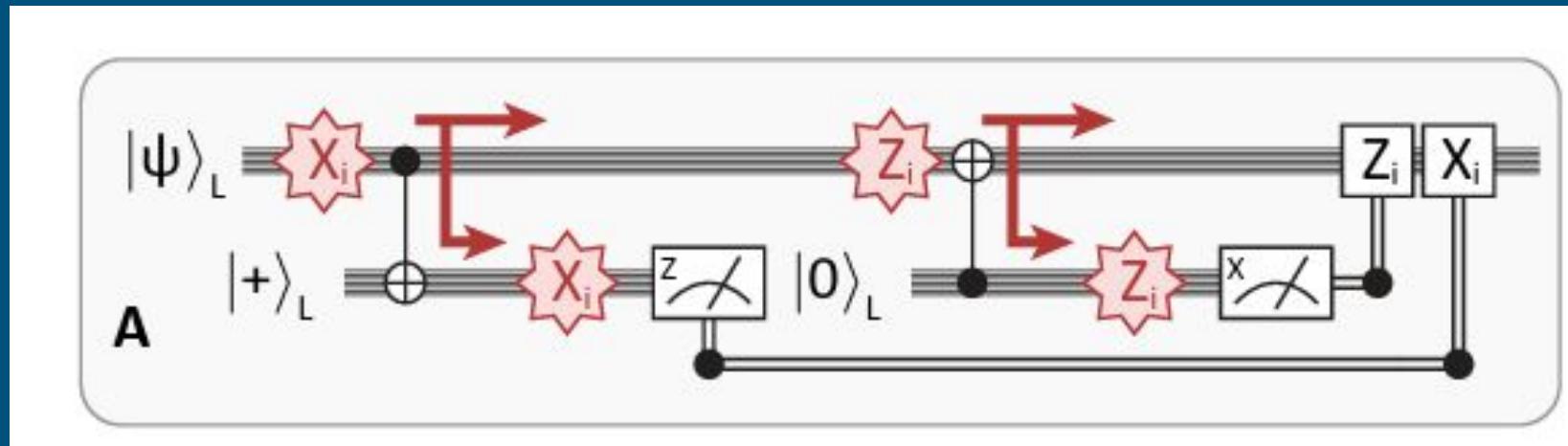
- The textbook Steane code circuit is kind of lengthy
- Bigger circuits are more expensive and provide more opportunity for decoherence
- Can we make a smaller circuit that implements the Steane code?

Here's one idea...

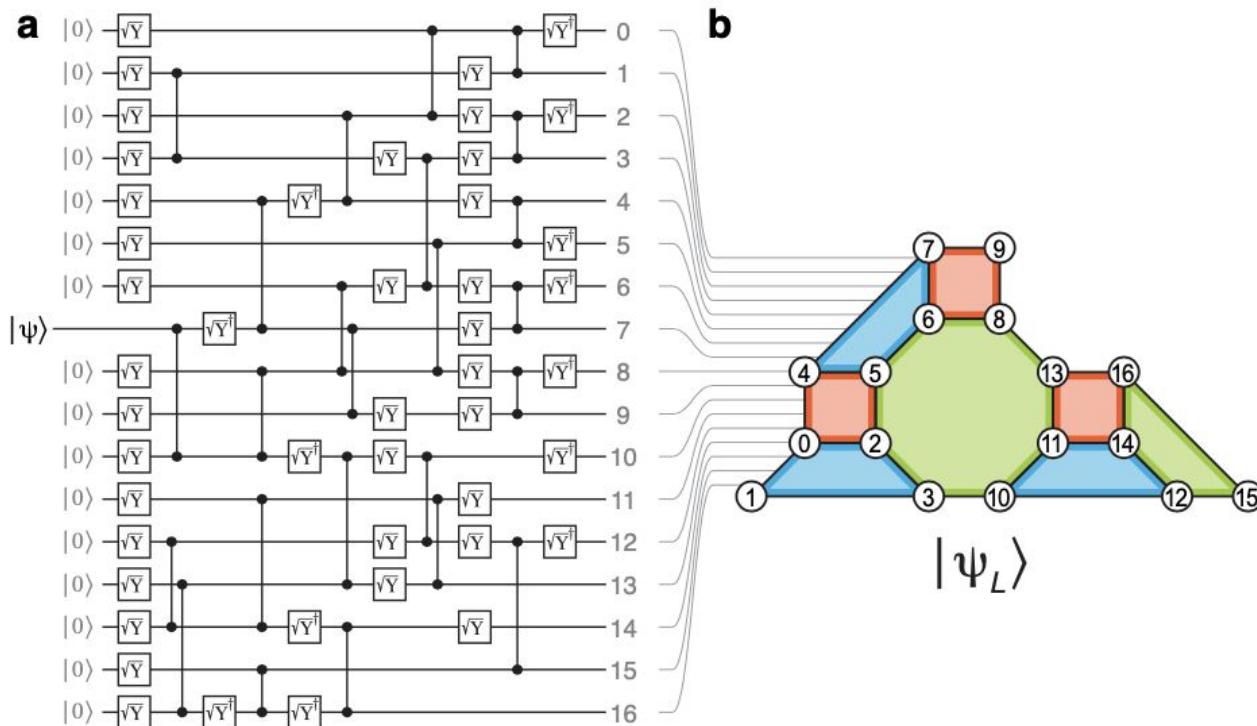


...How Do We Know?

