KINEMATICS: THE BOUNCING BALL

Lab Report

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Section: H4
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Purpose

To understand the graphical relationships between displacement, velocity, and acceleration, slope and derivative, area under a curve and integral.
To learn data acquisition techniques.
To understand connection between theoretical predictions and experimental data.

Readings

Here you can read the arguments about bouncing ball.
And you can learn the details from these videos:
Bouncing Ball – Elastic
Bouncing Ball - Inelastic

Some References to *Mathematica* Commands

Drawing Tools

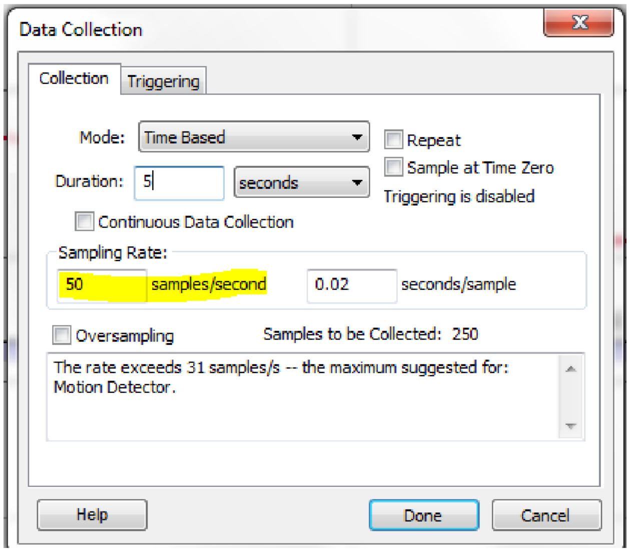
Open the drawing tool by typing Ctrl+D, choose the Line tool or the Freehand Line tool to draw lines, choose Text tool to make labels.

Procedure

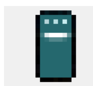
In this lab you will first use the data acquisition program LoggerPro to measure and analyze the one-dimensional motion of a bouncing ball. Then you will compare the data with solutions of the kinematic equations.

Setting Up the Data Collection Program LoggerPro

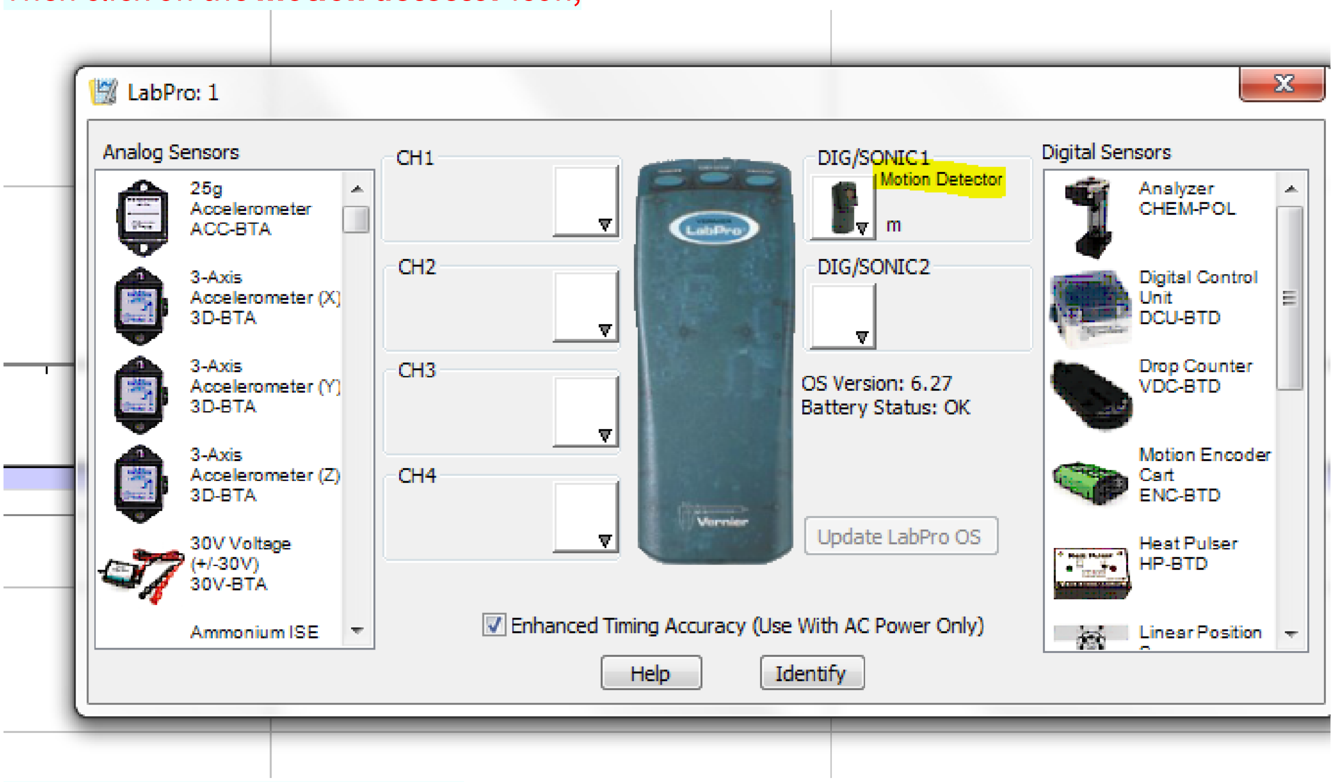
- Load the computer program **LoggerPro** by double clicking on its icon.
- Click on the clock icon, and set the data collection rate to 50 Hz.



