Electric Potential & Field

PreLab submission with a pass grade is required to begin the lab.
Must be submitted no later than a day before the lab.

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Section: H5
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Readings

Electric Field & Electric Potential

The electric vector field **E** at a point is the force per unit charge on a test charge located at that point. It is numerically equal to the force on a unit (one Coulomb) positive test charge. Because similar charges repel each other, a positive test charge placed near another positive charge experiences a repulsive force, and the field due to a positive charge points away from the charge. The field direction near a negative charged object is towards the object.

Electric-Field Lines:
Electric field lines are a way of visualizing the electric field near charged objects. Field lines define the direction of the force that a positive "test" charge experiences near other charges. The field lines are also used to define the magnitude of the electric field. The lines are drawn such that their density (the number of lines that cross a unit area) is proportional to the magnitude of **E**.

Rather than trying to measure electric fields directly, it is easier to measure the electrical potential near the charges. This is an easier measurement because electric potential does not have a direction (it's a scalar) and can be measured with a voltmeter.

Equipotential Lines:
Consider a test charge that is made to move in the presence of one or more other charges in such a direction that it does not experience any change in electrical potential energy. The path of this test charge is perpendicular to the electric field at every point of its motion, and is called an equipotential line. No work is done to move the test charge; its potential energy does not change. We define electrical potential as the electrical potential energy divided by the test charge. The unit of electrical potential is the Volt. Work against the electric field happens only when the charge is moved with a component parallel to the **E** field; thus, equipotential lines are perpendicular to the **E** field. Devices that measure electric potential are voltmeters, which are inexpensive and readily available. The equipotential lines, *V*, measured with the voltmeter can then be used to map the electric field lines, **E**.

From the definitions of **E** and *V* it follows that:
• Electric field lines at any location are perpendicular to the equipotential lines.

• Electric field lines are directed from higher to lower potential.
• Electric fields are vectors: they define the direction of force on test charge.

Some properties of electrically charged conductors in electrostatic equilibrium:
• All points on the conductor's surface are at the same potential.
• The electric field at a conductor's surface is perpendicular at each point to the surface (since the entire conductor surface is at the same potential).
• The electric field at the surface of a charged conductor is large at a sharp point (a convex region with small radius of curvature).

Example 1:

Consider the point P near the positive charge in Figure 1. The electric field lines and the associated equipotential lines are shown. At point P you measure E = 40 N/C pointing to the right and V = +200V. What do these quantities mean? If a charge of +1.0 nanocoulomb (1 nC = 10^{-9} C) is brought to this point, the electric force on the charge is $F_e = q\,E$ = 1.0×10^{-9} C × 40 $\frac{N}{C}$ to the right = 4.0×10^{-8} N to the right.

The value of the electric potential, V = +200 V, means that the work required to bring a charge of +1.0 x10^{-9} C from the location where the electric potential is zero up to 200 V is $W = q\,V$ = 1.0×10^{-9} C × 200 $\frac{J}{C}$ = +2.0 × 10^{-7} J.

An electric field can be expressed in units of Volts per meter, as well as in units of Newtons per Coulomb. The field can be written as E = 40 N/C = 40 V/m to the right. This means that if you make a small 1 cm displacement along the direction of E, the electric potential drops by 0.40 volt, from 200.0 V to 199.6 V. In general, E= -\[Laplacian]V/\[Laplacian]s, where \[Laplacian]s is a small displacement along the direction of E and \[Laplacian]V is the corresponding change in electric potential; the minus sign means that V drops along the direction of E.

