# Capacitance

PreLab submission with a pass grade is required to begin the lab.
Must be submitted no later than a day before the lab.

Name: Aryan Malhotra
Section: H5
Date: Feb 26th, 2025

## Readings

### Capacitors & RC Circuits, A Short Review

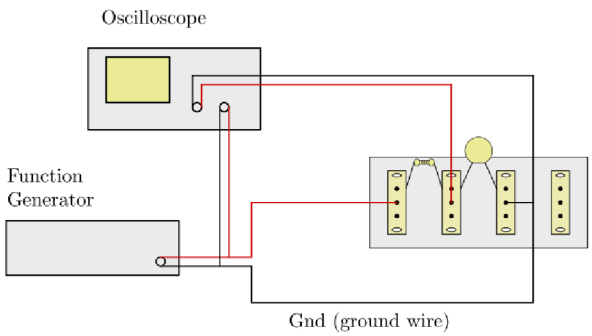
A capacitor is a device consisting of two very closely spaced conducting plates that are insulated from each other. When a charge $+Q$ flows onto one of the capacitor plates an equal and opposite amount of charge $-Q$ from away from the other plate, and a voltage V develops across the two plates. The capacitance C is the constant of proportionality between $C$ and $V$; $Q = \mathrm{CV}$.

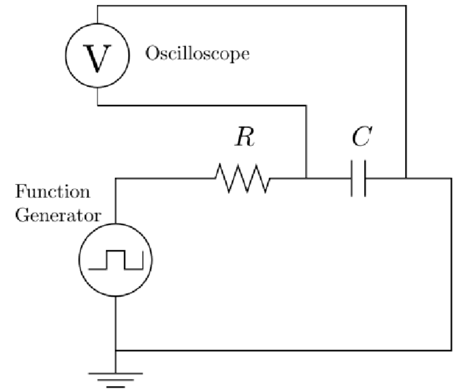
In this experiment you will study a simple circuit consisting of a capacitor in series with a resistor and a voltage source such as a function generator. The resistor acts to limit the rate at which current flows on or off the capacitor, which means that the voltage across the capacitor cannot instantly respond to a change in the voltage source. So you will see how it takes some time for the capacitor voltage to catch up with the source voltage.

We want to study the transient behavior of the RC circuit when we suddenly change the voltage across the circuit. In order to do this we generate a square wave and observe the response of the voltage across the capacitor using an oscilloscope.
One can also use a battery, a switch, a stopwatch, and a voltmeter to study the transient behavior of $V(t)$, but it would be a bit more tricky to do the experiment, and one would need large characteristic time constant.

The circuit shown in Figure 1 allows the capacitor to be charged through the resistor when the function generator changes from the low state of the square wave to the high state. The oscilloscope measures the voltage across the capacitor as a function of time. **Be careful to keep the grounds of the function generator and oscilloscope together (usually the ground is the black colored wire).**

**Figure 1. The setup of the RC circuit.**




To understand the behavior of the circuit, suppose the function generator output has been at ground for a long time so the capacitor is uncharged. Then when the function generator changes to its high state, the entire voltage $V_f$ will appear across the resistor (because the capacitor is uncharged) and a current $V_f / R$ will flow. But as this charge flows it will accumulate on the capacitor, a voltage drop $V(t)$ will build up across the capacitor and the amount of current flowing will decrease (because the voltage across the resistor, $V_f - V(t)$, will decrease). Eventually all current will stop flowing when $V(t)$ reaches $V_f$. Mathematically the behavior of the voltage across the leads of the capacitor is given by:

(1) $V(t) = V_f(1 - e^{-t/\tau})$ [while charging],

Where $\tau = R\,C$, the characteristic time constant and e is the base of the natural logarithms, e = 2.71828... . It is called the characteristic time because it determines (characterizes) how quickly or slowly the RC circuit responds. A way to derive the above formula is to write the loop rule for the above circuit,

(1') $R\,C \frac{d}{dt} V(t) + V(t) = V_f$, with initial condition $V(0) = 0$.

The charge on the capacitor is given by, $Q(t) = C\,V(t)$.

