cubical evaluation semantics

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I've been thinking a bit more carefully about the evaluation semantics and the semantic domain. What I think we want to do is take a term and an environment and return a family of values (indexed in restrictions); but crucially, I think it is important that the environment be a vector of *values* and not of value-families. By studying carefully the case for instantiating closures (where the rubber meets the road for variables), I think it becomes clear that this is the right way.

To interpret a variable, we just project from the environment and ignore the restriction. I think the way that we had been thinking of doing it would result in the restriction from inside the closure infecting the argument.

Colors: terms, value families, values, restrictions. I am writing $\phi^* F$ to mean the reindexing of a family by a restriction.

$$
\begin{aligned}
[\![(\lambda\{M\})]\!]_\rho &= [\phi]\ \lambda\langle M, \rho, \phi\rangle \\
[\![(\Pi\ A\ \{B\})]\!]_\rho &= [\phi]\ \mathbf{\Pi}\Big(\phi^* [\![A]\!]_\rho, \langle B, \rho, \phi\rangle\Big) \\
[\![(M\ N)]\!]_\rho &= \underline{\mathbf{app}}([\![M]\!]_\rho, [\![N]\!]_\rho) \\
[\![x_i]\!]_\rho &= [\_]\ \rho_i
\end{aligned}
$$

$$
\underline{\mathbf{app}}(F, G) = [\phi]\ \begin{cases}
F[\phi] \equiv \lambda\langle M, \rho, \psi\rangle & \Longmapsto\quad [\![M]\!]_{[\rho, G[\phi]]}[\psi] \\
F[\phi] \equiv \uparrow^C R, C[\cdot] \equiv \mathbf{\Pi}\big(A, \langle B, \rho, \psi\rangle\big) & \Longmapsto\quad \uparrow^{\psi^* [\![B]\!]_{[\rho, G[\phi]]}}\big([\chi]\ \mathbf{app}(\chi^* R, \chi \circ \phi^* G)\big) \\
F[\phi] \equiv \mathbf{coe}_{\langle\alpha\rangle C}^{r \rightsquigarrow r'}\ M, C[\cdot] \equiv \mathbf{\Pi}\big(A, \langle B, \rho, \psi\rangle\big) & \Longmapsto\quad \nu\beta.\ \nu\gamma.\ \underline{\mathbf{coe}}_{\langle\beta\rangle [\![B]\!]_{\left[\rho, \underline{\mathbf{coe}}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}^{r' \rightsquigarrow \beta}\ G[\phi]\right]}}^{r \rightsquigarrow r'}\ \underline{\mathbf{app}}(M, [\chi]\ \underline{\mathbf{coe}}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}^{\chi^* r' \rightsquigarrow \chi^* r}\ \chi \circ \phi^* G) \\
F[\phi] \equiv \mathbf{hcom}_C^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}], C[\cdot] \equiv \mathbf{\Pi}\big(A, \langle B, \rho, \psi\rangle\big) & \Longmapsto\quad \nu\beta.\ \underline{\mathbf{hcom}}_{\psi^* [\![B]\!]_{[\rho, G[\phi]]}}^{r \rightsquigarrow r'}\ \underline{\mathbf{app}}(M, \phi^* G)\ [\overline{\xi_i \hookrightarrow \langle\beta\rangle \underline{\mathbf{app}}((\alpha_i\ \beta) \cdot N_i, \phi^* G)}]
\end{cases}
$$

$$\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] = \begin{cases} r[\cdot] = r'[\cdot] & \Longmapsto \quad M[\cdot] \\ \exists_{min} k.\ \models \xi_k[\cdot] & \Longmapsto \quad N_k[\alpha_k = r'[\cdot]] \\ C[\cdot] \equiv \mathbf{\Pi}(A, B) & \Longmapsto \quad \mathbf{hcom}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] \\ C[\cdot] \equiv \mathbf{bool} & \Longmapsto \quad M[\cdot] \end{cases}$$