

# Project 2

## Fitting the LMT Pointing Model

### 1 Background

The Large Millimeter Telescope (LMT) is a 50m-diameter millimeter wavelength telescope located in the state of Puebla, Mexico. The LMT is the largest telescope of its kind in the world, and the UMass Astronomy Department has access to 30% of the observing time on the instrument. The telescope has been in scientific operation for about four years now, using the inner 32m-diameter of the antenna. During 2017, we hope to finish out the full 50m diameter dish surface.

The LMT is an example of an Azimuth-Elevation mounted telescope, and it points to arbitrary locations on the sky by turning around one axis which points up (the Azimuth Axis) and then rotating the dish about the orthogonal axis (the Elevation Axis) to point at different angles upward from the horizon. This kind of mounting system is favored for large structures.

The Azimuth axis is defined as pointing up so that as we rotate the antenna about this axis it can point in any direction, with the North direction as it's zero position. Azimuth increases as the telescope rotates towards the East, and so East is 90 degrees, South is 180 degrees, and West is 270 degrees.

The Elevation axis is orthogonal to the Azimuth axis, and therefore parallel with the ground. Zero elevation corresponds to the antenna pointing at the horizon and elevation increases as the telescope points upward. When pointing straight up, which we call the zenith, the Elevation is 90 degrees.

One of the key technical considerations in observing celestial sources with the LMT is using the Az and El axes to *point* the telescope. We are required to orient the antenna with a precision of about 1 second of arc in order to make best use of the telescope. The mechanical construction of the telescope was not nearly so precise that we could simply enter in the position of a point on the sky and go there to arc second precision. There are many defects that we need to consider:

- Knowledge of the precise location of 0 on the encoder which measures the Azimuth angle.
- Knowledge of the precise location of 0 on the encoder which measures the Elevation angle.
- Tilt of the Azimuth Axis with respect to zenith.
- Alignment of the telescope's optical axis with the mechanical structure.
- Non-orthogonality of the telescope's azimuth and elevation axes.
- Gravitational bending of the mechanical system under changing gravity load as it points to different elevation angles.
- Refraction by the Earth's atmosphere.

Even in a carefully built system (like the LMT) each of these effects is very large compared to our pointing requirement. What is required is a way to measure the effects and determine the values of the parameters that describe these effects.

### 2 Pointing Model

In order to improve the pointing of the telescope we fit a *Pointing Model* to observations of celestial sources at known positions in the sky. We make careful measurements of position that we read out from the antenna's



Figure 1: The Large Millimeter Telescope

angle encoders and then compare this to the known position. We call the difference between these angles the pointing error.

We may easily derive formulae for correction of the Azimuth and Elevation angles measured by their respective encoders to the actual Azimuth and Elevation of a point on the sky. Each of the effects listed in the previous section has a different behavior, and by making a lot of observations we can use these formulae to make a least squares fit to find the size of each of the possible errors. Presuming these are stable with time (and for the most part they are) we can then use the model to correct the antenna pointing.

## 2.1 Azimuth Pointing Model

For the azimuth pointing model, we fit FIVE terms at the LMT.

Function	Parameter	Explanation
1	$A_1$	Angle between telescope optical axis and mechanical structure
$\sin(El)$	$A_2$	term for nonorthogonality of the Az and El axes.
$\cos(El)$	$A_3$	Zero position of the Azimuth encoder.
$\sin(El) \sin(Az)$	$A_4$	Tilt of Azimuth axis from Zenith
$\sin(El) \cos(Az)$	$A_5$	Tilt of Azimuth axis from Zenith

We can put all these terms together to derive the azimuth pointing offset from our desired position,  $\delta Az$ , in terms of all these properties. (See Equation 1.)

$$\delta Az = A_1 + A_2 \sin El + A_3 \cos El + A_4 \sin El \sin Az + A_5 \sin El \cos Az \quad (1)$$

## 2.2 Elevation Pointing Model

For the elevation pointing model, we fit SIX terms at the LMT.

Function	Parameter	Explanation
1	E1	Zero position of the Elevation encoder
$\cot(El)$	E2	Refraction error
$\cos(El)$	E3	Gravitational Bending
$\sin(El)$	E4	Gravitational Bending
$\cos(Az)$	E5	Tilt of Azimuth axis from Zenith
$\sin(Az)$	E6	Tilt of Azimuth axis from Zenith

We can put all these terms together to derive the elevation pointing offset from our desired position,  $\delta El$ , in terms of all these properties. (See Equation 2.)

$$\delta El = E_1 + E_2 \cot El + E_3 \cos El + E_4 \sin El + E_5 \cos Az + E_6 \sin Az \quad (2)$$

### 2.3 Fitting the Pointing Model

Note that the model parameters in Equation 1 and Equation 2 are just constants multiplying some function of the Azimuth and/or Elevation. This means that the model we have adopted is *linear* with respect to our model parameters and we can go ahead and make use of our algorithm for doing a least squares fit.

