Electrostatics

Lab Report

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Section: H5

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Purpose

In this experiment we will investigate the properties of the electrostatic force. First we will verify the exponent in the inverse square law. Second we will show that the force is proportional to the product of the two charges.

Apparatus

The apparatus consists of two metal spheres supported by non-conducting rods. The bottom sphere rests on a scale, and the top sphere is supported near a meter stick, and can be moved vertically. We will charge the spheres by triboelectrification; i.e., by rubbing plastic rods with cloth to charge them, then transferring the charge to the metal spheres. Rubbing the plastic rod with the rabbit fur charges the rod because of the different electro-negativity of the two materials: one chemically takes electrons from the other.

Readings

Electric Charges & Coulomb's Law, A Short Review

By observing the attractive and repulsive forces between certain objects, we could conclude that there is not one, but two types of charge that can explain all the interactions. If you charge two same objects the same way, they repel, which means like charges repel.

By introducing the law of conservation of charge later on, we started calling one charge positive and the other negative. Namely we understood that charge exchanges between the objects, when certain objects touch, like fur and a piece of acrylic. The phenomena that you can charge certain materials by contact (like fur and acrylic) is called triboelectric effect.

Coulomb's law states that the interaction force between two point charges Q_1 and Q_2 is given by, (1) $F = k \frac{Q_1 Q_2}{r^2}$,

where $k = 8.99 \times 10^9 \,\mathrm{Nm^2}/C^2$ is a constant, the direction of the force F is along the line between the charges. Negative force means attractive force and vice versa. Meaning that the like charges repel and opposites attract. r is the distance between the point charges. A point charge approximation is valid if the size of the

charge carrying object is much smaller than r.

Capacitors, A Short Review

A capacitor is a device made of usually two conductors. The point of making a capacitor is to hold charge. The capacitance of a capacitor is defined by the amount of charge it holds when 1V voltage is applied to it,

(2) C = Q/V.

The unit for the capacitance is Farad denoted by F. A large capacitor in an electric circuit can be about μ F or 10^{-6} F.

The capacitance depends on capacitor's geometrical shape and the dielectric material used inside it.

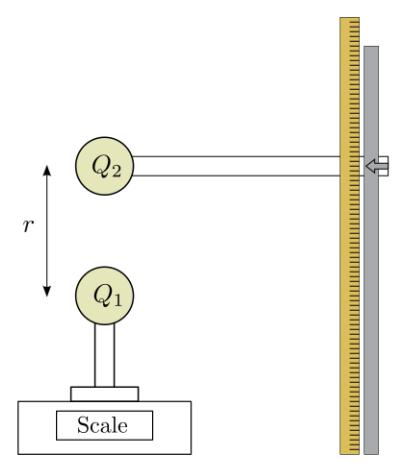
Intuitively speaking, capacitance shows how much room a single charge has when sitting on the capacitor. Lower capacitance means less room which is less comfortable. Remember that the like charges repel. A conducting sphere can be thought of as a capacitor which has one conductor which is sphere and another conductor which is at infinity. In any case, the capacitance of the spheres we will use in this lab are pretty small, about $R/k = 2.8 \, \text{pF}$. This means the charges are really packed on top of it and are quite unhappy. Now if you connect this sphere to a large $10 \, \mu\text{F}$ capacitor, almost all the charges will leave the sphere and sit on the larger capacitor. This would allow you to measure the charge on the sphere. You simply need to measure the voltage and use,

 $Q = C V = (10 \ \mu\text{F}) V.$ Remember that $(1\text{F}) \times (1\text{V}) = 1 \text{ C}.$

Procedure

The setup will be more or less like the one shown in fig. 1. You will use the scale to measure the electrostatic force. When trying to charge the spheres or when doing the experiment, your hands and any other conductor must be far apart from the apparatus (far apart means at least couple of feet). Otherwise the charge on the spheres will leak too quickly and you will not be able to do the measurements. Also try to use plastic gloves if provided, so that the moist in your hand does not transfer to the fur. The fur, rod, and the spheres must stay as dry as possible. You can use the dryer in the restroom to dry the fur if needed.

Figure 1. The setup of the experiment.



In today's experiment we will use two rather small conducting spheres as point charges. We will check to see how the force F depends on the distance and the charges. Optionally, using another fit equation, we will study the dipole effect. And finally we will try to estimate the constant k by measuring charges using a 10 μ F capacitor.