When the function generator changes back to its low state (ground), the voltage across the capacitor will exponentially decrease as the positive charge on the one plate flows backward through the resistor to neutral-ize the negative charge on the other plate. Mathematically,

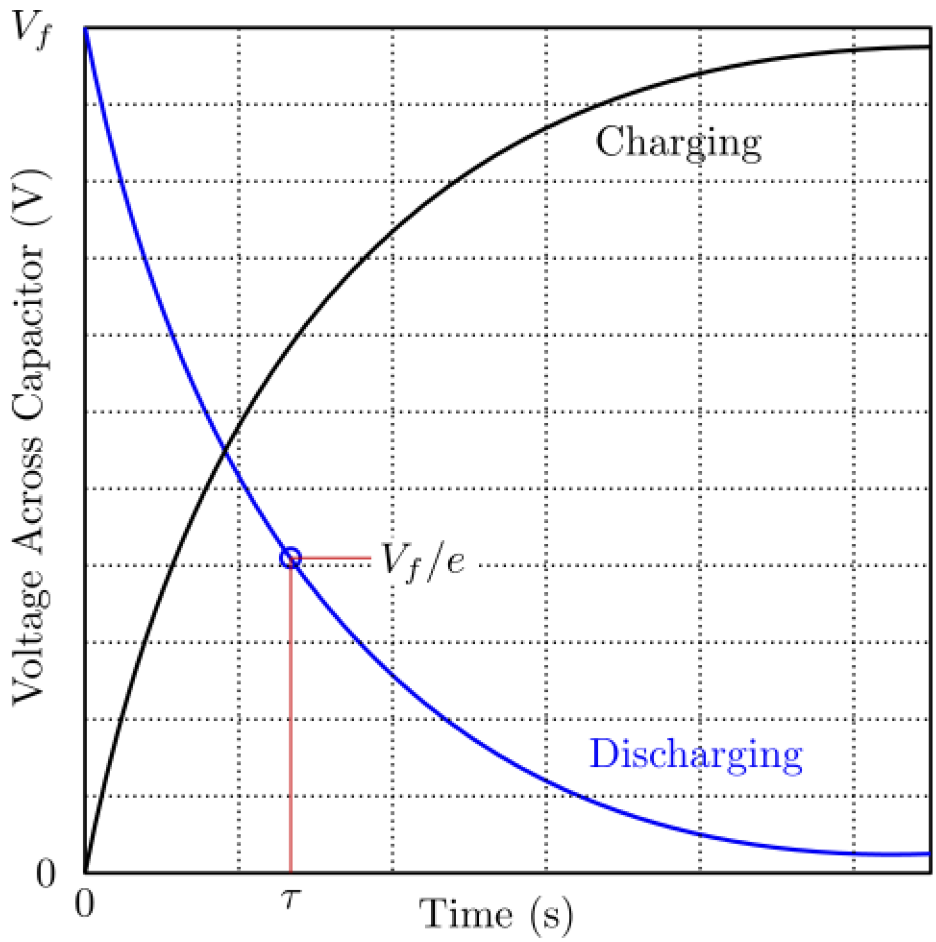
(2) $V(t) = V_f\, e^{-t/\tau}$, [while discharging].

When discharging, at $t = \tau$, $V(\tau) = V_f / e$. That is, $V$ is about one-third of the initial voltage ($1/e = 0.368...$). During the charging process the time constant is the time it takes for the capacitor to charge to about two thirds of its final value. The equation (2) can be derived from the loop rule for the circuit without any source term,

(2') $R\,C \frac{d}{dt} V(t) + V(t) = 0$, with initial condition $V(0) = V_f$.

The equations (1) and (2) are graphed in fig. 2. Study the graphs to be sure you understand the physics.

**Figure 2. The voltage of the capacitor in an RC circuit, charging and discharg-ing.**




## Understanding The Lab

At this point, it might seem to you that the charging and discharging of capacitors are different phenomena. But the difference is more about how the story is told, rather than begin mathematical. One can easily put both equations (1) and (2) in the same place using,

(3) $V(t) = V_f\left(1 - e^{-t/\tau}\right) + V_0\, e^{-t/\tau} = V_f + (V_0 - V_f)\, e^{-t/\tau}$, where, $V(t = 0) = V_0$ is the initial (zero time) value for capacitor's voltage.

