1 Analysis

- This paper describes the numerical solution of the Taylor-Bretherton bubble flow by a Lattice Boltzmann method. The simulations are two-dimensional with a flow driven by a body force and periodic boundary conditions.
- The Introduction discusses the interest of diffuse interface schemes such as the Lattice Boltzmann method in simulating numerically complex microfluidic flows involving trains of bubbles. The challenge of choosing a consistent discretization is emphasized. In addition, the use in the present work of binary liquids with uniform overall density is justified by the fact that inertia has negligible influence on the thickness of the film between the bubbles and the channel walls, this quantity being the main one that the present study focuses on. The manuscript is organized in four sections. In Section 2, the Authors present a synthetic state of the art in which the cited (experimental or numerical) works on bubble flows or on the related gas finger propagation are chosen with consistency. The Lattice Boltzmann method is decribed in Section 3. The challenges and difficulties in applying the method to bubble train flows are emphasized in Section 4 "Lattice Boltzmann benchmark". In the Section 5 "Results", the first Subsection is dedicated to the presentation of the numerical domain, the definition of the main variables and the procedure to initialize the numerical scheme. In Section 5-2, the Authors study the influence of grid refinement on the film thickness, on the dimensional speed of the bubble and on the thickness of the gaz-liquid interface. These quantities are measured at the center of one bubble. The influence of the phase gradient at the wall on the previous quantities is discussed in Section 5-3. These two Subsections allow to validate the numerical procedure and a parametric study for a range of targeted capillary numbers varying from 0.03 and 0.8 is done. The real measured capillary numbers vary in fact from 0.04 to 1.4 approximately. In Subsection 5-5, the variation along the bubble of the film thickness is discussed and compared qualitatively to another paper. At last, in Subsection 5-6, the Authors show the weak influence of different viscosity ratios.
- The principle of a simulation is as follows: A targeted capillary number is chosen and a corresponding mean flow speed is obtained; The assumption of an axisymmetric Poiseuille flow in the liquid surrounding the bubble leads to the calculation of the Poiseuille pressure gradient which is used to initialize the body force; At the same time, the film thickness at this capillary number is estimated with the help of results from the literature ("Bretherton's" equation - Equation 7, graphical results of Ref. 3 & 10); This allows the Authors to choose the number of nodes necessary to represent the diffuse interface accurately; The scheme is then initialized and run until a steady state is reached; The real capillary number and the film thickness among other quantities are eventually measured.
- This is an interesting work that uses a recent numerical method to simulate a classical fluid problem with a free interface. The choice of the model parameters is consistent and leads to film thicknesses in good agreement with the literature. The paper is equilibrated between the description of the numerical scheme and the physical results. In view of the potential application of the presented method to the microfluidic context, the paper fits well in a Journal of chemical engineering. Nevertheless, several points of the protocol need to be developped and the analysis of the results is not complete. The manuscript also suffers from the lack of rigorous definitions for some of the physical variables. At last, although the English writing is very correct, legibility would certainly be improved if some parts of the paper were re-organised.
- My recommendation is therefore to revise the manuscript and I strongly suggest the following remarks.

2 Main comments

Measurement of the film thickness

- The main quantity that the paper focuses on is the thickness of the film between the bubbles and the wall. As shown by Figure 8 and as mentioned by the Authors in Section 4, the film thickness varies along the bubble. However, the Authors do not explain the choice of the location of the measurement of the film thickness. This choice is of primary importance when comparing results with those of the literature and in particular with works on finger propagation in which the film thickness is measured "at infinity".
- It may be appropriate to introduce Figure 8 on thickness variation before the main results on film thickness.