Figure 1. Example 1, the electric field lines and the equipotential lines for a positive point charge, on the 2D plane.
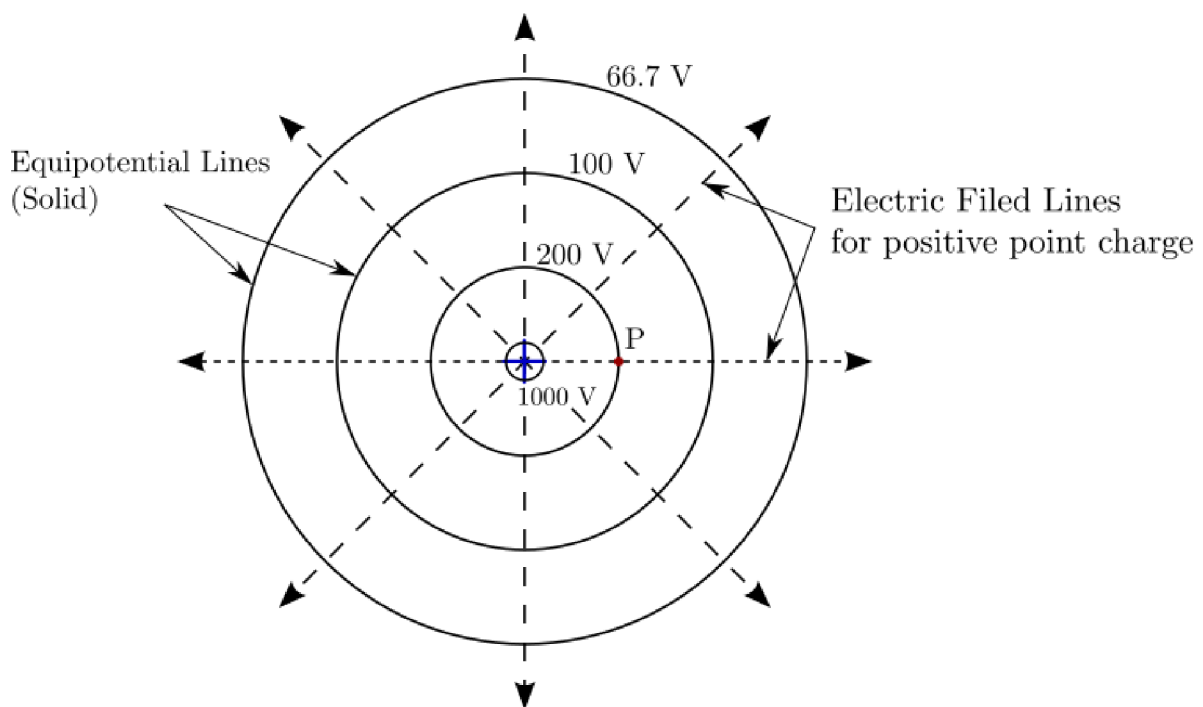
Calculating Electric Potential:

In class you have learned that the potential due to a number of charges can be found by adding up the potential at the point of interest due to each charge. In two dimensions (a plane in three dimensions) this can be

written as,

(1) $V(x_P, y_P) = \frac{1}{4\,\pi\epsilon_0} \sum_i \frac{Q_i}{\sqrt{(x_i-x_P)^2+(y_i-y_P)^2}}$,

where $V(x_P, y_P)$ means the potential at the point (x_P, y_P) and (x_i, y_i) is the location of the charge Q_i. This is simply Coulomb's law expressed in terms of potential rather than electric field.

Dialog:

Question 1.
- Draw the electric filed lines and equipotential lines for a parallel plate capacitor with and without a hollow conducting ring placed between the plates. Please read your manual for the properties of E-fields that determine the shape of the field lines. Pay special attention to the surface of circular conductor in the right figure. The upper plate is at positive potential, the lower one is grounded.
To draw with Mathematica you can use Ctrl+D to see the Drawing Tools. The easiest tool might be the freehand which has the shortcut key f.

(For at least a 2 dimensional view of the problem, the ring would behave like a faraday cage for the centre (at least for the 2d surface))

Figure 3. Plates are connected to a voltage.

Equipotential
Surface

E field

A
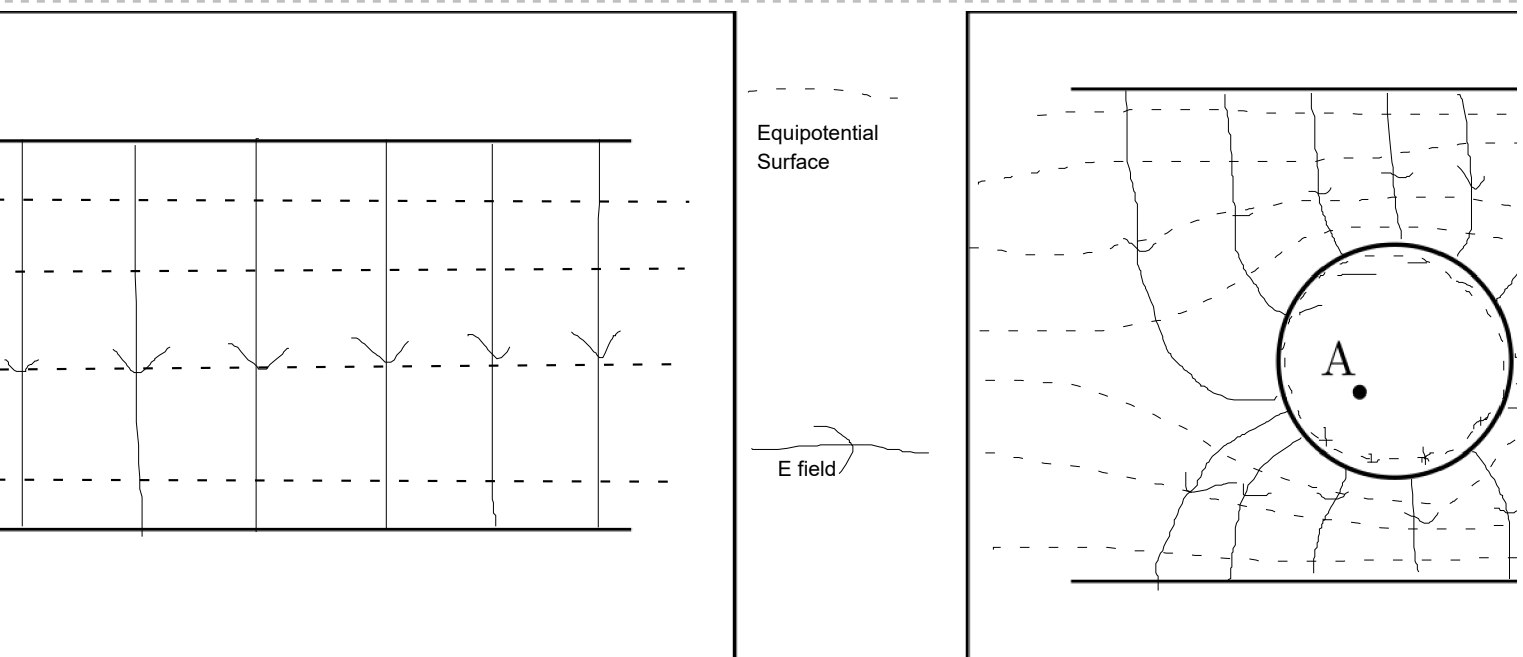
Question 2.
- Write down a general mathematical relationship between the vector electric field **E** and the potential V. Give a single equation.