Dialog:

Part I. Electrostatic Force & Distance

The goal of this part is to quantitatively study the dependence of electrostatic force to the distance. We want to see if we can measure $1/r^2$ part of the Coulomb's law using our setup.

Step 1. Measurements

- Turn on the scale and wait without touching it so it starts operating. Place the conducting sphere with the stand on the scale. Now support the other sphere from its other end and arrange the spheres as shown in fig. 1. Look directly from above and check if the spheres are vertically aligned.
- Charge the rod with the fur by rubbing them together. Use your fingers to hold the fur, not your palm so that the fur stays dry. When rubbing the rod with the fur you must easily hear cracking sounds when you hold it near your ear. If you are not hearing the cracking sound, you might need to dry the fur using a dryer or get a dry fur.
- Gently touch both spheres so that they are discharged. Now zero the scale. Scale must read zero when there

is no electrostatic force.

- Charge both spheres by touching them with rod. Use all the area on the rod to touch the spheres (rod is an insulator). Do few test runs to see how far, i.e. for which r values, you can have meaningful reading on the scale (at least about 0.05 grams). Decide on your range for r and properly pick the r values (at least five data points). Do not take any r < 3R. R is the radius of the spheres.

Question 1.

- Why when you touch the spheres they become discharged?
- Charge only one sphere and put the spheres at the distance r = 3 R. What do you observe? Explain.
- Why, in this experiment, we must not take *r* values that are really small and comparable to *R*?

Answer 1.

- Since humans are conductors, charges are free to move through them. When we touch a charged object, the charge moves through us and disperses into the ground. Or if it is positively charged, electrons move through us and into the object.
- When you charged only one sphere, you would get a negative weight reading on the scale. That is because the charged sphere induced a polarization in the other sphere. Since the opposite charges would be closer together, they would be attracted towards each other, attracting the mass on the scale away from the scale, hence reducing the reading.
- At small distances, you may not be able to approximate the forces between the spheres as forces between point charges. This is because at small distances, you may have significant polarization within the spheres such that the spherical symmetry is broken. You may also have significant boundary conditions for the imperfections in the geometry of the spheres. At larger distances, these imperfections in the geometry and distribution become less apparent and neglect-able.
- Now, we start taking the data. Gently touch both spheres. Zero the scale. Charge the spheres as much as you can, by repeatedly rubbing the rod on them multiple times. For the lowest *r* your reading must be well above 0.30 grams on the scale. While doing the measurements you must stay as far as possible from the setup to minimize charge leakage.
- Read your scale measurements for different r. The whole process must not take more than 15 seconds. Try to use less than 3 seconds per data point.
- Record your data on the table below. ΔM is the error in reading the scale.

Suggestion:

- You can use this idea to estimate the error ΔM . Start from the lowest r, move up and record your first readings, now move down and record your second readings on the same r values. This way you can estimate the error of the scale reading, ΔM , more reliably.

Step 2. Analysis

R = 2.5; (*cm*)

- Now we analyze the data. On the code block below change the units and define Fr variable to represent the electrostatic force readings in Newtons.
- Make a nonlinear fit of $F = a r^b$, where a and b are the fitting parameters. Report the values for $a \pm \Delta a$ and $b \pm \Delta b$.
- Plot the data with error-bars and on top of the it, plot the fitting curve.
- Explain the results and what parameters *a* and *b* are representing and if they match our theoretical understanding.
- Explain the main sources of error.

Answer:

- The non Linear Fit for $F = a r^b$ fits the reading of the extra weight (force) that is applied on the charged conducting sphere due to the other charged sphere. Therefore, b is the order of the distance that F is proportional to and a is the constant of proportionality

We know $F = kq1q2/r^2$ by coulomb's law. Hence, we expect b = -2 and a = kq1q2 (k^* product of charges on spheres)