- The circuits previously discussed allow us to encode seven physical qubits into one logical qubit
- We need another circuit leveraging properties of our encoding to actually detect and correct errors
- Luckily, Steane helps us here too



Extending to distance 5 -> [17, 1, 5]



Magic State Distillation -> Tsim

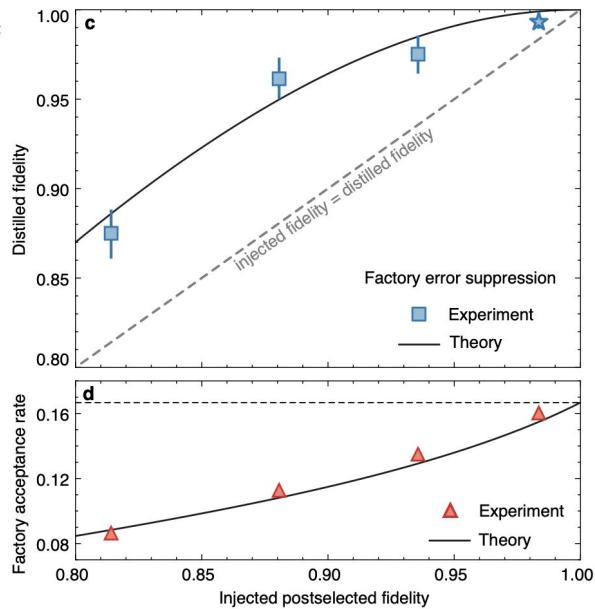
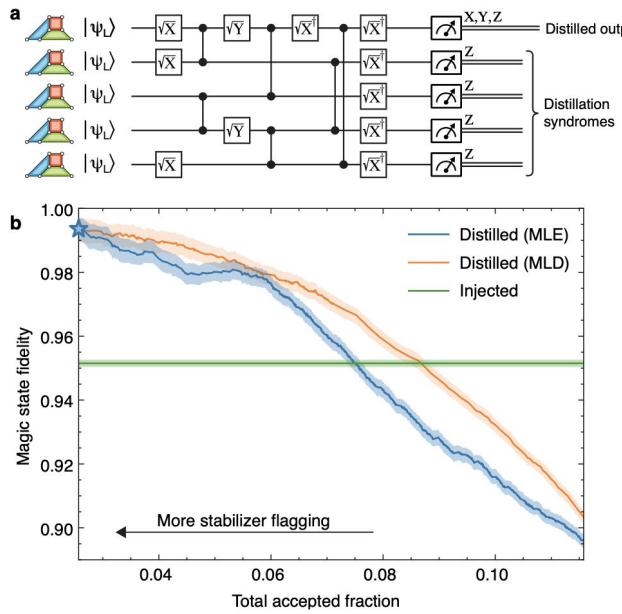


Figure 3. 5-to-1 magic state distillation. **a**, Magic state distillation circuit based on the $[[5,1,3]]$ code (distillation code).

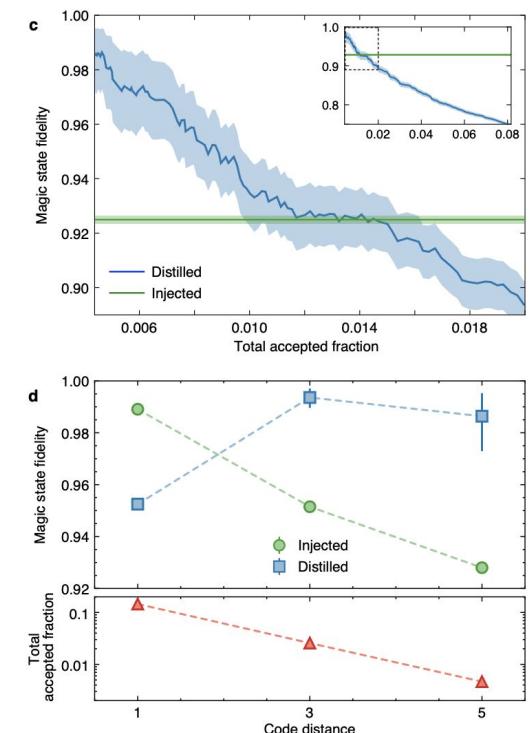


Figure 4. Scaling of distillation as a function of data code distance. **a**, $d=5$ encoding circuit. **b**, $d=5$ color code stabilizers. **c**, Output magic state fidelity for $d=5$ distillation (blue) as a function of the total accepted fraction, again showing improvement over the input magic state fidelity (green). Extended range shown in inset. **d**, Injected (green circles) and distilled (blue squares) magic state fidelity with total acceptance fraction (red), when performing full stabilizer and factory postselection, all as a function of code distance.

Did We Do Better?

- To test the fault-tolerance of our setup, we injected random gate flips (X, Y, Z) into our system and tried to correct them
- We can simulate introducing noise to our system using PyQrack and Stim
- Tried three different error rates: 5%, 25%, 60%; 200 shots per error rate
- Our error rate dropped from 20% to 2% with Steane QEC, just by guessing the error to be the most likely flip
- Without QEC, significant portion of runs would be wasted (scaling with error probability), but QEC allows to salvage most of them