- Set up LoggerPro such that up is the positive direction and the floor (not the sensor) is the position = 0.

Click on the LoggerPro icon  .

Then click on the **motion detector** icon,



and check **“reverse direction”**.
Now Calibrate **“zero”** so that you measure height from the floor level, i.e. y=0 must represent the floor.
You can run the sensor without anything in front of it so it echoes from the floor.

Some References to *Mathematica* Commands

Drawing Tools

Open the Drawing Tools by pressing Ctrl+D, choose the Line tool  or the Freehand Line tool  to draw

lines,
choose Text tool T≡ to make labels.

Dialog:

Part I. Data & Observations

Step 1. The data and labeling (U, T, D)

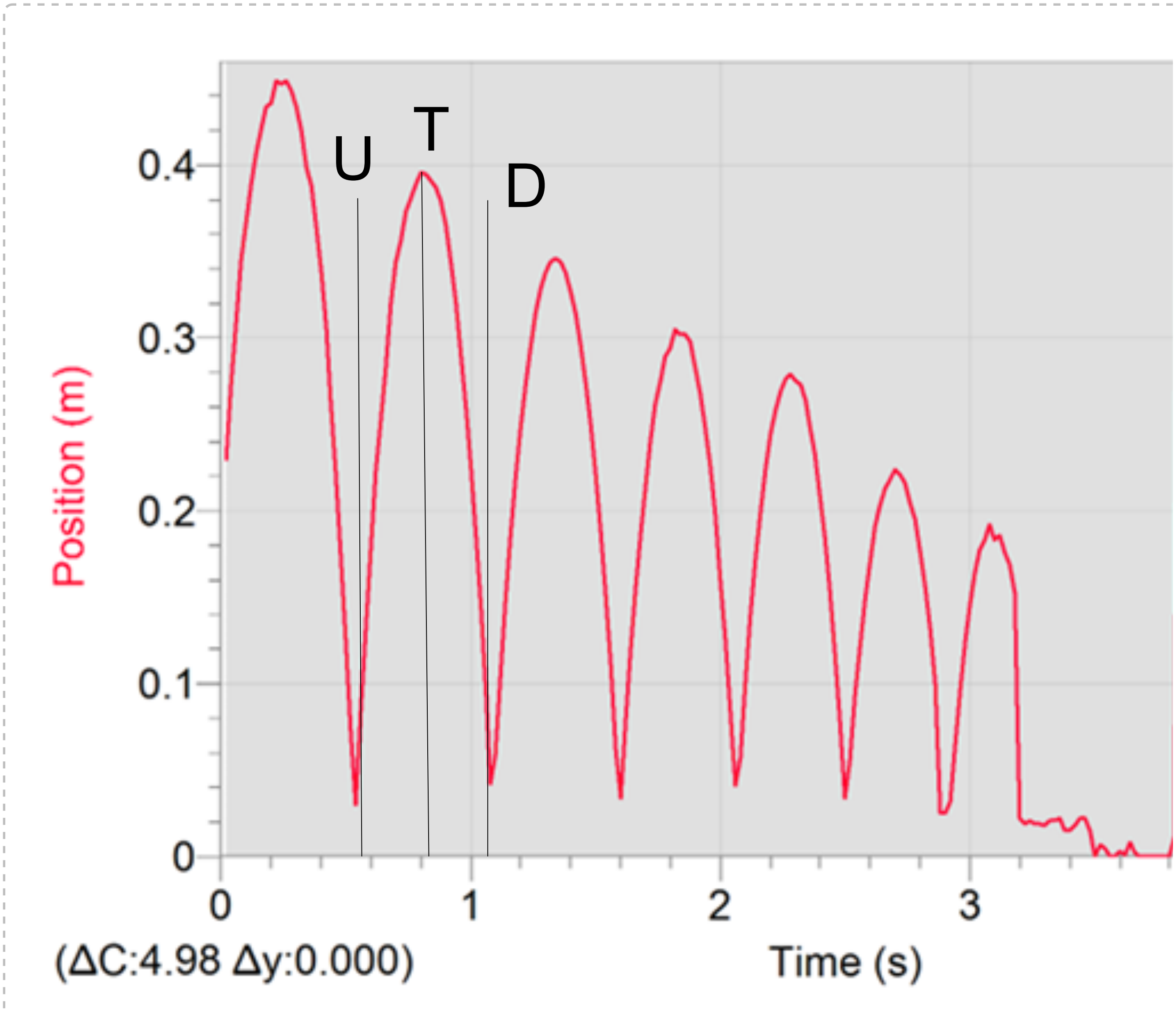
Start by taking data for a bouncing golf ball using the motion detector. Drop the ball right under the motion detector.
Do not let the ball get any closer than 4 inches to the motion detector or it will not function properly. REMEMBER: You are studying **one dimensional** motion so the ball must be bouncing **vertically** under the motion detector.
You will need to get at least two good bounces in your data. When you are satisfied with your data, be sure to save it (to prevent it from being lost if you restart taking data).

