# Ideal & Non-Ideal Gases

## Lab Report

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Section: H5
Date: February 2nd, 2025

## Purpose

- To measure how the pressure of a low-density gas varies with temperature, to determine the absolute zero of temperature by making a linear fit to your data assuming the gas is an ideal gas, and to investigate the difference in behavior between an ideal gas and a non-ideal (van der Waals) gas.
- To measure how volume varies with temperature.

## Apparatus

Ideal gas apparatus, pressure gauge, thermometer, hot plate, stand, piston, tubing, blue (or maybe white) valve in tubing.

## Readings

### The Ideal Gas, A short review

The ideal gas law states that $PV = nRT$ where $T$ is the absolute temperature measured in Kelvins. This equation is also called the equation of state of an ideal gas. It is a good description of most gases in the low-density regime where, on average, the gas molecules are far apart. The purpose of the experiment is to determine experimentally how the pressure $P$ of a gas varies with temperature when no change is made in either the volume $V$ or the number of moles of gas n. Using this information, you will determine $T_0$, the absolute zero temperature measured in degrees Centigrade, by linear extrapolation of the ideal gas equation to where the pressure $P$ of the gas is zero (no molecular bombardment of the walls of the gas container).
In this experiment you will measure $T_C$, the temperature in degrees Centigrade, which you will recall is related to $T$ (in Kelvins) by $T = T_C - T_0$ ($T_0$ is a negative number). The graphical technique you will use is to plot $T_C$ (vertical axis) vs. $P$ (horizontal axis). The equation for the straight line fitted to the data points will have the form of a linear equation,

(1)  $T_C = T_0 + \left(\frac{V}{nR}\right)P,$

where $V$ is the volume of the gas, $n$ is the number of moles, and $R = k_B N_A$ is the universal gas constant.
For $P = 0$ the vertical axis intercept, $T_0$, in degrees Centigrade, can be read directly from the equation.

### A Non-Ideal Gas, A short review of van der Waals gas

We noted above that the ideal gas law holds for low-density gases. Theoretically, for deriving the ideal gas law, it is necessary to make two assumptions

- the gas molecules are very small (they have no volume) and

- the molecules are non-interacting (there is no force between them).

In 1873 the Dutch physicist van der Waals derived an equation of state without these assumptions. It is known as the van der Waals equation of state:

(2)   $\left(P + \frac{a}{v^2}\right)(v - b) = R\,T$,

where $a$ and $b$ are constants chosen to agree with experiment, and $v$ is the molar specific volume, i.e. the volume of the container divided by the number of moles of gas inside. $b$ is also a molar specific volume and represents the total volume per mole of gas that is inaccessible to other molecules because it is already occupied by a molecule, considering you can't have two molecules at the same place at the same time. If the molecules in the gas have a radius $r$, then when two collide the centers can only come with a distance $2\,r$ of each other.  Thus if we want to consider the molecules to have no volume, we must subtract from the volume of the container the volume of a "sphere of exclusion" of radius $2\,r$ around each molecule,

(3)   $b = \frac{1}{2}\,N_A\,\frac{4}{3}\,\pi(2\,r)^3$,

where $N_A$ is the Avogadro's number, the number of molecules in a mole. The 1/2 occurs because only the hemisphere facing the colliding molecule is effective in excluding it.  The term $a/v^2$ that is added to the pressure of the gas in the container arises from a very short range attractive force between molecules known as the van der Waals force.  When a molecule is not near the wall of the container, it is surrounded by molecules and, on average, this force cancels out. But near the wall the molecule experiences a net force pulling it away from the wall, which reduces the force (pressure) it exerts on the wall when it collides. This is the force that makes some gases liquefy at low enough temperatures.

The values for $a$ and $b$ are determined experimentally for each gas.  For oxygen $a = 1.38 \times 10^5$ Pa.$m^6/(\text{kmol})^2$ and $b = 0.0319\,m^3/(\text{kmol})$.

[Pa = Pascal = $N/m^2$, where N is Newton.]

For the purposes of your calculations remember that $R = 8314$ J/(kmol K). Avogadro's number is $N_A = 6.02 \times 10^{23}$ molecules per mole, and the volume occupied by one kmole of a gas at standard temperature and pressure (0 °C and 760 torr = 1 atm = 1.01 x $10^5$ Pa) is 22.4 $m^3$.

[Note: the mole is defined to be the quantity of material whose mass in grams is equal to the molecular weight of the material. The proper SI unit is not the mole, but rather the kmole = 1000 moles. The mass of a kmole in kilograms equals the molecular weight of the material, the number of molecules in the kmole is $6.02 \times 10^{26}$ and the standard volume is 22.4 $m^3$.]

Equation (2) gives a fairly good description of a real gas. Other equations of state such as the Beattie-Bridgman equation (which uses five adjustable parameters instead of two) gives even better agreement with experiment.

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# Procedure

**SPECIAL PRECAUTIONS:**

**- Be careful of the hot water and hot bucket. KEEP ALL THE WIRES AND PIPES AWAY FROM THE HOT PLATE.**

**- The hot plates are not waterproof.  Do NOT spill water on them.**

**- Do NOT tamper with the valve on the bulb holding the gas.**

**- Do NOT allow the black plastic on the thermometer to touch water.**

**- Note the foam protecting the fragile glass-walled piston. Do NOT remove this foam.**
**- Do NOT exert too much force on the piston plunger, either pulling or pushing, or you may shatter the glass!**

Note: If the pressure gauge reads in units of mm Hg (torr), change it to Pa as the ideal gas equation uses SI units. If you forgot and your data is units of torr, convert the pressure to SI units, using 1 torr = 133.3 Pa.