Influence of inertia on the film thickness and on the flow

- The lines 54-55 indicating that "numerical simulations and experimental studies showed [...] Reynolds number effects on the film thickness for capillary numbers larger than 0.003" is in contradiction with the lines 32-33 which state "negligible Reynolds number effects on the film thickness for a relatively wide range of Reynolds numbers."
- Although inertia has been shown to have a minor effect on film thickness, inertia does influence some flow variables in the Bretherton's problem (see Ref. 3 for instance). To check whether inertia is important or not, and to discuss the assumption of uniform density, the Authors should provide the reader with the Reynolds numbers associated with each simulation. This is also required for the consistency of Figure 7 in which results are compared with other works of the literature.

The body force and estimations of the pressure gradient

- In this work, the diphasic flow is driven by a body force. The body force is therefore the control parameter in the simulations. The Authors should give more detail about this body force. Indeed, although the estimation of the body force from the pressure gradient is well described, it is not clear where this body force appears in the governing equations (Equations 2, 3 and 4). Does this body force correspond to the variable F_i in Equation 2? Should not this force or its equivalent appear in the Navier-Stokes equations (Equation 4)?
- I would suggest to calculate the Bond number to check if the shapes of the bubbles and the main results of the paper are not influenced.
- The use of the variable Ca_{lit} and the calculation of the corresponding pressure gradient yield estimations of the body force to apply and of the film thickness to choose grid resolution. The presentation of the corresponding procedure is repeated twice in Section 5-1 for Ca=0.005 and Ca=0.05. It is again described in a different way in Section 5-4. Similar equations for the calculation of the pressure gradient are thus given in Equations 6, 14 and 16. The Authors should summarize these equations into one unless they explain their differences. In fact, it would probably be straightforward not to detail the calculation of estimated variables since they are intermediary variables which allow the design of the benchmark but do not influence directly the results. Also, the Authors could possibly say they control and impose directly the body force without describing all the estimation procedure.
- Moreover, the estimations of the pressure gradient for the initialisation of the scheme are obtained under the assumption of an axisymmetric Poiseuille flow. However, among the "Key words", one can read the key word "Flow between plates" in agreement with the 2D non axisymmetric simulations that are done in this work. The Authors should discuss this choice of estimating the pressure gradient with the axisymmetric assumption instead of a Hele-Shaw flow (factor 1/12 instead of 1/8 in Equation 5).

Presentation of the Lattice Boltzmann method

- The Section on LBM is concise and this seems a reasonable choice when publishing such a work in a journal of applied science. The Authors would improve significantly the manuscript by defining more rigorously some of the variables of the model. For instance, the Authors refer to surface tension in lines 106, 111 and 123 via the parameter k, the weights w and the quantity $\sqrt{8kA/9}$ respectively. A suggestion is to emphasize within an array the links between physical parameters and the model parameters. The variables i, f_i^{eq}, g_i^{eq} should be defined and the Equations for f and g in Equation 3 should be described (which equation corresponds to continuity equation...). Equation 2 could perhaps be rewritten with the use of a generic variable in order to remove any ambiguity with the physical variable f of Equation 3. The notation $i = 1 \div 8$ is unusual, it would be clearer to use a classical notation such as $1 \leq i \leq 8$. The sound speed c_s^2 should be defined just after its introduction in line 107.
- The name of the Section "Lattice Boltzmann benchmark" lets the reader think that this Section would describe the numerical procedure (assumptions, choice of the geometry and the grid, boundary conditions, control parameters, initialization of the scheme). This is not the case because this Section develops the challenges and the difficulties of using the Lattice Boltzmann method to bubble flows. This Section should be renamed or reorganized.

Lattice units and dimensional/non dimensional units

- The Authors should make a distinction between variables in lattice units and dimensional or non dimensional variables. For instance, in line 195, the definition $H_{eff} = N_y - 2$ where N_y is the number of nodes in the vertical direction indicates that H_{eff} has lattice units. However, this is in contradiction with the use of H_{eff} in Equation 5. The Authors should define each variable without any ambiguity on its units. Besides, the unit of U_{bubble} should also be given. These improvements would fit well in the Subsection named "The nondimensionalization and initialization procedure" which does not in fact introduce much nondimensionalisation procedure.