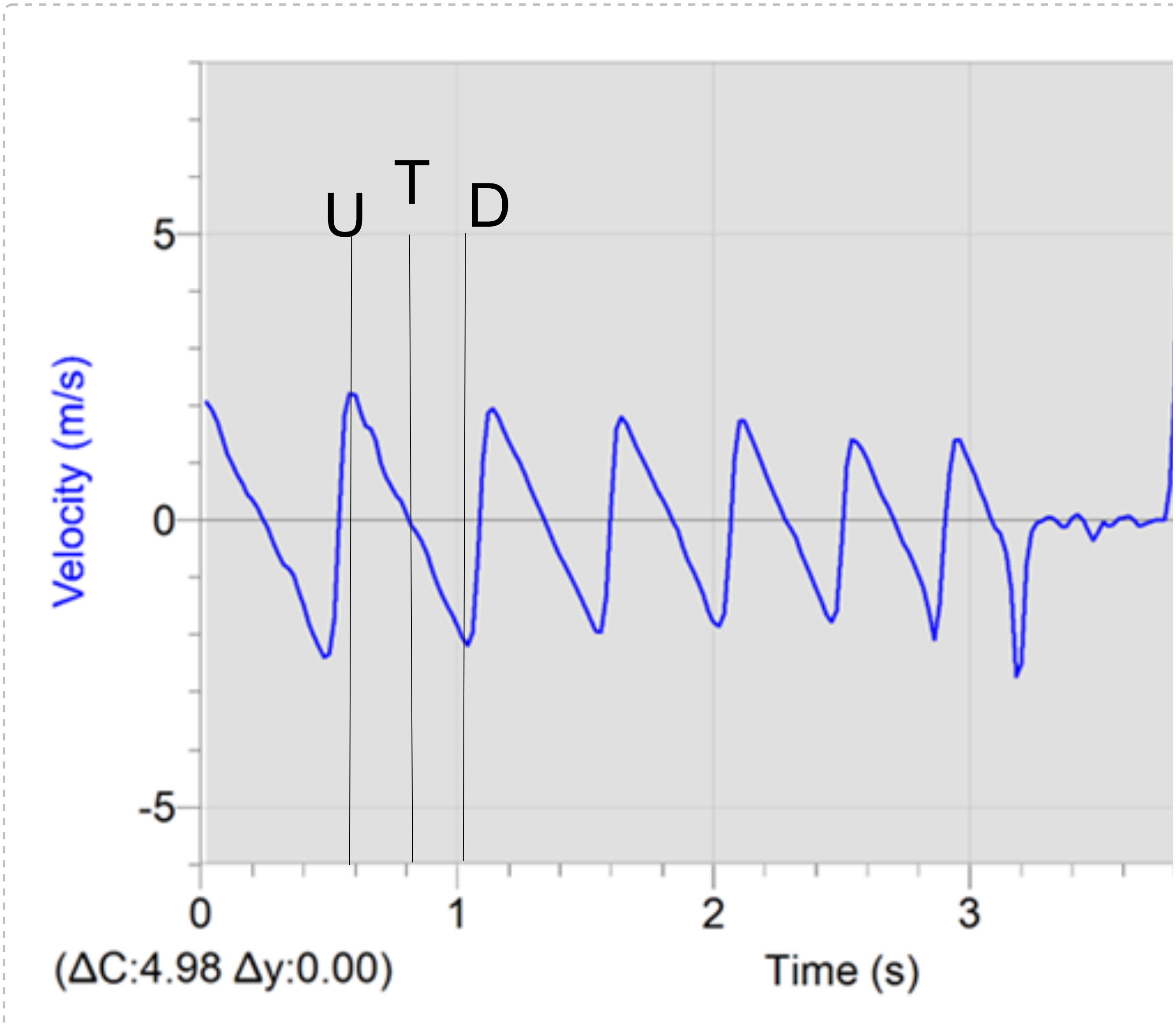
Now change the LoggerPro settings to display the velocity and acceleration curves in addition to the displacement. Follow these steps inside LoggerPro:
- Select **Graph** from **Insert** pull-down menu.
- Click on the y-axis label and choose velocity (acceleration).
- Select **Auto Arrange** from **Page** pull-down menu.