In part I, you will assemble the apparatus on the ring stand. See Figure 1 below.
- Make sure that the bulb does not touch the bottom or sides of the can.
- Fill the bucket with ice water.
- Make sure the water is at least 4 cm over the top of the bulb.
- Don't add a lot of ice; it'll take longer for your water to heat if you add too much.
For parts II and III, you will connect the bulb to the cylinder with the piston. See Figure 1 below.
In part II, you will change the temperature of the gas inside the bulb using the warm water you have, making the pressure to be a constant and study the V-T relation while P stays constant.
In part III, using the room temperature for the gas, cautiously you will change the pressure and study the P-V relation while T is held constant.

**Figure 1. The setup of the experiment for parts I, II, and III.**
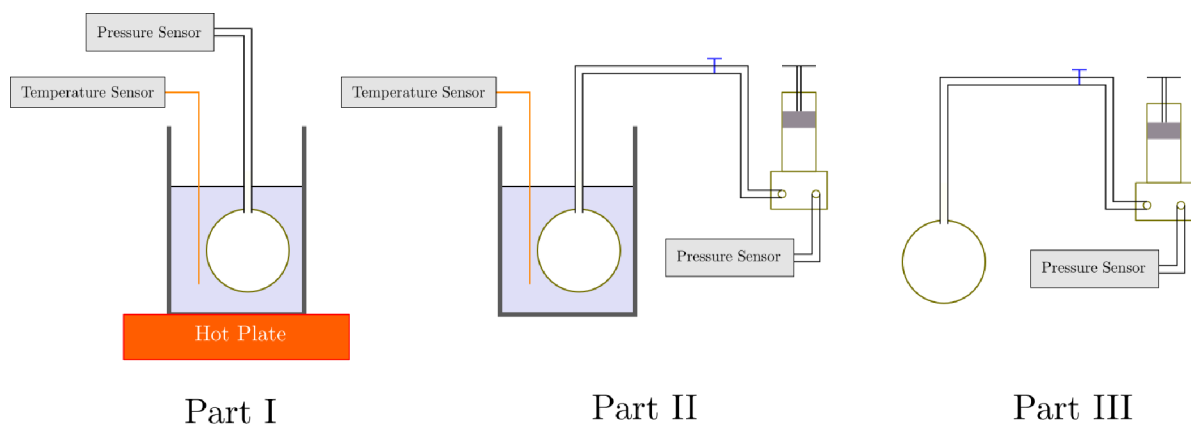


# Dialog:

## Part I. Ideal Gas Law, **Pressure vs Temperature**

### Step 1. Measurements (See Figure 1)

- Connect the setup as shown in Figure 1. If needed, check with the instructor. Do not turn on the hot plate before fully understanding the setup, the goal for part I, and reading all the warnings.
- Before fastening the screws on the pressure sensor which is connected to the bulb, put the bulb inside **the bath of 0°C water**. This way the density of the gas would be correctly matching standard temperature and pressure, aka **STP**. Fasten (or loosen and fasten) the screws when the bulb is submerged in ice water.
- The temperature of the bath is raised by turning on the hot plate to the highest temperature setting. Record in parallel columns the absolute pressure (mm Hg = torr) and temperature (°C). To collect the data open

"Logger Pro", click on "Data Collection" (a small clock icon) and set duration to **"90" minutes** (more than expected time needed to complete data collection) and sampling rate to "**6 samples/minute**".
- There may be a temperature gradient between the bottom and top surfaces, and the thermometer might not read exactly the bulb gas temperature. When you get to **about 88 C**, shut the hot plate off and allow it to cool. Don't let the water boil. **Be careful not to touch or spill the hot bucket. Keep yourself, wires, and pipes away from the hot plate.**
After the data is collected, export it as a CSV file.

Question 1.
Theoretically you write the equation,
$$P\,t = m\,c\,\Delta T,$$
and argue that the temperature must rise linearly with time. But when you look at the T vs t curve on "Logger Pro", it does not look like a linear curve. At the beginning the slope is really low, then it is really high and starts gradually decreasing.
How do you explain,
- slow rise at the beginning,
- after the slow rise, slope being high but gradually decreasing over long time.
For each case, specifically state what is missing in the equation above.

Answer 1.
- At the start, the entire system is not in Thermal equilibrium. Instead, before the system reaches a near steady state flow of heat, the heat is first absorbed by other components such as the container. The heat capacity of the container and heat loss to the environment is unaccounted for during the start.

After the container is heated, the slope becomes more linear and steep as a steady state of heat flow is achieved.

- As the heating increases and the water heats a lot, processes like convection and radiation make the system lose heat (since radiation is proportional to T^4!). As per the Newton's law of cooling, as the temperature of the system becomes sufficiently larger than the surrounding, our system tends to lose heat to the surrounding, hence decreasing the rate of increase of our heat while supplying constant power to the system.

## Step 2. Importing Data & Finding $T_0$ and $v$ for Ideal Gas

- Considering the data collection is finished, import the data to the notebook. In the code below, you need to change the file path. You might also need to swap $P$ and $T$ definitions. Check your csv file.
- Do a linear fitting for $T$ as a function of $P$ and find the intercept, slope, and their errors.

- Plot the temperature $T$ (in °C) on the vertical axis, as a function of pressure $P$ on the horizontal axis. Find sigmaT, $\sigma_T$, and include the error bars for $T$. Also include the fit line on the graph.
- Record your result for $T_0 \pm \sigma_{T_0}$ and compare it with the given value of -273.16 °C.
- Using the slope, find the molar specific volume of the air and its error. Compare this to the given values at standard temperature and pressure (STP).

Note that this line is a huge extrapolation into a region where the equation for the straight line does not apply. A real gas will condense into a liquid and then freeze to a solid as the average thermal energy per molecule decreases with falling temperature.

