FLEX PWR

Cross-market BESS Optimizer

This document shows the mathematical formulation of a cross-market optimization problem of a Battery Energy Storage System (BESS). This is a simple model of flexibility asset optimization, which is aimed at transparency and comprehensibility. Therefore, more complex system parameters like efficiency or grid connection constraints are omitted. This document is meant to accompany the Python implementation of the optimization available on our GitHub. This model is also used to calculate the FlexIndex.

Optimization Step 1 (Day-ahead Auction)

Parameter	Description	Value/Unit
H	List of hours	$\{0, 1, ..., 23\}$
Q	List of quarters	$\{1, 2, ..., 96\}$
Q^+	List of quarters plus 1	$\{1, 2, ..., 97\}$
cap_{energy}	Maximum state of charge	MWh
cap_{power}	Maximum charge/discharge rate	MW
n_{cycles}	Allowed daily equivalent full cycles	[-]
limit	Allowed daily cycle capacity	$\mathrm{cap}_{energy} \cdot \mathrm{n}_{cycles}$
λ_q^{DAA}	Perfect foresight day-ahead price at quarter q	[EUR/MWh]

Parameters of Optimization Step 1 (DAA)

Variable	Description	Value/Unit
SOC_q	State of charge at quarter q	MWh
$\mathrm{CHA}_q^{\mathrm{DAA}}$	Charge (buy on DAA) at quarter q	$0 \leq \mathbb{R} \leq 1$
$\mathrm{DIS}_q^{\mathrm{DAA}}$	Discharge (sell on DAA) at quarter q	$0 \leq \mathbb{R} \leq 1$

Variables of Optimization Step 1 (DAA)

Optimization Step 1: Objective Function

The objective is to find the charge-discharge-schedule and according set of DAA Trades, which generate the highest revenue:

$$\max_{\mathrm{CHA}_q^{\mathrm{DAA}}, \mathrm{DIS}_q^{\mathrm{DAA}}} \quad \sum_{q=1}^{96} \lambda_q^{\mathrm{DAA}} \cdot \frac{\mathrm{cap}_{power}}{4} \cdot (\mathrm{DIS}_q^{\mathrm{DAA}} - \mathrm{CHA}_q^{\mathrm{DAA}})$$

Optimization Step 1: Constraints

(1.1) State of charge in any quarter cannot be higher than the maximum capacity (which is cap_{energy}):

$$\text{SOC}_q \leq \text{cap}_{energy} \forall q \; in \; \{1, ..., 97\}$$

(1.2) State of charge in any quarter cannot be less than 0:

$$\text{SOC}_q \geq 0 \quad \forall q \; in \; \{1, ..., 97\}$$

(1.3) SOC at the starting quarter is fixed as 0:

$$\text{SOC}_1 = 0$$

(1.4) SOC at the ending quarter is fixed as 0. The ending quarter is not q96 but q97.

$$\text{SOC}_{97} = 0$$

(1.5) Step constraint. The SOC of each quarters is given as the SOC of the previous quarter plus charges minus discharges:

$$\text{SOC}_q = \text{SOC}_{q-1} + \frac{\text{cap}_{power}}{4} \cdot (\text{CHA}_q^{\text{DAA}} - \text{DIS}_q^{\text{DAA}}) \quad \forall q \; in \; \{2, ..., 97\}$$

(1.6) Total physical charges have to be less or equal to the daily physical volume limit

$$\frac{\text{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \text{CHA}_q^{\text{DAA}} \;\; \leq \;\; \text{limit}$$

(1.7) Total physical discharges have to be less or equal to the daily physical volume limit

$$\frac{\text{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \text{DIS}_q^{\text{DAA}} \;\; \leq \;\; \text{limit}$$

(1.8) Trades on the DAA have to be taken simultaneously in all 4 quarters of a given hour.

$$\text{CHA}_{4h+1}^{\text{DAA}} = \text{CHA}_{4h+2}^{\text{DAA}} = \text{CHA}_{4h+3}^{\text{DAA}} = \text{CHA}_{4h+4}^{\text{DAA}} \qquad \forall h \; in \; \text{H}$$

(1.9) Trades on the DAA have to be taken simultaneously in all 4 quarters of a given hour.

$$\text{DIS}_{4h+1}^{\text{DAA}} = \text{DIS}_{4h+2}^{\text{DAA}} = \text{DIS}_{4h+3}^{\text{DAA}} = \text{DIS}_{4h+4}^{\text{DAA}} \qquad \forall h \; in \; \text{H}$$

Optimization Step 2 (Intraday Auction)