Equation (3) is easy to think about and will be the one we will use for this lab. Basically, capacitor is starting with $V_0$ voltage and charging towards $V_f$. $V_f$ can be positive, zero, or negative. So instead of “charging” or “discharging”, we can use the term “charging towards a value, starting from an initial voltage”. For example, “discharging” usually refers to “charging towards zero, starting from a positive voltage”.

In this lab we will use square wave voltage for $V_f$. So $V_f$ will be a positive constant value for a half-cycle and same but negative constant value for the next half-cycle. This is the blue curve below. In each half-cycle, capacitor will try to reach the $V_f$ (charging towards it). One goal in this lab is to see how well equation (3) describes the voltage of the capacitor.

## Capacitors in Parallel & in Series

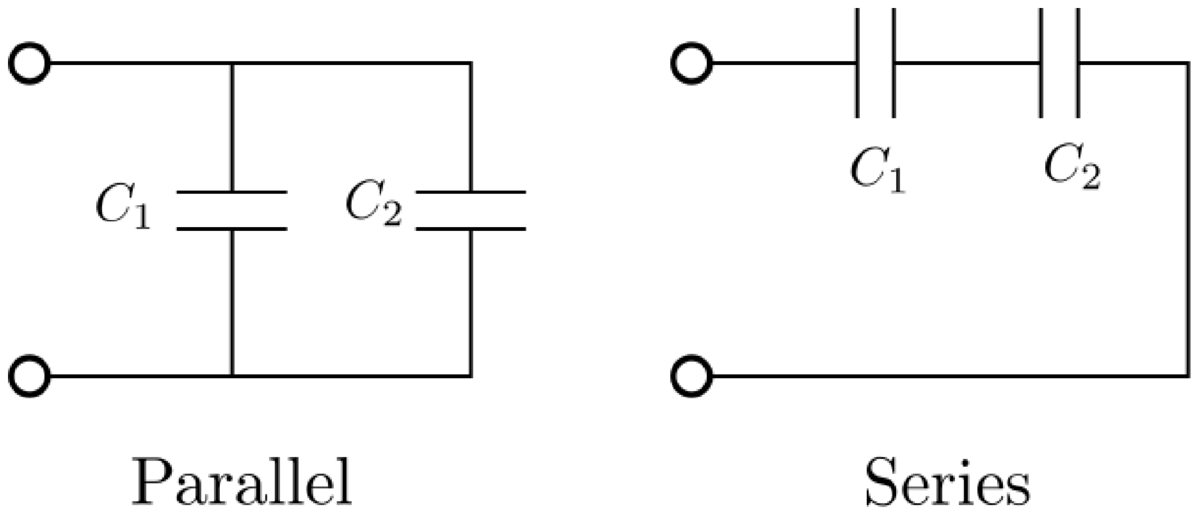
When we connect two capacitors, $C_1$ and $C_2$, in parallel, their voltages are the same. See Figure 3. So $Q_1 + Q_2 = C_1\, V + C_2\, V = (C_1 + C_2)\, V$, which means equivalently we can use a $C_{eq}$ given by,

(4) $C_{eq} = C_1 + C_2$, [capacitor in parallel].

When we connect two capacitors, $C_1$ and $C_2$, in series, their currents or charges are the same. See Figure 3. So $V_1 + V_2 = Q / C_1 + Q / C_2 = (1 / C_1 + 1 / C_2)\, Q$, which means equivalently we can use $C_{eq}$ given by,

(5) $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$, [capacitors in series].

**Figure 3. Capacitors in parallel and in series.**




# Dialog:

Question 1.
- For a resistance of 270 kΩ and a capacitance of 1.5 nF, what is the characteristic time for the RC circuit?
- If this RC circuit is discharged, how long will it take for the charge to drop to 50% of the original value?