**Answer:**
Approximating the behaviour of the gas as that of an ideal gas, PV=nRT at any point of the experiment (assuming the temperature change was consistent and there was a thermal equilibrium)
v = molar specific volume = V/n = RT/P
Hence, **v = R * slope of the line** (T/P is the slope of the line)

**T0** then is the temperature that corresponds to the point of the extrapolated line that corresponds to Zero Pressure (No movement of particles - **Absolute Zero**!)

Doing a linear fit for the data, we get the y-intercept (T0 - absolute zero), slope and their errors to be

**T0±sigmaT0 (Celsius)= -282.85±0.155768 Celsius**
**v±sigmav (Litres/mol or m^3/kmol)= 23.8108±0.0110456 m^3/kmol**
**(Or v±sigmav (m^3/mol)= 0.0238108±0.0000110456) m^3/mol**

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Comparison of experimental value of `v` with the theoretical value of `v` at STP:
The theoretical value of molar specific volume for a gas at **STP is 22.4 $m^3$/kmol**
The value we got is **23.8108±0.0110456 $m^3$/kmol**
That is a **6.29815% error** which is justifiable because **our setup was not under standard temperature and pressure conditions but instead, the temperature of the experiment's start was closer to room temperature**. Also, because given the conditions our setup was in, the gas was widely different from what an ideal gas would behave like. It was much more real. Due to the high temperature and more extreme conditions than that the experiment was conducted in as compared to the more controlled STP conditions, there can be a deviation from the theoretical value. Also, 6.3% is an acceptable measurement error as well, given that our apparatus was **not in a steady state of heating** throughout the experiment.
There is also a possibility of a **Zero Error Within the Temperature or Pressure measurement instrument calibration.**

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Comparison of experimental value of Absolute temperature with experimental value T0:
Our measured value of T0 (y intercept of the TvP graph for our gas) was **-282.85±0.155768** °C

The theoretical value is around **-273.15** °C

Hence the **error** is around **3.55%** which is totally acceptable and in fact, really good, considering that the value that we have is a wide extrapolation of the graph we got for state that wasn't entirely steady while heat flow (the slope was not totally linear). Also, our actual curve should have followed a more complicated equation for a real gas, not merely and ideal gas for better results.

In[542]:=

```
(* This is an Input cell in case you need *)
tPT =
  Rest[Import["E:\\Notes\\College-Notes\\Spring 2025\\Physics\\Classical Physics Lab II\\Lab
      2\\Part I.csv"]];
(*>>> - Change the filepath to point to the saved data file. <<<*)
t = tPT[[All, 1]];
P = tPT[[All, 2]];
P = P * 133.322 / 1000; (*Convert to kPa*)
T = tPT[[All, 3]];
P = P;
TvP = Table[{P[[i]], T[[i]]}, {i, 1, Length[P]}];

lineTvP = LinearModelFit[TvP, x, x]
lineTvP["ParameterConfidenceIntervalTable"]
slope = lineTvP[1] - lineTvP[0];

Show[
 Plot[lineTvP[x], {x, 0, 150}],
 ListPlot[TvP, PlotStyle → Red, PlotRange → Full],
 AxesLabel → {"P (Torr)", "T (°C)"}]

-282.8498032718738`   0.15576757313882045`
```
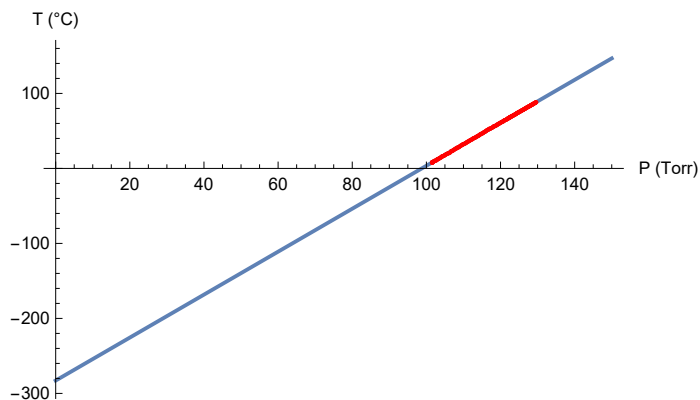
Out[549]=

FittedModel[ -283. + 2.86 x ]

Out[550]=

|   | Estimate | Standard Error | Confidence Interval |
|---|---|---|---|
| 1 | -282.85 | 0.155768 | {-283.156, -282.543} |
| x | 2.86394 | 0.00132855 | {2.86132, 2.86655} |

Out[552]=



Out[553]=

{{-282.85, 0.155768}}

In[554]:=

```
(*Calculations for T0*)
T0 = -282.8498032718738; (*Celsius*)
sigmaT0 = 0.15576757313882045;

(*Calculations for v - Molar Specific Volume*)
slope; (*T/P - C/kPa*)
sigmaSlope = 0.001328554976734153;

R = 8.314; (*m^3 Pa K^-1 mol^-1*)
v = R * slope; (*Litres/mol or m^3/kmol*)
sigmav = R * sigmaSlope; (*Litres/mol or m^3/kmol*)

Print["T0±sigmaT0 (Celsius)= ", T0, "±", sigmaT0]
Print["v±sigmav (Litres/mol or m^3/kmol)= ", v, "±", sigmav]

(*Conversion of volume from Litres to m^3*)
v = v * 0.001; (*m^3/mol*)
sigmav = sigmav / 1000; (*m^3/mol*)
Print["v±sigmav (m^3/mol)= ", v, "±", sigmav]

(*Comparisons with theoretical values*)
(v * 1000 - 22.4) / 22.4 * 100
(T0 - (-273.15)) / 273.15 * 100
```

T0±sigmaT0 (Celsius)= -282.85±0.155768

v±sigmav (**Litres/mol or m^3/kmol**)= 23.8108±0.0110456

v±sigmav (**m^3/mol**)= 0.0238108±0.0000110456

Out[566]=

6.29815

Out[567]=

-3.55109

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Question 2.
- Discuss the systematic uncertainties in the experiment. Make quantitative estimates for reasonable variations. For example, assume the thermometer calibration is incorrect, so that 1.00 °C actually corresponds to 1.01 °C, and calculate the error on absolute zero.

