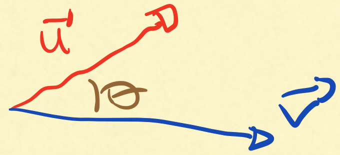
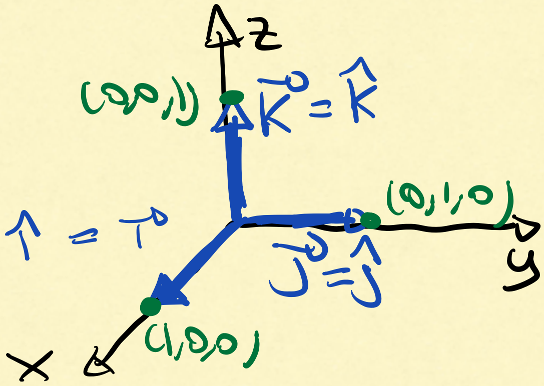
Lec 3 (1/29)



$$\vec{u}\cdot\vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta$$

$$\vec{u}\cdot(\vec{v_1}+\vec{v_2}) = \vec{u}\cdot\vec{v_1} + \vec{u}\cdot\vec{v_2}$$

standard unit vectors
(canonical)



Key properties of $\vec{i}, \vec{j}, \vec{k}$

① each has length one
(unit vector)

so
$\vec{i}\cdot\vec{i} = 1$ $\quad$ $\vec{k}\cdot\vec{k} = 1$
$\vec{j}\cdot\vec{j} = 1$

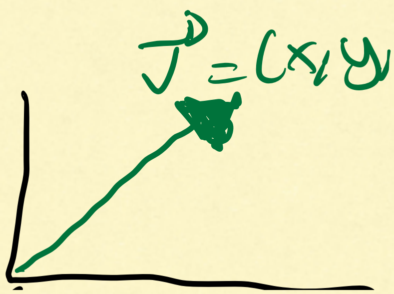
② they pairwise
perpendicular (orthogonal)

$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{i} = 0$

$\vec{i}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0$

$\vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{j} = 0$

③ Decomposition property
(basis in linear algebra)



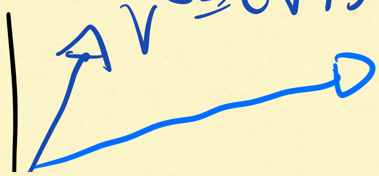
any vector can
be decomposed
in terms of
$\vec{i}, \vec{j}, \vec{k}$

$$\vec{v} = (x,y,z)$$
$$= (x,0,0) + (0,y,0) + (0,0,z)$$
$$= x\underbrace{(1,0,0)}_{\vec{i}} + y\underbrace{(0,1,0)}_{\vec{j}} + z\underbrace{(0,0,1)}_{\vec{k}}$$
$$= x\vec{i} + y\vec{j} + z\vec{k}$$

example

$$\vec{v} = (2,-3,5)$$
$$= 2\vec{i} - 3\vec{j} + 5\vec{k}$$

can use $\vec{i}, \vec{j}, \vec{k}$ to find an alternative formula for dot product



$\vec{v} = (v_1, v_2, v_3)$

$\vec{u} = (u_1, u_2, u_3)$

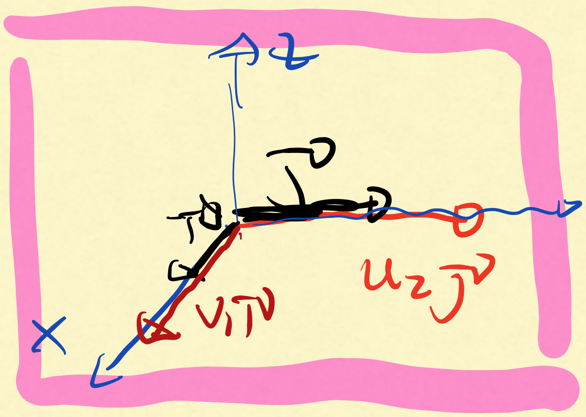
$$\vec{u} \cdot \vec{v}$$

$$= (u_1\vec{i} + u_2\vec{j} + u_3\vec{k}) \cdot (v_1\vec{i} + v_2\vec{j} + v_3\vec{k})$$