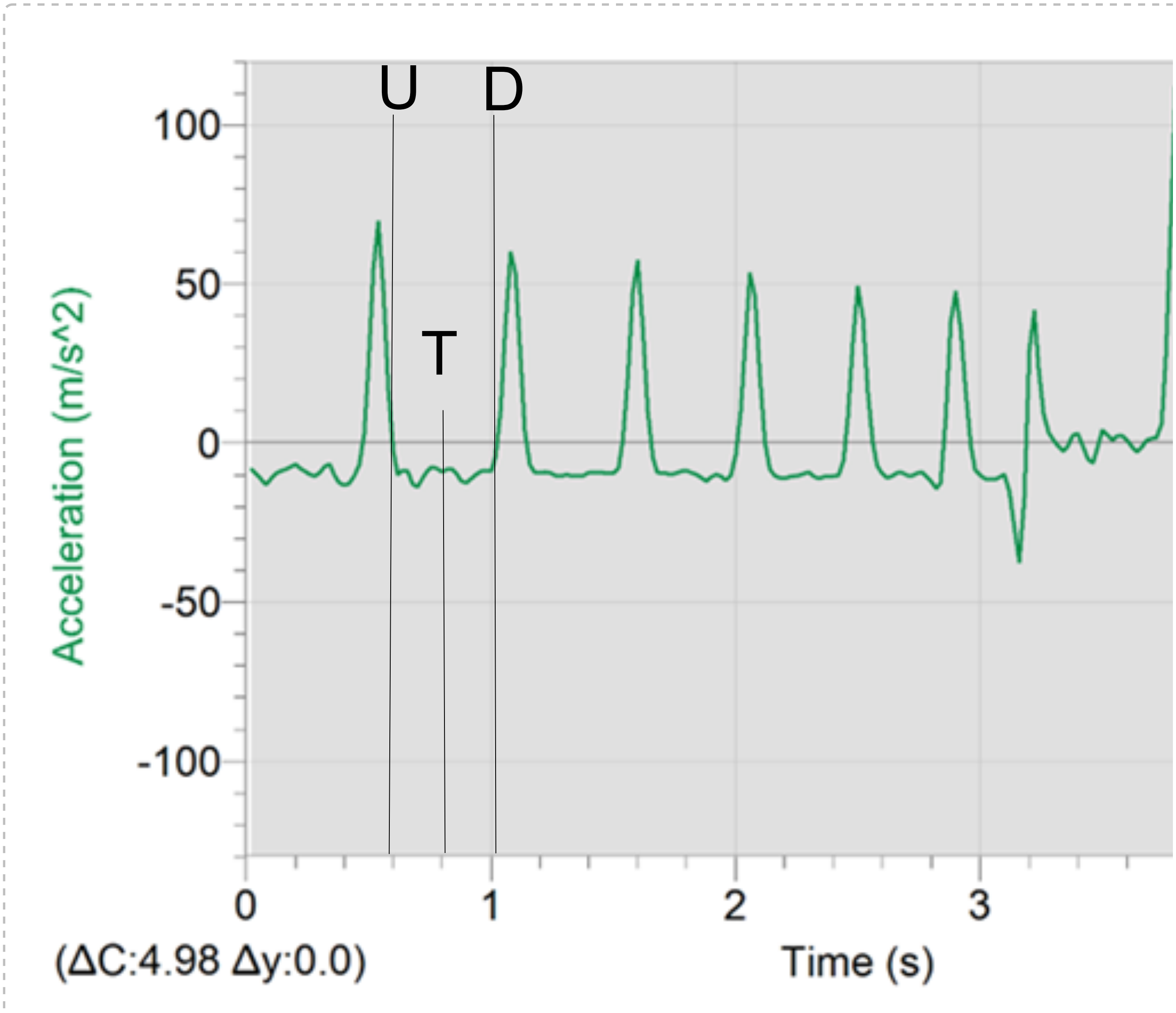
Compare your predictions with these curves. If you made an error, be sure to understand why.
My predictions in the pre-lab document were very similar to the actual data we just collected. The velocity-time plot had straight lines with negative gradient. At every bounce, the velocity would suddenly increase but to a lower value than the previous local maxima. The acceleration remains constant at around -10m/s^2 (g) but during every bounce, the curve rockets up to a very positive value.

The mistake I made was I thought the acceleration during bounce would have a lower magnitude than g since the ball reached a lower high. However, in reality it has a greater acceleration during that time. It does have a lower Impulse but the acceleration is indeed higher than the magnitude of g. It's just that the time duration of the acceleration is so short, the ball doesn't reach as high as it started.

Copy and paste the displacement, velocity, and acceleration vs. time plot frames below.


U
T
D
0.4
0.3
0.2
0.1
0
Position (m)
0
1
2
3
Time (s)
(ΔC:4.98 Δy:0.000)


U
T
D
Velocity (m/s)
5
0
-5
0
1
2
3
(ΔC:4.98 Δy:0.00)
Time (s)



Consider the region for **two bounces** and on the “Time” axis label **time just after the first bounce** with letter **U** (for moving up), the **top of the trajectory** with letter **T** (for top), the time **just before the next bounce** with letter **D** (for moving down). [Open the drawing tools by typing Ctrl+D, choose the Text tool]
- Choose the Line tool and draw vertical lines at U, T, and D on the all three plots.

Step 2. Focus on the interval T to D, during which the ball drops from the top of its trajectory until just before it hits the floor, and answer the following questions (remember that the up direction is positive):

Question 1:
For the velocity graph, what shows that the ball’s motion was away from the detector?

After reaching the top at point T, the velocity is negative, meaning that the ball was moving downwards and hence away from the detector.

Question 2:
- For the velocity graph, what shows that the ball was speeding up?
- How would a graph of motion with a constant velocity differ?

The velocity starts off at point T at v=0. It then continues to be more and more negative, meaning it is picking speed in the negative direction. Hence, it is evident that the ball is speeding up (the magnitude of the velocity is increasing).

A graph of motion with constant velocity would have no gradients. It would have a tangent slope of 0.

Question 3:
- Is the acceleration positive or negative when the ball is speeding up?
- What is the sign of the velocity while the ball is speeding up?
- What can you conclude about the relationship between the directions of the velocity and acceleration vectors when the ball is speeding up?

Since the change in velocity in the interval is negative, the acceleration is negative.
The sign of velocity while speeding up is also negative. It’s is increasing in magnitude but actually getting more and more negative.
If the ball is speeding up, the magnitude of it’s velocity is increasing. Meaning that acceleration is acting in the same direction as its velocity. Hence, both are in the negative direction.

Step 3. Now focus on the interval from U to T, just after the bounce until the ball reaches the top of its trajectory, and answer the following questions:

Question 4:
For the velocity graph, what shows that the ball’s motion was toward the detector?

