Standing Waves

Lab Report

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Section: H4
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Purpose

To show that standing waves can be set up in a string and in an air column, to determine the velocity of a standing wave, and to understand the differences between transverse and longitudinal waves.

Apparatus

A function generator, a speaker, a tube with a rod, buzzer board with a string, a meter stick, a weight holder, weights.

Readings

You can explore these concepts using these links or your favorite textbook,
Standing waves, Relationship between the speed of a wave on a string and the string’s tension,
Also watch this video which explains the sound waves inside tubes,
Standing waves in tubes.

Theory and concepts in short

A traveling wave is a disturbance of some physical quantity, such as air pressure in the case of a sound wave, that propagates through the medium with
a wave velocity c. A sinusoidal traveling wave appears as a series of maxima (or minima) separated by a distance λ (the wavelength). An observer will see these maxima pass by every T seconds (the period). The wave velocity c is λ/T . Because f (the frequency) is defined as 1/T , we can also write c = fλ. Light and sound are examples of traveling waves, albeit very different kinds of waves.

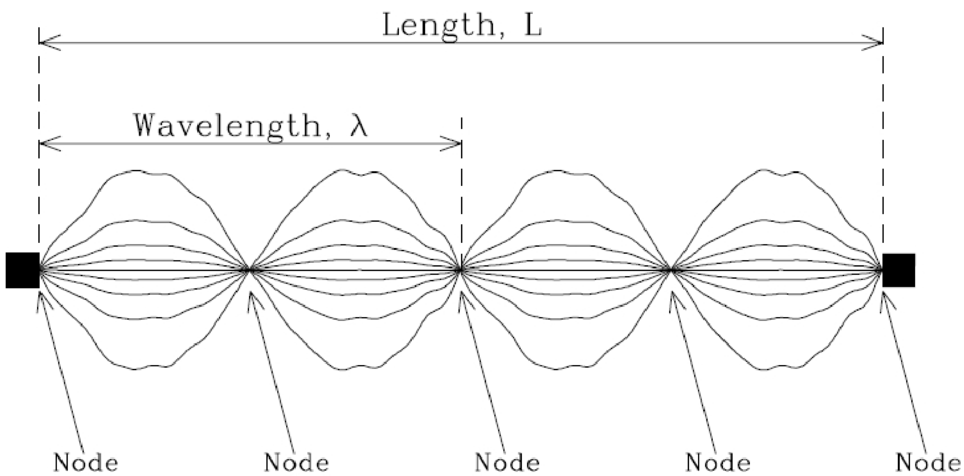


FIGURE 1. Standing waves on a vibrating string

A “standing” wave is produced by superposing two similar (same frequency and wavelength) traveling waves, moving in the opposite directions. Figure 1 above shows a vibrating string. At the fixed ends, the traveling waves are reflected. If the wavelength is “just right”, we will see nodes (positions where there is no motion at all). The “just right” condition is a condition when the wavelength is matching half-multiples of the string’s length:

$n/2 = L/\lambda$, n = 1, 2, 3, ...

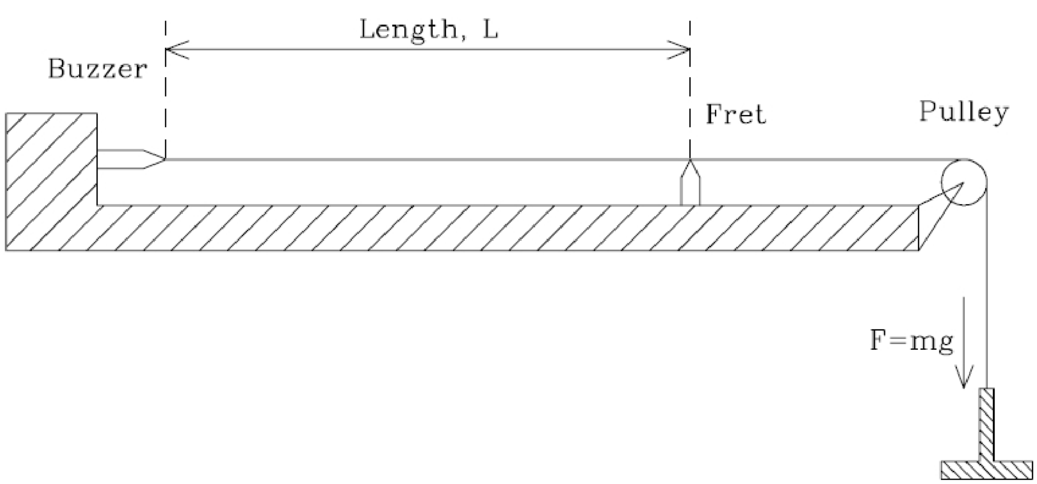
This comes from the constraint that *an integer number of half-waves must fit within the full length, L*.

