2. $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}_{3\times 1} \begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times 1}$

$A \cdot x$

The product is undefined.

The matrix has a single column while the vector has 2 elements.

The product is defined to be linear combination of columns of Matrix A with corresponding entries of x acting as weights.

There are not equal number of columns & entries.

4. $\begin{bmatrix} 8 & 3 & 1 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 8 \\ 5 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 8+3+1 \\ 5+1+2 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \end{bmatrix}.$

Row - vector rule :

$\begin{bmatrix} 8 & 3 & 1 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8(1) + 3(1) + 1(1) \\ 5(1) + 1(1) + 2(1) \end{bmatrix}$

$= \begin{bmatrix} 12 \\ 8 \end{bmatrix}.$

11. $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$

$\Rightarrow$ Aug matrix equation corresponding to $Ax = b$:

$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right]$

$r_3 + 2r_1 \to$ new r_3.

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{array}\right] \xrightarrow{\frac{r_3}{5} \to \text{new } r_3} \left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$r_1 - 4r_3 \to$ new r_1
$r_2 - 5r_3 \to$ new r_2.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$r_1 - 2r_2 \to$ new r_1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -3 \\ x_3 = 1 \end{cases}$$

$$\therefore Ax = b, \quad x = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$$

11. $A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

since A is already in REF, $Ax = 0$ cam mean.

$$1x_1 - 4x_2 - 2x_3 + 0x_4 + 3x_5 - 5x_6 = 0$$
$$0x_1 + 0x_2 + 1x_3 + 0x_4 + 0x_5 - 1x_6 = 0$$
$$0x_1 + 0x_2 + 0x_3 + 0x_4 + 1x_5 - 4x_6 = 0$$
$$0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 = 0$$

$$\Rightarrow x_1 = 4x_2 + 2x_3 \cancel{\mp} 3x_5 + 5x_6 = 4x_2 + 2x_6 - 12x_6 + 5x_6$$
$$x_3 = x_6$$
$$x_5 = 4x_6.$$

$\therefore \quad x_1 = 4x_2 +$

$$\boxed{\begin{aligned} \therefore\ x_1 &= 4x_2 - 5x_6. \\ x_3 &= x_6 \\ x_4 &= 0 \\ x_5 &= 4x_6. \end{aligned}}$$

Hence, the solution set $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$

$$= \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ 0 \\ x_6 \\ 4x_6 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 1 \\ 4 \end{bmatrix}$$