Parameter	Description	Value/Unit
H	List of hours	$\{0, 1, ..., 23\}$
Q	List of quarters	$\{1, 2, ..., 96\}$
Q^+	List of quarters plus 1	$\{1, 2, ..., 97\}$
cap_{energy}	Maximum state of charge	MWh
cap_{power}	Maximum charge/discharge rate	MW
n_{cycles}	Allowed daily equivalent full cycles	[-]
limit	Allowed daily cycle capacity	$\text{cap}_{energy} \cdot \text{n}_{cycles}$
λ_q^{IDA}	Perfect foresight IDA price at quarter q	[EUR/MWh]
$\text{CHA}_q^{\text{DAA}}$	CHA from step 1 at quarter q	$0 \leq \mathbb{R} \leq 1$
$\text{DIS}_q^{\text{DAA}}$	DIS from step 1 at quarter q	$0 \leq \mathbb{R} \leq 1$

Parameters of Optimization Step 2 (IDA)

Variable	Description	Value/Unit
SOC_q	State of charge at quarter q	MWh
$\text{CHA}_q^{\text{IDA}}$	Charge (buy on IDA) at quarter q	$0 \leq \mathbb{R} \leq 1$
$\text{DIS}_q^{\text{IDA}}$	Discharge (sell on IDA) at quarter q	$0 \leq \mathbb{R} \leq 1$
$\widehat{\text{CHA}}_q^{\text{IDA}}$	buy on IDA to flatten existing DIS at quarter q	$0 \leq \mathbb{R} \leq 1$
$\widehat{\text{DIS}}_q^{\text{IDA}}$	sell on IDA to flatten existing CHA at quarter q	$0 \leq \mathbb{R} \leq 1$

Variables of Optimization Step 2 (IDA)

Step 2: Objective Function

The objective is to find the charge-discharge-schedule and according set of IDA Trades, which generate the highest revenue:

$$\max_{\substack{\text{CHA}_q^{\text{IDA}},\text{DIS}_q^{\text{IDA}} \\ \widehat{\text{CHA}}_q^{\text{IDA}},\widehat{\text{DIS}}_q^{\text{IDA}}}} \sum_{q=1}^{96} \lambda_q^{\text{IDA}} \cdot \frac{\text{cap}_{power}}{4} \cdot (\text{DIS}_q^{\text{IDA}} - \text{CHA}_q^{\text{IDA}} + \widehat{\text{DIS}}_q^{\text{IDA}} - \widehat{\text{CHA}}_q^{\text{IDA}})$$

Optimization Step 2: Constraints

(2.1) State of charge in any quarter cannot be higher than the maximum capacity:

$$\text{SOC}_q \leq \text{cap}_{energy} \qquad \forall q \; in \; \{1, ..., 97\}$$

(2.2) State of charge in any quarter cannot be less than zero:

$$\text{SOC}_q \geq 0 \qquad \forall q \; in \; \{1, ..., 97\}$$

(2.3) SOC at the starting quarter is fixed as 0:

$$\text{SOC}_1 = 0$$

(2.4) SOC at the ending quarter is fixed as 0. The ending quarter is not q96 but q97.

$$\text{SOC}_{97} = 0$$

(2.5) Step constraint. The SOC of each quarters is given as the SOC of the previous quarter plus charges minus discharges:

$$\mathrm{SOC}_q = \mathrm{SOC}_{q-1} + \frac{\mathrm{cap}_{power}}{4}(\mathrm{CHA}_q^{\mathrm{IDA}} - \mathrm{DIS}_q^{\mathrm{IDA}} + \widehat{\mathrm{CHA}}_q^{\mathrm{IDA}} - \widehat{\mathrm{DIS}}_q^{\mathrm{IDA}} + \mathrm{CHA}_q^{\mathrm{DAA}} - \mathrm{DIS}_q^{\mathrm{DAA}})$$

$$\forall q \; in \; \{2, ..., 97\}$$

(2.6) Total physical charges have to be less or equal to the daily physical volume limit. Here, $\widehat{\mathrm{DIS}}_q^{\mathrm{IDA}}$ are subtracted, since these are quarters where we had a position on the DAA which we now close on the IDA.

$$\frac{\mathrm{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \left(\mathrm{CHA}_q^{\mathrm{IDA}} - \widehat{\mathrm{DIS}}_q^{\mathrm{IDA}} + \mathrm{CHA}_q^{\mathrm{DAA}} \right) \;\leq\; \mathrm{limit}$$