---

Answer 2.
Since our model is a linear regression fit, if each value we measure is just shifted by 0.01 degrees due to calibration error, it is as if our entire line of measurements had an additional offset of 0.01 degrees C.
Hence, the entire line of data points would be shifted up by 0.01 units. So would the value of T0.

Hence, sigmaT0 = sigmaT = 0.01 degree C in this case.

## Step 3. Air Being Non-Ideal

Using the Manipulate environment below, we are trying to see how sensitive $T_0$ is to the molar specific volume $v$, also $a$ and $b$ parameters of the van der Waals gas. This way we can find out how much error can be attributed to the assumption that the air is not ideal.

- To do a sanity check, put $a = b = 0$ (ideal gas law) and the same $v$ and $T_0$ from step 1 above and check that the green line matches the ideal gas law, the blue fit line.
- We start with oxygen values at STP. This the default setting for the sliders. Try to play with the sliders $a$ and $b$ and see how much they can change the extrapolation.
- Using the standard volume $v = 22.4 \frac{m^3}{kmol}$ and $T_0 = -273.16\,°C$, eyeball an estimate for $a$ and $b$, so that the van der Waals gas formula (green line) matches the data. Are these values of $a$ and $b$ realistic for air? Why?
(If you forgot to loosen and fasten the screws at 0°C, see part I step 1, the starting pressure would be lower and not STP. So use the value for $v$ based the room temperature not 0°C, which is about $24 \frac{m^3}{kmol}$.)
- Now setting $a$ and $b$ to be the values for oxygen and nitrogen, check the fit results for $v$ and $T_0$. The variable names on the fit equation are vvdW and T0vdW, respectively and fitting result is given on Table X2. Compare the results to the ideal gas results. Conclude with answering question 3.

In[568]:=

```
SetSystemOptions["CheckMachineUnderflow" → False];
R = 8314; (* J/(kmol K) *)
Manipulate[
 vdWline = NonlinearModelFit[TvP, 10^3 (p + a / vvdW^2) * (vvdW - b) / R + T0vdW, {vvdW, T0vdW}, p];
 Grid[{
   {Show[
     Plot[lineTvP[x], {x, 0, Min[P] - 10}, PlotStyle → Dashed],
     Plot[lineTvP[x], {x, Min[P] - 10, Max[P] + 10}],
     Plot[10^3 * (x + a / v^2) * (v - b) / R + t0, {x, Min[P] - 10, Max[P] + 10}, PlotStyle → Green],
     Plot[10^3 * (x + a / v^2) * (v - b) / R + t0, {x, 0, Min[P] - 10}, PlotStyle → {Green, Dashed}],
     ListPlot[TvP, PlotStyle → Red],
     AxesLabel → {"P (kPa)", "T (°C)"},
     PlotRange → All, ImageSize → Large
    ],
   Grid[{
     {Text[
       "TABLE 1. The values of a, b, v, and T₀ in T=(P+a/v²)(v-b)/R + T₀.\nThis is the green
          line. You can change all four parameters using sliders.\n"], SpanFromLeft},
     {"a (kPa.m⁶/(kmol)²)", "b (m³/(kmol))", "v (m³/(kmol))", "T₀ (°C)"},
     {a, b, v, t0}
    }, Frame → All]
   },
   {Show[
     Plot[vdWline[x], {x, Min[P] - 10, Max[P] + 10}, PlotStyle → {Black, Thick}],
     Plot[10^3 * (x + a / v^2) * (v - b) / R + t0, {x, Min[P] - 10, Max[P] + 10}, PlotStyle → Green],
     ListPlot[TvP, PlotStyle → Red, PlotMarkers → {"o", 5 }],
     AxesLabel → {"P (kPa)", "T (°C)"},
     PlotRange → All, ImageSize → Large
    ],
   Grid[{
     {Text["TABLE 2.\nFit values for v and T₀ using
           (P+a/v²)(v-b)=R(T-T₀).\nThe a and b parameters are given
           by sliders.\nThis is the black line.\n"], SpanFromLeft},
     {vdWline["ParameterConfidenceIntervalTable"]}
    }, Frame → True]
  }
 }],
 {{a, 1.38 * 10^2}, 0, 1.38 * 10^3, Appearance → "Labeled"},
 {{b, 0.0319}, 0, 0.319, Appearance → "Labeled"},
 {{v, 22.4}, 22.4 + 10, 22.4 - 10, Appearance → "Labeled"},
 {{t0, -273}, -350, -200, Appearance → "Labeled"}
]
```

Out[570]=

a    ⊞ 138.