A vibrating string is an example of a *transverse* wave: its oscillation is perpendicular to the direction of its velocity. On the other hand,

a sound wave is a longitudinal wave: its oscillation is in the same direction as its velocity. These waves have similar properties, although, as you will see, they are quite different in certain aspects.

Dialog:

Part I. Standing Waves on String



On the standing wave board shown schematically in Figure above, a buzzer vibrates a string with a frequency of 120 Hz. Because the motion is perpendicular to the direction of propagation, these are

transverse waves. Both ends of the string are fixed; thus, a standing wave will be set up, with each end being a node (the location of zero displacement).
This requires that L, the length of the string, must be equal to any whole number of half-wavelengths:
(1) L =n*(λ/2), n = 1, 2, 3, ...
Although the buzzer end of our string is vibrating, its amplitude is so small that the end is a good approximation of a node. Standing waves will appear between the buzzer end and the adjustable fret.
The wave velocity depends on the string's tension and its material (the mass density, as well as the diameter of the string). Their relation is given by:
(2) $c = \sqrt{F/\rho}$,
where F is the tension (in N) produced by the suspended mass (F = mg), and ρ is the linear density of the string in kg/m (mass per unit length, not volume). For a particular string (fixed ρ), changes in F by varying the mass, m, of the suspended weight enables us to verify the square-root dependence of the wave velocity on the tension. For a given string with a linear density ρ and tension F, the velocity c is fixed.
The frequency f is set by the buzzer (120 Hz). If both ends of the string are fixed, a standing wave will be produced when, and only when, the conditions given by Equation 1 above are met.

Step 1.

-Turn on the buzzer.
- Produce a large transverse standing wave by varying L (i.e., move the fret along the string).
- Using a meter stick, determine the distance between nodes by measuring the length between the most widely separated nodes and then dividing by the number of segments in that length.
- Find as many patterns as possible for a given tension. [**Don't let the weight swing!**]

Step 2.

- Record your data in the table below.
- Change the tension and repeat step 1. Use at least three different tensions. The hanger alone may give a nice standing wave pattern.
- For each value of tension determine λ/2 and thus your experimental value of c (for that particular value of tension). The most accurate results are obtained by using the longest length of string. [**For the tension do not forget the weight of the hanger**.]
- For each value of tension calculate the average value of c_exp and its standard deviation.
- Using equation (2), calculate the theoretical value of c. The linear density of the string is
rho = $\rho = (2.0 \pm 0.2) \times 10^{-4}$ kg/m.
- Compare the experimental and theoretical values of c for each tension value.

In[1]:=
```
rho = 2.0*10^-4; (*linear density of the string in kg/m*)
f = 120; (*frequency in Hz*)
```

In[12]:=
```
Ta1 = 0.050 * 9.8
cthe1 = Sqrt[Ta1 / rho] (*theoretical value of the wave velocity in the string in m/s*)
Table1 =
 Grid[{{Text["Table 1A1. Standing Waves on String, Tension value F1 = 0.49 N"], SpanFromLeft},
   {"L (m), Vibrating length", "n, # half wavelengths in L", "λ/2 (m)", "c_exp (m/s)"},
   {0.20, 1, 0.20, 0.20 * 2 * f}, {0.41, 2, 0.41 / 2, 0.41 * 2 / 2 * f},
   {0.61, 3, 0.61 / 3, 0.61 * 2 / 3 * f}, {0.81, 4, 0.81 / 4, 0.81 * 2 / 4 * f}}, Frame → All]
```