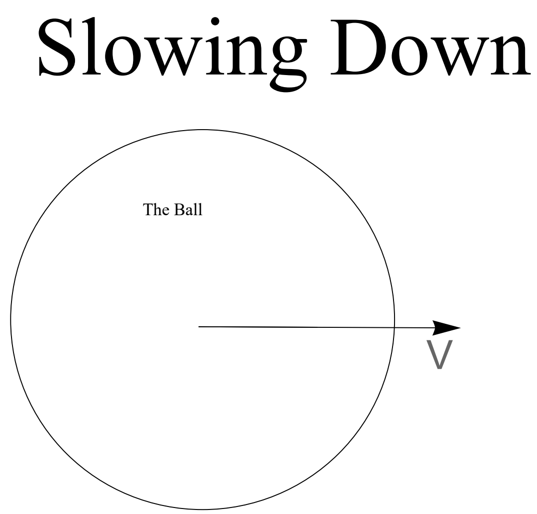
In the interval U to T, the ball has a positive velocity. It is hence moving upwards, towards the detector.

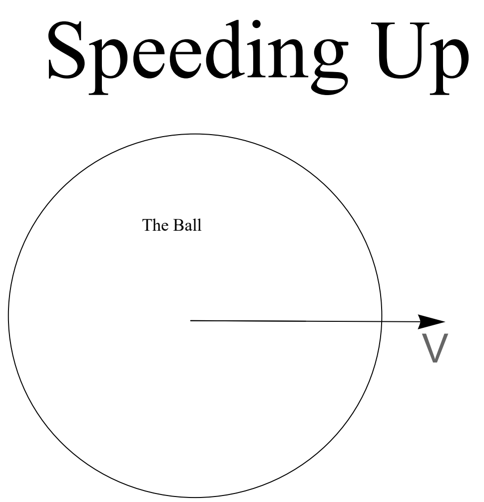
Question 5:
- For the velocity graph, what shows that the ball was slowing down?
- Is the acceleration positive or negative when the ball is slowing down?
- What is the sign of the velocity while the ball is slowing down?
- What can you conclude about the relationship between the directions of the velocity and acceleration vectors when the ball is slowing down?

The Velocity is changing and approaching 0 as it reaches the top. It is hence slowing down linearly with time.
Since the change in velocity is negative (it is slowing down in the positive direction), the acceleration is still

negative.
The sign of velocity when the ball is slowing down is positive. Even through the change is negative, the actual velocity is still positive and the ball is moving upwards.
We can conclude that if the ball’s velocity is decreasing in magnitude, the acceleration and velocity have an opposite direction. Hence, the acceleration is negatively impacting the velocity of the ball. The acceleration is negative, and the velocity is positive. It’s velocity is thus decreasing.

Step 4. Draw in the acceleration vectors for the following two cases:





(drawing tool not working in the graphic)
Slowing down Acceleration Vector
← (to the left, in the opposite direction as v)

Speeding up acceleration Vector
→ (to the right, in the opposite)

Step 5. Answer the Questions

Question 6:
What is the velocity of the ball at its highest point, T?

v=0 at the highest point T (it momentarily stops)

Question 7:
Is the acceleration at this point T positive, negative, or zero?

The acceleration is still negative.

Question 8:
Explain the observed sign of the acceleration at T.
(Hint: is the ball slowing down or speeding up just before it reaches the top? Just after? What are the directions of the velocity vector at these times?)

Even though the velocity is 0 at T, the velocity is constantly (linearly) decreasing around time T. Hence, the acceleration is a negative value (and constant)

Part II. Data Analysis Using **LoggerPro** Analysis Tools

Step 1. Measuring the tangents
Select Tangent from the Analyze pull-down menu in LoggerPro.

Step 1A. Measure the tangent to the displacement vs. time graph at the points U, T, and D.
Do not use points very close to the bounce; the slope there is not calculated correctly due to the averaging.

Tangent at U: 2.215
Tangent at T: -0.073
Tangent at D: -2.187

Question 1:
In what units are these slopes expressed?

The slope has units of that of displacement/time and hence m/s

Question 2:
What does the slope correspond to physically?

Physically, the slopes correspond to (almost) the instantaneous velocities at those specific points. (Our data was sampled in tiny bits but not instantaniously, hence, the velocity is still an average/approximation of the instantaneous velocity)

Step 1B. Measure the slope of the velocity vs. time graph at the points U, T, and D.

Tangent at U: -8.86
Tangent at T: -9.19
Tangent at D: -8.82

Question 3:
In what units are these slopes expressed?

The units of these slopes are m/(s^2) (velocity/time)

Question 4:
- What does the slope correspond to physically?
- Compare the measured slopes with the textbook value for the acceleration of gravity.
- Compare your results to the data on the acceleration vs. time graph. Do they agree?

The slope is (almost) the instantaneous acceleration at the point that the ball experiences.
The textbook value of acceleration due to gravity is 9.8m/s^2. Our measured slopes are slightly less steeper than what they should be.
The acceleration time graph has a lot of variation. On average, it does seem like on average, the value of acceleration is about equal to the value of our slopes or around -9.8m/s^2.

Question 5:
How would your velocity and acceleration graphs differ if this experiment were performed on the moon?

F=GMm/r^2 = mg
That means g = GM/r^2.

Considering the moon has roughly the same density as the earth, it’s mass is Mass decreases by a power of (1/3) (proportional to the volume) whereas the denominator decreases with a square.