Notations

- Some notations are unclear. For instance, the film thickness is denoted as h_∞ in Equation 7 whereas it is denoted δ in most of the paper. The notations w or δ_{sim} are also found before line 381 or in Table 3 respectively. The thickness of the interface is denoted ξ in Figure 1 but the same variable is denoted 5ξ in Table 1. The speed of the bubble is denoted U_{bubble} in some places and U_b in others. The variable U_{liq} in Equation 5 does not seem to be defined.

Plots

- Three figures in the manuscript are scanned from the litterature (Ref. 3, 10 and 16). This is unusual for an article which is not a review. I encourage the Authors to remove at least the two first ones which are not essential.
- In Figure 7, the points of data reported from the numerical work of Ref. 3 exhibit a surprisingly noisy trend that corresponds more to an experimental study. The Authors should explain how they obtain this data and plot them.
- The plots of Figure 8 are of less quality than the other figures of the paper.

Bibliography

- The work of Yang seems to be close from the present study since it involves the solution of Bretherton flow with a Lattice Boltzmann method. The Authors should discuss this paper and emphasize the differences between their work and this paper.

- It would be interesting if the Authors could mention a study in which a level-set method is used in order to complete the bibliography.

Miscellaneous

- A secondary quantity of interest is the bubble speed or in a non-dimensional form the capillary number. This quantity is measured at the center of the bubbles. The Authors should describe how this quantity as well as the center of the bubble are measured.
- If possible, the Authors should present results showing that steady state is reached. It would also be interesting if the Authors show some patterns of the flow by plotting for instance the vector field of velocity around the bubbles.
- In the Abstract, the Authors should give the range of real capillary numbers they measured instead of the targeted values (Ca_{lit}).
- The Authors should discuss more their results (Sections 5-4, 5-5, 5-6). Did the Authors measure the pressure drops at the front and rear interfaces of the bubbles?
- The Authors mention the shooting method. It is not clear if this method is really used or just mentioned as a part of the discussion.
- It seems more appropriate to introduce the variables of the Bretherton's problem as well as to plot Figure 1 elsewhere than in the Section "Results".

3 Minor corrections

- in Abstract: "two times" should be replaced by "twice"
- in Abstract: "liquids dynamic viscosity" should be replaced by "liquid dynamic viscosity"
- line(s) 12: "those kind" should be replaced by "those kinds"
- line(s) 16: "for correctly resolving" should be replaced by "to resolve correctly" (same remark for abstract)
- line(s) 17: add (LBM) after lattice Boltzmann method
- line(s) 35: "factor" should be replaced by "parameter"
- line(s) 35-38: unclear : "other simulations" are the same that the references cited five lines before
- line(s) 49: reformulate "deposition film thickness"
- line(s) 70: "cases" could be replaced by "geometries"
- line(s) 72: "other" could be replaced by "previous"
- line(s) 76: "works" should be replaced by "work"
- line(s) 78: "for a relatively should be replaced by "for relatively"
- line(s) 81-88: would be more appropriate in the Introduction in order to be emphasized
- line(s) 95: "in certain directions" could be replaced by "along some directions"
- line(s) 190-192: unclear
- line(s) 194: "grid number" should be replaced by "number of nodes" or something equivalent
- line(s) 217: "characteristic length of the interface" : unclear, which length, transversal, longitudinal?
- line(s) 229-230: add "As previously mentioned" or something equivalent
- line(s) 249-264: This paragraph does not seem well located.
- line(s) 268: Reynolds number 40 equals 0.12 –¿ add approximately
- line(s) 286-287: correct "is varies"
- line(s) 298-302: to locate in a more appropriate location
- line(s) 306-308: to locate in a more appropriate location (end of page 9?)
- line(s) 395: it would be more appropriate to call this section "The influence of different viscosity ratios"
- line(s) 454: "mulitphase" should be replaced by "multiphase"
- line(s) 469: "i. meniscus" should be replaced by "I- Meniscus"
- Equations with numerical calculations should be removed (third terms in Eq. 9, 13 and 17).