Out[12]=

0.49

Out[13]=

49.4975

Out[14]=

Table 1A1. Standing Waves on String, Tension value F1 = 0.49 N			
L (m), Vibrating length	n, # half wavelengths in L	λ/2 (m)	c_exp (m/s)
0.2	1	0.2	48.
0.41	2	0.205	49.2
0.61	3	0.203333	48.8
0.81	4	0.2025	48.6

```
(*Table⟦1⟧⟦3,4,5,6⟧⟦4⟧ access the c_exp*)
```

In[19]:=
```
cexpm1 = Sum[Table1⟦1⟧⟦3 + i⟧⟦4⟧, {i, 0, 3}] / 4 (*The mean experimental velocity*)
sigmacexp1 = Sqrt[Sum[(Table1⟦1⟧⟦3 + i⟧⟦4⟧ - cexpm1) ^2, {i, 0, 3}] / 3]
 (*Standard Deviation in experimental velocity*)
```

Out[19]=

48.65

Out[20]=

0.5

Hence, the theoretical value of velocity for T=0.49 N is 49.4975 m/s and our experimental value is 48.65 ± 0.5 m/s. Hence, our experiment is rather perfect with very little variance and close to theoretical results.

In[21]:=
```
Ta2 = 0.070 * 9.8
cthe2 = Sqrt[Ta2 / rho] (*theoretical value of the wave velocity in the string in m/s*)
Table2 =
 Grid[{{Text["Table 1A2. Standing Waves on String, Tension value F2 = 0.686 N"], SpanFromLeft},
   {"L (m), Vibrating length", "n, # half wavelengths in L", "λ/2 (m)", "c_exp (m/s)"},
   {0.23, 1, 0.23 / 1, 2 * 0.23 / 1 * f}, {0.46, 2, 0.46 / 2, 2 * 0.46 / 2 * f},
   {0.69, 3, 0.69 / 3, 2 * 0.69 / 3 * f}, {0.94, 4, 0.94 / 4, 2 * 0.94 / 4 * f}}, Frame → All]
```

Out[21]=

0.686

Out[22]=

58.5662

Out[23]=

Table 1A2. Standing Waves on String, Tension value F2 = 0.686 N			
L (m), Vibrating length	n, # half wavelengths in L	λ/2 (m)	c_exp (m/s)
0.23	1	0.23	55.2
0.46	2	0.23	55.2
0.69	3	0.23	55.2
0.94	4	0.235	56.4

In[24]:=
```
cexpm2 = Sum[Table2[[1]][[3 + i]][[4]], {i, 0, 3}] / 4 (*The mean experimental velocity*)
sigmacexp2 = Sqrt[Sum[(Table2[[1]][[3 + i]][[4]] - cexpm2)^2, {i, 0, 3}] / 3]
 (*Standard Deviation in experimental velocity*)
```

Out[24]=
```
55.5
```

Out[25]=
```
0.6
```

Hence, the theoretical value of velocity for T = 0.686 N is 58.5662 m/s and our experimental value is 55.5 ± 0.6 m/s. Hence, our experiment is a little less perfect but with very little variance and still decently close to theoretical results .

In[26]:=
```
Ta3 = 0.090 * 9.8
cthe3 = Sqrt[Ta3 / rho] (*theoretical value of the wave velocity in the string in m/s*)
Table3 = Grid[
  {{Text["Table 1A3. Standing Waves on String, Tension value F3 = 0.8829 N"], SpanFromLeft},
   {"L (m), Vibrating length", "n, # half wavelengths in L", "λ/2 (m)", "c_exp (m/s)"},
   {0.26, 1, 0.26 / 1, 2 * 0.26 / 1 * f}, {0.52, 2, 0.52 / 2, 2 * 0.52 / 2 * f},
   {0.80, 3, 0.80 / 3, 2 * 0.80 / 3 * f}, {1.07, 4, 1.07 / 4, 2 * 1.07 / 4 * f}}, Frame → All]
```

Out[26]=
```
0.882
```

Out[27]=
```
66.4078
```

Out[28]=

Table 1A3. Standing Waves on String, Tension value F3 = 0.8829 N			
L (m), Vibrating length	n, # half wavelengths in L	λ/2 (m)	c_exp (m/s)
0.26	1	0.26	62.4
0.52	2	0.26	62.4
0.8	3	0.266667	64.
1.07	4	0.2675	64.2

