

Note that as a matter of convention, I write $(s \boxplus)$ to mean “delete this tube if the resulting instantiation mentions \boxplus ” (it’s how I implement \forall).

Projecting the cap

COEF_{COM/CAP}

$$\frac{\begin{array}{c} (s \ r') = (s' \ r') \quad (s \ x) \neq (s' \ x) \quad (s_i \ x) \neq (s'_i \ x) \\ \hline (\text{coe}^{r \rightsquigarrow r'}_{[x] (\text{fcom}^{(s \ x) \rightsquigarrow (s' \ x)} (A \ x) [(s_i \ x) = (s'_i \ x) \hookrightarrow [z] (B_i \ x \ z)])}) M \end{array}}{\mapsto} \\ (\text{gcom}^{r \rightsquigarrow r'}_{[x] (A \ x)} M [(s_i \ \boxplus) = (s'_i \ \boxplus) \hookrightarrow [x] (\text{coe}^{(s' \ x) \rightsquigarrow (s \ x)}_{[z] (B_i \ x \ z)} (\text{coe}^{r \rightsquigarrow x}_{[x] (B_i \ x \ (s \ x))} M))]) [(s \ \boxplus) = (s' \ \boxplus) \hookrightarrow [x] (\text{coe}^{r \rightsquigarrow x}_{[x] (A \ x)} M)])$$

Projecting the tube

COEF_{COM/TUBE}

$$\frac{\begin{array}{c} (s \ r') \neq (s' \ r') \quad k \text{ min s.t. } (s_k \ r') = (s'_k \ r') \quad (s \ x) \neq (s' \ x) \quad (s_i \ x) \neq (s'_i \ x) \\ \hline O \triangleq (\text{hcom}^{(s' \ r') \rightsquigarrow (s \ r')}_{(A \ r)} (\text{cap}^{(s \ r) \rightsquigarrow (s' \ r')} M [(s_i \ r) = (s'_i \ r) \hookrightarrow [z] (B_i \ r \ z)]) [(s_i \ r) = (s'_i \ r) \hookrightarrow [z] (\text{coe}^{z \rightsquigarrow (s \ r)}_{[z] (B_i \ r \ z)} (\text{coe}^{(s' \ r) \rightsquigarrow z}_{[z] (B_i \ r \ z)} M))]) \\ P \triangleq (\text{gcom}^{r \rightsquigarrow r'}_{[x] (A \ x)} O [(s_i \ \boxplus) = (s'_i \ \boxplus) \hookrightarrow [x] (\text{coe}^{(s' \ r') \rightsquigarrow (s \ r')}_{[z] (B_i \ r' \ z)} (\text{coe}^{r \rightsquigarrow x}_{[x] (B_i \ x \ (s' \ x))} M))], [(s \ \boxplus) = (s' \ \boxplus) \hookrightarrow [x] (\text{coe}^{r \rightsquigarrow x}_{[x] (A \ x)} M)]) \end{array}}{\mapsto} \\ (\text{coe}^{r \rightsquigarrow r'}_{[x] (\text{fcom}^{(s \ x) \rightsquigarrow (s' \ x)} (A \ x) [(s_i \ x) = (s'_i \ x) \hookrightarrow [z] (B_i \ x \ z)])}) M) \\ \mapsto \\ (\text{gcom}^{(s \ r') \rightsquigarrow (s' \ r')}_{[z] (B_k \ r' \ z)} P [(s_i \ \boxplus) = (s'_i \ \boxplus) \hookrightarrow [z] (\text{coe}^{(s' \ r') \rightsquigarrow z}_{[z] (B_i \ r' \ z)} (\text{coe}^{r \rightsquigarrow r'}_{[x] (B_i \ x \ (s' \ x))} M))]) [r = r' \hookrightarrow [z] (\text{coe}^{(s' \ r') \rightsquigarrow z}_{[z] (B_k \ r' \ z)} (\text{coe}^{r \rightsquigarrow r'}_{[x] (B_k \ x \ (s' \ x))} M))])$$

Within this case, there are several sub-cases depending on what the dimensions involved are; but it is not yet clear to me whether it will be an advantage to expand them here.