LAB 5: Linear Transformations, Inverse matrices

(Math 250: Sections C3)

Name: Aryan Malhotra

Last 4 digits of RUID: 8256

Date: 11/25/2024

Learning Outcomes

- The standard matrix associated with a linear transformation on $\mathbb{R}^2$
- Properties of inverses and patterns among inverses

Table of Contents

Random Seed

Initialize the random number generator by typing the following code, where abcd are the last four digits of your RUID

```
rng('default');
rng(abcd, 'twister');
```

```
% Enter your code here
rng('default');
rng(8256, 'twister');
```

This will ensure that you generate your own particular random vectors and matrices.

The lab report that you hand in must be your own work. The following problems use randomly generated matrices and vectors, so the matrices and vectors in your lab report will not be the same as those of other students doing the lab. Sharing of lab report files is not allowed in this course.

Question 1. Linear Transformations in $\mathbb{R}^2$ (Total Points - 6)

In this question you will investigate the stadard matrices of certain linear transformations in 2-dimensional space.

First create a geometric figure of a house which will be our test object.

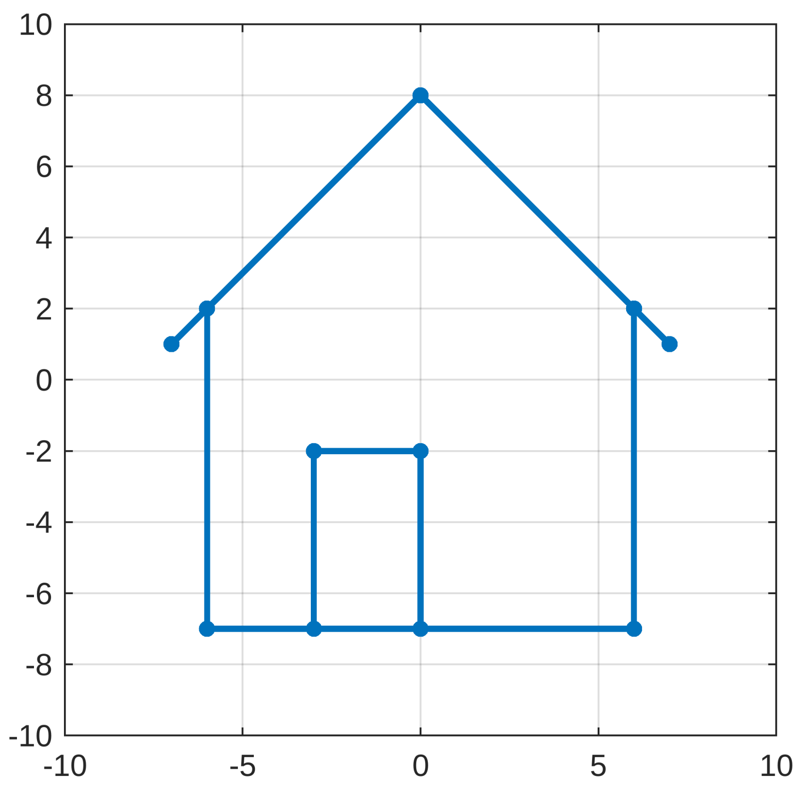
Use the following code to create a figure of a house.

```
x = [-6 -6 -7 0 7 6 6 -3 -3 0 0;-7 2 1 8 1 2 -7 -7 -2 -2 -7] % points on the house figure
dot2dot(x) % helper code dot2dot will connect the points
```

```
% Enter your code here
x = [-6 -6 -7 0 7 6 6 -3 -3 0 0;-7 2 1 8 1 2 -7 -7 -2 -2 -7] % points on the house figure
```

```
x = 2×11
    -6    -6    -7     0     7     6     6    -3    -3     0     0
    -7     2     1     8     1     2    -7    -7    -2    -2    -7
```

```
dot2dot(x) % helper code dot2dot will connect the points
```



(a) Recall (textbook section 1.9) that the columns of the standard matrix associated with a linear transformation are obtained by applying the transformation to the standard unit vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Example: The standard matrix for a counterclockwise rotation by 90 degrees (pi/2).