b    ⊞ 0.0319

v    ⊞ 22.4

t0    ⊞ −273
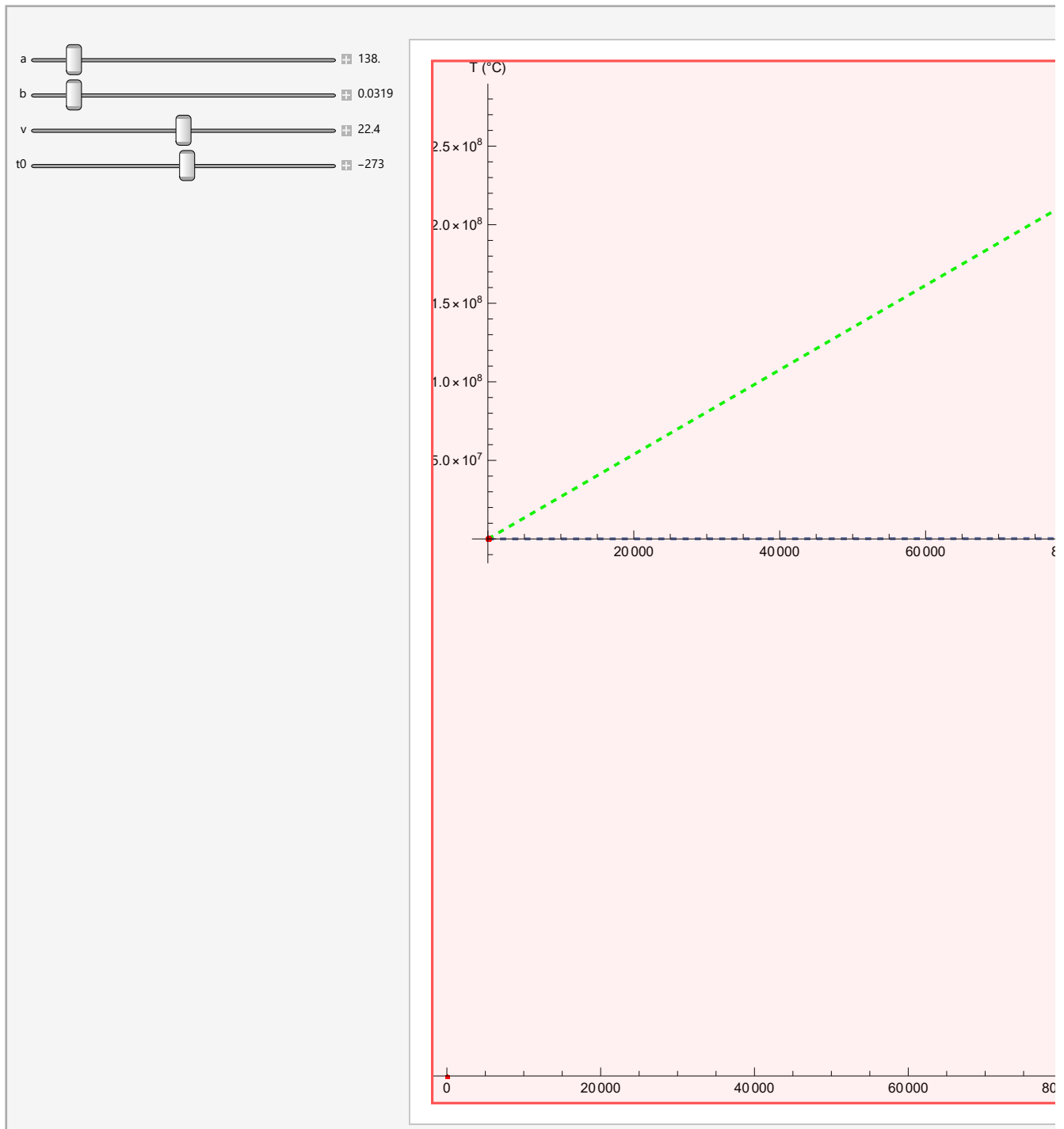


**Answer:**

The following is the best fit that I could get for the data:

| a ($kPa.m^6/(kmol)^2$) | b ($m^3/(kmol)$) | v ($m^3/(kmol)$) | $T_0$ (°C) |
|---|---|---|---|
| 1380. | 0. | 22.4 | −273 |

But this has major issues. Certainly, this does not reflect the true values for air as air is mostly comprised of Nitrogen and Oxygen.

It did feel like fitting the values of a and b to the wrong data. There seemed to be an offset for the data such that T0 and v values would have been deviated as non-ideal.

What truly fit the data was a curve with different T0 and v values, much closer to the Ideal Gas situation:

| a $(kPa.m^6/(kmol)^2)$ | b $(m^3/(kmol))$ | v $(m^3/(kmol))$ | $T_0$ $(°C)$ |
|---|---|---|---|
| 138. | 0.0319 | 23.85 | −283.5 |

---

This suggests that air can be approximated as an ideal gas fairly well, however, due to experimental error, we had a persistent offset in our data. It seems like we may have had an initial calibration error for the temperature and possibly the pressure (we may not have opened the connection when retrying our experiment for the 2nd time, causing a higher initial pressure than anticipated)
The offset in temperature calibration may also explain why the temperature did not drop any further than 7 degrees even though it was very stable and should have been closer to zero degrees in presence of an ice-water equilibrium. It also explains why our water seemed to be boiling close to 88 degree Celsius.

In[571]:=

```
(* This is an Input cell in case you need *)
12 / 273.15 * 100
```

Out[571]=

4.39319

Question 3.
- How much of the error that you found in your experimental value for absolute zero can be attributed to our assumption that air is an ideal gas? Explain briefly.
- Why do we need to use dashed line for extrapolation when the temperature is much colder? In other words, what does this dashed line implies? Explain briefly.

Answer 3.
- We had a 4.4% error in the experimental calculation of absolute zero. Since the line fit well, just with an offset, taking into account other it is fair to say that air is decent candidate to be considered an ideal gas and much of our error was probably due to experimental and calibration errors for the thermometer and pressure. *HOWEVER*, at low pressures, air's behaviour diverges from that of an ideal gas, hence extrapolating the linear fit to determine absolute zero may not be so accurate.
- Since air liquifies after a certain temperature, as temperature drops down, it is inaccurate to treat air with gas laws. However, we use a dashed line to extrapolate air's behaviour as if it remained in the gaseous state, and it just so happens that this interpolation for all gasses meets at absolute zero, even though the gases themselves are probably liquified until that point.

# Part II. Ideal Gas Law, **Volume vs Temperature**

In this part we will study volume vs temperature relation. You must have some hot water from the previous part (about 80°C).