In[29]:=
```
cexpm3 = Sum[Table3[[1]][[3 + i]][[4]], {i, 0, 3}] / 4 (*The mean experimental velocity*)
sigmacexp3 = Sqrt[Sum[(Table3[[1]][[3 + i]][[4]] - cexpm3)^2, {i, 0, 3}] / 3]
 (*Standard Deviation in experimental velocity*)
```

Out[29]=
```
63.25
```

Out[30]=
```
0.984886
```

Hence, the theoretical value of velocity for T = 0.8829 N is 66.4078 m/s and our experimental value is 63.25 ± 0.985 m/s . Hence, our experiment is a little less perfect but with very little variance and still decently close to theoretical results .

Step 3.

- Take the uncertainty in `rho` to be $2 * 10^{-5}$ kg/m (i.e., 10% of the nominal value) and find the uncertainty in the theoretical velocity.

- For each tension take the total uncertainty to be $\sigma_{tot} = \sqrt{\sigma_{th}^2 + \sigma_{exp}^2}$.
- Express the difference between the theoretical and experimental velocities in terms of σ_{tot}. A difference of less than $2\sigma_{tot}$ means they agree. Show your calculations and comment on the agreement of theory and experiment.

$\sigma_c = c_{the} \times \sigma_\rho/(2\rho)$

In[33]:=
```
sigmacthe1 = cthe1 * 2 * 10^-5 / (2 rho)
sigmacthe2 = cthe2 * 2 * 10^-5 / (2 rho)
sigmacthe3 = cthe3 * 2 * 10^-5 / (2 rho)
```

Out[33]=
```
2.47487
```

Out[34]=
```
2.92831
```

Out[35]=
```
3.32039
```

In[36]:=
```
Grid[{{Text["Table 1B. Standing Waves on String"], SpanFromLeft},
  {"F (N), Tension value", "c_the (m/s)", "c_exp (m/s)", "|c_exp-c_the| (m/s)", "σtot (m/s)"},
  {0.49, cthe1, cexpm1, Abs[cexpm1 - cthe1], Sqrt[sigmacexp1^2 + sigmacthe1^2]},
  {0.686, cthe2, cexpm2, Abs[cexpm2 - cthe2], Sqrt[sigmacexp2^2 + sigmacthe2^2]},
  {0.8829, cthe3, cexpm3, Abs[cexpm3 - cthe3], Sqrt[sigmacexp3^2 + sigmacthe3^2]}
 }, Frame → All]
```

Out[36]=

Table 1B. Standing Waves on String				
F (N), Tension value	c_the (m/s)	$\overline{\text{c\_exp}}$ (m/s)	|$\overline{\text{c\_exp}}$-c_the| (m/s)	σ_{tot} (m/s)
0.49	49.4975	48.65	0.847475	2.52488
0.686	58.5662	55.5	3.0662	2.98915
0.8829	66.4078	63.25	3.15783	3.46338

We conducted the experiment really well and for each experiment, we can see that the difference in the theoretical and experimental value of the velocity is well within the total standard deviation of our data.

We can however, notice that the error in the greater tension is ever so slightly greater. That may be because a higher tension pulls the string tighter, causing the density to change ever so slightly. There also seemed to be a slight zero error in the scale inherently which we didn't account for. But overall, the experiment was great!!

Part II. Standing Waves in Air Column

The waves in the setup for this part of the lab are made by a small loudspeaker inside a clear plastic tube and powered by a sine wave (function) generator. The speaker sets the air molecules into longitudinal vibration (molecular vibration in the direction of wave propagation). The oscilloscope shows two signals (traces). One trace is the sinusoidal signal applied to the speaker. The other is the output of a microphone measuring the amplitude of the standing sound wave at the microphone's position.

Step 1.

- Set the function generator to a frequency near 1000 Hz. The sine-wave generator **should not be set at the maximum-amplitude** (this will damage the loudspeaker).
- The signal from the function generator should be fed to the oscilloscope's input B and should trigger the scope, since its amplitude is constant as you slide the plunger.
- Input A should be connected to the microphone. Check that the microphone switch is on.
- Also check that the A and B inputs are set for AC or DC, but not Gnd.
- Check that the Mode is Dual, and the Trigger is B.
- Adjust the trace sweep speed and the A and B amplifier gains as needed for a clear, stable pattern.

