cubical evaluation semantics

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I've been thinking a bit more carefully about the evaluation semantics and the semantic domain. What I think we want to do is take a term and an environment and return a family of values (indexed in restrictions); but crucially, I think it is important that the environment be a vector of *values* and not of value-families. By studying carefully the case for instantiating closures (where the rubber meets the road for variables), I think it becomes clear that this is the right way.

To interpret a variable, we just project from the environment and ignore the restriction. I think the way that we had been thinking of doing it would result in the restriction from inside the closure infecting the argument.

Colors: terms, value families, values, restrictions. I am writing $\phi^* F$ to mean the reindexing of a family by a restriction.

$$
\begin{aligned}
[\![(\lambda\{M\})]\!]^{\pi}_{\rho} &= [\phi]\ \lambda\langle M, \rho, \pi, \phi\rangle \\
[\![(\Pi\ A\ \{B\})]\!]^{\pi}_{\rho} &= [\phi]\ \mathbf{\Pi}\Big(\phi^*[\![A]\!]^{\pi}_{\rho}, \langle B, \rho, \pi, \phi\rangle\Big) \\
[\![(M\ N)]\!]^{\pi}_{\rho} &= \underline{\mathbf{app}}([\![M]\!]^{\pi}_{\rho}, [\![N]\!]^{\pi}_{\rho}) \\
[\![x_i]\!]^{\pi}_{\rho} &= [\_]\ \rho_i
\end{aligned}
$$

We write $\phi \vDash F \Downarrow V$ as a shorthand for $F[\phi] \equiv V$.

$$
\frac{\phi \vDash F \Downarrow \lambda\langle M, \rho, \pi, \psi\rangle}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow [\![M]\!]^{\pi}_{[\rho, N[\phi]]}[\psi]}
\qquad
\frac{\phi \vDash F \Downarrow \uparrow^{C} R \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow \uparrow^{\psi^*[\![B]\!]^{\pi}_{[\rho, N[\phi]]}}([\chi]\ \mathbf{app}(\chi^* R, \chi \circ \phi^* N))}
$$

$$
\frac{\phi \vDash F \Downarrow \mathbf{coe}^{r \rightsquigarrow r'}_{\langle\alpha\rangle C}\ M \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow \nu\beta.\ \nu\gamma.\ \underline{\mathbf{coe}}^{r \rightsquigarrow r'}_{\langle\beta\rangle [\![B]\!]^{[\pi, (\beta\ \alpha)]}_{\left[\rho, \underline{\mathbf{coe}}^{r' \rightsquigarrow \beta}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}\ N[\phi]\right]}}\ \underline{\mathbf{app}}(M, \underline{\mathbf{coe}}^{r' \rightsquigarrow r}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}\ \phi^* N)}
$$

$$
\frac{\phi \vDash F \Downarrow \mathbf{hcom}^{r \rightsquigarrow r'}_{C}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, O) \Downarrow \nu\beta.\ \underline{\mathbf{hcom}}^{r \rightsquigarrow r'}_{\psi^*[\![B]\!]^{\pi}_{[\rho, O[\phi]]}}\ \underline{\mathbf{app}}(M, \phi^* O)\ [\overline{\xi_i \hookrightarrow \langle\beta\rangle \underline{\mathbf{app}}((\alpha_i\ \beta)\cdot N_i, \phi^* O)}]}
$$

$$
\phi \vDash \underline{\mathbf{coe}}^{r \rightsquigarrow r'}_{\langle\alpha\rangle A}\ M \Downarrow \underline{\mathbf{coe}}^{\phi^* r \rightsquigarrow \phi^* r'}_{\nu\beta.\ \langle\beta\rangle \phi^*(\beta\ \alpha)\cdot A}\ \phi^* M
$$

$$\frac{\cdot \vDash r = r'}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \equiv M[\cdot]}
\qquad
\frac{\cdot \vDash r \neq r' \qquad \exists_{min} k, \cdot \vDash \xi_k}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \equiv N_k[\alpha_k = r'[\cdot]]}
\qquad
\frac{\cdot \vDash r \neq r' \qquad \overline{\cdot \nvDash \xi_i} \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}(A, B)}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \equiv \mathbf{hcom}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}]}$$

$$\frac{\cdot \vDash r \neq r' \qquad \overline{\cdot \nvDash \xi_i} \qquad \cdot \vDash C \Downarrow \mathbf{bool}}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \equiv M[\cdot]}$$

$$\phi \vDash \underline{\mathbf{hcom}}_{A}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \Downarrow \nu\beta.\ \underline{\mathbf{hcom}}_{\phi^*A}^{\phi^* r \rightsquigarrow \phi^* r'}\ \phi^* M\ [\overline{\phi^*\xi_i \hookrightarrow \langle\beta\rangle \phi^*(\beta\ \alpha_i) \cdot N_i}]$$

$$\frac{\cdot \vDash r = r'}{\underline{\mathbf{coe}}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M \equiv M[\cdot]}
\qquad
\frac{\cdot \vDash r \neq r' \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}(A, B)}{\underline{\mathbf{coe}}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M \equiv \mathbf{coe}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M}
\qquad
\frac{\cdot \vDash r \neq r' \qquad \cdot \vDash C \Downarrow \mathbf{bool}}{\underline{\mathbf{coe}}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M \equiv M[\cdot]}
\qquad
\frac{\begin{array}{c}\cdot \vDash r \neq r' \qquad \cdot \vDash C \Downarrow \mathbf{fcom}^{s \rightsquigarrow s'}\ A\ [\overline{\xi_i \hookrightarrow \langle\beta\rangle B_i}] \qquad \phi \triangleq (r' = \alpha) \\ ?foo\end{array}}{\underline{\mathbf{coe}}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M \equiv \underline{\mathbf{box}}^{\phi^* s \rightsquigarrow \phi^* s'} \left(\underline{\mathbf{hcom}}_{\phi^* A}^{\phi^* s \rightsquigarrow \phi^* s'}\ P\ [\overline{T}]\right) [\overline{U}]}$$