- $a \pm \Delta a = 0.0000316916 \pm 3.12049 \times 10^{-6}$ and $b \pm \Delta b 2.08285 \pm 0.046164$ which seem to be really close to what we expect
- The minor error may be explained by the fact that since there was a little bit of charge lost during the entire process, there were differences in the charge in the 2 runs that we conducted this experiment to find ΔM . Also, close to small enough radial distances, polarization and other effects can also cause error.

```
(* This is an Input cell in case you need *)
         (* Defining Variables *)
         Fr = m*9.8*10^-3; (* Defines Fr in terms of the list
          variable m defined above. Calculating forces in terms of Newtons. *)
         dFr = dm * 9.8 * 10^-3;
         (* Calculates list of \Delta F_i for the purpose of showing errorbars on the plot. *)
         Clear[x, a, b]
         (* Power Law Fit *)
         Fvr = Table[{r[i], Fr[i]}, {i, 1, Length[r]}];
         (* Makes a list of (r_i, F_i) pairs and saves it in Fvr variable. *)
         line = NonlinearModelFit[Fvr, a (x^b), {a, b}, x]
         (* Does a nonlinear power law fitting to the data. *)
         line["ParameterConfidenceIntervalTable"]
         (* Shows the fitting parameters on a table with their errors. *)
         (* Plotting *)
         FvrErr = Table[{{r[i], Around[Fr[i]], dFr[i]]}}, {i, 1, Length[r]}];
         (* Makes a list of triples (r_i,F_i,\Delta F_i) and saves it in FvrErr variable. *)
         Show [
          \label{eq:listPlot} ListPlot[\texttt{FvrErr}, \ \texttt{PlotStyle} \rightarrow \texttt{Red}, \ \texttt{PlotRange} \rightarrow \texttt{Full}], \ (* \ \texttt{Plots} \ \texttt{FvrErr} \ \texttt{variable}, \\
          which includes the vertical axis data errorbars on the third column. *)
          Plot[line[x], {x, 0, Max[r]}],
           (* Plots the fitting line for r from 0 to the maximum of r based on data. *)
          AxesLabel \rightarrow {"r (m)", "F (N)"}]
Out[0]=
                        0.0000317
        FittedModel
Out[ = ] =
          Estimate
                    Standard Error Confidence Interval
        a 0.0000316916 \ 3.12049 \times 10^{-6} \ \{0.0000230277, 0.0000403555\}
       b -2.08285
                    0.046164
                             {-2.21102, -1.95467}
Out[ ]=
        0.004
        0.003
        0.002
        0.001
                                                                – r (m)
                     0.12
                                0.14
                                           0.16
                                                      0.18
         b = -2.08285;
```

Step 3. Further Analysis (Optional)

The small spheres are not point charges. There are certain ways one can theoretically study the force between two conducting spheres. A well-known method is the method of image charges. But usually one ends up with

infinite number of terms to add up. Take R to be the radius of the sphere. Considering R/r to be small, the first non-point-charge term comes from the dipole or polarization effect. This can cause attractive force when one of the spheres is not charged.

Considering both spheres are charged with more or less the same charge, the correction term is proportional to $1/r^5$, so one can try to do a nonlinear fit like,

(3)
$$F = k \frac{Q_1 Q_2}{r^2} \left(1 - \alpha \left(\frac{R}{r} \right)^3 \right)$$
,

where α and similarly $a = k Q_1 Q_2$ are the fitting parameters.

- Do a nonlinear fit for F based on the above equation. Report the values of $a \pm \Delta a$ and $\alpha \pm \Delta a$.
- Compare this fit to the point charge Coulomb's law fit on step 2. Write down your conclusion.

Answer:

```
"a = "0.0000362267" \pm "7.82711\times10<sup>-7</sup> "alpha = "-0.0407712" \pm "0.0198361 for the fit F = a/r^2 * (1-alpha*(R/r)^3)
```

Comparing this to the fit for approximating the charge as a point charge in step 2: For Step two, the proportionality constant for $F=a/r^2$ was $a \pm \Delta a = 0.0000316916 \pm 3.12049 \times 10^{-6}$

Which is rather close the the estimate we got in step 3. (there was a 13% error).

Since Taylor expansion is the best polynomial approximation for any polynomial function of the same order, we know that the actual value of a must be different from the value in step 2. Hence, we can conclude that our spheres can not actually be approximated to be point charges. We know this because our curve better fits the nonlinear fit (there is less error in our estimate) hence, a better fit must be the one NOT associating spheres as just point charges.

