# cubical evaluation semantics

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I've been thinking a bit more carefully about the evaluation semantics and the semantic domain. What I think we want to do is take a term and an environment and return a family of values (indexed in restrictions); but crucially, I think it is important that the environment be a vector of *values* and not of value-families. By studying carefully the case for instantiating closures (where the rubber meets the road for variables), I think it becomes clear that this is the right way.

To interpret a variable, we just project from the environment and ignore the restriction. I think the way that we had been thinking of doing it would result in the restriction from inside the closure infecting the argument.

Colors: terms, value families, values, restrictions. I am writing $\phi^* F$ to mean the reindexing of a family by a restriction.

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\begin{aligned}
[\![(\lambda\{M\})]\!]_\rho &= [\phi]\ \lambda\langle M, \rho, \phi\rangle \\
[\![(\Pi\ A\ \{B\})]\!]_\rho &= [\phi]\ \Pi\big(\phi^*[\![A]\!]_\rho, \langle B, \rho, \phi\rangle\big) \\
[\![(M\ N)]\!]_\rho &= \underline{\mathbf{app}}([\![M]\!]_\rho, [\![N]\!]_\rho) \\
[\![x_i]\!]_\rho &= [\_]\ \rho_i
\end{aligned}
$$

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\underline{\mathbf{app}}(F, G) = [\phi]\begin{cases}
F[\phi] \equiv \lambda\langle M, \rho, \psi\rangle & \Rightarrow\ [\![M]\!]_{\rho, G[\phi]}[\psi] \\
F[\phi] \equiv \uparrow^C R, C[\cdot] \equiv \Pi(A, \langle B, \rho, \psi\rangle) & \Rightarrow\ \uparrow^{\psi^*[\![B]\!]_{\rho, G[\phi]}} \mathbf{app}(R, \phi^* G) \\
F[\phi] \equiv \boxed{?coe} & \Rightarrow\ \boxed{?coe} \\
F[\phi] \equiv \mathbf{hcom}_C^{r \rightsquigarrow r'}\ M\ [\overrightarrow{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}], C[\cdot] \equiv \Pi(A, \langle B, \rho, \psi\rangle) & \Rightarrow\ \nu\beta.\ \underline{\mathbf{hcom}}_{\psi^*[\![B]\!]_{\rho, G[\phi]}}^{r \rightsquigarrow r'}\ \underline{\mathbf{app}}(M, \phi^* G)\ [\overrightarrow{\xi_i \hookrightarrow \langle\beta\rangle \underline{\mathbf{app}}((\alpha_i\ \beta)\cdot N_i, \phi^* G)}]
\end{cases}
$$

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\underline{\mathbf{hcom}}_C^{r \rightsquigarrow r'}\ M\ [\overrightarrow{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] = \begin{cases}
r[\cdot] = r'[\cdot] & \Rightarrow\ M[\cdot] \\
\exists_{min} k.\ \vDash \xi_k[\cdot] & \Rightarrow\ N_k[\alpha_k = r'[\cdot]] \\
C[\cdot] \equiv \Pi(A, B) & \Rightarrow\ \mathbf{hcom}_C^{r \rightsquigarrow r'}\ M\ [\overrightarrow{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \\
C[\cdot] \equiv \mathbf{bool} & \Rightarrow\ M[\cdot]
\end{cases}
$$