$$= (u_1\vec{i})\cdot(v_1\vec{i}) + \underline{(u_1\vec{i})\cdot(v_2\vec{j})} + \underline{(u_1\vec{i})\cdot(v_3\vec{k})}$$

$$+ \underline{(u_2\vec{j})\cdot(v_1\vec{i})} + (u_2\vec{j})\cdot(v_2\vec{j}) + \underline{(u_2\vec{j})\cdot(v_3\vec{k})}$$

$$+ \underline{(u_3\vec{k})\cdot(v_1\vec{i})} + \underline{(u_3\vec{k})\cdot(v_2\vec{j})} + (u_3\vec{k})\cdot(v_3\vec{k})$$



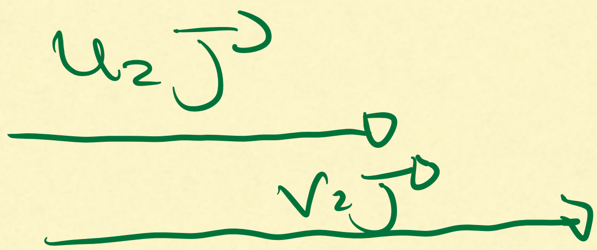
all the products <u>with</u> are 0

because the vectors are perpendicular

$$\vec{u} \cdot \vec{v}$$

$$= (u_1\vec{i})\cdot(v_1\vec{i}) + (u_2\vec{j})\cdot(v_2\vec{j}) + (u_3\vec{k})\cdot(v_3\vec{k})$$

$$= \boxed{u_1 v_1 + u_2 v_2 + u_3 v_3}$$



so we found

$$\vec{u} = (u_1, u_2, u_3)$$

$$\vec{v} = (v_1, v_2, v_3)$$

$$\boxed{\vec{u}\cdot\vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3}$$

↳ multiply corresponding entries and then add.

together with

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta}$$

makes the dot product useful

$$\cos\theta = \frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|\vec{u}|\,|\vec{v}|}$$

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example

$\vec{u} = (1, 2, 3)$

$\vec{v} = (7, -2, -1)$

$$\vec{u} \cdot \vec{v} = (1)(7) + (2)(-2) + (3)(-1)$$

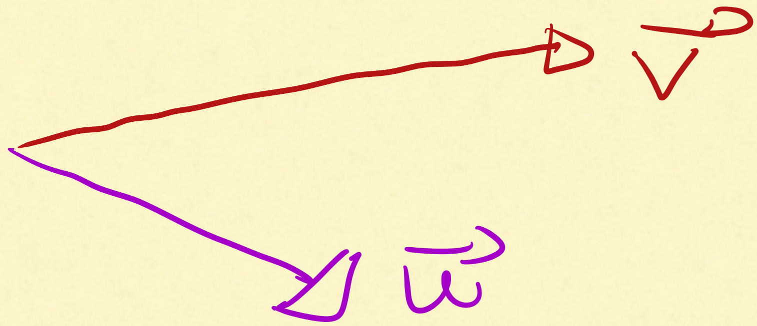
$$= 7 - 4 - 3$$

$$= 0$$

So $\theta = 90°$

Cross Product
(another way to multiply vectors)