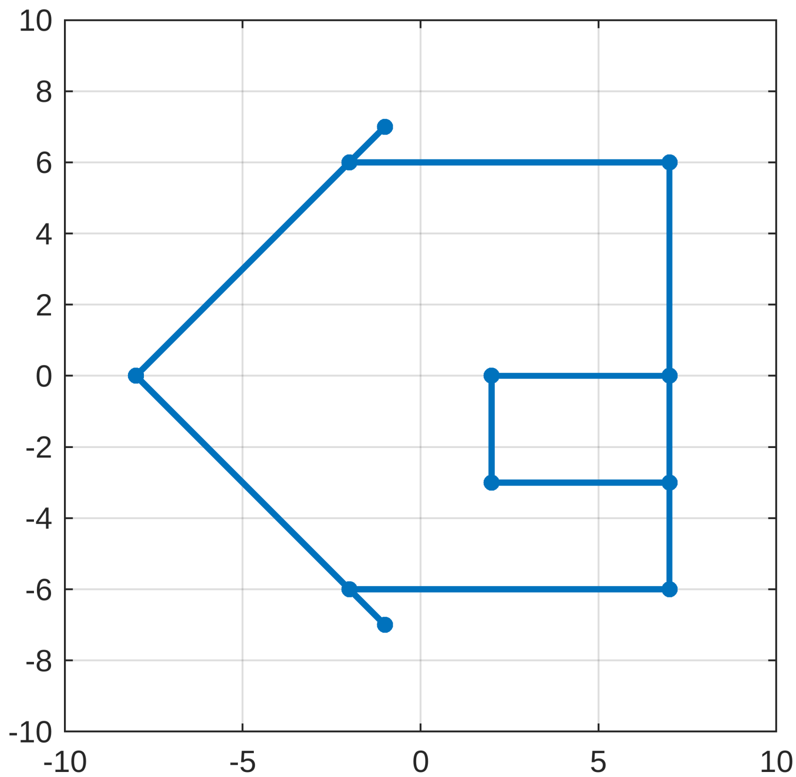
This will take e_1 to $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and e_2 to $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$. This leads to following code to create the matrix, and then to rotate the house figure counterclockwse by pi/2.

```
R_90 = [0 -1; 1 0] % create the 90 degree rotation matrix
dot2dot(R_90*x) % draw the figure on ratated points
```

```
% Enter your code here
R_90 = [0 -1; 1 0] % create the 90 degree rotation matrix
```

```
R_90 = 2×2
     0    -1
     1     0
```

```
dot2dot(R_90*x) % draw the figure on ratated points
```



(i) ***Exercise (Points - 2):*** Find the standard matrix R_30 for the linear transformation which rotates counterclockwise by 30 degrees (pi/6).

Hint: When e_1 and e_2 rotate, they stay on the unit circle. The end points of the rotated e_1 and e_2 will be of the form (cos(theta), sin(theta)) and (cos(theta+pi/2), sin(theta+pi/2))