Answer 1.
- $\tau$ = R C = **0.000405s or 406$\mu$s**
- $V(t) = V_f\, e^{-t/\tau}$, [while discharging]

Hence, $V(t) / V_f = 1/2 = e^{-t/\tau}$

Therefore, -ln(1/2) * $\tau$ = **t50%** = **0.000280725s or 280$\mu$s**

In[8]:=
```
(* This is an Input cell in case you need *)
tau1 = 270 * 1000 * 1.5 * 10^-9
t150 = -Log[1 / 2] * tau1
```

Out[8]= 0.000405

Out[9]= 0.000280725

Question 2.

Run the simulation below. If it crashes, select Abort, go to the Evaluation from top menu and check-mark “Dynamic Updating Enabled”. Or quit Mathematica and open it again.
What is the shape of the capacitor’s voltage curve, $V_C(t)$, considering the conditions below? Explain briefly why do you think that is.
- $\tau$ is really large.
- $\tau$ is really small.
- frequency is really large.
- frequency is really small.

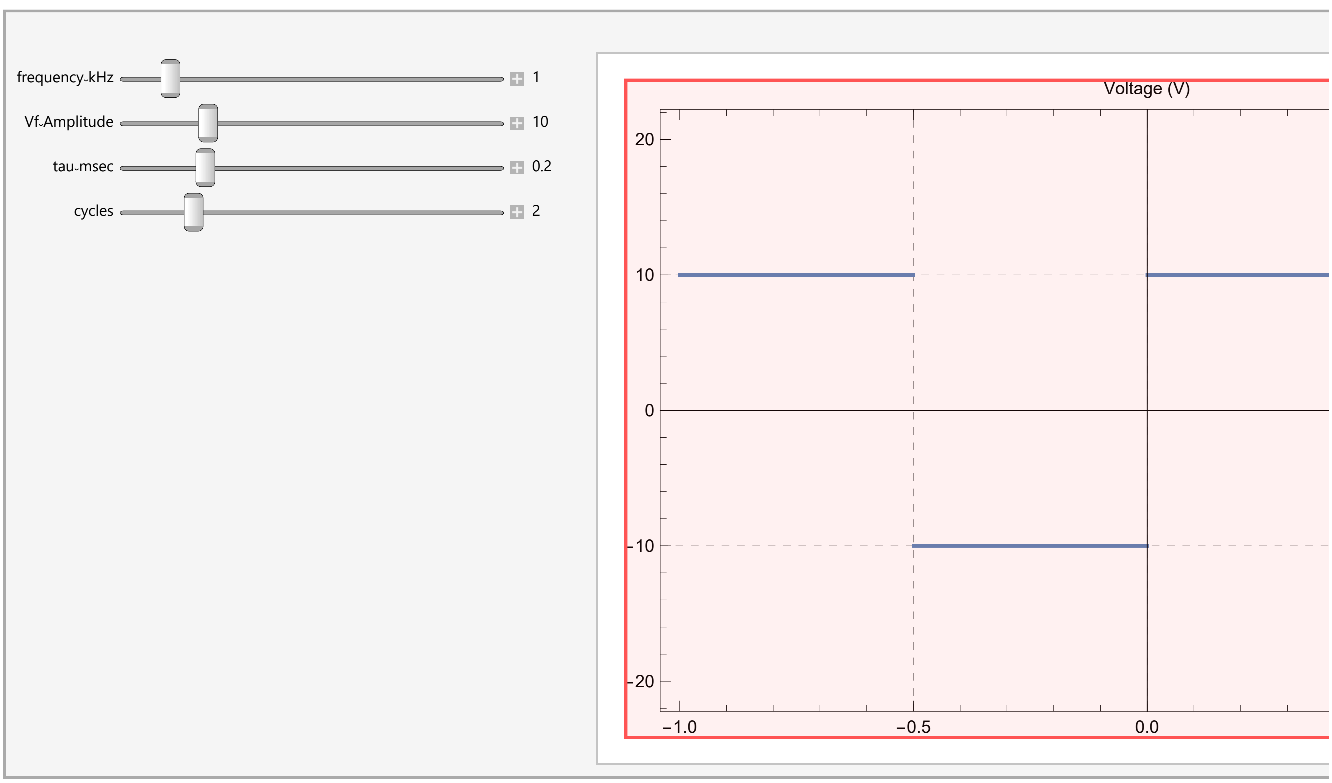
Answer 2.
- For large $\tau$, the capacitor’s voltage curve, $V_C(t)$ changes in a **zig zag pattern, with each bit being more or less linear**. This seems to be because the capacitor’s rate of charging is slow. So it is as if most impedance to the circuit current comes from the Resistor. Hence, the current flow is mostly linear ($I \approx V/R$), so is the build up rate of the Potential on the capacitor. **In other words, before the current flow rate and hence the potential build up rate gets to plateau, the current direction switches.**
- Here, the exact opposite happens compared to large tau. **The potential builds up quickly, plateauing for most of the half cycle of the current and then quickly flips and builds up in the opposite way.** Here, I would guess since the progression rate of the voltage buildup is quick, the potential builds up way before the current gets to switch signs.
- Again, similar to the case of large tau, the voltage curve has a **zig zag pattern, with each bit being more or less linear**. Again, this seems to be because since the frequency is high, time period of the oscillation of the current value is very small. Hence, the current flips well before the potential gets time to flatten out and reach a steady value.
- Again, similar to the case of small tau, **the voltage curve seems to be shooting high quickly and reaching a steady state for most of the half cycle of the current, only to flip signs and do the same and repeat.** Again, this would happen because now, the time period for the current to flip signs is so large that it is ample for the capacitor to reach a steady state voltage, even with a fairly slow charging rate.