- What is the electric field at a point INSIDE an electric conductor? For example, at point A inside the circular conductor in Figure 3.
- Use the equation you wrote to show that the potential inside the circular conductor is the same everywhere.
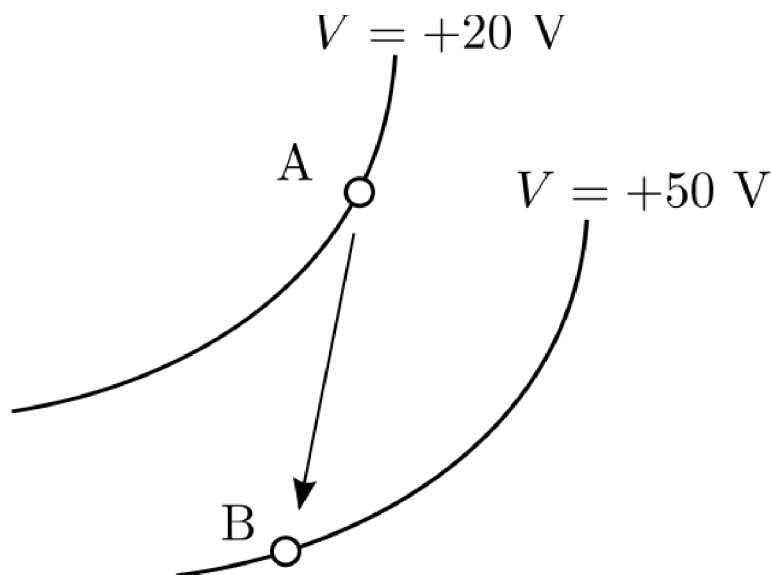
Answer 2.
- $\vec{E} = -\nabla V$ (Electric field is the negative gradient of Potential)
- The conductor's surface charges orient in a way to cause no field within the conductor. Assuming there is no charge within the conductor, **there would be NO field inside the conductor** since it acts as a faraday cage.
- since $\vec{E} = 0 = -\nabla V$, the gradient (hence the change in any direction) of V is zero. Which by definition means that the Potential isn't changing for any change in position as long as the E field is zero (That is the definition of the gradient! How much does the Potential change with the change in position?).

We could also define $\Delta V = \int_a^b \vec{E} \cdot d\vec{l} = 0$ for any point a and b within the conductor since E field is zero.

Question 3.
- Calculate the work done by moving a charge $Q = 1 \times 10^{-6}$ C from point A (on +20 V line) to point B (on +50 V line). See Figure 4.
- How does the choice of trajectory leading from A to B affect the result?

Figure 4. Equipotential lines.

Answer 3.
- Electric Potential energy $U = q\,V = W\,o\,r\,k = \Delta U = q\,\Delta V = 0.00003$ J

- The trajectory itself would make no difference since the work done is path dependent and only dependent on the Voltage of the initial and final position (Electric force is a conservative force).

If moving from B to A, the work done would be opposite.

In[3]:=
```
(* This is an Input cell in case you need *)
a = 1 * 10^-6 * (50 - 20)
N[a]
```

Out[3]= $\dfrac{3}{100\,000}$

Out[4]= 0.00003

Table X. Copy All Your Final Results.

In[5]:=
```
Grid[{{Text["Table X. Final Answers, Lab 4 Prerequisite"], SpanFromLeft},
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   {"E = -∇V ",
    "0",
    N[a]}
  }, Frame → All]
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Out[5]=

Table X. Final Answers, Lab 4 Prerequisite		
Q2. Equation	Q2. E inside conductor (V/m)	Q3. Work done (J)
$\vec{E} = -\nabla V$	0	0.00003

Rutgers 276 Classical Physics Lab

"Electric Potential & Field"

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