cubical evaluation semantics

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I've been thinking a bit more carefully about the evaluation semantics and the semantic domain. What I think we want to do is take a term and an environment and return a family of values (indexed in restrictions); but crucially, I think it is important that the environment be a vector of *values* and not of value-families. By studying carefully the case for instantiating closures (where the rubber meets the road for variables), I think it becomes clear that this is the right way.

To interpret a variable, we just project from the environment and ignore the restriction. I think the way that we had been thinking of doing it would result in the restriction from inside the closure infecting the argument.

Colors: terms, value families, values, restrictions. I am writing $\phi^* F$ to mean the reindexing of a family by a restriction.

$$\begin{aligned}
[\![(\lambda\{M\})]\!]^{\pi}_{\rho} &= [\phi]\ \lambda\langle M, \rho, \pi, \phi\rangle \\
[\![(\Pi\ A\ \{B\})]\!]^{\pi}_{\rho} &= [\phi]\ \mathbf{\Pi}\Big(\phi^* [\![A]\!]^{\pi}_{\rho}, \langle B, \rho, \pi, \phi\rangle\Big) \\
[\![(M\ N)]\!]^{\pi}_{\rho} &= \underline{\mathbf{app}}([\![M]\!]^{\pi}_{\rho}, [\![N]\!]^{\pi}_{\rho}) \\
[\![x_i]\!]^{\pi}_{\rho} &= [_]\ \rho_i
\end{aligned}$$

We write $\phi \vDash F \Downarrow V$ as a shorthand for $F[\phi] \equiv V$.

$$\frac{\phi \vDash F \Downarrow \lambda\langle M, \rho, \pi, \psi\rangle}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow [\![M]\!]^{\pi}_{[\rho, N[\phi]]}[\psi]}
\qquad
\frac{\phi \vDash F \Downarrow \uparrow^{C} R \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow \uparrow^{\psi^* [\![B]\!]^{\pi}_{[\rho, N[\phi]]}}\big([\chi]\ \mathbf{app}(\chi^* R, \chi \circ \phi^* N)\big)}$$

$$\frac{\phi \vDash F \Downarrow \mathbf{coe}^{r \rightsquigarrow r'}_{\langle\alpha\rangle C}\ M \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, N) \Downarrow \nu\beta.\ \nu\gamma.\ \underline{\mathbf{coe}}^{r \rightsquigarrow r'}_{\langle\beta\rangle [\![B]\!]^{[\pi, (\beta\ \alpha)]}_{\left[\rho, \underline{\mathbf{coe}}^{r' \rightsquigarrow \beta}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}\ N[\phi]\right]}}\ \underline{\mathbf{app}}(M, [\chi]\ \underline{\mathbf{coe}}^{\chi^* r' \rightsquigarrow \chi^* r}_{\langle\gamma\rangle(\gamma\ \alpha)\cdot A}\ \chi \circ \phi^* N)}$$

$$\frac{\phi \vDash F \Downarrow \mathbf{hcom}^{r \rightsquigarrow r'}_{C}\ M\ [\overline{\xi_i \hookrightarrow \langle\alpha_i\rangle N_i}] \qquad \cdot \vDash C \Downarrow \mathbf{\Pi}\big(A, \langle B, \rho, \pi, \psi\rangle\big)}{\phi \vDash \underline{\mathbf{app}}(F, O) \Downarrow \nu\beta.\ \underline{\mathbf{hcom}}^{r \rightsquigarrow r'}_{\psi^* [\![B]\!]^{\pi}_{[\rho, O[\phi]]}}\ \underline{\mathbf{app}}(M, \phi^* O)\ [\overline{\xi_i \hookrightarrow \langle\beta\rangle \underline{\mathbf{app}}((\alpha_i\ \beta) \cdot N_i, \phi^* O)}]}$$

$$\frac{\cdot \vDash r = r}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] \equiv M[\cdot]}
\qquad
\frac{\exists_{min} k, \cdot \vDash \xi_k}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] \equiv N_k[\alpha_k = r'[\cdot]]}
\qquad
\frac{\cdot \vDash C \Downarrow \mathbf{\Pi}(A, B)}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] \equiv \mathbf{hcom}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}]}$$

$$\frac{\cdot \vDash C \Downarrow \mathbf{bool}}{\underline{\mathbf{hcom}}_{C}^{r \rightsquigarrow r'}\ M\ [\overline{\xi_i \hookrightarrow \langle \alpha_i \rangle N_i}] \equiv M[\cdot]}$$

$$\underline{\mathbf{coe}}_{\langle \alpha \rangle C}^{r \rightsquigarrow r'}\ M = \begin{cases} \vDash r[\cdot] = r'[\cdot] & \Longmapsto \quad M[\cdot] \\ C[\cdot] \equiv \mathbf{\Pi}(A, B) & \Longmapsto \quad \mathbf{coe}_{\langle \alpha \rangle C}^{r \rightsquigarrow r'}\ M \\ C[\cdot] \equiv \mathbf{bool} & \Longmapsto \quad M[\cdot] \end{cases}$$