Step 2.

- Move the plunger to vary the length of the air column. When the plunger length is just right, an anti-node develops at the microphone, and a strong standing wave is set up in the tube. You can recognize the anti-node by the sharp maximum in the amplitude as the plunger is moved.
- Determine all positions of the plunger for maximum sound intensity. The distance between two adjacent positions of maximum intensity is equal to $\lambda/2$.
- From f and λ determine the speed of sound c for each pattern. Calculate the average speed c **for each of the three frequencies.** Compare your average experimental value with the theoretical value,
$c = (331.4 + 0.6T)$ m/s;
where T is the temperature in degrees centigrade (room temperature is about 22°C).

In[165]:=

```
T2A1 = Grid[{{Text["Table 2A1. Longitudinal Waves, frequency f1 = 997 Hz"], SpanFromLeft},
   {"", "Positions of intensity maxima (i.e., location of anti-nodes) (m)",
    "difference between successive maxima; λ/2 values (m)"},
   {"1", 0.18, 0.18 - 0}, {"2", 0.35, 0.35 - 0.18}, {"3", 0.53, 0.53 - 0.35},
   {"4", 0.70, 0.70 - 0.53}, {"5", 0.88, 0.88 - 0.70}}, Frame → All]
lambdaavg1 = 2 * Sum[T2A1[[1]][[3 + i]][[3]], {i, 0, 4}] / 5
(*Average Wave length is twice the average of half wavelengths*)
f1 = 997;
cavg1 = lambdaavg1 * f1


T2A2 = Grid[{{Text["Table 2A2. Longitudinal Waves, frequency f2 = 1354 Hz"], SpanFromLeft},
   {"", "Positions of intensity maxima (i.e., location of anti-nodes) (m)",
    "difference between successive maxima; λ/2 values (m)"},
   {"1", 0.13, 0.13 - 0}, {"2", 0.255, 0.255 - 0.13}, {"3", 0.38, 0.38 - 0.255},
   {"4", 0.51, 0.51 - 0.38}, {"5", 0.63, 0.63 - 0.51}}, Frame → All]
lambdaavg2 = 2 * Sum[T2A2[[1]][[3 + i]][[3]], {i, 0, 4}] / 5
f2 = 1354;
cavg2 = lambdaavg2 * f2


T2A3 = Grid[{{Text["Table 2A3. Longitudinal Waves, frequency f3 = 1693 Hz"], SpanFromLeft},
   {"", "Positions of intensity maxima (i.e., location of anti-nodes) (m)",
    "difference between successive maxima; λ/2 values (m)"},
   {"1", 0.11, 0.11 - 0}, {"2", 0.205, 0.205 - 0.11}, {"3", 0.31, 0.31 - 0.205},
   {"4", 0.41, 0.41 - 0.31}, {"5", 0.51, 0.51 - 0.41}}, Frame → All]
lambdaavg3 = 2 * Sum[T2A3[[1]][[3 + i]][[3]], {i, 0, 4}] / 5
f3 = 1693;
cavg3 = lambdaavg3 * f3
```

Out[165]=

Table 2A1. Longitudinal Waves, frequency f1 = 997 Hz		
	Positions of intensity maxima (i.e., location of anti-nodes) (m)	difference between successive maxima; $\lambda/2$ values (m)
1	0.18	0.18
2	0.35	0.17
3	0.53	0.18
4	0.7	0.17
5	0.88	0.18

Out[166]=

0.352

Out[168]=

350.944

Out[169]=

Table 2A2. Longitudinal Waves, frequency f2 = 1354 Hz		
	Positions of intensity maxima (i.e., location of anti-nodes) (m)	difference between successive maxima; $\lambda/2$ values (m)
1	0.13	0.13
2	0.255	0.125
3	0.38	0.125
4	0.51	0.13
5	0.63	0.12