However, it is a reasonable estimate for our experiment.

```
(* This is an Input cell in case you need *)
         R = 0.117; (* Radius of the spheres in meters. *)
         Clear[x, alpha, a]
         (* Nonlinear Fitting, Equation (3) *)
         line2 = NonlinearModelFit[Fvr, (a/x^2) * (1 - alpha (R/x)^3), \{a, alpha\}, x]
         line2["ParameterConfidenceIntervalTable"]
         (* Plotting *)
         FvrErr = Table[{{r[i], Around[Fr[i], dFr[i]]}}, {i, 1, Length[r]}];
         Show [
          ListPlot[FvrErr, PlotStyle → Red, PlotRange → Full],
          Plot[line2[x], {x, 0, Max[r]}],
          AxesLabel \rightarrow {"r (m)", "F (N)"}]
Out[0]=
       FittedModel
Out[0]=
             Estimate
                       Standard Error Confidence Interval
             0.0000362267\ 7.82711\times 10^{-7}\ \{0.0000340535,\ 0.0000383998\}
       alpha -0.0407712 0.0198361
                                {-0.0958451, 0.0143026}
Out[0]=
          F (N)
       0.004
       0.003
       0.002
       0.001
                                                                 r (m)
                    0.12
         params = line2["BestFitParameters"];
         errors = line2["ParameterErrors"];
         (* Assign values *)
         aVal = params[1][2];
         aErr = errors[1];
         alphaVal = params[2][2];
         alphaErr = errors[2];
         (* Print results *)
         Print["a = ", aVal, " ± ", aErr];
         Print["alpha = ", alphaVal, " ± ", alphaErr];
         alpha = aVal;
        \mathsf{a} \ = \ \mathbf{0.0000362267} \ \pm \ \mathbf{7.82711} \times \mathbf{10}^{-7}
       alpha = -0.0407712 \pm 0.0198361
```

The goal of this part is to quantitatively study the dependence of electrostatic force to the charges $Q_1 \times Q_2$.

Step 1. Measurements

- Before getting to the measurements you need to answer the question 1 below. Now you have a way to halve the charge on anyone of the spheres.

Question 1.

- How can you use the setup on your table to halve the charge on one of the spheres?

Answer 1.

- After one of the spheres is charged, we use the "grounding" sphere and touch it to the sphere which we would like to halve the charge of. The charge will move freely across both of them, to balance their electric potentials at the surfaces. Since the radii of the spheres are the same, charge will average out and since the net charge is the charge of the testing sphere, they will both have half the charge of the testing sphere.
- Now, to do the measurements, first set the spheres on a constant distance r. Using what you have learned from part I, choose an appropriate r(11.7 cm).
- Similar to part I, charge both spheres as much as possible.
- Record the scale reading. Then halve the charge on one of the spheres and record the scale reading again. Do this in a zigzag fashion, i.e. halve one then halve the other and so on.
- Fill out the table below. The units for the charges are the initial charges. So you record the fractions. Try to do the whole experiment as quick as you can.

```
"q<sub>2</sub> (Q<sub>20</sub>)"
                                             "M [±0.01] (grams)'
                 q_1 (Q_{10})
                                                        .46
                    1/2
                                    1
                                                        . 25
mvqq = Rest
                    1/2
                                   1/2
                                                        .13
                    1/4
                                   1/2
                                                        .08
                    1/4
                                   1/4
                                                        .04
                                   1/4
                                                       .02
                    1/8
q1 = mvqq[All, 1];
```

```
q2 = mvqq[All, 2];
m2 = mvqq[All, 3];
dm2 = 0.01;
```

- Explain how force depends on charges. Do a fit to show if it is the case. Calculate the correlation coefficient (R2) value as an evidence.
- Explain what each parameter of your fit represents. Can you find k constant using this fit? Explain.

Answer:

By coulomb's law, we know the force must be $F=kq1q2/r^2$, and must hence be linearly proportional to the product of the charges.