We will connect the piston to the bulb, so the volume of the gas will be,

(4)    $V = V_{piston} + V_{bulb} + V_{tubes} = h\,A + V_{bulb} + V_{tubes} \approx h\,A + V_{bulb}$,

where $h$ is the height and $A$ is the area of the piston, and we have neglected the volume of the air inside the tubes when writing $\approx$.

## Step 1. Measurements (See Figure 1)

- Disconnect the pressure sensor from the bulb. You will release the extra pressure this way and the pressure of the gas will be about 101 kPa at initial hot temperature (about 80°C). Connect the piston tube with the blue (or maybe white) valve to the bulb tube, by tightening the screw.
- Connect the other piston tube to the pressure sensor, by tightening the screw. Set the initial height of the piston to about 80mm. Do not start taking data before understanding equation (4) and the goal for part II. See Figure 1. The valve must be open.
- Now let the temperature fall down (from about 80°C) and record the height of the piston at different temperatures. This process will be slow, so you can speed it up using small ice pieces. Add a small piece, let the ice melt and water reach equilibrium (stir if possible) and read the height and temperature. Now add another piece of ice and continue. Try to record at least six data points before the height reaches 10mm on the piston.
- Record the value of the pressure on the variable Pii below.

In[439]:=

```
Tvh = Rest [
```

| "h [± 1] (mm)" | "T [± 0.1] (°C)" |
|---|---|
| 72 | 75 |
| 69 | 72.8 |
| 62 | 70.7 |
| 55 | 66.5 |
| 48 | 64 |
| 38 | 61 |

```
];

hii = Tvh[[All, 1]];
Tii = Tvh[[All, 2]];
dT = 0.1;
Pii = 101.93 (*kPa*);
```

## Step 2. Data Analysis

- Measure the diameter of the bulb. This is the **DiaBulb variable** in the code below. DiaBulb is in the range 80mm to 120mm depending which setup you got.
- Using the equation (4) calculate $V$ using $h$ data. This is the **Vii variable** in the code below. You can read, Piston Diameter = 32.5 ± 0.1 mm in front of piston apparatus.
- Do a linear fit on the data and explain what each fit parameter represents.

- Find another estimation for zero Kelvin $T_0^{ii}$ and its error.
- Using the slope of the line and measured pressure $P$, estimate the number of moles, $n^{ii} \pm \Delta n^{ii}$.

**Answer:**

Using $V = V_{piston} + V_{bulb} + V_{tubes} = h\,A + V_{bulb} + V_{tubes} \approx h\,A + V_{bulb}$,

we know that V for any measurement i is hA + V_bulb, which tends to be the following value for a sequence of measurements:

{631880., 629391., 623584., 617777., 611970., 603674.} (units are mm^3)

In the linear fit,

$\text{FittedModel}\left[\ -241. + 0.000499\,x\ \right]$

| | Estimate | Standard Error | Confidence Interval |
|---|---|---|---|
| 1 | −240.889 | 18.6716 | {−292.73, −189.049} |
| x | 0.000498977 | 0.0000301256 | {0.000415335, 0.000582619} |

The intercept -241 represents the temperature that is interpolated for when volume approaches zero... i.e. The **intercept represents Absolute Temperature**.
Propagating error within out value for the diameter
Intercept: -240.889 ± 18.672 °C
Slope: 0.000499 ± 0.000030°C/mm^3

(*Calculate the number of moles*)
R = 8.314; (*Gas constant in J/(mol·K)*)
P = Pii*1000; (*Pressure we measured in Pa*)

 (*Since PV=nRT, n = PV/RT = P/(R*slope)*)
nii = P/(R*Slope*10^9)/1000 (*kmol*)

Hence, there is quite a bit of uncertainty in our measurement inherently. That is **11.7% error**. But given that we were bound to have a sigma of 18 degrees, it is well within the acceptable error range while also considering that the least count for our measurements was 1 mm and that we ignored the volume within pipes and some air may have escaped during the experiment, changing the con-straints.

Physically, the slope here represents rate of change of temperature $(T)$ with respect to volume $(V)$.

In[572]:=

```
(* This is an Input cell in case you need *)

DiaBulb = 103; (* mm *)
DiaPiston = 32.5; (* 32.5±0.1 mm *)

Vii = (hii * Pi * (DiaPiston / 2) ^ 2) + ((4 / 3) * Pi * (DiaBulb / 2) ^ 3);

TvV = Table[{Vii〚i〛, Tii〚i〛}, {i, 1, Length[Vii]}];

line = LinearModelFit[TvV, x, x]
line["ParameterConfidenceIntervalTable"]


(* Plotting *)
TvVErr = Table[{{Vii〚i〛, Around[Tii〚i〛, dT]}}, {i, 1, Length[Vii]}] ;
Show[
 Plot[line[x], {x, 0, Max[Vii]}],
 ListPlot[TvVErr, PlotRange → Full, PlotStyle → Red] ,
 AxesLabel → {"V (mm^3)", "T (°C)"}]

N[(-241 + 273) / 273];

Intercept = -240.88907720193626
Slope = 0.0004989768633798615
DeltaIntercept = 0.00003012559790407891
DeltaSlope = 18.671562165700085

(*Calculate the estimation for zero Kelvin*)
VT0 = -Intercept / Slope

(*Calculate the propagated error for zero Kelvin estimation*)
DeltaVT0 = VT0 * Sqrt[(DeltaIntercept / Intercept) ^ 2 + (DeltaSlope / Slope) ^ 2]

(*Calculate the number of moles*)
R = 8.314; (*Gas constant in J/(mol·K)*)
P = Pii * 1000; (*Pressure we measured in Pa*)

 (*Since PV=nRT, n = PV/RT =  P/(R*slope)*)
nii = P / (R * Slope * 10 ^ 9) / 1000 (*kmol*)

(*Output the calculated values*)
Print["Zero Kelvin Estimation (VT0): ", VT0, " mm^3"]
Print["Error in Zero Kelvin Estimation (DeltaVT0): ", DeltaVT0, " mm^3"]
Print["Number of Moles (nii): ", nii, " kmol"]
```
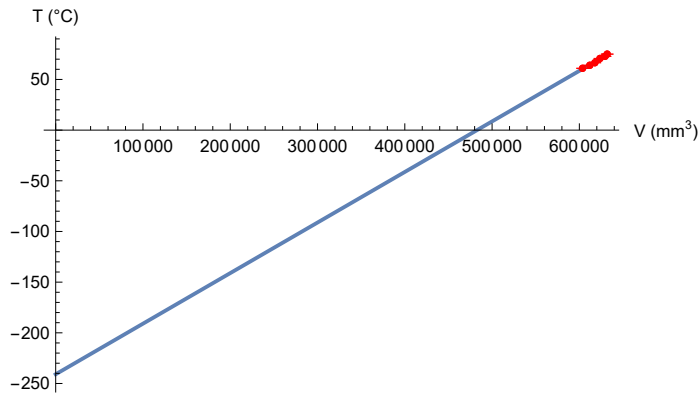
Out[576]=