Out[170]=

0.252

Out[172]=

341.208

Out[173]=

Table 2A3. Longitudinal Waves, frequency f3 = 1693 Hz		
	Positions of intensity maxima (i.e., location of anti-nodes) (m)	difference between successive maxima; $\lambda/2$ values (m)
1	0.11	0.11
2	0.205	0.095
3	0.31	0.105
4	0.41	0.1
5	0.51	0.1

Out[174]=

0.204

Out[176]=

345.372

In[141]:=

```
((350.944 + 341.208 + 345.372) / 3 - 331.4) / 331.4
```

Out[141]=

0.0435767

Although our values of the velocity of sound are slightly higher that the 331.4 + 0.6 by 4.35%, these can be due to the imperfect conditions. Possibly, the fact that the room was very hot due to the blower and hot air implies a greater average kinetic energy of air molecules and hence faster speed of sound. Possibly, other few error parameters can be a zero error, imperfect shape of apparatus or errors in detection apparatus.

In any case, the idea velocity falls nearly one standard deviation away from our mean and that means the experiment was conducted fairly well.

Step 3.

- Repeat the steps above for two other widely different frequencies. You will have two more tables 2A2 and 2A3 similar to the table above.
- For each frequency find the standard deviation in the wavelength and from it calculate the uncertainty in the velocity.
- Record all the results on the table 2B.

$\lambda f = c \Rightarrow \sigma_c = c * \sigma_\lambda / \lambda = f\, \sigma_\lambda$

In[192]:=

```
lambdasigma1 = Sqrt[Sum[(2 * T2A1[[1]][[3 + i]][[3]] - lambdaavg1)^2, {i, 0, 4}] / 4] ;
lambdasigma2 = Sqrt[Sum[(2 * T2A2[[1]][[3 + i]][[3]] - lambdaavg2)^2, {i, 0, 4}] / 4];
lambdasigma3 = Sqrt[Sum[(2 * T2A3[[1]][[3 + i]][[3]] - lambdaavg3)^2, {i, 0, 4}] / 4];

csigma1 = f1 * lambdasigma1;
csigma2 = f2 * lambdasigma2;
csigma3 = f3 * lambdasigma3;

Grid[{{Text["Table 2B. Longitudinal Waves"], SpanFromLeft},
  {"f (Hz)", "λ̄ (m)", "Δλ (m)", "c_exp (m/s)", "Δc_exp (m/s)"},
  {f1, lambdaavg1, lambdasigma1, cavg1, csigma1}, {f2, lambdaavg2, lambdasigma2, cavg2, csigma2},
  {f3, lambdaavg3, lambdasigma3, cavg3, csigma3}}, Frame → All]
```

Out[198]=

Table 2B. Longitudinal Waves				
f (Hz)	$\bar{\lambda}$ (m)	Δλ (m)	c_exp (m/s)	Δc_exp (m/s)
997	0.352	0.0109545	350.944	10.9216
1354	0.252	0.0083666	341.208	11.3284
1693	0.204	0.0114018	345.372	19.3032

Question 1.
- From the air column data, what can you conclude about the dependence of the speed of sound on frequency?
Make sure you have the evidence to support your conclusion.

The speed of sound should ideally remain constant across different frequencies, as it depends primarily on the medium (air) and its temperature, pressure, and composition—not on the frequency. Even in our experiment, there seems to be little to no correlation between the frequency and the speed of sound.

We do however see a greater deviation from the ideal experimental speed of sound for lower frequencies, that may be because the column was barely long enough for higher lengths and for such lengths, the boundary conditions may come into play as we extend the rods to those lengths.

But overall, all our values are around 345 m/s which, given the conditions of the room (high temperature, other measurement error possibilities) seems to be independent of the frequency and fairly close to the expected velocity. If c remains nearly constant within the experimental uncertainties, it indicates that the speed of sound is independent of frequency.

Question 2.
- From your data what can you say about whether the loudspeaker is a node or anti-node? Explain.
(Hint: Think carefully about this. The microphone detects pressure not displacement.)

The loudspeaker causes longitudinal pressure waves which them propagate out to the microphone. Hence, the loudspeaker must be an antinode (a moving trough and crest of pressure)

Nodes would be the point where the pressure is not varying and at equilibrium. But the entire premise of the working of a loudspeaker is the create varying pressures. Hence, it is an anti-node.