In[83]:=
```
Clear[Vf, V0, Vc, Vr, x]
Manipulate[
 half = 1 / (2 * frequency‿kHz); (* Duration of half a cyle, in msec. *)
 Vf[x_] = (1 - 2 * Floor[Mod[x / half, 2]]) * Vf‿Amplitude;
 (* The source voltage, blue curve, in V. *)
 V0[x_] = - (1 - 2 * Floor[Mod[x / half, 2]]) *
   Vf‿Amplitude * (1 - Exp[-half / tau‿msec]) / (1 + Exp[-half / tau‿msec]);
 (* Initial capacitor voltage for each cycle, in V. *)
 ctime[x_] = x - half * Floor[x / half];
 (* Time that passed from the starting point of a cycle, in msec. *)

 Vc[x_] = Vf[x] * (1 - Exp[-ctime[x] / tau‿msec]) + V0[x] * Exp[-ctime[x] / tau‿msec];
 (* Capacitor voltage, in V. *)
 Vr[x_] = Vf[x] - Vc[x];

 Show[
  Plot[{Vf[t], Vc[t], Vr[t]}, {t, -cycles * half, cycles * half},
   PlotLegends → {"Vf (Source)", "VC(t)", "VR(t)"}],
  AxesLabel → {"Time (ms)", "Voltage (V)"},
  PlotRange → {All, {-2 * Vf‿Amplitude, 2 * Vf‿Amplitude}}, ImageSize → Large,
  GridLines → Automatic, GridLinesStyle → Dashed, Frame → True
 ],
 {{frequency‿kHz, 1}, 0.1, 10, Appearance → "Labeled"},
 {{Vf‿Amplitude, 10}, 0, 50, Appearance → "Labeled"},
 {{tau‿msec, 0.2}, 0.01, 1, Appearance → "Labeled"},
 {{cycles, 2}, 0.5, 10, Appearance → "Labeled"}
]
```

Out[84]=




## Question 3.

- You connect two similar capacitors with $C = 100 \pm 10$ nF in parallel. Find the equivalent capacitance and its error.
- You connect two similar capacitors with $C = 100 \pm 10$ nF in series. Find the equivalent capacitance and its error.