FittedModel[ $-241. + 0.000499 x$ ]

Out[577]=

|   | Estimate | Standard Error | Confidence Interval |
|---|----------|----------------|---------------------|
| 1 | −240.889 | 18.6716 | {−292.73, −189.049} |
| x | 0.000498977 | 0.0000301256 | {0.000415335, 0.000582619} |

Out[579]=



Out[581]=

$-240.889$

Out[582]=

$0.000498977$

Out[583]=

$0.0000301256$

Out[584]=

$18.6716$

Out[585]=

$482766.$

Out[586]=

$1.8065 \times 10^{10}$

Out[589]=

$0.0000245704$

Zero Kelvin Estimation (VT0): 482766. mm^3

Error in Zero Kelvin Estimation (DeltaVT0): $1.8065 \times 10^{10}$ mm^3

Number of Moles (nii): 0.0000245704 kmol

In[593]:=

```
nii = nii (*kmol*)
T0ii = Intercept; (*°C*)
```

Out[593]=

$0.0000245704$

## Part III. Ideal Gas Law, **Pressure vs Volume**

In this part we will study pressure vs volume relation. Because the piston setup cannot tolerate too much pressure and either it would leak or the glass would break, we will only change the pressure about five percent. But if the data is analyzed carefully we can find the diameter of the bulb and compare it with the direct measurement of this diameter using a ruler. This way we can test the ideal gas law and see if $PV = $ const. gives us the correct answer.

Same as part II, we will connect the piston the the bulb, so the volume of the gas will be,

(4)  $V = V_{piston} + V_{bulb} + V_{tubes} = h\,A + V_{bulb} + V_{tubes} \approx h\,A + V_{bulb}$,

where $h$ is the height and $A$ is the area of the piston, and we have neglected the volume of the air inside the tubes when writing $\approx$.

## Step 1. Measurements (See Figure 1)

- Connect the piston tube without the blue (or maybe white) valve to the pressure sensor, by tightening the screw. Now open the blue (or maybe white) valve and adjust the piston so that it stands at 70mm mark position, and close the blue (or maybe white) valve.

- Now connect the tube with the blue (or maybe white) valve to the bulb tube, by tightening the screw, and **open the blue (or maybe white) valve**. The volume of the gas is given by equation (4). Both piston apparatus and the bulb must stay at room temperature the whole time. See Figure 1. You do not need the water bath anymore.

- Now read the height of the piston versus pressure and record your measurements on the table below. 70mm must read 101-to-102 kPa (if the pressure sensor is calibrated). We will do the assumption that the system thermalizes quickly and stays at the room temperature. This assumption will let us do measurements quickly. The reason we are trying to make it fast is to minimize the leaking. After each measurement, if the reference 70mm does not read the correct pressure (too much leaking happened), try to loosen the screw on the pressure sensor and adjust again. 0.10 kPa tolerance for the pressure is reasonable.

Another idea which might help is do the measurements in a zigzag fashion, for example, you can go below 70mm, measuring pressure for 65mm, then coming back up and measuring pressure for 75mm. So about the similar amount that leaks out, leaks in.

In[123]:=

hvP = Rest

| "P [± 0.10] (kPa)" | "h [± 1] (mm)" |
|---|---|
| 102.01 | 70 |
| 102.69 | 65 |
| 101.19 | 75 |
| 102.84 | 64 |
| 101.27 | 74 |
| 102.25 | 67 |
| 101.48 | 73 |
| 102.8 | 63 |
| 101.32 | 72 |
| 102.25 | 66 |
| 101.53 | 71 |
| 102.09 | 69 |

;

```
Piii = hvP[[All, 1]];
hiii = hvP[[All, 2]];
dh = 1; (* mm *)
Tiii = 299.2; (* K *)
```

## Step 2. Data Analysis

- Using the idea gas law and the equation (4) for the volume, do the non-linear fit $V_{piston} = k/P - V_{bulb}$, where $k$ and $V_{bulb}$ are the constant fit parameters.

- Using the fitting parameters, find the diameter of the bulb and its error, $D_{bulb} \pm \Delta D_{bulb}$. Compare with the

measured DiaBulb and explain the sources of error.

- Using the other fit parameter denoted as *k*, and the temperature (which is taken to be a constant), estimate the number of moles, $n^{iii} \pm \Delta n^{iii}$. Is this smaller or bigger than number of moles you found in part II, $n^{ii} \pm \Delta n^{ii}$? Is this expected? Why?

**Answer:**

Put your work here.