We begin by writing our problem as a linear system (illustrating below with the Azimuth equations although we will have to solve BOTH the Azimuth system and the Elevation system to get our complete pointing model). If I have  $N$  observations  $d_1$  through  $d_N$  at  $N$  Az-El positions  $Az_1, El_1$  through  $Az_N, El_N$ , then my set of equations of condition would be:

$$\begin{aligned} d_1 &= A_1 + A_2 \sin El_1 + A_3 \cos El_1 + A_4 \sin El_1 \sin Az_1 + A_5 \sin El_1 \cos Az_1 \\ d_2 &= A_1 + A_2 \sin El_2 + A_3 \cos El_2 + A_4 \sin El_2 \sin Az_2 + A_5 \sin El_2 \cos Az_2 \\ d_3 &= A_1 + A_2 \sin El_3 + A_3 \cos El_3 + A_4 \sin El_3 \sin Az_3 + A_5 \sin El_3 \cos Az_3 \\ &\dots \\ d_i &= A_1 + A_2 \sin El_i + A_3 \cos El_i + A_4 \sin El_i \sin Az_i + A_5 \sin El_i \cos Az_i \\ &\dots \\ d_N &= A_1 + A_2 \sin El_N + A_3 \cos El_N + A_4 \sin El_N \sin Az_N + A_5 \sin El_N \cos Az_N \end{aligned}$$

We can write this system of equations in vector form as follows:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ \dots \\ d_i \\ \dots \\ d_N \end{pmatrix} = \begin{bmatrix} 1 & \sin El_1 & \cos El_1 & \sin El_1 \sin Az_1 & \sin El_1 \cos El_1 \\ 1 & \sin El_2 & \cos El_2 & \sin El_2 \sin Az_2 & \sin El_2 \cos El_2 \\ 1 & \sin El_3 & \cos El_3 & \sin El_3 \sin Az_3 & \sin El_3 \cos El_3 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sin El_i & \cos El_i & \sin El_i \sin Az_i & \sin El_i \cos El_i \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sin El_N & \cos El_N & \sin El_N \sin Az_N & \sin El_N \cos El_N \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \quad (3)$$

or

$$\vec{d} = H_A \vec{A} \quad (4)$$

where  $\vec{d}$  contains the observations,  $\vec{A}$  contains the values of the parameters  $A_1$  through  $A_5$ , and  $H_A$  is a matrix with  $N$  rows and 5 columns.

Once the equations are written in this form, I hope it is simple to follow Exercise 13 to determine the least squares solution.

## 3 Project Steps

### 3.1 Step 1: Assess the Data

#### 3.1.1 Get the Data

First we have to get the data. The data are in the file `PointingData.csv`. The file consists of several columns, but the ones we are interested in are:

- Column 4 - the Azimuth of the observation [degrees].
- Column 5 - the Elevation of the observation [degrees].
- Column 6 - the Azimuth pointing offset measured [arcsec].

- Column 7 - estimate of error in Azimuth pointing offset [arcsec]
- Column 8 - the Elevation pointing offset measured [arcsec]
- Column 9 - estimate of the error in the Elevation pointing offset [arcsec]

To speed things up, I have also provided a short python script to read these values and create numpy arrays. The script also plots the Azimuth and Elevation of the data points for reference.

### 3.1.2 Graph the Data; Look at Histograms

Make graphs of the Azimuth and Elevation pointing offset measurements versus Azimuth and Elevation. Are there any obvious trends?

Make a histogram of the pointing measurements and compute the standard deviation of the data. Note that most of our error estimates were in the vicinity of 1 arcsec. How does the distribution of the measurements compare to a normal distribution with a standard deviation of one arcsec.

## 3.2 Step 2: Fit the Data

Next step is to actually do the least squares fit of the data to the pointing model. You have to do two fits: one for Azimuth and one for Elevation. Present your results for the parameters of the Azimuth model and the Elevation model, including an estimate of the errors in these quantities.

## 3.3 Step 3: Assess the Fit

Once the fit is complete, it is important to look at how well we did in fitting the data. Here are some things to do in order to assess the fit:

- Compute the residuals to the model fit
- Make graphs of the residuals versus Azimuth and Elevation.
- Create histograms of the residuals for Azimuth pointing and Elevation pointing.

With these figures, look carefully at residuals and address questions:

- Have you improved the pointing errors by fitting the model? (Compare the model residuals to the standard deviation of the data before the fit.)
- Do the residuals look like they follow a normal distribution?
- Are the residuals in Azimuth the same as the residuals in Elevation?
- Do points with larger measurement errors have larger residuals than ones with small errors?
- In a graph of the residuals versus Azimuth and Elevation, do you see any systematic trends that might indicate unmodeled effects?

## 3.4 Step 4: Your Conclusion

Our main goal of course is to predict how well we will be able to point the antenna. Considering the above, do you think the antenna can point well enough to support observations with an 8 arcsecond beam?

## 4 Write Up your Results in a Jupyter Notebook

The jupyter notebook was invented as a way to write up research results, involving calculations, in a way that documented both the result and the calculation. Please use this format to present both your calculation and your writeup. The writeup should include the following:

- **Introductory Remarks** - describe the physics of the system and introduce any special numerical approach that is required.
- **Describe the Numerical Method** - describe the algorithm that is used to solve the problem. Give example listings of your program to show how it is implemented. Sometimes various development steps are identified in an assignment. In this case, you should be sure to show the program as it appeared at these milestone points.
- **Program Verification** - tell what you did to verify that the program gives correct results.
- **Presentation of Results** - present the results of running the program to demonstrate the behavior of the system under different circumstances. Results might be presented in graphical form or as tables, as appropriate. Be sure that results that are presented are labeled properly, so that the reader can figure out what has been calculated and what is being displayed.
- **Analysis** - sometimes, we will take the results of many runs and derive relationships between the variables of the model and computed quantities. When this occurs, you should present this analysis and comment on the accuracy of the relationship.
- **Discussion** - present a discussion of the physical behavior of the system based on your simulations and answer any special questions posed in the assignment.