Answer 3.
- In parallel, the capacitances simply pile up linearly. Hence, $C_{eq} = (C + C) \pm (\Delta C + \Delta C) =$ **200 ± 20 nF**
- In series, the reciprocals of the capacitances add up. Hence, $1/C_{eq} = 1/C + 1/C \Rightarrow C_{eq} = C/2 = 50$ nF
The error in series is given by $\delta C/C^2 = \delta C_1/C_1{}^2 + \delta C_2/C_2{}^2$ Hence $\delta C =$ $C^2 * (\delta C_1/C_1{}^2 + \delta C_2/C_2{}^2) = 5$ nF
Hence $C_{eq}$ = **50 ± 5 nF**

In[12]:=
```
(* This is an Input cell in case you need *)
50^2 * (10 / 100^2 + 10 / 100^2)
```

Out[12]=
5

Question 4. (Optional)
You connect the RC circuit, with same values given in question 1, to a square wave potential with frequency 2kHz. The voltage of this square wave source changes between +10V and -10V.
- In each full cycle, the voltage of the capacitor goes from $-V_C$ to $V_C$ and back, with a charging and discharging (charging in opposite direction) curves. Calculate $V_C$.
- Calculate the maximum value of current passing through the circuit.

Hints:
> Find how long a half-cycle takes.
> Draw a qualitative graph for the voltage of the capacitor as a function of time to understand the problem better.
> Take the voltage of the capacitor to be $-V_C$ at the end of -10V half-cycle (beginning of +10V half-cycle), and $+V_C$ at the end of +10V half-cycle. So capacitor's voltage changes $2\,V_C$ at each half-cycle. Equation (1) and (1') are written when the capacitance starts uncharged. Using $V(0) = -V_C$ we get,
(1'') $V(t) = V_f\left(1 - e^{-t/\tau}\right) - V_C\, e^{-t/\tau}$,
where $V_f$ is the battery voltage, here 10V.

Answer 4.
- Half cycle takes 1/4000 s (say tHalf). Then the time to go from -V to V is tHalf.
$V(t) = V_i + \Delta V\, e^{-t/\tau}$ therefore, $V_c$ =-10+ 20 $e^{-t/\tau}$, t = tHalf and tau = tau1

Thus, $V_c$=0.78815V.
But that is contradicting to my assumption - since steady state is not reached.
Voltage must not be given by transient calculation.

Hence, actually, ΔV must be $V_{ext} - V_c$
$V_c$ = 10 - (10 – $V_c$)$e^{-t/\tau}$ (Since the external voltage was 10 and the change in capacitor voltage was 10 – $V_c$ so the potential difference between the external and capacitor voltage must be 10 - $V_c$)

So $V_c$ actually is equal to 3V

- $I_{max}$ is just at current flip where V = $V_{ext}$ + $V_c$ = 10+3 = 13V
Hence, $I_{max}$ = V/R = 0.0000481481 A

In[87]:=
```
(* This is an Input cell in case you need *)
f = 2 * 10^3
tHalf = N[1 / f / 2]
tau1
Vc = -10 + (20 * Exp[-tHalf / tau1])
Vc = 4.61 / 1.539

Imax = N[13 / (270 * 1000)]
```

Out[87]=
```
2000
```
Out[88]=
```
0.00025
```
Out[89]=
```
0.000405
```
Out[90]=
```
0.78815
```
Out[91]=
```
2.99545
```
Out[92]=
```
0.0000481481
```

## Table X. Copy All Your Final Results.

In[93]:=
```
Grid[{{Text["Table X. Final Answers, Lab 5 Prerequisite"], SpanFromLeft},
  {"Q1. τ (ms)", "Q1. time to reach 50% (ms)", "Q3. Ceq for parallel (nF)",
   "Q3. Ceq for series (nF)", "Q4. VC (V)", "Q4. Maximum Current (μA)"},
  {tau1 * 1000,
   t150 * 1000,
   PlusMinus[200, 20],
   PlusMinus[50, 5],
   Vc,
   Imax * 10^6}
 }, Frame → All]
```

Out[93]=

| Table X. Final Answers, Lab 5 Prerequisite | | | | | |
|---|---|---|---|---|---|
| Q1. $\tau$ (ms) | Q1. time to reach 50% (ms) | Q3. $C_{eq}$ for parallel (nF) | Q3. $C_{eq}$ for series (nF) | Q4. $V_C$ (V) | Q4. Maximum Current ($\mu$A) |
| 0.405 | 0.280725 | 200 ± 20 | 50 ± 5 | 2.99545 | 48.1481 |

---

Rutgers 276 Classical Physics Lab
“Capacitance”
Copyright 2017 Roshan Tourani and Girsh Blumberg