**Vb=491999 mm^3**

Vb=pi*r^3*4/3=pi*(Db/2)^2*4/3

Db=2(3Vb/4Pi)^1/3

**Dbulb=97.68 mm**

DeltaVb = 38915 mm^3

DeltaDb = N[DeltaVb*2/(Pi*97.68^2)] (Error in Diameter = error in Volume * 2/(Pi*D^2)) = **2.59649 mm (less than 3% error)**

 this measurement of the bulbs diameter is smaller than what we observed before, by about 5.5 millimeters, this does however fall within our calculated standard error of the diameter.

PV=nRT

let k represent PV (we know this to be true since  k/P would be in mm^3 since it shares the same units with the volume. Thus, the Ideal gas law using Kpa as it's unit for pressure means k must be in Kpa*mm^3. Knowing this we can replace PV with K in our Ideal gas Formula. Thus

n=k/RT

then plug in

n=2.25*10^-5 mols or 2.25181*10^-8 kmols

we get our uncertainty or error by using the same formula but using the error in k insetad of k:

(dke*(10^-9))/(R*T)=$1.59524 \times 10^{-6}$ mols or $1.59524 \times 10^{-9}$ kmols

This value is significantly smaller than what we found in Part II. This is most likely due to the air being lost while valves were being opened and other adjustments or leakage. This is also the reason for some difference in pressure. The escaping of the air results in a loss of particles inside the system.

In[595]:=

```
(* This is an Input cell in case you need *)


DiaPiston = 32.5; (* mm *)
Vpiston = hiii * Pi * DiaPiston^2 / 4 ; (*mm^3*)
dVpiston = dh * Pi * DiaPiston^2 / 4; (*mm^3*)


VpistonvP = Table[{Piii[[i]], Vpiston[[i]]}, {i, 1, Length[Piii]}];


(* Nonlinear Fit V =
 k/P + Vb. k and Vb are the fit parameters. k depends on temperature and number of moles,
and Vb is the volume of the bulb. *)
line = NonlinearModelFit[VpistonvP, k / x - Vb, {k, Vb}, x]
line["ParameterConfidenceIntervalTable"]


(* Plotting *)
hvPErr = Table[{{Piii[[i]], Around[Vpiston[[i]], dVpiston]}}, {i, 1, Length[Piii]}] ;
Show[
 Plot[line[x], {x, 70, 120}],
 ListPlot[hvPErr, PlotStyle → Red, PlotRange → Full],
 AxesLabel → {"P (kPa)", "V (mm^3)"}]


DeltaVb = 38 915
DeltaDb = N[DeltaVb * 2 / (Pi * 97.68^2)]
```
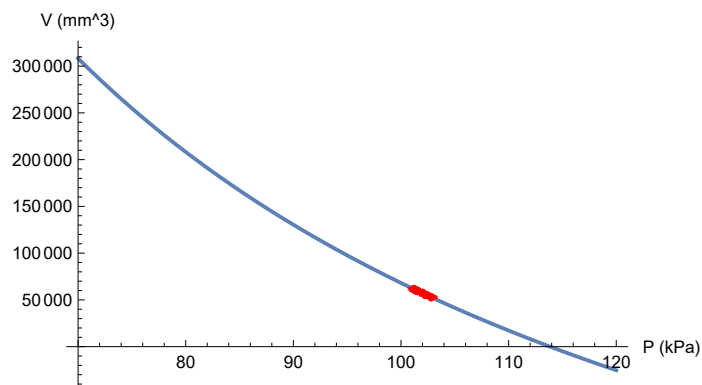
Out[599]=

$$\text{FittedModel}\left[\ -4.92 \times 10^5 + \frac{5.6 \times 10^7}{x}\ \right]$$

Out[600]=

|    | Estimate | Standard Error | Confidence Interval |
|----|----------|----------------|---------------------|
| k  | $5.60149 \times 10^7$ | $3.96823 \times 10^6$ | $\{4.71731 \times 10^7, 6.48566 \times 10^7\}$ |
| Vb | 491 999. | 38 915. | $\{405\,291., 578\,707.\}$ |

Out[602]=



Out[603]=

38 915

Out[604]=

2.59649

In[605]:=

```
DBulb = 97.68; (*mm*)
niii = 2.25181 * 10^-8; (*kmol*)
```

Table X. Copy All Your Final Results.

```
In[607]:=
Grid[{{Text["Table X. All Final Results Gathered"], SpanFromLeft},
   {"T₀ [I.2] (°C)", "v [I.2] (m³/kmol)", "T₀ii [II.2] (°C)",
    "nii [II.2] (kmol)", "Bulb Diameter [III.2] (mm)", "niii [III.2] (kmol)"},
   {T0, v, T0ii, nii, DBulb, niii}
  }, Frame → All]
```

Out[607]=

| Table X. All Final Results Gathered | | | | | |
|---|---|---|---|---|---|
| $T_0$ [I.2] (°C) | v [I.2] (m³/kmol) | $T_0^{ii}$ [II.2] (°C) | $n^{ii}$ [II.2] (kmol) | Bulb Diameter [III.2] (mm) | $n^{iii}$ [III.2] (kmol) |
| −282.85 | 0.0238108 | −240.889 | 0.0000245704 | 97.68 | $2.25181 \times 10^{-8}$ |

We thus see that our T0 value for part I is decently close to the theoretical value. There seems to be some offset in the value, probably due to measurement error and not releasing the pressure at close to standard temperature (0 degree Celsius) conditions.

T0 for Part II is much more error prone (11%) but since the error (at-least 8%) was bound to be there due to our least count of our piston, compared to our measured volume, the error was acceptable while keeping in mind other error causes such as leakage of air.

The moles in our final experiment seem to be much lower than in Exp II. This is possibly because a lot of the air may have leaked under pressure due to imperfections in the piston. Also, again, this experiment was very error prone when it came to number of moles since the measurements were imprecise due

# to the least count on the piston.

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"Ideal & Non-Ideal Gases"
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to the least count on the piston.