HOMEWORK 8.

1. • $-2A \Rightarrow$ Scaling every value by -2.

$$\begin{bmatrix} -2(2) & -2(0) & -2(-1) \\ -2(4) & -2(-3) & -2(2) \end{bmatrix} = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix}$$

$$\cancel{B-2A = \begin{bmatrix} 7+4 & -5-0 & 1-2 \\ 1+8 & -4-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 11 & -5 & -1 \\ 9 & -10 & 1 \end{bmatrix}}.$$

• $B-2A = B+(-2A) = \begin{bmatrix} 3 & -5 & 3 \\ -7 & 2 & -7 \end{bmatrix}$.

• AC. $\quad [A]_{2\times3}[C]_{2\times2} \rightarrow$ undefined since # columns of A $\neq$ # rows of C.

• $$CD = \left[\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix} \right]$$

$$= \left[\begin{pmatrix} 3-2 \\ -6-1 \end{pmatrix} \begin{pmatrix} 5+8 \\ -10+4 \end{pmatrix} \right] = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}.$$

7. $A_{5\times3}\, B_{m\times n} = C_{5\times7}$.

$m=3$ for valid multiplication, $7=n$

$\therefore$ B has dims 3×7.

8. $(BC)_{3\times4} = B_{m\times n}\, C_{r\times q} \qquad m=3, \quad q=4.$

$\therefore$ B has 3 rows.

9. $AB = \begin{bmatrix} 3\times5 + 4\times3 & 3\times-6 + 4\times k \\ -2\times5 + 1\times3 & -2\times-6 + 1\times k \end{bmatrix} = \begin{bmatrix} 27 & 4k-18 \\ -7 & k+12 \end{bmatrix}.$

$BA = \begin{bmatrix} 5\times3 + -6\times-2 & 5\times4 + -6\times1 \\ 3\times3 + -2\times k & 3\times4 + k\times1 \end{bmatrix} = \begin{bmatrix} 27 & 14 \\ 9-2k & 12+k \end{bmatrix}.$

$9-2k = -7$ & $4k - 18 = 14.$

$\Rightarrow 16 = 2k$ $\quad 4k = 32$

$\boxed{k=8}$ $\quad \boxed{k=8.}$

$\boxed{k=8}$

28. a_{ij} th ~~comp~~ element of AB is

~~[ith column of a] · [jth~~

[ith row of A] × [jth column of B].

since all 2nd columns have $j=2$,

all of them would be a multiple of the zero vector ($\vec{0}$)

& would hence be $\boxed{0.}$

1). $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix} = A \Rightarrow A^{-1} = \frac{1}{|A|} \operatorname{adj} A.$

$= \frac{1}{16-15} \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}.$

$$8x_1 + 3x_2 = 2$$
$$5x_1 + 2x_2 = -1$$
$$\Rightarrow \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$A \cdot X = b$$

$$\Rightarrow A^{-1} A \cdot X = A^{-1} b.$$

$$\Rightarrow X = A^{-1} b.$$

$$= \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2\times 2 + -3\times -1 \\ -5\times 2 + 8\times -1 \end{bmatrix}.$$

$$\therefore x_1 = 7$$
$$x_2 = -18.$$

45. $A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}.$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A.$$

$$= \frac{1}{|A|} \cdot (\text{co-factor matrix})^T.$$

hence, third column of the A^{-1} would correspond to transpose of co-factor matrix's third row (scaled by $\frac{1}{|A|}$).

$$\Rightarrow \frac{1}{|A|} \begin{bmatrix} -42+45 & -(-12+18) & -10+14 \end{bmatrix}^T$$

$$|A| = -2(20-18)$$
$$+7(8-6)$$
$$\cancel{-9(-10+14)}$$
$$-9(6-5)$$
$$\cancel{= -4+14-38.}$$
$$\cancel{= -26.}$$
$$= -4+14-9$$
$$= 1.$$

$$= \cancel{\begin{bmatrix} -3/26 \\ 6/26 \\ -4/26 \end{bmatrix}} = \begin{bmatrix} -3 \\ 6 \\ -4 \end{bmatrix}.$$