↳ specific to vectors in 2 or 3 dimensions

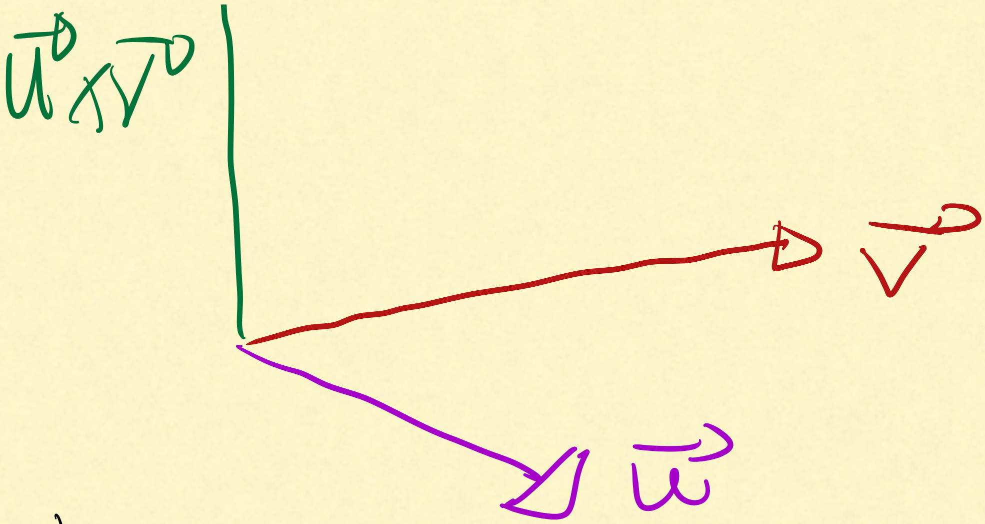


$\vec{u} \times \vec{v}$ = cross product of $\vec{u}$ with $\vec{v}$

= new vector! (not a number)

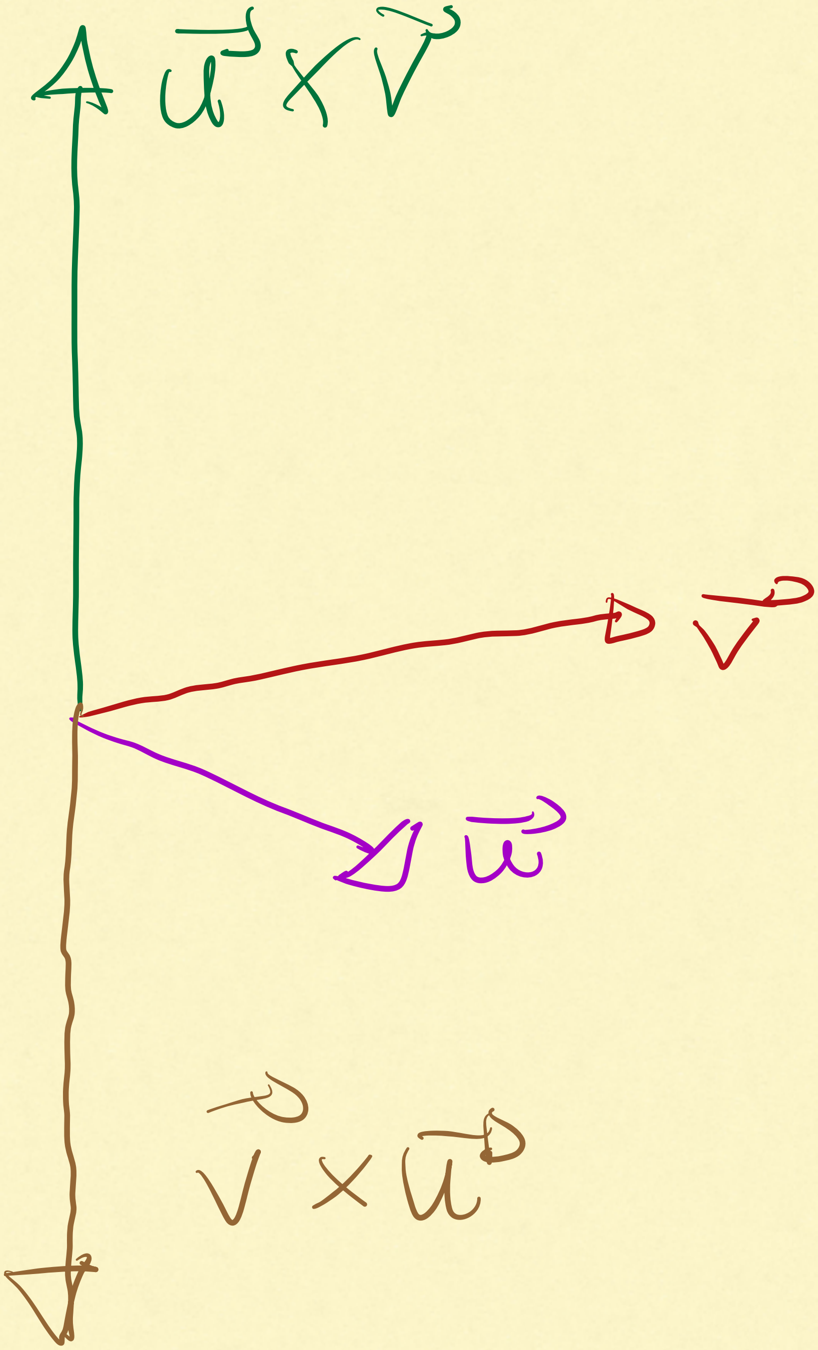
what is direction of $\vec{u} \times \vec{v}$?