(2.7) Total physical discharges have to be less or equal to the daily physical volume limit. Here, $\widehat{\mathrm{CHA}}_q^{\mathrm{IDA}}$ are subtracted, since these are quarters where we had a position on the DAA which we now close on the IDA.

$$\frac{\mathrm{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \left(\mathrm{DIS}_q^{\mathrm{IDA}} - \widehat{\mathrm{CHA}}_q^{\mathrm{IDA}} + \mathrm{DIS}_q^{DAA} \right) \;\leq\; \mathrm{limit}$$

(2.8) Since $\widehat{\mathrm{CHA}}_q^{\mathrm{IDA}}$ are meant to close existing positions, they can only be placed where a $\mathrm{DIS}_q^{\mathrm{DAA}}$ position exists.

$$\widehat{\mathrm{CHA}}_q^{\mathrm{IDA}} \leq \mathrm{DIS}_q^{\mathrm{DAA}} \qquad \forall q \; in \; \{1, ..., 96\}$$

(2.9) Since $\widehat{\mathrm{DIS}}_q^{\mathrm{IDA}}$ are meant to close existing positions, they can only be placed where a $\mathrm{CHA}_q^{\mathrm{DAA}}$ position exists.

$$\widehat{\mathrm{DIS}}_q^{\mathrm{IDA}} \leq \mathrm{CHA}_q^{\mathrm{DAA}} \qquad \forall q \; in \; \{1, ..., 96\}$$

(2.10) Sum of $\mathrm{CHA}_q^{\mathrm{IDA}}$ and $\mathrm{CHA}_q^{\mathrm{DAA}}$ has to be ≤ 1, s.t. the battery charges with cap_{power} as maximum rate.

$$\mathrm{CHA}_q^{\mathrm{IDA}} + \mathrm{CHA}_q^{\mathrm{DAA}} \leq 1 \qquad \forall q \; in \; \{1, ..., 96\}$$

(2.11) Sum of $\mathrm{DIS}_q^{\mathrm{IDA}}$ and $\mathrm{DIS}_q^{\mathrm{DAA}}$ has to be ≤ 1, s.t. the battery discharges with cap_{power} as maximum rate.

$$\mathrm{DIS}_q^{\mathrm{IDA}} + \mathrm{DIS}_q^{\mathrm{DAA}} \leq 1 \qquad \forall q \; in \; \{1, ..., 96\}$$

Optimization Step 3 (Intraday Continuous ID1)

Parameter	Description	Value/Unit
H	List of hours	$\{0, 1, ..., 23\}$
Q	List of quarters	$\{1, 2, ..., 96\}$
Q^{+}	List of quarters plus 1	$\{1, 2, ..., 97\}$
cap_{energy}	Maximum state of charge	MWh
cap_{power}	Maximum charge/discharge rate	MW
n_{cycles}	Allowed daily equivalent full cycles	[-]
limit	Allowed daily cycle capacity	$\text{cap}_{energy} \cdot \text{n}_{cycles}$
λ_q^{IDC}	Perfect foresight IDC QH ID1 price at quarter q	[EUR/MWh]
$\text{CHA}_q^{\text{DAAIDA}}$	CHA from step 1 and 2 combined at quarter q	$\text{CHA}_q^{\text{DAA}} + \text{CHA}_q^{\text{IDA}} - \widehat{\text{DIS}}_q^{\text{IDA}}$
$\text{DIS}_q^{\text{DAAIDA}}$	DIS from step 1 and 2 combined at quarter q	$\text{DIS}_q^{\text{DAA}} + \text{DIS}_q^{\text{IDA}} - \widehat{\text{CHA}}_q^{\text{IDA}}$

Parameters of Optimization Step 3 (IDC)

Variable	Description	Value/Unit
SOC_q	State of charge at quarter q	MWh
$\text{CHA}_q^{\text{IDC}}$	Charge (buy on IDC) at quarter q	$0 \le \mathbb{R} \le 1$
$\text{DIS}_q^{\text{IDC}}$	Discharge (sell on IDC) at quarter q	$0 \le \mathbb{R} \le 1$
$\widehat{\text{CHA}}_q^{\text{IDC}}$	buy on IDC to flatten existing DIS at quarter q	$0 \le \mathbb{R} \le 1$
$\widehat{\text{DIS}}_q^{\text{IDC}}$	sell on IDC to flatten existing CHA at quarter q	$0 \le \mathbb{R} \le 1$

Variables of Optimization Step 3 (IDC)