Hence, the numerator decreases more than the denominator. Hence, the value of g on the moon would be less than that on the Earth. (Which is why people can be seen with having a greater airtime on moon!)

If we were to conduct this experiment on the moon, we would get a less steep slope for v-t graph. That is to say, we would get an acceleration closer to 0 in interval U to D

Step 2. Measuring the areas
Select the “Integrate” function of LoggerPro .

Step 2A. Calculate the area under the velocity vs. time graph from point U to T.
Do not use points near the bounce, since the averaging will give a misleading value.

Initial Time = 0.593
Final Time = 0.813
x_U = 0.153
x_T = 0.400
Area under the curve = 0.240

Question 6:
- What are the units of this “area”? What does this area correspond to physically?
- Compare your results to the displacement data, do they agree?
- What would you expect this “area” to be equal to for the entire bounce? Explain and check your prediction.

The units of the area are that of Velocity * time = meters (m). It physically corresponds to the net displacement during this interval.
x_T-x_U = 0.247 which corresponds almost perfectly to the area. So yes, the 2 results agree.
Since after the entire Bounce (U to D) the ball returns to it’s initial position, the net displacement is 0. Hence the area must also nullify to 0.
The practical integral from U to D does approximate to about 0 (0.002)

Step 2B. Use the integrate function to calculate the “area” under the acceleration vs. time graph from U to T.
Again, stay away from the points near the bounce.

v_U = 2.19
v_T = -0.03

Area under the curve = -2.030

Question 7:
- What are the units of this “area”? What does this area correspond to physically?
- Compare your results to the velocity data, do they agree?

The units of this area are that of acceleration*time, and hence m/s and they physically correspond to the change in velocity within that interval (U to T).
The actual difference in the final and initial velocities is around -2.22 which almost corresponds correctly with the area under the curve that we calculated.

Part III. Kinematics Calculations

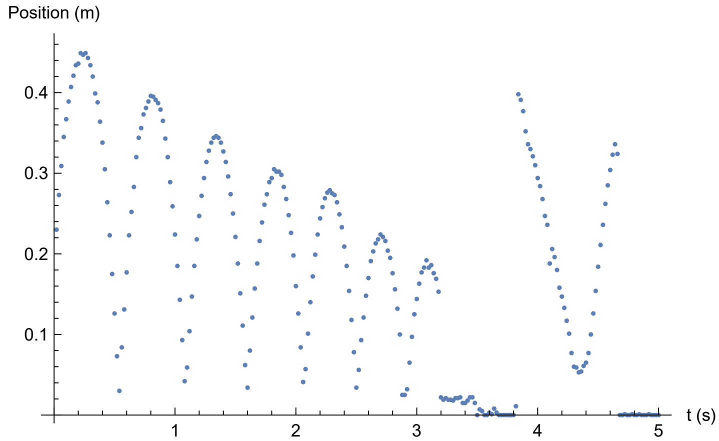
In this part you should perform a numerical integration to calculate displacement vs. time for the golf ball and compare calculation to experimental data. You should also find the coefficient of restitution and make a plot of energy dissipation vs. time.

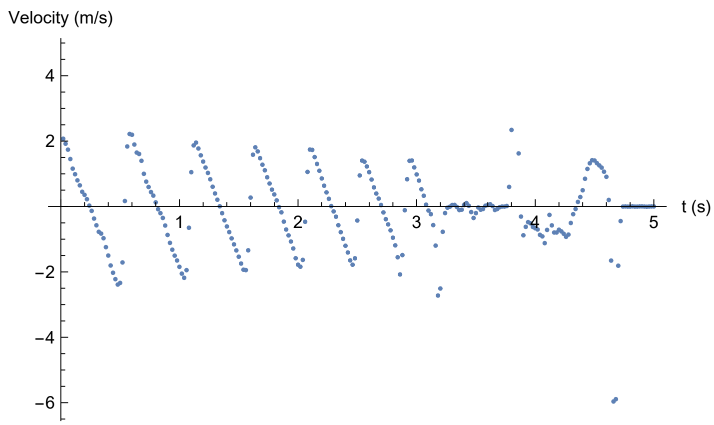
Step 1. Export from LoggerPro the displacement, velocity, and acceleration data in a file in spreadsheet format .csv.
Read the data in *Mathematica* and plot using the example file.

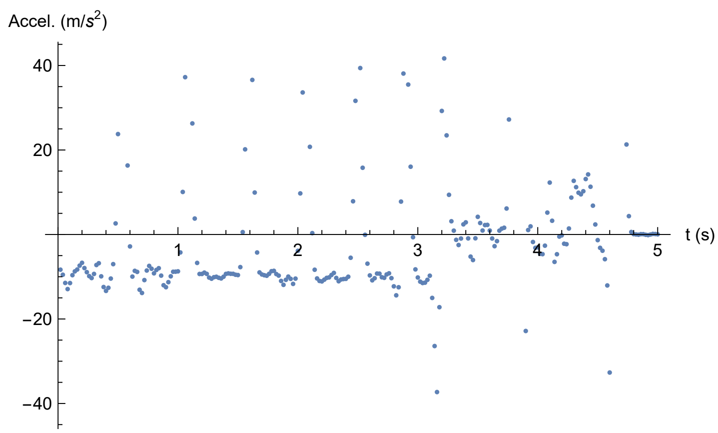
Import your data from .csv file. You can list all your data using the command:
data // TableForm
which display a list variable named data in a table form.
You can plot all your data as shown below,

```
In[ ]:= data = Import["E:\\BoxDrive\\Box\\Obsidian\\Mind\\Physics\\Classical
      Physics Lab I\\THE GOOD one data 3 - Lab 2.csv"];
dis = Rest[Part[data, All, {1, 2}]];
vel = Rest[Part[data, All, {1, 3}]];
acc = Rest[Part[data, All, {1, 4}]];
GraphicsColumn[{ListPlot[dis, AxesLabel → {"t (s)", "Position (m)"}],
  ListPlot[vel, AxesLabel → {"t (s)", "Velocity (m/s)"}],
  ListPlot[acc, AxesLabel → {"t (s)", "Accel. (m/s^2)"}]}]
```