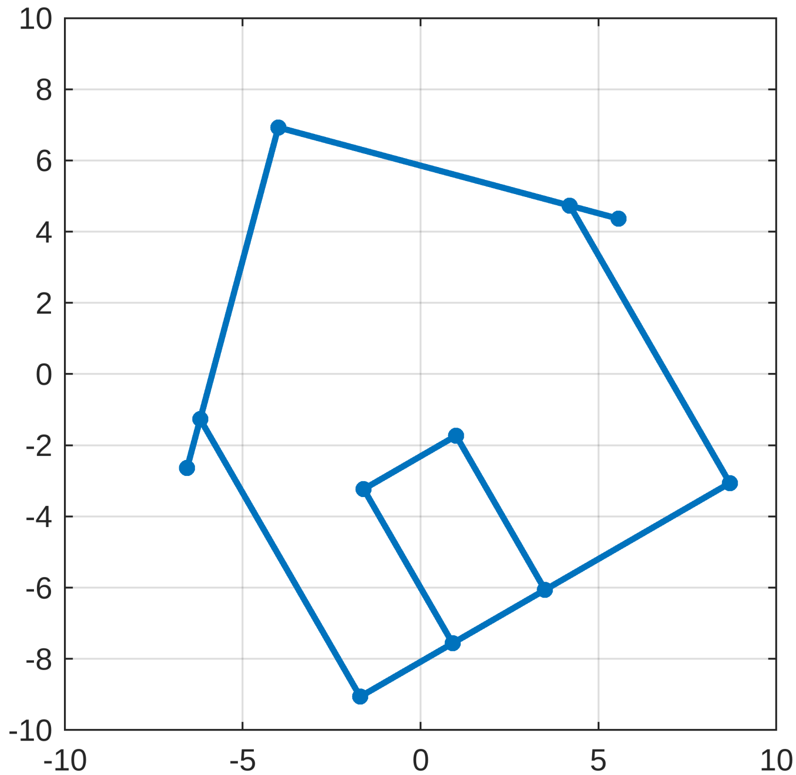
```
% Enter the code to create the matrix R_30 here
```

```
R_30 = [cosd(30) cosd(90+30); sind(30) sind(90+30)] % create the 90 degree rotation matrix
```

```
R_30 = 2×2
    0.8660   -0.5000
    0.5000    0.8660
```

Now check how your R_30 rotates the house figure. Enter and run the appropriate code in the box below.

```
% Enter the code to produce the rotated figure here
dot2dot(R_30*x) % draw the figure on ratated points
```



(ii) ***Exercise (Points - 2):*** Obtain the formula for a general rotation matrix that rotates counterclockwise by an angle theta. (The trig identities for cos(theta + pi/2) and sin(theta + pi/2) will be helpful).

Answer: R_theta = [cosd(theta) cosd(90+theta); sind(theta) sind(90+theta)]

Where theta is in degrees. Here, [a b;c d] creates row vectors [a b] and [c d] of a matrix.

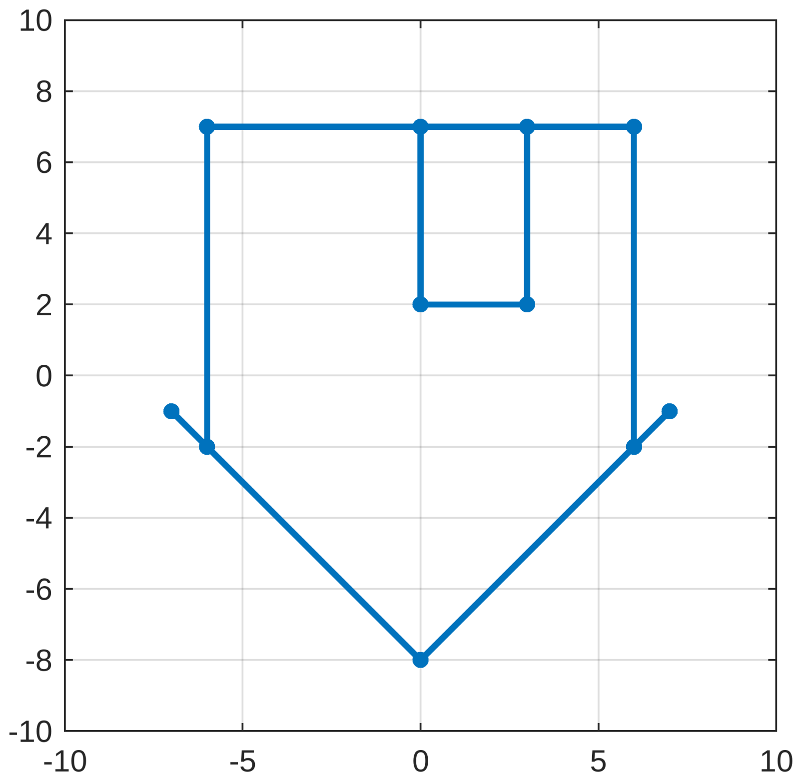
(b) ***Exercise (Points - 2):*** Find the standard matrix R_down for the linear transformation that will flip the figure upside down.

```
% Enter the code to create R_down here
R_upsideDown = [cosd(180) cosd(90+180); sind(180) sind(90+180)]
```

```
R_upsideDown = 2×2
    -1     0
     0    -1
```

