CSCI 301 Homework 1

Name _____   W#_____

1. True or False (10pts)
   1) Every element of a subset is also an element of the original set. True
   2) The statement (p → q) is false only when p is true, and q is false. False
   3) The empty set is a subset of every set. True
   4) If A ⊆ B and B ⊆ A, then A and B are equal. True
   5) The negation of the statement (p ∨ q) is logically equivalent to (~p ∧ ~q). True
   6) The intersection of two sets is always non-empty. False
   7) If sets A and B are disjoint, they have no elements in common. True
   8) The converse of the statement (p → q) is logically equivalent to (q → p). True
   9) The union of a set and its complement is the universal set. True
   10) Here is a list data structure: [1, 2, 2, 3]. When you convert this list to a set, the cardinality of that set is 4. False

2. (5pts) A survey was taken by 200 students on what courses they have taken. The results show that 90 students took CS, 110 took Mathematics, 60 took Physics, 20 took both CS and Math, 20 took both CS and Physics, and 30 took both Math and Physics. How many students have taken course in all three areas, including Mathematics, CS, and Physics? Please demonstrate your stepwise analysis. (You can assume that all students took at least one of the courses)

   With x being what we are trying to solve
   200 = 90+110+60-20-20-30+x
   200=260-70+x
   X= 200-190
   X=10

3. (5pts) A detective has interviewed four witnesses of a crime. From the stories of the witnesses the detective has concluded that:
   1) If the butler is lying, then so is the handyman.
   2) The cook and the butler cannot both be telling the truth.
   3) The cook and the gardener are not both lying.
   4) If the butler is telling the truth, then the gardener is lying.

   For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Use the truth table to explain your reasoning. Please use the following notations in your reasoning –

   B: The butler is telling the truth.
   C: The cooking is telling the truth.
   G: The gardener is telling the truth.
   H: The handyman is telling the truth.

b means h
not c or not b
c or g
if not b then g

assume b
not c; is h;

assume not b
not c; contradiction detected.
assuming the butler isnt lying it means the cook is lying
the cook and the gardner aren't both lying, so the gardner must be telling the truth
But that isn't possible since if the butler wasn't lying, then the gardner should be lying

b and h are lying

4. (5pts) Prove if $n \in Z$, then n^2+5n-3 is odd.
   $2a+1 = $ odd int
   $n^2+5n-3 \rightarrow n(n+5)-3$
   denote n as 2m - 5 (m being another int)
   using new applied n value into $n^2+5n-3 \rightarrow 4m^2-10m-3$

5. (5pts) Suppose a, b $\in$ Z where Z denotes all integers. Try to prove the following statement using **contraposition**:

   If ab is even, then a and b are not both odd.
   $a = 2k_1+1$; $b=2k_2+1 \rightarrow ab = (2k_1+1)(2k_2+1)$
   ➔ $4k_1k_2+2k_1+2k^2+1$

Submission

Please submit a PDF file to Canvas.