Out[ ]=


Position (m)
0.1
0.2
0.3
0.4
1
2
3
4
5
t (s)

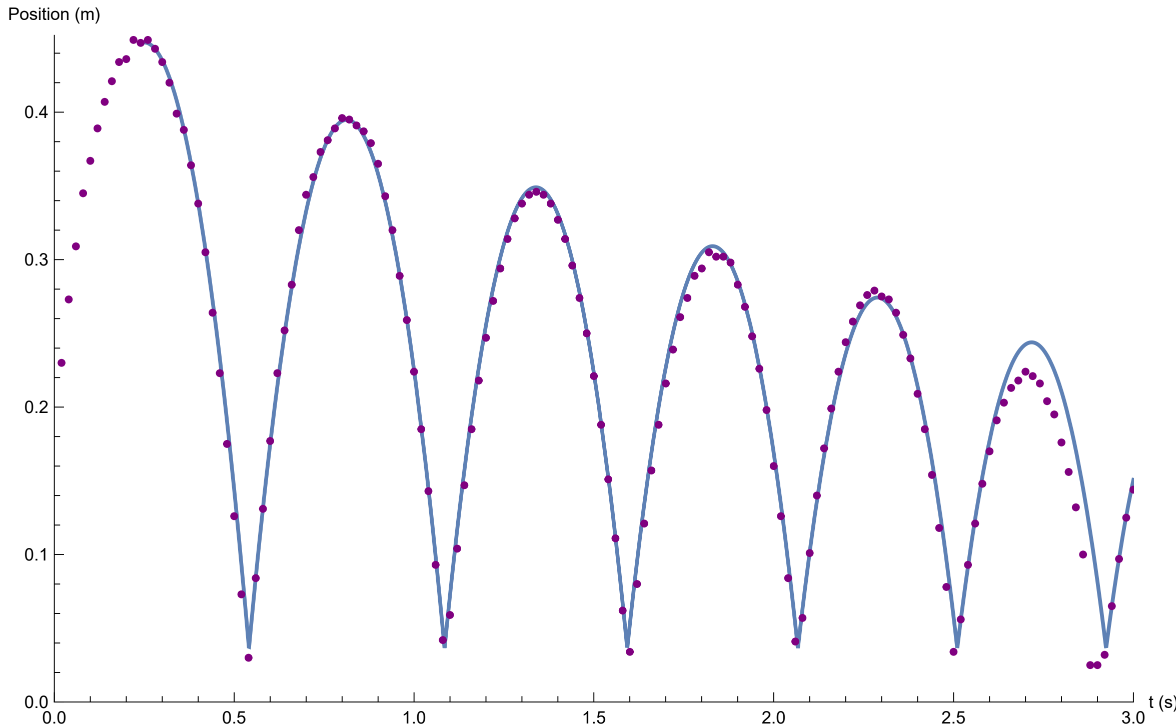

Velocity (m/s)
4
2
−2
−4
−6
1
2
3
4
5
t (s)


Accel. (m/s²)
40
20
−20
−40
1
2
3
4
5
t (s)

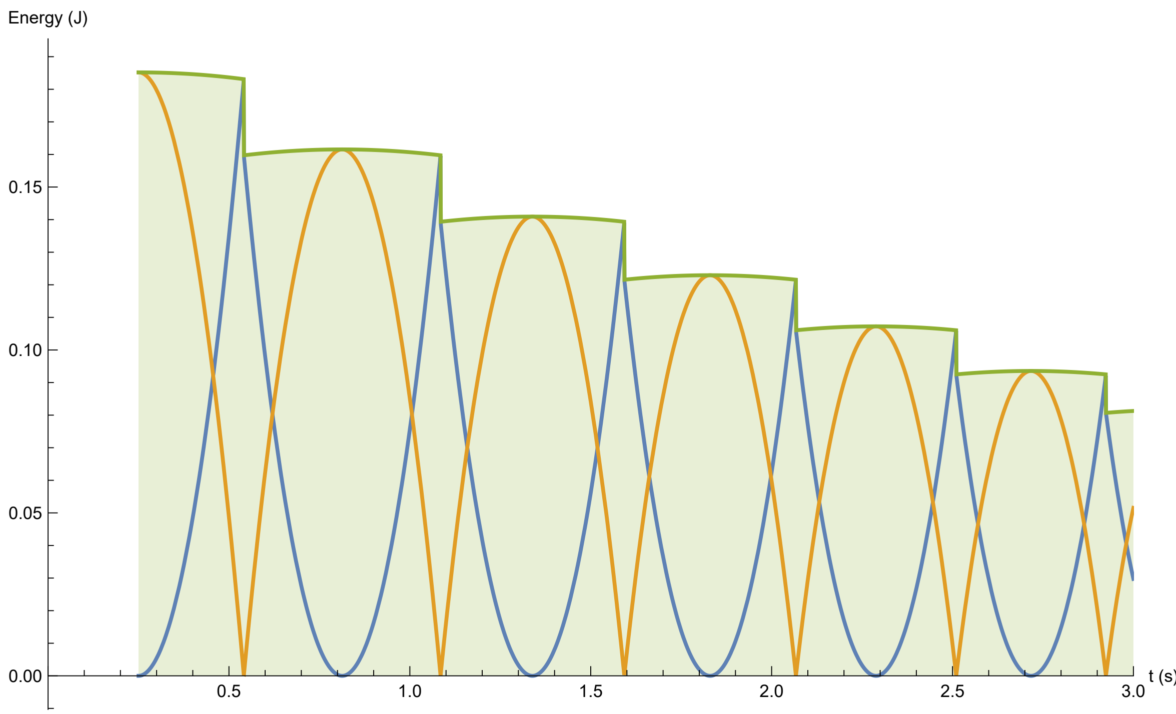
Step 2. Follow the example below:
- Perform numerical integration and plot calculated displacement against your data.
- First explain each variable and every line of the code below. Explain how we use each variable.
- Adjust the initial time, the height, and the coefficient of restitution to achieve the best fit.
- Plot potential, kinetic, and total energy as a function of time.