Enter and run the appropriate code in the box below to produce the upside down figure.

```
% Enter the code to produce the upside down figure here
dot2dot(R_upsideDown*x)
```



Question 2. Patterns among Inverses of Matrices (Total Points - 4)

We want to investigate some patterns among matrices and their inverses.

(a) Let A be the 3x3 matrix consisting of zeros along the off-diagonal and ones elsware. Find the inverse of A using the following code.

```
A = [1 1 0; 1 0 1; 0 1 1]
B = inv(A)
```

```
% Enter your code here
A = [1 1 0; 1 0 1; 0 1 1]
```

```
A = 3×3
     1     1     0
     1     0     1
     0     1     1
```

```
B = inv(A)
```

```
B = 3×3
    0.5000    0.5000   -0.5000
    0.5000   -0.5000    0.5000
   -0.5000    0.5000    0.5000
```

Notice the patterns among the entries of A and B.

(i) ***Exercise (Points - 1):*** Explain why this makes the product AB equal to I.

Answer: A has a 1 everywhere except the offdiagonals. (0 at the offdiagonals). B has 0.5 everywhere but the offdiagonals where it has -0.5.

Since makes perfect sence because due to this, each row vector pair for product AB will lead to a 0 (0.5-0.5) except on the diagonals where you have 1 (0.5+0.5).

Hence, leading to an identity matrix of dimensions 3x3.

(b) Another pattern present in the A and B above is that they are both symmetric matrices, i.e. $A = A^T$ and $B = B^T$.

Explore if the inverses of (invertible) symmetric matrices are also symmetric using the following matrices C and D

```
C= ones(3) + eye(3)
D0 = rmat(3,3); D = D0+D0'
```

```
% Enter your code here
C= ones(3) + eye(3)
```

```
C = 3×3
     2     1     1
     1     2     1
     1     1     2
```

```
D0 = rmat(3,3); D = D0+D0'
```

```
D = 3×3
    18    10    10
    10     8    12
    10    12    18
```

(i) ***Exercise (Points - 1):*** Find the inverses of C and D.

```
% Enter your code here
inv(C), inv(D)
```

```
ans = 3×3
    0.7500   -0.2500   -0.2500
   -0.2500    0.7500   -0.2500
   -0.2500   -0.2500    0.7500
ans = 3×3
   -0.0000    0.3000   -0.2000
    0.3000   -1.1200    0.5800
   -0.2000    0.5800   -0.2200
```

(ii) ***Exercise (Points - 1):*** Are the inverses of C and D symmetric?

Answer: Yes, it does turn out that the inverses of C and D are symmetic. (Their transpose is the same as the matrix itself)

(iii) ***Exercise (Points - 1):*** Prove that if A is a symmetric matrix then its inverse is also symmetric, i.e. show that if $A = A^T$, then $(A^{-1})^T$ is the same as A^{-1}.

Answer: We know that $(A^{-1})^T = (A^T)^{-1}$ (because (A−1)T=(A−1)TAT(AT)−1=(AA−1)T(AT)−1=idT(AT)−1=(AT)−1)

but since $A = A^T$, that implies that $(A^{-1})^T = (A^T)^{-1} = (A)^{-1}$

Hence proved.

Helper Function (Do Not Edit)

```
function A = rmat(m,n)
A = fix(10*rand(m,n));
end
```

```
function dot2dot ( X )
  X(:,end+1) = X(:,1);
  plot ( X(1,:), X(2,:), '.-', 'markersize', 18, 'linewidth', 2 );
  grid ( 'on' );
  axis ( 10 * [ -1, 1, -1, 1 ] );
  axis ( 'square' );
  return
end
```