$\vec{u} \times \vec{v}$ = vector perpendicular (orthogonal) to $\vec{u}$ and also to $\vec{v}$



to choose the direction use the right hand rule:

put your right hand along $\vec{u}$, twist your fingers towards $\vec{v}$ using the smaller angle and your thumb gives direction of $\vec{u} \times \vec{v}$



$$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$$

switching order of product gives you a negative sign.

(cross product is anti-commutative)

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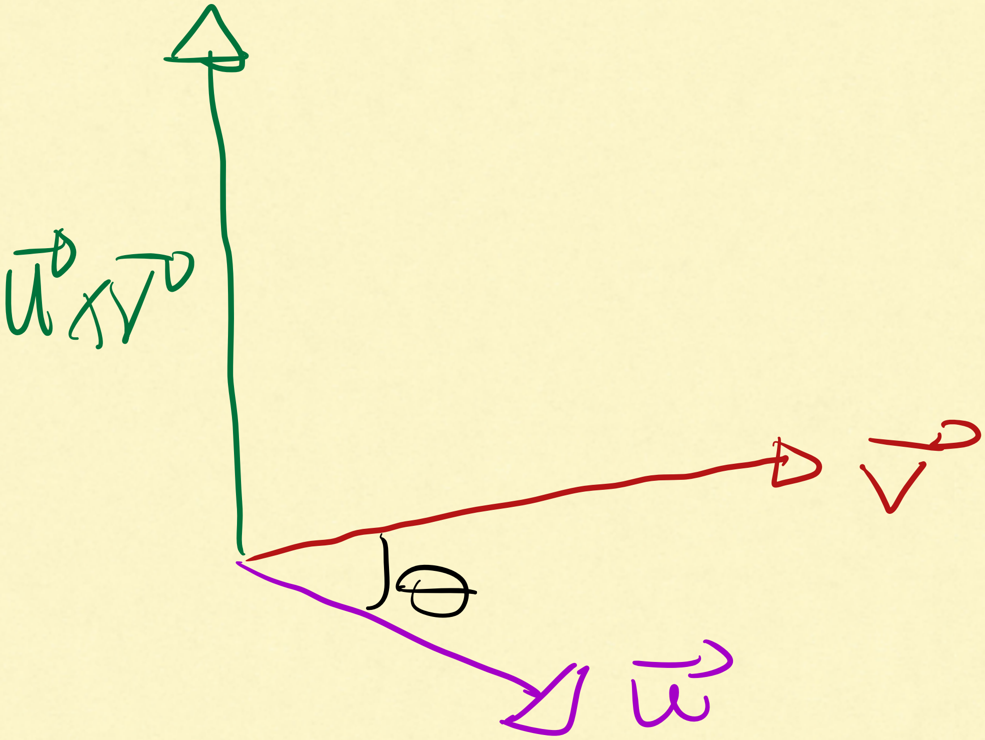
second question?

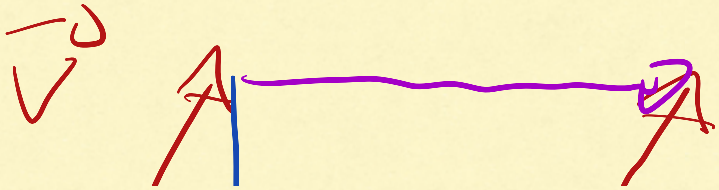
what is the length of cross product?