In[•]:=
```
(*The constant parameters.*)
a = -9.7;
bdiam = 0.04267;    (*diameter of the ball*)
bmass = 0.04593;    (*mass of the ball*)

(*Initial conditions and coefficient of restitution. Adjust these according to your data.*)
t0 = .25;             (*initial time*)
height = .411;      (*the height from which the ball is dropped, this is the height at t0*)
cor = .934;
(*right after each bounce the ball loses some of its velocity:y'[t]→-0.9065 y'[t]*)

(*NDSolve solves the equation y''[t]==
 a numerically using the initial conditions and the coefficient of restitution given above.*)
ball = NDSolve[{y''[t] == a, y[t0] == height,
    y'[t0] == 0, WhenEvent[y[t] == 0, y'[t] → -cor y'[t]]}, y, {t, t0, 5}];

kin[v_] := .5 bmass v^2;  (*kinetic energy*)
pot[y_] := 9.81 bmass y;    (*potential energy*)
energy[y_, v_] := kin[v] + pot[y];   (*Total energy*)

Show[Plot[0.85 bdiam + y[t] /. ball, {t, t0, 3}, PlotRange → {{0, 3}, {0, 1.1 height}},
  AxesLabel → {"t (s)", "Position (m)"}], ListPlot[dis, PlotStyle → Purple]]
Plot[Evaluate[{kin[y'[t]], pot[y[t]], energy[y[t], y'[t]]} /. ball], {t, t0, 3},
 Filling → {3 → 0}, PlotRange → {{0, 3}, All}, AxesLabel → {"t (s)", "Energy (J)"}]
```

Out[ ]=



Out[ ]=



Step 3.
Explain all the graphs above. There are total of five graphs.

The first 3 plots: position-time, velocity-time and acceleration-time

The position-time plot is the collection of raw data that had been sampled using a sensor at a sample rate of

50hz.
It shows what the distance of the ball with respect to time is. We can see how the ball bounces repeatedly and each time it reaches a lower height than what it does after the previous bounce.

The velocity-time plot has been created by the The Logger Pro software by numerically differentiating the graph at each point. We can see how velocity is always decreasing linearly except when the ball hits the ground, at which point the velocity shoots up again, slightly lower than the previous local maxima of the velocity. Hence, we can see how after the bounce, the ball is moving upwards, it is experiencing a downward acceleration. It momentarily stops at the top of the trajectory, and then continues moving down again, only to bounce back up.

The acceleration-time plot, again created numerically by the Logger Pro software has a lot of noise due to limitations in the apparatus precision. However, it has enough data to give us the insights that the ball largely experiences downward accelerations of around 9.8m/s^2 (the acceleration due to gravity). During the bounce however, the ball has a sudden positive spike in the acceleration where it shoots up to turn the velocity vector of the ball around and make it bounce back.

The 4th plot has 2 datasets. 1 is the actual position-time data collected and plotted in plot 1. The 2nd dataset is the simulated data set that we compute using physics equations (2nd order differential equations), given the initial conditions. We set the initial conditions so as to imitate the conditions in the lab and recreate the practical data (or atleast try to do so).

The 5th plot is an Energy-Time plot that represents 1. The Kinetic Energy of the system, 2. The Potential Energy of the system and 3. The total energy of the system. We can notice how within each bounce interval, the total energy stays constant while kinetic and potential energy curves are both quadratics. During the bounce, some of the energy is lost by the system, hence the total energy decreases.

Over time, the entire system is damping away it’s energy and approaching 0 total energy.

Question 1:
- What is the green curve in the bottom graph and why does it have a staircase shape?
- What is the ratio between two consecutive steps in this green graph? How does this ratio depends on the parameters above?

The green curve in the bottom graph is the curve for the total energy of the system. It decreases after every bounce because the coefficient of restitution of the system is no 1. Hence, some of the energy of the system is lost in each bounce.

The coefficient of restitution is the ratio of the velocities before and after the bounce. The energies (mv^2/2) is portional to the squares of the velocities. Hence, the ratio of two consecutive steps in this green graph is (cor)^2 {cor^2 = final_height/initial_height.}