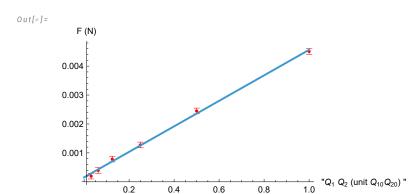
Doing a linear fit, we see that the correlation coefficient is about 0.998 which is really close to 1. Hence, it is evident that $F \propto q_1 q_2$

F = 0.00015407 + 0.00440868 qq (lets call that a + bqq)

The fit has an initial offset (a), encapsulating the errors and offsets in the measurement. b represents the slope of the fit (approximately $F/qq = k/r^2$)

We can not find k from the given data since we do have the unit $Q_{10}Q_{20}$ in terms of coulombs to actually know the slope of the line.

```
(* This is an Input cell in case you need *)
Fqq = m2 * 9.8 * 10^-3;
dFqq = Table[dm2 * 9.8 * 10^-3, {i, 1, Length[q1]}];
(* Linear Fitting *)
Fvqq = Table[{q1[i] * q2[i], Fqq[i]}, {i, 1, Length[q1]}];
line = LinearModelFit[Fvqq, qq, qq]
line["ParameterConfidenceIntervalTable"]
Print["Correlation Coefficient R^2 = ", line["RSquared"]]
(* Plotting *)
FvqqErr = Table[{{q1[i] *q2[i]], Around[Fqq[i]], dFqq[i]]}}, {i, 1, Length[q1]}];
Show [
 ListPlot[FvqqErr, PlotStyle → Red, PlotRange → Full],
 Plot[line[x], {x, 0, 1}],
 AxesLabel →
  {"}^{\}(\*SubscriptBox[\(Q\), \(1\)]\) \}(\*SubscriptBox[\(Q\), \(2\)]\)  (unit
     \!\(\*SubscriptBox[\(Q\),
     \(10\)]\)\!\(\*SubscriptBox[\(Q\), \(20\)]\)) \"", "F (N)"}]
```



R2 = .997869;

Part III. Electrostatic Force, Constant k

The goal of this part is to find the constant $k=\frac{1/4\pi\epsilon 0}{1}$. Again we will change the charge, but unlike part II, we will use a multimeter and a capacitor to measure Q_1 and Q_2 .

Step 1. Measurements

- To measure the charges on the spheres you will use the multi-meter. Put the multi-meter on the mode voltage dc with a sign V===. This multi-meter is connected to a capacitor which reads 10μ F. Review the Readings section above if needed.
- To read the voltage, first make sure the blue electrolyte 10μ F capacitor is fully discharged and voltmeter shows 0.0 mv. Now take the black wire on your hand (the metal) and gently touch the sphere with the red wire. Be careful not to get your hands near the sphere as you might lose some charge to the ground. The blue capacitor will capture almost all the charges on the sphere. Practice a few times reading the voltage, discharging the spheres. To get the charge you only need to use formula (2).
- Now that you are comfortable with the voltmeter, charge both of the spheres, again as much as you can. Read the scale, then discharge spheres one by one to the voltmeter and read the voltages. Repeat this procedure at least four times. Save your data on the table below.
- measure the value of the distance r between the two spheres and record it in meters below.

Step 2. Analysis

- Do the proper fit and report the parameters.
- Find the slope and its error
- Find an estimate for the value of $(1/4\pi\epsilon 0)$ from the slope. Is it close to the theoretical value?

Answer:

the slopes comes out to be 8.31348×10^{11} with an error of 1.03655×10^{11}

let $k=(1/4\pi\epsilon 0)$

 $F=kQ1Q2/r^2$

k=Fr^2/Q1Q2

k=Fvv*r^2/v1*v2

The value w are able to estimate is $k=1.15708 \times 10^{10}$. This value is approximately 29% times larger than the expect value of $8.99*10^9$. This is a somewhat large deviation from the expected value however (as we discuss in part IV), there is a large factor that accounts for a lot of error in this experiment. The leakage of charge likely contributes to the error in our calculated value.