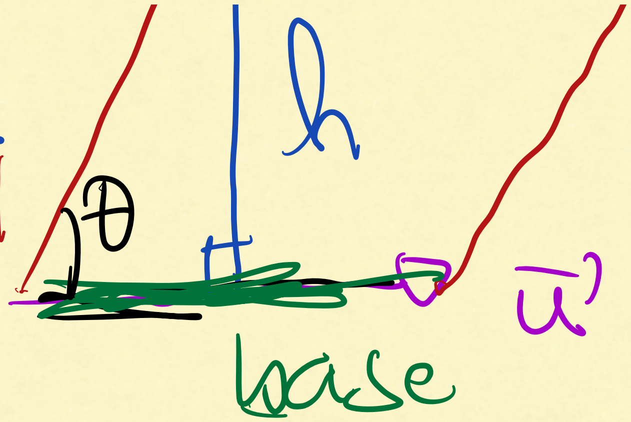
$$\boxed{|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta}$$

why this formula?

Parallelogram



$\sin\theta = \frac{h}{|\vec{v}|}$



area is

base · height

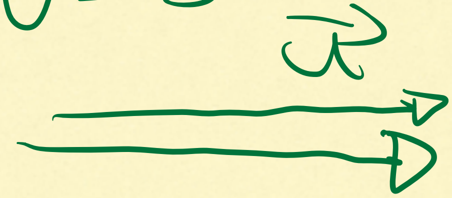
$= |\vec{u}||\vec{v}|\sin\theta$

$= |\vec{u} \times \vec{v}|$

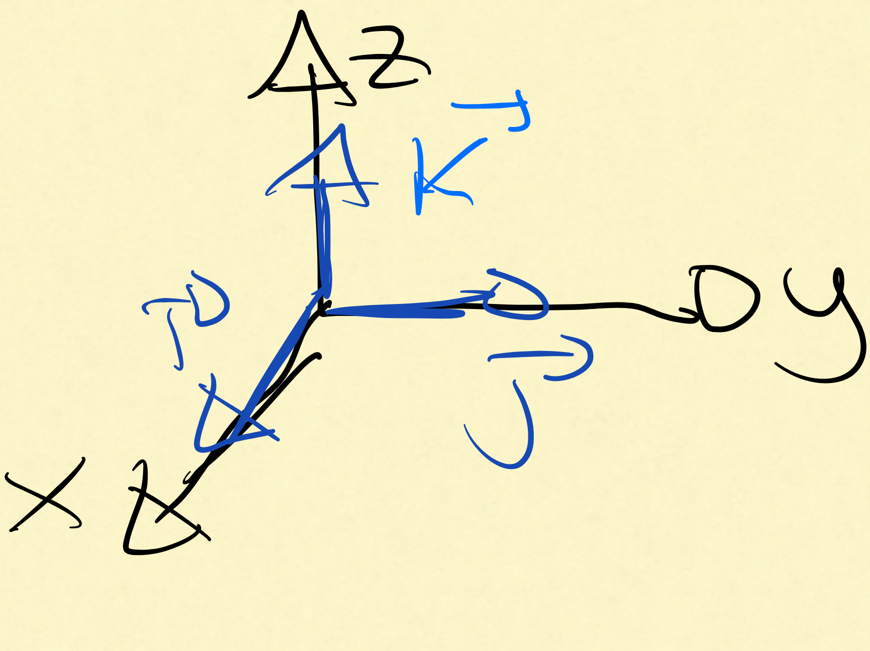
So the length of $\vec{u} \times \vec{v}$ numerically is the same as the area of the parallelogram whose sides are $\vec{u}, \vec{v}$.

$$\vec{u} \times \vec{u} = \vec{0}$$

$\theta = 0$



and $\sin\theta = 0$



$$\vec{i} \times \vec{j} = \vec{k}$$

$$\vec{j} \times \vec{k} = \vec{i}$$

$$\vec{k} \times \vec{i} = \vec{j}$$

$$\begin{tikzcd} & \vec{i} \arrow[dr, "\times"] & \\ \vec{k} \arrow[ur, "\times"] & & \vec{j} \arrow[ll, bend left, "\times"] \end{tikzcd}$$

Try it!

$\vec{u} \times \vec{v}$

$$= (u_1\vec{i} + u_2\vec{j} + u_3\vec{k}) \times (v_1\vec{i} + v_2\vec{j} + v_3\vec{k})$$

try to foil
(distributive)