Vectors True & False

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1. $u \cdot u = |u|$

   False.

   For instance, $(2, 0) \cdot (2, 0) = 4$

   While $|(2, 0)| = 2$

   ($u \cdot u = |u|^2$ instead)

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2. $u \cdot v > 0 \implies \text{angle between the vectors is acute}$

   True.

   Since $u \cdot v = |u||v|cos(\theta)$

   Hence, if $u \cdot v$ is positive, $\implies cos(\theta) > 0 \implies \theta \in [0, \pi)$

   ($|u||v|$ is positive by definition)

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3. $|u \times v| = |u||v|cos(\theta)$

   False.

   ($|u \times v| = |u||v|sin(\theta)$ instead)

   $(1, 0) \times (1, 0) = (0, 0)$ (since $u \times u = \vec{0}$)

   But as per the given formula, $(1, 0) \times (1, 0) = 1$ which is wrong!

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4. $u \times v = v \times u$

   False.

   ($u \times v = -v \times u$ instead)

   $(1, 0, 0) \times (0, 1, 0) = (0, 0, 1) \neq (0, 0, -1) = (0, 1, 0) \times (1, 0, 0)$

   (i.e., $i \times j = k \neq -k = j \times i$)

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5. u, v are unit vectors, so is $u \times v$

   False.

   $\hat{i} \times \hat{i} = \vec{0}$ and $\vec{0}$ is NOT a unit vector

6. $(u \times v) \cdot u = (u \times v) \cdot v = 0$
   True.
   Since by definition, $u \times v$ is perpendicular to both u and v, it's dot product with either vector would be 0 since they completely misalign (more formally, $\theta = \frac{\pi}{2}$)

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7. $|u + v| = |u| + |v|$
   False.
   A very simple Pythagorean theorem example!
   $|3\hat{i} + 4\hat{j}| = 5 \neq 3 + 4$

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8. $(u \times v) \times w = u \times (v \times w)$
   False.
   $(j \times j) \times i = \vec{0}$
   But $j \times (j \times i) = j \times -k = -i$

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9. $(u + v) \times (u - v) = 2(v \times u)$
   True.
   $LHS = (u \times u) - (u \times v) + (v \times u) - (v \times v)$
   $= (v \times u) + (v \times u)$ (since $u \times v = -v \times u$)
   $= 2(v \times u)$

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10. $(u + v) \cdot (u - v) = |u|^2 - |v|^2$
    True.
    $LHS = u \cdot u - u \cdot v + v \cdot u - v \cdot v$
    $= |u|^2 - |v|^2$