$$k = Mean[Fvv * r^2 / x]$$

 $k / (8.99 * 10^9)$

Out[@]=

Mean
$$\left[\frac{0.013689 \, \text{Fvv}}{x}\right]$$

Out[0]=

$$\textbf{1.11235} \times \textbf{10}^{-\textbf{10}} \, \textbf{Mean} \, \Big[\, \frac{\textbf{0.013689 Fvv}}{\textbf{x}} \, \Big]$$

```
(* This is an Input cell in case you need *)
         Fvv = mv * 9.8 * 10^-3;
         dFvv = Table[dmv * 9.8 * 10^-3, {i, 1, Length[v1]}];
         x = Table[v1[i] * v2[i] * (10^-3 * 10 * 10^-6)^2, {i, 1, Length[v1]}];
         (* Linear Fitting *)
         FvQQ = Table[{x[i], Fvv[i]}, {i, 1, Length[v1]}];
         line = LinearModelFit[FvQQ, QQ, QQ]
         line["ParameterConfidenceIntervalTable"]
         (* Plotting *)
         FvQQErr = Table[{{x[i], Around[Fvv[i], dFvv[i]]}}, {i, 1, Length[v1]}];
         Show [
          ListPlot[FvQQErr, PlotStyle → Red, PlotRange → Full],
          Plot[line[QQ], {QQ, 0, Max[x]}],
          AxesLabel \rightarrow {"Q<sub>1</sub>Q<sub>2</sub> (unit Q<sub>10</sub>Q<sub>20</sub>)", "F (N)"}]
Out[0]=
        FittedModel
                        0.0000337 + 8.31 \times 10^{11} QQ
Out[0]=
            Estimate
                      Standard Error Confidence Interval
           0.0000336656 0.000399128 {-0.00168364, 0.00175098}
        QQ 8.31348 \times 10^{11} \ 1.03655 \times 10^{11} \ \{3.85356 \times 10^{11}, \ 1.27734 \times 10^{12}\}
Out[0]=
        0.004
        0.003
        0.001
                  k = Mean[Fvv * r^2 / x]
Out[0]=
        1.15708 \times 10^{10}
```

Part IV. Time Scale of Charge Leakage (Optional)

In this part, we want to study the charge leaking effect. This will show how much of the error in the experiment is caused by charge leaking out while we are doing the experiment.

Step 1. Measurements

- Charge both the spheres as much as you can. Now read the scale value as times passes.
- Record the time points and the scale readings on the table below. You can stop taking data when the scale

reaches one-fifth of its starting point.

```
[±1 sec]
                                    (min)
                                              "M [±0.01]
                                                           (grams)'
                                0
                                                      0.45
                                1
                                                       .41
                                2
                                                       .37
In[@]:=
       mvt = Rest
                                                                     |;
                            4 + 1 / 12
                                                        .3
                              6.5
                                                       .23
                               13
                                                       .15
                             23.33
                                                       . 09
       t = mvt[[All, 1]];
       mt = mvt[All, 2];
        dmt = 0.01;
```

Step 2. Analysis

- The rate of which the charge leaks out is almost proportional to the amount of charge the sphere has. This is how you can get an exponential behavior. You can think about this system as a RC circuit which you will learn later on this course. The resistors from which the charge can leave the spheres are the insulators holding the spheres. Another way of charge leakage, and the main one during a humid day, is the ions in the air knocking to the sphere. If the charge is larger, the field would be stronger and would attract more ions from the air. So exponential decay is a plausible model to study the leakage of charge.
- The above explanation means you can define an effective rate or characteristic time which is almost a constant number. To do this fit an exponential curve $M = M_0 e^{-t/\tau}$ to the data. Find τ and its error.
- Compare τ to the time it took you to finish the experiments for the previous parts. What can you conclude? Was the charge leakage a major systematic error for your experiments? Explain.

Answer:

Tau=8.53253 and its error dTau=0.542125

all of the other experiments were done very quickly so that we could reduce the amount of interference due to the leakage of charge. For this reason all of the experiments were done in no more than one minute. However, at even one minute, assuming no other manipulation, the ratio of final force reading over initial would = to $e^{-1/8.5352}$ or 0.889441. This is a non negligible interference, this would mean that this is a major and largely unavoidable error and is an explanation for many errors that occur in the experiment.

```
In[*]:= Exp[-1/8.5352]
Out[*]:=
0.889441
```

Out[0]=

Out[0]=

Out[0]=

```
0.001
                                                                        - t (min)
                                10
                                              15
                                                             20
```

tau = "8.5325"; In[@]:=

Table X. Copy All Your Final Results.

Out[0]=

Table X. All Final Results Gathered				
b in $F=\setminus (ar^b\setminus)$ [I.2]	α [I.3 Optional]	Corrlation	k [III.2]	τ [IV.2] (min)
		Coefficient	(Nm^2/C^2)	
		R^2 [II.2]		
-2.08285	0.0000362267	0.997869	$\textbf{1.15708} \times \textbf{10}^{\textbf{10}}$	8.5325

Rutgers 276 Classical Physics Lab

"Electrostatics"

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