Step 3: Objective Function

The objective is to find the charge-discharge-schedule and according set of IDC Trades, which generate the highest revenue:

$$\max_{\substack{\text{CHA}_q^{\text{IDC}}, \text{DIS}_q^{\text{IDC}} \\ \widehat{\text{CHA}}_q^{\text{IDC}}, \widehat{\text{DIS}}_q^{\text{IDC}}}} \sum_{q=1}^{96} \lambda_q^{\text{IDC}} \cdot \frac{\text{cap}_{power}}{4} \cdot (\text{DIS}_q^{\text{IDC}} - \text{CHA}_q^{\text{IDC}} + \widehat{\text{DIS}}_q^{\text{IDC}} - \widehat{\text{CHA}}_q^{\text{IDC}})$$

Step 3: Constraints

(3.1) State of charge in any quarter cannot be higher than the maximum capacity:

$$\text{SOC}_q \le \text{cap}_{energy} \qquad \forall q \; in \; \{1, ..., 97\}$$

(3.2) State of charge in any quarter cannot be less than 0:

$$\text{SOC}_q \ge 0 \qquad \forall q \; in \; \{1, ..., 97\}$$

(3.3) SOC at the starting quarter is fixed as 0:

$$\text{SOC}_1 = 0$$

(3.4) SOC at the ending quarter is fixed as 0. The ending quarter is not q96 but q97.

$$\text{SOC}_{97} = 0$$

(3.5) Step constraint. The SOC of each quarters is given as the SOC of the previous quarter plus charges minus discharges:

$$\mathrm{SOC}_q = \mathrm{SOC}_{q-1} + \frac{\mathrm{cap}_{power}}{4}(\mathrm{CHA}_q^{\mathrm{IDC}} - \mathrm{DIS}_q^{\mathrm{IDC}} + \widehat{\mathrm{CHA}}_q^{\mathrm{IDC}} - \widehat{\mathrm{DIS}}_q^{\mathrm{IDC}} + \mathrm{CHA}_q^{\mathrm{DAAIDA}} - \mathrm{DIS}_q^{\mathrm{DAAIDA}})$$

$$\forall q \; in \; \{2, ..., 97\}$$

(3.6) Total physical charges have to be less or equal to the daily physical volume limit.

$$\frac{\mathrm{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \left(\mathrm{CHA}_q^{\mathrm{IDC}} - \widehat{\mathrm{DIS}}_q^{\mathrm{IDC}} + \mathrm{CHA}_q^{\mathrm{DAAIDA}} \right) \; \leq \; \mathrm{limit}$$

(3.7) Total physical discharges have to be less or equal to the daily physical volume limit.

$$\frac{\mathrm{cap}_{power}}{4} \cdot \sum_{q=1}^{96} \left(\mathrm{DIS}_q^{\mathrm{IDC}} - \widehat{\mathrm{CHA}}_q^{\mathrm{IDC}} + \mathrm{DIS}_q^{\mathrm{DAAIDA}} \right) \; \leq \; \mathrm{limit}$$

(3.8) $\widehat{\mathrm{CHA}}_q^{\mathrm{IDC}}$ positions can only be placed where a $\mathrm{DIS}_q^{\mathrm{DAAIDA}}$ position exists.

$$\widehat{\mathrm{CHA}}_q^{\mathrm{IDC}} \leq \mathrm{DIS}_q^{\mathrm{DAAIDA}} \qquad \forall q \; in \; \{1, ..., 96\}$$

(3.9) $\widehat{\mathrm{DIS}}_q^{\mathrm{IDC}}$ positions can only be placed where a $\mathrm{CHA}_q^{\mathrm{DAAIDA}}$ position exists.

$$\widehat{\mathrm{DIS}}_q^{\mathrm{IDC}} \leq \mathrm{CHA}_q^{\mathrm{DAAIDA}} \qquad \forall q \; in \; \{1, ..., 96\}$$

(3.10) Sum of $\mathrm{CHA}_q^{\mathrm{IDC}}$ and $\mathrm{CHA}_q^{\mathrm{DAAIDA}}$ has to be ≤ 1.

$$\mathrm{CHA}_q^{\mathrm{IDC}} + \mathrm{CHA}_q^{\mathrm{DAAIDA}} \leq 1 \qquad \forall q \; in \; \{1, ..., 96\}$$

(3.11) Sum of $\mathrm{DIS}_q^{\mathrm{IDC}}$ and $\mathrm{DIS}_q^{\mathrm{DAAIDA}}$ has to be ≤ 1.

$$\mathrm{DIS}_q^{\mathrm{IDC}} + \mathrm{DIS}_q^{\mathrm{DAAIDA}} \leq 1 \qquad \forall q \; in \; \{1, ..., 96\}$$