19. $\begin{bmatrix}5\\-3\end{bmatrix} = 5e_1 - 3e_2.$

$$\Rightarrow T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right) = 5T(e_1) - 3T(e_2)$$

$$= 5y_1 - 3y_2$$

$$= \begin{bmatrix}5\times 2 - 3\times(-1)\\ 5\times 5 - 3\times 6\end{bmatrix} = \begin{bmatrix}13\\7\end{bmatrix}.$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = x_1e_1 + x_2e_2. \Rightarrow T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = x_1T(e_1) + x_2T(e_2).$$

$$= x_1y_1 + x_2y_2.$$

$$\therefore T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}2x_1 - x_2\\ 5x_1 + 6x_2\end{bmatrix}.$$

for $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$,

5. $T(e_1) = e_1 - 2e_2 = \begin{bmatrix}1-2(0)\\0-2(1)\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}.$

$T(e_2) = e_2$

$\therefore$ Std. Matrix $= \begin{bmatrix}T(e_1) & T(e_2)\end{bmatrix} = \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}.$

6. $T(e_1) = e_1$

$T(e_2) = e_2 + 5e_1 = \begin{bmatrix}0+5(1)\\1+5(0)\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}.$

$\therefore$ Std. Matrix $= \begin{bmatrix}T(e_1) & T(e_2)\end{bmatrix} = \begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}$

22 $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ -x + 3x_2 \\ 3x_1 - 2x_2 \end{bmatrix}$.

$\Rightarrow$ for $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$T(e_1) = y_1 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ $\quad T(e_2) = y_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$.

$\therefore$ std. Matrix $= \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix} = A$.

To find $Ax = b$, we have to solve $[A|b]$

$\Rightarrow \left[\begin{array}{cc|c} 1 & -2 & -1 \\ -1 & 3 & 4 \\ 3 & -2 & 9 \end{array}\right]$.

$\Rightarrow$ $r_2 + r_1 \to$ new $r_2$, $r_3 - 3r_1 \to$ new $r_3$

$\left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 4 & 12 \end{array}\right]$.

$r_3 - 4r_2 \to$ new $r_3$.

$$\left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right].$$

$r_1 + 2r_2 \to$ new $r_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right].$$

$\therefore \; x_1 = 1, \quad x_2 = 3.$

$$\Rightarrow \; X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$